Optimal control of clarifier-thickeners

Sakari Teerikoski
Abstract

Optimal control of clarifier-thickeners

Sakari Teerikoski

A computer model for a general clarifier-thickener has been made and used for comparing different strategies for control of such dewatering units through simulations. The model builds on widely acknowledged theory for modelling clarifier-thickeners - discretizing the settling PDE into a set of ODEs so that concentrations of solids at discrete depths inside the unit become the states of the numerical model - and it was written in the Modelica language using the modelling and simulation environment Dymola. The work of estimating model parameters for the model and comparing different control strategies was made looking at one particular clarifier-thickener at the company Boliden's mineral processing plant in Garpenberg. Among the control strategies that were compared in simulations were simple PID control and cascade control as well as more advanced control schemes. The simulations showed that the optimal control strategy to use is cascade control using rake torque as a secondary variable, which makes sense in particular for large thickeners such as the one mainly considered in this work.
Sammanfattning

En datormodell för en generell förtyckare skapades med syfte att användas för att genom simuleringar jämföra olika reglerstrategier för hur man bäst ska reglera dylika avvattningsenheter. Modellen bygger på välkänd teori för modellering av förtyckare genom att diskretisera den partiella differentialekvationen för sedimentering till en uppsättning ordinära differentialekvationer så att koncentrationer av fast ämne på djup med jämna mellanrum inne i förtyckaren blir den numeriska modellens tillstånd. Programmeringsspråket Modelica och modellerings- och simuleringsmiljön Dymola användes i utvecklingen av förtyckarmodellen. Den del av projektet som utgjordes av parameterestimering av modellparametrar och av jämförandet av reglerstrategier skedde i form av en fallstudie där en specifik förtyckare i företaget Bolidens anrikningsverk i Garpenberg stod i fokus. Bland de reglerstrategier som provades i simulationerna fanns både enkel PID-reglering och kaskadreglering samt mer avancerade reglerstrategier. Simuleringarna visade att den optimala reglerstrategin är kaskadreglering med momentet hos förtyckarens raka som sekundär uppmätta variabel, vilket är ett vettigt resultat framför allt för större förtyckare såsom den som låg i fokus i fallstudien.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>z</td>
<td>Vertical space</td>
<td>m</td>
</tr>
<tr>
<td>N</td>
<td>Number of height levels in model</td>
<td>-</td>
</tr>
<tr>
<td>$\dot{V}_u$</td>
<td>Underflow flow rate</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>$\dot{V}_e$</td>
<td>Overflow flow rate</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>$\dot{V}_f$</td>
<td>Feed flow rate</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>$\dot{V}_{floc}$</td>
<td>Flocculant flow rate</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>$\phi_u$</td>
<td>Underflow solids fraction</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_e$</td>
<td>Overflow solids fraction</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_f$</td>
<td>Feed solids fraction</td>
<td>-</td>
</tr>
<tr>
<td>$c_u$</td>
<td>Underflow solids concentration</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$c_e$</td>
<td>Overflow solids concentration</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Feed solids concentration</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$h_B$</td>
<td>Bed height level</td>
<td>m</td>
</tr>
<tr>
<td>$p_C$</td>
<td>Cone pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$\tau_R$</td>
<td>Rake torque</td>
<td>Nm</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$v_{hs}$</td>
<td>Hindered settling velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$v_0$</td>
<td>Unhindered settling velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Mass flux for settling solids</td>
<td>kg/m$^2$s</td>
</tr>
<tr>
<td>$A$</td>
<td>Cross-section area of clarifier-thickener</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Feed level height</td>
<td>m</td>
</tr>
<tr>
<td>$h_e$</td>
<td>Total height minus feed height</td>
<td>m</td>
</tr>
<tr>
<td>d</td>
<td>Diffusion coefficient (function)</td>
<td>m$^2$/s OR kg/m$^2$/s</td>
</tr>
<tr>
<td>$r_E$</td>
<td>Settling function model parameter</td>
<td>-</td>
</tr>
<tr>
<td>$p$</td>
<td>Settling function proportionality parameter</td>
<td>-</td>
</tr>
<tr>
<td>$a_1$</td>
<td>Dispersion function model parameter</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>Dispersion function model parameter</td>
<td>s/m$^2$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Compression function model parameter</td>
<td>Pa</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Compression function model parameter</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_R$</td>
<td>Rake expression proportionality constant</td>
<td>Nm</td>
</tr>
<tr>
<td>$h_R$</td>
<td>Rake height</td>
<td>m</td>
</tr>
</tbody>
</table>
## List of figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Map showing some of Boliden’s sites</td>
<td>8</td>
</tr>
<tr>
<td>Figure 2</td>
<td>A typical outdoor CT</td>
<td>9</td>
</tr>
<tr>
<td>Figure 3</td>
<td>An overview of a mineral processing plant</td>
<td>10</td>
</tr>
<tr>
<td>Figure 4</td>
<td>A picture showing the working principle of a CT</td>
<td>11</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Conceptual picture of the batch settling process</td>
<td>12</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Figure indicating some important CT variables</td>
<td>13</td>
</tr>
<tr>
<td>Figure 7</td>
<td>A conceptual picture of continuous operation of a CT</td>
<td>14</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Yoshioka developed a graphical method for designing dimensions of a CT</td>
<td>15</td>
</tr>
<tr>
<td>Figure 9</td>
<td>The rake visible at the bottom of an empty thickener</td>
<td>16</td>
</tr>
<tr>
<td>Figure 10</td>
<td>The settling conditions vary from the top to the bottom in CTs</td>
<td>18</td>
</tr>
<tr>
<td>Figure 11</td>
<td>Plósz et al. divide the CT into different regimes based on settling conditions</td>
<td>19</td>
</tr>
<tr>
<td>Figure 12</td>
<td>A second model parameter for narrowing down the hindered settling function curve</td>
<td>24</td>
</tr>
<tr>
<td>Figure 13</td>
<td>High-capacity mode, in which the characteristic settling regimes are different</td>
<td>24</td>
</tr>
<tr>
<td>Figure 14</td>
<td>Drawing of what a CT looks like on the inside</td>
<td>25</td>
</tr>
<tr>
<td>Figure 15</td>
<td>Illustration of how the amount of flocculant used may affect the settling</td>
<td>29</td>
</tr>
<tr>
<td>Figure 16</td>
<td>A schematic illustration of how the thickener model is implemented in Dymola</td>
<td>31</td>
</tr>
<tr>
<td>Figure 17</td>
<td>The icon of the CT model in Dymola</td>
<td>31</td>
</tr>
<tr>
<td>Figure 18</td>
<td>The inventory of the model package in Dymola</td>
<td>32</td>
</tr>
<tr>
<td>Figure 19</td>
<td>The dimensions of the CT used for model calibration</td>
<td>33</td>
</tr>
<tr>
<td>Figure 20</td>
<td>How the concentration profile will change from a starting guess to steady-state</td>
<td>33</td>
</tr>
<tr>
<td>Figure 21</td>
<td>How the concentration profile will change with the aid of control</td>
<td>34</td>
</tr>
<tr>
<td>Figure 22</td>
<td>How the concentration profile will change when the compression term is nonzero</td>
<td>35</td>
</tr>
<tr>
<td>Figure 23</td>
<td>Step change schedule for the step tests with the Pb thickener at Garpenberg</td>
<td>37</td>
</tr>
<tr>
<td>Figure 24</td>
<td>Model calibration in both Dymola and Matlab</td>
<td>39</td>
</tr>
<tr>
<td>Figure 25</td>
<td>A simulation in Matlab where the parameters have been estimated</td>
<td>40</td>
</tr>
<tr>
<td>Figure 26</td>
<td>Model calibration window in Dymola</td>
<td>40</td>
</tr>
<tr>
<td>Figure 27</td>
<td>Model calibration fit in Dymola</td>
<td>40</td>
</tr>
<tr>
<td>Figure 28</td>
<td>Cascade control concept illustration</td>
<td>44</td>
</tr>
<tr>
<td>Figure 29</td>
<td>Block diagram illustrating the principle of using feed-forward information</td>
<td>45</td>
</tr>
<tr>
<td>Figure 30</td>
<td>Simple process chart for thickeners at the Kevitsa process plant</td>
<td>46</td>
</tr>
<tr>
<td>Figure 31</td>
<td>Betancourt’s control scheme, level 2</td>
<td>47</td>
</tr>
<tr>
<td>Figure 32</td>
<td>Betancourt’s control scheme, level 3</td>
<td>47</td>
</tr>
<tr>
<td>Figure 33</td>
<td>An ideal control scheme for a CT where MIMO control could be used</td>
<td>48</td>
</tr>
<tr>
<td>Figure 34</td>
<td>A control scheme by Segovia using a fuzzy controller</td>
<td>49</td>
</tr>
<tr>
<td>Figure 35</td>
<td>Xu’s visualization of CT state feedback for a rule-based control scheme</td>
<td>49</td>
</tr>
<tr>
<td>Figure 36</td>
<td>The model $CT_{\text{exterior}}$, including defined inputs and control</td>
<td>50</td>
</tr>
<tr>
<td>Figure 37</td>
<td>The default Dymola models for a PI controller and a PID controller</td>
<td>51</td>
</tr>
<tr>
<td>Figure 38</td>
<td>Controller block model where two controllers connect as a cascade pair</td>
<td>52</td>
</tr>
<tr>
<td>Figure 39</td>
<td>Sub-model block for the calculation of a reference value for $\dot{V}_u$</td>
<td>53</td>
</tr>
<tr>
<td>Figure 40</td>
<td>Dymola model for the control block using switching control</td>
<td>54</td>
</tr>
<tr>
<td>Figure 41</td>
<td>Showing the impact of the model being slower than the actual process</td>
<td>55</td>
</tr>
<tr>
<td>Figure 42</td>
<td>Simulation of PID control of a CT</td>
<td>57</td>
</tr>
<tr>
<td>Figure 43</td>
<td>Simulation of cascade control using cone pressure</td>
<td>58</td>
</tr>
<tr>
<td>Figure 44</td>
<td>Simulation of cascade control using rake torque</td>
<td>59</td>
</tr>
<tr>
<td>Figure 45</td>
<td>Simulation of cascade control using total mass of solids</td>
<td>60</td>
</tr>
<tr>
<td>Figure 46</td>
<td>Simulation of cascade control using bed height</td>
<td>61</td>
</tr>
<tr>
<td>Figure 47</td>
<td>Simulation in Matlab of model-predictive control</td>
<td>62</td>
</tr>
<tr>
<td>Figure 48</td>
<td>Simulation of switching control</td>
<td>63</td>
</tr>
<tr>
<td>Figure 49</td>
<td>Segovia’s MIMO scheme</td>
<td>65</td>
</tr>
<tr>
<td>Figure 50</td>
<td>Dewatering units in series</td>
<td>66</td>
</tr>
</tbody>
</table>
List of tables

<table>
<thead>
<tr>
<th>Figure description</th>
<th>page nr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1: Parameter and variables list for Bürger’s model</td>
<td>1</td>
</tr>
<tr>
<td>Table 2: Detailed walk-through of the $\dot{V}_a$ step tests carried out at Garpenberg</td>
<td>39</td>
</tr>
<tr>
<td>Table 3: The final model parameters</td>
<td>43</td>
</tr>
<tr>
<td>Table 4: PI controller parameters of the controllers in current use</td>
<td>45</td>
</tr>
<tr>
<td>Table 5: Parameter sensitivities of the three most important model parameters</td>
<td>57</td>
</tr>
<tr>
<td>Table 6: Settling times for different cascade control options</td>
<td>62</td>
</tr>
<tr>
<td>Table 7: Graphical areas between the output curve and setpoint for different cascade control</td>
<td>62</td>
</tr>
</tbody>
</table>
Contents

1 Introduction ............................................. 9
  1.1 Mineral processing .................................. 10
  1.2 Clarifier-thickener basics ......................... 11
    1.2.1 Sedimentation in a CT ......................... 12
  1.3 Continuous CT ..................................... 15
    1.3.1 Flocculation .................................. 16
    1.3.2 Rake torque and bed level .................... 17

2 Modelling the clarifier-thickener ...................... 19
  2.1 The five regimes model ............................. 19
  2.2 Functional regimes model ......................... 20
  2.3 Bürger’s discretized model ....................... 20
    2.3.1 The discretization ............................ 22
    2.3.2 The actual model ............................. 23
  2.4 Extended model .................................... 23
    2.4.1 Clarification and compression zones .......... 24
    2.4.2 Feed well, underflow cone and overflow gutter 26
  2.5 Final model to be implemented .................... 28
  2.6 Making the model in Dymola ....................... 31
    2.6.1 Simulating the model ......................... 33
    2.6.2 Modeling work - extended model ............. 36
  2.7 Problematic issues with the model ................. 37

3 Field tests (Garpenberg) ................................ 38
  3.1 Methods for taking samples ....................... 38
    3.1.1 Converting from mass fraction to volume fraction 38
  3.2 Step tests ......................................... 39

4 Parameter estimation .................................... 40
  4.1 Calibration approach ............................... 40
    4.1.1 Calibration difficulties ....................... 42
  4.2 Calibration result .................................. 42

5 Control of a clarifier-thickener ...................... 44
  5.1 Control using PI controllers ...................... 44
    5.1.1 Cascade control ............................... 44
  5.2 Current control configurations in Garpenberg .... 45
    5.2.1 Feed-forward control ......................... 46
  5.3 Current control configurations at other sites .... 46
    5.3.1 Aitik ........................................ 46
    5.3.2 Boliden ....................................... 46
    5.3.3 Kevitsa ....................................... 46
  5.4 Alternative control schemes ...................... 47
    5.4.1 Betancourt’s control schemes ................. 47
    5.4.2 Model-predictive control ...................... 49
    5.4.3 Segovia’s regulator ........................... 49
    5.4.4 Simple rule-based control .................... 50

6 Simulation of control scenarios ...................... 51
  6.1 Control scenario models in Dymola ................. 51
  6.2 Simple controller models ......................... 51
    6.2.1 PI and PID .................................. 52
    6.2.2 Cascade control .............................. 52
    6.2.3 Feed-forward block for reference value .... 52
  6.3 Dymola models for switching control ............... 54
  6.4 Model-predictive control .......................... 55


1 Introduction

Thickeners are process units that are very commonly used at mineral processing plants. There they make part of the dewatering process, where water is removed from wet slurries containing metal concentrates. In mineral processing, a lot of process water is used for the separation of different minerals from each other, and thus the dewatering process step plays a big role in the big picture of a mineral processing plant.

Thickeners, or more generally clarifier-thickeners (CT), produce both a thicker underflow and clearer overflow liquid. In this way it has two products; the thicker concentrate that goes forward in the process chain, and water that can be recycled and used once more in the previous process steps (mineral separation, etc.). The difference between a clarifier and a thickener is really just that the thick underflow is the main product from the thickener (as in mineral processing) and the clear water is the main product from the clarifier (as in water treatment). In this text the word "thickener" will be used when a thickener, and not a clarifier, is meant, whereas the letters "CT" will be used when it doesn’t matter which variant of the dewatering unit is meant.

![Figure 1: Map showing some of Boliden’s sites (2015)](image)

When a thickener is in operation, the thick underflow produced in the unit should follow a certain given setpoint. The mass percentage of solids (i.e. one-hundred percent minus the mass percentage of water) in the underflow should be as close to a certain value as possible, and preferably always kept within certain bounds. The setpoint depends on the thickener’s dimensions, but also on process demand. There can be several good ways to control a thickener so that this requirement is met. While it may be sufficient with a fairly simple control scheme to keep the underflow mass fraction following its setpoint, there are many advantages in applying more advanced control strategies. This is because there are many secondary variables involved in the thickening process that one can measure and make use of.

Different control strategies are well compared through simulations. To simulate control strategies for the control of CTs, a CT model will have to be made into the simulation environment. The modeling of a CT is a subject in constant development. There is, however, already a well-established theory for how to model these units and simulate their behaviour.
This work revolves around simulating control strategies for CT control and around making a suitable CT model to be used in these simulations. The model will be written for a very general CT, but the calibration of the model as well as simulations will be tackled as a case study on a specific thickener used in the production of lead concentrate at Boliden’s new mineral processing plant in Garpenberg (where the new plant has been in operation since 2014). Boliden’s production sites are shown in figure 1.

Prior to this work, Boliden has had a number of possible control methods on how to keep the mass percentage of solids in the underflow from its thickeners within a desired range. It has not been evaluated which of these control methods is the optimal one, and in order to evaluate this a reliable computer model of a CT is needed.

A simulation model in the modeling environment Dymola can be developed, and there the control strategies as well as new ideas for control can be tested in a safe simulation environment before implementation on site. The performance of suitable methods that seem to perform well in simulations can be further evaluated on site, and methods that work well on site can be compared with other methods in simulations. This work is about building a CT model in Dymola that can successfully be used for simulation of control scenarios. The work in Dymola is covered in the sections Simulation of control scenarios and Simulation results, but a model calibration add-on to Dymola also plays a large role in the calibration and validation of the CT model, which is covered in the Parameter estimation section. The ultimate goal of the project is to find an optimal control strategy and to even see it perform well on site, if there is time left for its implementation at the end of the project.

![Figure 2: A typical outdoor CT](image)

The project builds on much of the modeling work conducted by researchers in the field of dewatering technologies, as told in the Modelling the clarifier-thickener section. The control strategies central to this work are either strategies already implemented at one of Boliden’s mineral processing plants or control schemes suggested by researchers in thickening engineering. The strategies are covered in the Control of a clarifier-thickener section.

1.1 Mineral processing

Mineral processing is the name of the process chain in which metallic concentrate is produced from ore. The concentrate is the raw material for the smelting process, which is carried out at smelters. Mineral processing can therefore be seen as kind of the middle link in the production chain of the metals industry. First mining in the mines, then mineral processing in dedicated plants and finally smelting at the smelters. An overview of a mineral processing plant is shown in figure 3. Mineral processing can be roughly divided
into the following process steps:

1. Crushing: All boulders of ore are crushed into smaller pieces so that the ore will be easier to transport and grind. The size of the boulders affect the grinding performance [1].

2. Grinding: The crushed ore is grinded into very small particles of ore. Because the next step is to separate different types of minerals as well as plain rock from each other, the ideal case would be that the grinding produces only tiny bits of ore that consist of exactly one mineral (or just plain rock) each. This is of course not the real case, but still the grinded particles can come out with micrometer-scale diameters. Water can be used in the grinding process so that the grinded particles exit the process as a slurry [2].

3. Separation: The grinded ore particles are separated so that valuable minerals are separated from plain rock and also from each other. There are several kinds of separation processes. Magnetic separation is one of them. It uses magnets to separate metal from plain rock. This kind of separation could also be used prior to the grinding. Ultimately the minerals are most commonly separated from each other (and from plain rock) by froth flotation. This is a chemical process where air bubbles, hydrophobia and hydrophilia play key roles. More details won’t be gone into here, but the main point is that the flotation process requires the ore particles to be in a wet slurry (flotation doesn’t work on plain solid particles). Therefore, if the ore wasn’t mixed with water into a slurry in the grinding step, then at least before the flotation step it will be. That’s what eventually brings the metals concentrates to the dewatering step.

4. Dewatering: The slurries/concentrates that exit the flotation tanks finally need to lose water before being transported to the smelters. The water can be reused at the mineral processing plant and the concentrate is a much better raw material at the smelter when it is thicker. One very common dewatering solution is the sedimentation in CTs. There are also centrifugal techniques that have been developed [2].

