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Valuation and Optimal Strategies in Markets Experiencing Shocks

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Abstract

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This thesis treats a range of stochastic methods with various applications, most notably in finance. It is comprised of five articles, and a summary of the key concepts and results these are built on.

The first two papers consider a jump-to-default model, which is a model where some quantity, e.g. the price of a financial asset, is represented by a stochastic process which has continuous sample paths except for the possibility of a sudden drop to zero. In Paper I prices of European-type options in this model are studied together with the partial integro-differential equation that characterizes the price. In Paper II the price of a perpetual American put option in the same model is found in terms of explicit formulas. Both papers also study the parameter monotonicity and convexity properties of the option prices.

The third and fourth articles both deal with valuation problems in a jump-diffusion model. Paper III concerns the optimal level at which to exercise an American put option with finite time horizon. More specifically, the integral equation that characterizes the optimal boundary is studied. In Paper IV we consider a stochastic game between two players and determine the optimal value and exercise strategy using an iterative technique.

Paper V employs a similar iterative method to solve the statistical problem of determining the unknown drift of a stochastic process, where not only running time but also each observation of the process is costly.

Keywords: American options, optimal stopping, game options, jump diffusion, jump to default, free-boundary problems, early exercise premium, integral equation, parabolic pde, convexity, sequential testing, fixed-point approach

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To Stellan and Ester

List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

I DYRSSEN, H., EKSTRÖM, E. AND TYSK, J. Pricing equations in jump-to-default models, *Int. J. Theor. Appl. Finance 17*, 3 (2014).

II DYRSSEN, H. The perpetual American put option in jump-to-default models, *Stochastics 89*, 2 (2017), 510–520.

III DYRSSEN, H. AND VANNESTÅL, M. The integral equation for the American put boundary in models with jumps, (2017). Submitted.

IV DYRSSEN, H. AND VANNESTÅL, M. Optimal stopping games for a process with jumps, (2016). Submitted.

V DYRSSEN, H. AND EKSTRÖM, E. Sequential testing of a Wiener process with costly observations, (2017). Submitted.

Contents

1. Introduction

This thesis consists of five articles, a short summary of the papers, and the present introduction. The introduction gives a brief overview of the most important results within the fields of mathematical finance and stochastics that the rest of the thesis is built on.

All five articles included in the thesis treat questions about stochastic analysis. Common to all the problems studied is that they are classical, in the sense well-known and widely recognized as relevant, but the settings are new and create mathematical complications that require new solution methods and/or yield new properties of the solutions.

Papers I–IV all concern problems in financial mathematics or problems with clear applications in finance. The titular notion of *markets experiencing shocks* is considered in these four papers; shocks are here to be understood as sudden significant changes of asset values. Topics treated are pricing of European options, optimal exercise and valuation of American options, and optimal stopping games, all of it done in models with jumps. Through arbitrage arguments and the Feynman-Kac formula the problem of pricing European options boils down to solving certain parabolic partial differential equations. American option pricing is an optimal stopping problem, where the best time to exercise the option is to be found as well as the value resulting from following this optimal strategy. For optimal stopping games the problem at hand is to find the optimal strategies, i.e. the times at which to stop the game, for both players, and the value of the game when these strategies are followed. For both of these problems one part is to determine whether an optimal strategy exists.

Paper V also studies optimal stopping, though in a setting that is statistical rather than financial. The problem is to determine the drift of a Wiener process as accurately, quickly and cheaply as possible, under the assumption that each observation of the process is costly. The solution is not just one but a sequence of stopping times – the times at which to observe the process, together with a final time at which to deliver the answer, and a value (the minimal cost) resulting from using this optimal strategy.

The rest of this introduction is organized as follows. In Section 1.1 arbitrage-free pricing of European options is discussed, both in the classic setting of the Black-Scholes model and in incomplete markets, such as models with jumps. Section 1.2 reviews optimal stopping problems and methods for their solution in diffusion and jump-diffusion models. Finally, in Section 1.3 a classical optimal stopping problem in mathematical statistics is presented along with methods for its solution.

1.1 European options

The two main types of options discussed in this thesis are European options and American options, with most emphasis on the latter which is discussed in subsection 1.2.2 below.

European options are contracts written on an underlying security, for example a stock, and it gives the holder the right to buy or sell the underlying for a pre-specified price at a pre-specified time. To capture the uncertainty about future price movements, stock prices are naturally modeled using stochastic processes.

Let us say that the stock price at time t is S_t, and take as an example a call option, i.e. an option that gives the holder the right to buy the asset. Then if T is the exercise time, also called the *maturity*, of the option and K is the exercise price, also known as the *strike*, of the option, the holder of the option will receive the amount

$$\max\{S_T - K, 0\},$$

oftentimes abbreviated as $(S_T - K)^+$, at time T. This is because if $S_T > K$ the holder buys the stock for K and then immediately sells it for S_T, thus making a profit $S_T - K$. If on the other hand $S_T \leq K$ the holder chooses not to exercise the option and receives nothing.

Generally we consider options which pay $G(S_T)$ at maturity, and call the function $s \mapsto G(s)$ the payoff function. One question we are concerned with in this situation is: what is a fair price to pay for such an option? To answer this, we first need to specify what is meant by a fair price. The classical definition of a fair price is in terms of the notion of arbitrage, which is described further below.

1.1.1 Arbitrage pricing in the Black-Scholes model

Loosely speaking, an *arbitrage opportunity* is an investment strategy that allows one to make a risk-less profit for zero cost. More specifically it is an investment strategy that has initial value 0, and a terminal value that is almost surely non-negative and positive with a positive probability. We consider a price to be fair if it doesn't introduce any arbitrage opportunities in the market. How to price options in accordance with the no-arbitrage criterion was shown in the seminal papers [5] and [26]. The theory of the martingale approach to no-arbitrage pricing was introduced in [17] and further developed in [18], [21] as well as in e.g. [11], [12] and [31]. The rest of this subsection describes in short their results; for a thorough introduction to the subject see [4].

Consider a market consisting of two assets B and S, where B is a bank account – a risk-less asset paying a constant interest rate r, and S is the price of a stock, modeled as a geometric Brownian motion. B follows the deterministic differential equation

$$\frac{dB_t}{dt} = rB_t,$$

whereas S satisfies the stochastic differential equation (SDE)

$$dS_t = \mu S_t dt + \sigma S_t d\bar{W}_t,$$

where the constants μ and σ are, respectively, the rate of return and the volatility of the stock, and $\bar{W}$ is a standard Wiener process. This is the famous Merton-Black-Scholes model.

