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Letter

On the Action of the Radiation Field Generated by a Traveling-Wave Element and Its Connection to the Time Energy Uncertainty Principle, Elementary Charge and the Fine Structure Constant

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Abstract: Recently, we published two papers in this journal. One of the papers dealt with the action of the radiation fields generated by a traveling-wave element and the other dealt with the momentum transferred by the same radiation fields and their connection to the time energy uncertainty principle. The traveling-wave element is defined as a conductor through which a current pulse propagates with the speed of light in free space from one end of the conductor to the other without attenuation. The goal of this letter is to combine the information provided in these two papers together and make conclusive statements concerning the connection between the energy dissipated by the radiation fields, the time energy uncertainty principle and the elementary charge. As we will show here, the results presented in these two papers, when combined together, show that the time energy uncertainty principle can be applied to the classical radiation emitted by a traveling-wave element and it results in the prediction that the smallest charge associated with the current that can be detected using radiated energy as a vehicle is on the order of the elementary charge. Based on the results, an expression for the fine structure constant is obtained. This is the first time that an order of magnitude estimation of the elementary charge based on electromagnetic radiation fields is obtained. Even though the results obtained in this paper have to be considered as order of magnitude estimations, a strict interpretation of the derived equations shows that the fine structure constant or the elementary charge may change as the size or the age of the universe increases.

Keywords: traveling-wave element; radiation fields; momentum; action; elementary charge; time energy uncertainty principle; fine structure constant; age of the universe

1. Introduction

In Smith [1], a traveling-wave element is defined as follows: it is a thin wire of length l through which a current pulse travels with the speed of light in free space from one end to the other without attenuation. When the current pulse reaches the end of the wire, it is absorbed at the termination without reflection. The traveling-wave element, though with a reduced speed of current propagation, had been utilized by lightning researchers for some time as a vehicle to extract lightning current characteristics from the measured radiation fields [2]. It had also been utilized to study the effect of the speed of propagation of the current pulse on the radiation fields. For example, when the speed of propagation of the current pulse is less than the speed of light, the electric field generated by the traveling-wave element consists of three terms, namely electrostatic, induction and radiation. As the speed of the current pulse approaches the speed of light, the three terms merge into each other, making the total electric field pure radiation [3,4]. Furthermore, the electric fields of the traveling-wave element reduce to dipole fields when the time variations of the current are much longer than the travel time of the current from one end to the other [5]. Indeed, the traveling-wave element is a device that could be used to study features of electromagnetic radiation under different conditions.

In two recent publications, Cooray and Cooray [6,7] (referred to as Paper 1 and Paper 2 from here on) described the features of the radiation field generated by a traveling-wave element including the action and the momentum transported by this field. Another goal of the study was to investigate the features of radiation fields when the charge associated with the current pulse is reduced to its lower limit, namely the elementary charge. In Paper 1 it was shown that the condition $\Delta U \times \tau \geq h/4\pi$ leads to the fact that $q \geq e$ where ΔU is the uncertainty in the radiated energy, τ is the effective duration of the emission of radiation, q is the charge associated with the current pulse in the traveling-wave element, h is the Plank constant and e is the elementary charge. The mathematical condition given above is known in the literature as the time energy uncertainty principle. In Paper 2, by studying the momentum of the radiation, it was shown that the condition $\Delta U \times \tau \geq h/4\pi$ is indeed satisfied by the emitted radiation. The results presented in Paper 1 were based purely on the numerical solution of cumbersome equations for the energy and action pertinent to the radiation fields. However, in Paper 2 these equations were reduced to simple analytical expressions that can be manipulated rather conveniently. The goal of this Letter is to utilize these simple expressions together with the ideas presented in Paper 1 to make a conclusion concerning the connection between the time energy uncertainty principle and the elementary charge.