Figure 3: An overview of a mineral processing plant (source: Outotec). The thickeners can be seen at the back, flotation cells in the foreground and grinders to the left.

1.2 Clarifier-thickener basics

The CT is basically an open cylindrical tank where the solid mineral particles settle to the bottom and clear(er) water exits at the top. In this way, a much thicker concentrate can be pumped out from the bottom than the incoming concentrate (also known as feed). Multiple units can be used in series if one
single CT is not enough to achieve the desired solids fractions. The feed enters from a feed well, the outlet of which lies below the water surface. This surface is also the highest point of the CT, because the overflow flows over the edge into gutters on the sides of the CT. In most units there is a rake that goes around at the bottom that helps the settled solids exit and makes sure that the feed inlet and underflow outlet don’t get short-circuited. The bottom of the CT is usually also shaped as a cone for the same reasons.

Figure 4: A picture showing the working principle of a CT. A clarifier is essentially the same as a thickener. The difference is that when the underflow is the main product, then the unit is referred to as a thickener, but when the underflow is considered waste, then the unit is called a thickener.

1.2.1 Sedimentation in a CT

The separation principle in a CT is that different particles in a suspension fall down to the bottom with different velocities. Heavier particles are pulled downwards by gravity faster than light ones. The idea is thus that the concentrate from the previous mineral processing step enters the CT through the feed inlet and that the solids of the feed fall through the resulting suspension in the CT so that the heavier solid particles reach the bottom of the suspension first. The bottom of the suspension is called the bed level of the CT. It is not the same as the actual bottom of the CT, but the interface between the suspension in the CT and the sediment consisting of solid particles that have already fallen all the way down through the suspension and cover the CT bottom [3].

Early authors Coe and Clevenger (1916) determined the required surface area for when the material in the CT settles with a definite interface:

$$ A = \frac{(F - F_u)\dot{m}_f}{v_{hs,\text{liquid}}} $$

Here $F$ is the liquid-to-solid ratio as a function of space/length in the CT and $F_u$ is $F$ at the underflow outlet, $\rho_{\text{liquid}}$ is the liquid density, $v_{hs}$ the hindered settling velocity and $\dot{m}_f$ the feed mass flow [2].

We have for these quantities relations to other quantities such as volume fraction of solids $\phi$;

$$ \phi = \frac{1}{1 + F}, \quad F = \frac{1}{\phi} - 1, $$

$$ \phi_u = \frac{1}{1 + F_u}, \quad F_u = \frac{1}{\phi_u} - 1. $$

12
The hindered settling velocity $v_{hs}$ depends on material properties. The general expression of the stationary falling velocity of a single particle in a suspension (of that one particle with density $\rho_p$ and volume $V_p$ in a liquid with density $\rho$) is given by the expression

$$v_0 = \sqrt{\frac{2gV_p(\rho_p - \rho)}{C_D(Re)A_{proj}\rho}},$$

where $A_{proj}$ is the projection area of the particle in the falling directions (i.e. downwards due to gravity) and $C_D(Re)$ is a drag coefficient which is a function of the Reynolds number $Re$. A more common expression for the same quantity is

$$v_0 = \frac{gd_p^2(\rho_p - \rho)}{18\eta}, \quad (2)$$

which holds for a spherical particle with a velocity low enough to give $Re < 0.2$ and which contains the viscosity $\eta$ of the suspension liquid and the particle’s diameter $d_p$. This expression follows from Stokes Law that gives $C_D = 24/Re$. The rest is geometry.

**Batch settling for fine suspensions**

![Batch settling for fine suspensions](image)

Figure 5: Conceptual picture of the batch settling process. In a sample that started out with uniform composition the solids will settle downwards due to gravity until the clear liquid zone eventually touches the sediment zone of settled solids. (Source: [3])

For non-independently falling (i.e. somehow interacting) particles the expression can be written in a version for restrained sedimentation given by

$$v_0 = \frac{gd_p^2(\rho_p - \rho_{susp})}{18\eta_{susp}},$$

where $\rho_{susp}$ and $\eta_{susp}$ are the density and viscosity respectively of the suspension defined by the part $V_{susp}$ of the total volume $V$ where restrained sedimentation conditions apply. More about sedimentation conditions in later sections. For the suspension viscosity there is an expression given by

$$\eta_{susp} = \frac{\eta V_{susp}}{V} \left(1 + \frac{3}{2} \left(1 - \frac{V_{susp}}{V}\right)\right),$$

which also requires the assumption of spherical particles [3, 4].

The suitable quantity to describe the movement of solids in the CT is the mass flux $\psi$. The solid
mass flux in the CT is the mass of solid particles that moves (falls) through an area $A$ (i.e. the CT cross-section area) per time unit. The solids flux is the product of the solid concentration and the falling velocity (both being functions of vertical position and time). For a batch CT, the mass flux of solids is

$$\psi = c \cdot v(c),$$

(3)

where $v(c)$ is the falling velocity of the solid particles at a concentration of solids $c$ [6]. The original author of the equation above, Kynch, also developed a graphical method for determining $v(c)$ from experimental results. For the low-concentration cases considered so far, the solids mass flux would be $\psi = c \cdot v_0$ for a batch CT. In the continuous case it is important to note that the real falling velocity of a particle is not only the velocity given by the settling velocity equations, but there is also the velocity due to discharge of material from the CT added to that velocity. It is the batch falling velocity of solids at restrained sedimentation conditions that is the hindered settling velocity $v_{hs}$.

Further down in the CT, where the solid concentration is higher, physics becomes more complicated and compression effects start playing a role as well. The bottom of the CT is more or less covered by a sediment layer formed by solid particles that have already fallen all the way down through the suspension. The height of this sediment bed is called the bed level or bed height $h_B$. In the sediment layer the thickening happens due to compression. As mentioned before, the bottom of the thickener commonly has a conical shape so that there is a point in the middle where the pressure is the largest. This point is the ideal place for the underflow outlet from the CT and the pressure at this point is called the cone pressure $p_C$ of the CT.

Compression in the sediment layer doesn’t have to depend only on the compression due to gravity. As the CT is commonly equipped with a rotating rake that moves in the sediment, this also contributes a great deal to the overall thickening of solid material. There can be one rake arm or more of them, typically two. The rake arms have rake blades that move the sediment towards the center of the CT bottom as the rake rotates (which is good as the underflow outlet is typically located at the center as well). This frees liquid that is pressed upwards and towards the sides of the CT [2]. The mass flux at the bottom of the CT thus becomes a function of the rake’s attributes (in control contexts most notably the torque $\tau_R$ of the rotating rake), however that is something that hasn’t been studied very much [5]. According to Wills ([2]), the blades of the rake arms assist in the compactation of settled particles, helping producing a thicker underflow than can be achieved with just plain settling. How the rake helps producing a thicker underflow depends on the rake design, but this is something that has not been studied very well.
1.3 Continuous CT

For a continuous CT the mass flux gets another term:

\[
\psi = c (v_{\text{falling}}(c) + v_{\text{discharge}}),
\]

(4)

where \(v_{\text{discharge}}\) is the downward velocity due to underflow discharge at the bottom of the CT and continuous feed into it [3]. Material is taken out from the CT at a volume rate \(\dot{V}_u\), which makes the material inside the CT with cross-section area \(A\) move down at a rate

\[
v_{\text{discharge}} = \frac{\dot{V}_u}{A}.
\]

(5)

This implies the following for the solids mass flux at the outlet,

\[
\psi_u = c_u \cdot v_{\text{discharge}} = \frac{\dot{V}_u c_u}{A},
\]

(6)

where \(c_u\) is the concentration of the underflow. We have here the main control variable and the main controlled variable in the same expression. The ultimate goal of controlling a CT is to keep \(c_u\) at a desired value in a stable manner (or rather to keep the solids weight percentage of the underflow at a desired level, but this is related to \(c_u\)), and the main variable to vary is the underflow volume flow rate \(\dot{V}_u\) out from the CT as this is given by pumps after the outlet. One needs to be careful with the definition of \(v_{\text{discharge}}\) though, because above the feed inlet level the velocity due to (effluent/overflow) discharge points upwards and is instead given by \(v = \dot{V}_e/A\) [6].

Figure 7: A conceptual picture of continuous operation of a CT. The feed comes in through a feed inlet, and the underflow is pumped out (with powerful pumps) from the bottom of the unit. Clear liquid exits the CT via the gutters on the sides. The thicker the sediment is at the bottom of the CT, the harder the rake has to work in order to keep a constant speed of rotation.

A volume balance relates all the flows into and out from the CT:

\[
\dot{V}_f = \dot{V}_u + \dot{V}_e,
\]

(7)

where \(\dot{V}_f\) and \(\dot{V}_e\) are the feed and effluent (overflow) flow rates respectively [6]. In practice the feed flow is always greater than the underflow, so there is always liquid leaving the CT from above during continuous operation and the balance is thus always valid. This means that the water level in the thickener is fixed at the maximum value all the time. The steady-state mass balance for solid material is obtained by multiplying with the respective solids concentrations:

\[
\dot{V}_f c_f = \dot{V}_u c_u + \dot{V}_e c_e.
\]

(8)
The steady-state mass balance for liquid can be obtained similarly. If the clarifying is completely successful so that no solid particles leave the CT from the overflow outlet, then the mass balance equation loses one term because of $c_n = 0$ [10]. In this special case we get another useful equality:

$$\dot{V}_{fc} = Ac(v_{falling}(e) + v_{discharge}) = \dot{V}_uc_u.$$ 

In practice there will, for some $\dot{V}_f$ and $c_f$, always exist some reachable steady-state with corresponding underflow rate and concentration $\dot{V}_u^{ss}$ and $c_u^{ss}$ respectively. In that sense, from a control theory point of view, the CT system is both controllable and observable (when $\dot{V}_u$ is the system input and $c_u$ is the system output). When designed, a CT is dimensioned for a certain output underflow concentration, i.e. the setpoint for this quantity is more or less known already in the construction phase.

Figure 8: When built, each thickener is designed specifically for the material that is going to be fed into it and according to some guideline of what should be the optimal throughput and optimal solids mass fraction of the underflow. For the case when optimal throughput means maximal throughput, Yoshioka developed a graphical method, based on Kynch’s settling theory, for designing the dimensions of a CT. (source: [3])

1.3.1 Flocculation

In order for the thickener to produce clear water at the top, the settling rate of solids inside it must be greater than the rise rate of the water. Very fine particles can due to their small size easily escape through the upper outlet with the overflow that is supposed to be clear. This is a problem, but polymers can be used to help the fine particles agglomerate into larger particles. The larger the particles, the
faster they settle. If the underflow rate $\dot{V}_u$ for some reason is low, then it means that the rise rate of the suspension inside the CT will be relatively high and increasing the amount of flocculant can be done to compensate for this. By measuring the overflow turbidity one measures the amount of solids exiting the CT through the upper end. If the overflow turbidity is high, increasing the amount of flocculant can be done to compensate for this as well. The flocculant flow into the system is included in the feed flow, and thus the flocculant flow rate $\dot{V}_{floc}$ does not have to be added to the mass balance equations above.

1.3.2 Rake torque and bed level

During continuous operation of a CT, the rake and bed impose some constraints that have to be taken into account. It is especially the rake mechanism that can be very sensitive to drastic changes in concentrations of solids in the bed, which is where the rake operates. First and foremost, if the CT is not in steady-state then there will be a mass accumulation term in the mass balance:

$$\dot{V}_{f} = \dot{V}_u c_u + \dot{V}_e c_e + \frac{d m_{tot}}{dt}.$$ (9)

In practice, this accumulation of mass in the CT is the same as accumulation of mass in the bed, which in turn is the same as the bed level rising (or sinking, if $d m_{tot}/dt$ is negative). It is in general undesired that the bed level changes. If the total mass in the CT continues to grow, the CT will eventually be overloaded with solids and operation of the CT will have to be suspended until the excess amount of solids have been pumped out. Then again, if the total mass diminishes, the CT will eventually be short-circuited, i.e. the incoming solids will just fall to the low-lying bed and be pumped out from the CT before having time to thicken to the desired concentration. In other words, a too low bed level would cause too low residence times for the solids in the CT.

The rake is there to prevent uneven residence times among the solids in the CT. If the rake would not move the solids from the sides towards the center of the CT bottom, then in the worst case all the solids on the sides would just stay there permanently and the newly fed solids would be the ones exiting with the underflow after short residence times. This is also an example of short-circuiting the CT. The rake is the most vulnerable piece of the CT, and the measured rake torque is very important to take into account during operation. A too low rake torque either indicates a lack of solids in the CT (bed too low, short-circuit) or means that the rake is too high above the CT bottom and should be lowered to improve performance.

A too high rake torque can be a big problem, because that means that the rake is in a risk of getting stuck in the bed of solids. If the rake gets stuck, it can break the rake drive that keeps the rake moving. In the case that the rake gets stuck in the bed there will be no other remedy than to suspend the process.
and pump the CT empty so that rake will not be stuck anymore and so that any damage to the rake or rake drive can be repaired. A CT usually has a rake lifting mechanism, so if the torque on the rake rises, then the rake can simply be lifted to a height where the resulting torque is more suitable (provided that the bed level doesn’t rise too fast). This will then, of course, increase the residence times of solids at the sides of the CT bottom, as these parts will not be raked properly if the rake is higher up. The convenient rake height is the lowest possible from that point of view [7].

A rising bed level will also increase the torque on the rake that operates in it. However, the rake torque can be high even if the bed level is low. This is the case if a lot of flocculant is being added to the feed. The polymeric flocculant will make it harder for the rake to move if there is a lot of it among the solids in the bed. Then again, if the flocculation is low the rake torque can be relatively low even though the bed level is rising [8, 9].

For the reasons stated above, it is not only important to keep the concentration of solids in the underflow at a stable value, but it is also important to keep the bed level and rake torque stable. In practice the bed level is less of a worry than the rake, because the cross-section area of the CT, and thus the area of the bed surface, is so large that even big changes in incoming mass make just a small change in the bed surface height. For smaller thickeners it of course becomes an issue to worry more about.
2 Modelling the clarifier-thickener

The theory of settling and the balance equations of the CT give the basic ingredients for modeling the CT. Recalling the previous description of settling behaviour being different depending on the concentration, a good first model can be made by cutting up the CT along the vertical z-axis into layers/regimes with different concentration conditions. The layers can then be further modeled individually with increasing complexity.

2.1 The five regimes model

The five regimes model divides the CT into five layers after the concentration profile of the CT. This is a conceptual model rather than a mathematical model. The idea is that in different sections within the CT the material behaves differently, the conditions vary, and thus the equations for clarification/thickening vary between regimes.

The five regimes model can be summarized in figure 10, borrowed from [10].

![Figure 10: The settling conditions vary from the top to the bottom in CTs. The falling velocity of particles is slower when the environment is dense with solid particles.](image)

The 5 regimes model recognizes one clear regime free of solids at the top of the CT as well as four lower regimes where different settling conditions apply. In the discrete settling regime, the particles are scarce and thus fall through the liquid medium independently from other particles. The falling velocity for the particle is thus the one given by the most basic equation of settling.

In the flocculent settling regime the particles will somewhat influence each others' falling rates. In this regime there amount of particles is large enough for some to stick together to form flocs of multiple particles. Flocs settle faster than independent particles. The flocculant that is added to the feed before the feed comes in into the CT from the feed well is intended to aid the formation of flocs. This flocculant is usually some polymeric chemical.

The hindered settling regime is where the batch flux analysis best applies. Here the particles are so many that they majorly affect the falling of each other. The batch flux is given by an expression similar to \( \psi_{\text{batch}} = v_0 \phi (1 - \phi)^r \), where \( v_0 \) is the unhindered settling velocity, \( r \) is a positive parameter and \( \phi \) is the relative concentration of solids in the hindered settling regime.

Finally, the compression regime is where the final thickening to the underflow concentration \( \phi_u \) happens. Down here the solids have already settled into a thick bed. It is clear that shear stress and the weight of all the material in above regimes play a significant role in this regime. There is also a rake going around the central axis of the CT at the bottom. Both the rake and the shape of the bottom vary from case to case. The bottom of a CT is usually not flat, but shaped as a cone, and the main purpose of the rake is to move the solids from the periphery to the center of the tip of the cone where the underflow
outlet is located. All this affects the thickening process in the lowest regime, and that’s what makes it different from the hindered settling regime. It is common in mineral processing that the last two regimes are clearly distinguishable due to a discontinuity in the concentration of solids profile; in other words, the bed level (to which the solids from above settle) [6, 10, 11].

2.2 Functional regimes model

Plósz et al. made another type of regimes model for a CT, or rather a secondary CT as he more specifically puts it. His model doesn’t divide up regimes according to settling properties, but rather according to what occurs in the CT at different depths.

<table>
<thead>
<tr>
<th>Regimes</th>
<th>Effluent layer</th>
<th>Clarification zone</th>
<th>Feed layer</th>
<th>Thickening zone</th>
<th>Sludge withdrawal</th>
<th>Underflow layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Settling term</td>
<td>$-V_s \cdot c$</td>
<td>$V_{s,1} \cdot c$</td>
<td>$V_{s,2} \cdot c$</td>
<td>$V_{s,3} \cdot c$</td>
<td>$V_{s,4} \cdot c$</td>
<td></td>
</tr>
<tr>
<td>Convective movement</td>
<td>$V_{d,0} \cdot c_{,i}$</td>
<td>$-V_{d,0} \cdot c_{,i}$</td>
<td>$V_{d,1} \cdot c_{,i}$</td>
<td>$V_{d,2} \cdot c_{,i}$</td>
<td>$V_{d,3} \cdot c_{,i}$</td>
<td></td>
</tr>
<tr>
<td>Solids dispersion term</td>
<td>$d(c_{,i} \cdot c)$</td>
<td>$d(c_{,i} \cdot c) \cdot d(c_{,i})$</td>
<td>$d(c_{,i} \cdot c)$</td>
<td>$d(c_{,i} \cdot c)$</td>
<td>$d(c_{,i} \cdot c)$</td>
<td></td>
</tr>
<tr>
<td>CT inflows and outflows</td>
<td>$-V_{f,0} \cdot c_{,i}$</td>
<td>$V_{f,1} \cdot c_{,i}$</td>
<td>$-V_{f,2} \cdot c_{,i}$</td>
<td>$-V_{f,3} \cdot c_{,i}$</td>
<td>$-V_{f,4} \cdot c_{,i}$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 11: This figure shows how Plósz et al. divide the CT into different regimes depending on settling conditions and location of inflows and outflows.

The authors define a feed layer as the height at which the feed comes in into the CT from the feed well, above which there is a clarification zone and below which there is a thickening zone. The names of the two zones originate from the fact that above the feed level the concentration of solids is in general less than in the feed whereas it below is the other way around. This is a natural consequence of gravity; the solids do of course fall downwards. In Plósz’s model the top level of the CT is the one where the concentration of solids is the same as in the overflow. This level he calls the effluent layer. Accordingly he defines a regime in the bottom of the CT which he calls the layers of sludge withdrawal, and the very lowest one of these he calls the underflow layer. Each regime has its own set of terms in the equation for the rate of change in the concentration of solids. However, his expression for the settling velocity remains the same for all of the regimes [12]. In fact, the expressions for all the velocities are intended to be the same in the table above, regardless of the regime. However, no more detailed analysis will be given here because a more thorough analysis of CT equations follow in the next chapter, and the equations there will cover all the terms given in Plósz’s table.