In this market each contract written on the underlying S can be replicated by a self-financing portfolio of bonds and stocks, meaning that at maturity the value of the option and the value of the portfolio are equal. A market with this property is said to be *complete*. Having the same terminal value, the option and the replicating portfolio should also have the same initial value, and this is indeed the only option price that does not induce arbitrage possibilities.

There exists a unique measure $\mathbb{Q}$, equivalent to the real world probability, under which the discounted stock-price process $e^{-rt}S_t$ is a martingale. The option price is the discounted expected value of the payoff at maturity under this $\mathbb{Q}$. In other words S has $\mathbb{Q}$-dynamics

$$dS_t = rS_t dt + \sigma S_t dW_t,$$

where W is a $\mathbb{Q}$-Wiener process, and an option with payoff function G has the arbitrage-free price

$$P(t,x) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}_{t,x}\left[G(S_T)\right]. \tag{1.1}$$

Here $\mathbb{E}^{\mathbb{Q}}_{t,x}$ denotes expectation with respect to $\mathbb{Q}^{t,x}$ under which $S_t = x$. Sometimes we also use $S^{t,x}$ for the process S under $\mathbb{Q}^{t,x}$. The measure $\mathbb{Q}$ is often called a pricing measure, risk-neutral measure, or equivalent martingale measure (EMM). The result that a market is arbitrage-free if and only if there exists an EMM is known as the *first fundamental theorem of mathematical finance*.

Thanks to the Feynman-Kac theorem, see e.g. [28], we know that P given by (1.1) also solves the partial differential equation (PDE)

$$\frac{\partial}{\partial t}P(t,x) + rx\frac{\partial}{\partial x}P(t,x) + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2}{\partial x^2}P(t,x) - rP(t,x) = 0, \tag{1.2}$$

with the terminal condition $P(T,x) = G(x)$. The pricing PDE (1.2), known as the Black-Scholes equation, can be transformed into the heat equation by a change of variables and thus solved explicitly (in terms of the standard normal cumulative distribution function).

1.1.2 Arbitrage pricing in models with jumps

More recent examples of financial modeling often include jumps in the stock price process. As opposed to the geometric Brownian motion of the Black-Scholes model discussed above, such processes have discontinuous sample paths. Pricing in this kind of model was first studied in [27]. Two commonly studied classes of processes are jump diffusions and Lévy processes. A jump diffusion is a process that follows a diffusion between the jump times, which are typically modeled as the jump times of a Poisson process so that the jump intensity is finite. For Lévy processes, on the other hand, one allows for infinite activity of jumps, i.e. the process may experience infinitely many tiny jumps during a bounded time-interval. In this thesis we focus on jump processes of finite activity. See [8] for a thorough treatment of financial modeling with jump processes.

When jumps may occur in the underlying the pricing equation is no longer a PDE but becomes a partial integro-differential equation (PIDE), that is the equation contains nonlocal terms. As an example we let N and W be two independent processes on a given probability space, where N is a Poisson random measure with intensity $\lambda\nu(dz)$ for some probability measure ν, and W is a Wiener process. Consider now a process satisfying the SDE

$$dS_t = \mu(S_t)dt + \sigma(S_t)dW_t + \int_0^1 \phi(S_{t-},z)N(dt,dz). \tag{1.3}$$

At a jump-time t the process jumps from S_{t-} to $S_t = S_{t-} + \phi(S_{t-},Z)$, where $Z \sim \nu$. The pricing PIDE has the form

$$\begin{aligned}\frac{\partial}{\partial t}P(t,x) + \mu(x)\frac{\partial}{\partial x}P(t,x) + \frac{1}{2}\sigma^2(x)\frac{\partial^2}{\partial x^2}P(t,x) - rP(t,x)\\ + \lambda\int \big(P\big(t,x+\phi(x,z)\big) - P(t,x)\big)\nu(dz) = 0,\end{aligned} \tag{1.4}$$

see [8].

With two sources of risk (N and W) but only one risky asset (S) the market is not complete and by the *second fundamental theorem of financial mathematics*, which states that an arbitrage-free market is complete if and only if the EMM is unique, there are in fact infinitely many viable choices for $\mathbb{Q}$. One way to deal with this is to calibrate the model to actual prices, so one could say that the market has already chosen a $\mathbb{Q}$ for us. With this in mind we will most often consider the process to be modeled directly under the pricing measure. Thus, in (1.3) we require that $\mu(x) = rx - \lambda\int\phi(x,z)\nu(dz)$ to make $e^{-rt}S_t$ a martingale. Note, however, that determining how to actually pick the martingale measure is a highly relevant mathematical problem. A few different approaches to it in general incomplete models are studied in e.g. [1], [10], [14], [15] and [32].

Jump to default

Often one would assume that $\phi(x,z) > -x$ for all (x,z), to avoid the price to jump below zero. Paper I and Paper II of this thesis consider pricing in so-called jump-to-default models, which are models where the underlying may suddenly drop (jump) to zero (default), where it is absorbed. This can be considered as putting $\phi(x,z) = -x$ for all (x,z). However, the general jump-to-default model studied in this thesis allows for a level-dependent jump intensity, i.e. $\lambda = \lambda(x)$, so it is not of the form (1.3). This kind of model, which is described in further detail in the summary of Paper I below, has been studied in e.g. [6], [7] and [24].

1.2 Optimal stopping in finance

Optimal stopping problems are a broad class of problems whose solutions entail finding a stopping time at which the expected value of some stochastic process is maximal (or minimal). To make these notions precise we consider a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ and let $(X_t)_{t\geq 0}$ be a strong Markov process on this space. A *stopping time* is a random variable τ taking values in $[0,\infty]$ such that $\{\tau \leq t\} \in \mathcal{F}_t$, for all $t \geq 0$. That is, knowing all information up to now it is possible to discern whether the stopping time has occurred. We denote by $\mathcal{T}_{t,T}$ the set of all stopping times τ such that $t \leq \tau \leq T$. A typical optimal stopping problem is given by,

$$V(t,x) = \sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}_{t,x}\left[G(X_\tau)\right], \tag{1.5}$$

for some real-valued function G and a fixed time horizon $T \in [0,\infty]$. If $T = \infty$ the problem is said to be *perpetual*. A solution to this problem is a pair (V, τ^*), where V is the value defined by (1.5), and τ^* is a stopping time such that $V(t,x) = \mathbb{E}_{t,x}\left[G(X_{\tau^*})\right]$, i.e. the supremum in (1.5) is realized by τ^*.

