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Wave energy parks with point-absorbers of different dimensions

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Abstract

An analytical model for point-absorbing wave energy converters connected to floats of different geometries and topologies is presented. The floats can be truncated cylinder or cylinder with moonpool buoys and have different outer radius, inner radius, draft, mass and can be connected to linear generators of different power take-off constants. The model is implemented into a numerical code where the input is measured time-series of irregular waves. After validation against benchmark software, the model is used to study optimal configurations of wave energy arrays consisting of different wave energy devices. It is shown that the total power absorption can be improved if the wave energy array consists of devices of different dimensions, and that a higher power-to-mass ratio can be achieved.

Keywords: wave energy, floating bodies, arrays, multiple scattering, interactions, optimization

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1. Introduction

Ocean waves provide a renewable and clean energy source that has potential to contribute significantly to the world's electricity consumption. There are many different technology concepts for converting wave energy to electricity, but so far none has reached a large-scale commercial level. Reducing costs is of major importance for the commercial compatibility of wave energy technology. The produced power of a device must therefore be weighted against its cost of construction, installation and operation, and installing a smaller device might be more cost-effective than a large, even if it has a smaller power production. Due to reduction in material costs and deployment, smaller devices may generally be more economically beneficial, as has been discussed by De Andres et al. (2015). Further, O'Connor et al. (2013) found that multiple smaller devices of Pelamis and Wavestar type gave a higher total energy output and capacity factor than fewer larger ones, and De Andres et al. (2016) studied optimal sizes of a CorPower wave energy device, and found that a total installed park of 20 MW had a larger annual energy production if the park was composed of many smaller devices instead of fewer large. Based on these earlier studies, it is relevant to ask if it might be beneficial to install smaller WECs despite lower power absorption.

Further, Göteman et al. (2014) showed that an increase of the total power production can be obtained if WECs of *different* dimensions are deployed together. Hence, it might be cost effective to install some smaller and some larger WECs together in arrays.

Most papers on point-absorbing wave energy converters consider floats that are truncated cylinders. Engström et al. (2017) and Gravråkmo (2014) have indicated that torus buoys or truncated cylinders with moonpools may be beneficial for wave energy applications, due to less motion in surge and smaller line forces, two factors that are important for the survivability of the device. A relevant question is therefore how the performance of a wave energy park is affected by including WECs with floats not only of different sizes, but also of different topologies.

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The devices in a park will interact by scattered and radiated waves, which may affect the total performance in a positive or destructive way. The number of devices in a park, their separating distance, the global geometry of the park and other park parameters all have a large impact on the park performance (Falnes, 1980; Thomas and Evans, 1981; Linton and Evans, 1990; McIver, 1994; Maniar and Newman, 1997; Peter et al., 2006; Fitzgerald and Thomas, 2007; Ricci et al., 2007; Tissandier et al., 2008; Cruz et al., 2009; De Backer, 2009; Child and Venugopal, 2010; Child et al., 2011; Rahm et al., 2012; Borgarino et al., 2012; Vicente et al., 2013; Engström et al., 2013; Sjolte et al., 2013; Stratigaki et al., 2014; Babarit, 2013; Sarkar et al., 2014; Götteman et al., 2015c; Kara, 2016).

Some of the studies on optimal park configurations have been based on numerical software, others on analytical models and a few on experimental results. To perform optimization of wave energy parks with a large number of involved devices and parameters, (semi-)analytical methods are often the most suitable. Many papers have presented and used analytical models for single or multiple floating bodies. An iterative multiple scattering method was presented by Ohkusu (1974) and extended by Mavrakos and Koumoutsakos (1987) and Mavrakos (1991) to study different floating body problems, including arrays with individual float geometries. By combining the iterative multiple scattering method with a direct matrix method (Spring and Monkmeyer, 1975; Simon, 1982), a multiple scattering method for heaving bodies was presented by Kagemoto and Yue (1986). The method was later combined with the single body diffraction solution of Garrett (1971) by Yilmaz and Incecik (1998) and extended to independent radiation by Siddorn and Eatock Taylor (2008).

To study the questions posed above, namely if an improved power absorption and/or power-to-mass ratio can be obtained if wave energy arrays consist of devices of different dimensions, and how the performance is affected by including floats of different topologies, the analytical model presented by Götteman et al. (2015b,a) has here been extended to allow for wave energy converters of different dimensions. The devices are of point-absorbing type with a surface buoy that may be a truncated cylinder (CYL) or a truncated cylinder with moonpool (CWM), with individual inner and outer radius, draft and mass. The float is connected to a linear generator at the seabed, more details are given in section 2.2. The analytical method is implemented as a MATLAB code, validated against benchmark software and used to study and optimize arrays consisting of devices of different dimensions.

A similar analytical model has been presented by Konispoliatis and Mavrakos (2016) for oscillating water column devices, where the multiple body problem was solved by iteration in order of scattered waves. In this paper, the full multiple scattering problem with independent radiation from all floats is solved simultaneously as in the work by Siddorn and Eatock Taylor (2008), with the extension that all buoys may have individual dimensions, and that buoys may be cylindrical or cylindrical with a moonpool. The resulting model is computationally fast and further used in the paper to study optimal configurations of point-absorber wave energy arrays with different floats.

The paper is structured as follows. The analytical model is derived and presented in section 2.1 and its numerical implementation described in section 2.2. Lengthy expressions are defined in the appendix. The model is compared to WAMIT computations in section 3.1 and then used to study optimal array configurations in section 3.2. Conclusions from the results are summarized in section 4.

2. Method

2.1. Theory

Assume linear potential flow theory, i.e. the fluid domain is governed by the Laplace equation for a fluid velocity potential, $\nabla^2 \Phi = 0$, where $\bar{u} = \nabla \Phi$ is the fluid velocity, and the boundary constraints at the seabed, at the free surface and at any rigid boundary are linear, see, e.g., Linton and McIver (2001).

Consider an array of N point-absorbing wave energy converters labelled $i = 1, \dots, N$. The devices consist of a surface buoy connected to a linear generator at the seabed. The surface buoy may be cylindrical (CYL) or cylindrical with a moonpool (CWM), see figure 1, and is characterized by an outer radius R^i , an inner radius R_{in}^i (which is zero for cylinder buoys), a draft d^i and a position (x^i, y^i, z^i) . The vertical coordinate is defined such that $z = 0$ at the free surface at equilibrium, and $z = -h$ at the seabed at water depth h . Define a local cylindrical coordinate system (r, θ, z) with center at each buoy. Any point in the global coordinate system can be written in terms of the local coordinate system as $(x, y, z) = (x^i + r \cos \theta, y^i + r \sin \theta, z)$. At each buoy, the fluid domain is divided into two interior regions I-II beneath the buoy and an exterior region III outside $r > R^i$, see figure 1.

In general, all buoys are free to oscillate independently in heave with velocities V^i , and there are waves incident along the x -axis. Due to the linearity of the problem, the total fluid velocity potential will decompose into incoming

Figure 1: Illustration of the fluid domain and buoy geometries. The floats are characterized by an outer radius R^i and an inner radius R_{in}^i (which is zero for cylinder buoys) and a draft d^i . The floats are connected to a linear generator at the seabed as illustrated in the figure. The moving part of the generator is a translator moving vertically within a stator. The power take-off of the generator is characterized by the individual constant Γ^i . The fluid domain is divided into interior domains I and II beneath the moonpool and body surface, and exterior domains III outside $r > R^i$.

waves, scattered and radiated waves. For clarity, the multiple body problem is solved separately for the scattering problem (where all buoys are fix) and the radiation problem (where there is no incident wave).

