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A computerized method (UPPREC) for quantitative analysis of irradiated nuclear fuel assemblies with gamma emission tomography at the Halden reactor

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Abstract

The Halden reactor project (HRP) has recently developed a gamma emission tomography instrument dedicated for measurements of irradiated nuclear fuel in collaboration with Westinghouse and Uppsala University. This instrument is now assembled and the first experimental measurements have been performed on fuel assemblies irradiated in the Halden reactor. The objective of the instrument is to map the distribution of radioisotopes of interest in the fuel, e.g. ^{137}Cs or $^{140}\text{La/Ba}$, and for this purpose, a spectroscopic high-purity Germanium detector has been selected, which enables the identification and tomographic reconstruction of separate isotopes by their characteristic gamma rays.

To gain from the analysis of the data from this new instrument, and in the future from other gamma emission tomography instruments for nuclear fuels, various reconstruction methods are available that vary in the accuracy and the amount of detail obtainable in the analysis. This paper presents the details of the working principles of a new code for gamma emission tomography, the UPPREC (UPPsala university REConstruction) code. It is a development in MATLABTM code with the aim to produce detailed quantitative images of the investigated fuel.

In this paper, the methods assembled for the analysis of data collected by this novel instrument are described and demonstrated and a benchmark is presented using single rod gamma scanning. It is shown that the UPPREC methodology improves the accuracy of the reconstructions by removing the errors introduced by the presence of

highly attenuating fuel and structural material in the fuel assembly. With the introduction of UPPREC, detailed quantitative cross-sectional images of nuclide concentrations in nuclear fuel are now able to be obtained by nondestructive means. This has potential applications in both nuclear fuel diagnostics and in safeguards.

1. Introduction

In recent years, gamma emission tomography of irradiated nuclear fuel has been investigated for nuclear fuel examinations for various applications such as in safeguards (Jansson, et al., 2015), (Jacobsson Svärd, et al., 2015), (Peura, et al., 2016), (White, et al., 2015), (Smith, et al., 2016) and for various fuel performance applications (Caruso, et al., 2009), (Holcombe, et al., 2016), (Andersson, et al., 2016), (Holcombe, et al., 2016).

Since irradiated nuclear fuel contains a wide array of gamma emitting radioisotopes, the tomographic technique may also be combined with the gamma spectroscopy technique, where the gamma peaks of certain isotopes of interest are selected for quantitative analysis. A recent development of this technique is the gamma tomography system at the Halden Reactor Project (HRP) (Holcombe, et al., 2015), for examining experimental and driver fuel assemblies irradiated in the Halden research reactor. The HRP gamma tomography system uses a spectroscopic high-purity Germanium (HPGe) detector with high energy resolution, allowing for analysis of the contents of various isotopes of interest, and tomographic reconstruction of the spatial distribution of the selected isotopes.

In parallel with the use of the HRP gamma tomography system, there is ongoing development of the methods and analysis techniques to best take advantage of this novel non-destructive characterization capability. Previously, two methods have been used for the tomographic reconstruction of HRP tomography data; an analytical filtered back projection (FBP) based reconstruction method for qualitative image reconstruction (Holcombe, et al., 2015); and an algebraic reconstruction code for quantitative pin activity assessment, further described in (Jacobsson Svärd, 2004) and (Jacobsson Svärd, et al., 2015).

In this article, a new method is presented that is specially developed for spectroscopic gamma emission tomography of irradiated nuclear fuel. This method is a MATLAB development called UPPREC (UPPsala REConstruction). The objective of UPPREC is to be used with spectroscopic gamma emission tomography measurements of irradiated nuclear fuel to achieve quantitative, detailed reconstructions of the investigated fuel cross section. In the context of this work, it is helpful to make the distinction between quantitative vs. qualitative reconstruction and between detailed vs. pin-wise reconstruction.

1.1. Quantitative vs. qualitative reconstruction

Qualitative reconstruction methods are used to produce a visually interpretable image of the investigated fuel cross section, which may be used for (among other things) rod localization or for the verification of the presence of fuel rods in the investigated fuel (Davour A., 2016), (Davour, et al., 2015). These applications do not require highly accurate reconstruction of the spatial source distribution, and therefore idealizations in the methods may be implemented which maintain the qualitative information desired, while allowing for easier implementation and fast execution times in the reconstruction

software. For qualitative reconstructions, the spatial resolution should be substantially smaller than the characteristic sizes of the fuel, i.e. the diameter of the rods and the rod-to-rod pitch. A resolution of about one millimeter is suitable for this purpose.