2.3 Bürger’s discretized model

Bürger, Diehl, Farías, Nopens and Torfs present a numerical model [6] for secondary settling tanks without raking (SST), which can be further modified to hold for a more general CT. The model models the concentration of solids in 1D along the discretized $z$-axis as ODEs with respect to time. The modeling software Dymola only knows time derivatives and can’t solve PDEs, so this type of model is of the desired kind. First, a table of all variables and parameters of Bürger’s model will be listed below to facilitate reading this chapter.

The authors’ model, here to be called simply Bürger’s model for short, describes the whole settling process with the following equation for the interior of the SST:

$$\frac{\partial c}{\partial t} = -\frac{\partial \psi(c, z, t)}{\partial z} + \frac{\partial}{\partial z} \left( (d_e(c) + d_d(c, \tilde{V}_f)) \frac{\partial c}{\partial z} + \frac{\tilde{V}_f}{A} c \delta(z) \right). \quad (10)$$

The authors also define the same equation for the “exterior” of the SST, the meaning of which will be clarified later. In this case the two last terms on the right-hand side are left out. The $\psi$-term on the right-hand side of the equation is the flux term, and it represents both the batch flux and the flux due to discharge from the CT (or rather, the term is the sum of these two fluxes). Its sign is negative because the authors define the $z$-axis to have the positive direction downwards. They also define $z = 0$ to be at the feed inlet, hence the Kronecker delta $\delta(z)$ in the term that contains feed variables. On their $z$-axis, the overflow outlet level $h_{CT}$ is the negative $z = -h_e$ and the underflow level is the positive $z = h_u$. 
Variable | Description | Unit  
--- | --- | ---  
$t$ | Time | s  
$z$ | Height | m  
$c$ | Concentration of solids | kg/m\(^3\)  
$\phi$ | Volume fraction of solids | -  
$\psi$ | Flux of solids | kg/m\(^2\)s  
$\psi_{\text{batch}}$ | Batch flux of solids | kg/m\(^2\)s  
$d_{c}(c)$ | Compression function | kg/m\(^3\)s  
$d_{d}(z, \dot{V}_f)$ | Dispersion function | m\(^2\)/s  
$\dot{V}_f$ | Feed flow rate | m\(^3\)/s  
$V_u$ | Underflow flow rate | m\(^3\)/s  
$V_e$ | Overflow flow rate | m\(^3\)/s  
$A$ | Cross-section area of CT | m\(^2\)  
$h_f$ | Feed level height | m  
$h_e$ | CT height minus feed height | m  
$c_f$ | Feed conc. of solids | kg/m\(^3\)  
$c_u$ | Underflow conc. of solids | kg/m\(^3\)  
$c_e$ | Overflow conc. of solids | kg/m\(^3\)  
$c_{\text{crit}}$ | Critical conc. for compr. function | kg/m\(^3\)  
$\rho$ | Density of medium (water) | kg/m\(^3\)  
$\rho_s$ | Density of solids | kg/m\(^3\)  
$v_0$ | Unhindered falling velocity | m/s  
$v_{hs}$ | Hindered settling velocity | m/s  
$r_{\psi}$ | Parameter of batch flux function | m\(^3\)/kg  
$r_E$ | Parameter of batch flux function | -  
$c_{\text{max}}$ | Parameter of batch flux function | -  
$\alpha$ | Parameter of compression function | Pa  
$\beta$ | Parameter of compression function | kg/m\(^3\)  
$a_1$ | Parameter of dispersion function | m\(^{-1}\)  
$a_2$ | Parameter of dispersion function | s/m\(^2\)  
$b_1$ | Parameter involved in disp. func. | -  
$g$ | Gravity acceleration constant | m\(^2\)/s  
$\delta(z)$ | Kronecker’s delta | -  
$N$ | Number of height levels in discrete model | -  
$j, k$ | Index names in discrete model | -  
$\Delta z$ | Distance between height levels (discr.) | m  

Table 1: Parameter and variables list for Bürger’s model.

The interior of the tank is thus the range of $z$ where $-h_e \leq z \leq h_u$. The flux $\psi(c, z, t)$ is the same old familiar one, except that the batch flux term is left out in the "exterior" special case. As a reminder; $\psi = \dot{V}_u c/A + \psi_{\text{batch}}$ for $0 < z \leq h_u$, $\psi = -\dot{V}_e c/A + \psi_{\text{batch}}$ for $-h_e \leq z < 0$, $\psi = \dot{V}_u c/A$ for $z > h_u$ and $\psi = -\dot{V}_e c/A$ for $z < -h_e$. Down is the positive $z$-direction. Note that the model is kept general at this point with no introducing of $c_u$ and $c_e$. Due to the way the model will be made discrete later there is no need to define the solids mass flux at point $z = 0$, although it is not hard to figure out what would be. The batch solids mass flux $\psi_{\text{batch}}$ is the one familiar from before, i.e. $\psi_{\text{batch}} = c \cdot v_{\text{batch}}(c)$, where $v_{\text{batch}}$ is the concentration dependent hindered settling velocity. For high concentrations $v_{\text{batch}}(c)$ is not necessarily the same, however, since Bürger’s model should hold for an entire SST including the sediment layer at the bottom where naturally $v_{\text{batch}}$ should go towards zero. It is rather the compression function $d_{c}(c)$ that plays a bigger role at high solids concentrations, after some $c = c_{\text{crit}}$. This is the part of the model that needs to be further generalized to include raking effects. Finally, $d_{d}(z, \dot{V}_f)$ is a dispersion function that should capture any effects caused by turbulence near the feed inlet, and possibly even near the underflow outlet [6].

From the main equation of the model we deduce that it contains three functions that need models of their own. One that was already mentioned was the compression function $d_{c}(c)$. Without taking any
kind of rake into account, Bürger’s model proposes
\[ d_c(c) = \frac{\alpha \rho_s v_0 e^{-r_v c}}{g(\rho_s - \rho)(\beta + c - c_{\text{crit}})} \quad \forall c \geq c_{\text{crit}} \]  
and \( d_c(c) = 0 \) for lower concentrations than \( c_{\text{crit}} \). Here \( c_{\text{crit}} \) is a critical concentration basically at which the solid particles no longer fall independently through the liquid and thus it’s the same concentration as the one which defined the border between the flocculant regime and the discrete regime in the 5 regimes model. The parameters \( \alpha \) and \( \beta \) are model parameters. The parameter \( r_v > 0 \) is called Vesilind’s parameter, which originates from a model for \( v_{\text{batch}}(c) \) by an author with that name. Vesilind’s expression gives a model for the second unknown function, \( \psi_{\text{batch}}(c) \):
\[ \psi_{\text{batch}}(c) = c \cdot v_{\text{batch}}(c) = c \cdot v_0 e^{-r_v c}, \]  
where \( v_0 \) is the same as the good old familiar falling velocity \( w_T \). It can thus be seen that \( v_{\text{batch}}(c) \) is included in the expression for \( d_c(c) \). The exponential dependence on the concentration of solids makes the batch solids mass flux approach zero when the falling particles approach the sediment at the bottom. Another possible expression for the batch flux is
\[ \psi_{\text{batch}}(c) = c \cdot v_{\text{hs}}(c) = c \cdot v_0 (c_{\text{max}} - c)^{-r_e}, \]  
sometimes named after Richardson and Zaki [13, 14]. There is not much difference between the two expressions for \( \psi_{\text{batch}} \). Finally, Bürger’s model proposes the following model for the dispersion within a dispersion region \( |z| < a_2 V_f \) inside the CT:
\[ d_d(z, \bar{V}_f) = \alpha_1 V_f e^{-\frac{\beta_1(z, \bar{V}_f)^2}{2 \bar{V}_f^2}} \quad \forall \beta_1(z, \bar{V}_f) < 1 \]  
and \( d_d(z, \bar{V}_f) = 0 \) for \( \beta_1(z, \bar{V}_f) \geq 1 \), where \( \beta_1(z, \bar{V}_f) = |z|/a_2 \bar{V}_f \) and both \( \alpha_1 \) and \( \alpha_2 \) are positive model parameters. If the feed comes into the CT very slowly, then \( d_d(c) \) can be omitted. In general, the model gets harder to calibrate when there are more unknown parameters, and thus neglecting the dispersion can also prove to be a decision of convenience when it comes to model calibration. Bürger doesn’t go into any details regarding the calibration of his model [6].

2.3.1 The discretization

Discretizing the \( z \)-axis into points in such a way that one point is in \( z_0 = -h_c \), one point is in \( z_N = h_u \) and \( N - 1 \) points are in the interior of the CT, separated by \( \Delta z = (h_u + h_c)/N \), one defines the concentration of solids at each space between two points \( z_{j-1} \) and \( z_j \) as follows:
\[ c_j(t) = \frac{1}{\Delta z} \int_{z_{j-1}}^{z_j} c(z, t) dz \]  
Define also \( z_{k-1} \) and \( z_k \) as the two points between which the feed inlet is located. Lastly, define the "exterior" point \( z_{-1} \) above the CT and \( z_{N+1} \) and \( z_{N+2} \) under it. All the functions that depend on \( z \) and \( c(z, t) \) are discretized accordingly, including the primitive of \( d_c(c) \) that is given by
\[ D(c) = \int_{c_{\text{crit}}}^c d_c(c) dc. \]  
Here a short comment about the choice of \( N \) should be said. The number of discrete height levels will strongly influence the outcome of simulations and affect the time it takes to run them. A numerical model is generally more accurate when the resolution (in this case \( N \)) is high, whereas the resources (i.e. time) required to simulate it increases a lot with the complexity. This will play a big role in this work, as is told in later sections.

Before presenting the complete space-discretized model, it should at this point be explained why the "exterior" spaces are included in the model and where they come from. According to Bürger, concentration discontinuities across the two outlet locations make the intuitive assumptions \( c_{c_0}(t) = c_0(t) \) and \( c_{u}(t) = c_N(t) \) wrong. To correct for the mistake he introduces four ghost layers into the model so that the actual \( c_c \) and \( c_u \) will be given by \( c_{-1} \) and \( c_{N+2} \) respectively. The model equations treat these extra layers as if they were simply an extension of the CT, as shall be seen, and thus it is in a way convenient to think of the four extra points as "exterior". In fact, this thinking was formulated by Bürger already before he derived this model [15].
2.3.2 The actual model

From the top of the SST to the bottom, including the "exterior" layers, B"urer's model is given by the following equations. For the two top levels:

\[
\frac{dc_i}{dt} = \frac{V_e}{\Delta z}(c_0 - c_{i-1}),
\]
\[
\frac{dc_0}{dt} = \frac{V_e}{\Delta z}(c_1 - c_0) - \frac{\psi_{\text{batch},0}}{\Delta z} + \frac{1}{(\Delta z)^2}(D_1 - D_0).
\]

For \( j = 1, \ldots, k - 1 \) (with \( d_{d,0} = 0 \)):

\[
\frac{dc_j}{dt} = \left( \frac{V_e}{\Delta z} + \frac{d_{d,j}}{(\Delta z)^2} \right)(c_{j+1} - c_j) - \frac{d_{d,j-1}}{(\Delta z)^2}(c_j - c_{j-1}) - \frac{1}{\Delta z}(\psi_{\text{batch},j} - \psi_{\text{batch},j-1}) + \frac{1}{(\Delta z)^2}(D_{j+1} - 2D_j + D_{j-1}).
\]

For the feed level:

\[
\frac{dc_k}{dt} = \frac{V_{e,f} - V_{e,k}}{\Delta z} \frac{d_{d,k}}{(\Delta z)^2}(c_{k+1} - c_k) - \frac{d_{d,k-1}}{(\Delta z)^2}(c_k - c_{k-1})
\]
\[
- \frac{1}{\Delta z}(\psi_{\text{batch},k} - \psi_{\text{batch},k-1}) + \frac{1}{(\Delta z)^2}(D_{k+1} - 2D_k + D_{k-1}).
\]

For \( j = k + 1, \ldots, N \) (with \( d_{d,N} = 0 \)):

\[
\frac{dc_j}{dt} = -\left( \frac{V_e}{\Delta z} + \frac{d_{d,j-1}}{(\Delta z)^2} \right)(c_j - c_{j-1}) - \frac{d_{d,j}}{(\Delta z)^2}(c_{j+1} - c_j) - \frac{1}{\Delta z}(\psi_{\text{batch},j} - \psi_{\text{batch},j-1}) + \frac{1}{(\Delta z)^2}(D_{j+1} - 2D_j + D_{j-1}).
\]

For the two bottom levels:

\[
\frac{dc_{N+1}}{dt} = -\frac{V_e}{\Delta z}(c_{N+1} - c_N) + \frac{\psi_{\text{batch},N}}{\Delta z} - \frac{1}{(\Delta z)^2}(D_{N+1} - D_N),
\]
\[
\frac{dc_{N+2}}{dt} = -\frac{V_e}{\Delta z}(c_{N+2} - c_{N+1}).
\]

The discretized model is now almost in its complete ODE form. It still contains the primitives \( D(c) \), which are integrals with respect to concentration. These integrals need to be solved numerically before the ODE can be solved. B"urer’s model proposes a short algorithm to numerically compute all \( D_j(c) \) given some chosen maximum concentration \( c_{\text{max}} \):

\[
M = N^2
\]
\[
\Delta c = (c_{\text{max}} - c_{\text{crit}})/M
\]
\[
X_{i,0} = 0
\]
\[
\chi_{i,0} = d_c(c_{\text{crit}})
\]
\[
\text{for } i = 1, \ldots, M \{
\chi_{i} = d_c(c_{\text{crit}} + i \cdot \Delta c)
\}
\]
\[
X_{i,j} = X_{i,j-1} + \Delta c/2 \cdot (\chi_{i} + \chi_{i-1})
\]
\[
\text{for } j = 0, \ldots, N+1 \{
\text{if } c_{j} <= c_{\text{crit}} \{
D_{j} = 0
\}
\text{else } \{
\ i = \text{floor}((c_{j} - c_{\text{crit}})/\Delta c)
\ D_{j} = X_{i} + (X_{i+1} - X_{i}) \cdot ((c_{j} - c_{\text{crit}})/\Delta c)
\}\}
\]

B"urer’s model also recommends to approximate the discretized \( \psi_{\text{batch},j} \) with the Godunov numerical flux.

2.4 Extended model

There are many aspects in which B"urer’s model has to be extended in order to fulfill the requirements for simulating control scenarios. Most notably, the rake has to be included into the model. All other
measurable quantities such as bed level \( h_B \) and cone pressure \( p_C \) (and average density \( \rho_m \) in the CT) must also be included. For the bed level it rather easy; one just has to define a certain condition for what is and what is not regarded as part of the sediment. The bed level is then the layer between \( z_j \)-points where and below which all layers have a concentration \( c_j \geq c_B \) (where \( c_B \) would be the concentration at the current bed level). The cone pressure is \( p_C = \rho_m g h \), where \( \rho_m \) is the average density in the CT. The rake is a harder nut to crack. Ideally the rake would simply provide one extra term in the ODE expression for \( dc_j/dt \) for \( j = k + 1, \ldots, N \). The effects of the rake are more likely to be included in the model parameters though, for any kind of CT model.

To compare with the Bürger’s numerical model, Zhang et al. give a model which allows for taking into account the conic shape of the CT bottom as well as an even more general batch flux expression [17]. As Formulated by Zhang, the PDE describing the thickening takes the following form:

\[
\frac{\partial \phi}{\partial t} = -\frac{1}{A(z)} \frac{\partial (V_n \phi)}{\partial z} - \frac{1}{A(z)} \frac{\partial (A(z) \phi (1 - \phi) v_{rel})}{\partial z},
\]

where the authors define the \( z \)-axis to go from \( z = 0 \) at the bottom of the CT to \( z = L \) at the top, and \( v_{rel} \) is called the relative solid slip velocity (with respect to the water). By comparing the PDEs of Bürger and Zhang, this variable is easily obtained:

\[
v_{rel}(\phi) = \frac{\psi_{batch}}{\phi(1 - \phi)} \left(1 + \frac{d\sigma_e(\phi)}{d\phi} \frac{\partial \phi}{\partial z}\right).
\]

The small generalization to the batch flux function proposed by Zhang and others is to introduce a constraint variable \( \phi_{max} \) for the concentration of solids into the expression

\[
\psi_{batch}(\phi) = \nu_0 \phi (1 - \frac{\phi}{\phi_{max}})^{r_e},
\]

in case one doesn’t want to use \( \phi_{max} = 1 \). Here \( \phi \) denotes the volume of solids per total volume, so evidently \( \phi_{max} = 1 \) is the highest it can get. Zhang also proposes an alternative expression for the effective solid stress:

\[
\sigma_e(\phi) = \sigma_0 \left(\frac{\phi}{\phi_{crit}}\right)^k, \quad \phi > \phi_{crit}.
\]

Similarly as in Bürger’s expressions, the function values for these functions above \( \phi_{max} \) and below \( \phi_{crit} \) should be zero for the batch flux function and the compression function respectively [17].

In fact, Zhang’s expression for the batch flux can be generalized even further by introducing a factor \( f \) in front of \( \phi \) both in front of the parenthesis and inside it. In this way, the usual \( \psi_{batch} \)-curve can be deformed, more precisely narrowed or broadened. Clearly, by using the following equation,

\[
\psi_{batch} = \nu_0 f \phi (1 - f \phi)^{r_e},
\]

the factor \( f \) can be used to squeeze the curve so that it better corresponds to the settling behaviour in the CT in question (for the bed height and rake in question). This corresponds roughly to an introduction of \( \phi_{max} \neq 1 \) into the settling expression, just like was done by Zhang et al. In fact, it is exactly equivalent is \( f \) is only introduced inside the parenthesis. The introduction of this extra model parameter \( f \) can allow for model calibrations that better fit the model to real obtained data when simulated. Figure 12 shows how \( f \) can be used to squeeze the batch flux curve. Note that the function is only zero for \( \phi > 1/f \), since this was defined as the constraint \( \phi_{max} \).

### 2.4.1 Clarification and compression zones

As has been seen, Bürger’s model includes a hindered settling term for each level \( j \) in the CT. However, based on what has been argued before, it is clear that the settling conditions will be more or less favourable for single solid particles at different heights in the CT. Settling conditions will not correspond to that of hindered settling everywhere in the CT. Both Christian and Plósz mention a clear/clarification regime at the top and a compression/thickening regime down at the bottom. These zones will have to be taken into account in the model.
Figure 12: An extended model could make use of a second model parameter that would narrow down the hindered settling function curve as a function of solids concentration. This "squeezing factor" would favor the settling of the solids at some depths.

In the case of the clarification zone, it is rather simple to define where one should draw the line between easy settling and hindered settling. The settling conditions in the incoming feed are correspond to hindered settling for sure, and unless the CT is operated in high-capacity mode (in which case the CT is heavily loaded and the concentration of solids is high even above the feed level) there will not be solids reaching height levels above the feed level to such an extent that prevailing easy settling conditions would go over to hindered settling conditions. Thus the safe level where to draw a line between the clarification zone (above) and the hindered settling zone (below) is right above the feed level.

Figure 13: Betancourt claims that there exists another operating mode, called high-capacity mode, in which the characteristic settling regimes are different [14].

