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Warped D-Brane Inflation and Toroidal Compactifications

Master Degree Project

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Abstract

We set out on the ambitious journey to fuse inflation and string theory. We first give a somewhat extensive, yet free from the most complicated details, review of string inflation, discussing concepts as flux compactifications, moduli stabilization, the η-problem and reheating. Then, we consider two specific configurations of type II supergravity; type IIB on $\mathbb{T}^6$ with D3-branes and O3-planes, and type IIA on a twisted torus with D6-branes and O6-planes. In both cases, we calculate the scalar potential from the metric ansatzes, and try to uplift it to one of de-Sitter (dS) type. In the IIA-case, we also derive the scalar potential from a super- and Kähler potential, before we search for stable dS-solutions.

Sammanfattning

Vi tar oss an uppgiften att försöka förena kosmisk inflation och strängteori. Vi börjar med att ge en relativt grundlig, men inte allt för detaljerad genomgång av stränginflation, där vi behandlar koncept såsom flödeskompaktifikationer, modulistabilisering, η-problemet och återuppvärmning. Vi fortsätter med att i detalj betrakta två specifika konfigurationer av typ II supergravitation; typ IIB på $\mathbb{T}^6$ med D3-bran och O3-plan, och typ IIA på en vriden torus med D6-bran och O6-plan. I båda fallen beräknar vi den effektiva skalärpotentialen som uppkommer när vi kompaktifierar teorin, och vi försöker modifiera den så att den är av de-Sitter (dS) typ. Då vi betraktar IIA-supergravitation, härleder vi även den effektiva potentialen från en super- och Kählerpotential, varefter vi söker efter stabila dS-lösningar.

Contents

1 Introduction

1.1 Cosmology

The last century saw two major developments within theoretical physics - quantum mechanics and general relativity. Einsteins geometrically formulated theory, the general relativity, gave corrections to Newtonian gravity and changed our view on our World, joining time and space into one spacetime, describing e.g. gravitational force as a curved spacetime. However, contributing with corrections making predictions of e.g. planet orbits more exact, one eventually stumbled on new problems. Some of these have been solved, and some haven't.

Among the solved problems, we have the *flatness problem* and the *horizon problem*. Observations tell us that our Universe is flat, but the mathematics indicate that a flat Universe is an unstable solution, and there is no reason for the Universe to remain flat. Also, observations indicate that the Universe had to be extremely homogenous and isotropic early on, contradicting what one would expect. Patches not being in causal contact appeared to be very homogenous and isotropic. The concept solving these two problems is one of the things we will focus on in this work - namely *inflation* - an early era of accelerating expansion. If one assumes this, the mathematics agrees with the observations.

Anyhow, some problems remain unsolved. General relativity is a classical theory, and when it is necessary to take quantum effects into account, despite the many attempts, there appear to be no way of making sense of it. For instance, close to a black hole, spacetime is infinitely curved and the classical theory collapses. As of today, there exists no theory for quantum gravity. However, giving up the attempt of directly quantizing general relativity, string theory has emerged as a promising candidate.

1.2 String Theory and Compactifications

String theory is, at present, the most studied framework in which one hopes to be able to describe quantum gravity. When one first started to consider and develop string theory, the goal was to understand the strong nuclear interactions keeping neutrons and protons together in the atomic core. It wasn't until later one realized that string theory contained parts of general relativity, and hence predicted gravity. However, by considering objects extended in one dimension, and the observable particles as different oscillation modes, one hoped to describe various phenomena related to strong nuclear interactions. [2]

As the theory developed further, five different perturbative string theories emerged: Type I, Type IIA/IIB, $E_8 \times E_8$ and $SO(32)$. In 1995, Edward Witten proposed that all these theories can be viewed as different perturbative limits of one more general theory - he called it *M-theory* [3]. One common feature for all theories though, which might be rather puzzling when relating it to our World and Universe as we observe it, is that they all require extra dimensions. The different string theories require ten dimensions and M-theory eleven. How could this be consistent with our every-day observations?

In order to deal with this problem, one needs to rewrite the ten (eleven) dimensional theory as a four dimensional theory - one has to *compactify* the theory. By assuming that the six (seven) extra dimensions to be small and compact - such that they are only observable at very high energies - one might solve this problem. One issue remains though - it is very challenging to determine the exact details of the extra dimensions such that they result in realistic and interesting cosmologies. One specific feature to look for is positive vacuum energy, indicating a positive cosmological constant and hence, an accelerated expansion of our Universe. If one manages to obtain a description agreeing with cosmological observations, it would be possible to use such observations to indirectly test string theory.