The primary idealizations that simplify and speed up the analysis for qualitative reconstruction include neglecting the effects of inhomogeneous self-attenuation of the gamma-ray transport in the fuel assembly and instrument components, and assuming perfect collimation, i.e. gamma rays form an infinitesimally thin beam line from the fuel to each detector position. Despite these idealizations, the qualitative methods may successfully be used for locating and counting of the fuel rods in the assembly, as seen in (Holcombe, et al., 2016).

Differing from qualitative methods, quantitative methods are instead aimed at accurately reproducing the gamma-ray emission spatial distribution. This requires a higher level of detail in the model of the instrument response. Specifically, the attenuation of the gamma rays in the fuel assembly and instrument components (such as fuel pins, cladding and structural materials) need to be included in the analysis as well as a realistic collimator field-of-view model that considers the quasi-cone field-of-view through the collimator slit, and transmission of gamma rays through the collimator material itself. In this paper, the methods presented are of the quantitative type. For comparison, a qualitative reconstruction with the Filtered Backprojection method is also presented in section 2.7.

1.2. Detailed vs. pin-wise reconstruction

Based on the level of detail allowed by the shape and the number of reconstruction elements used (e.g. square image pixels), a distinction can be made between detailed vs. pin-wise reconstruction.

Detailed reconstructions, such as FBP can produce high spatial-resolution images with a large enough number of pixels to provide resolution much smaller than the typical features of the nuclear fuel, i.e., the fuel rod. Therefore, features of unknown size, location and gamma-ray emission intensity may appear in the reconstructed high-resolution images. Detailed reconstructions avoid obscuring or hiding features that are unexpected, provided that they are large enough to be observed in the measured data by a magnitude greater than the level of any noise and/or systematic errors that may also be present.

Pin-wise reconstruction codes utilize larger reconstruction elements that are tailor-made to fit the known components of the fuel object (e.g., the typical circular cross-section fuel rods). This enables using much fewer reconstruction elements for the complete assembly. As a result, the computational speed and the conditioning of the reconstruction problem is greatly improved; however, using only elements that correspond to known or expected features in the fuel means that unexpected features, or features that are not in the expected locations, will not be accounted for in the reconstructed images and data sets. Examples of features that may be overlooked are relocated fuel fragments in transient test fuel, or pin-internal variations of the isotopic concentration.

In this paper, the reconstruction methods presented are of the detailed type, although pin-activity can be achieved by simple summing of the pixels of each fuel rod.

2. Method

2.1. Measurement geometry

The methods presented in this work were developed for the HRP gamma tomography system; however, they are applicable for analysis of data from any gamma emission tomography system that can produce the equivalent measurement data. The measurements using the HRP gamma tomography system are performed as shown schematically in Figure 1.

Figure 1. Schematic illustration of measurements using the HRP gamma tomography system which are analyzed using the methods presented in this work. In this figure the nuclear fuel is a 13-rod Halden driver fuel assembly (with 4 tie rods located at 12, 3, 6, and 9 o'clock in the cross-section view of the fuel assembly).

The radiation field should be assessed with 360 ° rotation of the fuel relative to the detector. In principle 180 ° rotation may be sufficient but due to the very strong attenuation of the gamma rays in the nuclear fuel two opposing views are relatively different and this is not recommended. For each small angular increment of the fuel position, the detector also performs a complete lateral scan ranging over the full width of the inspected object. The sampling intervals of the rotational and the lateral motions should roughly be of the same order of magnitude, and the product of the two should be roughly in the same order of magnitude as the number of pixels desired in the reconstruction. However, the use of regularization in the reconstruction procedure allows for reconstruction of a larger number of pixels than the number of radiation measurements. For more information on the optimization of the sampling intervals we refer to other literature from the field of Gamma Emission Tomography in general, such as (Groch, et al., 2000).

2.2. Description of the measured data.

For the UPPREC analysis, the data should contain the net peak size, I , [number of counts], the total peak size, P , [number of counts] and an estimate of the Compton continuum background, B , [number of counts] of the selected characteristic gamma ray for each position. It may be noted that this is a redundancy, where $I = P - B$. In addition, the position coordinates are needed, i.e., the lateral position L and the angular position of the fuel relative to the collimator α , and the dead-time corrected measurement time, T , of the recorded intensity data.

The HRP tomography instrument was manufactured with relatively small tolerances in order to ensure high precision control of the relative positions of the fuel and detector/collimator (Holcombe, et al., 2015) during the measurements. These high-precision components include the mechanisms that move the fuel and detector/collimator, as well as the fixtures in the instrument which hold the top and bottom of the measured fuel assembly. The high precision of the fixtures and instrument components reduces the uncertainty in the measured data that would otherwise arise from e.g. positioning uncertainty. Positioning uncertainty is described further in (Holcombe, et al., 2015).