Physically the hindered settling conditions are defined so that there are so many solid particles around that the settling of individual particles is slowed down due to collisions and interaction with the others. The settling becomes hindered. Mathematically the easy settling is a special case of hindered settling where the hindrance factor is 1. In other words, the exponent $r_V$ or $r_E$ becomes zero in $v_{hs} = v_0 e^{-r_V c}$ or $v_{hs} = v_0 (1 - \phi)^{r_E}$ respectively, depending on which hindered settling expression is picked. The implementation in the model would then simply be $r_V = 0/r_E = 0$ for height levels above the feed level.

The compression zone is much more problematic. There is no proper universal theory for describing the settling in the lowest parts of the CT, because that is where the rake is and thus the settling behaviour varies from case to case. However, the compression function is included into the model precisely to help represent the behaviour in the compression zone in cases where the plain hindered settling equations aren’t enough.
2.4.2 Feed well, underflow cone and overflow gutter

The only geometry factors that Bürger’s model considers are the total height, feed level and the cross-section area of the CT. The geometry and functionality of the inlet and the outlets are not specifically taken into account in the model, but the effects they may have on the thickening process are instead included in the model parameters. For example, in the cone the thickened slurry is simply pumped out and thus forwarded to a storage tank further down the process line. This is, partly, why there is no settling term, compression term or even dispersion term included in the ODE for the lowest level in the model (i.e. the lowest ghost level). The same goes for the gutters at the top. The gutter goes all the way around the edge of the CT, and the overflow will end up there and be led away from the process. For the liquid that leaves the CT to the gutter (at the highest height level in the model) there is also no settling, compression or dispersion included in the ODE.

The feed well is meant to reduce the kinetic energy of the incoming slurry. That is why the feed enters the CT through the feed well and not straight from an inlet pipe. The idea is that the incoming solids immediately should start to settle when it enters the CT tank, instead of entering with a high pressure and causing a lot of unnecessary mixing at height levels close to the feed level. There is, however, inevitably some turbulence caused at this height by the incoming feed. The dispersion term is foremost included in the model to capture this behaviour. The more the feed well fails to reduce the kinetic energy of the incoming feed (for some $\dot{V}_f$ and $c_f$ that the feed well has not been dimensioned for), the more turbulence is caused around the feed inlet. As in the dispersion function expression proposed by Bürger,

$$d_d(z, \dot{V}_f) = a_1 \dot{V}_f e^{-\frac{(z/a_2 \dot{V}_f)^2}{1-(z/a_2 \dot{V}_f)^2}} \forall (|z/a_2 \dot{V}_f| < 1),$$

the model needs one model parameter $a_1$ to define the strength of the turbulence effect and another model parameter $a_2$ to define how deep down in the CT the effects of turbulence are present. Here some caution is required, though. The dispersion expression of Bürger is for the turbulence effects around the feed entrance, but any turbulence effects due to the rake or around the discharge exits are not taken into account. The rake continues to be an important factor in the thickening process and in a CT model where the rake’s effect on the settling/thickening is not specifically taken into account the model functions that are included will have to have parameter values that compensate for the lack of the rake in the model.

Figure 14: Drawing of what a CT looks like in the inside. The following things have been numbered: feed inlet (1), inlet pipe to the feed well for feed and flocculant (2), feed well (3), thick bed of solids (4), underflow outlet (5), overflow rising into gutters (6) and overflow outlet (7). (Source: 911 Metallurgist).

The geometry of the cone and the underflow exit also plays a second role in the model. The pressure in the cone is namely a very useful variable for the control of the thickening process. While in Bürger’s
model the cone pressure would be simply calculated as the hydrostatic pressure at the lowest depth of the CT tank, the true geometry in the CT results in a different pressure (and an important factor could also be where the pressure sensor is placed). Therefore some thickener type specific correction factor will have to be introduced into the model when calculating the cone pressure. Figure 14 shows a more accurate description of the geometry at the inlet and the outlet. For a large thickener, the feed well is not as massive in comparison to the CT size as the figure indicates.
2.5 Final model to be implemented

The model of a CT with a rake takes the following parameters: the number of discretization points \( N \) for discretization of the \( z \)-axis inside of the CT, the horizontal cross-section area \( A \) as well as the height \( h_{\text{rot}} \) of the thickener, the latter of which is a sum of the height of the feed level \( h_f \) plus the height above the feed level \( h_u \) of the CT. In addition to this, the materials properties of the feed that do not vary have to be given as parameters, including at least the density of solids and density of liquids in the suspension that will be in the CT. Last but not least is the initial concentration distribution of solids in the CT. The simulations of CT operation and control depend tremendously on the starting conditions.

The model outputs all the possible quantities that are measured in the actual thickeners used at Boliden’s site in Garpenberg (e.g., solids bed level \( h_B \), CT cone pressure \( p_C \) and rake torque \( \tau_R \)). The model takes as inputs the volume flow rate of the underflow \( \dot{V}_u \) (i.e. the main control variable considered) and all the properties of the feed that are interesting (unintentional variations in which are mainly considered as control disturbances). These feed properties include the volume flow rate of feed into the thickener \( \dot{V}_f \) and the concentration of solids of the feed \( \phi_f \), and can include the feed particle size distribution as well as the impact of adding flocculant. When the rake is included in the model, the variable rake height \( h_R \) is also an input to the model. So far the model assumes that the water level in the CT is always the same as the total height and that the volume flow rate of overflow \( \dot{V}_e \) is thus never zero (because after all clear overflow is a desired output of the thickening process).

The model only calculates the concentration profile of solids in 1D (vertically). The space domain in the CT is discretized so that there are \( N \) equally sized \( (\Delta z) \) spaces between \( z_0 = -h_e \) and \( z_N = h_u \). The \( z \)-axis is thus defined as pointing downward, because after all that is where the settling solids are going. The feed level falls somewhere in between two points \( z_{k-1} < 0 \) and \( z_k \geq 0 \). This is the discretization proposed in [16] by Betancourt et al. This is the domain in which the concentrations \( \phi_j \) \( (j = 1,...,N) \) in the spaces between the points \( z_j \) and \( z_{j-1} \) should fulfill the following PDE:

\[
\frac{\partial \phi}{\partial t} = -\frac{\partial \psi(\phi,z,t)}{\partial z} + \frac{\partial}{\partial z} \left[ (d_c(\phi) + d_d(z,\dot{V}_f)) \frac{\partial \phi}{\partial z} \right] + \frac{\dot{V}_f}{A} \phi_f \delta(z). \tag{30}
\]

The terms are all discussed above in section 2.3. According to Bürgér in [6], the model can be improved further by adding two extra "ghost" spaces above and below the CT where \( d_d \) and \( d_c \)-terms can be dropped from the equations. This discretization was discussed in section 2.3 about Bürgér’s discretized model.

With the vertical \( z \)-axis discretized into \( N + 4 \) points, the above PDE is solved as \( N + 4 \) ODEs. In the model in Dymola the system of ODEs is written in vectorized form more or less as

\[
\frac{d \phi}{dt} = T_1 + T_2 + T_3 + T_4 + T_5, \tag{31}
\]

where each of the five vectors \( T_i \) originate in the four terms of the above PDE, in which the first term is actually two terms due to the flux \( \psi \) being the sum of the batch settling flux of solids \( \psi_{\text{batch}} \) and the additional materials flux due to discharge and the top and bottom of the CT. The symbol \( \phi \) denotes the vector of mass fractions at the different levels in the CT, i.e. basically the vector is the same as the CT concentration profile. The mass fraction vector \( \phi \) is thus a sum of five vectors \( T_1 - T_5 \), or even more specifically the sum of five functions with vector valued outputs (compare the equation with figure 51, referred to in section 4).

**Term one** The first term is the term corresponding to the discharge at the CT top and the CT bottom. This is the only term in the ODEs for the outer ghost levels. From the feed level, the suspension flows both ways (up and down):

\[
T_{1,k} = -\left( \frac{\dot{V}_e}{A\Delta z} + \frac{\dot{V}_u}{A\Delta z} \right) \phi_k. \tag{32}
\]

For above the feed level \( (j < k) \) the discharge upwards gives the following term:

\[
T_{1,j<k} = \frac{\dot{V}_e}{A\Delta z} (\phi_{j+1} - \phi_j). \tag{33}
\]
For below the feed level \((j > k)\) the discharge downwards gives the following term:

\[
T_{1,j>k} = \frac{\dot{V}_a}{A\Delta z}(\phi_{j-1} - \phi_j).
\] (34)

**Term two** The second term is the term corresponding to the dispersion function. The function itself is first given by

\[
d_{d,j} = a_1\dot{V}_fe^{-\left(\frac{d_j}{\phi}\right)^{1-b}}
\] if \(b < 1\), otherwise \(d_d = 0\), and where \(b = |z_j/(a_2 \dot{V}_j)|\). This expression for the dispersion function has been suggested by Bürger [6]. The constants \(a_1\) and \(a_2\) are parameters. The second one should be chosen so that the value of \(b\) passes 1 at some two points within the thickener (recalling from the definition of the \(z\)-axis that \(z_b\) and thus \(b\) are close to zero at the feed level). This way, we get the physically intuitive \(d_{d,0} = d_{d,N} = 0\), i.e. no dispersion at the bottom and at the top of the CT. The actual term two then becomes

\[
T_{2,j} = d_{d,j}(\phi_{j+1} - \phi_j) - d_{d,j-1}(\phi_j - \phi_{j-1})
\] (36)
for all \(j = 1, ..., N\), but recalling that \(d_{d,0} = d_{d,N} = 0\). The dispersion term is zero for the four ghost levels.

**Term three** The third term is the term corresponding to the batch settling flux. Hear the model gets interesting when taking into account impact of flocculation and particle size distribution. Central in the third term is the batch settling flux, given by

\[
\psi_{\text{batch}} = \rho_s \phi
\] (37)

where \(\rho_s\) is the density of the solid particles and the hindered settling velocity \(v_{\text{batch}}\) is given by

\[
v_{\text{batch}}(\phi) = v_0(d_p, K_{\text{floc}}) \cdot (1 - \phi)^e
\] (38)
as presented by Bürger, where \(r_v\) is a parameter called Vegilind’s parameter ([6]) and \(K_{\text{floc}}\) is a multiplication factor variable which represents the impact of flocculation on the settling, as presented by Betancourt [16]. \(K_{\text{floc}}\) can also be chosen simply as a constant parameter. The falling velocity \(v_0\) of a free particle in the suspension can also be chosen as a constant parameter, but it is actually dependent on the particle size distribution. In the model a (variable) value for the mean particle size \(d_{p,\text{mean}}\) will be used to calculate \(v_0\) according to the familiar formula (see section 1.2):

\[
v_0 = \frac{gd_{p,\text{mean}}(\rho_s - \rho_1)}{18\eta}.
\] (39)

Recalling that the settling direction, i.e. downwards, was chosen as the positive \(z\)-direction. The sub-terms of the ODE term three are then negative for solids falling down to levels below while positive for solids falling from above. In other words, for \(j = 1, ..., N\), the settling equations are as follows:

\[
T_{3,j} = \frac{\psi_{\text{batch}}(\phi_{j-1}) - \psi_{\text{batch}}(\phi_j)}{\rho_s \Delta z}.
\] (40)

As there is nothing falling from one ghost lever to another, the equations for the extra levels are simply \(T_{3,N+1} = \psi_{\text{batch}}(\phi_N)/\rho_s \Delta z\), \(T_{3,0} = \psi_{\text{batch}}(\phi_0)/\rho_s \Delta z\) and \(\psi_{\text{batch}}(\phi_{-1}) = \psi_{\text{batch}}(\phi_{N+2}) = 0\).

Using the mean particle diameter \(d_{p,\text{mean}}\) to represent the particle size distribution is quite straightforwards. The impact of flocculation is, however, more complicated. Betancourt suggested that the variable \(K_{\text{floc}}\) should have values between 0 and 1 and thus be a measure or a degree of the success of flocculation in terms of impact on the falling velocity. How the value of \(K_{\text{floc}}\) depends on the amount of flocculant pumped to the system is not straightforward, and the dependence should be either assumed or studied and then added to the model. The optimal \(K_{\text{floc}} = 1\) conditions fall somewhere between insufficient addition of flocculant and overdose of flocculant [16].
Term four The fourth term is the term corresponding to the compression function. This term contains, as suggested by Bürger and Betancourt, the primitive function $D = \int d_c(\phi) d\phi$ of the compression function $d_c$ [6, 16], which can, as suggested by Betancourt, be described by the expression

$$D_j = \frac{\alpha \beta \gamma K_{floc}}{g(\rho_s - \rho_l)} \left( e^{\phi_j/\gamma} - e^{\phi_{crit}/\gamma} \right),$$

(41)

for all $j$ for which $\phi_j$ is greater than a certain critical concentration $\phi_{crit}$ (a model parameter) except for the outer ghost levels, otherwise $D_j = 0$. Here $\alpha$ and $\gamma$ are also model parameters. The expression is a result of modeling the effective stress function of the settling process as $\sigma_e(\phi) = \beta_1 e^{\beta_2 \phi}$ and the following definition of the compression function:

$$d_c(\phi) = \frac{v_{\text{batch}}(d\sigma_e(\phi)/d\phi)}{g(\rho_s - \rho_l)} g(\rho_s - \rho_l),$$

(42)

where $\beta_1$ and $\beta_2$ are some parameters. The fourth term then becomes, for $j = 0, ..., N + 1$, the following:

$$T_{4,j} = \frac{D_{j+1} - 2D_j + D_{j-1}}{\rho_s (\Delta z)^2}.$$  

(43)

For the outer ghost levels, $T_{4,-1} = T_{4,N+2} = 0$, and also $D_{-1} = D_{N+2} = 0$.

Term five This term is zero in all ODEs except the one for the feed level. In other words, the last term in the system of ODEs is the exact same term as the last term in the original PDE:

$$T_5 = T_5(\dot{V}_f, A, \phi_f) = \frac{\dot{V}_f}{A \Delta z} \phi_f \delta(j - k).$$

(44)

Finally, the whole system can be represented in a state representation of the following form:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t),$$

$$y(t) = Cx(t),$$

(45)

where the states $x(t)$ are the vector $\phi$, $g(x(t))u(t)$ corresponds to term one and $f(x)$ corresponds to the rest of the terms. The measured variable is simply $\phi_u$, so we get the following easy relations between the model and the state representation:

$$x(t) = \phi(t),$$

$$u(t) = \dot{V}_u(t),$$

$$g(t) = \phi_u(t),$$

$$g(x(t))u(t) = \dot{T}_1,$$

$$f(x) = T_2 + T_3 + T_4 + \dot{T}_5,$$

$$C = [0 \ldots 0 1],$$

(46)

where $C$ has the dimensions of $\phi$. It is important to note that the disturbances $\dot{V}_f$ and $\phi_f$ are contained in $f(x)$. Also, not all elements of the first term have $u(t) = \dot{V}_u(t)$ in them, but some have $\dot{V}_e = \dot{V}_f - \dot{V}_u$.
instead. Thus the division into $\tilde{T}_1$ and $\tilde{T}_5$ instead of $T_1$ and $T_5$. The relations between these are the following:

$$
\tilde{T}_{1,k} = 0,
\tilde{T}_{1,j<k} = -\frac{\dot{V}_u}{A\Delta z} (\phi_{j+1} - \phi_j) = T_{1,j<k} - \frac{\dot{V}_f}{A\Delta z} (\phi_{j+1} - \phi_j),
\tilde{T}_{1,j>k} = T_{1,j>k},
\tilde{T}_{5,k} = T_{5,k},
\tilde{T}_{5,j<k} = \frac{\dot{V}_f}{A\Delta z} (\phi_{j+1} - \phi_j) = T_{1,j<k} - \tilde{T}_{1,j<k},
\tilde{T}_{5,j>k} = 0.
$$

Two quantities that don’t intuitively follow from the concentration profile in the CT are the bed level and the rake torque. The bed level is the level $h_B$ in the thickener where the settling particles land, i.e. there is supposed to be a remarkable discontinuity in the concentration profile. Below the bed level, the settling velocity should be practically zero. In the model, it is not always evident where the bed level lies, especially if $N$ is small. It could be defined as the height of the level $j$ where the concentration reaches a certain percentage of the underflow concentration or where the batch settling velocity reaches a certain percentage of the batch settling velocity at the feed level, for example. The rake torque $\tau_R$ can be thought of as proportional to the concentration of solids at the height where the rake is, so a first model for the rake is simply the following:

$$
\tau_R = \alpha_R \phi_{j_R},
$$

where $\alpha_R$ is a parameter and $j_R$ is an index integer given by

$$
j_R = \lceil \frac{h_u + h_u - h_R}{\Delta z} \rceil,
$$

where $h_R$ is the height level of the rake.

When validating the model, it should be remembered that no parameter affects the model as much as the given initial concentration profile in the CT. To get a reasonable simulation output, the initialization must also be reasonable! Particularly good to know would be $\phi$ at height $h_B$ at time 0.

### 2.6 Making the model in Dymola

To model and simulate control scenarios with clarification-thickeners the key model component is the model of the CT itself. The model of the CT should be as general as possible so that as many control scenarios as possible can be tested. The model should comprise the theory of settling, materials- and mass balances, the concentration profile of solids within the CT unit as well as equations for how different measurable quantities such as cone pressure and rake torque depend on other quantities and variables. Bürger’s discretized model, where the main PDE describing the settling is spatially discretized into a large number of ODEs with respect to time, provides a good basis for modeling the CT in Dymola. The CT model is therefore built on that model.

Different control scenarios will be simulated with models all containing the CT model as the key component. These models will here be called ”CT-external models” to make clear that the CT model itself is a sub-model of these. More details about the CT-external models will come later.

A structural overview of the whole model can be seen in figure 18. It can be seen that some alternative CT models and CT-external models have been made. The model also contains multiple functions, controller models for some controllers and some model building blocks (these are not relevant for the CT model itself, but will be introduces in the section 6).

The CT model has in total ten inputs/outputs to the outside world. Five of them are disturbance inputs. Two of these represent the feed variables. Two of the remaining outputs are the overflow variables. These
variables are, of course, the volume flow rate and concentration of solids in the flow. The feed variables \( \dot{V}_f, c_f \) are always inputs to the model. This pretty much implies that the overflow variables \( \dot{V}_e, c_e \) will be outputs of the model because all control scenarios studied will involve the control of the underflow concentration of solids \( c_u \) (or related variables, an output) by varying the underflow rate \( \dot{V}_u \) (an input). The three remaining disturbances are for rake height, mean diameter of solid particles entering with the feed (assuming the particles are spherical) and an efficiency coefficient describing the efficiency of flocculation on the thickening process.

The final model output is a common one for all the additional measured variables, such as the cone pressure and the rake torque. This last output is called multi-sensor and it is a specifically defined connector (which is the Dymola name for input/output) for the CT model.

The main feature in the CT-model is of course the settling PDE discretized according to Bürger’s recipe. This is represented as a vector ODE with five terms: the main mass balance term, a dispersion function term, a settling function term, a compression function term and a feed term. The last one is a term which is zero for all vector elements in the ODE except for the feed level element. Dymola supports the use of functions in numerical calculation very well, so each of the term vectors are generated by a function of their own. The top level concentration of solids in the concentration vector is the same as the overflow concentration output and the bottom level concentration is the same as the underflow concentration output. For the feed level it is a bit different, since the concentration in the tank at the feed level height is not necessarily the same as the feed concentration. Other important functions in the model are the ones in the original PDE model, i.e. the CT model also contains function for the batch settling and the mass flux it results in as well as, naturally, the compression and dispersion functions. Functions to compute all the values outputted by the multi-sensor block are also included.
Figure 18: The inventory of the model package in Dymola. The whole model makes up a model package which in turn has sub-packages and models/sub-models in it.