2.1.1. Multiple scattering problem

Consider the problem where all buoys are fix and there is an incident wave propagating along the x -axis. The wave velocity potential describing a propagating free wave can be written as (see Linton and McIver (2001))

$$\Phi(x, t) = \frac{gA}{\omega} \frac{\cosh(k(z+h))}{\cosh(kh)} \sin(kx - \omega t), \quad (1)$$

where A is the amplitude, g the gravitational acceleration, k the wave number and ω the angular frequency, which relates to the wave number by the dispersion relation $\omega^2 = gk \tanh(kh)$. The surface elevation is then given by

$$\eta(x, y, t) = -\frac{1}{g} \frac{\partial \Phi}{\partial t} \Big|_{z=0} = A \cos(kx - \omega t), \quad (2)$$

which describes a propagating harmonic wave. By Fourier transform, the velocity potential can be considered in the frequency domain. Using properties of Bessel functions, the wave incident on a buoy i can be written in terms of the local coordinate system of the buoy as

$$\phi_{0,n}^i(x) = -\frac{igA}{\omega} \frac{\cosh(k(z+h))}{\cosh(kh)} e^{ikx^i} = \sum_{n=-\infty}^{\infty} Z_0(z) A_{0,n}^i J_n(kr) e^{in\theta}, \quad (3)$$

where $Z_0(z)$ and $A_{0,n}^i$ are defined in equations (A.1)-(A.2) in the appendix, and $J_n(kr)$ are Bessel functions.

A general scattered wave in the exterior region can be written in terms of propagating Hankel functions $H_n(kr)$ and evanescent modified Bessel functions $K_n(k_m r)$ as

$$\phi_S^{i(III)} = \sum_{n=-\infty}^{\infty} \left[\alpha_{0n}^i Z_0(z) \frac{H_n(kr)}{H_n(kR^i)} + \sum_{m=1}^{\infty} \alpha_{mn}^i Z_m(z) \frac{K_n(k_m r)}{K_n(k_m R^i)} \right] e^{in\theta} \quad (4)$$

where the vertical functions are defined in equation (A.1). The wave numbers k_m solve the dispersion relation $\omega^2 = -gk_m \tanh(k_m h)$. By Graf's addition theorem for Bessel functions, the scattered wave off a buoy j can be written as an

incident wave in the local coordinates of buoy i , and the total diffracted wave in the exterior region of buoy i is given as a superposition of scattered waves, incident waves from $x = -\infty$, and incident waves that are scattered off the other buoys,

$$\begin{aligned}\phi_D^{i(\text{III})} &= \phi_{0,n}^i + \phi_S^{i(\text{III})} + \sum_{j \neq i} \phi_S^{j(\text{III})} \Big|_i \\ &= \sum_{n=-\infty}^{\infty} \left[Z_0(z) \left(\alpha_{0n}^i \frac{H_n(kr)}{H_n(kR^i)} + J_n(kr) (A_{0,n}^i + \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{0ln}^{ij} \alpha_{0l}^j) \right) \right. \\ &\quad \left. + \sum_{m=1}^{\infty} Z_m(z) \left(\alpha_{mn}^i \frac{K_n(k_m r)}{K_n(k_m R^i)} + \frac{I_n(k_m r)}{I_n(k_m R^i)} \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{mln}^{ij} \alpha_{ml}^j \right) \right] e^{in\theta}.\end{aligned}\quad (5)$$

The functions T_{mln}^{ij} are defined in equation (A.4) in the appendix, and $I_n(k_m r)$ are modified Bessel functions. In the interior region I beneath the moonpool, a general diffracted wave can be written as

$$\phi_D^{i(\text{I})} = \sum_{n=-\infty}^{\infty} \left[Z_0(z) d_{0n}^i J_n(kr) + \sum_{m=1}^{\infty} Z_m(z) d_{mn}^i \frac{I_n(k_m r)}{I_n(k_m R_{\text{in}}^i)} \right] e^{in\theta} \quad (6)$$

and in the interior region II beneath the bottom surface of the cylinder, the potential is

$$\phi_D^{i(\text{II})} = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{\infty} \left[\gamma_{mn}^i R_{mn}^i(r) + \hat{\gamma}_{mn}^i \hat{R}_{mn}^i(r) \right] \cos(\lambda_m^i(z+h)) e^{in\theta}, \quad (7)$$

where $\lambda_m^i = \pi m / L^i$ and $L^i = h - d^i$, and the functions $R, \hat{R}$ are defined in equations (A.5) in the appendix. In the case that the float is a cylinder, i.e. the inner radius vanishes $R_{\text{in}}^i = 0$, the potential (7) holds in the entire region beneath the cylinder and is simplified as $\hat{R}_{mn}^i(r) = 0$ and $R_{0n}^i(r)$ and $\hat{R}_{mn}^i(r)$ simplify as shown in equation (A.5).

Continuity throughout the water depth $-h \leq z \leq 0$ at the domain boundaries $r = R_{\text{in}}^i$ and $r = R^i$ at all floats for the potentials imply that the coefficients in region II depend on the coefficients in the neighbouring domains as

$$\begin{aligned}\gamma_{sn}^i &= \sum_{m=0}^{\infty} \left(\alpha_{mn}^i + J_n(kR^i) A_{0,n}^i \delta_{m0} + ((J_n(kR^i) - 1) \delta_{m0} + 1) \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{mln}^{ij} \alpha_{ml}^j \right) c_{sm}^i \\ \hat{\gamma}_{sn}^i &= \sum_{m=0}^{\infty} d_{mn}^i ((J_n(kR_{\text{in}}^i) - 1) \delta_{m0} + 1) c_{sm}^i\end{aligned}\quad (8)$$

where the constants c_{sm}^i are defined in (A.3). Further, continuity for the radial derivatives of the fluid potentials throughout the water depth at the domain boundaries imply that the coefficients in the potentials are related as

$$\begin{aligned}\sum_{m=0}^{\infty} \left[\tilde{D}_{smn}^i d_{mn}^i - \tilde{L}_{smn}^i \left(\alpha_{mn}^i + ((J_n(kR^i) - 1) \delta_{m0} + 1) \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{mln}^{ij} \alpha_{ml}^j \right) \right] &= \tilde{L}_{s0n}^i J_n(kR^i) A_{0,n}^i \\ \sum_{m=0}^{\infty} \left[D_{smn}^i \alpha_{mn}^i + \tilde{D}_{smn}^i \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{mln}^{ij} \alpha_{ml}^j - \hat{L}_{smn}^i d_{mn}^i ((J_n(kR_{\text{in}}^i) - 1) \delta_{m0} + 1) \right] &= -\tilde{D}_{s0n}^i A_{0,n}^i\end{aligned}\quad (9)$$

where all matrices are defined in the appendix. By truncating the infinite matrices, the analytical method becomes semi-analytical, and we can solve for the coefficients α_{mn}^i and d_{mn}^i from the system of equations in (9). The cut-off for the angular sums $\sum_n e^{in\theta}$ and the vertical sums $\sum_m Z_m(z)$ in equation (4)-(7) may be chosen independently and are denoted Λ_θ and Λ_z , respectively. A cut-off $\Lambda_z \geq 1$ means that evanescent modes are included in the computations, and $\Lambda_z = 0$ is equivalent to the plane-wave, or wide-spacing approximation. McIver (1994) showed that substantially different results are obtained with the plane-wave approximation for separation distances $4R$ and $8R$, where R is the device radius. Since in this paper we consider separation distances that typically lie in the range $3R - 10R$, evanescent modes are included in all computations.