2.3. Solution strategy of the reconstruction problem in UPPREC

UPPREC uses an algebraic approach to solving the tomographic reconstruction problem. This means that the reconstruction is the solution to the system of linear equations, $I = WA$, where the column vector I is the measured intensity data [number of counts] in each investigated position, A is the number of gamma rays emitted per second per axial millimeter inside the square pixels and the W is the system matrix, which describes the relationship between A and I . For the reconstructions made so far using UPPREC to reconstruct data from measurements on HRP fuel rigs, typically in the order of 100x100 pixels have been used, and in the order of 10000 positions have been used in the measurements, giving a system matrix with in the order of 1E8 elements.

In principle, performing a reconstruction constitutes performing two actions; 1) calculation of the system matrix corresponding to the selected gamma-ray energy and measurement geometry of the fuel and instrument and 2) solving the matrix equation using a linear solver. In UPPREC an iterative approach is used to solve these two tasks (1 and 2), and the two tasks are therefore intertwined.

Using UPPREC, a system matrix is first calculated based on a zero-attenuation model. The reconstructed emission distribution that results from using this simplified model constitutes a naïve solution. As stated in the preceding section, this may cause the reconstruction to suffer from inaccuracies and artifacts as a result of the simplified description of the radiation transport, but the reconstruction can still be used as a qualitative image to identify and to locate the main structures and features of the nuclear fuel assembly, e.g. the fuel rods may be located by applying a template matching technique. Using the positional information derived from the naïve reconstruction, the calculation of the system matrix is performed a second time while including the gamma-ray attenuation in the fuel and cladding of each rod. This new reconstruction is visibly better. Subsequently, further improvements of the accuracy are obtained by identifying tie rod positions in the new reconstruction by their apparent shadows (which appear to be negative emission intensities) in the images, and accounting for their gamma-ray attenuation.

The solution strategy implemented in UPPREC for use on the reconstruction of gamma emission tomography data of irradiated nuclear fuel assemblies is summarized in Figure 2 and it is described in more detail in the following sections.

Figure 2. A schematic layout of the principles of the UPPREC analysis. 1) The input data is the sinogram containing the measured intensity (number of counts) for each lateral and rotational positions examined. 2) The naïve reconstruction achieved by neglecting the attenuation of gamma rays inside the object. 3) Rod localization using template matching. 4) Second reconstruction using gamma-ray attenuation properties of fuel in the response function. 5) Tie rod localization by using the shadow due to the attenuation of gamma rays in their locations. Alternatively, activation products may be used for this step. 6) A final reconstruction based on a response function that includes all the important physics for the transport of gamma to the detector, i.e. the gamma-ray emission distribution, the field-of-view of the detector and the attenuation distribution corresponding to the investigated gamma-ray energy. All plots use the Parula colormap, where blue corresponds to low and yellow to high intensity.

2.4. Naïve System matrix generation

The system matrix, W , is a matrix where each element describes the expected signal [number of counts registered] at a certain position per gamma rays emitted per mm axial length inside the quadratic cross sectional area of a certain pixel. The elements in W depend on instrument related parameters, such as dimensions of the tomographic setup and the collimator used, which can be termed the instrument response. Additionally, there is a dependence of the objects own properties, i.e., the attenuation distribution inside the fuel assembly. Both the attenuation and the instrument response are $m \times n$ matrices, where m is the number of measurements performed and n is the number of pixels used in the reconstructed 2D image. The system matrix can be described as the entrywise (Hadamard) product of the instrument response, U , and the attenuation response of the object, V , so that $W = U \circ V$. For the naïve reconstruction the attenuation, V , is neglected. This is the equivalent of setting all elements of V to unity and thus W equals U .

The elements of the instrument response, U , are thus estimates of the number of counts in a measurement per the number of gamma rays per mm and second emitted from a pixel, neglecting the impact of attenuation in the fuel assembly. The instrument response (expressed in eq. 1.) is defined here as the product of the dead-time corrected measurement live-time (T), which is provided by the Data Acquisition system (DAQ), the energy dependent detector efficiency (ϵ), which may be obtained by

using calibration, and the probability for a gamma emitted in the pixel to reach the detector position through the collimator, p_{ij} .

$$U_{ij} = \varepsilon T_i p_{ij} \quad (\text{eq. 1})$$

The probability per emitted gamma to enter the detector, p_{ij} , where i and j are the measurement position and the pixel index respectively, may be obtained by Monte Carlo simulations in order to achieve a highly accurate response function of the instrument system. This allows for taking into account the exact dimensions of the collimator (which may have a nonrectangular slit) and the transmission through wall materials for the gamma energy of interest.