Even if we had done all of that, there would still be a lot to check. For example, does the potential sourcing the vacuum energy allow for inflation? Mathematically, inflation is a rather delicate process, not to mention the processes taking place immediately after inflation. First of all, inflation must appear naturally, and it has to last for a sufficiently long time in order for it to solve the above problem. Then, there must be a natural ending of inflation, and that ending must correspond to a stable vacuum energy point in the potential. Furthermore, the potential needs to increase sufficiently in order to decrease the probability for quantum effects ruining the stable vacuum. Lastly, the energy must be distributed correctly in order for the processes following inflation to occur. In whatever configuration, the cosmological constant will eventually take the overhand. The important thing is that the cosmological constant is small enough, i.e. that a tiny fraction of the energy end up as a cosmological constant contribution. Finding a string theoretical configuration fulfilling all these criteria, would for sure revolutionize theoretical physics, and will, as mentioned, provide an indirect test of how well string theory describes Nature. This is an open field of research called *string cosmology*.

In this work, our focus is to find such realistic cosmologies. We try to find configurations leading to a stable solution with positive vacuum energy. In order to do this, we first give a brief review on string inflation.

2 String Inflation

The last few decades have provided observational cosmological results changing the theoretical views on cosmology and string theory. The complete game changer was, of course, the observations indicating that the expansion of our Universe is accelerating, a result that had impact on the theory of compactifications. Until then, compactifications using Calabi-Yau manifolds had proven very promising, but all of a sudden the obtained results didn't make that much physical sense any more.

Even though Calabi-Yau compactifications didn't result in the desired physics, the techniques could be used and slightly modified in order to match observation to a higher extent. For example, one considered certain spaces by using a Calabi-Yau manifold as a bulk space, and gluing so-called throats to it. Certain such configurations actually allow for analytical solutions. We will give a review on this topic, called conifold compactifications, and discuss it's ups and downs.

One could also imagine that the actual Calabi-Yau manifolds appear slightly deformed, which was suggested by Scherk and Schwarz in [4], making it possible to use the very same techniques. We will treat examples of ordinary Calabi-Yau compactifications and of such inspired by Scherk and Schwarz. Ultimately, we will look for stable solutions with appropriate cosmology and relate the different parts.

2.1 On Our Universe, Generic Inflation and Its Limitations

In order to settle notation and conventions, we will briefly review on why inflation is desired in general relativity and how we have to improve the theory. Our reasoning is based on [5], [6]. On large scales, we describe our Universe as homogenous and isotropic, which results in the Robertson-Walker metric,

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega^2_{(2)}\right), \tag{2.1}$$

and the Friedmann equations,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}, \tag{2.2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{2.3}$$

where $a(t)$ is the dimensionless scale factor. The Friedmann equations are derived from the Einstein equations, modeling the matter in the Universe[1] as a perfect fluid/gas. These equations describe our Universe well, up to certain issues.

First of all, observations indicate that our Universe is flat. We introduce $\Omega = \frac{8\pi G}{3H^2}\rho$, with $H = \frac{\dot{a}}{a}$, enabling us to write (2.2) as,

$$\Omega - 1 = \frac{\kappa}{H^2 a^2}, \tag{2.4}$$

where κ is the curvature. This means that Ω should be close to 1, since the curvature is nearly vanishing. But Ω is time-dependent, and $\Omega = 1$ is an *unstable* fix point, so why is $\Omega \sim 1$, and not anything else? This intuitive contradiction is called the *flatness problem*.

An additional problem arise when looking into the cosmic microwave background (CMB). Since the age of the Universe is finite, photons can only have travelled a finite distance between two certain times. We define the *comoving horizon* to be this maximum distance,

$$\tau = \int_0^t \frac{dt'}{a(t')}, \tag{2.5}$$

where we have used that light rays satisfy $ds^2 = 0$, and that the Universe is isotrpic. This can be rewritten as,

$$\tau = \int_0^a \frac{da}{Ha^2} = \int_0^a d(\ln a)\left(\frac{1}{aH}\right), \tag{2.6}$$

[1] I will refer to our Universe as *the* Universe without putting to much focus on what this actually means. The formulation should not be thought of as me stating that our Universe is the only existing.