This Letter is constructed as follows. First, the equations which were derived in Paper 1 and Paper 2 on the energy and action transported by the radiation fields are presented. Second, these equations are utilized to show how the action associated with radiation fields varies as a function of the charge associated with the current and its possible limits. Third, the expression for the action is combined with the theory outlined in Paper 2 to derive an expression for the minimum charge that could be detected by the electromagnetic radiation fields. This section is followed by a discussion and conclusions.

2. Analysis

2.1. Traveling-Wave Element

Expressions for the radiation fields generated by a traveling-wave element and various features associated with those radiation fields are presented in Paper 1 and Paper 2. In these studies, the current waveform associated with the traveling-wave element is represented by a Gaussian current pulse. In the sections to follow, a description of this current waveform and expressions for the energy and action associated with the radiation fields of the traveling-wave element are presented. It is important to mention here that the expressions to be given for the radiation fields of a traveling-wave element are valid when the effective wavelength associated with the time variation of the radiation emission is much longer than the radius of the wire. Moreover, since they are radiation fields, the distance from the center of the element to the point of observation, R, has to be much larger than the length of the traveling-wave element [6,7]. Observe that similar constraints are applied when deriving expressions for the electromagnetic fields generated by dipoles.

2.2. Current Waveform Associated with the Traveling-Wave Element

In Paper 1 and Paper 2, the current associated with the traveling-wave element is assumed to be a Gaussian current pulse. It is represented mathematically by

$$i(t) = i_0 \exp\left(-\frac{t^2}{2\sigma^2}\right) \tag{1}$$

In the above equation, i_0 is a constant and σ is the Gaussian RMS (Root Mean Square) width. The total charge associated with this current pulse is given by

$$q = i_0 \int_{-\infty}^{\infty} e^{-t^2/2\sigma^2} dt \quad (2)$$

After performing the integration, we find $q = i_0\sigma\sqrt{2\pi}$. Observe that one can construct current waveforms of different durations (i.e., having different values of σ) but having the same charge by appropriately adjusting the product $i_0\sigma$. In other words, the current waveform can be described mathematically as

$$i(t) = \frac{q}{\sigma\sqrt{2\pi}} \exp\left(-\frac{t^2}{2\sigma^2}\right) \quad (3)$$

The current waveform given in Equation (3) is associated always with charge q irrespective of the value of the parameter σ.

2.3. The Effective Duration of the Radiation Emission

As shown in Paper 1, the power emitted by the traveling-wave element has a temporal shape corresponding to the square of the current waveform. Thus, the temporal variation of the power emission by the radiation fields of the traveling-wave element has the form

$$W(t) \sim e^{-t^2/\sigma^2} \quad (4)$$

The effective duration τ of the energy emission (or power) is defined as the Full Width at Half Maximum (FWHM) of the power curve as given in Equation (4). Since the FWHM of the power curve is about 1.5 σ, one can write

$$\tau = 1.5\sigma \quad (5)$$

Thus, one can change the effective duration of the energy emission by changing the value of σ of the current waveform. In order to make the analysis valid for current waveforms of different durations, and hence for different FWHM values of the power curve, a parameter β is defined in Paper 1 as follows:

$$\tau = \beta l/c \quad (6)$$

In the above equation, l is the length of the traveling-wave element and c is the speed of light. According to this definition, β is equal to the ratio of the effective duration of the energy emission to the time necessary for the current pulse to propagate across the length of the traveling-wave element.

2.4. The Energy Dissipated by the Traveling-Wave Element

In Paper 2 it was shown that the condition $\Delta U \times \tau \geq h/4\pi$ is valid for the emitted radiation only when $\beta << 1$. They also showed that the cumbersome expression for the energy dissipation, which had to be solved numerically, reduces to a simple expression when $\beta \leq 10^{-6}$. That simple expression will be utilized here. The expression for the total energy dissipated by the traveling-wave element when $\beta \leq 10^{-6}$ is given by (this can be obtained by performing the integral in Equation (27) of Paper 2)

$$U = \frac{q^2}{4\pi^{3/2}\varepsilon_0 c\sigma} \ln\frac{1}{\beta} \quad (7)$$

In the above equation, ε_0 is the permittivity of the vacuum. Note that the energy emitted by the radiation is both a function of charge and σ.