Shown in figures 51 and 52 in the appendix are the main code of the CT-model and a structural overview of the functions belonging to the model.

The reason why there are two functions for the batch flux is that one gives a scalar output and the other gives a vector output. The functions named “bigD” and "smallD" are for the compression term and dispersion function respectively. The "burgerZeta" function is for calculating the height value corresponding to the discrete height levels.

It is important that the model is calibrated so that the model parameters a given sensible values. A simulation with an uncalibrated model can arrive at the specified underflow density for example by just arriving at a steady state where the underflow density would stay at the specified value at \( \dot{V}_u = 0 \) and then just shuts the underflow.

Some parameters are given by geometry. Figure 19 below shows the basic dimensions of the thickener used for model calibration. The three parameters that can be deduced from the geometry are \( A = 113.1 \text{ m}^2 \), \( h_e = 0.5 \text{ m} \) and \( h_u = 3.033 \text{ m} \).

2.6.1 Simulating the model

A first simulation result with and without control is presented here (in figures 20 - 22 below) for illustrative purposes. Shown there is the concentration profile as a function of time when the system moves to a steady state from a given initial starting configuration (with or without being controlled towards that state). Each curve in the concentration profile corresponds to the concentration of solids at a specific height level in the CT model. Worth noticing is that the concentration profile really does look like the profile of a settling process in the simulation without control, as the concentration curves increase in height when moving from the top towards the bottom of the CT. It is also worth pointing out that all parameters chosen at this point were chosen so that the example given in the graphs would be as illustrative as possible. This also goes for the CT dimensions.

The typical effect of the dispersion function can be seen in the very beginning at times close to zero.
There the concentration curves spread out a bit even though they start from same start values. What this corresponds to physically is that solids are spread out around the feed level (due to the kinetic energy of the incoming fluid and resulting turbulence). The absence of this effect would be that all the feed particles start at one horizontal level and hover there before being moved down (or up) by the model equations.

The compression function accounts for stabilizing settling behaviour so that the batch settling term doesn’t make the underflow concentration curves drift off to unreasonable values. The rest is just mechanic movement of material in the CT (which would typically result in settling curves where adjacent curves just lack behind each other like in some kind of heat profile).

Figure 20: This is how the concentration profile will change in time from a starting guess to steady-state. Curves higher up represent volume fractions of solids on height levels $j$ further down in the CT, whereas curves lower down represent the solids fractions at higher height levels. The simulated scenario is settling only, i.e. no control, but with a fix underflow volume flow rate.
This nice feature of the profile is completely absent in the graph from the simulation with control. The reason for this is that one of the functions used for computing the compression function term in the $d\phi/dt$ expression will crash as soon as the solids fraction value in the simulation reaches an unrealistic value above 1.0, which is bound to happen when an untuned controller outputs something unreasonable. Therefore the compression function term has at this point been set to zero in the model script. In the simulation, without there being any intended step change in the underflow rate, the underflow rate, $V_u$, makes a jump from 0.06 m$^3$/s to 0.04 m$^3$/s at time $t = 2.8 \cdot 10^4$ s (i.e. after 7.8 hours, note that thickening is a slow process).

As the lowest height level before the pumping out of the underflow is the last one with a batch flux term, settled solids tend to accumulate there. The CT model, as it is implemented, will output a value for the bed height $h_B$, but the settling term in the ODE doesn’t get that information. Therefore the solids in will keep on settling to the bottom of the CT model, rather than settle onto a bed of solids somewhere on the way. The settling of solids will, however, be much slower at the lower height levels, where the concentration of solids is higher. This behaviour is governed by the parameter(s) of the batch flux function $\psi_{\text{batch}}(\phi)$. The accumulation of solids at the lowest height levels will largely dominate the convergence towards a new steady state when simulating control scenarios. Figure 21 below shows an extreme case of this for illustrative purposes.

![Concentration profile in CT controlled with simple PID](image)

Figure 21: This is how the concentration profile will change in time from a starting guess to steady-state with the aid of control, when the compression term is set to zero. Curves higher up represent volume fractions of solids on height levels $j$ further down in the CT, whereas curves lower down represent the solids fractions at higher height levels. Note how the concentration profile becomes a physically unreasonable one where the solids fractions at the two lowest height levels are much greater than the rest! This is the typical result of a simulation with the wrong choice of parameters. Note that the parameters and CT dimensions for this simulation have been chosen to illustrate model behaviour and the no parameters have been estimated yet at this stage!
A similar control simulation can be seen in figure 22, where a compression term has been included (but not the same compression term as in previous figures). The compression term basically creates the bed of solids by spreading out solids among the height levels where the solids fraction is higher than a certain critical minimum value $\phi_{\text{crit}}$.

Figure 22: This is how the concentration profile will change in time from a starting guess to steady-state with the aid of control, when the compression term is nonzero. Curves higher up represent volume fractions of solids on height levels $j$ further down in the CT, whereas curves lower down represent the solids fractions at higher height levels. Note how the concentration profile becomes more uniform than in the case without the compression function! Note also that the lowest curves, representing solids fractions in the clear overflow, stay close to zero, which is of course characteristic for the CT. Once again, note that the parameters and CT dimensions for this simulation have been chosen to illustrate model behaviour and the no parameters have been estimated yet at this stage! (However, later in section 4 it will be seen that the model calibration yielded parameter values for which the compression term was indeed zero).

2.6.2 Modeling work - extended model

To include the effect of flocculation into the model, the code can be updated so that the settling velocity and the batch settling flux depend on a coefficient $K_{\text{floc}}$, which corresponds to some varying property of the feed. This could, among other things, be the amount of flocculant used per unit volume. For example, one could define the coefficient to have values in $0 \leq K_{\text{floc}} \leq 1$ so that $K_{\text{floc}} = 1$ corresponds to (somehow) ideal flocculation of the feed and $K_{\text{floc}} = 0$ corresponds to extremely non-ideal flocculation preventing any kind of settling.

Here also the falling velocity parameter $v_0$, corresponding to the falling velocity of an independent solid particle in a suspension in the batch falling velocity expression, has been changed to a function $v_0(d_p)$ of the mean solid particle diameter (with new parameters such as densities and fluid viscosity), according to equation (it's 39 now but might change).
2.7 Problematic issues with the model

A problem with the model is that the batch flux movement can only transfer particles from one layer to an adjacent layer per time step. This could probably create some physically weird behaviour in the model, especially when particles are heavy and relatively large, and thus should be able to fall quite rapidly. This can arise for example after a not so good initial concentration profile has been given to the model at the beginning of a simulation – some concentration curves can rise up quite high without affecting the adjacent levels too much. For smaller and thus lighter solid particles in the slurry we see the behaviour of a lot of solid particles leaving the CT with the overflow due to a much higher overflow rate compared to the underflow rate so that the small/light particles didn’t have time to fall downwards fast enough to settle on the bed. This is of course not a physically weird thing, but it can appear in physically weird proportions in the simulations.

Another issue comes with the model expression for the hindered settling velocity. In a real CT the solids will stop settling when the solids reach the bed surface. There the rake will cause some very different behaviour, including rotational movement of solids in the bed around the CT mid-axis, pushed by the rake (which a one-dimensional model doesn’t take into account). In the numerical model with a discrete space domain it can happen (in simulations) that physically unreasonable amounts of solids accumulate at the very lowest height levels in the CT.

The batch flux approaches zero when the concentration goes towards 100 % solids, i.e. the flux approaches zero when the concentration of solids is so high that the solids in question are down in the bed. The batch flux function never reaches zero, of course, because the solids fraction can never be as high as 100 %. This is all fine as long as a calibrated model is used for simulating continuous operation of a CT. However, some care is needed especially in simulations of steady states. It might be that the settling goes on until it looks like almost all the solids are at the lowest few height levels of the CT. This is caused by a numerical error in which the batch flux term actually "blows up" (i.e. concentration curves for deep levels suddenly diverge toward infinity), but the effect is reduced by other terms in the ODE.

The CT model behaves slower and more aggressively than the actual CT against which it is calibrated. For the model it takes longer to react to changes in the inputs. Also, when the model does react to changing inputs the response can be very extreme, resulting in steady states where the concentration of solids is nearly zero everywhere (if the input change was of the kind that would lower the underflow concentration). The opposite case can be equally problematic, resulting in steady states where the CT model is filled up with a lot of solids all the way up to the feed level (or even all the way to the top if hindered settling conditions are assumed all the way to the top and no clarification zone is included). This behaviour becomes especially evident in control simulations where the resulting control is unstable and where the controller ends up alternating between the minimum and maximum of the allowed interval of input values. The behaviour is also evident in the model calibration.

Considering that the model for low resolutions (low \(N \)) reacts slower to input step changes than the corresponding real CT, it is rather clear that when the model is calibrated against process data from a relatively short time period (the order of a few hours) it will arrive at parameter values for which the model is prone to quickly empty the CT one height level after the other, especially if the input step change caused the output underflow solids percentage to decrease during the time period in question. The fact that the model takes more time to react is also reflected in all the control simulations. The time constant in the PI-regulators will consequently be greater than the actual time constant (usually referred to as the integral time parameter \(T_i \) of the PI-controllers in the existing process control. How much more simulation time the process requires to reach steady states and how much greater the controller time constants need be depends on the discretization resolution. This is the main effect of \(N \) that can be seen in the CT model.

The main issue of the model is, however, the simple fact that it is just a 1D approximation of a process where all three dimensions of space play a significant role. The model will never be a completely accurate representation of a CT. There will always be a trade-off between model exactness and relative simplicity that is good for simulations of control schemes. The lack of established theory for how the impact of the rake on settling and thickening should be modeled is also an important factor to always keep in mind.
3 Field tests (Garpenberg)

The results from the step experiments were used to validate parameters in the model. Step tests were carried out according to the following test plan.

Figure 23: Step change schedule for the step tests with the Pb thickener at Garpenberg. The three step sequences were carried out on separate tests.

<table>
<thead>
<tr>
<th>Underflow rate:</th>
<th>4.5 m³/h</th>
<th>5.0 m³/h</th>
<th>4.0 m³/h</th>
<th>5.5 m³/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>2 h</td>
<td>2 h</td>
<td>2 h</td>
<td>2 h</td>
</tr>
<tr>
<td>Flocculant</td>
<td>112 l/h</td>
<td>85 l/h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>flow rate:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time</td>
<td>2.5 h</td>
<td>3 h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rake height:</td>
<td>70 mm</td>
<td>75 mm</td>
<td>85 mm</td>
<td>100 mm</td>
</tr>
<tr>
<td>time</td>
<td>By default</td>
<td>5 min</td>
<td>20 min</td>
<td>25 min</td>
</tr>
</tbody>
</table>

### 3.1 Methods for taking samples

During the step tests, samples were taken from three different locations. One sample was taken from the slurry entering the feed well to find out the mass percentage of the incoming feed. A sample of the overflow was taken from a gutter where there was clear liquid that had just left the thickener. Finally, the underflow sample could be taken out through a tap in the pipe leading the concentrate away from the thickener to the dedicated storage tank of the concentrate. The feed sample was straightforward to take, but the two others came with slight complications. The overflow sample took a long time to take, because the gutter was relatively hard to reach from the platform over the thickener where a person could stand and take the sample. The sample could only be obtained with a long scoop many times to fill a one liter bucket. The volume of one liter was necessary because the sample was weighed using a specific scale that could directly tell the solids weight percentage of a sample of exactly one liter. The fact that the scale immediately revealed the weight fraction of the solids was a huge advantage. The only information that was needed for these results was the bulk density of solids, which had to be known in advance in order to set the scale right. This was found to be 6.74 g/cm³.

The underflow sample couldn’t be taken out from the tap directly into a one liter bucket because of the fast flow rate. Therefore a hose was used in the sample-taking. The underflow then had to flow through the hose for a while into the drain before taking the sample, so that the composition in the hose was right and not influenced by previous samples. All issues stated above were potential sources of errors in the experiment, and it was not often that measurement results coincided with data in the process monitoring system. The overflow samples were also dried and weighed so that the mass of solids per liter of sample was obtained.

An attempt to take samples from various depths in the CT was also made. This was attempted both by pumping up the contents of the thickener from different depths into a bucket and by hoisting down a sample jar with a lid that could be closed at the desired depth. Both attempts to get up samples from the CT failed due to the high mass of the sample. Although it was possible to take a sample into the sample jar, it wasn’t possible to lift up the sample from the CT. Pumping up samples didn’t work either due to too weak pumps.

#### 3.1.1 Converting from mass fraction to volume fraction

Before presenting step test results, something should be said about unit conversions. When the bulk density of the solids is known (\( \rho_{\text{solid}} = 6.74 \text{ g/cm}^3 \), see above), the conversion from mass fraction to
volume fraction is easily obtained through

\[ \phi_v = \frac{\rho_{\text{liquid}} \phi_m}{\rho_{\text{solid}} - (\rho_{\text{solid}} - \rho_{\text{liquid}}) \phi_m}, \]

where \( \phi_v \) and \( \phi_m \) denote volume and mass fraction respectively. Naturally, \( \rho_{\text{liquid}} = 1 \text{ g/cm}^3 \) in water.

### 3.2 Step tests

Three variables were varied, each in its own step test sequence. The first one to be varied was the rake height. This was done to see how the system and the rake torque in particular reacted on raising the rake. The second variable to be varied was the flocculant flow rate into the system. The hypothesis was that this would mainly affect the measured bed height. Finally and most importantly, the third variable to be varied was the underflow rate, and here the impact on the underflow solids fraction was the main target.

<table>
<thead>
<tr>
<th>Clock</th>
<th>step time (min)</th>
<th>total time (min)</th>
<th>( \dot{V}_u ) (m³/h)</th>
<th>mass-% feed</th>
<th>mass-% ovf</th>
<th>mass-% unf</th>
</tr>
</thead>
<tbody>
<tr>
<td>08:15</td>
<td>0</td>
<td>0</td>
<td>4.5 (const.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>09:15</td>
<td>0</td>
<td>0</td>
<td>4.5 (const.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>09:23</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10:00</td>
<td>37</td>
<td>37</td>
<td>5</td>
<td>14</td>
<td>2.5</td>
<td>70.5</td>
</tr>
<tr>
<td>10:30</td>
<td>67</td>
<td>67</td>
<td>5</td>
<td>15</td>
<td>2.2</td>
<td>70.5</td>
</tr>
<tr>
<td>11:00</td>
<td>97</td>
<td>97</td>
<td>5</td>
<td>14</td>
<td>3</td>
<td>68.7</td>
</tr>
<tr>
<td>11:30</td>
<td>127</td>
<td>127</td>
<td>5</td>
<td>17</td>
<td>3</td>
<td>70.5</td>
</tr>
<tr>
<td>11:36</td>
<td>0</td>
<td>133</td>
<td>Step change!</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12:00</td>
<td>24</td>
<td>157</td>
<td>4</td>
<td>17</td>
<td>3.6</td>
<td>71</td>
</tr>
<tr>
<td>12:30</td>
<td>54</td>
<td>187</td>
<td>4</td>
<td>11</td>
<td>3.1</td>
<td>71.5</td>
</tr>
<tr>
<td>13:00</td>
<td>84</td>
<td>217</td>
<td>4</td>
<td>11</td>
<td>2.5</td>
<td>70.5</td>
</tr>
<tr>
<td>13:30</td>
<td>114</td>
<td>247</td>
<td>4</td>
<td>19</td>
<td>3</td>
<td>70.5</td>
</tr>
<tr>
<td>13:35</td>
<td>0</td>
<td>252</td>
<td>Step change!</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:00</td>
<td>25</td>
<td>277</td>
<td>5.5</td>
<td>17</td>
<td>2.7</td>
<td>70.2</td>
</tr>
<tr>
<td>14:30</td>
<td>55</td>
<td>307</td>
<td>5.5</td>
<td>13</td>
<td>5</td>
<td>72.5</td>
</tr>
<tr>
<td>15:00</td>
<td>85</td>
<td>337</td>
<td>5.5</td>
<td>15</td>
<td>3.6</td>
<td>72.5</td>
</tr>
<tr>
<td>15:15</td>
<td>100</td>
<td>352</td>
<td>5.5</td>
<td></td>
<td></td>
<td>64</td>
</tr>
<tr>
<td>15:30</td>
<td>115</td>
<td>367</td>
<td>5.5</td>
<td>15</td>
<td>3.1</td>
<td>64</td>
</tr>
</tbody>
</table>

Table 2: Detailed walk-through of the \( \dot{V}_u \) step tests carried out at Garpenberg.
4 Parameter estimation

After finding a suitable starting guess based on the results from the Garpenberg tests with trial and error in Dymola, a parameter estimation was carried out in MATLAB to find the parameters for which the model would best adjust to the obtained data. The parameter estimation uses the MATLAB command \textit{lsqcurvefit} and a subroutine in which the model ODE problem is solved. A model calibration add-on was also acquired for Dymola so that more time efficient parameter estimation could be made.

![Diagram of model calibration in Dymola and Matlab]

The MATLAB code (including subroutines) that was produced for this purpose can be used to estimate one or several model parameters while keeping the rest (such as those defined by the CT geometry) constant. Which parameters should be estimated (while keeping the others constant) can be specified with only a few modifications to the code. The parameter estimation code structure can be seen in figure 24.

4.1 Calibration approach

The parameters were not attempted to be estimated all at once, but rather in groups corresponding to one ODE term (out of five) at a time. However, in some attempts to estimate only some parameters at a time, starting from the parameters corresponding to the batch flux term, the parameter estimation algorithm would find parameter values for which the corresponding terms would be very close to zero, thus influencing the outcome as little as possible. This has a natural explanation, however, as the task of finding the optimal parameter values becomes hard for the solver when the variable pair to be estimated are partly in competition with each other. This means that the effect of raising one of the parameters will be more or less cancelled out by decreasing the other.

If the correction constant $p$ for the hindered settling velocity is included, all three ODE terms that include model parameters contain exactly two model parameters that the parameter estimation algorithm can find to be in conflict together. This causes the calibrated values of the parameters to escalate away in pairs during parameter estimation runs. The only remedy is then to fix one of the two model parameters and only calibrate the other one. Such runs of parameter estimation are of interest, because already Bürger states that depending on the case a user may keep the compression and dispersion terms equal to zero in simulations [6]. If batch flux parameters would already fit the model to data rather nicely, then the rest of the parameters could be estimated after that to complete the fit.
One major problem when estimating the parameters is the choice of $N$. For low $N$ the estimation algorithm can finish relatively fast, but for high $N$ it may take very long to complete a run. This creates a situation of trade-off; either one is satisfied with a low $N$ and can run and repeat runs quite fast, or one has a high $N$ and is satisfied with the parameter values that correspond to the best fit that could be achieved in a limited time. In addition, a higher $N$ of course gives a more accurate simulation. The problem lies not only in this trade-off, but in the fact that the model can go from a good fit to an unstable solution when simulated in Dymola just by changing $N$, but for otherwise same parameters. This makes sense because after all the solver always finds the best fit of $\phi_{N+2}(t_{data})$ to $\phi_{N+2}^{data}$. The fit has therefore naturally a strong dependence on $N$.

A calibration plot from Dymola is shown in figure 26 below.

Even in the most sophisticated parameter estimation we are forced to some sort of a guess of the initial
concentration profile. That is a one sole decision that has a big impact on the final obtained parameters.