with $aH = H_0^{-1} a^{\frac{1}{2}(1+w)}$, where w appears in the equation of state for perfect fluids,

$$p = w\rho. \tag{2.7}$$

This means that τ increases with time,

$$\tau \propto a^{\frac{1}{2}(1+3w)}. \tag{2.8}$$

So, the fraction of the Universe in causal contact *increases* monotonically. But, observations incicate that patches of the CMB not in causal contact have the same temperature to high precision. How can the Universe be homogenous at that time even though most of the regions were causally disconnected? The is called the *horizon problem.*

In order to address these problems, something needs to be done. If we modify the picture and consider an early era of inflation (accelerated expansion), we can actually solve both the flatness- and the horizon problem. The idea is to say that τ got large contribution at early times. This means that $(aH)^{-1}$ (which is the Hubble radius, indicating how far apart particles can be today in order to communicate) was smaller early on than it is now.Thus, regions which were in causal contact at CMB, might not be so today. Therefore the CMB can be homogenous without contradictions. Also, note that if $(aH)^{-1}$ decreases with time, (2.4) goes to 0,

$$\Omega - 1 = \frac{1}{(aH)^2} \to 0. \tag{2.9}$$

It drives the Universe towards flatness. Thus, we have solved both the flatness- and the horizon problem!

Inflation gives us certain constraints on our Universe. The Hubble radius must shrink,

$$\frac{d}{dt}\left(\frac{1}{Ha}\right) < 0, \tag{2.10}$$

which gives us an accelerated expansion,

$$\frac{d^2a}{dt^2} > 0, \tag{2.11}$$

resulting in negative pressure,

$$\rho + 3p < 0 \Rightarrow p < -\frac{1}{3}\rho. \tag{2.12}$$

The most straight-forward way of receiving an early inflationary phase, is to consider a Universe dominated by a spatially homogenous scalar field, the *inflaton.* The action is (given that the scalar is minimally coupled to gravity),

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \tag{2.13}$$

resulting in the following equations of motion,

$$\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}\partial^\mu\phi + \frac{\partial V}{\partial\phi} = \nabla^2\phi + \frac{\partial V}{\partial\phi} = 0. \tag{2.14}$$

For a Friedmann-Robertson-Walker Universe we get,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{2.15}$$

with $\kappa \to 0$, since the inflation flattens the Universe. The energy-momentum tensor for this action is,

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right], \tag{2.16}$$

giving us the Friedmann equations for our specific case,

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] := \frac{1}{3M_{\rm pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]. \tag{2.17}$$

Inflation occurs if the kinetic energy of ϕ is much smaller than the potential energy,