Since we can always choose $\tau = t$ in (1.5) we have $V(t,x) \geq G(x)$ and the domain of V splits into two regions: $\mathcal{C}$, which is called the *continuation region*, and $\mathcal{D}$, the *stopping region*, as defined below.

$$\mathcal{C} = \{(t,x) : V(t,x) > G(x)\},$$
$$\mathcal{D} = \{(t,x) : V(t,x) = G(x)\}.$$

If G and X are continuous and $\mathbb{E}\left[\left|\sup_{t\leq u\leq T} G(X_u)\right|\right] < \infty$, the supremum in (1.5) is attained at

$$\tau^* = \inf\{u \geq t : V(u,X_u) = G(X_u)\},$$

i.e. τ^* is the first entry time of $(u, X_u^{t,x})$ into $\mathcal{D}$, see Appendix D in [19].

There are two main approaches to finding V. The first is an iterative approach that builds on dynamic programming, reminiscent of the method described in subsection 1.2.4 below, see e.g. [34]; the second is the free-boundary approach discussed in the next subsection.

1.2.1 Free-boundary problems

The optimal stopping problem (1.5) can be reformulated as the following *free-boundary problem*, see for example [20], [25], [35].

$$\begin{cases} \mathcal{L}V = 0 & \text{in } \mathcal{C}, \\ V|_{\mathcal{D}} = G|_{\mathcal{D}}, \end{cases} \tag{1.6}$$

where $\mathcal{L} := \partial_t + L_X$ and L_X is the infinitesimal generator of the process X. If the discounted problem,

$$V(t,x) = \sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}_{t,x}\left[e^{-r(\tau-t)}G(X_\tau)\right], \tag{1.7}$$

where r is the discount rate, is considered instead, which is often the case in a financial setting, the operator $\mathcal{L}$ in (1.6) is instead $\mathcal{L} = \partial_t + L_X - rI$, with I the identity operator.

If $\partial\mathcal{C}$ is regular for X, meaning that after starting at $\partial\mathcal{C}$ the process (t, X_t) will immediately enter $\mathcal{D}$ (this holds e.g. when X is a diffusion with a non-zero Brownian part and $\partial\mathcal{C}$ is Lipschitz continuous), the *smooth fit* condition,

$$\left.\frac{\partial V}{\partial x}\right|_{\partial\mathcal{C}} = \left.\frac{\partial G}{\partial x}\right|_{\partial\mathcal{C}}, \tag{1.8}$$

will also hold. In the case of processes with jumps, [3] and [22] gives precise, both necessary and sufficient, conditions for smooth fit. When smooth fit doesn't hold there will instead be *continuous fit*,

$$V|_{\partial\mathcal{C}} = G|_{\partial\mathcal{C}}. \tag{1.9}$$

Now, solving the optimal stopping problem (1.5) or (1.7) amounts to solving (1.6) together with (1.8) (or (1.9) depending on the situation). Note that both V and $\mathcal{D}$ are unknown and thus the boundary $\partial\mathcal{C}$ is undetermined (free) and has to be found as a part of solving the problem.

1.2.2 American options

An American option is defined in a similar way as its European counterpart with one important difference: it can be exercised at any time up to maturity. That is, the holder of the contract can choose to exercise the option at any time $\tau \leq T$ and receive the amount $G(S_\tau)$. The arbitrage-free price, or value, V of the option is given by

$$V(t,x) = \sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}_{t,x}\left[e^{-r(\tau-t)}G(S_\tau)\right], \tag{1.10}$$

where r is the interest rate and the expectation is taken with respect to an EEM $\mathbb{Q}$ (cf. subsection 1.1.1).

Papers II and III concern questions about the American put option in different jump-diffusion models, so here we will limit our study to the put option, i.e. $G(x) = (K-x)^+$, in the model specified by (1.3).

There exists, for each $t < T$, a value $b(t)$ such that $V(t,x) > G(t,x)$ for all $x > b(t)$ and $V(t,x) = G(t,x)$ for all $x \leq b(t)$ and thus

$$\mathcal{C} = \{(t,x) : x > b(t)\} \quad \text{and} \quad \mathcal{D} = \{(t,x) : x \leq b(t)\}.$$

This b is known as the optimal-exercise boundary or critical price. Moreover $x \mapsto V(t,x)$ is decreasing and convex for each fixed t, V is $C^{1,2}$ and satisfies the PIDE (1.4) in $\mathcal{C}$. See for example [2], [23] and [30].

Perpetual option

In the case of a perpetual option, i. e. an option that has no expiration date, which corresponds to putting $T = \infty$, the price V is given by

$$V(x) = \sup_{\tau} \mathbb{E}_x[e^{-r\tau} G(S_\tau)], \tag{1.11}$$

and does not depend on the initial time t. Thus (1.4) reduces to the ordinary integro-differential equation

$$\begin{aligned} &rxV'(x) + \frac{1}{2}\sigma^2 x^2 V''(x) - rV(x) \\ &+ \lambda \int \Big(V\big(x + \phi(x,z)\big) - V(t,x) - \phi(x,z)V'(x)\Big)\, \nu(dz) = 0. \end{aligned}$$

The perpetual American put option is studied in a jump-to-default model in Paper II.

The variational inequality

Since $b(t) = \inf\{x : V(t,x) > G(x)\}$ once we know V we know b. Of course, as b does not appear in the definition (1.10) of V, this is not surprising. However a more constructive formula than (1.10) that also does not contain b is the *variational inequality*

$$\max\{\mathcal{L}V, G - V\} = 0,$$

satisfied by V. The variational inequality leads to numerical methods for solving the free-boundary problem using e.g. finite differences.

On the other hand, since

$$V(t,x) = \mathbb{E}_{t,x}\left[e^{-r(\tau_b - t)} G(S_{\tau_b})\right]$$

where

$$\tau_b = \inf\left\{u \geq t : S_u^{x,t} \leq b(u)\right\},$$

once we know b we know V, i.e. all the information about the solution is contained in the boundary. Given this it would of course be interesting to find b directly. Such an approach has turned out to be possible; indeed there exists an integral equation that characterize the optimal stopping boundary b.

1.2.3 Boundary equations

Since one can always choose to hold the option until maturity, the value V of an American option dominates the value P of its European counterpart. The overshooting value $V-P$ is called the early-exercise premium and V can in many cases (in *nicely behaved* models) be written as

$$V(t,x) = P(t,x) - \mathbb{E}_{t,x}\int_t^T \mathcal{L}V(u,S_u)du, \tag{1.12}$$

with $\mathcal{L} = \partial_t + L_S - rI$. This expression is known as the *early exercise premium representation*. Since V is known at $\partial\mathcal{C}$ letting $x = b(t)$ in (1.12) leads, at least heuristically, to an equation for b. For a continuous underlying stock-price process S the generator L_S is a local operator and

$$\mathcal{L}V(t,x) = \mathcal{L}G(x)\mathbb{1}_{\{x\leq b(t)\}} = -rK\mathbb{1}_{\{x\leq b(t)\}},$$

thus (1.12) becomes

$$K - b(t) = P(t,b(t)) + rK\int_t^T \mathbb{Q}^{t,b(t)}(S_u \leq b(u))du. \tag{1.13}$$

In the Black-Scholes model, i.e. when S is a geometric Brownian motion, equation (1.13) is known to characterize the free boundary b, see [29]. In Paper III of the present thesis this equation is studied in jump-diffusion models.