The scattering coefficients α are obtained from the matrix equation

$$\begin{bmatrix} \mathbb{1} & -B_M^1 T^{12} & \dots & -B_M^1 T^{1N} \\ -B_M^2 T^{21} & \mathbb{1} & \dots & -B_M^2 T^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -B_M^N T^{N1} & -B_M^N T^{N2} & \dots & \mathbb{1} \end{bmatrix} \begin{bmatrix} \alpha^1 \\ \alpha^2 \\ \vdots \\ \alpha^N \end{bmatrix} = \begin{bmatrix} B_{\text{diff}}^1 \\ B_{\text{diff}}^2 \\ \vdots \\ B_{\text{diff}}^N \end{bmatrix} \quad (10)$$

where the involved expressions are defined in the appendix. The diffraction matrix on the left-hand side is a quadratic matrix of size $N(\Lambda_z + 1)(2\Lambda_\theta + 1)$, each block $B_M^i T^{ij}$ and the identity matrices $\mathbb{1}$ being of quadratic size $(\Lambda_z + 1)(2\Lambda_\theta + 1)$. Inverting this diffraction matrix is the most computationally extensive part of the computation. In the method of Göteman et al. (2015a), this was improved by introducing an interaction distance cut-off, which allows the user to define a distance for which the hydrodynamical interactions are to be included. In very large parks with several hundred devices, this may speed up the computations significantly since the resulting diffraction matrix will be sparse, and efficient methods exist for computing inverses of sparse matrices. The same method with interaction distance cut-off is included also in this extended model, but in all the simulations presented in section 3 an infinite interaction distance has been used, implying full hydrodynamical interaction between all devices.

After the scattering coefficients have been solved from equation (10), the coefficients d_{mn}^i in the interior region I beneath the moonpool are obtained from (9) and the coefficients in region II from equation (8). The excitation force in the vertical direction can then be computed as

$$\begin{aligned} F_{\text{exc}}^i &= i\omega\rho \int_0^{2\pi} d\theta \int_{R_{\text{in}}^i}^{R^i} r dr \phi_D^{i(\text{II})}(r, \theta, -d^i) \\ &= 2\pi i\omega\rho \left[\gamma_{00}^i \left(\frac{(R^i)^2}{2} - \frac{(R^i)^2 - (R_{\text{in}}^i)^2}{4 \ln(R^i/R_{\text{in}}^i)} \right) - \hat{\gamma}_{00}^i \left(\frac{(R_{\text{in}}^i)^2}{2} - \frac{(R^i)^2 - (R_{\text{in}}^i)^2}{4 \ln(R^i/R_{\text{in}}^i)} \right) \right. \\ &\quad \left. + 2 \sum_{m=1}^{\infty} (-1)^m (\gamma_{m0}^i \int_{R_{\text{in}}^i}^{R^i} r dr R_{m0}^i(r) + \hat{\gamma}_{m0}^i \int_{R_{\text{in}}^i}^{R^i} r dr \hat{R}_{m0}^i(r)) \right] \end{aligned} \quad (11)$$

where the integrals of the radial functions can be solved and expressed analytically. Note that this expression is valid both for CYL and CWM buoys, as the asymptotic expression when $R_{\text{in}}^i \rightarrow 0$ is

$$F_{\text{exc}}^i \xrightarrow{R_{\text{in}}^i \rightarrow 0} 2\pi i\omega\rho \left[\gamma_{00}^i \frac{(R^i)^2}{2} + 2 \sum_{m=1}^{\infty} (-1)^m \gamma_{m0}^i \int_{R_{\text{in}}^i}^{R^i} r dr \frac{I_0(\lambda_m^i r)}{I_0(\lambda_m^i R)} \right] \quad (12)$$

which is the expected heave excitation force for a cylinder.

2.1.2. Multiple body radiation problem

In the multiple body radiation problem, all buoys are free to oscillate independently in heave with velocity V^i and there is no incident wave. Note that multiple scattering will occur also in the radiation problem, since radiated and scattered waves will be diffracted in the array. As in the multiple scattering problem described above, an ansatz that satisfies the boundary constraints at the seabed, body surface and free ocean surface can be made. In the domains beneath the buoys, the ansatz can be written as

$$\begin{aligned} \phi_D^{i(\text{I})} &= \sum_{n=-\infty}^{\infty} \left[Z_0(z) f_{0n}^i J_n(kr) + \sum_{m=1}^{\infty} Z_m(z) f_{mn}^i \frac{I_n(k_m r)}{I_n(k_m R_{\text{in}}^i)} \right] e^{in\theta}, \\ \phi_D^{i(\text{II})} &= \frac{V^i}{2L^i} \left((z+h)^2 - \frac{r^2}{2} \right) + \sum_{n=-\infty}^{\infty} \sum_{m=0}^{\infty} \left[a_{mn}^i R_{mn}^i(r) + \hat{a}_{mn}^i \hat{R}_{mn}^i(r) \right] \cos(\lambda_m^i (z+h)) e^{in\theta}. \end{aligned} \quad (13)$$

Note that the potential in the domain beneath the body surface has received a term required for the inhomogeneous boundary constraint for an oscillating body. The total incident wave on a body i is the sum of scattered and radiated

waves from the other bodies, and the total diffracted wave in the exterior domain is a superposition of scattered, radiated and incident waves,

$$\begin{aligned}\phi_D^{i(\text{III})} &= \phi_R^{i(\text{III})} + \phi_S^{i(\text{III})} + \sum_{j \neq i} [\phi_R^{j(\text{III})} + \phi_S^{j(\text{III})}]_i \\ &= \sum_{n=-\infty}^{\infty} \left[Z_0(z) \left(\tilde{\alpha}_{0n}^i \frac{H_n(kr)}{H_n(kR^i)} + J_n(kr) \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{0ln}^{ij} \tilde{\alpha}_{0l}^j \right) \right. \\ &\quad \left. + \sum_{m=1}^{\infty} Z_m(z) \left(\tilde{\alpha}_{mn}^i \frac{K_n(k_m r)}{K_n(k_m R^i)} + \frac{I_n(k_m r)}{I_n(k_m R^i)} \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{mln}^{ij} \tilde{\alpha}_{ml}^j \right) \right] e^{in\theta}.\end{aligned}\quad (14)$$