In addition, to allow for faster calculation than the typically rather slow convergence of Monte Carlo methods, an alternative approach has been developed for the calculation of p based on geometrical considerations. In this approach, shown in eq. 2, the probability of a gamma ray to reach the detector is calculated based on the area of the detector visible through the collimator from the source point (A_{det}) and the photon flux intensity which is inversely proportional to the square distance to the back end of the collimator, where the detector is situated. The (x,y) point source response is integrated over the pixel area, after transformation of the pixel location to a coordinate system rotated with the detector, as shown in Figure 3.

$$p_{ij} = \iint_{\text{pixel } j} \frac{A_{det}(x,y) H(x)}{4\pi ((x+l_c)^2 + (y-L_i)^2)} dx dy \quad (\text{eq. 2})$$

Here, x is the distance from the front end of the collimator to the source point in the object and y is the distance in the orthogonal direction from the zero position of the lateral motion motor to the source point. These coordinates are integrated over the surface of the pixel corresponding to the element of W . The distance l_c is the length of the collimator and L_i is the position of the lateral scan motor in measurement i . $H(x)$ is the axial height in the fuel object that the collimated detector is sensitive to, this is estimated according to an increased size of the field-of-view cone with the distance from the collimator according to eq. 3.

$$H(x) = \frac{h_c(x+l_c)}{l_c} \quad (\text{eq. 3})$$

Finally, $A_{det}(x,y)$ is the Area of the detector seen from position (x,y) , which is calculated based on eq. 4.

$$A_{ij} = \begin{cases} c_w c_h & \text{in the central region, where } |y - L| < y_1 \\ c_w c_h \left(1 - \frac{|y-L|-y_1}{y_2-y_1}\right) & \text{in the penumbra, where } y_1 < |y - L| < y_2(x) \\ 0 & \text{outside, i. e. where } |y - L| > y_2(x) \end{cases} \quad (\text{eq. 4})$$

Where c_w is the width of the collimator and c_h is the height. The coordinates x and y and the region borders (y_1 and y_2) are defined according to the schematic drawing of Figure 3.

Figure 3. Schematic image of the area of the detector seen from different regions of the object. It may be noted that in the HRP tomography system the fuel object is rotated and the detector-collimator system is fixed, which is equivalent to performing the rotation on the detector-collimator system as indicated above.

2.5. Linear solver

A weighted least squares approach is used to obtain a maximum likelihood solution. 2D total variance (TV) regularization is used to suppress the noise sensitivity that is characteristic for inverse problems such as tomographic reconstruction.

Weighting

After evaluation of the naïve system matrix, U , according to section 2.3, measured data can be analyzed to give a qualitative image of the fuel cross section. For this purpose the system of equations $I=WA$ is solved using a weighted 2D-TV regularized Least Squares Solver. TV regularization is used to reduce noise while preserving edge locations (Strong, et al., 2003).

The system matrix U is weighted, where the weights, w , are set to the inverse of the estimated variance of the measurement (eq. 3). The variance, in turn, can be estimated in the case of a radiation counting experiment using the assumption of Poisson statistics, by the number of counts collected in each position. However, except for the highest energy peaks there is typically a Compton continuum background in the spectrum that has been subtracted prior to this analysis by the DAQ. The DAQ provides both the total number of counts in the peak, P , and the background level as obtained by a straight line fit to the background, B , that is preferentially acquired by using two samples of the background, symmetrically placed on both sides of the peak, using a number of channels for the two that equals the number of channels in the peak. If this is performed, the variance of B is simply equal to the number of counts in the background samples (Parker, 1991) and the variance of the net number of counts, I , is thus $P + B$. It can be noted that the net number of counts, I , are used as the input to the

linear solver further described in the paragraphs below, and thus the measured data is not normalized by the measurement time to a count rate [cps] prior to the continued analysis. Instead the live-time of the measurement is a part of the response function, according to eq. 1.

To handle the possibility of low intensities in both the background, B , and in the total peak area, P , an additional count offset, δ , is added to the Poisson variance estimate. This is performed in order to avoid the variance estimate being equal to zero which could otherwise make the weight infinitely large. The parameter δ was set to 10 in the reconstructions demonstrated in this work. The offset is discussed further in the section 5.

$$w_i = \frac{\frac{1}{\text{Var}(I_i)}}{\sum_i \frac{1}{\text{Var}(I_i)}} = \frac{\frac{1}{P_i+B_i+\delta}}{\sum_i \frac{1}{P_i+B_i+\delta}} \quad (\text{eq. 3})$$

At the time of the measurements presented in this paper, only the net peak area, I , was provided by the DAQ. Therefore, the Poisson variance estimate of $\text{Var}(I) = P + B + \delta$, was substituted to $\text{Var}(I) = I + \delta$. Thus the influence of the Compton continuum background on the variance was neglected. Note, however, that the background estimate, B , has been subtracted from the total number of counts registered, P , by the DAQ to achieve the net number of counts in the peak, I . Neglecting the background only affects the weights of the linear equations.