1.2.4 Optimal stopping for jump diffusions

When pricing American options in jump-diffusion models the equation $\mathcal{L}V = 0$ satisfied by V in the continuation region has, as mentioned above, non-local (integral) terms. This makes the use of finite-difference methods to find the value much less efficient, thus a search for different approaches is warranted. One such approach is the iterative one, developed for piecewise deterministic Markov processes in [16], (see also [9]). The idea is the following.

Consider the perpetual problem defined in (1.11) with S given by the SDE (1.3), and let Z be the no-jump counterpart of S with dynamics

$$dZ_t = \mu(Z_t)dt + \sigma(Z_t)dW_t.$$

Note that the drift μ and the diffusion coefficient σ of Z are the same as for S. Let T_1 denote the first jump time of S, then, by the dynamic programming principle, one would expect that

$$V(x) = \sup_\tau \mathbb{E}_x\left[e^{-r\tau}G(S_\tau)\mathbb{1}_{\{\tau<T_1\}} + e^{-rT_1}V(S_{T_1})\mathbb{1}_{\{\tau\geq T_1\}}\right],$$

i.e. either it is optimal to stop before T_1 or it is optimal to wait until T_1 and perform optimally from then on. Furthermore, since $T_1 \sim \text{Exp}(\lambda)$, for any stopping time τ and any (regular enough) function f, we have

$$\mathbb{E}_x\left[e^{-r\tau}G(S_\tau)\mathbb{1}_{\{\tau<T_1\}}+e^{-rT_1}f(S_{T_1})\mathbb{1}_{\{\tau\geq T_1\}}\right]=$$
$$\mathbb{E}_x\left[e^{-(r+\lambda)\tau}G(Z_\tau)+\lambda\int_0^\tau e^{-(r+\lambda)t}Sf(Z_t)dt\right],$$

where $Sf(x)=\int f(x+\phi(x,z))\nu(dz)$. Now, if we define an operator $\mathcal{J}$ through its action on a test function f, given by

$$\mathcal{J}f(x)=\sup_\tau\mathbb{E}_x\left[e^{-(r+\lambda)\tau}G(Z_\tau)+\lambda\int_0^\tau e^{-(r+\lambda)t}Sf(Z_t)dt\right],$$

we would expect V to be a fixed point of $\mathcal{J}$. One way to construct the fixed point is by defining the sequence $\{V_n\}_{n=0}^\infty$ iteratively by

$$\begin{cases}V_0=G,\\ V_n=\mathcal{J}V_{n-1}\quad\text{for}\quad n\geq 1.\end{cases}$$

Then V_n is increasing in n and converges uniformly and exponentially fast to V, see [2]. Thus the problem has been reduced to a sequence of optimal stopping problems for a diffusion. A similar operator is employed in Paper IV of this thesis to find the value of an optimal stopping *game* in a jump-diffusion model.

1.2.5 Optimal stopping games

Optmial stopping games, also known as Dynkin games, are extensions of optimal stopping problems where the conflicting objectives of two parties (players) are to be optimized simultaneously. The game is a financial contract between the two players contingent on some underlying asset. We consider here only zero-sum games, meaning that the sum of the players' payoffs is zero.

Let the underlying asset be modeled by a strong Markov process S. Given payoff functions $G_1\leq G_2$ the game is specified as follows. Each player chooses a stopping time, say τ_1 and τ_2 respectively; at the earliest of these two times Player 1 receives from Player 2 the amount

$$G_1(S_{\tau_1})\mathbb{1}_{\{\tau_1\leq\tau_2\}}+G_2(S_{\tau_2})\mathbb{1}_{\{\tau_1>\tau_2\}}.$$

Thus it is the objective of Player 1 (2) to maximize (minimize) the expected discounted payoff

$$R_x(\tau_1,\tau_2):=\mathbb{E}_x\left[e^{-r(\tau_1\wedge\tau_2)}\left(G_1(S_{\tau_1})\mathbb{1}_{\{\tau_1\leq\tau_2\}}+G_2(S_{\tau_2})\mathbb{1}_{\{\tau_1>\tau_2\}}\right)\right].$$

The upper and lower values of the game are defined as

$$V^*(x)=\inf_{\tau_2}\sup_{\tau_1}R_x(\tau_1,\tau_2)\quad\text{and}\quad V_*(x)=\sup_{\tau_1}\inf_{\tau_2}R_x(\tau_1,\tau_2).$$

In this context one distinguishes between two different types of equilibria. The first is the *Stackelberg equilibrium*, which is said to hold if $V^*(x) = V_*(x)$ for all x, and implies that the value $V := V^* = V_*$ is well-defined. The second is the *Nash equilibrium*, which is defined as the existence of a saddle point (τ_1^*, τ_2^*), i.e. stopping times τ_1^* and τ_2^* such that

$$R_x(\tau_1, \tau_2^*) \leq R_x(\tau_1^*, \tau_2^*) \leq R_x(\tau_1^*, \tau_2).$$

Furthermore, the Nash equilibrium implies the Stackelberg equilibrium with $V(x) = R_x(\tau_1^*, \tau_2^*)$. It is shown in [13] that if S is right-continuous and the payoff functions satisfy the integrability condition

$$\mathbb{E}[\sup_t |G_i(S_t)|] < \infty \quad \text{for} \quad i = 1, 2,$$

then the game has a Stackelberg equilibrium. Furthermore, if S is also left-continuous over stopping times then the Nash equilibrium holds and the optimal stopping times are given by

$$\tau_1^* = \inf\{t \geq 0 : S_t \in \mathcal{D}_1\} \quad \text{and} \quad \tau_2^* = \inf\{t \geq 0 : S_t \in \mathcal{D}_2\},$$

where $\mathcal{D}_1 = \{V = G_1\}$ and $\mathcal{D}_2 = \{V = G_2\}$ are the stopping regions of Players 1 and 2 respectively. In Paper IV of this thesis we study optimal stopping games for jump diffusions and show how to find the value of the game using an iterative approach similar to the one discussed in subsection 1.2.4 above.

1.3 Optimal stopping in statistics

Many different types of optimal stopping problems with a wide range of applications turn up in statistical analysis. This is by no means an exhaustive or even extensive presentation of such problems, but merely an introduction to one type that is studied in Paper V of this thesis. In modeling statistical problems there are two main approaches, one that assumes a priori knowledge of the sought quantity and one that does not assume such knowledge. In this thesis we adopt the former, known as the *Bayesian* approach.

