BARGAINING AND STRATEGIC DISCRIMINATION

JONAS BJÖRNERSTEDT AND ANDREAS WESTERMARK
Bargaining and Strategic Discrimination\footnote{Our work has benefitted greatly from discussions with Sven-Olof Fridolfsson and Johan Stennek and from participants at seminars/workshops at The Research Institute for Industrial Economics, Uppsala University, ESEM 2002 in Venice, Games 2004 in Marseilles and the International Conference on Game Theory 2002 at Stony Brook. Björnerstedt gratefully acknowledges financial support from the Jan Wallander and Tom Hedelius Foundation and Westermark from the Swedish Council for Working Life and Social Research.}

Jonas Björnerstedt\footnote{The Research Institute for Industrial Economics, P.O. Box 55665, SE-102 15 Stockholm, Sweden. E-mail: jonas@iui.se.} \quad Andreas Westermark\footnote{Corresponding author. Department of Economics, Uppsala University, Box 513, SE-751 20 Uppsala, Sweden. E-mail: andreas.westermark@nek.uu.se.}

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Abstract

In bargaining between two sellers and one buyer on prices and quantities, strategic inefficiencies arise. By reallocating between the last agreement and the first, the buyer can increase it’s share of the surplus. With symmetric sellers producing substitutes, the quantities in the first agreement will be higher than the efficient, and lower than the efficient in the last, implying that sellers are strategically discriminated. In equilibrium when the sellers produce substitutes, the buyer agrees first with the seller with lowest marginal cost. Efficiency is decreasing in the symmetry of the sellers and in the relative bargaining power of the sellers.

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JEL Classification : C78, J22, J71, L10.
1 Introduction

Negotiations are often interdependent. When a firm bargains with more than one worker, for example, it cannot reasonably be argued that the two negotiations are independent, since the surplus of hiring one worker usually depends on the characteristics of the other worker. Several models such as Horn & Wolinsky (1988) and Stole & Zweibel (1996a,b) extend Rubinstein (1982) to interdependent bargaining. One strong assumption in these models, however, is that the quantities agreed upon are fixed, i.e., there is only bargaining on how to split a given surplus. It seems reasonable that the payoffs also can be affected by varying quantities.

In this paper, we analyze a bargaining model, when bargaining is on both prices and quantities. We show that in equilibrium we have strategic discrimination, i.e. an outcome where identical sellers are treated asymmetrically. In general this results in an inefficient allocation.

Since the surplus in price quantity bargaining might depend on the order of agreement, previous work such as Horn & Wolinsky (1988) is not applicable. Therefore, we extend the work of previous authors by analyzing a model where the total surplus depends on the order of agreement.1 Thus, we begin by studying a general bargaining model, with one buyer and two sellers, that can be used to study not only bargaining in prices and quantities, but also for example bargaining over tasks.

The main focus of this paper is on bargaining over prices and quantities. As examples, we can think of one firm that bargains simultaneously with two workers or a downstream firm that bargains with two upstream firms. We show how equilibrium quantities depend on the degree of substitutability and the asymmetry between the sellers.

When goods are substitutes, the buyer has a strategic incentive to increase the quantity in the first agreement, thereby decreasing the quantity and thus the price in the second agreement. Conjecture an agreement on the efficient quantities and consider a small increase in the first quantity. By the envelope theorem, the effect on the total surplus is small. The change in the quantity also decreases the payment in the last agreement, since the surplus in the last agreement decreases when the first quantity increases. Since the price effect is a first-order effect, while the effect on the total surplus is a second order effect, it is optimal for the buyer to increase the first quantity. When goods are complements, both quantities are inefficiently low.

When sellers are completely symmetric, they will be treated symmetrically in the sense that the equilibrium will prescribe immediate agreement with either seller. The quantities agreed upon will be asymmetric, however. Small asymmetries between sellers are sufficient to determine

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1 The model is related to bargaining with externalities along the lines of Jehiel & Moldovanu (1995a and 1995b). For a complete discussion, see Björnerstedt & Westermark (2006a).
equilibrium order of agreement, with the low cost seller agreeing first. This is related to the issue of discrimination. From a theoretical standpoint this question boils down to: "Under what conditions will essentially identical goods have different prices in competitive markets?" (Cain 1986). We provide an explanation why a small difference in the willingness to work between men and women results in large differences in actual hours worked leading to price differences.

Under certain circumstances, the outcome will be efficient. If goods are independent the negotiations will also be independent, with efficiency as a result. Efficiency also holds if supply is completely inelastic, as the scope for strategic discrimination disappears. We also get efficiency in the special case when goods are perfect substitutes and sellers have constant marginal costs.

Efficiency is also affected by the relative bargaining power of the buyer and the sellers, i.e. the relative probability of being the proposer. The outcome is efficient when the buyer makes all the bids. As all the surplus in both the first and the last agreement will be captured by the buyer, there is no strategic incentive to distort quantities. When sellers make all the bids, incentives to reallocate surplus to the first period is the largest, as all the surplus in the last agreement is captured by the seller. Reinterpreting the model in terms of two buyers and one seller, our model can be thought of as an auctions under full information with multiple objects and interdependent valuations; see Krishna (2002). Our result is in contrast with Bulow & Klemperer (1996), who argue that auctions are more efficient than negotiations.

With linear marginal cost and revenue, we find that the degree of strategic discrimination increases, the flatter supply is relative to demand, the more homogeneous goods are and the more symmetric sellers are. Similarly, efficiency increases the steeper supply is relative to demand, the less homogeneous goods are and the more asymmetric sellers are.

The result that quantities are inefficient has implications for how to view e.g., union formation. As in Horn & Wolinsky (1988), workers form a union if they are substitutes. When a union forms, the firm has only one counterpart. Then, from standard results in bargaining theory, quantities are efficient. Thus, forming a union leads to an increase in aggregate welfare.

Applying this model to intermediate goods markets, the more asymmetric the sellers are, the less inefficient is the outcome. We find a justification for the idea of countervailing power – high concentration on one side of the market can justify increased concentration on the other. Increasing the seller concentration reduces the ability of the buyer to strategically discriminate, with higher efficiency as a result.

Beyond potential applications of this mechanism, there are two reasons why the results are important. Firstly, it shows that the relationship between bargaining and efficiency can be more problematic than is commonly assumed. In the labor literature for instance, bargaining in wage
and employment is often called “efficient bargaining”. Here, we show that efficient bargaining is inefficient when there is more than one worker.

Secondly, there is a literature that implicitly assumes that this strategic possibility is not used. In Björnerstedt & Stennek (2004), the outcome is efficient because firms are assumed to have representatives negotiating, each bargaining with only one other firm. If for some reason firms cannot strategically coordinate negotiations we use the model of Björnerstedt & Stennek.

In a series of papers, Segal (1999), Segal (2003) and Segal & Whinston (2003) analyze bilateral contracting with externalities. It is assumed that total surplus only depends on aggregate quantities. We have a more general payoff structure as we allow for the distribution of trades to affect welfare. In the special case when welfare only depends on aggregate quantities, the allocation is efficient, in stark contrast with the result of these papers.

In section 2 the general bargaining model is analyzed. Section 3 shows how bargaining over prices and quantities can be analyzed in terms of the general bargaining model, section 4 analyzes the equilibria of price and quantity bargaining and finally section 5 concludes.

2 The General Model

We will consider asynchronous simultaneous bargaining between a buyer $A$ and two sellers 1 and 2. As examples consider two workers who sell their labor to a firm or an upstream firm $A$ selling an intermediate good to downstream firms 1 and 2, with bargaining over prices and quantities. If the downstream firms interact in the final goods market, the quantity bought by firm 1 imposes externalities on firm 2 and vice versa. The bargaining model in this section is completely general. For example, it covers the case when player $A$ has two objects that are sold to 1 and 2, respectively. As we will see in section 3, the setup above also covers the case when bargaining is over a divisible good, i.e., we have e.g. a firm bargaining with two workers over both wages and work hours.

The buyer $A$ bargains with both sellers 1 and 2 simultaneously. In each period, however, only one bid and response will be made, observed by all. Agreement is immediate and binding. We assume that bids alternate between $(A, 1)$ and $(A, 2)$, with the buyer $A$ making the bid with probability $\lambda$, and the seller $i$ with probability $1 - \lambda$. The exact bargaining protocol is not important; all bids can also be random or bids can alternate in a deterministic order.  

We assume that the surplus generated can depend on the order of agreement. If $(A, 1)$ agree first, the surplus in this first agreement is given by $\Pi_1$. Similarly, $\Pi_2$ is the surplus when $(A, 2)$

\footnote{Furthermore, when bids are made simultaneously, results are similar as can be seen in section 4.3.}
agree first. Given that \((A, 1)\) have come to agreement, the additional surplus generated by \((A, 2)\) agreeing is given by \(\pi_2\), with \(\pi_1\) similarly defined. This general formulation also allows us to study models with divisible goods.

Let \(V_i\) and \(W_i\) denote the value to \(A\) of bidding and receiving a bid from seller \(i\), and \(v_i\) and \(w_i\) denote the value to seller \(i\) of bidding and receiving a bid. Let \(\sigma_{Ai}\) be the probability that \(A\) gives an acceptable bid to \(i\) when bidding and \(\sigma_{iA}\) the probability that \(i\) gives an acceptable bid. Defining \(\sigma_i = (\sigma_{Ai} + \sigma_{iA})/2\), the value equations are given by:

\[
V_i = (1 - \sigma_{Ai}) W_i + \sigma_{Ai} (\Pi_i + \delta \lambda \pi_j - w_i), \\
W_i = \delta (\lambda W_j + (1 - \lambda) W_j), \\
v_i = (1 - \sigma_{iA}) w_i + \sigma_{iA} (\Pi_i + \delta \lambda \pi_j - W_i), \\
w_i = \delta^2 ((1 - \lambda) \pi_i \sigma_j + (1 - \sigma_j) ((1 - \lambda) v_i + \lambda w_i))
\]

for \(i = 1, 2\) with \(j \neq i\). To understand (1), if \(A\) and \(i\) have come to agreement and \(A\) bargains with \(j\), by standard reasoning in expectation the firm gets \(\lambda \pi_j\) and \(j\) gets \((1 - \lambda) \pi_j\). Thus in the first negotiating with \(i\), the total amount at stake is \(\Pi_i + \delta \lambda \pi_j\). In giving an acceptable offer (with probability \(\sigma_{Ai}\)) it is sufficient to offer \(w_i\) to \(i\). Since \(W_i\) is the continuation value conditional on disagreement, the value \(V_i\) in (1) follows. By similar reasoning \(v_i\) is determined. When rejecting a proposal by \(i\), \(A\) gets \(V_j\) with probability \(\lambda\) and \(W_j\) with \(1 - \lambda\), giving \(W_i\) in (1). When rejecting a proposal, \(i\) will receive \((1 - \lambda) \pi_i\) if \(A\) and \(j\) agree in the next period. With probability \(1 - \sigma_j\) they do not, giving \((1 - \lambda) v_i + \lambda w_i\).

The equilibrium outcome will be shown to depend upon on which order of agreement is more efficient and the degree of substitutability. To capture these notions we let, for \(i = 1, 2\),

\[
\varepsilon_i (\delta) = \delta (\Pi_i + \delta \pi_j) - (\Pi_j + \delta \pi_j) \quad (2)
\]

\[
\gamma_i (\delta) = \Pi_i - \delta^2 (1 - \lambda) \pi_i \quad (3)
\]

The expression \(\varepsilon_i (\delta)\) is interpreted as the difference in total surplus of waiting to agree with \(i\) in the next period, instead of agreeing with \(j\) today. Similarly \(\gamma_i (\delta)\) is related to the degree of substitutability of \(i\) and \(j\). In the symmetric case \((\Pi = \Pi_1 = \Pi_2\) and \(\pi = \pi_1 = \pi_2)\) when \(\delta = 1\) and \(\lambda = \frac{1}{2}\), equation (3) simplifies to \(\gamma_1 = \Pi - \frac{\pi}{2}\). Thus, \(\gamma_1\) depends on the relative size of the gains of the first and second agreement, which is related to substitutability. Also, we let \(\gamma_i = \gamma_i (1)\) and

\[
\varepsilon = \Pi_1 + \pi_2 - (\Pi_2 + \pi_1).
\]

5
Note that $\varepsilon_1 (\delta) \rightarrow \varepsilon$, $\varepsilon_2 (\delta) \rightarrow -\varepsilon$ as $\delta \rightarrow 1$. One might think that the order of agreement do not matter for total surplus. The example below illustrates why $\varepsilon \neq 0$ is natural in applications.

