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Mathematical modelling approach to collective decision-making

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Abstract

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In everyday situations individuals make decisions. For example, a tourist usually chooses a crowded or recommended restaurant to have dinner. Perhaps it is an individual decision, but the observed pattern of decision-making is a collective phenomenon. Collective behaviour emerges from the local interactions that give rise to a complex pattern at the group level. In our example, the recommendations or simple copying the choices of others make a crowded restaurant even more crowded. The rules of interaction between individuals are important to study. Such studies should be complemented by biological experiments. Recent studies of collective phenomena in animal groups help us to understand these rules and develop mathematical models of collective behaviour. The most important communication mechanism is positive feedback between group members, which we observe in our example. In this thesis, we use a generic experimentally validated model of positive feedback to study collective decision-making.

The first part of the thesis is based on the modelling of decision-making associated to the selection of feeding sites. This has been extensively studied for ants and slime moulds. The main contribution of our research is to demonstrate how such aspects as "irrationality", speed and quality of decisions can be modelled using differential equations. We study bifurcation phenomena and describe collective patterns above critical values of a bifurcation points in mathematical and biological terms. In the second part, we demonstrate how the primitive unicellular slime mould *Physarum Polycephalum* provides an easy test-bed for theoretical assumptions and model predictions about decision-making. We study its searching strategies and model decision-making associated to the selection of food options. We also consider the aggregation model to investigate the fractal structure of *Physarum Polycephalum* plasmodia.

Keywords: collective behaviour, collective decision-making, communication mechanisms, positive feedback, mathematical modelling, bifurcation phenomena, steady state solutions, symmetry breaking, symmetry restoring, diffusion-limited aggregation, fractal dimension

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Dedicated to my sons Nikita and Joseph

List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

- I Nicolis SC, **Zabzina N**, Latty T, Sumpter DJT (2011). Collective Irrationality and Positive Feedback. PLoS ONE 6(4): e18901. DOI:10.1371/journal.pone.0018901
- II Sumpter DJT, **Zabzina N** and Nicolis SC (2012). Six Predictions about the Decision Making of Animal and Human Groups. Manage. Decis. Econ., 33: 295-309. DOI:10.1002/mde.2553
- III **Zabzina N** (2015). A gradient flow approach to the model of positive feedback in decision-making. Chaos, Solitons & Fractals, vol.77, pp.215-224 DOI:10.1016/j.chaos.2015.05.027
- IV **Zabzina N**, Dussutour A, Mann RP, Sumpter DJT, Nicolis SC (2014). Symmetry Restoring Bifurcation in Collective Decision-Making. PLoS Comput Biol 10(12): e1003960. DOI:10.1371/journal.pcbi.1003960
- V Gaydashev D and **Zabzina N** (2016). Fractal dimensions of the conformal diffusion-limited aggregation. Manuscript.

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1. Introduction

The present thesis deals with the mathematical modelling of collective decision-making. How can we link the behaviour of an individual to collective decision-making? Is it possible to neglect individual differences and consider decision of group of human as a solution of differential equations? Can we explain our own decision through mathematics?

Individual differences are affected by the development of an individual, where the social relationships play a crucial role. Considering the behaviour of an independent person is interesting from the point of psychoanalysis, medicine, criminology and related areas. Mathematicians, on the other hand, have often models and tools describing behaviour of a whole population. An individual frequently acts by standards and rules given by society or situations. To make a decision simple common sense is normally used. Common sense is known as a basic ability to perceive, understand, and judge things that is shared by nearly all people. In other words, consistent decision-making of one individual is often based on judgements of others. As it may be, there is a need to be a part of a group, community, share similar interests and a life style. Perhaps everyone wants to be a part of society and has a fear of being alone. Walking in the same direction, avoiding collisions in the streets, solving conflicts in life are examples of social relationships, which we recognise as collective behaviour.

Now we have placed an individual among other individuals and will consider the collective behaviour pattern of the group of individuals. In our work we consider how the process of collective decision-making can be examined in the following way: first we observe what individuals do and how they respond to their environment, then how they interact with each other and what are the consequences of these interactions. Indeed, there are common needs, dreams and there are also resembling strategies to achieve aimed goals among all individuals. Education is one of the examples of strategies to achieve a well-paid job and success in life. An everyday situation can serve as another illustration: a tourist is in a foreign country choosing a place to have a dinner. The most common choice is to go to a crowded restaurant. Perhaps it is an individual decision, but the observed pattern is a collective phenomenon. This example of a crowded restaurant becoming more crowded can be studied from the point of collective decision-making.

Collective behaviour emerges from the local interactions between individuals that give rise to a complex pattern at the group level. It is important to investigate the rules of interaction between individuals, their communication mechanisms and complement the investigation by biological experiments.

Collective behaviour is all around us. Walking in the forest one may notice a flock of birds flying above ones head. Should one have, at the same time, mistakenly put a foot in a wasp nest, one can observe rapid and active collective animal behaviour around the intruder. Of course, one does not have to venture into the forest to observe these phenomena. If one spots an ant in the kitchen, one knows that there are many more soon to follow. Recent studies of collective phenomena in animal groups shed light on the understanding of the interacting rules between individuals. For example, ants live in a communal nest which provides opportunity for the information transfer among group members. Thus a steady trail of ants walking in the kitchen can be explained by a solution of differential equations representing a mathematical model of collective decision-making. There are many models of collective behaviour that have been used extensively by many researchers over the last years to study communication mechanisms between individuals and to make theoretical predictions for collective decision-making. We, human beings, view ourselves as being quite different from each other with our private goals and ambitions. It is therefore surprising that many observations and studies show how our collective behaviour closely resembles that of animals. Thus it is possible to apply model experimentally tested on animals to explain human collective decision-making.

To study the link between individual behaviour and collective decision-making we regard decision-making of a group of individuals as a solution of a mathematical model. One of the main contributions of this thesis is to demonstrate how aspects of collective decision-making such as "irrationality", decision quality and speed can be modelled using differential equations. We also make a thorough study of bifurcation phenomena corresponding to situations where a small change in the biological system results in radically different collective patterns. Finally we demonstrate that such ostensibly primitive organisms as slime mould, *Physarum Polycephalum*, can provide a good test-bed for a study of collective decision-making.

This thesis consists of two parts: the first is based on the modelling of decision-making associated to the selection of feeding sites. In the second part, we study searching strategies and model the decision-making of slime mould. Recent studies show that *Physarum Polycephalum* plasmodia have a fractal structure. This motivates us to use an aggregation model to investigate this fractality.

2. Positive feedback is the most common mechanism in decision-making

In every day situations individuals make decisions that, from a mathematical perspective, should maximize some utility function but can also relate to simple satisfaction. To make a good decision one should invest effort to search for more information. The picture that emerges from a large number of decision-making experiments indicates that even when there is no explicit time limit, there is implicit time pressure because of the cost of processing time. In fact individuals do trade off accuracy for effort [11, 39]. There are beliefs that irrational behaviour appears due to use of heuristics that require little effort. Indeed, use of common sense, stereotypes, an intuitive judgment or a rule of thumb are mental shortcuts that ease the cognitive load of making a decision. There are studies that show that decision-making mechanisms in brains and in colonies of social insects are related to each other. Moreover, social insect colonies may achieve optimal collective decision-making in a very similar way to primate brains [48, 10]. Is there a perfect model of reality which can be used to describe brain activity and collective decision-making? To search for such perfection can take a life time.