1.3.1 Sequential testing of a Wiener process

A classical problem in this area is sequential testing of the drift of a Brownian motion. The problem is set up as follows.

On the probability-statistical space $(\Omega; \mathcal{F}; \mathbb{P}_\pi, \pi \in [0,1])$ let μ be a random variable such that

$$\mathbb{P}_\pi(\mu = \mu_2) = 1 - \mathbb{P}_\pi(\mu = \mu_1) = \pi,$$

for some $\mu_2 > \mu_1$. Note that $\mathbb{P}_\pi$ is the a priori probability distribution of the unknown μ that distinguishes this approach from the non-Bayesian one. The variational formulation of the problem, also known as the fixed error probability formulation, due to [36], assumes no probabilistic knowledge of μ.

Assume that we observe a stochastic process X of the form

$$X_t = \mu t + \sigma W_t,$$

where W is a standard $\mathbb{P}_\pi$-Wiener process, and we want to determine as quickly and with as much certainty as possible the unknown drift of X.

The a posteriori probability process Π, defined as

$$\Pi_t = \mathbb{P}_\pi(\mu = \mu_2 | \mathcal{F}_t^X),$$

satisfies the SDE

$$d\Pi_t = \frac{\mu_2 - \mu_1}{\sigma} \Pi_t (1 - \Pi_t) d\hat{W}_t,$$

where the innovations process

$$\hat{W} = \frac{1}{\sigma} \left(X_t - (\mu_2 - \mu_1) \int_0^t \Pi_s ds - \mu_1 t \right)$$

is a standard Wiener process. The problem reduces to the optimal stopping problem

$$U(\pi) = \inf_\tau \mathbb{E}_\pi \left[a\Pi_\tau \wedge b(1 - \Pi_\tau) + c\tau \right], \tag{1.14}$$

see [33]. The infimum in (1.14) is taken over all $\mathcal{F}^\Pi$-stopping times and the positive constants a, b, and c represent the costs of the two types of wrong guesses for the drift, and the cost of running time, respectively.

The continuation region takes the form of an open interval (A, B) and, with $\mathcal{L}$ being the infinitesimal generator of Π, the free-boundary problem solved by the value function U has the form

$$\begin{array}{rcll} c + \mathcal{L}U(\pi) & = & 0 & \text{for} \quad \pi \in (A, B), \\ U(\pi) & < & g(\pi) & \text{for} \quad \pi \in (A, B), \\ U(\pi) & = & g(\pi) & \text{for} \quad \pi \notin (A, B), \\ U'(A) & = & a & (smooth fit), \\ U'(B) & = & b & (smooth fit), \end{array} \tag{1.15}$$

where $g(\pi) := a\pi \wedge b(1 - \pi)$. The formulation (1.15) yields the free boundary $\{A, B\}$ as the unique solution to a pair of transcendental equations and, given these, an explicit formula for the value U.

Paper V of this thesis studies this problem when each observation of the underlying entails a cost, and thus observations are necessarily made at discrete times.

2. Summary of papers

2.1 Paper I

In this paper we study pricing of European options in a general jump-to-default model. It is assumed that the pre-default stock price Y has the dynamics

$$dY_t = (r - \delta + \lambda(Y_t))Y_t dt + \sigma(Y_t) dW_t,$$

where λ is the default intensity, which is allowed to explode at zero. The default event is modeled as the the first time the so-called hazard process hits a random, exponentially distributed level θ, or the stock price diffuses to zero: The hazard process A is given by

$$A_t = \begin{cases} \int_0^t \lambda(Y_s) ds & t < \tau_0^Y, \\ \infty & t \geq \tau_0^Y, \end{cases}$$

where $\tau_0^Y = \inf\{t \geq 0 : Y_t \leq 0\}$. Now, the time of default is

$$\zeta := \inf\{t \geq 0 : A_t \geq \theta\} \wedge \tau_0^Y = \inf\{t \geq 0 : A_t \geq \theta\} \text{ a.s.},$$

and the defaultable stock price process X is given by

$$X_t = \begin{cases} Y_t & t < \zeta, \\ 0 & t \geq \zeta. \end{cases}$$

Consider a claim written on the stock X with payoff g at maturity in case of no default and time-dependent rebate ϕ in case of default (paid out at the time of default). The value of this claim is

$$u(x,t) = \mathbb{E}_x \left[e^{-rt} g(X_t) \mathbb{1}_{\{t<\zeta\}} + e^{-r\zeta} \phi(t-\zeta) \mathbb{1}_{\{\zeta \leq t\}} \right],$$

where $t \geq 0$ denotes time to maturity. The main result of the paper is that, under a technical condition related to whether default occurs if the process is close enough to zero and the continuity condition $\phi(0) = g(0)$, the value function u is the unique (polynomially bounded) classical solution to the partial differential equation,

$$\begin{cases} u_t(t,x) = \mathcal{L}u(t,x) + \lambda(x)\phi(t), & (t,x) \in (0,\infty)^2, \\ u(0,x) = g(x), & x > 0, \\ u(t,0) = \phi(t), & t > 0, \end{cases}$$

where $\mathcal{L}u = \alpha u_{xx} + \beta u_x - (r+\lambda)u$ with

$$\alpha(x) = \sigma^2(x)/2 \quad \text{and} \quad \beta(x) = x(r-q+\lambda(x)). \tag{2.1}$$

Furthermore we prove that $x \mapsto u(t,x)$ is convex for convex payoff-functions g if λ is convex and the function $t \mapsto e^{rt}\phi(t)$ is increasing. Moreover we prove that in convexity-preserving models prices are monotone in the risk parameters σ and λ. More precisely if either of the models specified by (σ_1,λ_1) and (σ_2,λ_2) is convexity preserving and if $\sigma_1 \le \sigma_2$ and $\lambda_1 \le \lambda_2$ then the corresponding prices u_1 and u_2 satisfy $u_1 \le u_2$. This is an important property for studying model-robustness in that it provides information on how possible mis-specifications of parameters affect option prices.

2.2 Paper II

In this paper we study the perpetual American put option in the same jump-to-default model as in Paper I. We solve the associated free-boundary problem and with a verification argument we prove that the solution is indeed the value of the option, arriving at an explicit expression for the value function and an equation that characterizes the optimal stopping boundary.

The value of the perpetual American put option is, with $g(x) = (K-x)^+$,

$$V(x) = \sup_{\tau} \mathbb{E}_x \left[e^{-r\tau} g(X_\tau) \right], \tag{2.2}$$

where the supremum is taken over all $\mathcal{F}^X$-stopping times τ.