The coefficients $\tilde{\alpha}_{mn}^i = \alpha_{mn}^i + V^i b_{mn}^i$ include contributions from both scattered and radiated waves. As before, continuity at the domain boundaries $r = R^i$ and $r = R_{\text{in}}^i$ for the potentials implies that the coefficients are related as

$$\begin{aligned}a_{sn}^i &= V^i L^i \delta_{0n} \left[\left(\left(\frac{R^i}{2L^i} \right)^2 - \frac{1}{6} \right) \delta_{s0} - \frac{(-1)^2}{\pi^2 s^2} (1 - \delta_{s0}) \right] + \sum_{m=0}^{\infty} \left(\tilde{\alpha}_{mn}^i + ((J_n(kR^i) - 1)\delta_{m0} + 1) \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{mln}^{ij} \tilde{\alpha}_{ml}^j \right) c_{sm}^i \\ \hat{a}_{sn}^i &= V^i L^i \delta_{0n} \left[\left(\left(\frac{R_{\text{in}}^i}{2L^i} \right)^2 - \frac{1}{6} \right) \delta_{s0} - \frac{(-1)^2}{\pi^2 s^2} (1 - \delta_{s0}) \right] + \sum_{m=0}^{\infty} f_{mn}^i ((J_n(kR_{\text{in}}^i) - 1)\delta_{m0} + 1) c_{sm}^i.\end{aligned}\quad (15)$$

Continuity for the radial derivatives of the potentials across the domain boundaries implies a similar relationship between the coefficients as in (9),

$$\begin{aligned}\sum_{m=0}^{\infty} \left[\tilde{D}_{smn}^i f_{mn}^i - \tilde{L}_{smn}^i \left(\tilde{\alpha}_{mn}^i + ((J_n(kR^i) - 1)\delta_{m0} + 1) \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{mln}^{ij} \tilde{\alpha}_{ml}^j \right) \right] &= V^i \delta_{0n} \tilde{B}_R^i \\ \sum_{m=0}^{\infty} \left[D_{smn}^i \tilde{\alpha}_{mn}^i + \tilde{D}_{smn}^i \sum_{j \neq i} \sum_{l=-\infty}^{\infty} T_{mln}^{ij} \tilde{\alpha}_{ml}^j - \hat{L}_{smn}^i f_{mn}^i ((J_n(kR_{\text{in}}^i) - 1)\delta_{m0} + 1) \right] &= V^i \delta_{0n} B_R^i\end{aligned}\quad (16)$$

where the right-hand sides of the equations are given by the radiation matrices B_R^i and $\tilde{B}_R^i$, defined in equation (A.11) in the appendix. The coefficients $\tilde{\alpha}$ are solved from the matrix equation

$$\begin{bmatrix} \mathbb{1} & -B_M^1 T^{12} & \cdots & -B_M^1 T^{1N} \\ -B_M^2 T^{21} & \mathbb{1} & \cdots & -B_M^2 T^{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -B_M^N T^{N1} & -B_M^N T^{N2} & \cdots & \mathbb{1} \end{bmatrix} \begin{bmatrix} \tilde{\alpha}^1 \\ \tilde{\alpha}^2 \\ \vdots \\ \tilde{\alpha}^N \end{bmatrix} = \begin{bmatrix} V^1 B_{\text{rad}}^1 \\ V^2 B_{\text{rad}}^2 \\ \vdots \\ V^N B_{\text{rad}}^N \end{bmatrix}\quad (17)$$

where the right-hand side is a radiation vector as defined in (A.12), and the diffraction matrix is identical to the one in equation (9) in the multiple scattering problem. When the coefficients $\tilde{\alpha}$ have been solved, the coefficients f are obtained from equation (16) and finally a and $\hat{a}$ from equation (15). The radiation force can then be computed as

$$\begin{aligned}F_{\text{rad}}^i &= i\omega\rho \int_0^{2\pi} d\theta \int_{R_{\text{in}}^i}^{R^i} r dr \phi_D^{i(\text{III})}(r, \theta, -d^i) \\ &= 2\pi i\omega\rho \left[V^i L^i \left(\frac{(R^i)^2 - (R_{\text{in}}^i)^2}{4} - \frac{(R^i)^4 - (R_{\text{in}}^i)^4}{16(L^i)^2} \right) \right. \\ &\quad \left. + a_{00}^i \left(\frac{(R^i)^2}{2} - \frac{(R^i)^2 - (R_{\text{in}}^i)^2}{4 \ln(R^i/R_{\text{in}}^i)} \right) - \hat{a}_{00}^i \left(\frac{(R_{\text{in}}^i)^2}{2} - \frac{(R^i)^2 - (R_{\text{in}}^i)^2}{4 \ln(R^i/R_{\text{in}}^i)} \right) \right. \\ &\quad \left. + 2 \sum_{m=1}^{\infty} (-1)^m \left(a_{m0}^i \int_{R_{\text{in}}^i}^{R^i} r dr R_{m0}^i(r) + \hat{a}_{m0}^i \int_{R_{\text{in}}^i}^{R^i} r dr \hat{R}_{m0}^i(r) \right) \right].\end{aligned}\quad (18)$$

Again, note that the expression is valid both for CYL and CWM buoys, as the asymptotic expression when $R_{\text{in}}^i \rightarrow 0$ becomes the expected expression for the heave radiation force of a cylinder.

From the radiation problem, there is an implicit dependency of the oscillating velocities V^i in the coefficients a^i and $\hat{a}^i$, stemming from the fact that the scattering coefficients contain contributions from both the diagonal and non-diagonal elements of the diffraction matrix,

$$\tilde{a}^i = \left[\text{DiffMatrix}^{-1} \right]_i^i V^i B_{\text{rad}}^i + \sum_{j \neq i} \left[\text{DiffMatrix}^{-1} \right]_j^i V^j B_{\text{rad}}^j. \quad (19)$$

All velocities from all buoys will thus be present in the radiation force, where the non-diagonal elements represents interaction by radiated waves between different buoys. The radiation force is commonly divided into added mass proportional to the velocity of the buoy and radiation damping proportional to the acceleration, $F_{\text{rad}}(t) = -m_{\text{add}}\ddot{z} - B\dot{z}$, which in the frequency domain corresponds to

$$F_{\text{rad}}^i = [i\omega m_{\text{add}} - B]_i^i V^i + \sum_{j \neq i} [i\omega m_{\text{add}} - B]_j^i V^j. \quad (20)$$

For clarity, the multiple scattering and radiation problem were here solved independently. They could just as well have been solved in one problem, where there is both an incident wave and all floats are free to oscillate independently. The solution would be the same, where the scattering coefficients would contain contributions both from scattered waves and radiated waves, and where the right hand side of equation (17) would also contain the right hand side of equation (10).

2.1.3. Time-domain motion and power

With the hydrodynamical forces computed in equations (11) and (18), the motion and absorbed power of the wave energy device can be computed given input waves. The waves used for the simulations in this paper are irregular waves measured offshore, see section 2.2.

Implicit in expression (11) for the excitation force is the product with the frequency domain amplitude $A^i(\omega)$ of the incident waves, and we can write $F_{\text{exc}}^i(\omega) = A^i(\omega)f_{\text{exc}}^i(\omega)$ where $f_{\text{exc}}^i(\omega)$ is the amplitude independent part of the expression. The equations of motion for the wave energy converter in the frequency domain then take the form

$$\left[-\omega^2(m^i + m_{\text{add}}^i(\omega)) - i\omega(B^i(\omega) + \Gamma^i) - \rho g \pi ((R^i)^2 - (R_{\text{in}}^i)^2) \right] z^i(\omega) = A^i(\omega)f_{\text{exc}}^i(\omega), \quad (21)$$

where $m^i = m_b^i + m_t^i$ is the total mass of the buoy and translator in the generator at the seabed, characterized by the constant power take-off coefficient Γ^i , and z^i is the vertical position of the buoy. A stiff connection between the buoy and translator is assumed, such that their vertical displacements are equal. The equations of motion can equivalently be expressed as $\tilde{z}^i(\omega) = A^i(\omega)H^i(\omega)$ where we defined a transfer function $H^i(\omega)$. By computing the inverse Fourier transform $\tilde{z}^i(t) = \tilde{z}^i(\omega)$, the position of the buoy in the time-domain can be obtained.