Decision-making associated to the selection of feeding sites or shelters has been extensively studied in many social [12, 25, 65] and gregarious [3] living organisms, including vertebrates [74, 17, 75]. These decisions were studied in terms of their mechanisms and fitness benefits to group members. There are rules of thumb in the context of group decision-making and the most important of these is positive feedback between group members [24, 25, 8, 2, 7]. Positive feedback occurs when the commitment to a particular option increases as a function of the number of individuals already committed to it. This phenomenon is the basic principle underlying so-called pheromone trails. In social insects, particularly in some species of ants, the formation of pheromone trail is used to inform nest mates where food is located or to find the shortest path to it. After finding food and feeding, an ant returns to the nest. On its way it leaves chemicals, known as pheromones to mark the route from the food to the nest. Other nest mates follow the trail, find food and reinforce this trail. Through such reinforcement the pheromone trail builds up and after a short time there is a steady trail of ants walking between the food and the nest. For collective decision-making, the key heuristic is positive feedback.

2.1 The model of positive feedback

We assume that there is a constant flow ϕ of decision makers who make a decision one after another. In biological terms, we think of a number of ants per time step leaving their nest or fish swimming up a channel. Each of these individuals approaches a choice point where one makes a decision as a function of the intensity of a signal left by the number of individuals that have already committed to the option i with quality encoded by variable q_i . This flow is inexhaustible, but over time, the effect of the decision of previous choices fades with rate ν . The evolution of the commitments x_i (which can be a number of individuals or amount of pheromone left by individuals) to option i can be cast in the form [55]

$$\frac{dx_i}{dt} = \phi q_i f_i(x_1, \dots, x_n) - \nu x_i, \quad i = 1, \dots, n, \quad (2.1)$$

where n is the number of the options and the choice function $f_i(x_1, \dots, x_n)$ expresses how a future commitment to an option i is affected by the current commitment both to i and other. The specific forms of choice function depend on the biological system one wants to describe. For *Temnothrax* ants, f_i can be considered linear [62] but for *Physarum* it is a nonlinear saturating function [76]. However, f_i should provide a quorum-like response, so that above a threshold the rate of increase in commitment becomes significantly large.

We consider one of the first functional forms that has appeared in mathematical modelling of positive feedback which has been proposed and tested experimentally [34, 23, 5, 6] for ant species using trail-laying recruitment. Here the choice function is assumed to take the form

$$f_i(x_1, \dots, x_n) = \frac{(x_i + k)^l}{\sum_{j=1}^n (x_j + k)^l}, \quad (2.2)$$

where the parameter k is the threshold number beyond which the choice of option begins to be effective and l stands for the sensitivity of the response to the process of choice of a particular option. The cooperativity parameter l has been measured experimentally for *Lasius niger* as $l = 2$ [5] and for Argentine ants *Linepithema humile* as $l = 4$ [80].

The Model (2.1) with the Functional Form (2.2) describes positive feedback loops and each loop measures how the buildup of commitment to a particular option evolves in time. We consider the model of positive feedback in the form

$$\frac{dx_i}{dt} = \phi q_i \frac{(x_i + k)^l}{\sum_{j=1}^n (x_j + k)^l} - \nu x_i. \quad (2.3)$$

The decision making depends on a parameter measuring the "quality" q_i of the option and the number of individuals already committed to each of the options,

thus the rate of commitment depends on both a private evaluation of the option and the social information. This model was extended and well studied for homogeneous environments and for the case where one of the options is better than others.

We have introduced the model of positive feedback and described the extensive theoretical and experimental work which has been done for many years in different fields of research. What can we add to these studies? In this thesis we introduce some other ideas and approaches to the classical modelling of collective decision-making. In the following subsections we consider the model of positive feedback to highlight "irrationality" in decision-making. We discuss how concepts of speed-accuracy trade-off, increasing quality with group size and "irrational" decisions are correlated to the number of options. Then we propose a geometrical reformulation of this model in terms of gradient flow equations to show how the tools of Riemannian geometry and topology can be used to explore biological phenomena described by the model of positive feedback. Finally we apply positive feedback phenomena to the modelling of slime mould, *Physarum Polycephalum*, and particularly to its searching strategies.

2.1.1 Irrationality

In Papers I and II, we argue that positive feedback is a heuristic which often produces fast and accurate group decision-making, but is always susceptible to apparent irrationality when studied under particular experimental conditions. We apply the model of positive feedback to the problem of choosing between multiple options to show that the probability of taking the best $q_1 > q_j$ of n options depends on initial conditions and the strength of feedback.

The main analytical results are related to the study of steady state solutions, their existence and stability [55, 56]. Only physically acceptable solutions (i.e., real and positive solutions) are considered to test the stability in order to determine the state that will actually be chosen by the system. The steady state condition means that the system does not change over time. If a system is in a steady state then the recently observed behaviour of the system will continue into the future. To study system behaviour depending on the parameters we find the steady state solutions of the Model (2.3) and draw a bifurcation diagram. The bifurcation diagram shows the possible long-term commitment values of a system, in our case commitment values of steady state solutions as a function of a bifurcation parameter ϕ . The summary of the results are displayed in Figure 2.1. We show how the steady state level of commitment to the best option changes with the flow ϕ for $n = 2$, $n = 3$ and $n = 4$. As the flow rate increases the system switches from having one stable state to having two steady states with stronger commitment to one of the options. In Paper I, we describe how the bifurcation point, i.e. the critical value of the parameter ϕ , increases with the number of options thus showing how the preferences change

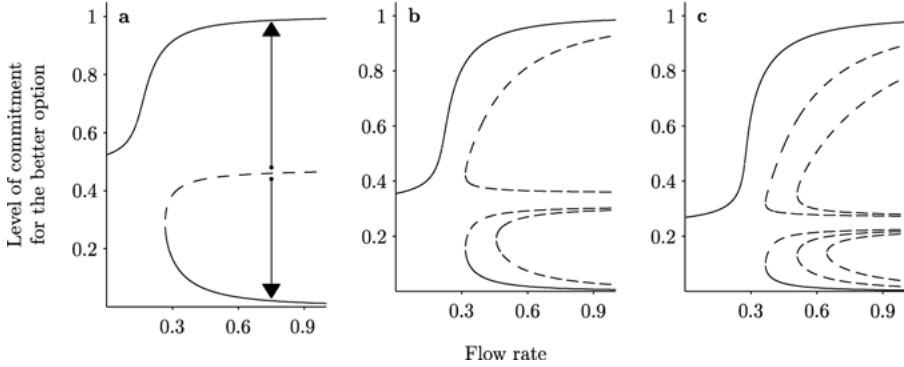


Figure 2.1. Bifurcation diagrams of $x_1 / \sum_{j=1}^n x_j$ corresponding to the steady state level of commitment for the better option with respect to the flow rate ϕ . a) case $n = 2$, b) $n = 3$ and c) $n = 4$. Full and dashed lines correspond to stable and unstable solutions respectively. The arrows indicate the evolution of initial conditions on the two sides of a threshold value corresponding to the intermediate unstable state. The parameter values are $k = 1$, $q_1 = 0.11$, $q = 0.1$, $l = 2$ and $\nu = 0.01s^{-1}$.

with the number of options. Indeed, there are known patterns in human behaviour where the relative attractiveness of two options often depends on the presence or absence of the third one. Such preference changes violate independence from irrelevant alternatives (abbreviated as IIA), because a new option of lesser value apparently alters the value of the two superior options, and can be classified as "irrational" [47]. But we do not label the situation "irrational" when the choice of the lower quality option is forced by the initial condition. In Figure 2.1a the arrows show how different initial commitment levels will evolve. If a commitment is initially strong for the lower quality option then this option will be chosen.