With the notation

$$E(x) = \exp\left\{ \int_1^x \frac{\beta(y)}{\alpha(y)} dy \right\} \quad \text{and} \quad u(x) = x \int_x^\infty \frac{1}{y^2 E(y)} dy,$$

where α and β are as defined in (2.1), the boundary b is the unique positive solution to the equation

$$rK \int_b^\infty \frac{E(y)}{\alpha(y)} u(y) dy = 1,$$

if it exists, and if no such solution exists $b = 0$. The pair (V,b) satisfies the free-boundary problem

$$\begin{cases} \alpha V_{xx} + \beta V_x - (r+\lambda)V + \lambda g(0) = 0, & x > b, \\ V(x) = g(x), & x \le b. \end{cases}$$

If $b > 0$ the smooth-fit condition $V'(b) = -1$ holds as well.

The value function is

$$V(x) = \begin{cases} K - rK\Big(x \int_x^\infty \frac{E(y)}{\alpha(y)} u(y) dy + u(x) \int_b^x \frac{E(y)}{\alpha(y)} y dy \Big), & x > b, \\ g(x), & 0 \le x \le b, \end{cases}$$

and $\tau_b = \inf\{t \geq 0 : X_t \leq b\}$ is optimal in (2.2).

Moreover, we prove that the option price is convex if the default intensity λ is nonincreaing, (in fact a slightly weaker, but more technical condition is sufficient for convexity). Also, we find that in the convexity-preserving case prices are monotone (increasing) in σ and λ. To conclude we study an example where our condition for convexity is violated and show that the option value in this case indeed fails to be convex.

2.3 Paper III

We study the integral equation for the optimal stopping boundary for the American put option in a jump-diffusion model, that also allows for a continuous dividend yield, δ. The dynamics of the underlying are specified as

$$dS_t = \mu S_t dt + \sigma S_t dW_t + S_t \int_{\mathbb{R}} (e^z - 1) N(dt, dz),$$

were N is a Poisson random measure on $\mathbb{R}_+ \times \mathbb{R}$ with mean measure $\lambda \nu(dz)dt$, with ν a probability measure on $\mathbb{R}$ and λ a nonnegative constant.

The option value V, defined by (1.10), satisfies the early exercise premium representation (1.12). For a spectrally negative underlying process inserting $x = b(t)$ into the early exercise premium representation yields, as described in subsection 1.2.3, the equation

$$K - b(t) = P(t, b(t)) + \int_t^T \mathbb{E}_{t,b(t)} \left[(rK - \delta S_u) \mathbb{1}\{S_u \leq b(u)\} \right] du. \tag{2.3}$$

The first main result of the paper is that, in the present case of only downward jumps, the optimal stopping boundary b is the unique solution to (2.3) in the class of nonnegative continuous functions on $[0, T)$ bounded by K.

In a model allowing upward jumps the early exercise premium representation is still valid but letting $x = b(t)$ leads to an equation that contains both the unknown value function V and the boundary b. However, in a spectrally positive model, we have the following result:

For any $\varepsilon > 0$ consider the equation

$$\begin{aligned} K - c(t) = P(t, c(t)) + rK \int_t^T e^{-r(u-t)} \mathbb{P}_{t,c(t)} \{S_u \leq c(u)\} du \\ - \mathbb{E}_{t,c(t)} \left[\int_t^T e^{-r(u-t)} \hat{\Psi}(u, S_u; c) \mathbb{1}\{S_u \leq c(u) + \varepsilon\} du \right], \end{aligned} \tag{2.4}$$

where

$$\hat{\Psi}(t, x; c) = \lambda \int_{\mathbb{R}} (xe^z - c(t)) \mathbb{1}\{c(t) < xe^z\} \nu(dz).$$

If $c = \hat{c}_\varepsilon$ satisfies (2.4) for a continuous $\hat{c}_\varepsilon : [0, T) \to (0, K]$, then $\hat{c}_\varepsilon \geq b$.

2.4 Paper IV

In this paper we study an optimal stopping game for a jump diffusion X, with dynamics given by (1.3). The game is a zero-sum Dynkin game, specified as in subsection 1.2.5.

With the short-hand notation

$$G_X(t_1,t_2) = e^{-r(t_1\wedge t_2)}\left(G_1(X_{t_1})\mathbb{1}_{\{t_1\le t_2\}} + G_2(X_{t_2})\mathbb{1}_{\{t_1>t_2\}}\right),$$

the value of the game is

$$V(x) = \sup_{\tau_1\in\mathcal{T}_X}\inf_{\tau_2\in\mathcal{T}_X}\mathbb{E}_x\left[G_X(\tau_1,\tau_2)\right].$$

We prove that the value V is the unique fixed point of the associated operator J defined, for any f in a class of test functions, by

$$Jf(y) = \sup_{\tau_1\in\mathcal{T}_Y}\inf_{\tau_2\in\mathcal{T}_Y}\mathbb{E}_y\left[e^{-\lambda(\tau_1\wedge\tau_2)}G_Y(\tau_1,\tau_2) + \lambda\int_0^{\tau_1\wedge\tau_2} e^{-(r+\lambda)t}Sf(Y_t)\,dt\right],$$

where Y is the corresponding diffusion with dynamics as in (1.2.4) of subsection 1.2.4, and $Sf(x) = \int_0^1 f(x+\phi(x,z))\nu(dz)$.

Moreover, we show that the sequence $\{v_n = J^n G_1\}_{n=0}^{\infty}$ converges to V monotonically, uniformly and exponentially fast. We also show that the function v_n is the value of the game stopped at the nth jump in the underlying X.

Furthermore, an example with a callable perpetual American put option in a spectrally positive model is treated and an explicit expression for v_n is found. Here $G_1(x) = (K-x)^+$ and $G_2 = G_1 + \varepsilon$. For this problem we find that if ε, the cost of canceling the option, is less than or equal to the perpetual put price P at K then stopping at K is always optimal for the minimizer, and if $\varepsilon > P(K)$ then the minimizer (the seller of the contract) should never exercise their right to cancel the option.

2.5 Paper V

In this paper we study the problem of sequential testing of the drift of a Wiener process when each observation is costly. As in subsection 1.3.1 the underlying process X is assumed to be given by

$$X_t = \mu t + \sigma W_t,$$

where σ is a known constant, W a standard Wiener process and the drift μ is an unknown constant modeled as a random variable taking values in $\{\mu_1,\mu_2\}$. We assume the a priori probabilities $\mathbb{P}_\pi(\mu=\mu_2) = \pi = 1-\mathbb{P}_\pi(\mu=\mu_1)$, and let Π dentoe the a posteriori probability process $\Pi_t = \mathbb{P}_\pi(\mu=\mu_2|\mathcal{F}_t^X)$.