With the position of the buoy determined, the absorbed wave power can now be computed in time as $P^i(t) = \Gamma^i(\dot{z}^i(t))^2$. The instantaneous power absorbed by the full park is then given by the sum

$$P_{\text{total}}(t) = \sum_{j=1}^N P^j(t). \quad (22)$$

The absorbed power presented in section 3 is the time averaged power absorbed during a 1 hour sea state.

2.2. System and numerical implementation

2.2.1. Wave energy system

Point-absorber wave energy converters with floats of different geometries and topologies are considered in this paper. The device is similar to the wave energy converter that has been developed at Uppsala University since 2006 (Leijon et al., 2009). The WEC consists of a surface buoy connected by a stiff line to a linear generator at the seabed. A constant power take-off (PTO) coefficient of the generator is assumed for each device. A priori, the parameters of the

Geometry	Topology	Radius	Inner radius	Draft	PTO constant	Mass
# 1	CYL	2.0 m	0 m	0.5 m	70 kNs/m	6440 kg
# 2	CYL	2.5 m	0 m	0.5 m	110 kNs/m	10063 kg
# 3	CYL	3.0 m	0 m	0.6 m	140 kNs/m	17389 kg
# 4	CYL	3.5 m	0 m	0.6 m	200 kNs/m	23668 kg
# 5	CWM	2.0 m	1.0 m	0.5 m	55 kNs/m	4830 kg
# 6	CWM	3.0 m	1.0 m	0.6 m	135 kNs/m	15457 kg

Table 1: Dimensions of the buoy geometries and topologies used. The first four represent CYL buoys, and the geometries # 5-6 represent CWM buoys. The values of the constant PTO dampings are chosen to correspond with the geometry and mass of the buoy. The mass includes both the mass of the buoy and the attached translator mass.

WEC system may take any values. Here, six different WECs are considered, with sizes and topologies listed in table 1. The different geometries represent realistic dimensions of full-scale WEC devices developed at Uppsala University. Geometries # 1-4 represent cylinder (CYL) floats, whereas geometries # 5-6 represent cylinder with moonpool (CWM) floats. Geometries # 1 and 5 share the same outer radius and draft, and the same holds for geometries # 3 and 6. For each buoy geometry, a suitable PTO constant has been chosen.

In this paper, the floats are assumed to move in heave only, and surge motion is neglected. In full-scale offshore experiments on a point-absorber device as the one studied in this paper, it was found that the inclination angle between the float and the generator at the seabed was less than 8% (Savin et al., 2012). The time-series of incident irregular waves are measured at the same site as the experiment was performed, which implies that the surge motion can be assumed to be small. Also, a numerical model based on heave motion only was validated with full-scale experiments for the same wave energy converter system by Eriksson et al. (2007), again showing that the impact of surge, sway or pitch is small.

The irregular waves that are used as input in the model have been measured at the Lysekil test site at the west coast of Sweden by a commercial Datawell Waverider buoy with a sampling rate of 2.56 Hz. The waves are characterized by significant wave height $H_s = 1.53$ m and energy period $T_e = 5.01$ s, and the measured time-series of 20 min has been repeated twice to model irregular waves of 1 hour.

2.2.2. Numerical implementation

The theory and method described in section 2.1 has been implemented as a MATLAB code with the time-series of measured waves as input. The system of equations in (10) and (17) have been truncated at the vertical and angular cut-offs $\Lambda_z = 30$ and $\Lambda_\theta = 3$. The inversion of the diffraction matrix is computed using the object-oriented factorization algorithm of Davis (2013).

The computations of arrays of WECs were executed on a standard desktop PC with four parallel Intel® Xeon® 3.07-GHz processors and 6 MB RAM, both for the analytical code and the WAMIT simulations. The computations of 1000 configurations of 12 WEC arrays were executed on 8 parallel Opteron 6220 cores running at 3 GHz with 4 GB RAM each on the high performance computer cluster UPPMAX.

3. Results

3.1. Validation of the hydrodynamical model

3.1.1. Arrays of different CYL buoys

The hydrodynamic model presented in section 2 is validated with the software WAMIT, an established commercial software based on solving potential flow theory using the boundary element method. The simulations with WAMIT were computed with the higher-order method for the body geometry and solution, and the first order linear potential approximation was taken. The panel method was used to solve for the velocity potential and fluid pressure on the submerged surfaces of the bodies, and the output was given in terms of the hydrodynamic forces. To model cylinder buoys of different dimensions in WAMIT, the Fortran library file GEOMXACT was first modified to allow cylinders of different dimensions, and the subroutine CIRCCYL was used.

The validation was carried out for the array with four WECs shown in figure 2 a). The WECs 1-4 have the dimensions given by geometry # 1-4 shown in table 1, respectively. The radiation force, presented in terms of the

Figure 2: Six of the many array configurations studied. The upper row a-c) show layouts for CYL buoys only, which corresponds to geometries #1-4 in table 1. Specifically, in the figures a-c), WEC number 1, 2, 3, 4 has geometry a) #1, #2, #3, #4; b) #1, #4, #1, #4; and c) #4, #1, #4, #1. The lower row d-f) show configurations where both CYLs and CWMs are allowed, i.e. there may be four CWMs, four CYLs, or a mix of CYL and CWM geometries. Specifically, WEC 1, 2, 3, 4 has geometry d) #5, #6, #5, #6; e) #3, #3, #3, #3; and f) #5, #5, #5, #1.

Figure 3: Validation of the radiation force components (added mass and radiation damping) computed with the analytical tool, as compared with the software WAMIT. The validation is performed for the layout in figure 2 a), i.e. with all four CYL dimensions present. The notation “WEC i - j ” means the radiation coupling terms between WEC i and WEC j .

added mass and the radiation damping as in equation (20), is shown in figure 3. The figure shows the diagonal terms for the four cylinder buoys, as well as one cross-coupling term between WEC 1 and 4. The agreement between the analytical method and WAMIT is excellent for all angular frequencies $\omega \in [0.1, \dots, 16]$ rad/s. Likewise, the real and imaginary parts of the excitation force in equation (11) computed with the analytical method and WAMIT are shown in figure 4. As for the radiation force, the agreement for the excitation force is excellent. Only two small peaks of the WAMIT computation for the radiation force at $\omega \approx 5$ rad/s and for the excitation force at $\omega \approx 16$ rad/s are not reproduced by the analytical code, but since these peaks do not correspond to physical processes, the peaks are non-physical and the absence of them is a wanted result.

3.1.2. Arrays of different CWM buoys

For the validation of the arrays with different CWM buoys, the subroutine CYLMP of the Fortran library file GEOMXACT in WAMIT was used. The validation was carried out for an array of four WECs with CWM geometries of table 1, where WEC 1 and 3 had identical geometry # 5 and WEC 2 and 4 geometry # 6. Two separate WAMIT simulations were used for the two different buoy geometries.