4.1.1 Calibration difficulties

The fact that the model reacts slowly and overly-aggressively (as was described in section 2.7) sets a trap to the model calibration algorithm. When the model is calibrated against a data series taken during a relatively short time period, it is not sure whether the found parameters do actually accurately describe the process or not. If the calibration data concentration curve bends downwards, the model calibration process may have found a parameter combination for which the measured increase in the underflow rate rapidly drains the model’s thickener empty of solids, but that this behaviour of the model is still too slow to be observed during the short time period of the calibration data. If the model indeed reacts slowly to the changes is $\dot{V}_u$, the parameter estimation is likely to find a parameter combination for which the model’s CT is starting to be drained empty during the relatively short time period in question. The parameter estimation algorithm cannot see that the concentration curves might point straight downwards after a little more time. Another alternative is that the parameter estimation results in very low model parameter values so that the discharge term in the ODE becomes largely dominating. The outcome of this is then that the model’s concentration curves at each height level approaches the feed concentration as material keeps getting pumped out. This will also result in an immediate downhill curve for the underflow concentration.

The model calibration is the part where the starting guess of the initial concentration profile impacts the outcome the most. Later in the control scenario simulations, it is possible to let the model reach a steady state (after starting from some given initial conditions) before the controllers are switched on. For the model calibration this option as such doesn’t exist, because the steady state reached would still need to have an underflow mass percentage that coincides with the initial value in the data series used for model validation as well as some model parameters that yield that steady state. In other words, accurate model parameters would be needed before they are acquired. The conclusion from this is that the final estimated parameter values will always depend to some extent on the starting configuration.

4.2 Calibration result

Finally, a chosen parameter configuration was picked for the simulation of control scenarios. This is shown in Table 3.

The model parameter values listed in table 3 provided a good fit to the data series obtained when
Table 3: Table showing the final model parameters. The parameters listed were used in simulations of control strategies. As a conclusion to the model calibration, the compression function was left out in the simulations. (The value of $\gamma$ is irrelevant if $\alpha$ is zero, as the whole compression term will then be zero, and has therefore been left without any specific value in the table.)

<table>
<thead>
<tr>
<th>model parameter</th>
<th>parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_E$ (sett. fun.)</td>
<td>50</td>
</tr>
<tr>
<td>$p$ (sett. fun.)</td>
<td>1</td>
</tr>
<tr>
<td>$a_1$ (disp. fun.)</td>
<td>0.1</td>
</tr>
<tr>
<td>$a_2$ (disp. fun.)</td>
<td>230</td>
</tr>
<tr>
<td>$\alpha$ (compr. fun.)</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma$ (compr. fun.)</td>
<td>-</td>
</tr>
</tbody>
</table>

doing the $\dot{V}_{floc}$ step test (see figure 27), and a somewhat good fit to the data obtained when doing the $\dot{V}_u$ step test. There is one thing worth pointing out about both of these fits, however. The flocculation itself was actually not taken into account when the fit to the $\dot{V}_{floc}$-test was made. There was an unexplained disturbance occurring during the test period of the $\dot{V}_u$-test. Towards the end of the data sequence, the underflow solids mass fraction takes an upturn instead of reaching a steady state even though the underflow rate is kept constant. This final upturn is actually larger than what is shown in figure 26 (at $t = 3 \cdot 10^4$ s), but the data sequence was cut there so that the non-explainable upturn wouldn’t influence the parameter calibration process. That said, there is uncertainty in that data sequence and thus the fit given by the chosen parameters can still be considered relatively reliable even though the fit is far from perfect. The listed parameters were chosen considering both curve fits. The rejection of the compression term, evident due to the choice of $\alpha = 0$, is also a matter of simplicity when analyzing the model and the simulation results.
5 Control of a clarifier-thickener

This chapter presents the present-day control scheme used to control real CTs at Boliden as well as different slightly more complicated theoretical suggestions for more advanced control. In section 8 later on there will be evaluations of these control strategies, or if some control strategy lacks evaluation it will be a subject for further study.

The controllers used at present for controlling the lead (Pb) thickener that has been looked upon in this work are all PI controllers (i.e. a proportional-integral controllers) that output a value that states how much the control variable \( u \) should change from some reference value in order to satisfy the setpoint.

The purpose of the Dymola model is to be used in simulations of the different control schemes. It should be said that the discretization of the model affects the stability of the CT model. The simulated time it takes to reach steady states varies with the number \( N \), and while the system would rather quickly reach steady-state with a high \( N \), the system might never reach a steady state for a low \( N \). So does the number of states vary with \( N \) too, because each of the \( N + 4 \) elements of the mass fraction/concentration vector will be a state of its own (because the states of the system are the concentration profile, i.e. the unknowns of the ODE).

5.1 Control using PI controllers

The very most basic control scheme for controlling a CT is to use one single PI-controller. This gives the simple control law

\[
\dot{V}_u(t) = K \left( \phi_u(t) - \phi_{u,sp} \right) + \frac{1}{T_i} \int_0^t \left( \phi_u(t) - \phi_{u,sp} \right) dt + \dot{V}_{u,ref},
\]

where \( K \), and \( T_i \) are constants as tabled above, \( \phi_{u,sp} \) is the \( \phi_u \)-setpoint and \( \dot{V}_{u,ref} \) is the reference value for the underflow rate (i.e. the value to which the traditional PI operator’s output value is compared). The thickening process is a very slow one, and the time constant for this simple scheme would quite naturally be big. It is more convenient to use a control scheme based on cascade control [17].

5.1.1 Cascade control

In any kind of control it is good to have as much information as possible about the values of important variables. In the case of a thickeners, there are several quantities that can be measured. As has been mentioned in earlier sections, these are for example rake torque, bed level, cone pressure, volume flow rates in and out, as well as solid weight percentages in the flows. Therefore it is a good idea to consider control strategies that make use of extra measurements. One such control strategy is cascade control. In this control strategy a secondary controlled variable is introduced into the control loop in the form of an inner, secondary loop. The primary controller that would otherwise have given the control signal to the system will instead give a setpoint signal to a secondary controller used to control the secondary variable by altering the main control variable \( \dot{V}_u \), and the secondary variable is then in turn used to control the primary controlled variable \( \phi_u \).

The secondary variable should be such that it is affected by some disturbance that causes problems in the control of \( y \) (and can’t be easily handled by just the primary controller). The feed flow rate \( \dot{V}_f \) can be such a disturbance. The secondary variable should also be directly affected by the variable that is primarily used to control the CT. For thickeners this can be taken for granted, because everything inside depends on the underflow rate either directly or indirectly [18].

In cascade control it is important that the dynamics of the secondary variable should not be slower than the dynamics of the primary output variable. Otherwise the control scheme might slow control performance down instead of improving it. This is something that must be evaluated from case to case. In a wide thickener the bed height will be a slowly responding internal variable. If the cross-section area is smaller, the bed height will respond faster. Also, for example, the rake is situated a few centimeters above the bottom of the CT, but it does not necessarily mean that the rake torque responds faster than the underflow composition.
One positive advantage with using cascade control for the control of thickeners is that then there is always at least one extra variable that the control scheme will be keeping within bounds. In operation of a thickener there is always a risk that extra variables start to deviate too much and drift off from allowed value intervals so that the process gets ruined or the thickener somehow harmed. To keep all the relevant variables within allowed bounds at the same time, while still ensuring optimal control of the underflow solids percentage, is something that many are striving to achieve when studying thickener control strategies [8, 9, 19, 20]. Keeping the overflow solids percentage as low as possible is an example of keeping a secondary variable within bounds.

5.2 Current control configurations in Garpenberg

Here’s a table over the currently available controllers for controlling the same lead thickener on which step tests were carried out during the testing at Garpenberg.

<table>
<thead>
<tr>
<th>Name</th>
<th>direct/reverse</th>
<th>$K$</th>
<th>$T_i$</th>
<th>input/sp variable</th>
<th>output variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1</td>
<td>direct</td>
<td>1</td>
<td>200 s</td>
<td>cone pressure</td>
<td>underflow rate</td>
</tr>
<tr>
<td>AC2</td>
<td>reverse</td>
<td>0.05</td>
<td>400 s</td>
<td>underflow weight-%</td>
<td>cone pressure</td>
</tr>
<tr>
<td>MC2</td>
<td>direct</td>
<td>1</td>
<td>1500 s</td>
<td>rake torque</td>
<td>underflow rate</td>
</tr>
<tr>
<td>AC3</td>
<td>reverse</td>
<td>1</td>
<td>500 s</td>
<td>underflow weight-%</td>
<td>rake torque</td>
</tr>
<tr>
<td>LC1</td>
<td>reverse</td>
<td>0.5</td>
<td>15 s</td>
<td>bed height</td>
<td>underflow rate</td>
</tr>
<tr>
<td>AC1</td>
<td>reverse</td>
<td>0.5</td>
<td>1500 s</td>
<td>underflow weight-%</td>
<td>bed height</td>
</tr>
</tbody>
</table>

Table 4: Table of PI controller parameters of the controllers that are actually in use for controlling the same thickener for which the model was calibrated.

The six controllers in the table make up three distinct pairs of controllers used for cascade control of the CT. As such, one (or none, if the CT is operated in manual mode) of the variables bed height $h_B$, cone pressure $p_C$ or rake torque $\tau_R$ is used as an extra measurement (in addition to $\phi_u$) to allow for cascade control. The main control variable is always $\dot{V}_u$ in all of the three cascade loops, and thus it is straightforward to see which PI controllers form the cascade pairs by looking at the table above (the first two together, the middle two together and finally the last two together). The secondary controller outputs the change in setpoint for the primary controller. This is a very natural choice of control strategy, as $\phi_u$ is always the slowest variable to respond to any changes in the main control inputs (possibly the second slowest after $\phi_e$).

At the facility in Garpenberg, one of the three cascade pairs is always used for controlling the underflow density, unless the underflow rate is kept in manual mode in the case of for example the step tests that were carried out as part of this work. In practice, though, the chosen pair is always the cone pressure pair, i.e. using $p_C$ as the extra measured variable. This is because the two other controller pairs haven’t been tuned after some changes to instrumentation was made fairly recently. The final model will be used to simulate all the three control configurations and determine which one is the best as well as find out if there are other improved control strategies not yet thought of that would perform even better. All six controllers in the table above were included in the Dymola model as building blocks to choose from when
testing different control strategies.

It should be stated that the currently used cascade control strategy used with the lead thickener at Garpenberg is not a bad one. This is in the sense that the currently used scheme with cone pressure as a secondary variable does keep the underflow solids fraction stable, following its setpoint. In that sense, it is not an urgent matter at the time of this project to find a replacement for the existing control strategy, but there are many ways one could start to think of how to improve the control of the CT from here.

5.2.1 Feed-forward control

As information about the feed is already available well in advance before the feed enters the CT (\(\dot{V}_f\) is already used to calculate the amount of flocculant to be added and \(\phi_f\) is measured already in the flotation step), the use of feed-forward information could be considered. One could for example think that if \(\dot{V}_f\) experiences a step change due to some occurrence in the flotation process, then that change could be compensated by a step change in \(\dot{V}_u\) without having to wait for the dynamics in the CT, or by a step change in the addition of flocculant [18]. The use of feed-forward information about the disturbances could be combined with any control strategy.

![Diagram](image)

Figure 29: Block diagram illustrating the principle of taking advantage of measurable disturbances in a feed-forward manner when controlling a process by using for example feed-back control. (Source: [18])

5.3 Current control configurations at other sites

Boliden is not limited to the mineral processing plant at Garpenberg, and neither is the control of CTs. The control strategies used in dewatering varies from site to site within the company. Here follows a short walkthrough of how the control of thickeners is done at other sites.

5.3.1 Aitik

The dewatering control at the site in Aitik does not make use of cascade control or any further advanced control strategy. There the control of thickeners is done using one single PID that takes the underflow solids percentage deviation as an input (and outputs the underflow rate).

5.3.2 Boliden

Cascade control has been implemented at the Boliden site. There the rake torque can be used as a secondary variable. In other words, there aren’t as many control options as in Garpenberg, but the control scheme is not as basic as the one in Aitik.

5.3.3 Kevitsa

At the mineral processing plant in Kevitsa, a kind of model-predictive control (MPC) implementation has been made for the control of thickeners. The process chart below shows the main variables of the
model-prediction system. The Kevitsa mine and the associated mineral processing plant are so new in Boliden that no data on the control performance of this implementation was available for this thesis project. In fact, the Kevitsa site was included into Boliden during the time period when this work was carried out.

![Figure 30: Simple process chart for thickeners at the Kevitsa process plant. Notice that the overflow turbidity is measured and can be used as an input to the control scheme. (Source: Outotec)](image)

In the process chart it is not indicated that the flocculant flow rate depends on the feed flow, which is the case in Garpenberg. Thus the flocculant flow rate can be regarded as an independent control variable, and thus the control scheme as a MIMO scheme.

### 5.4 Alternative control schemes

The subsections below present some control schemes that have been proposed in literature for controlling CTs. Each of the control suggestions is purely theory and not based on already acquired data from actual implementations.

#### 5.4.1 Betancourt’s control schemes

A reasonable control strategy candidate is Betancourt’s mass balance based controller which takes a set-point for \( \phi_u \) (or alternatively the mass concentration of solids \( \phi_u \)) and controls the total mass of solids in the thickener with the underflow rate so that the underflow concentration follows the given setpoint. The disadvantage here is that the control requires the computation of the steady-state total mass of solids and underflow rate under some assumption for the overflow variables (such as \( \phi_e = 0 \)).

Betancourt makes three control scheme proposals in one, each of the three being pretty much the same thing except that they are three difficulty levels of the same thing. The levels 1 and 2 can, in theory, be implemented to improve the current control schemes using cascade control for the control of the thickener. The third level is a bit more advanced than that, and goes beyond the current control schemes.

Betancourt’s proposed controller (level 2) is shown in the block diagram below. The controller takes feed-forward information about the disturbances \( \dot{V}_f \) and \( \phi_f \) and adds it to the PI output, yielding a \( \dot{V}_u \)-output that steers the total mass in the thickener towards the given setpoint \( m^{sp} \). There could be another controller that controls underflow solids percentage through the total mass setpoint, thus working as a cascade control pair similarly as the primary controllers in the table above.

Betancourt’s equations for the controller are as follows. First, here is the equation for the PI strategy that doesn’t include the feed-forward information (level 1). Note from the block diagram that the PI controller is non-linear:

\[
\dot{V}_u(t) = K_p(t)(m(t) - m^{sp}) + K_i(t) \int_0^t (m(t) - m^{sp})dt,
\] (49)
where the time dependence of \( K_p(t) \) and \( K_i(t) \) depend on \( \phi_u(t) \). Note that these two time-dependent coefficients aren’t the usual PI constants \( K \) and \( K/T_i \), but rather depend on them and other variables according to the expressions

\[
K_p(t) = \frac{K}{\rho_{\text{solid}} \phi_u(t)}, \\
K_i(t) = \frac{K}{T_i} \frac{1}{\rho_{\text{solid}} \phi_u(t)}.
\]

In other words, the controller is linear for a control variable \( u(t) = \rho_{\text{solid}} \phi_u(t) \dot{V}_u(t) \). When the feed-forward term is included, the control law for the controller in the block diagram becomes

\[
u(t) = \rho_{\text{solid}} \dot{V}_f(t) \phi_f(t) + K(m(t) - m^{\text{sp}}) + \frac{K}{T_i} \int_0^t (m(t) - m^{\text{sp}}) dt \tag{50}
\]

for the same \( u(t) \). Why the measured disturbances are added in this way is a result from the steady-state mass balance, which states \( \dot{V}_f \phi_f = \dot{V}_u \phi_u \) for the special case when \( \phi_e \) is always equal to zero. For a primary CT from which the effluent will continue to a secondary CT, this is obviously not the case, and so the control law would have to be modified for such a CT.

A block diagram for Betancourt’s proposed scenario (level 3) is shown in figure 32. Note how the scheme is very similar to a cascade control scheme if one just changes the steady-state estimation block (SS) into a PI controller with no disturbance feed-forward. The idea behind this controller relies on the theory that for a certain region in the space of the variables \( \phi_f, \dot{V}_f, \phi_u^{\text{sp}} \) there exists a reachable steady state with corresponding underflow rate \( \dot{V}_u^{\text{ss}} \) and \( m^{\text{ss}} \) (given \( \phi_e \), or assumed that \( \phi_e = 0 \)).

The key block in the controller is the SS-block in which the steady-state variables are computed numerically. At this point any details of this computation are left out, but the point is that enough has to be known about the real process in order for the right corresponding steady-state values to be obtained. Here we are already touching the domain of model-predictive control (MPC), which will be the next control strategy to cover.

While Betancourt focuses on the total mass in the CT as the key variable in his controller, the total
mass could naturally be replaced by other related quantities such as rake torque $\tau_R$, bed height $h_B$ or cone pressure $p_C$. Just like for the total mass, there are steady-state values for these variables as well corresponding to steady states given by $\phi_{sp}$, $\dot{V}_f$ and $\phi_f$.

### 5.4.2 Model-predictive control

As this work revolves around building a simulation model as well as control of a thickener, it is clear that something has to be said also about model-predictive control of a thickener. This control scheme is conventionally written down as an optimization problem:

$$
\min \int_{t_k}^{t_k+\Delta k} \left( W_x (\tilde{x}(t) - x_{ss})^2 + W_u (u(t) - u_{ss})^2 \right) dt, \tag{51}
$$

where the state equation has the form

$$
\dot{x}(t) = f(\tilde{x}(t)) + g(x(t))u(t) \tag{52}
$$

(compare with the model ODE), where $\tilde{x}$ denotes prediction of $x$, where $\Delta k$ is called prediction horizon, where $W_x$ and $W_u$ are weights and where the $u$-function that minimizes the optimization problem should be in the set of piece-wise continuous function (with the sampling time that is the time difference between two sampling instances $t_k$ and $t_{k+1}$). The steady-state input $u_{ss}$ corresponds to the steady state $x_{ss}$. In the context of the CT model, it is clear from the above equation that the state vector $x$ is the same as the vector $\phi$ in the model. As there is no way to measure each $\phi_j$ at all the depths in the CT model, these values have to come from somewhere else (e.g. a state estimator such as a Kalman filter) in order for MPC to be used in this way. \[17, 20\]

### 5.4.3 Segovia’s regulator

Segovia has proposed the following control scheme for controlling a CT [19]:

![Figure 33: An ideal control scheme for a CT where two outputs could be controlled using two inputs, one of which is flocculant flow rate [19].](image)

From the previous discussion about Betancourt’s $K_{floc}$, it is evident that this control scheme suggested by Segovia is nothing but trivial to implement (without any known $K_{floc} = f(\dot{V}_{floc}, d_p)$). Segovia realizes this, and proposes another scheme based on a fuzzy controller using fuzzy rules to compute the output. A simple case of such a fuzzy controller could be implemented using two PI-controllers and a switch. Other authors too have suggested similar more or less intelligent rule-based control of the thickening process [8, 9, 21, 22].
5.4.4 Simple rule-based control

Xu et al. have suggested a simple rule-based control scheme that is built on PI controllers, but where the tuned constants vary depending on a set of rules which depend on the measured states of the thickener [8]. In this particular case, the states also include the bed level and the rake torque, both of them measured. The authors also assume that the overflow concentration of solids $\phi_e$ is a measured output, so in total it makes four output variables all to be controlled in a rule-based fashion.