By multiplying the weights, w_i , according to eq. 3. to each corresponding row of U and I we obtain the weighted system of equations of eq. 4.

$$\begin{bmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & & \vdots \\ \vdots & & \ddots & \\ 0 & \dots & & w_m \end{bmatrix} \begin{bmatrix} U_{11} & \dots & U_{1n} \\ \vdots & & \vdots \\ U_{m1} & \dots & U_{mn} \end{bmatrix} \begin{bmatrix} A_1 \\ \vdots \\ A_n \end{bmatrix} = \begin{bmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & & \vdots \\ \vdots & & \ddots & \\ 0 & \dots & & w_m \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_m \end{bmatrix} \quad (\text{eq. 4})$$

This is expressed more simply in eq. 5.

$$wUA = wI \quad (\text{eq. 5})$$

Regularization

The 2D TV regularization is implemented by appending additional rows to the matrix wU and additional elements to the array I , corresponding to the constraints of all neighboring pixels in x and y directions being equal. This adds two constraints per pixel. (Also the border pixels are set equal to the opposite side pixel to avoid diverging values at the edge of the reconstructed image.) The added constraints are weighted by a regularization parameter, λ , which expresses the importance of the expectation of a smooth solution relative to the objective of achieving a good fit to the measured data. After the regularization, the linear problem can be expressed as in eq. 6.

$$\begin{bmatrix} [wU]_{m \times n} \\ [\lambda S]_{2n \times n} \end{bmatrix} [A]_{n \times 1} = \begin{bmatrix} [wI]_{m \times 1} \\ \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{2n \times 1} \end{bmatrix} \quad (\text{eq. 6})$$

Accounting for background in the measurement environment

Some background radiation from long-lived radioisotopes has been observed in the facility where the HRP tomography system is located. This may interfere with the measurements of radioisotopes in the inspected fuel. Note that there are two major sources of background that are affecting the measurements at HRP:

- 1) Background from the inspected fuel itself, and
- 2) Background from other sources in the ambient.

Background 1) is primarily Compton continuum, although there could be multiple peaks at interfering energies or Rayleigh scattered gamma rays that have lost negligible amounts of energy. While 2) can be both Compton continuum or full energy peaks.

The Compton continuum background is subtracted from full energy peaks using a linear fit by the DAQ as described in the earlier sections. However, in the full energy peaks the contribution from leakage through the collimator material or from other sources in the ambient, such as stored fuel and/or contamination in the facility also need to be accounted for, if such background is present. Therefore, one extra explanatory variable is added to the column matrix A , which is solved for, to account for the constant background level, b , across all detector positions. Correspondingly, the matrix wU is also given an extra column, where all elements are equal to unity (all detector positions are considered equally sensitive to the background), and the appended matrix λS is provided with an extra column where all elements are zero. This reflects that while each measurement is assumed to have a constant background term, the regularization is not affected by the inclusion of the background in the model.

After adjusting the linear problem to include a constant background level, the resulting system of equations is the following, expressed in eq. 7.

$$\begin{bmatrix} [wU]_{m \times n} & \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{m \times 1} \\ [\lambda S]_{2n \times n} & \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{2n \times 1} \end{bmatrix} \begin{bmatrix} [A]_{n \times 1} \\ [b]_{1 \times 1} \end{bmatrix} = \begin{bmatrix} [wI]_{n \times 1} \\ [0]_{2n \times 1} \end{bmatrix} \quad (\text{eq. 7})$$

This system of equations is solved using the standard linear solver `mldivide` of MATLABTM, resulting in the gamma-ray emission intensity of each pixel as well as the constant background level.

2.6. The attenuation response matrix generation

The attenuation response matrix, V , is calculated based on the structure and fuel material identified in the naïve reconstructions of the inspected object. For this purpose, the fuel may be reconstructed using any of the dominant gamma energies in the fuel. The rod positions are localized using template matching, i.e. searching for the best fit of the reconstructed image with an image of the feature of known size and shape. The template matching procedure has been described earlier for localization of

fuel rods in gamma emission tomography in (Davour A., 2016). In UPPREC, the rod positions are localized as follows:

- 1) First the location of a rod center in each pixel center is evaluated by a figure of merit, which is simply the sum of all pixel values for pixels that have their center inside the rod radius from the center of the evaluated pixel.
- 2) The pixel corresponding to the maximal value of the figure of merit is localized.
- 3) Using the 5x5 pixels in the closest neighborhood of the maximum, a 2D second order polynomial is fitted to the figure of merit as a function of the pixel center coordinates, see Figure 4 for demonstration example.
- 4) The local optimum of the polynomial function is solved and used as the best fit rod location.
- 5) The figure of merit is set to zero in the pixels located inside the identified rod to avoid overlapping rod localizations.