The result for the added mass and radiation damping of the radiation force is shown in figure 5, and the real and imaginary parts of the excitation force are shown in figure 6. Since the array is symmetric, only results for WEC 1 and 2 are shown, plus the coupling terms between WEC 1 and 3. The resonance peaks in the added mass and radiation damping are captured, but have a slightly differing magnitude than WAMIT. The resonance peaks originate from neglecting viscous damping associated with flow separation at the outer and inner corners of the cylinder (Lee, 1995), hence an exaggerated resonance peak magnitude in any linear inviscid model can be expected. Since the vertical motions of the floats are small, this damping can be neglected, but still the behaviour of the CWM buoys close to resonance has to be considered with some care. However, the small sizes of the floats imply that the operational mode

Figure 4: Validation of the excitation force computed with the analytical tool, as compared with the software WAMIT. The validation is performed for the layout in figure 2 a), i.e. with all four CYL dimensions present.

Figure 5: Validation of the radiation force components (added mass and radiation damping) computed with the analytical tool, as compared with the software WAMIT. The validation is performed for the layout in figure 2 d), i.e. with the CWM geometries #5 and #6. The notation “WEC i - j ” means the radiation coupling terms between WEC i and WEC j .

of the WECs will not be close to resonance.

For the excitation force, the resonance peak deviates, but the agreement is still acceptable. As for the radiation force, an angular frequency up to $\omega = 16$ rad/s was modelled, but figure 6 has been cut off at $\omega = 8.5$ rad/s for clarity. The agreement for $\omega = 8.5, \dots, 16$ rad/s was excellent.

The analytical method is computationally fast, which allows for park optimization studies. As a comparison, the computational time of the analytical method for the four CYL buoys of different dimensions was 12 % of WAMIT’s, and 5 % for the four CWM buoys. For the CWM buoy, the CPU time for the analytical method can be estimated to 3.5 s per frequency, and 69 s per frequency for the WAMIT computations.

3.2. Optimization of array with different buoys

Performance of wave energy arrays or parks can be evaluated using the q -factor, defined as the ratio between the total time averaged absorbed power of the array P_{tot} and the time averaged power absorbed by the individual WECs in isolation P^i ,

$$q = \frac{P_{\text{tot}}}{\sum_{i=1}^N P^i}. \quad (23)$$

The q -factor is sometimes denoted the interference effect factor or the interaction factor since it reflects the interactions in an array: a value $q > 1$ reflects that there is positive interaction in an array, whereas $q < 1$ shows a destructive interaction. In realistic, irregular sea states a destructive interaction factor is more likely to occur, and optimization involves minimizing the destructive interaction.

As discussed in the introduction, studies indicate that installing WECs of smaller dimensions might be cost effective despite lower power absorption (De Andres et al., 2015; O’Connor et al., 2013; De Andres et al., 2016), and that a positive net absorption can be obtained if arrays consist of WECs with different dimensions (Göteman et al., 2014).

Figure 6: Validation of the excitation force computed with the analytical tool, as compared with the software WAMIT. The validation is performed for the layout in figure 2 d), i.e. with the CWM geometries #5 and #6.

Figure 7: a) Annual energy absorption as a function of the total device masses for four devices with CYL buoys as shown in figure 2 a-c). The result for all 256 configurations are shown. The lowest energy absorption corresponds to the configuration where all WECs have the smallest dimension with radius $R = 2$ m, whereas the highest corresponds to all four WECs with largest dimension $R = 3.5$ m. b) Annual energy absorption as a function of the total masses for four devices with CYL or CWM buoys as shown in figure 2 d-f). The result for all 256 configurations are shown. A line between the configurations with lowest and highest mass is drawn.

To study this, the analytical tool that has been presented in section 2 has further been used to study arrays consisting of WECs of different dimensions.

3.2.1. Arrays with 4 CYL devices in fixed layout

First, an array consisting of 4 WECs in the layout shown in figure 2 a-c) was studied. The radius of each cylinder buoy was allowed to take a value in the set $R \in \{2, 2.5, 3, 3.5\}$ m. A corresponding draft, power take-off and mass of the WEC for each radius was chosen as in table 1. The mass includes both the mass of the buoy and the translator, as discussed in section 2.2.

Assigning one of the four geometries to the four WECs gives a total of $4^4 = 256$ configurations. A parameter sweep of the configurations was performed and is shown in figure 7 a). Here, the averaged power over one hour irregular waves was multiplied with the number of hours in a year (8760 h) to give an estimate of the annual absorbed energy, which is plotted as a function of the total mass of the four WECs. As expected, the lowest amount of energy is absorbed by the configuration with the lowest mass, i.e. when all four WECs have the geometry # 1, and analogously the highest amount of energy is absorbed by the configuration with the highest mass consisting of four WECs of geometry # 4.

More interesting are all the configurations in between with mixed geometries. Since the mass can give a crude estimate of the construction costs of the devices, a low mass and high annual absorbed power is desired for a cost effective wave energy system. In other words, given a certain mass, a high power absorption is wanted, which corresponds to points as far above the marked line in figure 7 a) as possible. As seen in the figure, most of the configurations lie above the marked line and represent a better power-to-mass, or energy-to-cost, ratio than the two arrays of only the smallest or largest WEC dimension.

The configuration with four WECs of the smallest geometry # 1 have an average power of 24.3 kW and a q -factor of 0.96, i.e. some destructive interaction occurs between the WECs. The configuration with four WECs of the largest geometry # 4 absorbs a total power of 51.7 kW which gives a q -factor of 0.94, i.e. more destructive interaction occurs between the larger buoys. Based on this, a park of two small and two large WECs should have total power of 38.01 kW, but in fact the array shown in figure 2 b) absorbs a power of 39.1 kW which gives a q -factor of 0.98 and the park with layout shown in figure 2 c) absorbs 38.84 kW which corresponds to a q -factor of 0.97. In other words, if four small and four large WECs were to be deployed, installing them in mixed arrays of two small and two large WECs would give 2.9 % more absorbed power, than in separate arrays of small and large WECs. For a large park with hundreds of devices, this would amount to a large amount of increased annual energy absorption.

3.2.2. Arrays with different CYL and CWM devices in fixed layout

Similar to the parameter sweep of the cylinder geometries # 1-4 shown in figure 7 a), a parameter sweep of 4 WECs with either CYL or CWM buoys has been performed. The allowed geometries were # 1, 3, 5, 6 shown in table 1, corresponding to outer radius $R = 2$ m or $R = 3$ m and inner radius $R_{in} = 0$ m or $R_{in} = 1$ m. The result for the $4^4 = 256$ configurations is shown in figure 7 b) in terms of annual absorbed energy as function of total mass of the array, where the mass is the sum of the buoy and translator mass. For clarity, a line has been drawn between the configurations of lowest and highest mass.

As discussed above, a high power absorption, but also a low total mass is desired for a cost effective wave energy system. Thus, depending on the tariff price per electricity kWh and the construction cost per mass of the device, it might not be the array with highest power absorption that is optimal. In fact, when comparing the configurations in 7 b), the configuration with lowest and highest absorbed power-total mass ratio is the ones shown in figures 2 e) and 2 f), respectively, i.e., the array with smaller WECs have a better power-to-mass ratio.

Note that the annual absorbed energy has here been estimated by multiplying the average absorbed energy during one hour by the number of hours in a year. This assumption would hold if the wave climate at a site consisted of the same sea state during the whole year, which is obviously not the case. A more precise model would take into account the different sea states occurring during a year at a site and model the system for each sea state. However, the sea state chosen represents typical conditions at the Lysekil test site at the west coast of Sweden (Waters et al., 2009), and the method still gives a rough estimate of the annual energy absorption to compare the different configurations.