$$\dot{\phi}^2 \ll V(\phi). \tag{2.18}$$

Also, in order for inflation to maintain sufficiently long,

$$|\ddot{\phi}| \ll |V'|. \tag{2.19}$$

Schematically, this looks like ,

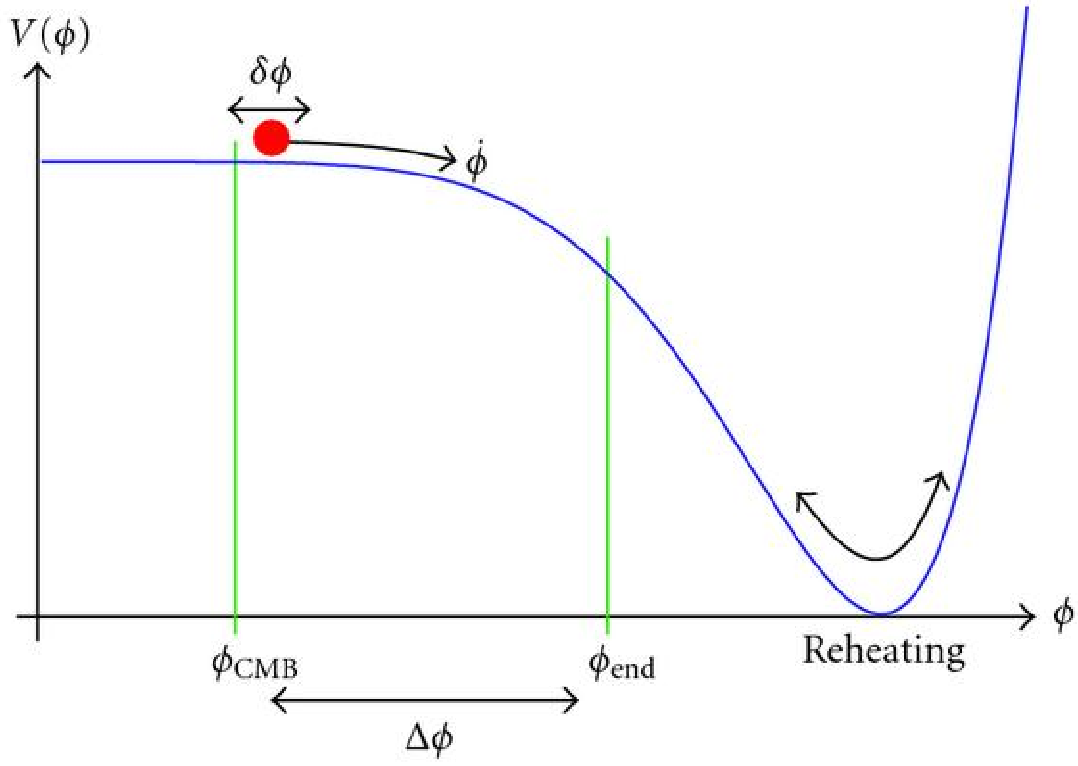

Figure 1: An example of how a slow-roll inflation potential might look like. Figure credits: https://www.hindawi.com/journals/aa/2010/565248.fig.001.jpg (2017-07-28).

Inflation ends when $\frac{1}{2}\dot{\phi}^2 \sim V(\phi)$. We introduce the *slow-roll parameters* to summarize these demands,

$$\epsilon = \frac{1}{2}M_{\text{pl}}^2\left(\frac{V'}{V}\right)^2, \tag{2.20}$$

$$\eta = M_{\text{pl}}^2\frac{V''}{V}, \tag{2.21}$$

and inflation occurs if $\epsilon, |\eta| \ll 1$. During inflation, the Hubble parameter H^2 is roughly constant, $\dot{\phi} \sim -\frac{V'}{3H}$, and our spacetime is approximately de-Sitter,

$$a(t) \sim e^{Ht}. \tag{2.22}$$

Inflation ends when $\epsilon \sim 1$.

Even though this model seems nice, it doesn't solve all the problems. In particular, it is not UV-complete. Thus, we have to consider another type of framework. This is where string theory enters the stage. If we can prove that inflation arise naturally from string theory, it can be tested in all regimes, and might even serve as a measure of how good a theory string theory is.

2.2 Idea and Limitations

The main motivation for trying to incorporate inflation in string theory is the provided knowledge of UV-physics. Our aim can be summarized in one rather simple expression,

$$S_{10}[\mathcal{C}] \mapsto S_4[\Phi(t)], \tag{2.23}$$

where $\mathcal{C}$ consist of ten-dimensional geometrical data, fluxes, localized sources and quantum effects. $\Phi(t)$ is a time dependent configuration of scalar fields in the four-dimensional effective theory. We want to specify the compactification data, giving an effective theory with interesting cosmology. We have to demand that S_4 have positive vacuum energy contribution and that the light moduli Φ describe a controlled instability of the vacuum.

Observations in the CMB indicate that we have probe energies of order the expansion rate H when modes cross the horizon and freeze. Higher energies gives a four-dimensional effective action parametrizing UV-physics, which is both computable and comprehensible in string theory.

The ideal case related to string inflation is to be able to derive the inflaton action from first principles. We would like to start with a configuration $\mathcal{C}$ for some compactification, solve the equation of motion for the ten-dimensional supergravity theory (order by order in α' and g_s) and then integrate out the massive fields, obtaining the four-dimensional effective theory. Unfortunately, this cannot be done analytically, so we have to use approximations. One such is the α'-expansion, valid when the gradients of the background fields are small in units of α'. One problem occurs though, related to our compactification. Upon compactifying, Kaluza-Klein modes appear, the mass of which we want to be larger than the expansion rate. Thus, the compactification volume is limited and the approximation is not working well. One could also use the g_s-expansion, assuming the coupling $g_s = e^{\Phi} \ll 1$. But this assumption disrupts the balance of energies responsible for moduli stabilization, since the dilaton Φ couples to most of the fields and the localized sources. An approximation that we actually will use occasionally, is the noncompact approximation. It will become clear that we want the compactification space to be compact. But we cannot derive analytic expressions for the metric on Calabi-Yau threefolds. Instead, we consider Calabi-Yau cones, which are noncompact, and approximate our space by a finite portion of such a space.

2.3 Compactifications

In order to make everything as clear as possible and to introduce our conventions, we will spend some time introducing various techniques often used in string inflation.