In case of a search for many rods, the procedure 1-5 is repeated until the selected number of rods has been localized. The fuel rods are searched for using either the as-manufactured radius or by accounting for swelling according to e.g. reference (Schrire, et al., 1998).

One methodological difference between the method used in UPPREC and the method described in (Davour A., 2016) can be noted. In (Davour A., 2016), the precision better than the pixel size is achieved by resampling the reconstructed image with interpolated values, while with UPPREC the polynomial fit is used as described above.

Figure 4. The figure of merit values for the pixels of a 5x5 matrix surrounding the pixel of with the highest value. A second order 2D polynomial surface has fitted to the values. The rod is localized by solving for the maximum of the polynomial.

The rod localization results from a 13 pin driver fuel are shown in Figure 5. In a) the fuel rods are located using 662 keV gamma rays, in b) a new reconstruction has been performed, including the attenuation of

gamma rays in the 13 fuel rods and in this image the 4 tie rods are identified by the negative artifacts caused by the attenuation of gamma rays in their material. It has been observed that lower energy gamma rays can be advantageous to use when locating attenuating structures (Jacobsson Svård, et al., 2015), (Holcombe, et al., 2016). However, as seen here, the tie rod locations appear also with 662 keV gamma rays. Note that a new reconstruction was made after the localization of the 13 fuel rods, prior to the localization of the 4 tie rods.

Figure 5. The locations of the fuel rods (a) and tie rods (b) as identified with the UPPREC analysis. The red circles show the identified rod positions.

After the localization of rods, the transmission of gamma rays through the fuel and cladding is determined by Beer's law for each pixel,

$$I/I_0 = e^{-\mu l} = e^{-\frac{\mu}{\rho} l \rho}$$

Where I/I_0 is the transmission through the rod, μ is the linear attenuation coefficient, which is determined based on NIST data tables (Hubbell, 2004) of the mass attenuation coefficient, μ/ρ , and the density, ρ , which may be either as-manufactured or may account for the solid swelling as described above.

The distance travelled through a rod, l , is determined based the distance between the intersections of the circular rods with the line from the center of the pixel to the center of the collimator slit. This is achieved by first performing a change of basis by translation and rotation to a coordinate system with the origin in the pixel center and the detector directed along one of the coordinate axes. The resulting two coordinates of the rods may be called u and v , and l is calculated based on the two according to a set of conditions; these are shown in Figure 6.

Figure 6. Four example rod locations and their estimation of path length, l , through fuel rod. The blue fuel rods are attenuating the rays from the pixel while the red are outside of the path to the detector.

In addition to fuel rods, the attenuation of tie rods and cladding material, and other cylindrical rods or pipes are also accounted for in the same manner.

2.7. Full system matrix calculation and final reconstruction

After deriving the attenuation distribution and the system response to it, V , the final reconstruction of the fuel gamma-ray emission distribution is performed as described in section 2.4., but now with the full system matrix W , which is the Hadamard product of U and V , as described in 2.4. The result from the 13 rod driver fuel is demonstrated in a 2D surface plot in Figure 7. In this case, the attenuation of the 13 rods as well as the 4 structural tie-rods has been accounted for in the analysis. The reconstruction shows the increased emission rate in the discs corresponding to the locations of the 13 fuel rods and the artifacts that were visible between the fuel rods in the naïve reconstruction have clearly been reduced. For comparison between the step-by-step quantitative approach used by UPPREC and qualitative reconstructions where attenuation is neglected, the first naïve reconstruction of UPPREC as well as a Filtered Backprojection reconstruction using a ramp filter have been added to Figure 7. In addition, two cut-through slices of the final reconstruction and, for comparison, the naïve reconstruction are shown in Figure 8.

Figure 7. Left: Quantitative reconstruction of the gamma emission distribution in a 13-rod HBWR driver fuel. Several features may be seen. Most prominently the 13 fuel rods, also the suppression of the activity in the centers of each rod is seen. In addition, the peripheral fuel rods appear to have a higher activity towards the outside of the assembly. Middle: The original naïve reconstruction for comparison. Due to neglecting gamma-ray attenuation this reconstruction has many artifacts and the emission intensity in the central rod and in the rod centers are deceptively low. Right: For comparison a qualitative Filtered-Backprojection reconstruction was performed. This reconstruction contains the same artifacts as the naïve reconstruction due to neglecting attenuation.