**Example 1** Task allocation of workers in a firm. We assume that the firm can employ two workers for different tasks, $t_x$ and $t_y$. The payoff of the firm is

$$R(t_1, t_2) - p_1 - p_2$$

where $p_i$ is the amount paid to $i$ and $R$ revenue to $A$. The payoff of seller $i$ is given by:

$$p_i - c_i(t_i)$$

where the function $c_i$ represents the cost of performing task $t_i$. When no agreement has been reached, the proposed task when the buyer and seller 1 bargain is

$$t_F^1 = \arg \max_{t_1 \in \{t_x, t_y\}} R(t_1, 0) - c_1(t_1) + \delta \lambda (R(t_1, t_2) - R(t_1, 0) - c_2(t_2))$$

(5)

where $t_2 \neq t_1$. Then

$$\Pi_1 = R(t_F^1, 0) - c_1(t_F^1)$$

with $\Pi_2$ similarly defined. Letting $t_{i}^{F}$ denote the $i$'s task in the last agreement, $\pi_2$ is given by

$$\pi_2 = R(t_{i}^{F}, t_{2}^{F}) - R(t_{1}^{F}, 0) - c_2(t_{2}^{F})$$

with $\pi_1$ similarly defined. If the solution for the problem above is $t_F^1 \neq t_F^2$, e.g. $t_F^1 = t_x$ and $t_F^2 = t_y$ then the tasks are allocated in the same way, irrespectively if $A$ agrees with 1 or 2 first. We then get

$$\Pi_1 + \pi_2 = R(t_x, t_y) - c_1(t_x) - c_2(t_y) = \Pi_2 + \pi_1$$

and hence $\varepsilon = 0$. However, if $t_F^1 = t_F^2$ i.e., $t_F^2 = t_x$ we get

$$\Pi_2 + \pi_1 = R(t_y, t_x) - c_1(t_y) - c_2(t_x)$$

which is different from $\Pi_1 + \pi_2$, unless $R$ is symmetric and $c_1$ and $c_2$ are identical, implying $\varepsilon \neq 0$.

There are several types of equilibria. There are equilibria with immediate agreement (denoted $I$), where $A$ always agrees with the seller it meets. In equilibria $P_i$, $A$ always agrees with seller
i first. There are also mixed equilibria, denoted $M_i$, where $A$ agrees with probability 1 when meeting $i$, and with probability $0 < \sigma_j < 1$ when meeting $j$, and mixed equilibria, denoted $H$, where $0 < \sigma_i < 1$ for $i = 1, 2$. We have the following equilibrium characterization.

**Proposition 2** As $\delta \to 1$, the stationary subgame perfect equilibria when $\varepsilon > 0$ are characterized as follows.

<table>
<thead>
<tr>
<th>$-\lambda \pi_2$</th>
<th>$\gamma_1 \leq -\lambda \pi_2$</th>
<th>$\gamma_1 &lt; -\frac{\lambda}{1-\lambda} \varepsilon$</th>
<th>$-\frac{\lambda}{1-\lambda} \varepsilon \leq \gamma_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_2 \leq -\lambda \pi_1$</td>
<td>$P_1, P_2, H$</td>
<td>$P_1, P_2$</td>
<td>$P_2, M_1$</td>
</tr>
<tr>
<td>$-\lambda \pi_2 &lt; \gamma_1 &lt; -\frac{\lambda}{1-\lambda} \varepsilon$</td>
<td>$P_1, P_2, M_2$</td>
<td>$P_1, P_2, M_2$</td>
<td>$P_2, M_1, M_2$</td>
</tr>
<tr>
<td>$-\frac{\lambda}{1-\lambda} \varepsilon \leq \gamma_1$</td>
<td>$P_1$</td>
<td>$P_1$</td>
<td>$M_1$</td>
</tr>
</tbody>
</table>

When $\lambda \geq \frac{1}{2}$ we have $-\lambda \pi_2 < \gamma_1$.

The last condition makes it significantly easier to characterize equilibria when $\lambda \geq \frac{1}{2}$. Then one can restrict attention to whether $\frac{\lambda}{1-\lambda} \varepsilon \leq \gamma_2$ and $-\frac{\lambda}{1-\lambda} \varepsilon \leq \gamma_1$, since the first row (two rows) can be eliminated if $-\lambda \pi_2 < -\frac{\lambda}{1-\lambda} \varepsilon$ ($-\lambda \pi_2 \geq -\frac{\lambda}{1-\lambda} \varepsilon$).

The case when $\varepsilon = 0$ is more complicated to illustrate than in the proposition above, as we end up with more cases, i.e., alternative orderings of columns. When $\lambda \geq \frac{1}{2}$ however, there is only one case and, using (28) in the appendix, the equilibria are characterized as follows;

<table>
<thead>
<tr>
<th>$\gamma_2 &lt; 0$</th>
<th>$\gamma_1 &lt; 0$</th>
<th>$0 \leq \gamma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I, P_1, P_2, M_1, M_2$</td>
<td>$I, P_2, M_2$</td>
<td>$I, P_1, M_1$</td>
</tr>
</tbody>
</table>

Although $\varepsilon = 0$ is non-generic, it is still interesting, as it covers some standard models, e.g., Horn & Wolinsky (1988). For example, a buyer bargaining with two sellers when quantities are fixed implies $\varepsilon = 0$. This can be seen from noting that the fixed quantity model can be analyzed in terms of Example 1 with $t_1^F = t_1^L$ and $t_2^F = t_2^L$. Also note that in the case of fixed quantities, the condition that $\gamma_1 \leq 0$ is identical to the condition in Horn & Wolinsky (1988). ³

³To see this, note that we have, in the terminology of Horn & Wolinsky, $\Pi = x$ and $\pi = y$. Then $\gamma_1 = \frac{x}{2} - \frac{y}{2} \leq 0$.
In the previous proposition, the equilibria are characterized. For the remainder of the analysis of this paper, only the outcomes matter however. The next proposition characterize the order of agreement and payoffs for $\delta$ close to 1.

**Proposition 3** As $\delta \rightarrow 1$, the order of agreement and payoffs for the equilibria in Proposition 2 are given by

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Limit Probabilities} & \text{Profit to } A & \text{Profit to } 1 & \text{Profit to } 2 \\
\hline
I & (1,1) & \Pi_1 + \lambda \pi_2 - (1 - \lambda) \pi_1 & (1 - \lambda) \pi_1 & (1 - \lambda) \pi_2 \\
P_1 & (1,0) & \lambda (\Pi_1 + \lambda \pi_2) & (1 - \lambda) (\Pi_1 + \lambda \pi_2) & (1 - \lambda) \pi_2 \\
P_2 & (0,1) & \lambda (\Pi_2 + \lambda \pi_1) & (1 - \lambda) \pi_1 & (1 - \lambda) (\Pi_2 + \lambda \pi_1) \\
M_1 & (1,0) & \Pi_1 + \lambda \pi_2 - (1 - \lambda) \pi_1 - \varepsilon & (1 - \lambda) \pi_1 + \varepsilon & (1 - \lambda) \pi_2 \\
M_2 & (0,1) & \Pi_2 + \lambda \pi_1 - (1 - \lambda) \pi_2 + \varepsilon & (1 - \lambda) \pi_1 & (1 - \lambda) \pi_2 - \varepsilon \\
H & (0,0) & 0 & \Pi_1 + \lambda \pi_2 & \Pi_2 + \lambda \pi_1 \\
\hline
\end{array}
\]

**Proof:** Follows immediately by taking limits in Lemmas 9 - 12 (in the appendix).

Note that the mixed equilibria $M_i$ have the property that the probability $\sigma_j$ converges to zero. Thus, probabilities in the mixed equilibrium $M_i$ converge to the pure equilibrium $P_i$. The equilibrium payoffs do not converge, however. If goods are enough substitutable, then there are only equilibria where $A$ and 1 agree first in the limit. Thus all equilibria are efficient. If goods are sufficiently complementary, there are multiple equilibria, some where the buyer agrees with seller 1, and some where 2 comes first. Hence there are also inefficient equilibria.

We will now provide some intuition for the payoffs and the conditions under which the different equilibria exist. In all cases, the payoff in the last agreement is given by standard Rubinstein-Ståhl reasoning. Assuming for expositional purposes that $\lambda = \frac{1}{2}$ and $\delta \rightarrow 1$, then if $(A, i)$ agree last, then $i$ gets $\frac{\pi_i}{2}$.

Consider first the immediate agreement equilibrium $I$, where $A$ agrees when meeting either seller. If $(A, i)$ do not agree, equilibrium play implies that $i$ will be last. Thus, $A$ can threaten both sellers with being last and can then force down the payoff of both sellers $i$ to $\frac{\pi_i}{2}$. When agreeing with 1 first, $A$ gets the remainder of the gains of trade: $\Pi_1 + \pi_2 - \frac{\pi_1}{2} - \frac{\pi_2}{2} = \Pi_1 + \frac{\pi_2}{2} - \frac{\pi_1}{2}$. Similarly, when agreeing with 2 first, $A$ gets $\Pi_2 + \frac{\pi_1}{2} - \frac{\pi_2}{2}$. Note that this equilibrium only exists when $\varepsilon = 0$. To ensure that $A$ does not have an incentive to disagree with either seller, the payoff for $A$ when agreeing first with 1 or 2 have to be equal, implying $\varepsilon = 0$.

---

or $x \leq 2y$ is identical to the cutoff condition in their Proposition 1.
To understand the pure strategy equilibrium $P_1$, note that

$$\gamma_2 < \varepsilon \iff \Pi_2 + \frac{\pi_1}{2} - \frac{\pi_2}{2} < \frac{1}{2} \left( \Pi_1 + \frac{\pi_2}{2} \right). \quad (6)$$

In the second agreement, $(A, 2)$ split $\pi_2$ equally, resulting in a payoff of $\frac{\pi_2}{2}$ to $A$ from this agreement. This is similar to a situation where $A$ only bargains with 1 and the surplus consists of $\Pi_1$ and the net gain from hiring 2, given by $\frac{\pi_2}{2}$. Then both $A$ and 1 get $\frac{1}{2} \left( \Pi_1 + \frac{\pi_2}{2} \right)$ each. In order for this to be an equilibrium, $A$ should not want to agree with 2 first. In doing so, $A$ would have to pay $\frac{\pi_2}{2}$ to 2, as this is the equilibrium payoff to 2. Conditional on acceptance, 1 is last, getting $\frac{\pi_2}{2}$. Then $A$ gets $\Pi_2 + \frac{\pi_1}{2} - \frac{\pi_2}{2}$. In order for this deviation not to be profitable, $\Pi_2 + \frac{\pi_1}{2} - \frac{\pi_2}{2} \leq \frac{1}{2} \left( \Pi_1 + \frac{\pi_2}{2} \right)$ corresponding to condition (6).

If $\varepsilon > 0$, no pure strategy equilibria exist when $\gamma_2 > \varepsilon$ and $\gamma_1 > -\varepsilon$, i.e., when the degree of substitutability is high. The payoffs in the mixed strategy equilibria can best be understood in relation to bargaining with outside options. The share of the surplus of the agreement that $A$ gets in negotiating with 1, depends on whether the equal split payoff $\frac{1}{2} \left( \Pi_1 + \frac{\pi_2}{2} \right)$ is greater or less than the "outside option" agreeing with 2 first: $\Pi_2 + \frac{\pi_1}{2} - \frac{\pi_2}{2}$. The condition (6) thus shows that the outside option is not binding if there are strong complementarities for 2, i.e., if $\gamma_2 < \varepsilon$. Furthermore, the equilibrium payoff of $1$ in $M_1$ is $\frac{\pi_2}{2} + \varepsilon$. Thus, compared with being last, 1 captures the entire gain in efficiency $\varepsilon$ from being first.