2.3.1 Flux Compactifications

One problem we have to deal with when trying to fuse string theory and inflation is how to obtain our four observable dimensions from the ten or eleven dimensions imposed by string theory or M-theory. For example, one could imagine a world on a Dp-brane, from which we can consider local models by decoupling gravity. One could also think that a Dp-brane wraps a p-cycle, i.e. a p-dimensional topologically non-trivial (not simply connected) submanifold of our geometry, resulting in (p+1)-dimensional gravity.

An assumption often made regardless, is that our ten dimensional geometry is of the form,

$$\mathcal{M}_{(10)} = \mathcal{M}_{(1,3)} \times \mathcal{M}_{(6)}, \tag{2.24}$$

where $\mathcal{M}_{(1,3)}$ is our four dimensional spacetime and $\mathcal{M}_{(6)}$ is a *compact* internal space. Below, we introduce an example of something similar that was attempted by Kaluza and Klein in the beginning of the last century.

Example 2.1 (Kaluza-Klein Truncation - Fusing Gravity and Gauge Theories)**.** The purpose of the Kaluza-Klein truncation was to unify gravity and gauge theories, using $U(1)$ gauge theory as a model. They wanted to compactify five dimensional Einstein gravity on S^1_R, a circle of radius R. The 5D-metric can be written as,

$$ds_5^2 = e^{2\alpha\phi} ds_4^2 + e^{2\beta\phi}(dz + A)^2, \tag{2.25}$$

where $(dz + A)^2 = dz^2 + A_\mu A_\nu dx^\mu dx^\nu + 2A_\mu dx^\mu dz$, and $\alpha, \beta \in \mathbb{R}$ (resulting in the Einstein frame for the metric and canonical kinetic terms for ϕ). The five dimensional metric g_{MN}, $M = \mu, z$, gives rise to three four dimensional objects; the four dimensional metric $g_{\mu\nu}$, a four dimensional vector field $g_{\mu z}$, usually denoted A_μ and a four dimensional scalar g_{zz}, usually denoted ϕ. Since the compactification is on S^1_R, we can Fourier expand the metric (we have that $z \sim z + 2\pi R$),

$$g_{MN}(X, z) = \sum_{k=0}^{\infty} g^{(k)}_{MN}(X) e^{i\frac{kz}{R}}, \tag{2.26}$$

giving us the so called KK-tower, $g^{(k)}_{\mu\nu}, A^{(k)}_\mu, \phi^{(k)}$, resulting in a Klein-Gordon equation for the dilaton modes,

$$\left(\Box - \frac{k^2}{R^2}\right)\phi^{(k)} = 0, \tag{2.27}$$

where $\frac{k^2}{R^2}$ acts as a mass term. We see that the $k = 0$ mode is massless. Therefore, we want to compactify using a small R, in order to get really massive modes for $k > 0$. Then we can integrate them out and obtain a good and rather accurate effective description. What we actually do is neglecting z-dependence if R is small enough. This gives us,

$$-g_5 = -g_4 e^{8\alpha\phi + 2\beta\phi}, \tag{2.28}$$

and thus the five dimensional action,

$$S^{(5D)} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g_5}\, \mathcal{R}^{(5)}, \tag{2.29}$$

gives us the effective four dimensional action,

$$S^{(4D)} = \int d^4x\sqrt{-g_4}e^{(4\alpha+\beta)\phi}\left[\frac{2\pi R}{2\kappa_5^2}\,\mathcal{R}^{(4)}\,e^{a\phi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}e^{b\phi} - \frac{1}{2}(\partial\phi)^2 e^{c\phi}\right], \tag{2.30}$$

where a, b, c are some powers to be determined, the exact form of them is not important for our reasoning. The four dimensional Newton term can be identified as,

$$\frac{1}{2\kappa_4^2} = \frac{2\pi R}{2\kappa_5^2}, \tag{2.31}$$

and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the gauge field, $F_{(2)} = dA_{(1)}$. Now, we want to get rid of the ϕ-dependence multiplying $\mathcal{R}^{(4)}$. Our Einstein equations are,

$$G_{\mu\nu} = \kappa_4^2 T_{\mu\nu}^{(\phi+\mathrm{Max})}, \tag{2.32}$$

with,

$$T_{\mu\nu}^{\phi} \sim \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}(\partial\phi)^2, \tag{2.33}$$

$$T_{\mu\nu}^{\mathrm{Max}} \sim F_{\mu\rho}F_\nu^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}. \tag{2.34}$$

The equations of motion are,

$$\nabla^\mu F_{\mu\nu} = 0, \tag{2.35}$$

$$\Box\phi = e^{a'\phi}F_{\mu\nu}F^{\mu\nu}. \tag{2.36}$$

with a' again an, for the reasoning, unimportant constant. Here the problems start. ϕ is massless, and just setting $\phi \equiv 0$ is inconsistent, we would have to kill electromagnetism as well, contradicting the main assumption.