For each of the four system outputs, define $e_i$ and $\Delta e_i$, $i = 1, 2, 3, 4$. The variable $e_i$ is the deviation of output $y_i$ (i.e. of $\phi_u$, $\phi_e$, $h_B$ or $\tau_R$) from its setpoint value (or rather, from the steady-state value corresponding to the setpoint of $\phi_u$). The other variable, $\Delta e_i$, is the difference between two subsequent most recent measurements of $e_i$. It goes without saying that all measurements are sampled, but also the control scheme will only read measurement values at the times between the rule-checks. In that sense it works a bit like a processor in a computer. The rule-based controller will receive the values of $e_i$ and $\Delta e_i$. From $e_i$ it will learn whether the corresponding variable is less than, equal to or greater than the given setpoint. From $\Delta e_i$ it will learn whether the variable is steady, increasing or decreasing. There are nine possible combinations of this information; if $y_i$ is steadily at the setpoint the controller does not have to do anything, but in the eight other cases it will have to do something. The control output, i.e. the change in underflow rate and possibly flocculant rate, will be determined by the controller based on a set of rules defining what the output should be given $e_1$, $e_2$, $e_3$, $e_4$, $\Delta e_1$, $\Delta e_2$, $\Delta e_3$ and $\Delta e_4$ (or, in practice, what the gain and time constant of the underlying PI controller(s) should be) [8].
6 Simulation of control scenarios

For the simulation of control scenarios the true advantage of using Dymola and the Modelica language comes into play. A "CT exterior" model where the CT model was included, together with everything else relevant, was made, i.e. a Dymola model containing sources of input signals, controllers, etc. This model, describing the entire system, was then simulated for different controllers and control schemes. The model is shown in figure 36 below.

![CT exterior model](image)

Figure 36: The model $CT_{exterior}$, including defined inputs and control.

The biggest difference between simulation are in the controller block at the bottom of the diagram view of the CT exterior model. The other difference lies in which output from the multisensor block (lower right) is connected to the controller block and which are not. The $Disturbances$ block’s four outputs are constants or step inputs corresponding to $\dot{V}_f$, $\phi_f$, $K_{floc}$ and $d_{pmean}$. The block right under the disturbance block is the rake height input (which in practice also is a constant or step input). The upper left source block ($SP$) is the underflow mass fraction setpoint input. The boolean step is included in the model to make it possible to switch on the controller at a later time than $t = 0$ of the simulation. The $Ref$ block will be explained below in section 6.2.3.

6.1 Control scenario models in Dymola

Here follows a short walkthrough of the controller models that have been used to produce the simulation results in section 7. The simulations include control of $\phi_u$ through $\dot{V}_u$ with one PI controller (like at the Aitik site), with two PI controllers in cascade using rake torque, cone pressure and bed level as extra measurements (like at the Garpenberg site) as well as using the total mass of solids in the CT as an extra measurement, and finally the impact of derivative action is checked for each case by replacing the PI controllers with PID controllers. Additionally more advanced control schemes were simulated as well.

6.2 Simple controller models

The simple controller models correspond to PI and PID controllers, individually and in cascade pairs. The submodels for existing controllers are based on documentation for those controllers.
6.2.1 PI and PID

Figure 37 below shows the basic model blocks for a PI and a PID controller respectively. Both models are by default available in Dymola’s model libraries.

Also, alternative models were built where the output was instead given as a change in percent that should be applied to the control variable. In these models, the original models were completed with an addition block and a source block with the constant output $y = 1$ to which the PI output could be added, resulting in a percentage change output (providing that the PI gain was chosen so that the final output would not go below zero).

Setpoint weighing was not considered in this work. The existing controllers do support setpoint weighing according to $e(t) = w_{sp} \cdot y^{sp} - y(t)$, but it is not used. Implementing setpoint weighing in Dymola is trivial.

6.2.2 Cascade control

When pairing up two controllers like the ones seen above into cascade pairs, the model looks like the one shown in figure 38 below. Both PI controllers take the deviation in its respective controllable variable as inputs and their outputs are added to reference values for the respective control variables. This model was used for rake torque $\tau_R$, bed level $h_B$, cone pressure $p_C$ and total mass of solids $m_{solid}$ as secondary control variables to choose between.

The first input DefaultVu is for the reference underflow rate $\dot{V}_u^{ref}$. It is called the default $\dot{V}_u$ because in this controller model it also serves as the output underflow rate should the controller be switched off. This in turn depends on the boolean input. If the signal boolean is true, the controller is switched off. When the signal changes to false, the controller is switched on. Until this happens, the deviation inputs $e(t) = y^{sp} - y(t)$ to the two PI controllers are constant and zero. The second and fourth inputs are for the primary and secondary controllable variables. The measured value of the secondary variable is also the reference value for the primary PI controller, so there is no pre-recorded steady-state value for the secondary variable that would correspond to the primary variable’s setpoint used in this model. However, the computation of $\dot{V}_u^{ref}$ takes the underflow mass fraction setpoint $\phi_u^{sp}$ into the calculation. See the section right below for details. The final input is for $\phi_u^{sp}$.

6.2.3 Feed-forward block for reference value

The common way to write the expression for a PI controller is

$$U(s) = K(1 + \frac{1}{Ts}) (Y_{sp} - Y(s)) + U_{ref},$$

where $U_{ref}$ is some reference value of the control variable to which the PI output $G(s)(Y_{sp} - Y(s))$ is compared. As can be deduced from the block diagrams in the Dymola models for the controllers shown above, this is the case even in this work. For the control variable $\dot{V}_u(t)$ there exists some reference value $\dot{V}_u^{ref}$. By using feed-forward information from the disturbances $\dot{V}_f$ and $\dot{\phi}_f$, this reference value can be cleverly chosen.
Figure 38: The version of the controller block model where two controllers are connected as a cascade pair.

In figure 39 it can be seen that the $\textit{Ref}$ block multiplies the measured values of the overflow rate $\dot{V}_e$ and the overflow percentage of solids $\phi_e$ with each other as well as the feed flow rate $\dot{V}_f$ and the feed solids percentage $\phi_f$ with each other. Their difference is then divided by the setpoint value for the underflow solids percentage. The result of this calculation is roughly equal to the steady-state value $\dot{V}_u^{\text{ss}}$ that corresponds to the given setpoint $\phi_u^{\text{sp}}$ for the current values of $\dot{V}_f(t)$ and $\phi_f(t)$, considering that there are almost no solids exiting the CT model from above (and the $\phi_e\dot{V}_e$ term in the mass balance therefore is negligible). The equation that the $\textit{Ref}$ block represents is the mass balance of the thickener.

The output of the $\textit{Ref}$ block is used in the controller block as the reference value for $\dot{V}_u$ because it is a smart choice of a reference value, but there is also a more practical reason for using this particular reference value in the simulations instead of using the current measured value of $\dot{V}_u$, as for the secondary variable. Dymola will not accept a model where the measurement value of the control variable is used to compute the control variable itself, but rather conclude that the model has one unknown variable too much compared to the number of equations. This is the case even when the controller is switched on at later simulation time than at $t = 0$.

A practical comment for the use of the $\textit{Ref}$ block at sites where the overflow turbidity isn’t measured: In cases where the overflow solids fraction is low and close to zero, the term $\dot{V}_e\phi_e$ in the mass balance can be considered negligible. If the overflow quantities aren’t measured, the $\phi_e$ input to the $\textit{Ref}$ block can simply be set to zero, and the block will instead calculate the steady-state value $\dot{V}_u^{\text{ss}}$ corresponding to the ideal case where $\phi_e$ would be zero anyway. It is a just as good reference value for the control of the thickener. From a modelling perspective it is true that the CT model does not let much solids through its clarification zone if the settling velocity is such that the settling is efficient enough. This will reduce the overflow turbidity given by the model compared to the real case. The expression of the settling velocity contains the mean particle diameter of the solid particles as a parameter. However, the particles that escape a CT form above are likely smaller in diameter than the average solid particle entering with the feed. Not only are the smaller particles more likely to escape flocculation in the feed well, but they also
fall slower. The model only sees solid particles that are all of the same size (mean particle diameter variation is included as a disturbance).

6.3 Dymola models for switching control

Implementing switching control models for simulations was attempted based on the idea that the control mechanism should be able to deduce which cascade loop out of more than one available option it should activate in order to get the best output. The switching rules that were implemented follow the reasoning of Xu et al. The corresponding Dymola model for the controller block is shown in figure 40.

This controller block goes beyond the usual one in the sense that it takes three secondary variables as inputs. Thus there will be three connections going from the secondary outputs’ block to the controller block instead of just one. The block in turn consists of four submodels, where one is essentially a model containing three separate cascade pairs of PI controllers (instead of just one, as is the usual case). This model, named **ControllersSW** in figure 40, has six additional outputs, which are the inputs/conditions to the switching rules as defined by Xu. The relations to the setpoints and the derivatives of the secondary signals are computed in the **ControllersSW** block, and the inputs to the **Rule** block are the signs corresponding to all these signals. Based on whether the secondary variables are above or below setpoint, or whether they are increasing or decreasing, the **Rule** block outputs the switching signals, i.e. values that indicate which cascade controller pair should be switched on. This pair’s output signal is then the whole controller block’s control output. The switching itself happens in the **Switch** block in the upper right of figure 44.

Even here the option to switch the entire control block on or off has been implemented. The **On/Off Switch** block takes the incoming boolean value as input, and it also takes the **Rule** block’s outputs as its inputs. In this way, the switching on and off inside the **ControllersSW** block can be managed.
6.4 Model-predictive control

An attempt to simulate control using an MPC strategy was made using Matlab. The program consists of a chain of calling Matlab functions, in which the *ode45*-solver is used to solve the model ordinary differential equation in which all model inputs are known except $\dot{V}_u$, which is in turn returned by the *fminsearch*-solver when applied to a copy of the original ODE function but with a shorter time span. The cost function given to the minimiser is in principle

$$J(\phi_u) = \sum_{t=0}^{N_{PH}} (\dot{\phi}_u^p - \phi_u(t))^2,$$

except that the times $t \in [0, N_{PH}]$ are not linearly spaced but whatever times the ODE solver outputs $\phi_u$-data for when solving the model problem for times from zero to the prediction horizon $N_{PH}$. This is of course the pretty much easiest imaginable MPC cost function for control of the thickener. A proper advanced MPC that takes into account more states and/or even includes internal variables as states could be implemented without much effort. However, Matlab requires much time to compute the MPC strategy even with the given cost function, so that becomes a trade-off matter.
7  Simulation results

The simulation results are discussed here in the same order as the underlying models were presented in the previous section. It should be said that the main focus was put on the simulation performance of the CT model and on comparing the different cascade scenarios. The MPC implementation, which was implemented in Matlab and not in Dymola, did not work well with the Matlab version of the CT model (i.e. the same Matlab code that was used for parameter estimations, see section 4). The simulations in Dymola for the switching control case didn’t yield results with any advantage over cascade control simulation results either.

7.1 Model performance

The model performs rather well during simulations, but the performance depends much on the discretization in space. In general, higher resolution always gives more accurate simulation results while the simulations then require more time. In the case of the CT model, the inaccuracy for low resolutions become very evident in simulations. The whole simulated process is much slower in simulated time if the resolution is low compared to higher resolutions (i.e. higher values of \( N \)). That is all a result of the inaccuracy due to bad discretization. For higher resolutions, more precisely for \( N \) on the order of a couple of thousands, the simulated time it takes for the system to reach a steady state converges to a certain value. Figure 41 clearly shows the problematics around a too small choice of \( N \).

![Figure 41: Simulation of cascade control with rake torque used as the extra variable. In both graphs the same situation is simulated, except that the time constants \( T_i \) in the PI controllers used for cascade control are 10 times higher in the left figure compared to the right figure. The values used in the right figure are used in existing process control. It is seen that the control is unstable for the default values and stable for the higher values. This is to illustrate the impact of the model being slower than the actual process. At time 0, the simulation is initialized from a starting guess that yields a steady state. At time \( 1.5 \times 10^6 \) s, the controllers are switched on. At time \( 4 \times 10^6 \) s, a \( \phi_u \) setpoint change occurs. At time \( 5 \times 10^6 \) s, a \( \dot{V}_f \) step occurs and, at time \( 6 \times 10^6 \) s, a \( \phi_f \) step change occurs.](image)

The CT model is much more sensitive to setpoint changes than to disturbance step changes. This is something that is to be expected; disturbances happen all the time but the thickeners are usually dimensioned for certain underflow solids fractions (which also depends on high concentrations much the process plant equipment, such as pumps, can handle) and setpoint changes are therefore hardly ever done. It is also clear that the model is much more sensitive for setpoint changes downwards (i.e. from a higher underflow solids fraction to a lower one) than for setpoint changes upwards (i.e. the other way around). The control of the model would always alternate between a maximum rate and a minimum rate for pumping out material from the CT model in the cases when the setpoint change was too high for to be handled in the system, resulting in diverging control behaviour. The other extreme case was when the setpoint change was from a lower concentration of solids to a higher one and when the control reached a steady state where the underflow solids percentage converged to a value that was between the new setpoint value and the old value.

The drawback for having a model that was sensitive towards setpoint changes was that the main difference between control schemes was seen in how they handled those step changes, rather than the disturbance...
step changes, which were much easier for the controllers to handle. Also, every simulation would have to contain at least one setpoint challenge, namely reaching an initial setpoint from the initial staring configuration of the CT model.

As for sensitivity for parameter changes, the hindered settling exponent $r_E$ and the dispersion horizon parameter $a_2$ are most critical. Considered here are the parameters with the following values used for simulations; $r_E = 50$, $a_1 = 0.1$, $a_2 = 230$, $\alpha = 0$. Simulations showed that a steady-state with a desired underflow solids mass fraction of 0.72 could be reach when the values were kept within limits shown in table 5, when using one of the cascade control strategies presented in the previous section. For parameter values outside of these intervals the system was not controllable with that strategy. The table is to give a hint about what parameters the CT model is the most sensitive to.

<table>
<thead>
<tr>
<th>parameter</th>
<th>lower limit</th>
<th>upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_E$</td>
<td>40</td>
<td>52</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>170</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Table 5: Parameter sensitivities of the three most important model parameters.

While the CT model is most sensitive to changes in the exponent parameter $r_E$, the model can tolerate ever higher values of $a_1$ and $a_2$ up to infinity. In practice, the dispersion horizon can only be at most the depth of the CT, so there is no practical meaning in an dispersion horizon parameter with the value infinity. Even though the system doesn’t become unstable when $a_1$ is raised to high values, it will still yield non-physical behaviour in the process in the form of overly oscillating concentration curves (which makes sense, as $a_1$ is a parameter related to mixing of material due to turbulence).

### 7.2 Simple control scenarios

The following graphs show simulation results of the simplest control scenario simulation.
Figure 42: Simulation of PID control of a CT. In all graphs the same situation is simulated. At time 0, the simulation is initialized from a starting guess that yields a steady state. The time between step changes is longer in the first graph in order to clearly show that stable steady states are reached (quite fast) after each step change. At time $2 \times 10^4$ s, the controllers are switched on. At time $4 \times 10^4$ s, a $\phi_u$ setpoint change occurs. At time $5 \times 10^4$ s, a $V_f$ step occurs and, at time $6 \times 10^4$ s, a $\phi_f$ step change occurs.

Values for the PID controller: $K = -0.05$, $T_i = 400$ s, $T_d = 100$ s. Parameter values: $r_E = 50$, $a_1 = 0.1$, $a_2 = 230$, $\alpha = 0$.

### 7.3 Cascade control scenarios

The cascade pairs can be compared in the following figures 43 - 46.
Figure 43: Simulation of cascade control using cone pressure. In all graphs the same situation is simulated, except that the time between input step changes has been reduced in the second and third graph. At time 0, the simulation is initialized from a starting guess that yields a steady state. The time between step changes is longer in the first graph in order to clearly show that stable steady states are reached (quite fast) after each step change. At time $1.5 \cdot 10^6$ s, the controllers are switched on. At time $4 \cdot 10^6$ s, a $\phi_u$ setpoint change occurs. At time $5 \cdot 10^6$ s, a $V_f$ step occurs and, at time $6 \cdot 10^6$ s, a $\phi_f$ step change occurs. In the second and third graph the last two step changes occur instead at times $4.1 \cdot 10^6$ s and $4.2 \cdot 10^6$ s, respectively. Values for primary PI controller: $K = 0.05$, $T_i = 400$ s. Values for secondary controller: $K = -1$, $T_i = 200$ s. Parameter values: $r_E = 50$, $a_1 = 0.1$, $a_2 = 230$, $\alpha = 0$. 
Figure 44: Simulation of cascade control using rake torque. In all graphs the same situation is simulated. At time 0, the simulation is initialized from a starting guess that yields a steady state. The time between step changes was chosen long enough to demonstrate how stable steady states are reached (quite fast) after each step change. At time $1.5 \cdot 10^6$ s, the controllers are switched on. At time $4 \cdot 10^6$ s, a $\phi_u$ setpoint change occurs. At time $4.5 \cdot 10^6$ s, a $V_f$ step occurs and, at time $5 \cdot 10^6$ s, a $\phi_f$ step change occurs. Values for primary PI controller: $K = 0.05$, $T_i = 400$ s. Values for secondary controller: $K = -1$, $T_i = 200$ s. Parameter values: $r_E = 50$, $a_1 = 0.1$, $a_2 = 230$, $\alpha = 0$. 
Figure 45: Simulation of cascade control using total mass of solids. In all graphs the same situation is simulated. At time 0, the simulation is initialized from a starting guess that yields a steady state. The time between step changes was chosen long enough to demonstrate how stable steady states are reached (quite fast) after each step change. At time 1.5 × 10^6 s, the controllers are switched on. At time 4 × 10^6 s, a φ_u setpoint change occurs. At time 4.5 × 10^6 s, a V_f step occurs and, at time 5 × 10^6 s, a φ_f step change occurs. Values for primary PI controller: K = 0.05, T_i = 400 s. Values for secondary controller: K = −1, T_i = 200 s. Parameter values: r_E = 50, a_1 = 0.1, a_2 = 230, α = 0.

Worth noticing is how rapidly the spikes in the lowest height level concentrations rise and re-stabilize when the step changes in V_u and φ_f occur. After all, these spikes are nothing to be expected from a model that’s characterized as slow compared to reality. The observation points at the control scheme rather than at the CT model. These spikes are only present because the control reference V_u^{ref} also experiences a step change, since the computation of it takes feed-forward information from the disturbances. The control scheme is, however, very fast in correcting this mistake as well. For a constant V_u^{ref}, the spikes did not appear. The relatively slow model has enough time to correct for the step changes before any resulting deviation from the setpoint is ever seen in the target variable.
Figure 46: Simulation of cascade control using bed height. In all graphs the same situation is simulated. At time 0, the simulation is initialized from a starting guess that yields a steady state. The time between step changes was chosen long enough to demonstrate how stable steady states are reached (quite fast) after each step change. At time $1.5 \cdot 10^6$ s, the controllers are switched on. At time $4 \cdot 10^6$ s, a $\phi_u$ setpoint change occurs. At time $4.5 \cdot 10^6$ s, a $\dot{V}_f$ step occurs and, at time $5 \cdot 10^6$ s, a $\phi_f$ step change occurs. Values for primary PI controller: $K = 0.05$, $T_i = 400$ s. Values for secondary controller: $K = -1$, $T_i = 200$ s. Parameter values: $r_E = 50$, $a_1 = 0.1$, $a_2 = 230$, $\alpha = 0$.