To understand why $\sigma_1 = 1$ and $0 < \sigma_2 < 1$ in the equilibrium $M_1$, consider the cases where $\sigma_2 = 1$ or $\sigma_2 = 0$. From the discussion of the $I$ equilibrium above, we know that if $\sigma_2 = 1$, if $\varepsilon > 0$, $A$ will never want to agree with 2, as $\varepsilon > 0$ implies that $\Pi_1 + \frac{\pi_2}{2} - \frac{\pi_1}{2} > \Pi_2 + \frac{\pi_1}{2} - \frac{\pi_2}{2}$. Thus he gains by reducing the probability $\sigma_2$. In the case where $\sigma_2 = 0$, the payoff to $A$ is $\frac{1}{2} \left( \Pi_1 + \frac{\pi_2}{2} \right)$. As (6) is violated, $A$ gains by agreeing with 2 to obtain $\Pi_2 + \frac{\pi_1}{2} - \frac{\pi_2}{2}$. Thus to ensure that neither of these deviations are profitable, by continuity we have $0 < \sigma_2 < 1$.

The hold-up equilibrium $H$ does not exist for $\lambda \geq \frac{1}{2}$. From Proposition 3 we see that both agreement probabilities converge to zero as $\delta \to 1$. The condition for existence of $H$ can be rewritten as $\Pi_1 + \lambda \pi_j \leq (1 - \lambda) \pi_i$. Thus, the total amount at stake in the first agreement is less than the surplus buyer $i$ would get in the last agreement, generating a hold-up problem Delay is significant in the sense that the total expected surplus as $\delta \to 1$ is given by $(\Pi_1 + \lambda \pi_2) + (\Pi_2 + \lambda \pi_1)$, which is less than the total surplus of the inefficient order of agreement $\Pi_2 + \pi_1$ by the condition $\gamma_1 \leq -\lambda \pi_2$.

The existence of multiple equilibria when there are strong complementarities can be seen as a coordination failure. Consider for example the case when $\gamma_1 < -\varepsilon$ and $\gamma_2 < \varepsilon$, where both $P_1$ and $P_2$ exist. Although $P_2$ is an inefficient outcome with agreement between $(A, 2)$ first, this
equilibrium can exist. When there are strong complementarities, \( \pi_i \) is large. In order for \( i \) to switch to agreeing first, \( A \) has to give him at least \( \frac{\pi_1}{2} \). In addition \( 2 \) will increase his payoff to \( \frac{\pi_1}{2} \) by now being last. If complementaries are strong enough, the gain in efficiency cannot compensate \( A \) for paying \( 2 \) more.

Example 1 (continued). It is clearly possible with inefficient equilibria. This is e.g. the case when \( \lambda \) is zero and
\[
R(0, t_x) - c_2(t_x) > R(0, t_y) - c_2(t_y)
\]
Then inefficient equilibria can exist if complementarity is large enough, since we have \( \varepsilon \neq 0 \).

3 Bargaining over price and quantity

Now we will use the general setup of the previous section to analyze bargaining over prices and quantities.4 To do this, we need to specify cost and revenue functions and derive \( \Pi_i \) and \( \pi_i \) in terms of these functions. We assume that the buyer has the payoff function
\[
R(q_1, q_2) - p_1 q_1 - p_2 q_2,
\]
where \( p_i \) and \( q_i \) are the price and quantity sold by \( i \) to \( A \). We say that 1 and 2 produce substitutes when \( \frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} < 0 \), produce complements if \( \frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} > 0 \) and that goods are independent if \( \frac{\partial^2 R(q_1, q_2)}{\partial q_1 \partial q_2} = 0 \). The payoff of seller \( i \) is given by:
\[
p_i q_i - c_i(q_i)
\]
where \( c_i(q_i) \) represents the cost of supplying good \( q_i \). We assume that \( R \) is concave and \( c_i \) convex. If \( R \) is strictly concave or \( c_i \) strictly convex, we say that production is strictly convex.

Let \( q_1^e \) and \( q_2^e \) denote the efficient quantities, i.e., the quantities that solve5
\[
\max_{q_1, q_2} R(q_1, q_2) - c_1(q_1) - c_2(q_2).
\]

---

4 Banerji (2002) has quantity choices in a bargaining setting with one buyer/firm bargaining with two sellers/unions. Their results, in particular Lemma 1 and the following corollary in that paper are incorrect; there exists a deviation from the proposed equilibrium profile where there is agreement on identical Cournot quantities with the two sellers. In the model, the sellers have zero marginal cost of supplying additional units. The firm then has the following profitable deviation. Offer, say, union 1 the same share of the surplus plus some small amount and a quantity equal to the sum of proposed equilibrium quantities. Since there is no marginal cost of supplying additional units, union 1 accepts the proposal. That the deviation is profitable can be seen from the fact that the firm only pays out half of the surplus it paid in the proposed equilibrium.

5 The analysis can be applied to a model where \( A \) sells goods to 1 and 2, where there is interdependence of buyer profits on the quantity on the other agreement. This could be due to the buyers being competitors in a final goods market. The payoff to \( A \) is then \( p_1 q_1 + p_2 q_2 - c(q_1, q_2) \) and to \( i \) is \( R(q_i, q_i) - p_i q_i \).
We now show that we can determine equilibrium quantities in both the first and last agreement, on and off the equilibrium path. In characterizing the subgame perfect equilibrium, we begin by looking at the last agreement.

### 3.1 Last agreement

Suppose the buyer and seller 1 already have agreed upon \( q_1 \). Any subgame following this agreement has \( A \) bargaining with only one seller. From Binmore (1987) we know that the quantity is chosen to maximize total surplus of the agreement between \( A \) and 2. Thus, given an agreement between \( A \) and 1 on \( q_1 \), the equilibrium agreement between \( A \) and 2 will maximize total profits to \( A \) and 2 conditional on this agreement. Thus

\[
\max_{q_2} R(q_1, q_2) - R(q_1, 0) - c_2(q_2) \tag{7}
\]

is the surplus \( \pi_2 \) from an agreement between \( A \) and 2. Similarly, given agreement on \( q_2 \) between \( A \) and 2, one can characterize the surplus from an agreement between \( A \) and 1. The optimal quantity is denoted \( q_2(q_1) \). For \( \delta = 1 \), the price is determined as the price solving,

\[(1 - \lambda)(R(q_1, q_2(q_1)) - R(q_1, 0) - p_2q_2) = \lambda(p_2q_2 - c_2(q_2(q_1))) \]

in the last agreement. Then

\[
p_2(q_1) = \frac{(1 - \lambda)(R(q_1, q_2(q_1)) - R(q_1, 0)) + \lambda c_2(q_2(q_1))}{q_2(q_1)}. \tag{8}
\]

Given that production is strictly convex, for any initial agreement on \( q_1 \) or \( q_2 \) there exists a unique \((p_2(q_1), q_2(q_1))\) or \((p_1(q_2), q_1(q_2))\) maximizing last period total surplus.

### 3.2 First agreement

In negotiations between \( A \) and \( i \), the amount at stake is \( \Pi_i + \delta \lambda \pi_j \), regardless of who is proposer. Thus, by Binmore (1987), the quantity in the first agreement will maximize \( \Pi_i + \delta \lambda \pi_j \). For \( A \) and 1 we thus have

\[
q_1^F = \arg \max_{q_1} R(q_1, 0) - c_1(q_1) + \delta \lambda \left(R(q_1, q_2(q_1)) - R(q_1, 0) - c_2(q_2(q_1))\right). \tag{9}
\]

Lemma 13 in the appendix shows that the maximand is strictly concave. Then

\[
\Pi_1 = R(q_1^F, 0) - c_1(q_1^F). \tag{10}
\]
with $\Pi_2$ similarly defined. Note that continuity and strict concavity ensures that $q_1^F$ is a continuous function of $\delta$. The last quantity $q_2$ only depends on $\delta$ indirectly through $q_1^F$. By the maximum theorem, as the surplus in the last agreement is strictly concave in $q_2$, $q_2$ is a continuous function of $q_1^F$ and hence of $\delta$. Hence, $\pi_i$ and $\Pi_i$ are also continuous in $\delta$. A straightforward modification of the proof of Proposition 2 establishes that the conditions in the proposition hold when $\pi_i$ and $\Pi_i$ depend continuously on $\delta$. Hence, $\pi_i$ and $\Pi_i$ are also continuous in $\delta$. A straightforward modification of the proof of Proposition 2 establishes that the conditions in the proposition hold when $\pi_i$ and $\Pi_i$ depend continuously on $\delta$. From now on, we focus on limit equilibrium quantities, i.e., the quantities evaluated at $\delta = 1$. Since $q_1^F$ (and $q_2(q_1^F)$) are continuous functions of the parameters, equilibrium quantities for $\delta$ close to one are close to the limit quantities.

We let $q_i^L$ and $p_i^L$ denote the quantity and price of good $i$ in the last agreement, evaluated at $q_j^F$, i.e., $q_i^L = q_i(q_j^F)$. Also, $p_i^F$ is the first period price. As quantities in the first agreement are uniquely defined, $\pi_2$ is given by

$$\pi_2 = R(q_1^F, q_2^L) - R(q_1^F, 0) - c_2(q_2^L)$$

$$q_2^L = \arg \max_q R(q_1^F, q_2) - R(q_1^F, 0) - c_2(q_2)$$

with $\pi_1$ and $q_1^L$ similarly defined.

### 4 Equilibrium

Before analyzing the equilibria, we need to define strategic discrimination. A natural way of doing this when sellers are symmetric, is to say that it is present when sellers are asymmetrically treated in equilibrium. Production is more asymmetric than in an integrated firm producing efficient quantities. However, to define strategic discrimination when sellers are asymmetric is not as straightforward, since sellers should be asymmetrically treated even in an efficient allocation. We say that sellers are discriminated if they are asymmetrically treated with quantities differing from the efficient. An equilibrium exhibits strategic discrimination, if sellers are discriminated.

The next proposition describes the relationship between equilibrium and efficient quantities.

**Proposition 4** Suppose the buyer agrees first with seller 1. For $\delta$ sufficiently close to 1, if production is strictly concave and goods are substitutes (complements), $q_1^F$ is decreasing (increasing) in $\lambda$ and $q_2^L$ is increasing in $\lambda$ with $q_1^F = q_1$ and $q_2^L = q_2$ for $\lambda = 1$. If goods are independent then $q_1^F = q_1^e$ and $q_2^L = q_2^e$.

Thus, if goods are substitutes, the seller that agrees first produces too much, and the last too little. The intuition is the following. The buyer uses the quantity in the first agreement to affect the price in the second agreement. Suppose the equilibrium agreement is on the efficient
quantities and consider a small increase in the first equilibrium agreement \( q_1 \). This offer is accepted if the surplus offered to the seller is marginally larger than in equilibrium. By the envelope theorem, the effect on the total surplus is small. The change in \( q_1 \) also affects the price in the last agreement. Since the revenues in the last period is half of the surplus in the last period, the effect can be analyzed using (7). The effect is

\[
\frac{\partial R(q_1, q_2)}{\partial q_1} - \frac{\partial R(q_1, 0)}{\partial q_1}
\]

This expression is decreasing in \( q_2 \), leading to a smaller optimal level of \( q_2 \). Hence, when \( q_1 \) increases the surplus in the last agreement decreases and also the payoff for the last seller. Also, the amount paid to the first seller is unaffected by the deviation. Since the price effect is a first-order effect, while the effect on the total surplus is a second order effect, it is optimal for the buyer to increase \( q_1 \). If goods are complements, a similar argument shows that both sellers produce too little. The firm uses quantities strategically to increase profits.

The model has a potential to explain some of the differences in work hours and pay between men and women, assuming that men prefer to work a little more than women. In this setting, employees are the sellers and the firm is the buyer. Introducing small asymmetries in a symmetric setup leads to potentially large asymmetries in treatment of workers. From above, we know that \( q_1^F > q_1^e \) and \( q_2^L < q_2^e \). Hence, the worker that is more/less inclined to work increases/decreases work hours more than what is motivated by efficiency considerations. Thus, the large difference in work hours between men and women could at least partially be caused by strategic effects.

In Proposition 4, the order of agreement is not determined. The following proposition shows that the most efficient order of agreement prevails given that the sellers are almost symmetric. Specifically, let \( c_1(q_1) = \varphi c_1(q_1) \) and \( c_2(q_2) = c(q_2) \) with \( \varphi < 1 \). Let \( \hat{p}(\varphi, \delta) \) be the probability of agreeing with 1 first. Then

**Proposition 5** If goods are substitutes, \( c_1(q_1) = \varphi c_1(q_1) \) with \( \varphi < 1 \) and \( c_2(q_2) = c(q_2) \), then there exists a \( \bar{\varphi} < 1 \) such that \( \lim_{\delta \to 1} \hat{p}(\varphi, \delta) = 1 \) for \( \varphi > \bar{\varphi} \).