Even though the result of this example is negative, we will use similar methods in our attempts to fuse string theory and inflation. Inspired by the Kaluza-Klein truncation above, we in the next example consider how a compactification from six to four dimensions could look like. This is a somewhat extended (i.e. some details are included) version of what they deal with in [7].

Example 2.2. We start with the six dimensional Einstein/Yang Mills action,

$$S_6 = \int d^6X\sqrt{-g_6}\left(M_6^4\,\mathcal{R}^{(6)} - M_6^2|F_{(2)}|^2\right), \tag{2.37}$$

with the metric ansatz,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + R^2\hat{g}_{mn}dy^m dy^n, \tag{2.38}$$

where the metric $\hat{g}_{mn}$ has unit determinant. Here, μ, ν are 4D spacetime indices, and m, n are compact indices, $m = 4, 5$. We thus assume that our manifold is of the form $\mathcal{M}_{(6)} = \mathcal{M}_{(4)} \times \mathcal{M}_{(2)}$. We will deal with the different terms in the action one-by-one.

First, we rescale the four dimensional metric by introducing,

$$h_{\mu\nu} = R^2 g_{\mu\nu}, \tag{2.39}$$

which gives us,

$$ds^2 = R^{-2} h_{\mu\nu} dx^\mu dx^\nu + R^2 \hat{g}_{mn} dy^m dy^n. \tag{2.40}$$

The Ricci scalar can be written as,

$$\mathcal{R}^{(6)} = R^2 \mathcal{R}^{(4)}[h] + R^{-2} \mathcal{R}^{(2)} + ..., \tag{2.41}$$

where $\mathcal{R}^{(4)}[h]$ indicates that we have used $h_{\mu\nu}$ when constructing the Ricci scalar. The dots indicate that we omit the dependence of any derivatives of the rescaling factor R, which appear when writing the Ricci scalar in terms of a rescaled metric. The determinant of the metric goes as,

$$\sqrt{-g_6} = \sqrt{-g_4 R^4} = \sqrt{-h R^{-4}}. \tag{2.42}$$

Combining (2.41) and (2.42) yields,

$$\int d^6X \sqrt{-g_6} M_6^4 \mathcal{R}^{(6)} = \int d^6X \sqrt{-h} R^{-2} M_6^4 \left(R^2 \mathcal{R}^{(4)}[h] + R^{-2} \mathcal{R}^{(2)} \right). \tag{2.43}$$

Introducing $M_4^2 = M_6^4 R^2$, and remembering that the Euler characteristic of the surface described by $\hat{g}_{mn}$ is,

$$\int_{\hat{\mathcal{M}}_{(2)}} K = 2\pi\chi(\hat{\mathcal{M}}_{(2)}), \tag{2.44}$$

where $K = \frac{\mathcal{R}^{(2)}}{2}$ is the Gaussian curvature, we get,

$$\int d^6X \sqrt{-g_6} M_6^4 \mathcal{R}^{(6)} = \int d^4x \sqrt{-h} \left[M_4^2 R^{-2} \mathcal{R}^{(4)} \times \text{vol}(\mathcal{M}_{(2)}) + M_6^4 R^{-4} \cdot 4\pi\chi \times \text{vol}(\mathcal{M}_{(2)}) \right], \tag{2.45}$$

which can be put on the form,

$$\int d^6X \sqrt{-g_6} M_6^4 \mathcal{R}^{(6)} = \int d^4x \sqrt{-h} M_4^2 \left[\mathcal{R}^{(4)}[h] - \tilde{V}(R) \right], \tag{2.46}$$

with,

$$\tilde{V}(R) = -4\pi \frac{\chi}{R^4}. \tag{2.47}$$

The term including $|F_{(2)}|^2$ will be dealt with analogously. One important thing is the quantization of $F_{(2)}$,

$$\int_{\hat{\mathcal{M}}_{(2)}} F_{(2)} = n, \quad n \in \mathbb{Z}, \tag{2.48}$$

which can be written as,

$$\int_{\mathcal{M}_{(2)}} |F_{(2)}|^2 = \frac{n^2}{R^4} \times \text{vol}(\mathcal{M}_{(2)}). \tag{2.49}$$