2.5. The Action of the Radiation Fields of the Traveling-Wave Element

The action of the radiation field for $\beta \leq 10^{-6}$ is given in Paper 2 by

$$A = \tau \times U = \frac{\tau q^2}{4\pi^{3/2}\varepsilon_0 c\sigma} \ln \frac{1}{\beta} \tag{8}$$

Substituting for τ from Equation (5), we obtain

$$A = \frac{3q^2}{8\pi^{3/2}\varepsilon_0 c} \ln \frac{1}{\beta} \tag{9}$$

Note that even though the energy emitted by the traveling-wave element depends directly on both the charge and σ, the action depends only on the charge. Note also that the behavior of action is different to the behavior of energy as $1/\beta$ increases. This difference is due to the fact that the energy is directly proportional to $1/\sigma$ (which indeed is related to β through Equations (5) and (6)).

One can see from the expression given in Equation (9) that the action increases with increasing $1/\beta$. So the upper limit of the value of the action corresponding to a given charge is decided by the largest value of $1/\beta$ that can be achieved in nature. In order to study this further, let us investigate more closely the physical meaning of the parameter β which is defined by Equation (6). As pointed out in Paper 2, the effective frequency, ν_{eff}, is given by

$$\nu_{eff} = 1/\tau \tag{10}$$

Recall that τ is the effective duration of the energy emission. With $\tau = 1.5\sigma$, ν_{eff} is at about 1% spectral amplitude of the power spectrum. The above equation shows that β can be expressed as a function of this effective frequency as

$$\beta = c/(\nu_{eff} l) \tag{11}$$

If λ_{eff} is the wavelength associated with this frequency, Equation (11) can also be written as

$$\beta = \lambda_{eff}/l \tag{12}$$

Using this expression for β in Equation (9), we obtain

$$A = \frac{3q^2}{8\pi^{3/2}\varepsilon_0 c} \ln \frac{l}{\lambda_{eff}} \tag{13}$$

As we mentioned at the beginning of Section 2, the expressions for the electromagnetic fields, and hence the expressions for energy and action, are valid when $\lambda_{eff} \gg a$ and $R \gg l$, where a is the radius of the conductor and R is the distance to the point of observation or the radius of the sphere over which the Poynting vector is integrated to obtain the energy. In order to satisfy these constraints, let us set $\lambda_{eff} = ka$ and $l = R/k$ with $k \approx 10$. Thus, the upper bound of the action, denoted by A_{ub}, for a given length of the conductor and charge q is given by

$$A_{ub} \leq \frac{3q^2}{8\pi^{3/2}\varepsilon_0 c} \ln \frac{l}{ka} \tag{14}$$

Note that we have replaced λ_{eff} by ka because this is the ultimate lower bound for the effective wavelength. This equation shows that for a given charge, the absolute maximum value of the action is decided by the smallest radii and the longest length of the conductor that can be achieved in nature. The maximum radius of the sphere that can be realized in nature is equal to the radius of the universe, which is denoted here by l_u. That is, the maximum value of R is equal to the radius of the universe and, therefore, the maximum value of l that can be plugged into the equation is l_u/k. The smallest radius of the conductor that can be realized in nature is the size of an atom. The size of an atom is given approximately by the Bohr radius a_0. Thus, the absolute maximum action, denoted by $A_{\max}$, that can be achieved in nature for a given charge satisfies the relation

$$A_{\max} \leq \frac{3q^2}{8\pi^{3/2}\varepsilon_0 c} \ln \frac{l_u}{k^2 a_0} \quad (15)$$