Generally, we consider the case when the parameter $l = 2$, corresponding to the minimal setting of co-operativity. The larger l the sharper the choice and the higher nonlinearity involved in the process. We define a strong positive feedback, corresponding to some $l > 1$, to be the condition for a quorum-like response in the positive feedback. If the parameter $l = 1$ we speak of a weak positive feedback. There are models of decision-making that assume that choice is a linear process ($l = 1$), in other words with absence of co-operativity. Here, there is only one steady state solution, which corresponds to the better option being selected in all of the cases. The proportion of commitment increases linearly with ϕ for a particular number of options and decreases again smoothly with the number of options for a particular value of ϕ . By using Model (2.3) with a co-operativity parameter $l > 1$ (in Paper I) we show outcomes, arising naturally from underlying dynamics, violating IIA.

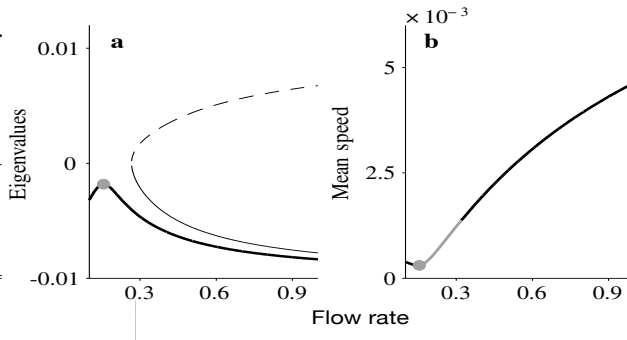


Figure 2.2. a) The corresponding eigenvalues of the linearised equations around the steady-state solutions of Model (2.3) for $n = 2$. b) The time needed to reach the steady state corresponding to the better option as a function of the flow rate ϕ . The parameter values are the same as in Figure 2.1.

2.1.2 Speed

Decision-making is frequently associated with the speed-accuracy trade-off. To achieve an accurate decision, time should be invested, but as we discussed previously, many decisions are made by using a heuristic approach. Due to that, "irrational" behaviour is observed in biological systems. To address the question of speed in decision-making in our system, we introduce the rate at which a decision is reached as we increase the flow ϕ (Paper I and II). This rate is determined by the largest eigenvalue λ for the steady state corresponding to the better option. Figure 2.2 gives two measurements of the speed of decision-making when the Model (2.3) is applied to the case of two different quality options. The first one is the value of the eigenvalues associated with each of the steady states. The eigenvalues show the rate of convergence to the steady state. The time to reach a decision is proportional to the inverse of the magnitude of λ . Thus a larger magnitude implies a faster approach to that steady states. As can be seen in Figure 2.2a, above the maximum point, as we increase ϕ , the absolute value of λ increases and thus the time to reach a decision decreases.

The second measure of the decision-making speed is taken directly from the numerical solution of the Model (2.3). Figure 2.2b exhibits the time needed to reach the steady state corresponding to the better option as a function of the parameter ϕ . There is a minimum value of $\phi = 0.156$ at which decisions are slowest. Thereafter the speed of decision-making increases with the flow rate. This confirms our eigenvalue analysis.

2.1.3 Quality

To discuss the quality of decisions, we consider how many individuals choose the better option. For example, in Figure 2.1b when $\phi = 0.01$, only 36% choose the best option. This is to be compared to $\phi = 0.6$ where 96% choose the best option. In other words, the quality of the option should be confirmed by the quantity of the individuals, choosing the best option. We say that the biological system makes a good quality decision if the best option is chosen by the majority. We define the quality of decision as a combination of the proportion of individuals committed to the better option (Figure 2.3a) and the proportion of cases where this option is selected over a less favourable one (Figure 2.3b). This latter quantity depends on initial conditions and/or random factors. To calculate how many individuals prefer the better option we run a Monte Carlo simulation and count the mean level of commitment to the better option (Figure 2.3a). To calculate how often this option was chosen we use a Monte Carlo simulation again and count the number of cases in which the better option was preferred over the total number of the Monte Carlo realisations (Figure 2.3b). Figure 2.3c exhibits how quality of a decision depends on the flow ϕ and how it changes with number of options. To get the upper curve, corresponding to the quality of decision between two options, we multiply the curves from Figure 2.3a and 2.3b pointwise. To get the other two curves we performed calculations for the cases $n = 3$ and $n = 4$ (see Paper I). As can be seen in the Figure 2.3c the maximal decision quality occurs near the bifurcation point, where the system goes from one to more steady states and the position of the bifurcation point increases linearly with the number of options. In Paper I we show that the quality of decision is not just a matter of population size but is affected by the strength of positive feedbacks along with the number of options presented.

2.1.4 How to use topology to explore biology

As we saw in the previous sections, the bifurcation point, where the system goes from one to more steady states is an interesting point from many perspectives. We consider the region around the bifurcation point, the critical value of ϕ and discuss the phenomenon of system flexibility. Particularly the region of coexistence of the stable steady state solutions around the critical value is intriguing. There is a set which contains initial conditions that lead the system to different steady states. For example, a population of ants chooses a particular food source and a predator appears on the way to it. If the population is big enough it will explore another food source without starvation of the colony. The flexibility of the system will be determined by how fast the population will switch to the exploitation of another available food source with a minimal loss. In our theoretical studies we assume that in the region around the critical value of ϕ , the system can easily switch from one steady state to another.

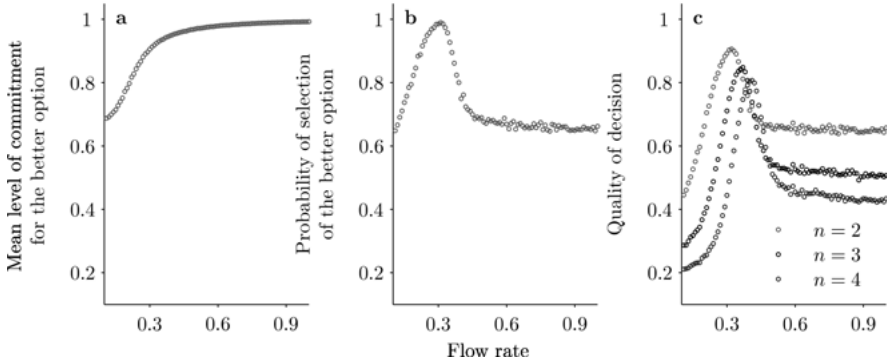


Figure 2.3. Quantitative view of the quality of decision as a function of the flow rate ϕ as obtained by Monte Carlo simulations. a) Mean level of commitment for the better option q_1 , b) Probability of selection of the better option and c) Quality of decision for three different number of options. The parameter values are as in Figure 2.1. The number of realisations is 5000.

Figure 2.4 summarises the work performed with the Model (2.3) in the case of multiple equal quality options. As can be seen, the steady states level of commitment changes within flow rate, and the bifurcation point increases with the number of options. We display Figure 2.4 to show an example of how a region of coexistence between three stable steady states increases with the number of options, which reflects an increasing plasticity of the system. Indeed, in the case of four equal options (Figure 2.4c) the region of coexistence is bigger: ϕ values range from 0.35 to 0.4. This should be compared to the region of three options (Figure 2.4b), where ϕ ranges from 0.27 to 0.3.