Using this, we get,

$$\int d^6X\sqrt{-g_6}M_6^2|F_{(2)}|^2 = \int d^4x\sqrt{-h}R^{-2}M_6^2\frac{n^2}{R^4}\times \mathrm{vol}(\mathcal{M}_{(2)})$$
$$= \int d^4x\sqrt{-h}M_4^2\frac{n^2}{M_6^2R^6}. \tag{2.50}$$

So, in total we have,

$$S = \int d^6X\sqrt{-g_6}\left(M_6^4\,\mathcal{R}^{(6)} - M_6^2|F_{(2)}|^2\right) = \int d^4x\sqrt{-h}M_4^2\left(\mathcal{R}^{(4)}[h] - V(R)\right), \tag{2.51}$$

with,

$$V(R) = -4\pi\frac{\chi}{R^4} + \frac{n^2}{M_6^2R^6}. \tag{2.52}$$

For our purpose here, we are only interested in the scaling and not the pre factors. Thus, we say, roughly,

$$V(R) \sim -\frac{\chi}{R^4} + \frac{n^2}{R^6}, \tag{2.53}$$

and the potential has a minimum for $R \sim n\chi^{-\frac{1}{2}}$. The interesting thing here is that the potential, and hence gravity, is a topological object, determined by the Euler characteristics!

2.3.2 Moduli

As mentioned in the previous section, we will consider solutions with a geometry of the form

$$\mathcal{M}_{(10)} = \mathcal{M}_{(1,3)} \times \mathcal{M}_{(6)}, \tag{2.54}$$

where $\mathcal{M}_{(6)}$ consist of moduli fields, describing the size and the shape of the extra dimensions. In order to deliver a picture as clear as possible, we will consider a toy-model example.

Example 2.3 (Compactification on a Calabi-Yau three-fold in $\mathcal{N} = 2$ supersymmetry)**.** Before 1997, we didn't know that the cosmological constant Λ was non-zero, and we furthermore didn't know of the AdS/CFT correspondence. Thus, compactifications of this kind where very successful. Some kind of moduli fields arise naturally from Calabi-Yau manifolds. For example, we have the Kähler moduli, coming from those deformations of the metric keeping it Hermitian (with respect to *some* complex structure, not necessarily the original one). These are realized as,

$$\delta\Omega = i\delta g_{ij}dz^i \wedge d\bar{z}^j, \tag{2.55}$$

with $\delta\Omega \in H^{1,1}_{\bar{\partial}}(\mathcal{M}, \mathbb{C})$. Also, we have moduli fields arising from similar deformations of the almost complex structure, realized as,

$$\chi := \omega_{ijk}\delta J^k_{\bar{l}}dz^i \wedge dz^j \wedge dz^{\bar{l}} = \omega_{ijk}\delta g_{\bar{h}\bar{l}}g^{k\bar{h}}dz^i \wedge dz^j \wedge dz^{\bar{l}}, \tag{2.56}$$

where $\chi \in H^{2,1}_{\bar{\partial}}(\mathcal{M}, \mathbb{C})$ and ω is the nowhere-vanishing $(3,0)$-form of $\mathcal{M}$. Such a deformation is called a complex structure moduli. These deformations should be thought of the change in the complex structure induced by the change in the metric, such that the new metric is Hermitian with respect to the new complex structure.

These two moduli fields can be parametrized [1], and thus seen in another way. If we introduce α^I, $I = 1, ..., h^{1,1}$, where $h^{1,1}$ is the Hodge number, i.e. the dimension of $H^{1,1}_{\bar{\partial}}(\mathcal{M}, \mathbb{C})$. The α^I consist a basis of $H^{1,1}_{\bar{\partial}}(\mathcal{M}, \mathbb{C})$. The Kähler form can then be written as,

$$\Omega = t^I(x)\alpha_I, \tag{2.57}$$

with $t^I(x)$ being $h^{1,1}$ real four dimensional scalar fields. Similarily, introducing $\zeta^A(x)$, and writing $\delta J^k_{\bar{l}}$ in terms of variations in the metric, we get,

$$\delta g^A_{\bar{i}\bar{j}} = c\zeta(x)^A (\chi_A)_{kl\bar{i}} \bar{\omega}^{kl}_{\bar{j}}. \tag{2.58}$$

The $\zeta(x)^A$ are in fact the complex structure moduli, and the $\chi^A \in H^{2,1}_{\bar{\partial}}(\mathcal{M}, \mathbb{C})$, $A = 1, ..., h^{2,1}$ consist a basis of $H^{2,1}_{\bar{\partial}}(\mathcal{M}, \mathbb{C})$.