In conjunction with Proposition 4, equilibria exhibit strategic discrimination when sellers are sufficiently symmetric. Proposition 4 shows that quantities differ from the efficient outcome when production is strictly convex both when goods are substitutes and complements. If convexity is not strict, efficiency can be restored.

---

6It will be shown in the next section that with linear marginal cost, the buyer always agrees first with the low cost seller.
Proposition 6 If goods are perfect substitutes, $R$ strictly concave in aggregate quantities and sellers have constant marginal costs $d_1$ and $d_2$, for $\delta$ sufficiently close to 1, the equilibrium outcome is efficient with positive quantities only in the first agreement. If $d_1 \leq d_2$, agreement is with 1 first, giving 1 a unit profit of at most $d_2 - d_1 \geq 0$.

The proposition implies that if supply is perfectly elastic, we get a result similar to Bertrand, with two symmetric sellers. If asymmetric, the low cost seller receives positive profits. At most, the low cost seller can capture the entire difference in cost between the two sellers, as in Bertrand competition. When sellers are symmetric, both receive zero profit.

In Segal (1999) analyzing bilateral contracting with externalities, it is shown that aggregate quantities are too high. It is assumed that welfare depends only on aggregate quantities, implying that goods are perfect substitutes and marginal costs linear. As Proposition 6 shows, our results are in stark contrast to Segal (1999).

The propositions above can be used to derive some qualitative comparative statics results. More specifically, we want to study how quantities, profits and efficiency varies with substitutability, relative slope of supply and demand and the degree of asymmetry between sellers.

In Proposition 6, we have efficiency with completely elastic supply. Also, efficiency holds if supply is completely inelastic, as there is no scope for strategic discrimination through reallocating purchases from one seller to the other. By Proposition 2, we see that for intermediate values however, we do not get efficiency.

To see that efficiency is not monotonic in the degree of substitutability in general, consider constant marginal cost of production, Proposition 6 shows that the outcome is efficient with perfect substitutes. Also, we have efficiency when goods are independent. If goods are not perfect substitutes, however, inefficiency arises.

Elasticity of supply affects the distribution of surplus. When goods are perfect substitutes and supply is completely elastic, Proposition 6 shows that the firm obtains all the gains of trade (with symmetric sellers). If supply is completely inelastic, however, the standard results of Horn & Wolinsky hold: sellers are treated equally with both obtaining a positive share of the surplus.

To obtain results on how efficiency and unequal treatment varies with the degree of substitutability, seller asymmetries and relative slopes, we have to make more specific assumption on functional forms. In section 4.1, such specific assumptions are made.

4.1 Strategic Discrimination and Inefficiency

The qualitative results of the previous section can be strengthened by more specific assumptions on revenue and cost functions. In the special case of linear marginal cost functions, comparative
statics of the determinants of strategic discrimination can be analyzed.

We wish to study how strategic discrimination depends on exogenous parameters, such as marginal costs, substitutability and asymmetries between sellers. As relative bargaining power will not be the focus in the following, we will assume that \( \lambda = \frac{1}{2} \). In light of the definition in section 4, we say that strategic discrimination increases in a parameter \( \xi \) if \( q^F_1(\xi) / q^e_1(\xi) \) is increasing in \( \xi \) and \( q^L_2(\xi) / q^e_2(\xi) \) is decreasing in \( \xi \).

To study how equilibrium surplus depends on parameters is slightly more complicated than just analyzing the effect on the equilibrium surplus \( \Pi_1 + \pi_2 \), as changes in parameters affect the efficient surplus as well. Therefore, we look how the ratio of equilibrium to efficient surplus

\[
\rho_e = \frac{\Pi_1 + \pi_2}{\Pi^e_1 + \pi^e_2}
\]

varies as we change parameters.

Let revenue and cost functions be given by

\[
R(q_1, q_2) = r(q_1 + q_2) - \frac{1}{2} \left( q_1^2 + 2s q_1 q_2 + q_2^2 \right)
\]

and

\[
c_1(q_1) = \frac{c}{1 - \theta} q_1^2
\]

\[
c_2(q_2) = \frac{c}{\theta} q_2^2.
\]

As \( \theta \to 0 \), only seller 1 is selling, with \( \theta = 1/2 \), sellers are symmetric. The relative slope of supply and demand is given by \( c \). Solving for equilibrium quantities gives, when 1 is first,

\[
q^F_1 = r \frac{(2c + (2 - s) \theta)(1 - \theta)}{2c + 2c^2 + \theta (2 - s^2)(1 - \theta)}
\]

\[
q^L_2 = 2r \frac{(c + (1 - s)(1 - \theta)) \theta}{2c + 2c^2 + \theta (2 - s^2)(1 - \theta)}.
\]

The next proposition shows that, with linear marginal cost, we can say more than in Proposition 4. Agreement will be first with the seller with lower marginal cost and the quantity in the first agreement is higher than the efficient and the last quantity smaller. Strategic discrimination is high if supply is elastic, goods are easily substitutable and sellers relatively symmetric.

**Proposition 7** If goods are substitutes and \( \theta < \frac{1}{2} \) A and 1 agree first as \( \delta \to 1 \). Also:

1. Strategic discrimination increases
(a) as the slope of supply relative to demand \( c \), decreases.

(b) as the homogeneity of goods \( s \) increases for \( c > 1 \).

(c) as the symmetry of sellers \( \theta \) increases.

2. Efficiency increases

(a) as the slope of supply relative to demand \( c \) increases, for \( c > 1 \).

(b) as the homogeneity of goods \( s \) decreases, for \( c > 1 \).

(c) as the symmetry of sellers \( \theta \) decreases.

In the appendix, only the result on the order of agreement is proved. As the proof of the rest of the proposition involves tedious algebra, it is available separately in Björnerstedt & Westermark (2006b).

The degree of strategic discrimination decreases with the slope of supply relative to demand. The slope of supply relative to demand can be interpreted as the disutility of effort for the sellers. Reallocation from one seller to the other costs more. With linear marginal costs, for sufficiently inelastic supply, efficiency increases in the slope of supply relative to demand.

Strategic discrimination increases and efficiency decreases in the degree of substitutability if the slope of supply relative to demand is high enough. The higher the degree of substitutability, the lower the cost of using quantities strategically, implying more strategic discrimination. Also, strategic discrimination increases and efficiency decreases in the symmetry of sellers. If \( \theta = 0 \) there is just one seller and standard results shows, e.g. Binmore (1987) that we get efficiency. If sellers are symmetric, we get inefficiencies. As the proposition shows, the relationship is monotonic.

4.2 Do workers want to work less?

With quadratic production, in the context of firm/worker interaction, further implications of the model can be drawn. If workers are perfect substitutes, the wage paid to 1 is higher than the wage paid to 2. Although 2 would obviously prefer to obtain the higher wage, she would not prefer \((p_1, q_1)\) to \((p_2, q_2)\). In addition, if workers are relatively symmetric, given the hourly wages \( p_1 \) and \( p_2 \), worker 1 will want to work less.

First we show that \( p_1^F > p_2^L \). Using (8), we have

\[
p_2 = \frac{c r (3 c + \theta)}{2 (2 c + 2 c^2 + \theta - \theta^2)}.
\]
Using Proposition 3 and the payoff function of worker 1 and using $\Pi_1$, $\Pi_2$ and $\pi_2$ from (45) in the appendix gives
\[
p_1 = \frac{cr \left( 6 c^2 + (1 - \theta) \theta + 2 c (1 + \theta) \right)}{2 \left( 2 c + \theta \right) \left( 2 c + 2 c^2 + \theta - \theta^2 \right)}.
\] (14)

Combining (13) and (14), we get
\[
p_1 - p_2 = \frac{cr}{2} \left( \theta + 2c \right) \left( 1 - 2\theta \right) + c\theta > 0.
\]

Also, 1 gets a larger share of the surplus. Using Proposition 2, the payoff to 1 is given by $\frac{\pi_1}{2} + \varepsilon$ compared with $\frac{\pi_2}{2}$ for 2.

An implication of the model when the workers are sufficiently symmetric, is that, given the hourly wages $p_1$ and $p_2$, worker 1 will want to work less. Worker 2 will want to work more or less at $p_2$, depending on whether the relative slope of aggregate demand $c$ is smaller or greater than $\theta$. To see this, note that as $\theta < \frac{1}{3}$, the firm agrees first with worker 1 in equilibrium. The effect of an increase in work hours on the utility of 1, taking prices as given, is $p_1 - \frac{c}{1-\theta}q_1$. Using (12) and (14) gives
\[
p^{F}_1 - \frac{c}{1-\theta}q^{F}_1 = -cr \frac{2c^2 + (2c + \theta) (3\theta - 1)}{2 \left( 2 c + \theta \right) \left( 2 c + 2 c^2 + \theta - \theta^2 \right)}.
\]

Thus, for $\theta \geq \frac{1}{3}$ the expression above is negative. Hence, an increase in work hours $q_1$ taking wages $p_1$ as given leads to a decrease in payoff. Hence, worker 1 would prefer to work less at $p_1$ for $\theta \geq \frac{1}{3}$. The effect on the utility of worker 2 of a change in working time, taking wages $p_2$ as given, is $p_2 - \frac{c}{\theta}q_2$. Similar calculations using (12) and (13) gives
\[
p^{L}_2 - \frac{c}{\theta}q^{L}_2 = -cr \frac{c - \theta}{2 \left( 2 c + 2 c^2 + \theta - \theta^2 \right)}.
\]

The sign of this expression depends on the relationship between $c$ and $\theta$. If $\theta > c$ the expression above is positive and hence worker 2 would prefer to work more and if $\theta < c$ worker 2 would prefer to work less at $p_2$.

Although worker 2 works too much or too little depending on whether $\theta > c$, it seems reasonable to expect that, in a large economy, a share of the type 2 workers perceive that they work too much. Then, in the economy as a whole, more workers perceive that they work too much rather than too little.

This is in line with empirical evidence in Bell & Freeman. Evidence from German GSOEP data indicates that workers feel that the actual working time is larger that the desired working time, taking the effect of reduced pay into account. The workers were asked the following
question: “If you could choose the extent of your hours at work, taking into account that your earnings would change corresponding to the time, how many hours would you work?” The difference between actual and desired hours is significant. The average difference for all workers is approximately four to five hours per week, with a slightly larger difference for men and smaller for women.

4.3 Robustness - Simultaneous proposals

One potential problem with the previous analysis is that agreement on contracts are assumed to be made sequentially. The Stackelberg outcome would then follow from the sequentiality of agreements. We show that this is not the case and the same result holds with simultaneous bids, when the sellers are symmetric and $\lambda = \frac{1}{2}$. By considering this special case, we see that the result is not simply due to the assumed timing of the model.

Consider two sellers negotiating with a buyer $A$. We assume that in odd periods, $A$ proposes to 1 and 2 proposes to $A$, and in even periods 1 proposes to $A$ and $A$ proposes to 2. Both recipients are assumed to observe both bids.

We first argue that there cannot be an equilibrium where the buyer agrees on the efficient quantities with the two sellers. Too see this, consider an equilibrium on the efficient quantities with $\delta$ close to one. Note that at least one of the sellers must get a payoff that is at least equal to $\frac{\pi_i}{2}$. Then consider the following deviation by strategy by $A$. In the subgame where $A$ proposes to seller $j \neq i$, $A$ offers the continuation payoff of the seller plus $\epsilon$, i.e., $w_j + \epsilon$, combined with the Stackelberg leader quantity $q^L_j$. The buyer rejects the proposal of the other seller. In the other type of subgame, $A$ makes unacceptable proposals and rejects any offer. Following acceptance of the leader quantity, we have a standard Rubinstein-Ståhl game where the parties then must agree on the follower quantity. Then in the limit seller $i$ gets $\frac{\pi_i}{2}$. The payoff of the buyer is

$$\Pi_j + \frac{\pi_i}{2} - \frac{\pi_i}{2} - (w_j + \epsilon)$$

compared with the equilibrium payoff of at most

$$\Pi_i^c + \frac{\pi_i^c}{2} - \frac{\pi_i^c}{2} - w_j,$$

where we use that $\pi_i^c = \pi_j^c$. By choice of $q^L_j$ and $q_i^F$ we have $\Pi_j + \frac{\pi_i}{2} > \Pi_i^c + \frac{\pi_i^c}{2}$ and hence the deviation is profitable.