Further we investigate the regions around the critical value by proposing a geometrical reformulation of the model of positive feedback (Paper III). We associate the parameter h with population size and define $h_i = \phi q_i$. We introduce the change of variables in the following way

$$x_i + k = h_i \xi_i^2(t), \quad (2.4)$$

where ξ_i is a time dependent function. We substitute the new variables into (2.3) and rewrite in the form of gradient flow equations as

$$\frac{d\xi_i}{dt} = -\frac{\partial U}{\partial \xi_i}, \quad (2.5)$$

where the function $U(\xi_1, \xi_2, \dots, \xi_s)$ is called a potential. It decreases along the flow. Any solution of the System (2.5) gives rise to the solution of (2.3) via the change (2.4). This generalises the steepest decent equation and thus it is related to the problem of minimising the potential function U . The important feature of the gradient flow is that the steady state stable solutions of the gradient flow equations correspond to minima of U , while the unstable ones correspond to

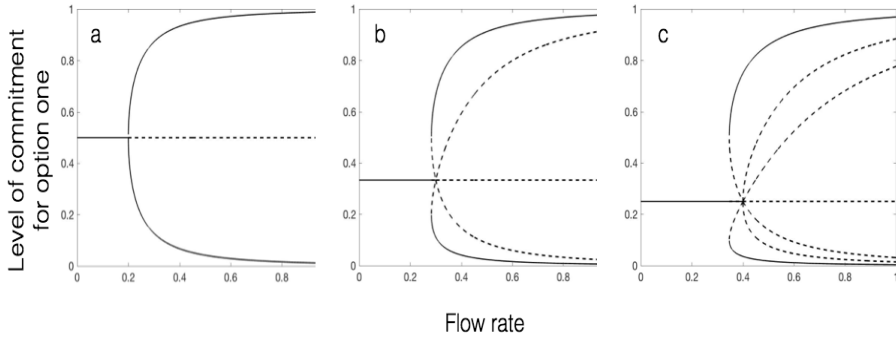


Figure 2.4. Bifurcation diagrams of $x_1 / \sum_{j=1}^n x_j$ corresponding to the steady state level of commitment for the better option with respect to the flow rate ϕ . a) case $n=2$, b) $n=3$ and c) $n=4$. Full and dashed lines correspond to stable and unstable solutions respectively. The parameter values are $q_1 = q_2 = 1$, other parameters are the same as in Figure 2.1.

maxima and saddle points. We mark out these minima as the attractors of our dynamical system, the set of all initial conditions converging to an attractor is its basin of attraction.

As we discuss in detail in Paper III, U is a smooth function on a smooth manifold with boundary, where the Riemannian metric and thus equilibrium solutions and their stabilities are governed by the topology of the graph of this smooth function. As an example of our new method to explore biological phenomena, we consider the Model of Positive Feedback (2.3) with two options. Figure 2.5 exhibits the behaviour of potential U around the attractors. By inspecting the level sets of U which enclose minima we can investigate the basin of attraction for given attractors. The size of a basin of attraction depends on the concrete parameters of the system. When two minima are surrounded by a common level set of U , we assume that there is at least one point for which any neighbourhood has points in both domains of attraction. In the case when the level set encloses only one minimum the particular initial conditions will ensure the system's fate to this minimum and it will be reached by steepest descent.

Figure 2.5a shows one minimum and the level sets enclose only this minimum. Here, we consider the bifurcation point, which occurs at $h = 2$ and any initial condition will lead to the unique minimum. Figure 2.5b exhibits two minima in a neighbourhood of the bifurcation point $h = 2.67$, but we inspect the level sets and conclude that there is a possibility to reach any of these minima with the same initial condition. This reflects the system flexibility around the bifurcation point. When $h = 3$ (Figure 2.5c) there are two minima and they are surrounded by two separated level sets.

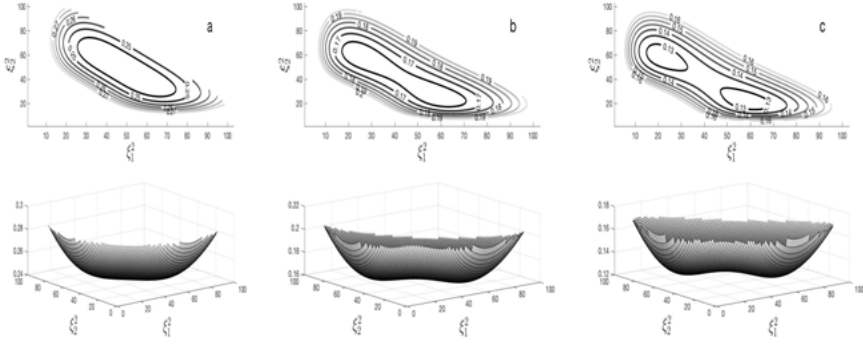


Figure 2.5. Graph and the level sets of potential function in a neighbourhood of steady states solutions. We vary the values of ξ_1^2 and ξ_2^2 from 0.5 to 2 to study the behaviour of potential U around the values corresponding to critical value of $h = 2$. The parameter values are $k = 1, \nu = 1, q_1 = q_2 = 1, l = 2$, a) $h = 2$, b) $h = 2.67$ and c) $h = 3$.

Figure 2.6a shows the level sets of potential U around the attractor for $h = 3$ and $q_1 > q_2$. As can be seen, the attractor corresponding to the better option has more level sets enclosing it than the another one. In other words, the attractor corresponding to the better option has a bigger basin of attraction.

In Paper I we discuss the influence of initial preferences on the decision-making. We do not label the case "irrational" where individuals are not aggregating at the better option in every experimental trial. In some experimental trials the biological system has initial preferences to the poor option and the system commitment will be towards this option. To highlight this phenomena we investigate the level sets exhibited on Figure 2.6 and level of commitment to the better option shown in Figure 2.1a. As can be seen on Figure 2.1a, the level of commitment to the better option is higher for small values of flow rate ϕ , which in that case corresponds to a population size. As ϕ increases the system chooses another option more often as we discuss in Paper I. Here, by false cascades we mean the choice of the option of poor quality. With the help of the new approach we can increase population size and investigate the basins of attraction for two different attractors. Figure 2.6b shows that these basins become almost equal for the value of $h = 20$.

2.2 Discussion

We summarise our theoretical work with the Model of Positive Feedback (2.3) which was proposed and tested experimentally for an ant species using trail-laying recruitment. We showed that the selection of the option depends strongly on the size of the group and on the initial preferences, which will entrain the majority of individuals to focus on a particular option. The "rational" deci-

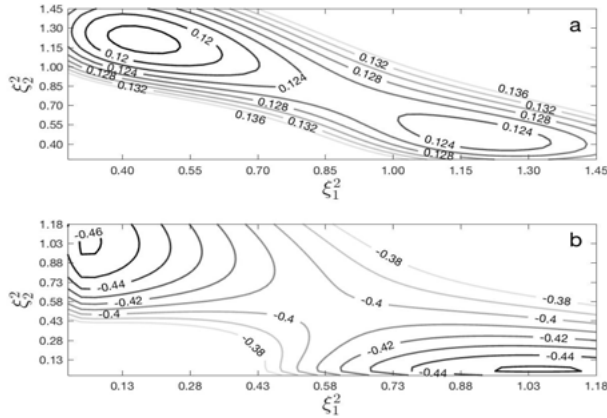


Figure 2.6. The level sets of potential function U in neighbourhood of attractors. The parameter values are $q_1 = 1.05$, $q_2 = 1$, a) $h = 3$ and b) $h = 20$ and other parameters are the same as in Figure 2.5.

sion depends on the initial conditions and the strength of positive feedback. When the choice is linear, with a weak positive feedback, we consider the system making a "rational" decision, for example species of house hunting ants [62, 63]. For strong positive feedback we observe "irrational" decision making in ants and slime moulds [76, 38]. How fast a decision is made, is shown by the eigenvalues magnitude. But when we compare speed with the quality of decision we notice that there is no trade-off between speed and accuracy, which perhaps is counterintuitive. What we detect, is an optimal parameter, where both are maximised. Perhaps these results hold only for our Model of Positive Feedback (2.3). As we discuss in Paper II, increasing the flow rate leads to an increased speed but decreased accuracy. For the flow rate indicated with a light grey line in Figure 2.2b, we see that both speed and accuracy increase with the flow rate. After $\phi = 0.32$ the quality of decision (Figure 2.3b) starts to fall, whereas speed continues to increase (Figure 2.2b). The maximal decision quality occurs near the bifurcation point. The optimal parameter ϕ is different for any given number of options. Figure 2.3c shows that the value of ϕ that is optimal for choosing between two options is not optimised for choosing between three or more options. We expect the system based on positive feedback to do fast and accurate decision, but it is important not to exclude the possibility of false cascades arising. We conclude that the basic properties of positive feedback and quorum response heuristics are captured in this model.