If we substitute l_u = 4.4 × 10^{26} m and a_0 = 5.3 x 10^{-11} m we obtain l_u/a_0 = 0.83 × 10^{37}, and for the elementary charge (i.e., $q = e$) we obtain $(A_{\max})_e \approx h/4\pi$ where h is the Planck constant. This fact, namely that the action associated with the electromagnetic fields approaches $h/4\pi$ when the charge in the current is equal to the elementary charge, is pointed out in Paper 1. The differences in the results presented here and in Paper 1 are due to the fact that the definition of τ was different in Paper 1, i.e., $\tau = 4\sigma$ (full-width of the current) instead of $\tau = 1.5\sigma$ (half-width of the energy emission) used here. The correct definition that should be used for τ became apparent from the research work presented in Paper 2. This difference made the action for a given charge and $1/\beta$ larger by a factor 8/3 in Paper 1 in comparison to the results presented here. Moreover, since the equations were solved numerically, the results presented in Paper 1 could not be extended for extremely high values of $1/\beta$.

The interesting point here is that the action corresponding to $q = e$ reduces to $h/4\pi$ when the extreme values of l and a are plugged into the equations. Observe that the above results are based on pure classical electrodynamics. One can question whether the results are unique for dipole-like structures represented by the traveling-wave element or whether the results obtained are general. In order to answer this question, we analyzed the radiation fields generated by a bi-conical antenna, another unit that is being used to study radiation fields both theoretically and practically [8], using the same approach. Interestingly, similar results can be obtained when the length of the bi-conical antenna reaches l_u and when the maximum radius of the antenna is about a_0 (i.e., $\tan\alpha \approx a_0/l_u$ where α is the half-cone angle of the antenna). Let us now consider the possible physical interpretation of these results.

3. Physical Interpretation and Discussion

In Paper 2 it was shown that the electromagnetic fields generated by the traveling-wave element satisfy the inequality

$$\tau \times \Delta U \geq h/4\pi \quad (16)$$

In the above equation, ΔU is the uncertainty associated with the energy measurement. The next question is: what is the uncertainty in the energy measurement? We follow the argument introduced in Paper 1 as follows. The parameter that can introduce uncertainty into the energy measurement is the uncertainty in the charge associated with the current. Assume that the charge comes in elementary units of, say, $q_{\min}$. When the charge associated with the current pulse approaches this limit, the uncertainty in the measurement of energy becomes comparable to the energy itself. Thus, the minimum uncertainty associated with the energy measurement is given by

$$(\Delta U)_{\min} = \frac{q_{\min}^2}{4\pi^{3/2}\varepsilon_0 c\sigma} \ln \frac{1}{\beta} \quad (17)$$

Since the minimum possible uncertainty for a given value of $1/\beta$ is given by Equation (17), one can safely write Equation (16) as

$$\frac{\tau q_{\min}^2}{4\pi^{3/2}\varepsilon_0 c\sigma} \ln \frac{1}{\beta} \geq \frac{h}{4\pi} \quad (18)$$

Substituting for τ from Equation (5), we obtain

$$\frac{3q_{\min}^2}{8\pi^{3/2}\varepsilon_0 c}\ln\frac{1}{\beta} \geq \frac{h}{4\pi} \tag{19}$$

From this, an expression for the minimum detectable charge can be obtained and it is given by

$$q_{\min}^2 \geq \frac{h}{4\pi}\frac{8\pi^{3/2}\varepsilon_0 c}{3\ln\frac{1}{\beta}} \tag{20}$$

One can see from this expression that the minimum detectable charge $q_{\min}$ depends on the value of $1/\beta$ and it decreases with increasing $1/\beta$. Since the upper bound of this parameter is equal to l_u/k^2a_0, we can write

$$q_{\min}^2 \geq \frac{h}{4\pi}\frac{8\pi^{3/2}\varepsilon_0 c}{3\ln\frac{l_u}{k^2a_0}} \tag{21}$$