Now consider the set of equilibria of the game. In equilibrium $A$ makes an acceptable bid on the Stackelberg leader contracts in each period. Subsequently, in the next period $A$ makes
an acceptable bid to the remaining seller. The outcome is thus the same as in the sequential analysis.

The following proposition shows that there is a stationary SPE that is similar in flavour to the equilibrium in the original model.

**Proposition 8** With symmetric sellers and $\lambda = \frac{1}{2}$, for $\delta$ close to one, there is a SSPE where $A$ always makes an accepted bid and the other negotiation is concluded in the following period. As $\delta \to 1$, equilibrium payoffs correspond to the immediate agreement equilibrium $I$ in Proposition 3 and quantities to Proposition 4.

Thus, when sellers are identical, the buyer offers the leader quantity to the first seller it makes a proposal to and rejects the offer by the other seller. In the next period, the buyer and the remaining seller agree on the follower quantity. Non-agreement with 2 can be sustained, as if 2 makes an acceptable bid, it is profitable for 1 to reject the bid received from $A$. If 2 offers $q_2^L$, and $A$ accepts, then by rejecting 1 gets an half of the bilateral surplus given $q_2^L$. Since $q_2^L$ is small, half of the bilateral surplus is larger than the equilibrium payoff.

## 5 Concluding remarks

With simultaneous negotiations and observable bids, we show that the buyer can obtain higher payoffs by strategically discriminating sellers. Whether firms actually do so is an open question.

The outcome implies treating symmetric sellers differently. Concerns of social norms, team building etc. might in practice put limits on the scope of such strategies.

The possibility of arbitrage might also affect the results. In the model it is assumed that 1 and 2 cannot agree upon letting 2 do some of the work of 1. However, arbitrage between workers seems to be uncommon. The employer normally decides not only what work is to be done, but also who does it. Arbitrage can thus be hindered. In applying the theory to questions of industrial organization this might be of greater concern.

It should be noted that the buyer will not have an incentive to renegotiate the contract with 1 to reduce the quantity agreed upon. Although quantities would be more efficient, in bargaining with seller 1 again, the buyer has to split surplus equally. This will not be beneficial for the buyer. The equilibrium is thus renegotiation proof in some sense.
A Proofs

To show Proposition 2, we first characterize the different equilibrium types in Lemmas 9 - 12. The proposition then collects the conditions of these Lemmas, letting $\delta \to 1$.

A.1 Pure equilibria

We first analyze pure strategy equilibria and then turn to mixed strategy equilibria. We focus on stationary subgame perfect equilibria (SSPE). Let

$$
\varepsilon_i(\delta) = \delta (\Pi_i + \delta \pi_j) - (\Pi_j + \delta \pi_i) \quad (15)
$$

$$
\gamma_i(\delta) = \Pi_i - \delta^2 (1 - \lambda) \pi_i \quad (16)
$$

Note first that there is no equilibrium where $\sigma_1 = 0$ and $\sigma_2 = 0$. Using this in the value equations implies that $V_i = W_i = v_i = w_i = 0$ for $i = 1, 2$. Then any proposer has a profitable deviation by offering slightly more than zero. Similarly, there is no equilibrium with $0 < \sigma_i < 1$ and $\sigma_j = 0$. Using this in the value equations for $i$ gives $V_i = W_i = v_i = w_i = 0$. Indifference when $i$ proposes gives $w_i = \Pi_i + \delta \lambda \pi_j - W_i = \Pi_i + \delta \lambda \pi_j$, contradicting $w_i = 0$.

Let

$$
A_i(\delta) = \frac{\delta \lambda (\varepsilon_i(\delta) - \delta (1 - \delta^2) (1 - \lambda) \pi_i)}{1 - \delta^2 (1 - \lambda)^2}
$$

$$
a_i(\delta) = \frac{\delta \lambda \varepsilon_i(\delta) + (1 - \delta^2) ((\Pi_i + \delta \lambda \pi_j) - \delta^2 \lambda (1 - \lambda) \pi_i)}{1 - \delta^2 (1 - \lambda)^2}
$$

Note that, when $\varepsilon = 0$, we have $\lim_{\delta \to 1} A_i(\delta) = \lim_{\delta \to 1} a_i(\delta) = 0$ as $\lim_{\delta \to 1} \varepsilon_i(\delta) = 0$.

Lemma 9 There is an equilibrium with immediate agreement between $A$ and sellers 1 and 2 in any subgame if $(1 - \delta^2) (1 - \lambda) \gamma_1(\delta) \geq \lambda \varepsilon_2(\delta)$ and $(1 - \delta^2) (1 - \lambda) \gamma_2(\delta) \geq \lambda \varepsilon_1(\delta)$. We have

$$
V_i = \Pi_i + \delta \lambda \pi_j - \delta^2 (1 - \lambda) \pi_i, \quad (17)
$$

$$
W_i = \frac{\delta^2 \lambda (2 - \lambda)}{1 - \delta^2 + \delta^2 \lambda (2 - \lambda)} (\Pi_i + \delta \lambda \pi_j - \delta^2 (1 - \lambda) \pi_i) - A_i(\delta),
$$

$$
v_i = \frac{\delta^2 \lambda (2 - \lambda)}{1 - \delta^2 + \delta^2 \lambda (2 - \lambda)} \delta^2 (1 - \lambda) \pi_i' + a_i(\delta),
$$

$$
w_i = \delta^2 (1 - \lambda) \pi_i
$$

Proof: If an agreement is reached in any meeting between $A$ and the sellers then we have $\sigma_{Ai} = \sigma_{iA} = \sigma_i = 1$. Using this in the value equations (1) and some algebra gives (17).
In order for it to be an equilibrium, no player should have an incentive to set the probability \( \sigma_{Ai} \) or \( \sigma_{iA} \) less than 1. This is true if \( W_i \leq V_i \) and \( w_i \leq v_i \) for \( i = 1, 2 \) or, since \( v_i - w_i = V_i - W_i \),

\[
V_i \geq \frac{\delta \lambda}{1 - \delta^2 (1 - \lambda)} V_j \\
V_j \geq \frac{\delta \lambda}{1 - \delta^2 (1 - \lambda)} V_i
\]

Using the solution for \( V_i \) and \( V_j \) gives

\[
(1 - \delta^2) (\Pi_i - \delta^2 (1 - \lambda) \pi_i) \geq \frac{\lambda}{1 - \lambda} (\delta (\Pi_j + \delta \pi_i) - (\Pi_i + \delta \pi_j))
\]

This condition holds if \( (1 - \delta^2) (1 - \lambda) \gamma_i (\delta) \geq \lambda \varepsilon_j (\delta) \) for \( i = 1, 2 \).■

**Lemma 10** There is an equilibrium where an agreement is reached first between the buyer and seller \( i \) if \( \lambda \varepsilon_i (\delta) \geq (1 - \lambda) \gamma_j (\delta) \). We have

\[
V_i = (1 - \delta^2 (1 - \lambda)) (\Pi_i + \delta \lambda \pi_j) \\
W_i = \delta^2 \lambda (\Pi_i + \delta \lambda \pi_j) \\
v_i = (1 - \delta^2 \lambda) (\Pi_i + \delta \lambda \pi_j) \\
w_i = \delta^2 (1 - \lambda) (\Pi_i + \delta \lambda \pi_j) \\
v_j = w_j = \delta^2 (1 - \lambda) \pi_j.
\]

**Proof:** Using the value equations, setting \( \sigma_i = 1 \) and \( \sigma_j = 0 \) gives (18). Since \( \frac{\delta^2 \lambda}{1 - \delta^2 (1 - \lambda)} < 1 \) and \( \frac{\delta^2 (1 - \lambda)}{1 - \delta^2 \lambda} < 1 \) we have \( V_i \geq W_i \) and \( v_i \geq w_i \). Thus, both the buyer and seller \( i \) find it profitable to make acceptable offers. In order for \( A \) not to want to bid to \( j \), the value to \( A \) has to be less than waiting, i.e.\( \Pi_j + \delta \lambda \pi_i - w_j \leq W_j \) or, using that \( W_j = \delta \lambda (\Pi_i + \delta \lambda \pi_j) \):

\[
\delta \lambda (\Pi_i + \delta \pi_j) - \lambda (\Pi_j + \delta \pi_i) \geq (1 - \lambda) (\Pi_j - \delta^2 (1 - \lambda) \pi_j).
\]

This condition holds if \( \lambda \varepsilon_i (\delta) \geq (1 - \lambda) \gamma_j (\delta) \). As the deviation condition when \( j \) is the proposer is identical, it is satisfied in this case also.■

### A.2 Mixed equilibria

The following lemmas state conditions for existence of mixed equilibria. Note that \( \lim_{\delta \to 1} \sigma_i = 0 \).
Lemma 11 There is a SSPE with $0 < \sigma_1 < 1$ and $0 < \sigma_2 < 1$ if

$$(1 - \delta^2) (1 - \lambda) \pi_i - \delta \lambda \pi_j > \gamma_i (\delta)$$

for $i = 1, 2$. We have

$$\sigma_i = (1 - \delta^2) \frac{\Pi_j + \delta \lambda \pi_i}{\delta^2 \left( (1 - \lambda) \pi_j - (\Pi_j + \delta \lambda \pi_i) \right)}.$$  \hspace{1cm} (19)

and

$$V_i = W_i = 0 \hspace{1cm} (20)$$

$$v_i = w_i = \Pi_i + \delta \lambda \pi_j.$$

Proof: Both sellers are indifferent between making an acceptable offer or not, and thus

$$w_1 = \Pi_1 + \delta \lambda \pi_2 - W_1$$
$$w_2 = \Pi_2 + \delta \lambda \pi_1 - W_2.$$  

Solving the value equations (1) for $V_i$ and $W_i$,

$$V_1 = W_1 = \delta \left( \lambda V_2 + (1 - \lambda) W_2 \right),$$
$$V_2 = W_2 = \delta \left( \lambda V_1 + (1 - \lambda) W_1 \right),$$

which implies that $V_i = \delta^2 V_i$, which cannot hold, unless $V_i = 0$. Using this and the other equations in (1) gives (19) and (20). If the denominator in the last equation is positive, i.e.

$$(1 - \delta^2) (1 - \lambda) \pi_i - \delta \lambda \pi_j > \Pi_i - \delta^2 (1 - \lambda) \pi_i = \gamma_i (\delta)$$

for $i = 1, 2$ and $j \neq i$ then a mixed equilibrium exists.