Figure 8. Cross-sectional slices of the reconstructed emission intensity. Left: Locations of slices. Right: Cross-sectional slice intensities, final reconstruction in filled line and naïve reconstruction in dashed line for comparison.

It can be noted that with the fuel rod and structural positions known, it is a relatively fast operation to extract and reconstruct additional gamma-ray energies available in the fuel spectrum. Thereby, any additional fission or activation products of interest in the fuel assembly may readily be extracted at this point. The only change needed is regarding the attenuation response matrix V , which needs to be adjusted for the new linear attenuation coefficient corresponding to the gamma energy. However, in the case that a Monte Carlo based instrument response is being used, this needs to be replaced with new transport simulations with the appropriate energy. In the calculations here, the faster method described in section 2.3 was used.

3. Validation by comparison with single rod gamma scanning

In order to validate the performance of the UPPREC reconstruction methods, two of the prominent peaks in the gamma spectra of the 13-rod Halden driver fuel assembly were used to reconstruct the rod-wise activity distribution and compare to the rod-wise distribution as determined through measurements using the established technique of single rod gamma scanning. For this purpose, the 724 keV peak of ^{95}Zr and the 766 keV peak of ^{95}Nb were selected, their emission distributions were reconstructed by tomography and the relative activity of each rod was determined by summation of the pixel values inside each rod. The values were normalized using the average rod activity.

The agreement with the tomographic reconstruction and the single-rod gamma scanning results is shown in Table 1 below. The average absolute error and median error were less than 3 % for both isotopes.

It can be noted that the tomographic reconstructions show an asymmetrical activity distribution within each rod, as seen in Figure 7. Considering this asymmetry, the level of agreement between the gamma tomography and single-rod gamma scanning results is as good as may be expected, since the single rod gamma scans may have a random error depending on which side of the fuel rod was measured. This problem is not expected to affect tomographic measurements, where the rods are examined from all directions (Holcombe, et al., 2016).

Table 1. Benchmarking results against single rod gamma scans using ^{95}Zr and ^{95}Nb .

Rod ID	Tomography		Gamma scanning				
	724 keV	766 keV	724 keV	766 keV			
1	1.016	1.022	0.980	0.977			
2	0.995	1.001	0.936	0.948			
3	0.975	0.978	0.982	0.984			
4	1.002	1.012	1.026	1.028			
5	1.043	1.053	1.080	1.089			
6	1.071	1.071	1.076	1.071			
7	1.091	1.093	1.046	1.035			
8	1.085	1.088	1.114	1.109			
9	0.942	0.932	0.902	0.914			
10	0.941	0.922	0.931	0.930			
11	0.944	0.938	0.938	0.942			
12	0.980	0.967	1.068	1.055			
13	0.917	0.922	0.920	0.918			
						Deviations [%]	
							724 keV 766 keV
						Average absolute	2.998 2.768
						Median	2.971 1.878

4. Conclusions

The UPPREC reconstruction method for gamma emission tomography of irradiated nuclear fuel has been developed and used for reconstructions of nuclear fuel assemblies measured using the HRP gamma tomography instrument. This reconstruction technique takes into account the strong and inhomogeneous radiation attenuation within a fuel rig/assembly and in addition it makes use of a

realistic field-of-view cone in order to provide quantitative images of the emission distribution of the investigated gamma rays. The resulting images are quantitative and detailed (i.e. high-resolution with regard to the characteristic spatial length scales of the fuel, such as the rod diameter).

It was demonstrated that the iterative approach, where firstly the attenuation was completely neglected in order to localize the fuel rods and subsequently the structure materials, was a successful step-by-step strategy to improve the quality of the reconstruction, and finally the attenuation of all important structures was accounted for. As seen in a comparison between the naïve and quantitative reconstructions of a 13 rod fuel in Figure 7 and 8, the method is efficient in suppressing the artifacts created by the attenuating material in the object.

Using pixel summation in the reconstructed images, the rodwise activity can be obtained for any radioisotope of interest that is visible in the gamma spectra. The relative rod activity distributions of ^{95}Zr and ^{95}Nb were used to benchmark the technique against single-rod gamma scans for a 13-rod Halden fuel assembly. The agreement was better than 3% for both isotopes, both in terms of mean absolute deviation and median deviation.