Now, for an increasing sequence of random times, $\hat{\tau} = \{\tau_k\}_{k=0}^{\infty}$, with $\tau_0 = 0$ we define the filtration

$$\mathcal{F}_t^{\hat{\tau}} = \sigma\left\{\Pi_{\tau_1}, \Pi_{\tau_2}, .., \Pi_{\tau_k} \text{ where } k = \sup\{j : \tau_j \leq t\}\right\}.$$

A pair $(\hat{\tau}, \tau)$ is called an admissible strategy if τ_{k+1} is $\mathcal{F}_{\tau_k}^{\hat{\tau}}$-measurable for each $k \geq 0$ and τ is an $\mathcal{F}^{\hat{\tau}}$-stopping time, such that $\tau = \tau_k$ for some k a.s.

For nonnegative constants a, b, c and d, let $g(\pi) := a\pi \wedge b(1-\pi)$ and define

$$V(\pi) = \inf_{(\hat{\tau},\tau)} \mathbb{E}_\pi \left[g(\Pi_\tau) + c\tau + d\sum_{k=1}^{\infty} \mathbb{1}_{\{\tau_k \leq \tau\}} \right], \tag{2.5}$$

with the infimum taken over all admissible strategies $(\hat{\tau}, \tau)$. Here a and b represent the costs of the two types of wrong guesses for μ, c represents the cost of running time and d is the cost per observation, thus V is the minimal expected total cost.

Via an associated operator we construct a sequence V_n that converges monotonically to V, and by characterizing the value V as the unique fixed point the associated operator we identify an optimal strategy, i.e. a strategy for which the infimum in (2.5) is attained.

More precisely, the operator $\mathcal{J}$ is defined on a suitable class of concave functions by

$$\mathcal{J} f(\pi) = \min\left\{ g(\pi), d + \inf_{t\geq 0}\{ct + \mathbb{E}_\pi[f(\Pi_t)]\}\right\}.$$

We prove that the decreasing sequence $\{V_n = \mathcal{J}^n g\}_{n=0}^{\infty}$ converges to V, and that V is the unique fixed point of $\mathcal{J}$.

Moreover, the continuation set $\{\pi : V(\pi) < g(\pi)\}$ is an open interval (A, B). For $\pi \in (A, B)$ we let

$$t(\pi) = \inf\{t \geq 0 : cs + \mathbb{E}_\pi[V(\Pi_s)] \text{ attains its minimum at } s = t\},$$

and define the sequence $\hat{\tau}^* = \{\tau_k^*\}_{k=0}^{\infty}$ recursively by $\tau_0^* = 0$ and

$$\tau_{k+1}^* = \begin{cases} \tau_k^* + t(\Pi_{\tau_k^*}), & \text{for} \quad k = 0, ..., n^* - 1 \\ \infty, & \text{for} \quad k \geq n^*, \end{cases}$$

where $n^* = \inf\{k : \Pi_{\tau_k^*} \notin (A, B)\}$. With $\tau^* = \tau_{n^*}^*$ the strategy $(\hat{\tau}^*, \tau^*)$ is optimal in (2.5). Our numerical results suggest that the optimal strategies for V_n converges.

3. Sammanfattning på svenska

Denna avhandling bygger på fem vetenskapliga artiklar inom ämnet sannolikhetsteori och matematisk statistik, alla med olika tillämpningar, främst inom finans.

Avhandlingen inleds med en introduktion till området, vari de olika problem och lösningsmetoder som återfinns i artiklarna beskrivs tillsammans med tidigare kända resultat dessa bygger på. Introduktionen åtföljs av en kort sammanfattning av de viktigaste resultaten i avhandlingen.

Gemensamt för alla frågeställningarna i denna avhandling är att de kan sägas vara klassiska, i betydelsen välkända och erkänt relevanta, men att de här studeras i en ny kontext eller under nya förutsättningar.

De första två artiklarna behandlar båda en så kallad jump-to-default modell, det vill säga en modell där någon kvantitet, till exempel ett aktiepris, representeras av en stokastisk process i kontinuerlig tid som hoppar till noll med en viss sannolikhet. Förutom det eventuella hoppet till noll, som representerar en betalningsinställelse eller konkurs (eng. *default*) och vid vilket processen absorberas, antas processens trajektorier vara kontinuerliga.

Artikel I behandlar prissättning av europeiska optioner i denna modell. Under ett par tekniska, men naturliga villkor visas att den associerade partiella integro-differentialekvationen karakteriserar optionspriset. Vidare studeras modellens konvexitets- och monotonicitetsbevarande egenskaper, vilka är viktiga för att uppskatta eventuella felaktigheter i prissättningen härrörande från att modellens parametrar inte valts korrekt.

Artikel II handlar om optioner av amerikansk typ, mer bestämt den perpetuala amerikanska säljoptionen, i samma modell. Genom att associera det optimala stopptidsproblemet som beskriver optionens värde med ett så kallat frirandsproblem härleds explicita formler för priset samt en ekvation för den optimala säljnivån – den fria randen. Även här presenteras villkor för konvexitet och monotonicitetsbevarande.

Artiklarna III och IV behandlar båda frågeställningar om hoppdiffusionsmodeller, dvs modeller som kombinerar de kontinuerliga brusartade fluktuationerna hos till exempel Wienerprocessen eller geometrisk Brownsk rörelse (diffusion) med plötsliga, relativt både stora och sällsynta, förändringar (hopp).

I Artikel III studeras den fria randen, det vill säga den optimala säljnivån, för en amerikansk säljoption, här med ändlig tidshorisont. En integralekvation som uppfylls av randen härleds och studeras, och i fallet med enbart nedåtgående (negativa) hopp ges ett bevis för att denna ekvation karakteriserar randen. Vidare studeras ekvationer för diverse approximationer av randen i det fall den underliggande processen har positiva hopp.

Artikel IV behandlar optimala stopptidsspel, även kända som Dynkin-spel, i hoppdiffusionsmodeller. Sådana spel är kontrakt mellan två parter (spelare) som båda kan stoppa spelet. När spelet stoppas betalas en summa, vilken beror dels på värdet av den underliggande processen vid denna tidpunkt och även på vem av spelarna som stoppat, till spelare 1 av spelare 2. Den ena spelaren har alltså som motiv att maximera ett visst väntevärde och den andra vill minimera detsamma. För att finna spelets värde och de optimala strategierna, vilka utgörs av en Nashjämvikt, har en känd iterativ teknik som tidigare använts till rena maximerings- eller minimeringsproblem anpassats till denna situation där vi har ett kombinerat supremum-infimum-problem.