Thus, the smallest detectable charge associated with the current in the traveling-wave element is given by

$$q_{\min} \approx \pm\sqrt{\frac{2\varepsilon_0\sqrt{\pi}}{3}\frac{hc}{\ln\left(l_u/k^2a_0\right)}} \tag{22}$$

If we substitute l_u = 4.4 × 10^{26} m, a_0 = 5.3 × 10^{-11} m and $k = 10$, we obtain

$$q_{\min} \approx \pm 1.6 \times 10^{-19} C \tag{23}$$

This shows that the smallest detectable charge is almost equal to the elementary charge. Note that the conditions used to derive this value are ideal but not practical. Under conditions related to practical lengths and practical radii of conductors, the minimum detectable charge is much larger than the elementary charge.

It is important to mention here that, due to the difference in the definition of τ, the charge estimated in Paper 1 differs from the above by a factor of $\sqrt{8/3}$. However, even with this difference, the estimated minimum charge is still on the order of the elementary charge. Another important point pertinent to the analysis presented in this paper is the following. As mentioned previously, the equations for the radiation fields used here are valid when the effective wavelength of the radiation is much greater than the radius of the conductor. Of course, one can derive equations for the radiation fields which are valid when the effective wavelength is less than the radius of the conductor. However, due to phase differences at the point of observation associated with the radiation emanating from different locations on the cross-section of the wire, the net energy emitted for a given charge would be less in this case than when the wavelength is much larger than the cross-section of the wire where the radiation fields coming from different locations on the cross-section of the wire are in phase at the point of observation. Thus, the case studied here is the one that gives rise to the largest energy (and the largest action) and the smallest detectable charge.

In the calculations presented in this paper we have considered ideal conditions by assuming that the radiating system can be represented by a traveling-wave element. In reality, the current will attenuate and disperse as it travels along a conductor and the radiation resistance and other dissipation losses will make the speed of propagation less than the speed of light [1]. As mentioned in Paper 1, these effects will reduce the energy dissipation for a given current signature and the effect of this is to decrease the magnitude of the radiated energy and hence to increase the value of the minimum charge estimated. Thus, the minimum charge estimated in this paper can be considered as the absolute minimum that can be obtained under either ideal or real conditions.

Another point that one has to discuss concerning the analysis presented in this paper is the following: Since we are considering atomic dimensions and electromagnetic waves with comparable wavelengths, the question arises as to the possible modification of the results due to quantum effects. Observe that the minimum wavelength that is being considered in the paper is in the range of X-rays. As far as the radiation from accelerating charges is concerned, the relativistic Larmor formula, which is derived from classical electromagnetic theory [1], predicts correctly the radiation produced by acceleration charges (synchrotron radiation) in the X-ray region. Thus, there is no reason to doubt the validity of Maxwell's equations and their solutions pertinent to accelerating charges at these frequencies. Having said that, even if we have decreased the wavelength to the upper limit of radio frequencies (i.e., 3000 GHz) in Equation (22) (i.e., replace ka with the wavelength corresponding to 3000 GHz), the minimum charge that we will obtain will still be on the order of the elementary charge i.e., $1.2e$. Actually, the main problem is that when we reduce the charge associated with the current pulse to the elementary charge, we will not be able to neglect the grainy nature (or quantum nature) of the electromagnetic radiation. This is the case because at high frequencies, the radiation may consist of only a few photons and the actual structure of the energy dissipation may not adhere to the smooth energy distribution predicted by the classical electrodynamics. In this situation, a single experiment may not produce either the correct energy or the energy distribution in space as predicted by Maxwell's equations. In this case, the results obtained here have to be interpreted as resulting from the average of a large number of identical experiments conducted with identical traveling-wave elements. This indeed is the correspondence principle of Bohr [9]. As the number of experiments approaches infinity, the average of the results reduces to the predictions based on classical equations. A somewhat similar situation exists in the double slit experiment. If the experiment is conducted with a single photon, one will find only a dot on the interference screen. However, if the same identical experiment is repeated n times with identical photons, the interference pattern predicted by the classical electrodynamics appears on the screen as n becomes very large.