There are benefits in the present geometrical reformulation: first it makes it easier to study certain qualitative aspects of the model by studying the smooth monotone potential function. The bias to initial conditions and the amplification of response can be seen in the gradient flow pictures. A second benefit of the reformulation is that many geometrical tools in Riemannian geometry and

topology can be used to explain biological phenomena described by the model of positive feedback. The consideration of gradient flow equation brings the natural analogy with physical systems, namely the particle moving according to a potential U on the Riemannian manifold with x_i as the coordinates of the particle.

3. Studies and modelling of slime mould searching strategies

In Paper IV we analyse decision-making at the cellular level, for the paradigmatic case of *Physarum Polycephalum*. *Physarum Polycephalum* is a unicellular, true slime mould, typically yellow in colour and inhabits shady, cool, moist areas such as decaying leaves and logs. It belongs to the supergroup Amoebozoa (see Figure 3.2). Its vegetative phase is a single cell plasmodium consisting of networks of protoplasmic veins and many nuclei. During this stage the organism searches for food. It explores the environment by forming a network of tubules (pseudopodia) where cytoplasm streams back and forth circulating nutrients and chemical signals. The plasmodium is able to sense various stimuli from a distance and move toward them. In the wild, *Physarum Polycephalum* eats bacteria and dead organic matter, in the laboratory they are fed oat flakes. It provides an easy test-bed for theoretical assumptions and model predictions about decision-making.

3.1 Symmetry restoring

For our Model (2.3) the critical value of the parameter ϕ is a symmetry breaking bifurcation point. The classical example is shown in Figure 2.4a, known as pitch fork bifurcation. There is a transition from a homogeneous exploitation of the options to a focus on a particular one that occurs above a certain critical value of a parameter. The phenomenon of symmetry restoring is also observed, specially when the size of the system becomes very large. For example, direct contacts resulting from crowding in foraging ants lead to the exploitation of the two options, despite the fact that only one route is chosen when there is no crowding [28]. Here the symmetry is restored due to active regulations of the traffic by individuals. Our study is focused on symmetry restoring without an active regulatory feedback. In Paper IV we show that a symmetry restoring bifurcation may arise depending on the mass of the slime mould *Physarum Polycephalum*.

We perform mathematical modelling of decision-making of *Physarum Polycephalum* confronted with two equal options (see Paper IV for description of experiment). Contrary to the classical paradigm of symmetry breaking, we show that symmetry is restored for the system, in our case an individual organism, of sufficiently large size. This arises from a combination of positive feedback and a regulation of the flow by the fraction of the system's mass already committed to the options.

3.1.1 The positive feedback model with new assumptions

We consider the general Model of Positive Feedback (2.1). We apply the model to two identical options ($n = 2$ and $q_1 = q_2$). We define the parameter M as the system size or the total number of system units in which the system is measured. Then $M - X_1 - X_2$ is the size of pool of uncommitted units, where X_1 and X_2 is the number of units committed to Option 1 and 2 respectively. Considering the finite number of units within the system leads to an expression for the flow rate parameter in the following form

$$\phi = \Phi(M - X_1 - X_2), \quad (3.1)$$

where Φ is the rate of choosing one of the options per individual unit of time. This form of the flow of units going towards the options per unit of time works as a passive regulation of the flow in our research. It can be also viewed as passive negative feedback of X_1 and X_2 , whereby depletion of units reduces the rate of recruitment. In other words, for any given value of the initial mass M , the portion of the system not yet committed to the option is decreasing as X_1 and X_2 are increasing.

The dominant idea is based on the following: the number of units having already committed to a particular option is independent of the number of units choosing the other option. Thus the choice function takes the form [71]

$$f(X_1) = \frac{X_1^l}{k^l + X_1^l}, \quad (3.2)$$

where the parameter k is the threshold flow for the tube construction and here, $l = 2$ stands for the sensitivity of the response. We assume that decisions are local in the sense that the choice function depends only on the fraction of the mass of the system attracted to a particular option.

We modify the model of positive feedback to the case of decision-making in slime mould *Physarum Polycephalum* with two equal food options in the following way:

$$\begin{aligned} \frac{dX_1}{dt} &= \Phi(M - X_1 - X_2) \frac{X_1^2}{k^2 + X_1^2} - \nu X_1, \\ \frac{dX_2}{dt} &= \Phi(M - X_1 - X_2) \frac{X_2^2}{k^2 + X_2^2} - \nu X_2. \end{aligned} \quad (3.3)$$

Figure 3.1 shows the bifurcation diagram of steady state solutions of Equation (3.3) with respect to the parameter mass M . There are three bifurcation points. Before the first one there is one trivial stable steady state, $X_1 = 0$. In terms of the behaviour of *Physarum Polycephalum* this state describes a situation where the plasmodium did not make a decision; it failed to find food or never moved from the starting point. Above the first bifurcation point, the system has three stable states. One is the trivial and the others two correspond to

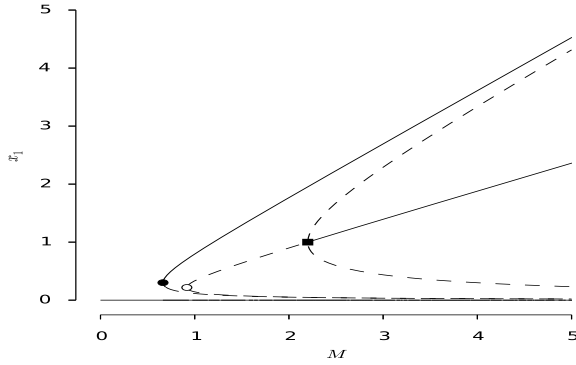


Figure 3.1. Bifurcation diagram of steady state solutions of Equation (3.3) with respect to the parameter mass M . Full and dashed lines correspond to stable and unstable solutions respectively. The black circle shows the first bifurcation, the white circle corresponds to the second bifurcation and the black square labels the third bifurcation. The parameter values are $\Phi = 1$, $k = 1$ and $\nu = 0.1$.

the exploration of one of the two available options. As can be seen, the level of commitment to an option changes with the size of the initial system. We continue to increase the initial mass M and observe the second bifurcation point, above which unstable homogeneous solution appears. In biological terms, the instability of this symmetric solution means that the plasmodium does not have enough mass to exploit two options at the same time and thus moves to just one. The third bifurcation point appears when the initial mass achieves a critical value M^* , above which the homogeneous solution becomes stable. There are three stable states: no decision will be made, one of the two options will be chosen or both options will be exploited. The bifurcation at M^* is a symmetry restoring bifurcation, since a stable, nontrivial symmetric solution occurs at this point.