Lemma 12 If

$$(1 - \delta^2) \gamma_j (\delta) < \frac{\lambda}{1 - \lambda} \varepsilon_i (\delta) \hspace{1cm} \text{and} \hspace{1cm} \gamma_j (\delta) > \frac{\lambda}{1 - \lambda} \varepsilon_i (\delta) > 0 \hspace{1cm} (21)$$

$$(1 - \delta^2) \gamma_j (\delta) > \frac{\lambda}{1 - \lambda} \varepsilon_i (\delta) \hspace{1cm} \text{and} \hspace{1cm} - \delta \lambda \pi_i \leq \gamma_j (\delta) < \frac{\lambda}{1 - \lambda} \varepsilon_i (\delta) < 0$$

there is a mixed SSPE where the buyer agrees with probability one with seller $i$ and with probability
\[ \sigma_j \in (0,1) \text{ with seller } j \text{ where} \]
\[ \sigma_j = \frac{1 - \delta^2 (1 - \lambda) \gamma_j(\delta) - \lambda \varepsilon_i(\delta)}{\delta^2 \lambda \varepsilon_i(\delta)} \]

If \( \gamma_j(\delta) = \varepsilon_i(\delta) = 0 \) for any probability \( \sigma_j \in (0,1) \) there is a mixed SSPE where the buyer agrees with probability one with seller \( i \) and with \( \sigma_j \) with seller \( j \). We have

\[ V_i = W_i + \frac{1 - \delta^2}{\delta^2 \lambda} W_i, \quad (22) \]
\[ W_i = \delta V_j = \delta W_j = \delta (\Pi_j + \delta \lambda \pi_i - \delta^2 (1 - \lambda) \pi_j), \]
\[ v_i = \Pi_i + \delta \lambda \pi_j - W_i, \]
\[ w_i = \Pi_i + \delta \lambda \pi_j - W_i - \frac{1 - \delta^2}{\delta^2 \lambda} W_i, \]
\[ v_j = w_j = \delta^2 (1 - \lambda) \pi_j. \]

**Proof:** In order for a \( A \) to mix with \( j \), we must have

\[ V_j = W_j = \Pi_j + \delta \lambda \pi_i - w_j \]

Similarly we have \( v_j = w_j \). Using this in the expression for \( W_i \) gives

\[ W_i = \delta (\lambda V_j + (1 - \lambda) W_j) = \delta W_j = \delta (\Pi_j + \delta \lambda \pi_i - w_j) \]

Then, using \( \sigma_i = 1 \) and solving for the values in (1), we have (22). Note that

\[ ((1 - \lambda) \pi_i - ((1 - \lambda) v_i + \lambda w_i)) = -\frac{1}{\delta} \varepsilon_i(\delta) \quad (23) \]
\[ \delta^2 \left( ((1 - \lambda) v_i + \lambda w_i) - w_i \right) = (1 - \delta^2) \frac{1}{\delta \lambda} \left( (1 - \lambda) \gamma_j(\delta) - \lambda (\varepsilon_i(\delta)) \right) \quad (24) \]

We can solve for \( \sigma_j \), using the expression for \( w_i \) in (1). If \( \varepsilon_i(\delta) \neq 0 \) we have

\[ \sigma_j = \frac{1 - \delta^2 (1 - \lambda) \gamma_j(\delta) - \lambda \varepsilon_i(\delta)}{\delta^2 \lambda \varepsilon_i(\delta)} \]

If \( \varepsilon_i(\delta) > 0 \), to ensure \( \sigma_j > 0 \) we require \( (1 - \lambda) \gamma_j(\delta) > \lambda \varepsilon_i(\delta) \). Also, for \( \sigma_j < 1 \), we must have

\[ (1 - \delta^2) (1 - \lambda) \gamma_j(\delta) < \lambda \varepsilon_i(\delta) \quad (25) \]

A similar argument takes care of the case when \( \varepsilon_i(\delta) < 0 \).
Assume now that \( \varepsilon_i(\delta) = 0 \). Then, using (23),

\[
w_i = \delta^2 \left( \sigma_j \left( (1 - \lambda) \pi_i - ((1 - \lambda) v_i + \lambda w_i) \right) + (1 - \lambda) v_i + \lambda w_i \right) \]

\[
= \delta^2 \left( (1 - \lambda) v_i + \lambda w_i \right)
\]

Then any \( \sigma_j \in (0, 1) \) satisfies the above expression. Also, from (24) we have \( \gamma_j(\delta) = 0 \).

Acceptable offers between \( A \) and \( i \) will be preferred, since \( \frac{1 - \delta^2 (1 - \lambda)}{\delta^2 \lambda} > 1 \) implies \( V_i > W_i \) and \( v_i > w_i \) whenever \( W_i > 0 \) and \( w_i > 0 \) from (22). Also from (22) we have

\[
\frac{W_i}{\delta} = \gamma_j(\delta) + \delta \lambda \pi_i,
\]

and \( W_i \geq 0 \) iff

\[
\gamma_j(\delta) + \delta \lambda \pi_i \geq 0
\]

If \( \varepsilon_i(\delta) \geq 0 \), then from (21) \( \gamma_j(\delta) \geq 0 \), and if \( \varepsilon_i(\delta) < 0 \), then \( \gamma_j(\delta) \geq -\delta \lambda \pi_i \) by assumption. To see that \( w_i > 0 \), consider

\[
w_i = \frac{\delta}{\delta^2 \lambda} \left( \lambda \varepsilon_i(\delta) - (1 - \lambda) (\gamma_j(\delta)) \right) + \frac{\delta^2 (1 - \lambda)}{\delta^2 \lambda} (\delta (\gamma_j(\delta) + \delta \lambda \pi_i))
\]

\[
= \frac{\delta}{\delta^2 \lambda} \left( \lambda \varepsilon_i(\delta) - (1 - \delta^2) (1 - \lambda) \gamma_j(\delta) \right) + \delta^2 (1 - \lambda) \pi_i.
\]

If \( \varepsilon_i(\delta) < 0 \) then \( (1 - \lambda) \gamma_j(\delta) < \lambda \varepsilon_i(\delta) \) and hence \( w_i > 0 \) if \( W_i \geq 0 \). If \( \varepsilon_i(\delta) \geq 0 \) then \( (1 - \delta^2) (1 - \lambda) \gamma_j(\delta) \leq \lambda \varepsilon_i(\delta) \) and hence \( w_i > 0 \). Thus in all cases, neither \( A \) in bidding to \( i \) or seller \( i \) have incentives to deviate. ■

When \( \lambda \geq \frac{1}{2} \) the condition \( -\delta \lambda \pi_i \leq \gamma_j(\delta) \) in the statement of the Lemma is redundant, as

\[
\gamma_j(\delta) + \delta \lambda \pi_i = -\lambda \varepsilon_i(\delta) - \delta^2 (1 - 2\lambda) \pi_j + (1 - \lambda) \Pi_j + \lambda \delta \Pi_i
\]

To simplify expressions, let \( \Delta = \frac{1}{1 - \delta^2} \) and \( \Lambda = \frac{\lambda}{1 - \lambda} \).

**Proof of Proposition 2.**

**Step 1:** Collecting results Lemmas 9 - 12.

Rewriting \( \varepsilon_i(\delta) \) as polynomials in \( \delta \)

\[
\varepsilon_1(\delta) = \pi_2 \delta^2 + (\Pi_1 - \pi_1) \delta - \Pi_2
\]

\[
\varepsilon_2(\delta) = \pi_1 \delta^2 + (\Pi_2 - \pi_2) \delta - \Pi_1
\]

24
As $\varepsilon_2(\delta)$ is strictly convex with $\varepsilon_2(0) = -\Pi_1$ and $\varepsilon_2(1) = -\varepsilon$, we have $\varepsilon_2(\delta) < 0$ for all $\delta$. Similarly, $\varepsilon_1(0) = -\Pi_2$ and $\varepsilon_1(1) = \varepsilon$. Also, for any $\Pi_i$ and $\pi_i$ such that $\varepsilon > 0$ there exists a unique $\delta < 1$ such that $\varepsilon_1(\delta) = 0$ and $\varepsilon_1(\delta) > 0$ for all $\delta > \delta$.

Let

$$a_i = -\delta \lambda \pi_i$$

$$b_i = a_i + (1 - \delta^2)(1 - \lambda) \pi_j$$

$$c_i = \Lambda \varepsilon_i(\delta)$$

$$d_i = \Delta c_i$$

**Case 1:** Suppose $\lambda \geq \frac{1}{2}$.

We can summarize the equilibria in Lemmas 9 - 12 in terms of the following table, using that $\varepsilon_i(\delta) > 0$ implies $\Delta \varepsilon_i(\delta) > \varepsilon_i(\delta)$ and $\varepsilon_i(\delta) < 0$ implies $\Delta \varepsilon_i(\delta) < \varepsilon_i(\delta)$,

<table>
<thead>
<tr>
<th>$\gamma_1(\delta)$</th>
<th>$\gamma_2(\delta)$</th>
<th>$\gamma_2(\delta)$</th>
<th>$\gamma_2(\delta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\in (-\infty, c_1)$</td>
<td>$\in (c_1, d_1)$</td>
<td>$\in [d_1, \infty)$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_1(\delta) \in (-\infty, d_2)$</td>
<td>$P_1, P_2$</td>
<td>$P_2, M_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>$\gamma_1(\delta) \in [d_2, c_2]$</td>
<td>$P_1, P_2, M_2$</td>
<td>$P_2, M_1, M_2$</td>
<td>$I, M_2, P_2$</td>
</tr>
<tr>
<td>$\gamma_1(\delta) \in (c_2, \infty)$</td>
<td>$P_1$</td>
<td>$M_1$</td>
<td>$I$</td>
</tr>
</tbody>
</table>

**Case 2:** Suppose $\lambda < \frac{1}{2}$.

Since $\varepsilon > 0$, there exists a $\delta < 1$ such that

$$-\delta \lambda \pi_1 + (1 - \delta^2)(1 - \lambda) \pi_2 < \Lambda \varepsilon_1(\delta) < \Delta \Lambda \varepsilon_1(\delta)$$

and, if $-\delta \lambda \pi_2 < \Lambda \varepsilon_2(\delta)$ then

$$\Delta \Lambda \varepsilon_2(\delta) < -\delta \lambda \pi_2 < -\delta \lambda \pi_2 + (1 - \delta^2)(1 - \lambda) \pi_1 < \Lambda \varepsilon_2(\delta)$$

and, if $-\delta \lambda \pi_2 \geq \Lambda \varepsilon_2(\delta)$ then

$$\Delta \Lambda \varepsilon_2(\delta) < \Lambda \varepsilon_2(\delta) \leq -\delta \lambda \pi_2 + (1 - \delta^2)(1 - \lambda) \pi_1.$$
We can summarize the equilibria in Lemmas 9 - 12 in terms of the following tables

If $-\delta \lambda \pi_2 < \Lambda \varepsilon_2 (\delta)$

<table>
<thead>
<tr>
<th>$\gamma_1 (\delta)$</th>
<th>$\gamma_2 (\delta)$</th>
<th>$\gamma_2 (\delta)$</th>
<th>$\gamma_2 (\delta)$</th>
<th>$\gamma_2 (\delta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\in (-\infty, d_2)$</td>
<td>$P_1, P_2, H$</td>
<td>$P_1, P_2$</td>
<td>$P_2, M_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>$\in [d_2, a_2)$</td>
<td>$P_1, P_2, H$</td>
<td>$P_1, P_2$</td>
<td>$P_2, M_1$</td>
<td>$I, P_2$</td>
</tr>
<tr>
<td>$\in [a_2, b_2]$</td>
<td>$P_1, P_2, M_2, H$</td>
<td>$P_1, P_2, M_2$</td>
<td>$P_2, M_1, M_2$</td>
<td>$I, P_2, M_2$</td>
</tr>
<tr>
<td>$(c_2, \infty)$</td>
<td>$P_1, P_2, M_2$</td>
<td>$P_1, P_2, M_2$</td>
<td>$P_2, M_1, M_2$</td>
<td>$I, M_2, P_2$</td>
</tr>
</tbody>
</table>

If $-\delta \lambda \pi_2 \geq \Lambda \varepsilon_2 (\delta)$

<table>
<thead>
<tr>
<th>$\gamma_1 (\delta)$</th>
<th>$\gamma_2 (\delta)$</th>
<th>$\gamma_2 (\delta)$</th>
<th>$\gamma_2 (\delta)$</th>
<th>$\gamma_2 (\delta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\in (-\infty, d_2)$</td>
<td>$P_1, P_2, H$</td>
<td>$P_1, P_2$</td>
<td>$P_2, M_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>$\in [d_2, c_2]$</td>
<td>$P_1, P_2, H$</td>
<td>$P_1, P_2$</td>
<td>$P_2, M_1$</td>
<td>$I, P_2$</td>
</tr>
<tr>
<td>$\in (c_2, b_2)$</td>
<td>$P_1, H$</td>
<td>$P_1$</td>
<td>$M_1$</td>
<td>$I$</td>
</tr>
<tr>
<td>$(b_2, \infty)$</td>
<td>$P_1$</td>
<td>$M_1$</td>
<td>$I$</td>
<td></td>
</tr>
</tbody>
</table>

**Step 2:** Taking limits. When $\varepsilon > 0$, as $\delta \to 1$ we have $\Delta \to \infty$, and hence $d_1 = \Delta \varepsilon_1 (\delta) \to \infty$, $d_2 = \Delta \varepsilon_2 (\delta) \to -\infty$ and $a_2 \to b_2$, eliminating some intervals.