3.2.3. Arrays with 12 CYL devices in a random layout

To study the influence of layout in an array of devices of mixed geometries, an array of 12 WECs consisting of six devices of the smallest CYL geometry # 1 and six devices of the largest CYL geometry # 4 (see table 1) is considered. The 12 WECs were allowed to take random positions on a grid where the 6×6 grid points were separated by 10 m. The grid size was chosen small enough so that the total area is restricted, which will usually be the case in realistic offshore installations, but large enough so that the devices will not collide. In total, 1000 random configurations were modelled.

The three best and three worst configurations with respect to absorbed power are shown in figure 8. The upper row shows the three configurations among the 1000 random layouts that absorbed most power during the 1 hour sea state, and the lower row shows the three arrays that absorbed least. The average absorbed power in all the 1000 configurations is denoted P_{mean} . The three best configurations absorbed 8-9 % more than the average configuration, and the worst absorbed 10-11 % less. Expressed differently, the best layouts absorbed 23 % more power than the worst.

From figure 8, it is clear that there is more shadowing effect in the worst layouts, whereas the WECs are more spread out along the y-coordinates in the best layouts. Also, there seems to be a tendency that the larger WECs are situated *behind* the smaller WECs in the direction of wave propagation.

Since each array consists of six devices of geometry # 1 and six devices of geometry # 4, the total mass will be equal in all configurations. Hence, the power-to-mass ratio corresponds directly to the computed power absorption in each array.

4. Conclusions

A hydrodynamical model for point-absorber wave energy devices of different dimensions was presented and validated with the numeric benchmark tool WAMIT. The floats were either cylindrical or cylindrical with moonpools, all with individual outer and inner radius, draft, mass and connected to generators with different power take-off constants. The agreement with WAMIT was very good and the computational efficiency high. The hydrodynamical model was then used to study and optimize wave energy arrays with devices of different dimensions.

When studying arrays of four devices connected to cylinder floats of different dimensions (given in table 1), it was found that deploying WECs of different dimensions could improve the total power absorption. Specifically, two arrays of mixed WECs of geometry # 1 and # 4 would give 2.9 % more power absorption, than one array each of the two geometries.

For commercially competitive wave energy technology, a high power production must be achieved simultaneously as a low cost for construction and maintenance. A crude estimate for this is the power-to-mass ratio. The estimated

Figure 8: The three best (upper row) and three worst (lower row) configurations for a 12 WEC array, out of 1000 random layout on grid points. In each array, six of the WECs have the smallest geometry # 1 (radius 2 m) and six have the largest # 4 (radius 3.5 m). The title above each subplot shows the total absorbed power of the array expressed as the ratio of the average of the power taken over all 1000 configurations.

annual absorbed power of all of the studied configurations of four devices of CYL or CWM floats was plotted as a function of the total device masses in figure 7, and it was shown that the configurations that had the highest power production were not the one with highest power-to-mass ratio. Hence the optimal array might not correspond to the array with highest power absorption, which confirms earlier related studies by De Andres et al. (2015, 2016) and O'Connor et al. (2013).

Arrays consisting of 6 WECs of the smallest CYL geometry # 1 and 6 WECs of the largest CYL geometry # 4 were studied as a function of layout. 1000 arrays with random layouts were studied. The difference in power absorption between the best and worst layout was as large as 23 %. In the layouts with best power absorption, the devices were more spread out along the wave crest, and there also seemed to be a tendency for the larger devices to align behind the smaller ones in the direction of the wave propagation.

The comparison between different array configurations has in this paper been performed by parameter sweep. A chosen number of arrays with fixed or random buoy geometries and/or array layouts have been modelled and compared. Since several hundreds of configurations are compared, the results give qualitative insight into optimal design for wave energy layouts. However, an even more systematic approach is to use a genetic algorithm, as by Child and Venugopal (2010) or Giassi and Götteman (2017). The hydrodynamical model developed and presented in this paper is perfectly suited for implementation in such an optimization scheme.

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Appendix A. Defined functions and parameters

The vertical functions in the fluid velocity potential are defined as

$$\begin{aligned} Z_0(z) &= \frac{1}{N_0} \cosh(k(z+h)), & N_0^2 &= \frac{1}{2} \left(1 + \frac{\sinh(2kh)}{2kh} \right), \\ Z_m(z) &= \frac{1}{N_m} \cos(k_m(z+h)), & N_m^2 &= \frac{1}{2} \left(1 + \frac{\sin(2k_m h)}{2k_m h} \right), \end{aligned} \quad (\text{A.1})$$

and the incident wave coefficients are defined as

$$A_{0,n}^i = -\frac{igA^i(\omega)}{\omega} \frac{1}{Z_0(0)} e^{ikx^i} t^n. \quad (\text{A.2})$$

Integrating the vertical coordinate along the fluid column gives rise to the constants

$$c_{s0}^i = \frac{(-1)^s k}{k^2 + \lambda_s^i} \frac{1}{L^i N_0} \sinh(kL^i), \quad c_{sm}^i = \frac{(-1)^s k_m}{k_m^2 - \lambda_s^i} \frac{1}{L^i N_m} \sin(k_m L^i) \quad (\text{A.3})$$

where the distance between the float bottom and the seabed at equilibrium is $L^i = h - d^i$ and $\lambda_m^i = \pi m / L^i$. The expressions needed for Graf's addition theorems are

$$T_{0ln}^{ij} = \frac{1}{H_l(kR^j)} H_{l-n}(kR_{ij}) e^{i\theta_{ij}(l-n)}, \quad T_{mln}^{ij} = \frac{I_n(k_m R^i)}{K_l(k_m R^j)} K_{l-n}(k_m R_{ij}) e^{i\theta_{ij}(l-n)} (-1)^n, \quad (\text{A.4})$$

where R_{ij} and θ_{ij} are the distance and the angle between the two floats, respectively. In notation inspired by Mavrakos (2004) and Konispoliatis and Mavrakos (2016), the radial functions in the diffraction potential and their values as $R_{in} \rightarrow 0$ are defined as