Settling times after a disturbance step change are listed in the table below for the different control strategies. For each of the four cases, the settling time after a $\dot{V}_f$ step change of $+25\%$ is what is included in the table.

Another metric that can be used for comparing the control performance is the graphical area between the output curve and the setpoint line in figures 42 - 46. The corresponding values for the middle graph of each case are shown in table 7.
<table>
<thead>
<tr>
<th>secondary variable</th>
<th>settling time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_C$</td>
<td>9700 s</td>
</tr>
<tr>
<td>$\tau_R$</td>
<td>3063.4 s</td>
</tr>
<tr>
<td>$h_B$</td>
<td>3060 s</td>
</tr>
<tr>
<td>$m_{solid}^{tot}$</td>
<td>10040 s</td>
</tr>
<tr>
<td>Single PID</td>
<td>4000 s</td>
</tr>
</tbody>
</table>

Table 6: Settling times for the different cascade control options. The settling time in the previous chapter’s single PID case is included for comparison.

<table>
<thead>
<tr>
<th>secondary variable</th>
<th>area between curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_C$</td>
<td>1.213 s</td>
</tr>
<tr>
<td>$\tau_R$</td>
<td>1.120 s</td>
</tr>
<tr>
<td>$h_B$</td>
<td>1.131 s</td>
</tr>
<tr>
<td>$m_{solid}^{tot}$</td>
<td>1.113 s</td>
</tr>
<tr>
<td>Single PID</td>
<td>0.719 s</td>
</tr>
</tbody>
</table>

Table 7: Graphical areas between the output $\phi_u$ curve and the setpoint line for the different cascade control options and the single PID case for simulations where the following step changes have occurred: $V_f$ from 0.014 m$^3$/s to 0.0175 m$^3$/s at time 0 and $\phi_f$ from 2.6 vol-% to 1.9 vol-% at time 10000 s. Note that all the graphs show the mass fraction profile of solids as a function of time, and that this is the reason why the graphical area is given in seconds (actually unitless fraction units times seconds).

### 7.4 Model-predictive control scenarios

This section shows the results from a simulation case where a very simple MPC implementation was tested in Matlab.

First of all, it should be noted that the simulation time is very short here. The Matlab implementation using a combination of the `fminsearch` and `ode45` commands worked very slowly, and thus the model-predictive control was simulated only for a short time period and with only $(\phi_u^{sp} - \phi_u)^2$ to be minimized. The result can be seen in figure 47 below.

![Simulation in Matlab of model-predictive control with the same disturbance inputs that were used in the model calibration process. Parameter values: $r_E = 50$, $a_1 = 0.1$, $a_2 = 230$, $\alpha = 0$, $N = 50$.](image)

What the result actually shows is that while the code of the MPC implementation in itself worked, the performance was rather poor and didn’t give much data for drawing conclusions for the short simulation time that the control was simulated for. In the left graph it can be seen that the control scheme does keep the underflow volume fraction close to the setpoint, but that the volume fraction also deviates quite much already during the short simulation time of 3000 seconds. The left graph shows how the whole concentration profile changes during the simulation, and how the main event in the simulation is the transition from a linear initial concentration profile to a more realistic one. This behaviour in the beginning of a simulation is a property of the CT model itself.
7.5 Other control scenarios

The remaining control strategy is the one with the rule-based switching control. Here the result is actually a very slow simulation whenever the switching control simulation reaches a step change or a disturbance, possibly because of some unknown technical issue in Dymola. The simulation graph shown here shows how the $\phi_u$ and $\phi_{N+1}$ curves jump up and down when the controller switches between cascade loops. The implementation in Dymola didn’t result in simulation results that was any better than an individual cascade strategy’s result, but the scenarios took much longer to simulate.

Figure 48: Simulation of switching control. In this particular graph it is shown how that switching itself works. No responses to disturbances or step changes are shown here.
8 Discussion

The work with the CT model and the control strategies ended up consisting of three main parts - the model, its calibration, and the comparing of the control strategies through simulations. Here the CT model is evaluated followed by a short comment about the parameter estimation part. This section also covers which of the schemes is the best control strategy for controlling a CT (as in optimal based on simulation results).

8.1 Reflecting upon the model

The CT model successfully follows the behaviour of an actual CT. It is a complex implementation that builds on a very general partial differential equation describing a CT. The numerical implementation in Dymola builds on a widely accepted theory (by Bürger et al.) for how to model and simulate a CT. However, even though the theory behind the implementation is widely accepted, it does not provide any proper model equations for the rotating rake’s torque or other variables that depend on the specific design and type of the CT. The CT model and the simulation results ought to be seen in the light of that fact.

The CT model has quite many model parameters that need to be calibrated for the model to make sense. It also has a fairly notable sensitivity to changes in these parameters, and the model performance highly depends on the number of height levels $N$ in the numerical approximation of the model. The main issue with the CT model is that it with the calibrated parameters and low $N$ gives a simulation result where the concentration profile changes with time slower than in the real thickener. It is rather obvious that the correct behaviour of the CT is reached when the model resolution $N$ goes to high values, but it is interesting that a low $N$ creates an effect of lagging behind. For low $N$ the sensitivity of the model is also higher. These are issues to keep in mind when looking at the simulation results, but in particular they affected the process of estimating the model parameters where long simulation (or model evaluation) times were not desired and low $N$ thus a certain advantage.

Last but not least, the CT model is much more sensitive to step changes in the $\phi_u$ setpoint than to changes in the model disturbances. In case of a setpoint change upwards that goes too high, the CT model will reach a steady state that is somewhere on the way between the old setpoint and the new one. In the case of a setpoint change downwards that goes too low, the CT model will go unstable and start alternating between two types of states; one where the CT is drained almost empty of solids and one where it is almost back where it started. On the other hand, the model handles disturbances exceptionally well. This behaviour of the model is to be seen in the light of how a real CT is operated; there is almost never a setpoint change for $\phi_u$, but the disturbances are there all the time. As was stated in section 1 of this text, a CT is dimensioned for a certain output. That includes the setpoint. It seldomly varies, and when it does it’s not shifted by much. Simulations would always include at least one setpoint change when allowing the controller to be switched on in the middle of a simulation, as has been done in the figures shown in section 7.

8.1.1 Parameters and calibration

As was stated in section 4, finding suitable parameter values was one of the more challenging parts of the project. While the toolbox in Dymola for model calibration is very advanced and the Matlab code also was of much help, the data series and the model itself were tricky to use in model calibration process. Issues related to simulation time and model resolution (i.e. the number of height levels $N$) and most notably the starting profile of solids concentrations in the model provided obstacles for the calibration process. Finally, though, the parameter values that were obtained worked very well in simulations. Although the model was sensitive to changes in the model parameters, the model still behaves like the real thickener against which it is calibrated. If the calibration data series would have spanned more time, the impact of the model initial configuration could have been reduced. Perhaps then the impact of the compression term would also have been more clear. As it was in this case, the calibration went well without including any compression effects at all. It would surely have been interesting to see how the compression term could affect simulation results and if the conclusion drawn based on simulation results would have been different.
8.2 Reflecting upon control strategies

One thing to note is that the dewatering process is at the very end of the mineral processing chain of process steps and is therefore very exposed to process disturbances. A disturbance in the feed flow rate $V_f$ or the feed properties (such as the solids fraction $\phi_f$) could be caused by pretty much anything that has occurred in the previous process steps. A disturbance could come from a fault in the flotation process. It could even be due to a rock containing an exceptionally high amount of a certain mineral that enters the grinders. This exceptionally high amount of the mineral will give a peak in the output flow of slurry of this particular mineral from the flotation process, resulting in a peak in $\dot{V}_f$ to the corresponding thickener. The high grade of exposure to disturbances from previous process steps became evident during the step tests in Garpenberg when strange behaviour could occur at surprising times.

This is one of the reasons why it is of interest to consider a combination of the existing cascade control scheme and feedforward control from a measured disturbance as a candidate control strategy. For example, if a rapid change in $\phi_f$ would occur, the control could respond in the secondary controller (the one outputting a secondary variable setpoint change), and in the case of a rapid change in $\dot{V}_f$, the control scheme could answer in the primary controller (the one outputting a $\dot{V}_u$ setpoint change). These are, however, only ideas until the model becomes so good that the feedforward part could be perfectly tuned in simulations. With a poor knowledge of the system an introduction of the feedforward control into the already used feedback loop could mess up the system more than help it control $\phi_u$. However, it was seen during simulation that the Ref block, where feed-forward information was processed, was indeed helpful.

One problem when using the rake torque cascade control pair is that the rake torque is also controlled elsewhere. For proper operation of the thickener it is important that the rake has to be saved from getting stuck, and therefore the control system can raise the rake a level where the concentration of solids is not as high in order to make the torque lighter (providing that the controller that takes care of this is activated). Now, if the control system can lower the rake torque both by altering $\dot{V}_u$ and by raising the rake there is a conflict. This makes the control of $\phi_u$ using the rake torque as an extra measurement slightly more complicated than when the cone pressure is used. Seen the other way around, it is more convenient to keep the rake torque at a suitable level, simply by raising and lowering the rake itself, when the $\tau_R$ is not taking part in a cascade loop for control of $\phi_u$. It should be remembered that a rake stuck in the bed of solids would mean that a long shutdown of a thickener has to be made in order to free the rake and let the process run normally again. Having the rake stuck is harmful for the rake itself, but it is obviously not good from a dewatering aspect either. So one thing that could be thought of is to introduce some more advanced MIMO control with $\dot{V}_u$ and rake height $h_R$ as control variables and $\tau_R$ and $\phi_u$ as the variables to control. However, if the rake mechanism is strong enough and the thickener is dimensioned so that the rake is rather safe from getting harmed at a certain known depth, then case is different. In simulations where the rake height was kept constant the rake torque actually proved to be an excellent secondary variable for cascade control. However, in the CT model in Dymola the rake of course doesn’t know that it can break if the torque gets too high. There was no breaking limit torque added to the rake function. This should be kept in mind when claiming that the rake torque $\tau_R$ was the best secondary variable to be used for cascade control.

---

Figure 49: Segovia’s MIMO scheme for controlling both the underflow solids fraction and the bed height using the underflow and flocculant flow rates as control variables.
As for the bed height, there was the MIMO control suggestion by Segovia that the $h_B$ and $\phi_u$ were controlled using $V_u$ and $V_{floc}$ as control variables [19] (see figure 49). However, there was no clear trend of how $h_B$ would respond to a step in flocculant flow rate when such a step test was made in Garpenberg. This supports the theory that this way of controlling $\phi_u$ while also keeping the bed within boundaries will not work at least for the Pb thickener at the site. Prior to the step tests, there had been recorded process data hinting that the bed level would somewhat linearly follow the flocculant flowrate in the thickener in question, but this behaviour could not be reproduced in the tests. The thickener in question is rather large and wide, so any response in $h_B$ to a change in any of the flow rates would naturally be barely noticeable on short time frames. In a smaller CT the situation could be different and such a MIMO scheme could show a nice performance.

There are sensors measuring the amount of solids in the overflow in the form of overflow turbidity, but in the considered plants this information is not used for the control. It is not for all CTs that one can assume that the overflow solids fraction is negligible, especially not in cases when clarifiers are connected in series. One thing in particular that would be interesting to look at would be the variable $\phi_{diff} = \phi_u - \phi_e$. One could consider using this variable as a controlled variable instead of using $\phi_u$ and see if that has a positive impact on thickener performance.

Figure 50: Dewatering units in series. The upper pair shows two thickeners in series and the lower pair is two clarifiers in series. They are practically the same setup, but the flows from one CT to the other are taken from different outlets.

At this point, it should be mentioned that the flocculant flow rate is currently not a free control variable for the Garpenberg Pb-thickener at all. In fact, it is controlled by a ratio controller that keeps it proportional to the feed flow rate $V_f$. Here one can in principle talk about feed-forward control input based on one of the disturbances.

Just by looking at the settling times (listed in table 6) and comparing the graphs, one can deduce that cascade control using cone pressure or total mass of solids as the secondary variable does not give an advantage over using just one PID controller. This is at least what the simulations with the CT model show. It is interesting that cone pressure falls into the less promising category of secondary variables as the plant actually uses cascade control via cone pressure for the thickener in question. Simulation results give rake torque a much better settling time, even though the curves look rather similar in both cases. It should be said that both the rake torque and the cone pressure were given functions that are natural for an open-tank thickener. In reality, this function for $p_C$ could fail, because the thickener differs from a simple open tank in that the rake is rotating in it, and also the geometry in the placement of the pressure sensor can affect the measurement.

The same reasoning largely goes for cascade control using bed level $h_B$. The settling time obtained
in simulations was very near the settling time of using rake torque $\tau_R$. The most reasonable explanation to this is that the model functions for $\tau_R$ and $h_B$ are related. One is not depending on the other, but both are written in such a way that both control schemes produce similar output (unintentionally). What can be said about the bed is that it is for sure harder to measure than the rake torque, and the bed responds more slowly than the rake if the thickener is wide. In the case of the CT model, it is the concentration profile that responds to changes, and the two variables $\tau_R$ and $h_B$ are functions of the concentration profile. Both are, put simply, essentially a function of the concentration of solids at some depth. As a conclusion of this reasoning, the rake torque was deemed the best choice for cascade control. It should be remembered, however, that the thickener mainly considered in this work is rather wide and that it has a heavy substance inside (lead concentrate). For a narrower thickener with less heavy content another one of the secondary variables could be a better choice.

The other metric, i.e. the graphical area tabled in table 7, turns things around a bit. There it is the single PID case that gives the best result in terms of area between the $\phi_u$-curve and the setpoint line. Of course the optimal would be if the $\phi_u$-curve would just follow the setpoint line in a horizontal manner. The graph area between these two make up a deviation measure of how well the control makes $\phi_u$ follow its setpoint. This indicates that, as far as the CT model is concerned, the alternative to just use a single PID controller for control of a CT can give good results as well (and indeed some plants do use this option, as was stated in section 5.3). As for all the cascade control scenarios, the area deviations are close to the same value in all four cases. Cascade control with rake torque only makes the third place, but still comes before both bed height and cone pressure. A bit surprisingly, the control scenario where the total mass of solids is the secondary variable gives an area deviation that’s smaller than the one with rake torque being the secondary variable, even though the settling time for the total mass of solids case is so much longer.

The MPC and switching control implementations did not provide satisfying simulation results. It could be a thinkable MPC implementation approach to just set the CT model aside and start from the beginning, develop a black-box model for the CT and just use that model in MPC simulations instead. This could be done using for example Matlab’s system identification and model-predictive control toolboxes. However, a black-box model would lack all the information about the inner concentration profile. All the secondary variables would have to be primary black-box model outputs as well, just like $\phi_u$ and $\phi_e$. A black-box model is also a model for just the one specific thickener from which the data that was used to create the model originates. Hence it is much less general than the CT model that was created in Dymola.
9 Conclusion

Based on results from simulations carried out using the CT model, the control scheme that works best for controlling a CT is cascade control with rake torque $\tau_R$ as the secondary measured variable. This conclusion is drawn mainly by comparing settling times of cascade control schemes and concluding that implementations of the more advanced control strategies - model predictive control and intelligent switching control - failed to provide satisfying simulation results using the Dymola model for the CT. A multiple-input multiple-output strategy was ruled out when step tests didn’t show that any secondary variable of the CT clearly respond to the flocculant flowrate $\dot{V}_{floc}$, which was considered a potential second control variable (in addition to the underflow rate $\dot{V}_u$). The choice of cascade control using cone pressure $p_C$ as the secondary variable, which is the current way Boliden’s thickener considered in this work is controlled, proved to also be a sensible choice of strategy.

The CT model was, however, tricky to calibrate, and it’s considerably greater sensitivity to $\phi_u$ set-point changes than to any of the disturbances provided some difficulty in comparing the control schemes. Dymola was a very useful modelling environment to use, especially for simulations, and the CT model with all its functions, all based on acknowledged theory for modelling such a unit, turned out quite advanced.

The work was carried out mainly as a case study of one particular thickener. It is possible that other control strategies are better suited for other thickeners. An advantage with the CT model is that it is very general, and thus it can be used to repeat the simulations for other units in future work.
References


Appendix

The main code of the CT model:

```plaintext
equation
Ds = (u + hu)/E; //Calculating Ds
k = integer(cell(hu/E)); //Calculating k
flock = u/E; // flock equals the input u/E
mean_dp = u_dp * l_unit; // mean_dp equals the input u_dp times 1 meter

// The value of Va is given by the input u_Da. Here Va is given its unit m3/s.
// Va must be within specific bounds. It cannot be smaller than zero and it
// cannot be larger than the feed flow rate (because then the model will not
// work right).
Va = max(min(u_Da,1.0*0.0000001),0.01*5.5655/10000)*V_unit;

// The overflow flow rate should always be greater than or equal to zero. The
// output y_Va is the Va without unit (m3/s).
y_Va = max(u_Va - Va/V_unit,0)*V_unit;
y_Va = y/plot_unit;

// For information about what is going on in the DPhil portion below, please see
// the comments in the function file.

// The discrete PD, i.e. set of ODEs, in a packed format:
Dphi = Functions. firstTerm(Ua/h,Da,E,phi1,phi2,phi3)
+ Functions. secondTerm(Ei,xi,phi1,phi2,phi3);

// Calculating mean density in the CT
rho = Functions. averageDensity(rho*phi1,1:23);

// Calculating core pressure according to the formula p = rho*g*N
pC = rho*9.81*8 + 0.1*101325;

// Calculating bed level
hb = hu - Functions. bedlevel4(1,Functions. burger2Beta(N,k,ha,hu));

// Calculating total mass of solids in the CT
mct = Ds*Arho*sum(phi1);

// Calculating the rate torque as tauR = alphaA * phi[ha] + betaR
tauR = betaR + alphaA * phi[integer(cell((Da+hu/u_DA)*U_unit))/8];

// Packing all non-flow related outputs into one single model output.
multiSensor. rho = rho;
multiSensor. pc = pC;
multiSensor. hb = hb;
multiSensor. mct = mct;
multiSensor. tauR = tauR;

// Converting from volume fractions into mass fractions
phi_m = Functions. massPercentage(phi1,phi2);
phi_m = Functions. massPercentage(phi3,phi4);

initial equation

// Initial concentrations are defined here.
phi1[N+4] = zeros(1); //0*1.0/9.2665
phi2[N+2] = (1:1:2)*0.5765/101325;
```

Figure 51: Showing some of the main model code in Dymola. Compare the line beginning with der (phi) with equation 31.
A list of the CT model’s functions:

The first five functions in figure 52 correspond to the five terms in the main ODE.

Average density refers to the average density in the whole CT. The batch quantity functions \( batch\text{Vel} \) and \( batch\text{Flux} \) are for the hindered settling velocity and the corresponding flux function. There are multiple alternatives for how to compute the bed level, and one for computing cone pressure (see figure 51 for the rake torque expression). The function \( burger\text{Zeta} \) is a \( z \)-coordinate transformation function. The functions \( smallD \) and \( bigD4 \) are for the dispersion function and for the compression function’s primitive function, respectively (the former used in the dispersion term and the latter in the compression term). Dymola did not have any function for finding the maximum and minimum values in vectors element-wise, so these functions were added (\( maxV \) and \( minV \)). The function \( areas \) is for cross-section area. There are some additional variants of the first five functions.