We need to find what we converge to if $\lambda \varepsilon = (1 - \lambda) \gamma_2$ or $\lambda \varepsilon = - (1 - \lambda) \gamma_1$ above, i.e., the boundary cases in proposition 2. If $\lambda \varepsilon = (1 - \lambda) \gamma_2$ then $(1 - \lambda) \gamma_2 (\delta) > \lambda \varepsilon_1 (\delta)$ for all $\delta < 1$ as

$$(1 - \lambda) \gamma_2 (\delta) - \lambda \varepsilon_1 (\delta) = (1 - \lambda) \left( (1 - \delta^2) (1 - \lambda) \pi_2 \right) + \lambda (1 - \delta) (\Pi_1 + \delta \pi_2) > 0.$$  

By the exact same argument, we get $(1 - \lambda) \gamma_1 (\delta) > \lambda \varepsilon_2 (\delta)$ for all $\delta < 1$. Also, when $-\lambda \pi_1 = \gamma_2$ we get

$$\gamma_2 (\delta) - ( - \lambda \pi_1 + (1 - \delta^2) (1 - \lambda) \pi_2 ) = \Pi_2 - (1 - \lambda) \pi_2 - (1 - \delta) \lambda \pi_1 - \lambda \pi_1 = - (1 - \delta) \lambda \pi_1 < 0$$

for all $\delta < 1$. A similar argument establishes the corresponding inequality when $-\lambda \pi_2 = \gamma_1$

When $\lambda \geq \frac{1}{2}$ we have $-\delta \lambda \pi_2 \leq \gamma_1 (\delta)$ since, using $\varepsilon_2 (\delta) < 0$, we have

$$\gamma_1 (\delta) + \delta \lambda \pi_2 = - \lambda \varepsilon_2 (\delta) - \delta^2 (1 - 2\lambda) \pi_1 + (1 - \lambda) \Pi_1 + \lambda \delta \Pi_2 > 0$$

When $\varepsilon = 0$ the Lemmas 9 - 12 can be used for a characterization. It is complicated to summarize the results in tables, since when $\lambda < \frac{1}{2}$ there are four different cases, as compared
to the two in Proposition 2. The complexity when \( \lambda < \frac{1}{2} \) is due to the fact that we can have \( \Delta \Lambda \varepsilon_i (\delta) \leq \delta \lambda \pi_i \) for \( i = 1, 2 \). This leads to four possible interval configurations. The case when \( \lambda \geq \frac{1}{2} \) can be more conveniently illustrated. Using a similar argument as in the proof of Proposition 2 we have \( \varepsilon_i (\delta) < 0 \) for \( \delta < 1 \), \( \varepsilon_i (1) = 0 \) and hence \( \Delta \varepsilon_i (\delta) < \varepsilon_i (\delta) \) when \( \delta < 1 \) for \( i = 1, 2 \). Also, when \( \varepsilon = 0 \), \( \lim_{\delta \to 1} \Delta \varepsilon_i (\delta) = -\frac{\Pi_i + \pi_i}{\delta} \) and \( \gamma_i (\delta) > \Delta \Lambda \varepsilon_j (\delta) \) for \( \delta \) close to 1, as

\[
\lim_{\delta \to 1} \gamma_i (\delta) - \Delta \Lambda \varepsilon_j (\delta) = \frac{1}{1 - \lambda} \left( \frac{1 - \lambda}{2} \right) \Pi_i + \left( \frac{\lambda}{2} - (1 - \lambda)^2 \right) \pi_i > 0.
\]

We can summarize the equilibria in Lemmas 9 - 12 as follows, using (27)

<table>
<thead>
<tr>
<th>( \gamma_1 (\delta) )</th>
<th>( \gamma_2 (\delta) )</th>
<th>( \gamma_3 (\delta) )</th>
<th>( \gamma_4 (\delta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \in (\infty, d_1) )</td>
<td>( \in (d_1, c_1) )</td>
<td>( \in (c_1, \infty) )</td>
<td>( \in (\infty, d_1) )</td>
</tr>
<tr>
<td>( P_1,P_2 )</td>
<td>( P_1,P_2,M_1 )</td>
<td>( P_2 )</td>
<td>( P_1,M_2 )</td>
</tr>
<tr>
<td>( \in (d_2,c_2) )</td>
<td>( \in (d_1,c_1) )</td>
<td>( \in (c_1,c_2) )</td>
<td>( \in (c_2,\infty) )</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>( I,P_1,M_1 )</td>
<td>( I )</td>
<td>( I )</td>
</tr>
</tbody>
</table>

A.3 Price quantity agreements

The following lemma is used in Section 3.2.

**Lemma 13** The maximand of (9)

\[
(1 - \delta \lambda) \ R (q_1,0) - c_1 (q_1) + \delta \lambda \ (R (q_1, q_2 (q_1)) - c_2 (q_2 (q_1)))
\]

is strictly concave.

**Proof:** We have a sum of concave functions of the last term in (29) is concave in \( q_1 \). Using the Envelope theorem, the second derivative of of the last term in (29) is

\[
R_{11} (q_1, q_2 (q_1)) + R_{12} (q_1, q_2 (q_1)) \frac{dq_2}{dq_1}
\]

Using

\[
\frac{dq_2}{dq_1} = - \frac{\partial^2 R (q_1,q_2)}{\partial q_1 \partial q_2} - c''_2 (q_2)
\]

and that, by concavity, the denominator of (31) is negative, we have

\[
-R_{11} (q_1, q_2 (q_1)) c''_2 (q_2) + R_{11} (q_1, q_2 (q_1)) R_{22} (q_1, q_2 (q_1)) - R_{12}^2 (q_1, q_2 (q_1)) \geq 0
\]
The first term is positive by the concavity of \( R \) and convexity of \( c_2 \). The latter two terms are positive as the Hessian of \( R \) is positive. Thus the last term in (29) is concave in \( q_1 \). Thus, (29) is strictly concave, as one of first two the terms in (29) is strictly concave.

**Proof of Proposition 4: Step 1.** Assume that the buyer and seller 1 have come to agreement on \( q_1 \). The first-order condition to the problem in (7) is

\[
\frac{\partial R (q_1, q_2)}{\partial q_2} - c'_2 (q_2) = 0
\]

The denominator in (31) is negative by the convexity of production. Thus, the sign is determined by the numerator \( \frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} \). If \( \frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} < 0 \) then \( \frac{\partial R}{\partial q_1} < 0 \) and if \( \frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} > 0 \) then \( \frac{\partial R}{\partial q_1} > 0 \).

**Step 2.** Now, let us analyze bargaining between the buyer and seller 1.

Suppose \( \lambda < 1 \). The first-order condition to the problem in (9) is, using the expression above,

\[
\frac{\partial R (q_1, q_2 (q_1))}{\partial q_1} - (1 - \lambda) \left( \frac{\partial R (q_1, q_2 (q_1))}{\partial q_1} - \frac{\partial R (q_1, 0)}{\partial q_1} \right) - c'_1 (q_1)
\]

Using (33) the above expression is,

\[
\frac{\partial R (q_1, q_2 (q_1))}{\partial q_1} - (1 - \lambda) \left( \frac{\partial R (q_1, q_2 (q_1))}{\partial q_1} - \frac{\partial R (q_1, 0)}{\partial q_1} \right) - c'_1 (q_1)
\]

Evaluating (34) at \( q_1^* \), we have \( q_2 (q_1^*) = q_2^* \) and, using that \( \frac{\partial R (q_1^*, q_2)}{\partial q_2} = c'_1 (q_1^*) \),

\[
\frac{\partial R (q_1^*, q_2^*)}{\partial q_1} - (1 - \lambda) \left( \frac{\partial R (q_1^*, q_2^*)}{\partial q_1} - \frac{\partial R (q_1^*, 0)}{\partial q_1} \right) - c'_1 (q_1^*)
\]

\[
= - (1 - \lambda) \left( \frac{\partial R (q_1^*, q_2^*)}{\partial q_1} - \frac{\partial R (q_1^*, 0)}{\partial q_1} \right) > 0
\]

when goods are substitutes. When evaluated at \( q_1^E \) and \( q_2^E \), expression (34) is zero. As (9) is strictly concave in \( q_1 \), (34) is decreasing in \( q_1 \). Thus \( q_1^E > q_1^* \). Since \( q_1^E > q_1^* \) and \( q_2^E (q_1) < 0 \), we have \( q_2^E < q_2^* \). A similar argument establishes the results when goods are complements and independent, respectively.

When \( \lambda = 1 \) it is easy to see from (34) that the efficient quantities solve the first-order condition. Also, the objective (9) is equal to social surplus.

**Step 3. Monotonicity in \( \lambda \).
Consider $\lambda, \hat{\lambda}$ with $\lambda < \hat{\lambda}$ and let $(q_1^F, q_2^F)$ and $(\hat{q}_1^F, \hat{q}_2^F)$ be the corresponding equilibrium quantities. Define

$$\Theta (q_1^F, \hat{\lambda}) = \frac{\partial R (q_1^F, q_2^F, q_1^F)}{\partial q_1} - (1 - \hat{\lambda}) \left( \frac{\partial R (q_1^F, q_2^F, q_1^F)}{\partial q_1} - \frac{\partial R (q_1^F, 0)}{\partial q_1} \right) - c_1'(q_1^F)$$

We have $\Theta (q_1^F, \lambda) = 0$ from the first-order condition. When $\frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} < 0$ then $\Theta (q_1^F, \hat{\lambda})$ is decreasing in $\hat{\lambda}$ and we have $\Theta (q_1^F, \hat{\lambda}) < 0$ and when $\frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} > 0$ then $\Theta (q_1^F, \hat{\lambda})$ is increasing in $\hat{\lambda}$ and we have $\Theta (q_1^F, \hat{\lambda}) > 0$. Since $\Theta (q_1^F, \hat{\lambda})$ is decreasing in $q_1^F$ it follows that $q_1^F > \hat{q}_1^F$ when $\frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} < 0$ and $q_1^F < \hat{q}_1^F$ when $\frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} > 0$. Also, since we have $\frac{dq_2}{dq_1} < 0$ when $\frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} < 0$ then $q_2^F > \hat{q}_2^F$ and since we have $\frac{dq_2}{dq_1} > 0$ when $\frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} > 0$ then $q_2^F < \hat{q}_2^F$. The relationship with efficient quantities follows from Proposition 4.

**Proof of Proposition 5:**

**Step 1.** Show that $\varepsilon > 0$.

$$\frac{d\varepsilon}{d\varphi} = \frac{d (\Pi_1 + \pi_2)}{d\varphi} - \frac{d (\Pi_2 + \pi_1)}{d\varphi} = \left( \frac{\partial R (q_1^F, q_2^F, q_1^F)}{\partial q_1} - \varphi c' (q_1^F) \right) \frac{dq_1}{d\varphi} - c (q_1^F)$$

$$- \left( \frac{\partial R (q_1 (q_2^F), q_2^F)}{\partial q_2} - c' (q_2^F) \right) \frac{dq_2}{d\varphi} + c (q_1 (q_2^F))$$

Evaluating at $\varphi = 1$, by symmetry, we have, using the first-order conditions

$$\frac{d\varepsilon}{d\varphi} = -c (q_1^F) + c (q_1 (q_2^F))$$

As workers are substitutes, by Proposition 4, we have $q_1^F > q_1 (q_2^F)$ and thus $\frac{d\varepsilon}{d\varphi} \big|_{\varphi=1} < 0$. For $\varphi = 1$ we have $\varepsilon = 0$. As the derivative is continuous in $\varphi$, we have $\varepsilon > 0$ for $\varphi$ sufficiently close to 1, and since $\varepsilon$ is continuous in $\varphi$, there is a $\hat{\varphi}$ such that for all $\varphi > \hat{\varphi}$ we have $\varepsilon > 0$.

**Step 2.** We need to show that

$$(1 - \lambda) \gamma_1 > -\lambda \varepsilon \quad (35)$$

$$\gamma_1 > -\lambda \pi_2$$

We have

$$\gamma_1 = \Pi_1 - (1 - \lambda) \pi_1 = R (q_1^F, 0) - \varphi c (q_1^F) - (1 - \lambda) \left( R (q_1^F, q_2^F) - R (0, q_2^F) - \varphi c (q_1^F) \right) \quad (36)$$
The first-order condition for $q_1^F$ can be rewritten as

$$\lambda \frac{\partial R (q_1^F, q_2^L)}{\partial q_1} + (1 - \lambda) \frac{\partial R (q_1^F, 0)}{\partial q_1} - \varphi c' (q_1^F) = 0.$$ 

When workers are substitutes, we have $\frac{\partial R (q_1^F, q_2^L)}{\partial q_1} < \frac{\partial R (q_1^F, 0)}{\partial q_1}$ and hence we have

$$\frac{\partial R (q_1^F, 0)}{\partial q_1} - \varphi c' (q_1^F) > 0. \quad (37)$$

Note that, by using that, since $\frac{\partial^2 R (q_1, q_2)}{\partial q_1 \partial q_2} < 0$ we have

$$R (q_1, 0) > R (q_1, q_2) - R (0, q_2)$$

and hence (36) is

$$\Pi_1 - (1 - \lambda) \pi_1 > R (q_1^F, 0) - \varphi c (q_1^F) - (1 - \lambda) (R (q_1^L, 0) - \varphi c (q_1^L)).$$

Also, (37) implies that $R (q_1^F, 0) - \varphi c (q_1^F) > R (q_1^L, 0) - \varphi c (q_1^L)$ and hence $\gamma_1 > 0$. For $\varphi > \bar{\varphi}$ we have, by continuity of $\varepsilon$, $\Pi_1$ and $\pi_1$ that (35) holds.