In Figure 3.1 we observe, that when M increases, the upper branch of a non-homogeneous stable solution becomes close to a branch of an unstable one, occurring above the bifurcation value M^* . As we discuss in Paper IV, an asymptotic analysis of these solutions shows that for a large mass these two solutions are approximately equal. The stable upper branch can never be reached in the sense that the set of initial conditions in its attraction basin decreases in size with M . In terms of the *Physarum Polycephalum* decision-making it means that a plasmodium of a very large mass nearly always chooses two options rather than moving to one.

To learn more and capture different type of the exploitation pattern of the plasmodium, we conducted a series of experiments with three different strains of *Physarum Polycephalum* (for detailed experiment description see Paper IV). We refer to these slime mould strains by the name of continent or country they come from: Japanese, Australian and American, respectively (Figure 3.2). The

Japanese strain forms a few thick tubes to explore the environment while the Australian spreads in all direction by forming multiple thin tubes. The American one combines both exploration patterns. We notice quantitative differences of exploration patterns between the strains, and assume that decision-making by *Physarum Polycephalum* depends not only on the size of the plasmodium but also on the different exploration patterns of the strains.

In order to compare the predictions of the model to the experimental outcome, we identified the best fit model in terms of the parameters Φ and k for each strain. We view Φ as the speed of displacement of the plasmodium and let k reflect a threshold beyond which a tube can be built. It is related to the way the different strains are moving: a small value of k means that a tube is more easily constructed, even with a low mass. In our Model (3.3) this is reflected by the Choice Function (3.2) which saturates more quickly and favours the homogeneous solutions. On the contrary, a large value of k implies that a large initial mass will be needed to build a tube and that the choice function saturates more slowly, thus system is favouring the non-homogeneous solution. The best-fit parameters identified for each strain of *Physarum Polycephalum* are shown in Table 1, along with the position of the symmetry-restoring bifurcation point. We notice that critical value of the mass M^* is different for the three strains. These differences can be explained in biological terms as the exploration patterns of the slime mould. A large mass is needed for the Japanese strain to be able to exploit the two options, because it forms thick tubes. In contrast, the Australian strain forms thin tubes thus a smaller mass is needed to exploit the two options. The Japanese strain is fast, covers a long distance but a small surface while the Australian spreads in all directions, covering a large surface but a small distance. These exploration pattern differences are taken into account by the differences between two parameters Φ and k that we use in our model, thus the value of M^* is different for the three strains.

<i>strains</i>	Φ	k	M^*
<i>Japanese</i>	2.5	1.5	3.12
<i>Australian</i>	1.5	0.9	1.92
<i>American</i>	1.5	1.2	2.56

Table 3.1. Best-fit parameters and value of the symmetry-restoring bifurcation point for different strains of *Physarum Polycephalum*.

The experimental results are qualitatively consistent with the predictions of the model. In the Paper IV we fit the model parameters by using the model outcomes and experimental data. As a result, the model correctly describes experiment in nearly 90%.

In our research we predict a symmetry restoration at large population size. The critical value of the mass M^* corresponds to the bifurcation point after which we observe coexistence of multiple steady states. The coexistence of



Figure 3.2. Exploration patterns of the three different strains of *Physarum Polycephalum* tested. A is Australian strain, Am is American strain and J is Japanese.

multiple steady states enhances flexibility. In the region of coexistence a population may quickly switch between available options. Previously this phenomenon was shown in Paper I in the presence of more than two options. In Paper V we suggest that flexibility may happen in an open environment in the presence of only two options and a regulation of traffic of the kind we consider in this paper.

3.2 Growth modelling approach to biological system.

During the series of experiments performed to determine foraging decision process in *Physarum polycephalum*, we observe naturally formed fractal structures (see Figure 3.2). In the non-reproductive stage of the slime mould, the *Physarum polycephalum* plasmodium is a single multinucleate cell consisting of networks of protoplasmic veins and many nuclei. These beautiful fractal structures yield themselves to box-counting. The studies of the *Physarum polycephalum* plasmodial fractal dimension were performed on a network of tubules [45]. These studies show that the plasmodial weight does not have a significant effect on the fractal dimension. The food quality makes the plasmodium cover the food source in a more compact, space-filling manner. A mean fractal dimension of plasmodia is $D = 1.37 \pm 0.02$, but it increases linearly with increasing food quality. This research inspires a study of *Physarum polycephalum* with of one of the growth models (Paper V).

Fractal dimensions are crucial in characterising clusters and shapes, to the extent that some researches claim their all importance: " What do we actually do? A simple answer to this question is as follows. If you are an experimentalist, you try to measure the fractal dimension of things in nature. If you are a theorist, you try to calculate dimension of model chosen to describe experimental situations; if there is no agreement then you try another model. " [70]

In Paper V we choose the DLA (diffusion-limited aggregation) model which was proposed in 1981 by Witten and Sander with application to a metal-particle

aggregation process [82]. They proposed to use the DLA model to study the time-dependent growth rate of real random aggregates, as well as their mechanical and transport properties. The features observed in the growth process may appear in many processes, including gelation, condensation, polymerisation and agglutination of biological molecules. There are biological patterns which seem to be self-similar, such as trees, bronchial trees and networks of nerves and blood vessels. In 1990 Matsushita and Fujikawa presented results on the fractal growth in the formation of *Bacillus subtilise* bacterial colonies [49]. They showed that the growth of bacterial colonies on agar plates is governed by DLA processes of the nutrient contained in the agar plates. The DLA model has many extensions and has become one of the most important particle-cluster aggregation model. In 1998 Hasting and Levitov formulated a model of growth by using conformal mappings [36]. The growth in the conformal DLA model is represented as a random sequence of conformal maps with memory.

3.2.1 Conformal DLA model.

We consider the unit circle as a seed of our model. The cluster grows by adding slits (see Figure 3.3). The shape of a cluster depends on the model parameters: ϵ is the size of the slit, m is the number of points uniformly distributed on the unit circle and n is the number of iterations. Let Ω be a domain in $\mathbb{C}^* = \mathbb{C} \cup \{\infty\}$. The interval $e^{i\theta}(1, 1 + \epsilon]$ will be called a *slit*. A univalent function is analytic and one-to-one, denoted as $f : \Omega \rightarrow \Omega \setminus \text{slit}$, called a slit mapping. To each of m points we apply a function f_n defined in the following way

$$f_{n+1} := f_n \circ \Phi_{\epsilon_n}^{\theta_n} = f_0 \circ \Phi_{\epsilon_0}^{\theta_0} \circ \dots \circ \Phi_{\epsilon_n}^{\theta_n}, \quad n \in \mathbb{Z}_+, \quad (3.4)$$

where $f_0 := id$ and $\Phi_{\epsilon_n}^{\theta_n}$ are conformal mappings (see Paper V). Here θ_n is randomly drawn from a uniform distribution on $[0, 2\pi)$ and ϵ_n is chosen so that the length of the image of the new slit equals $\delta > 0$, a small fixed number independent of n , in particular, $\epsilon_0 = \delta$. We implement our conformal DLA growth model and grow random fractals. Each outcome of our model is unique, the cluster shape depends on the initial and randomly chosen model parameters.

Estimating dimension is the way to detect and quantify the self-organisational properties of natural and artificial complex systems. To characterise the properties of our clusters we define $\partial S(n)$ as a set where the slit lands almost always (see Figure 3.4) and estimate its dimension. In Paper V we apply different methods to our different clusters. The upper bound for Hausdorff dimension of $\partial S(n)$ is smaller than or equal to one [13]. In our case it is also the dimension of the support of the harmonic measure on $\partial S(n)$.