I Artikel V används en liknande iterativ teknik för att finna värdet av, och en optimal strategi för, det statistiska problemet att bestämma den okända driften hos en underliggande process där inte bara den löpande tiden, som är fallet i den klassiska problemformuleringen, utan även varje observation av processen är kostsam. Denna artikel skiljer sig något från de övriga då detta problem inte främst motiveras av finansiella frågeställningar utan snarare av tillämpningar inom optimal design av experiment.

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References

[1] AVELLANEDA, M. The minimum-entropy algorithm and related methods for calibrating asset-pricing models. In *Proceedings of the International Congress of Mathematicians* (Berlin, 1998), vol. Extra III, Doc. Math., pp. 545–563.

[2] BAYRAKTAR, E. A Proof of the Smoothness of the Finite Time Horizon American Put Option for Jump Diffusions. *SIAM J. Control Optim. 48*, 2 (2009), 551–572.

[3] BAYRAKTAR, E., AND XING, H. Regularity of the optimal stopping problem for jump diffusions. *SIAM J. Control Optim. 50*, 3 (2012), 1337–1357.

[4] BJÖRK, T. *Arbitrage Theory in Continuous Time*, 3 ed. Oxford University Press, 2009.

[5] BLACK, F., AND SCHOLES, M. The pricing of options and corporate liabilities. *J. Polit. Econ. 81*, 3 (1973), 637–654.

[6] CARR, P., AND LINETSKY, V. A jump to default extended CEV model: an application of Bessel processes. *Finance Stoch. 10*, 3 (2006), 303–330.

[7] CARR, P., AND MADAN, D. B. Local volatility enhanced by a jump to default. *SIAM J. Financial Math. 1*, 1 (2010), 2–15.

[8] CONT, R., AND TANKOV, P. *Financial modelling with jump processes*. Chapman & Hall/CRC Financial Mathematics Series. Chapman & Hall/CRC, Boca Raton, FL, 2004.

[9] DAVIS, M. H. A. *Markov models and optimization*, vol. 49 of *Monographs on Statistics and Applied Probability*. Chapman & Hall, London, 1993.

[10] DAVIS, M. H. A. Option pricing in incomplete markets. In *Mathematics of Derivative Securities*, M. A. H. Dempster and S. R. Pliska, Eds. Cambridge University Press, Cambridge, 1997, pp. 216–226.

[11] DELBAEN, F. Representing martingale measures when asset prices are continuous and bounded. *Math. Finance 2*, 2 (1992), 107–130.

[12] DELBAEN, F., AND SCHACHERMAYER, W. A general version of the fundamental theorem of asset pricing. *Math. Ann. 300*, 3 (1994), 463–520.

[13] EKSTRÖM, E., AND PESKIR, G. Optimal stopping games for Markov processes. *SIAM J. Control Optim. 47*, 2 (2008), 684–702.

[14] FÖLLMER, H., AND SCHWEIZER, M. Hedging of contingent claims under incomplete information. In *Applied stochastic analysis (London, 1989)*, vol. 5 of *Stochastics Monogr.* Gordon and Breach, New York, 1991, pp. 389–414.

[15] FÖLLMER, H., AND SONDERMANN, D. Hedging of nonredundant contingent claims. In *Contributions to mathematical economics*. North-Holland, Amsterdam, 1986, pp. 205–223.

[16] GUGERLI, U. S. Optimal stopping of a piecewise-deterministic Markov process. *Stochastics 19*, 4 (1986), 221–236.

[17] HARRISON, J. M., AND KREPS, D. M. Martingales and arbitrage in multiperiod securities markets. *J. Econom. Theory 20*, 3 (1979), 381–408.

[18] HARRISON, J. M., AND PLISKA, S. R. Martingales and stochastic integrals in the theory of continuous trading. *Stochastic Process. Appl. 11*, 3 (1981), 215–260.

[19] KARATZAS, I., AND SHREVE, S. E. *Methods of mathematical finance*, vol. 39 of *Applications of Mathematics (New York)*. Springer-Verlag, New York, 1998.

[20] KOTLOW, D. B. A free boundary problem connected with the optimal stopping problem for diffusion processes. *Trans. Amer. Math. Soc. 184* (1973), 457–478 (1974).

[21] KREPS, D. M. Arbitrage and equilibrium in economies with infinitely many commodities. *J. Math. Econom. 8*, 1 (1981), 15–35.

[22] LAMBERTON, D., AND MIKOU, M. The smooth-fit property in an exponential Lévy model. *J. Appl. Probab. 49*, 1 (2012), 137–149.

[23] LAMBERTON, D., AND MIKOU, M. A. The critical price for the American put in an exponential Lévy model. *Finance Stoch. 12*, 4 (2008), 561–581.

[24] LINETSKY, V. Pricing equity derivatives subject to bankruptcy. *Math. Finance 16*, 2 (2006), 255–282.

[25] MCKEAN, H. P. On optimal stopping and free boundary problems. *Indust. Manag. Rev. 6* (1965), 32–39.

[26] MERTON, R. C. Theory of rational option pricing. *Bell J. Econom. and Management Sci. 4* (1973), 141–183.

[27] MERTON, R. C. Option pricing when underlying stock returns are discontinuous. *J. Financial Econ. 3*, 1–2 (1976), 125–144.

[28] ØKSENDAL, B. *Stochastic differential equations*, 5 ed. Springer-Verlag, Berlin Heidelberg, 1998.

[29] PESKIR, G. On the American option problem. *Math. Finance 15*, 1 (2005), 169–181.

[30] PHAM, H. Optimal stopping, free boundary, and American option in a jump-diffusion model. *Appl. Math. Optim. 35*, 2 (1997), 145–164.

[31] SCHACHERMAYER, W. Martingale measures for discrete-time processes with infinite horizon. *Math. Finance 4*, 1 (1994), 25–55.

[32] SCHWEIZER, M. On the minimal martingale measure and the Föllmer-Schweizer decomposition. *Stochastic Anal. Appl. 13*, 5 (1995), 573–599.

[33] SHIRYAEV, A. N. Two problems of sequential analysis. *Cybernetics 3*, 2 (1967), 63–69 (1969).

[34] SHIRYAEV, A. N. *Optimal stopping rules*, vol. 8 of *Stochastic Modelling and Applied Probability*. Springer-Verlag, Berlin, 2008. Translated from the 1976 Russian second edition by A. B. Aries, Reprint of the 1978 translation.

[35] VAN MOERBEKE, P. On optimal stopping and free boundary problems. *Arch. Rational Mech. Anal. 60*, 2 (1975/76), 101–148.

[36] WALD, A. *Sequential Analysis*. John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London, 1947.