**Step 3.** Since $\varepsilon > 0$, $(1 - \lambda) \gamma_1 > -\lambda \varepsilon$ and $\gamma_1 > -\lambda \pi_2$ then, by Proposition 1, the first agreement is with seller 1 with probability 1 as $\delta \to 1$. ■

**Proof of Proposition 6:** When goods 1 and 2 are perfect substitutes and marginal costs are constant, we have

$$R (q_1, q_2) = Q (q_1 + q_2)$$

$$c_i (q_i) = d_i q_i.$$ 

Without loss of generality we assume that $d_1 \leq d_2$. In order for the problem to be non-trivial, we also assume that $Q' (0) > d_2$.

**Step 1.** The efficient quantities.

When $d_1 < d_2$, it is easy to see that the efficient quantities satisfy

$$Q' (q_1^e) - d_1 = 0 \quad (38)$$

and $q_2^e = 0$. When $d_1 = d_2$, we have

$$Q' (q_1^e + q_2^e) - d_1 = 0. \quad (39)$$
There is a multiplicity of solutions where all $q_1$ and $q_2$ satisfying $Q' (q_1 + q_2) = d_1$ are solutions.

**Step 2.** The equilibrium quantities.

Suppose the firm agrees first with seller 1. The first-order condition in the last bargain is

$$Q' (q_1 + q_2) - d_2 \leq 0.$$  \(40\)

In the first bargain, the first-order condition is

$$Q' (q_1 + q_2) - (1 - \lambda) (Q' (q_1 + q_2) - Q' (q_1)) - d_1 \leq 0.$$  \(41\)

If $q_2 > 0$, then (40) holds with equality, and (41) is

$$Q' (q_1) \leq \frac{d_1 - \lambda d_2}{1 + \lambda}.$$  \(42\)

Then

$$Q' (q_1) \leq Q' (q_1 + q_2),$$

a contradiction. We thus have $q_1^F$ solving $Q' (q_1^F) = d_1$ and $q_2^L = 0$.

Suppose the firm agrees first with seller 2. Similar arguments as above establishes that if $d_1 < d_2$ we have $q_2^F > 0$ and $q_1^F > 0$. The aggregate quantity solves $Q' (q_2^F + q_1^F) = d_1$. If $d_1 = d_2$ we have $q_2^F$ solving $Q' (q_2^F) = d_2$ and $q_1^F = 0$.

If $d_1 < d_2$ agreeing with 1 first gives a total surplus of

$$\Pi_1 + \pi_2 = Q (q_1^F) - d_1 q_1^F$$  \(42\)

and agreeing with 2 first gives

$$\Pi_2 + \pi_1 = Q (q_2^F + q_1^F) - d_1 q_1^F - d_2 q_2^F$$  \(43\)

$$\gamma_1 = \Pi_1 - (1 - \lambda) \pi_1$$

Note that the aggregate quantities are the same in the two cases. Hence, we have $\varepsilon > 0$. Also, when agreeing with 1 first we have $\pi_2 = 0$ ensuring that $\Pi_1 - \pi_1 > \Pi_2 > 0$ and hence $(1 - \lambda) \gamma_1 > -\lambda \varepsilon$. Using Proposition 2 establishes that all equilibria prescribe agreement with 1 first. Since $Q' (q_1^F) = d_1$ we have, from expression (38) that $q_1^F = q_1^*$ and $q_2^L = q_2^* = 0$.

If $d_1 = d_2$ and the firm agrees first with 1, we have $q_1^F$ solving $Q' (q_1^F) = d_1$ and $q_2^F = 0$. If the firm agrees first with 2, we have $q_2^F$ solving $Q' (q_2^F) = d_2$ and $q_1^F = 0$. Clearly, both of these
candidates satisfy expression (39).

**Step 3.** Equilibrium payoffs.

When agreement is with 2 first, we have:

\[ Q'(qF_2) = \frac{d_2 - \lambda d_1}{1 + \lambda}. \]  \hspace{1cm} (44)

The payoff to 1 is \((1 - \lambda)\pi_1 + \varepsilon\). Using (42) and (43) we have \(\varepsilon = (d_2 - d_1)qF_2\), the efficiency loss of agreeing with 2 first. Since \(Q'(q) \leq d_2\) from (44) and \(Q'\) is decreasing,

\[
\pi_1 = Q(qF_2 + qL_1) - Q(qF_2) - d_1 qF_2 = \int_{qF_2}^{qF_2 + qL_1} Q'(q) \, dq - d_1 qF_2 < (d_2 - d_1) qF_2.
\]

Then \((1 - \lambda)\pi_1 + \varepsilon\) is at most \((d_2 - d_1)(qF_2 + (1 - \lambda)qL_1) < (d_2 - d_1)qF_2\). \(\blacksquare\)

**A.4 Strategic Discrimination and Inefficiency**

Let \(u_1 = \frac{\varepsilon}{r - \theta}\) and \(u_2 = \frac{\gamma}{r}\). Using the equilibrium quantities we get

\[
\Pi_1 = r^2 \frac{(2 - s + 2u_2)(2 + s(1 - 2s) + 2u_2 + u_1(2 + s + 2u_2))}{2(2 - s^2 + 2u_2 + 2u_1(1 + u_2))^2} \hspace{1cm} (45)
\]

\[
\pi_2 = \frac{2r^2(1 - s + u_1)^2 (1 + u_2)}{(2 - s^2 + 2u_2 + 2u_1(1 + u_2))^2}
\]

Again, when 2 is first \(\Pi_2\) and \(\pi_1\) are similar with indices interchanged.

**Proof of first part of Proposition 7:** From Proposition 2, if \(\varepsilon > 0\) and \(\gamma_1 > -\varepsilon\) and \(\gamma_2 > \varepsilon\), then the unique equilibrium is \(M_1\). Using (45) gives

\[
\varepsilon = \frac{r^2 s^2 (u_2 - u_1)}{2(2 - s^2 + 2u_2 + 2u_1(1 + u_2))^2}
\]

Thus \(\varepsilon > 0\) for \(u_1 < u_2\). Again using (45) gives

\[
\gamma_1 = \frac{r^2}{2} \frac{2 + 4s - 7s^2 + 2s^3 + 4(1 + s - s^2)u_2 + 2u_2^2 + u_1(2 + 4s - 3s^2 + 4(1 + s)u_2 + 2u_2^2)}{(2 - s^2 + 2u_2 + 2u_1(1 + u_2))^2}
\]

\[
\gamma_2 = \frac{r^2}{2} \frac{2 + 4s - 7s^2 + 2s^3 + (2 + 4s - 3s^2)u_2 + 2u_2^2(1 + u_2) + 4u_1(1 + s - s^2 + (1 + s)u_2)}{(2 - s^2 + 2u_2 + 2u_1(1 + u_2))^2}
\]

Some tedious algebra shows that we have \(\gamma_1 > -\varepsilon\) and \(\gamma_2 > \varepsilon\) for all \(u_1\) and \(u_2\). \(\blacksquare\)
A.5 Simultaneous proposals

**Proof of Proposition** 8: Consider the following strategy profile. When the buyer is assigned to make a proposal to 1, it offers \((\bar{p}^F, \bar{q}_1^F)\) where

\[
\bar{q}_1^F = \arg \max_{q_1} R(q_1, 0) - c_1(q_1) + \frac{\delta}{1 + \delta} (R(q_1, q_2(q_1)) - R(q_1, 0) - c_2(q_2(q_1))).
\]

The price \(\bar{p}^F\) is chosen such that the seller is indifferent between accepting \((\bar{p}^F, \bar{q}_1^F)\) and rejecting.

The other seller makes an unacceptable proposal, say \((\hat{p}_2, \hat{q}_2)\) where \(\hat{p}_2\) and \(\hat{q}_2\) are chosen such that the total surplus between A and 1 is negative for any possible \(q_1\). When responding, each seller accepts any offer giving them at least \(\delta^2 \frac{\pi(\bar{q}_1^F)}{1 + \delta}\) where

\[
\pi(q_1) = \max_{q_2} R(q_1, q_2) - R(q_1, 0) - c_2(q_2)
\]

\[
\bar{q}_2^F = \arg \max_{q_2} R(\bar{q}_1^F, q_2) - R(\bar{q}_1^F, 0) - c_2(q_2).
\]

Also, A accepts any offer giving a continuation payoff of at least \(\delta V\), where \(V\) is the continuation payoff of A. The equilibrium payoff of A is, in the limit,

\[
R(\bar{q}_1^F, 0) - c_1(\bar{q}_1^F) + \frac{1}{2} (R(\bar{q}_1^F, \bar{q}_2^F) - R(\bar{q}_1^F, 0) - c_2(\bar{q}_2^F)) - \frac{\pi(\bar{q}_1^F)}{2}.
\]

We claim that this strategy profile is a stationary SPE.

By construction of \((\bar{p}^F, \bar{q}_1^F)\) and \((\hat{p}_2, \hat{q}_2)\), it cannot be optimal for A to deviate. Thus, it remains to consider deviations by 2. Obviously a deviation resulting in a rejection cannot be profitable.

Consider any deviation \((\bar{p}'_2, \bar{q}'_2)\) by 2 such that A accepts.

If \(q'_2 < \bar{q}_1^F\) then 1 rejects, since \(\pi(q'_2) > \pi(\bar{q}_1^F)\). If 2 gains from this deviation, the payoff of A is smaller than \(\delta V\) since seller 2 gets at least \(\delta^2 \frac{\pi(\bar{q}_1^F)}{1 + \delta}\) and seller 1 \(\frac{\delta}{1 + \delta} \pi(q'_2) > \frac{\delta}{1 + \delta} \pi(\bar{q}_1^F)\).

Now suppose \(q'_2 \geq \bar{q}_1^F\).

First, suppose \(q'_2 = \bar{q}_1^F\) and A accepts but 1 rejects the offer by A. Then the payoff of 2 is at most \(\frac{\delta}{2} \frac{\pi(\bar{q}_1^F)}{1 + \delta}\), since otherwise, A looses and would reject. Hence 2 does not gain by the deviation.

Second, suppose \(q'_2 \geq \bar{q}_1^F\) and that 1 accepts the offer by A. The payoff of A when accepting the deviating offer is, in the limit, assuming that 2 does not loose by proposing,

\[
R(\bar{q}_1^F, q'_2) - c_2(q'_2) - c_1(\bar{q}_1^F) - \pi(\bar{q}_1^F).
\]
Since the equilibrium payoff is \( R(q_1^F, 0) - c_1(q_1^F) \), the difference between the payoff when 2 deviates and the equilibrium payoff is

\[
R(q_1^F, q_2') - c_2(q_2') - c_1(q_1^F) - (R(q_1^F, q_2^L) - R(q_1^F, 0) - c_2(q_2^L)) - (R(q_1^F, 0) - c_1(q_1^F))
\]

\[
= R(q_1^F, q_2') - R(q_1^F, 0) - c_2(q_2') - (R(q_1^F, q_2^L) - R(q_1^F, 0) - c_2(q_2^L))
\]

Since \( q_2^L \) maximizes the bilateral surplus when \( A \) and 2 bargain, the expression above is negative. Hence, the equilibrium payoff is larger than when accepting \( q_2' = q_1^F \) from 2. Thus, \( A \) rejects any offer \( q_2' \geq q_1^F \), a contradiction.

Thus, there is no profitable deviation for 2.

Note that, as \( \delta \to 1 \), \( q_1^F \) converges to \( q_1^F \) in (9) when \( \lambda = \frac{1}{2} \) and hence \( q_2^L \) converges to \( q_2^L \) in (11). ■
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