The calculations presented in this paper were conducted for a Gaussian current pulse, which is a symmetrical function. Analysis done with other monopolar transient current signatures such as exponential and rectangular shapes shows that the Gaussian predicts the smallest charge provided that the same relative spectral amplitude is used to define ν_{eff} for other current wave-shapes. This information when combined together with the results presented in this paper leads to the following generalization. In nature there exists different types of radiators that launch electromagnetic radiation into space. However, for a given current signature and a radiator length, the traveling-wave element is the one that generates the maximum action and hence it is associated with the smallest detectable charge. Based on this one can conclude that the smallest detectable charge associated with any electromagnetic radiating system in nature is on the order of the elementary charge or larger.

It is interesting to observe that a strict interpretation of Equation (22) shows that the elementary charge may decrease as the size of the universe increases. The question of whether the fundamental constants of nature change as the universe ages was raised by Dirac [10] in a paper where, based on the ratios of atomic and cosmological parameters expressed in natural atomic units, he suggested that some of the universal constants must be regarded as parameters which vary with the size or the age of the universe. While objecting to some of the suggestions made by Dirac, Teller [11] made the suggestion that the fine structure constant α, which is given by $e^2/2\varepsilon_0 hc$, is related to the logarithm of the age of the universe when this age is expressed in natural time units. The fine structure constant is the parameter that defines the strength of the electromagnetic force. Let us consider the equations we have derived in this paper. Observe that the Bohr radius is the atomic unit of distance and the quantity l/a_0 in Equation (22) is the radius of the universe expressed in natural units. The fine structure constant as predicted by Equation (22) is given by

$$\alpha_d = \frac{\sqrt{\pi}}{3} \frac{1}{\ln\left(R_u/k^2\right)} \tag{24}$$

In the above equation, α_d is the fine structure constant as predicted by Equation (22) and R_u is the radius of the universe expressed in natural units (i.e., l_u/a_0). Equation (24) shows that the fine structure constant varies as the inverse of the logarithm of the radius of the universe. Since the size of the universe is increasing with the age of the universe, Equation (24) is in agreement with the suggestion made by Teller [11]. If we substitute the current value of the radius of the universe into the above equation, we obtain

$$\frac{1}{\alpha_d} \approx 136 \quad (25)$$

The value obtained above is close (within 1%) to the experimentally established value. Note that the results obtained in this paper would not change significantly even if we had assumed that $k = 1$. Even though the results obtained here are interesting, it is important to point out that the discussion on whether the fundamental constants of nature can change with the age of the universe is still going on in the current literature and no conclusions on this topic can be made at the present time. On the other hand, recall that the time energy uncertainty principle will give only an order of magnitude estimation of the smallest detectable charge. Since the size of the universe appears inside the logarithmic term, it has to change by many orders of magnitude to make a change in the order of magnitude of the smallest detectable charge. Finally, it is important to stress that the calculations presented in this paper are based purely on classical electrodynamics and these results motivate a thorough analysis of the same problem using quantum electrodynamics.

4. Conclusions

In this Letter we have gathered together the information presented in the two papers published previously in this journal by Cooray and Cooray [6,7] and utilized it to obtain an expression for the minimum charge that could be detected by the radiation fields generated by a traveling-wave element. The results confirm the previous assertion made in Paper 1 that the application of the time energy uncertainty principle to the classical electromagnetic radiation leads to the conclusion that the smallest detectable charge in nature is the elementary charge. Based on the results, an expression for the fine structure constant is obtained and it agrees reasonably well with the experimentally obtained value. This is the first time that an order of magnitude estimation of the elementary charge and the fine structure constant based on electromagnetic radiation fields is obtained. Even though the results obtained in this paper have to be considered as order of magnitude estimations, a strict interpretation of the derived equations shows that the fine structure constant or the elementary charge is related to the size or the age of the universe.

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