UPPREC is the first Gamma Emission Tomographic reconstruction code for nuclear fuels that is both quantitative and detailed. This added ability allows for imaging of radionuclide distributions of a higher level of detail than previously achievable by non-destructive means. This may be used for studies of fuel behavior under normal operation conditions (e.g. for evaluation of power and burnup distributions, rod bow, migration of volatile fission products or fission gas release) or for investigation of the performance of nuclear fuels in special conditions, such as fuels that have undergone transient tests or that have been exposed to actual accidents. In addition, gamma emission tomography is a viable tool for nuclear safeguards inspections, where the methods assembled in UPPREC may also be useful (e.g.) for verification of the presence of single fuel rods.

The ability to non-destructively obtain this type quantitative information does not exist with other methods and being able to perform rodwise characterization of nuclear fuel assemblies without dismantling them is a significant advantage with regard to time and cost savings when performing fuel performance measurements at e.g. research or commercial reactors and with regard to safeguards measurements where fuel should preferably not be dismantled.

5. Discussion

5.1. Accuracy of the benchmark test

In the benchmark, single rod gamma scanning was used. It may be noted that several of the fuel rods appeared to have asymmetrical rod-internal burnup distributions, which may have inhibited the accuracy of the single rod gamma scan validation data, where the rods were assessed only from one side, thereby giving rise to a random error which may have affected the accuracy. Therefore, improved agreement may be seen if the radial activity distribution is taken into account in the benchmark measurements.

5.2. Accounting for the possibility of zero registered counts.

The base of the UPPREC methods is the WLS solver that solves the linear system of equations presented by the tomographic reconstruction. Here, the weights are based on the inverse variance estimate of the measured number of counts of each interrogated position, which constitutes the Maximum Likelihood solution to the linear system of equations. It is important to note that this is valid only under the assumption that the errors follow a Gaussian distribution. In reality, the often dominating source of error in radiation experiments are the Poisson errors of counting statistics. Poisson errors are to a good approximation substituted by Gaussian only if the number of counts is 10-20 or higher.

In the data collected so far, this condition on the number of registered counts has often been satisfied. The measurement positions with the highest number of registered events in the peak had in the order of 10^4 counts. Even in the measurement positions where no fuel rod has been located inside the collimators field of view, there may be background supplying enough registered counts in the full peak channels. This background constitutes of a combination of Compton continuum from higher energy gamma, gamma rays that are Compton scattered in structures surrounding the fuel, Rayleigh scattered gamma rays that have changed direction but lost only a negligible amount of energy, and background from the ambient or leakage through the collimator material. The lowest number of registered events in any position in this work was 15 counts.

If, however, the analysis would be performed on data with no background, the potential of zero counts in some position will present a problem for the WLS-solver with weights inversely proportional to the Poisson variance. Such data has been analyzed using simulated data, showing that if not dealt with, it causes infinite weights to the affected measurements, which in turn cause the reconstruction to fail. Because of this eventuality, we presented the offset, δ , to the number of counts used in the weight calculation of section 2.5. In simulated data, this has proven to improve the performance of the reconstruction with no background and low numbers of registered events.

Further studies are needed to optimize the parameter δ . It may also be considered to replace the use of the χ^2 statistic with the Cash statistic, the minimization of which corresponds to the maximum likelihood solution in Poisson fitting, if zero counts are registered.

5.3. Comments on the ambient background model

The presented data was measured in a setup where rotation of the fuel assembly relative to the detector system was achieved by applying the rotation on the fuel. Thereby, the detector system is nearly stationary during the measurements, apart from the lateral step interval spanning less than 10 cm. Thereby the background, in the measurement presented in this article, is believed to largely origin from ^{137}Cs (and ^{60}Co) contamination in the surrounding facility, can be expected to be nearly constant in all the measurements, and consequently, a constant background is added as an explanatory variable in section 2.4.

In the case of a different measurement setup, with fixed fuel and a rotating detector, the larger span of position and direction of the detector may be expected to cause a more complex background with a dependence on the position, which may require attention. If UPPREC is applied in this scenario, the

background from ambient may be better modeled by performing a careful experimental characterization of the background in each assessed position when no fuel is present.

5.4. Comments on the linear attenuation of the fuel material

In section 2.6, the response of the attenuation of gamma rays in the localized fuel rods is described. One may note that UPPREC considers the gamma energy dependent attenuation of fuel and cladding. The code can consider the fuel and cladding dimensions as-manufactured, or consider a swelling (affecting the density and radii) according to a swelling model such as (Schrire, et al., 1998). It is thus important to note that the fuel material and dimensions and the burnup are treated as known inputs in the approach described in section 2.6. If UPPREC or sets of equivalent methods were to be applied in the field of nuclear safeguards, one may not with certainty have access to the same detailed information from the fuel manufacturing and the burnup history. Or if such information is available, it may need to be verified by the inspection.

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