The box-counting dimension of our cluster is $D_b = 1.2 \pm 0.05$ and the pointwise dimension is $D_p = 1.4 \pm 0.15$. Figure 3.5 summarises our results, where a full line represents the box-counting dimension and dashed one corresponds to the pointwise dimension with respect to the number of iterations n . The

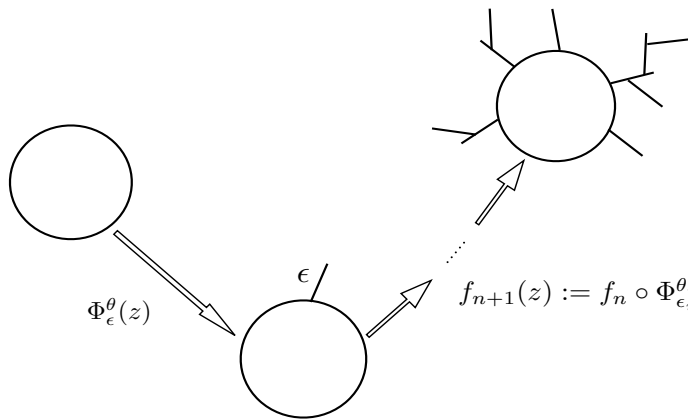


Figure 3.3. Schematic cluster growth. The cluster is generated by compositions of slit mappings.

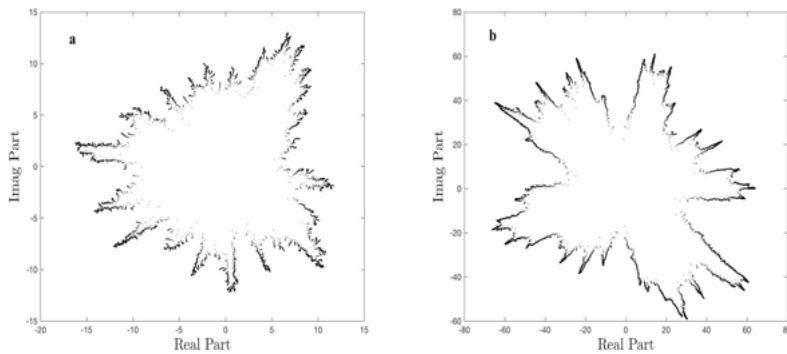


Figure 3.4. The shape of the clusters are generated by random compositions of slit mappings. The parameter values are $m = 100000$, $\epsilon = 0.1$ a) $n = 10000$ and b) $n = 100000$.

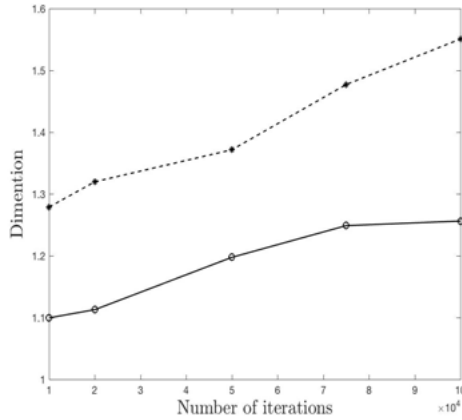


Figure 3.5. Approximations of dimensions for the cluster generated by random compositions of slit mappings with respect to n . Full and dashed lines correspond to the Box-counting Dimension and Pointwise Dimension respectively. The parameter values are $m = 100000$, $\epsilon = 0.1$.

number of points m and the number of iterations n influence computations of the dimension. The more points we have, the smaller the grid we can choose for our calculations. If n is large then the conformal mapping is applied many times thus the geometry is expected to be more complicated.

We notice that our cluster dimensions are not so different from the measurements of the mean fractal dimension of *Physarum polycephalum* plasmodia. We believe that studies of conformal DLA models can highlight the creation of many shapes and forms one observes in nature.

4. Paper summary

4.1 Paper I

Collective Irrationality and Positive Feedback.

Nicolis SC, **Zabzina N**, Latty T, Sumpter DJT (2011), PLoS ONE 6(4): e18901. DOI:10.1371/journal.pone.0018901

In Paper I we use a generic experimentally validated model of positive feedback to consider the problem of choosing between multiple options. We argue that positive feedback is a heuristic which often produces fast and accurate group decision-making, but it is susceptible to apparent "irrationality". We observe "irrationality" as the preferences change with the number of options. We assume that it is optimal to choose the option closer to an initial preference.

We base our theoretical studies on experimental data from work performed with slime moulds and ants. We study parameters and the bifurcation diagrams of steady state solutions of the model of positive feedback. We define strong and weak positive feedback to explain non-linearity and linearity of the choice. We consider speed-accuracy trade-off and quality of decision. We show that there is no trade-off between speed and accuracy but there is an optimal parameter where both are maximized. Maximal decision quality occurs near the bifurcation point. The bifurcation point increases with the number of options, thus for any given number of options the decision-making outcome is different.

We highlight the importance of symmetry breaking bifurcation point after which more than one stable steady state appears. We have shown that concepts such as the increasing accuracy with the group size, speed-accuracy tradeoffs and "irrationality" are strongly correlated to the coexistence of multiple stable states.

Contribution: I performed the simulation experiments. I analysed the experimental data and participated in writing the paper.

4.2 Paper II

Six Predictions about the Decision Making of Animal and Human Groups.

Sumpter DJT, **Zabzina N** and Nicolis SC (2012), Manage. Decis. Econ., 33: 295-309. DOI:10.1002/mde.2553

In this paper, we show how, in the context of group decision-making, mathematical models allow one to achieve a unifying view of the concepts of choice and "rationality". We review theoretical and empirical studies of how groups make decisions using combination of private information and preference copying. We consider models of animal decision-making developed from empirical observations. These models were used to make predictions in further experiments, but in the present paper we discuss our approach in the context of human organisations. First we study the Condorcet model, which represents the optimal strategy for a group. Then we consider the BHW (Bower-Herringer-Williamson) model, which is based on a bounded rationality for single selfish individual. The Deneubourg Model is the model of positive feedback, which implies that the more individuals that have already chosen one option, the greater the rate of commitment to that option. Finally we discuss the Quorum Model, based on the idea that individuals observe the choices of others with respect to only one option at a time. We present last three dynamic models of decision-making and provide background on their relevance to human and animal decision-making.

We give six testable predictions arising from models: 1) group decisions involve both true and false cascades; 2) a small number of leaders can heavily influence a decision; 3) individuals in groups make accurate and fast decisions; 4) irrelevant alternatives change preferences; 5) optimal group size depends on the number of options; 6) pairs of higher quality options are more difficult to distinguish than lower quality options.

We study how a group produces decision that benefits individuals within group, focusing on the interplay between experiments and models. Through a heuristic approach we link heuristics and optimality. We find out what individuals do, and then we look at the consequences of their actions.

Contribution: I analysed the models and performed some simulations.

4.3 Paper III

A gradient flow approach to the model of positive feedback in decision-making.
Zabzina N (2015), *Chaos, Solotons & Fractals*, vol.77, pp.215-224. DOI: 10.1016/j.chaos.2015.05.027

The idea of this paper is to apply an alternative method to study collective decision-making. It comes from recent studies on social dynamics, which use tools and methods of physics and economics. Social behaviour arises out of local interactions with different peers, but on a global scale it exhibits a regular pattern. A statistical physics approach to study the social dynamics is to consider the contacts with a limited number of individuals within the group in order to understand the transition from a disorder state to a configuration

that displays an order. While the evolutionary game theory considers the behaviour, which involves the interaction of individuals, to explore successful mechanisms that identify the efficient strategy. We want to point out that behind the details of the different approach and models there could be a level where the mathematical structures are similar.