$$\begin{aligned} R_{00}^i(r) &= \ln\left(\frac{r}{R_{in}^i}\right) / \ln\left(\frac{R_{in}^i}{R_{in}^i}\right) \rightarrow 1, & R_{0n}^i(r) &= \left[\left(\frac{r}{R_{in}^i}\right)^{|n|} - \left(\frac{R_{in}^i}{r}\right)^{|n|} \right] / \left[\left(\frac{R_{in}^i}{R_{in}^i}\right)^{|n|} - \left(\frac{R_{in}^i}{R_{in}^i}\right)^{|n|} \right] \rightarrow \left(\frac{r}{R_{in}^i}\right)^{|n|} \\ \hat{R}_{00}^i(r) &= \ln\left(\frac{R_{in}^i}{r}\right) / \ln\left(\frac{R_{in}^i}{R_{in}^i}\right) \rightarrow 0, & \hat{R}_{0n}^i(r) &= \left[\left(\frac{R_{in}^i}{r}\right)^{|n|} - \left(\frac{r}{R_{in}^i}\right)^{|n|} \right] / \left[\left(\frac{R_{in}^i}{R_{in}^i}\right)^{|n|} - \left(\frac{R_{in}^i}{R_{in}^i}\right)^{|n|} \right] \rightarrow 0 \\ R_{mn}^i &= \left[I_n(\lambda_m^i r) - \frac{I_n(\lambda_m^i R_{in}^i)}{K_n(\lambda_m^i R_{in}^i)} K_n(\lambda_m^i r) \right] / \left[I_n(\lambda_m^i R_{in}^i) - \frac{K_n(\lambda_m^i R_{in}^i)}{K_n(\lambda_m^i R_{in}^i)} I_n(\lambda_m^i R_{in}^i) \right] \rightarrow \frac{I_n(\lambda_m^i r)}{I_n(\lambda_m^i R_{in}^i)} \\ \hat{R}_{mn}^i &= \left[\frac{I_n(\lambda_m^i R_{in}^i)}{K_n(\lambda_m^i R_{in}^i)} K_n(\lambda_m^i r) - \frac{K_n(\lambda_m^i R_{in}^i)}{K_n(\lambda_m^i R_{in}^i)} I_n(\lambda_m^i r) \right] / \left[I_n(\lambda_m^i R_{in}^i) - \frac{K_n(\lambda_m^i R_{in}^i)}{K_n(\lambda_m^i R_{in}^i)} I_n(\lambda_m^i R_{in}^i) \right] \rightarrow 0. \end{aligned} \quad (\text{A.5})$$

Derivatives of the Bessel functions at the domain boundaries are defined as

$$\begin{aligned} q_{0n}^i &= h \frac{d}{dr} \left(\frac{H_n(kr)}{H_n(kR^i)} \right) \Big|_{r=R^i} & \tilde{q}_{0n}^i &= h \frac{d}{dr} (J_n(kr)) \Big|_{r=R^i} & \tilde{\tilde{q}}_{0n}^i &= h \frac{d}{dr} (J_n(kr)) \Big|_{r=R_{in}^i} \\ q_{mn}^i &= h \frac{d}{dr} \left(\frac{K_n(k_m r)}{K_n(k_m R^i)} \right) \Big|_{r=R^i} & \tilde{q}_{mn}^i &= h \frac{d}{dr} \left(\frac{I_n(k_m r)}{I_n(k_m R^i)} \right) \Big|_{r=R^i} & \tilde{\tilde{q}}_{mn}^i &= h \frac{d}{dr} \left(\frac{I_n(k_m r)}{I_n(k_m R_{in}^i)} \right) \Big|_{r=R_{in}^i} \end{aligned} \quad (A.6)$$

and

$$\begin{aligned} p_{mn}^i &= \frac{\varepsilon_n L^i}{2} \frac{d}{dr} R_{mn}^i(r) \Big|_{r=R^i}, & \tilde{p}_{mn}^i &= \frac{\varepsilon_n L^i}{2} \frac{d}{dr} R_{mn}^i(r) \Big|_{r=R_{in}^i}, \\ \hat{p}_{mn}^i &= \frac{\varepsilon_n L^i}{2} \frac{d}{dr} \hat{R}_{mn}^i(r) \Big|_{r=R^i}, & \tilde{\hat{p}}_{mn}^i &= \frac{\varepsilon_n L^i}{2} \frac{d}{dr} \hat{R}_{mn}^i(r) \Big|_{r=R_{in}^i} \end{aligned} \quad (A.7)$$

where the Neumann symbol is defined as $\varepsilon_0 = 1$ and $\varepsilon_n = 2$ for $n \neq 0$. The matrices of the multiple diffraction problem are defined as

$$\begin{aligned} L_{smn}^i &= 2 \sum_{r=0}^{\infty} p_{rn}^i c_{rm}^i c_{rs}^i, & \tilde{L}_{smn}^i &= 2 \sum_{r=0}^{\infty} \tilde{p}_{rn}^i c_{rm}^i c_{rs}^i, \\ \hat{L}_{smn}^i &= 2 \sum_{r=0}^{\infty} \hat{p}_{rn}^i c_{rm}^i c_{rs}^i, & \tilde{\hat{L}}_{smn}^i &= 2 \sum_{r=0}^{\infty} \tilde{\hat{p}}_{rn}^i c_{rm}^i c_{rs}^i \end{aligned} \quad (A.8)$$

and

$$\begin{aligned} D_{smn}^i &= q_{mn}^i \delta_{ms} - L_{smn}^i, \\ \tilde{D}_{smn}^i &= \tilde{q}_{mn}^i \delta_{ms} - \tilde{L}_{smn}^i ((J_n(kR_{in}^i) - 1) \delta_{m0} + 1), \\ \tilde{\tilde{D}}_{smn}^i &= \tilde{\tilde{q}}_{mn}^i \delta_{ms} - \tilde{\tilde{L}}_{smn}^i ((J_n(kR_{in}^i) - 1) \delta_{m0} + 1). \end{aligned} \quad (A.9)$$

Using a short notation for the matrix product $(\hat{L} \tilde{D}^{-1} \tilde{L}) = [\hat{L}_{smn}^i ((J_n(kR_{in}^i) - 1) \delta_{m0} + 1)] [\tilde{D}_{smn}^i]^{-1} [\tilde{L}_{smn}^i]$, the diffraction matrix in equation (10) contains the expressions

$$B_M^i = - \left[D^i - (\hat{L} \tilde{D}^{-1} \tilde{L})^i \right]^{-1} \left[\tilde{D}^i - (\hat{L} \tilde{D}^{-1} \tilde{L})^i ((J_n(kR^i) - 1) \delta_{m0} + 1) \right] \quad (A.10)$$

which simplifies to the single body diffraction matrix for a truncated cylinder $B = -D^{-1} \tilde{D}$ in case there is no moonpool, and $B_{diff}^i = B_M^i A_{0,n}^i$ is the vector for the incident wave.

The right-hand side of the multiple scattering equation of the radiation problem is given by the radiation matrices

$$\begin{aligned} [B_R^i]_s &= -\frac{1}{2} R_{0s}^i c_{0s}^i \left(p_{0n}^i \left(\left(\frac{R_{in}^i}{2L^i} \right)^2 - \frac{1}{6} \right) + \hat{p}_{0n}^i \left(\left(\frac{R_{in}^i}{2L^i} \right)^2 - \frac{1}{6} \right) \right) - 2L^i \sum_{m=1}^{\infty} (p_{mn}^i + \hat{p}_{mn}^i) c_{ms}^i \frac{(-1)^m}{\pi^2 m^2}, \\ [\tilde{B}_R^i]_s &= -\frac{1}{2} R_{in}^i c_{0s}^i \left(\tilde{p}_{0n}^i \left(\left(\frac{R_{in}^i}{2L^i} \right)^2 - \frac{1}{6} \right) + \tilde{\tilde{p}}_{0n}^i \left(\left(\frac{R_{in}^i}{2L^i} \right)^2 - \frac{1}{6} \right) \right) - 2L^i \sum_{m=1}^{\infty} (\tilde{p}_{mn}^i + \tilde{\tilde{p}}_{mn}^i) c_{ms}^i \frac{(-1)^m}{\pi^2 m^2} \end{aligned} \quad (A.11)$$

and

$$B_{rad}^i = \left[D^i - (\hat{L} \tilde{D}^{-1} \tilde{L})^i \right]^{-1} \left(B_R^i + (\hat{L} \tilde{D}^{-1})^i \tilde{B}_R^i \right) \delta_{n0}. \quad (A.12)$$

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