We consider a generic experimentally validated model describing one of the major interaction mechanism between group members, the model of positive feedback. We propose a geometric reformulation of this model in terms of gradient flow equations on a Riemannian manifold. We show how the geometric tools can be used to explore biological phenomena described by the model of positive feedback. We consider the continuous version of steepest decent method, which is known as gradient flow equations. The crucial biological features such as bias to initial conditions and amplification of response in decision-making process can be seen very clearly in the gradient flow pictures. We introduce an analogy of our biological system with a physical system, namely a particle moving according to a potential on a Riemannian manifold.

Contribution: I analysed the model and wrote the paper.

4.4 Paper IV

Symmetry Restoring Bifurcation in Collective Decision-Making.

Zabzina N, Dussutour A, Mann RP, Sumpter DJT, Nicolis SC (2014), PLoS Comput Biol 10(12): e1003960. DOI:10.1371/journal.pcbi.1003960

Extensive experimental work and mathematical modelling suggest that the basic feature underlying effective decision-making is a symmetry-breaking bifurcation. A key factor is the critical system size after which from evenly distributed exploitation of all options the system moves to a focussed exploration of just one. In this paper we develop a model in which symmetry-breaking is followed by a symmetry restoring bifurcation, whereby a very large system returns to an even distribution of exploitation among options. The results are based on mathematical modelling along with experiments carried out on three strains of the unicellular plasmodial organism *Physarum polycephalum* choosing between two feeding sites. The model assumes local positive feedback, coupled with a negative feedback regulating the flow toward the feeding sites.

We show, contrary to the classical paradigm, that symmetry is eventually restored for an individual of a sufficiently large size. We report that the model is consistent with the different types of exploration patterns of three different strains of the slime mould. We argue that the symmetry-restoring phenomenon is resulting from the coexistence of positive and negative (regulatory) feed-

backs, this allows the organism to react flexibly to the environmental changes. We predict a symmetry restoration above a large critical population size, where the coexistence of multiple steady states in our model enhance flexibility of the system.

Contribution: I participated in conceiving and designing the experiments. I performed the experiments and analysed the data. I created the model and performed model simulation and analysis. I was responsible for writing the paper.

4.5 Paper V

Fractal dimensions of the conformal diffusion-limited aggregation.

Gaydashev D and **Zabzina N** (2016), Manuscript.

In this paper we study the diffusion-limited aggregation model (DLA), particularly the conformal DLA model. The growth in a conformal DLA model is prescribed by a random sequence of conformal maps. We generate the clusters by compositions of "slit mapping", thus the cluster grows by adding slits. The initial state of our model is a unit circle. To each of the points, uniformly distributed on the circle, we apply a "slit mapping". The shape of a cluster depends on the length of the slit, number of iterations and the randomness of the process.

We compute several fractal dimensions of our clusters: the lower bound for the Hausdorff dimension, the box-counting and pointwise dimension. We compare the results with studies of the *Physarum polycephalum* plasmodial fractal dimension. The approximated dimensions of our different cluster shapes are similar to mean fractal dimension of plasmodia.

Contribution: I create and analysed the model. I participate in performing model simulations and wrote the paper.

5. Summary in Swedish

Denna avhandling handlar om matematisk modellering av kollektivt beslutsfattande. I vårt arbete ser vi hur processen för kollektivt beslutsfattande kan undersökas. Först observerar vi vad individer gör och hur de reagerar på sin omgivning, hur de kommunicerar med varandra och vilka konsekvenserna är av dessa interaktioner. Kollektivt beteende utgår från de lokala interaktionerna mellan individer som ger upphov till ett komplext mönster på gruppnivå. Vi studerar kommunikationsreglerna mellan individer och kompletterar med biologiska experiment. Det finns olika modeller av kollektivt beteende som har använts i stor utsträckning av många forskare under de senaste åren för att studera kommunikationsmekanismer och göra teoretiska förutsägelser för kollektiva beslutsfattande. Det största bidraget med denna avhandling är att visa hur aspekter av kollektivt beslutsfattande som "irrationalitet", besluts kvalitet och hastighet kan modelleras med hjälp av differentialekvationer.

Avhandlingen består av två delar: den första är baserad på modellering av beslutsfattandet i samband med valet av utfodringsplatser. Den viktigaste kommunikationsmekanismen är positiv feedback mellan gruppmedlemmarna, som uppstår när engagemang för ett visst alternativ ökar som en funktion av antalet individer som redan förbundit sig till det. Vi använder en generisk experimentellt validerad modell för positiv feedback för att betrakta problemet att välja mellan flera alternativ. Vi baserar våra teoretiska studier på experimentella data från arbeten som utförts för slemsvampar och myror. Vi visar att positiv feedback är en heuristisk som ofta ger snabbt och exakt gruppbeslutsfattande. Vi genomför en detaljerad studie av bifurkationsfenomen som motsvarar situationer där en liten förändring i det biologiska systemet resulterar i radikalt olika kollektiva mönster. Vi observerar och studerar "irrationalitet", dvs system där införande av underlägsna alternativ förändrar beslutsfattandet. Detta fenomen har vi upptäckt med vår modell när värdet av bifurkationspunkten ökar med antalet alternativ. Vi diskuterar också kvaliteten på besluten: systemet gör ett beslut av god kvalitet om det väljer bättre alternativ oftare än andra tillgängliga alternativ. Maximal besluts kvalitet inträffar nära bifurkationspunkten. Vi visar att det finns en optimal parameter där både hastighet och beslutsnoggrannhet maximeras. Vidare presenterar vi en geometrisk omformulering av modellen av positiv feedback. Vi undersöker biologiska fenomen med hjälp av en potential. Vi vågar göra en analogi mellan ett biologiskt system och ett fysikaliskt system, där partikeln rör sig i enlighet med en potential på en Riemannsk mångfald.

I den andra delen studerar vi sökstrategier och modellering av beslutsfattandet hos slemsvampen *Physarum polycephalum*. Vi visar att sådana till synes

primitiva organismer, som slemsvamp, kan ge en bra testbädd för studier av kollektivt beslutsfattande. Vi utför matematisk modellering och observerar att de experimentella resultaten är kvalitativt förenliga med de matematiska förutsägelser från modellen. Under arbetet med *Physarum polycephalum* observerade vi naturligt bildade fraktala strukturer av dess plasmodier. Nyligen utförda studier visar att Plasmodiumsfraktalstrukturer kan uppskattas med hjälp av box-räkningsmetoden. Vi studerar en aggregationsmodell och dess tillämpningar. I dessa fall kan vi beräkna fraktala dimensioner för våra kluster och jämföra våra resultat med de senaste studierna av *Physarum polycephalum* plasmodier.

6. Acknowledgements

I was born a girl and named Natasha. My personality is the way I live my identity and represent it. For many years I lived my identity believing in love and trust. I have been a good team worker devoted to the glorious future. I was afraid to be alone, to sleep alone, to work alone. As I grew older each moment I needed to feel my life partner, his energy, passion, sadness and happiness. I manage to devote myself to work in harness for somebody else's future. My own personality almost evaporated in my dreams and beliefs before my cloud castle fell down as rain.

Now I understand that the main contribution of this thesis is to express my own personality. During my PhD years I develop many qualities and traits of my personal character. My personality is rediscovered and changed over this time. My individual differences are affected by the development of my values, attitudes, habits and skills. For this breakthrough, I am grateful to my social relationships and that is because I am surrounded by my family, many wonderful friends and colleagues.

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