Using this, we can expand the p-forms in the NS-NS- and R-R sector of string theory. Without loss of generality, we will focus only on type IIB string theory, and we will only consider scalar contributions. The relevant forms are $B_{(2)}$, $C_{(2)}$ and $C_{(4)}$, expanded as,

$$B_{(2)} = B_{(2)}(x) + b^I(x)\alpha_I(x), \tag{2.59}$$

$$C_{(2)} = C_{(2)}(x) + c^I(x)\alpha_I(x), \tag{2.60}$$

$$C_{(4)} = \vartheta^I(x)\tilde{\alpha}_I(x), \tag{2.61}$$

where $B_{(2)}$ and $C_{(2)}$ represents a ten dimensional two form, and $B_{(2)}(x)$ and $C_{(2)}(x)$ a four dimensional two form. $\tilde{\alpha}_I$ is a basis of $H^{2,2}_{\bar{\partial}}(\mathcal{M}, \mathbb{C})$. We get two more scalars from the dilaton Φ and $C_{(0)}$.

This serves as an example on how moduli fields appear. Some of such moduli fields might be massless - these we want to get rid of! This procedure is called *moduli stabilization* and it will be the topic of the next section.

2.3.3 Moduli Stabilization

One of the hardest problems regarding moduli, and especially in string inflation, is to determine (and deal with) the potential. In this section, we will discuss three things contributing to the moduli-part of the potential.

Example 2.4 (Flux Contribution to the Effective Potential)**.** A flux is associated to p-form gauge fields, $C_{(p)}$. These give rise to fields strengths, $F_{(p+1)} = dC_{(p)}$ on $\mathcal{M}_{(6)}$.[2] To have a

[2]We will see that this definition of the field strength has to be modified in some cases. Note though that the Bianchi identity $dF_{(p+1)} = 0$ always holds.

flux on $\mathcal{M}_{(6)}$, it has to be topologically non-trivial. Also, we must have $(p+1)$-dimensional submanifolds ($(p+1)$-cycles), denoted $\mathcal{C}_{(p+1)}$. The flux generated by $F_{(p+1)}$ is then,

$$\int_{\mathcal{C}_{(p+1)}} F_{(p+1)} \Rightarrow F_{(p+1)} = f_{(p+1)} \mathrm{vol}_{\mathcal{C}_{(p+1)}}, \tag{2.62}$$

so $F_{(p+1)}$ is moduli dependent. Taking the ten dimensional string theory action,

$$S^{(10)} = \int d^{10}x\sqrt{-g}\frac{1}{2\kappa_{10}^2}\left[\mathcal{R}^{(10)}\, e^{-2\Phi} + \frac{9}{4}(\partial\Phi)^2 e^{-2\Phi} - \frac{1}{12}|H_{(3)}|^2 e^{-2\Phi} \right.$$
$$\left. -\frac{1}{2}\sum_p \frac{1}{(p+1)!}|F_{(p+1)}|^2\right] + (\text{Chern-Simons}), \tag{2.63}$$

we see that $f_{(p+1)}^2 \mathrm{vol}(\sigma)$ is acting as an effective potential,

$$f_{(p+1)}^2 \mathrm{vol}(\sigma) \equiv V_{\text{eff}}, \tag{2.64}$$

representing the flux contribution to the potential.

Example 2.5 (The Contribution from the Geometry to the Effective Potential)**.** Now, we assume that $\mathcal{M}_{(6)}$ is curved. We will use the following ansatz for the metric,

$$ds_{10}^2 = \tau ds_4^2 + \rho ds_6^2, \tag{2.65}$$

with τ, ρ as four dimensional scalars. ρ normalizes the determinant of the six dimensional metric to 1 and τ is basically the four dimensional Planck mass, making sure we end up in Einstein frame. Since the metric is split, the Ricci tensors will be as well,

$$\mathcal{R}_{MN}^{(10)} = \mathcal{R}_{\mu\nu}^{(4)} \oplus \mathcal{R}_{mn}^{(6)}, \tag{2.66}$$

giving us a Ricci scalar of the form,

$$\mathcal{R}^{(10)} = \tau^{-1}\,\mathcal{R}^{(4)} + \rho^{-1}\,\mathcal{R}^{(6)}\,. \tag{2.67}$$

The determinant of the ten dimensional metric will go as,

$$-g_{10} = -g_4 g_6 \tau^4 \rho^6. \tag{2.68}$$

The gravity term in the action will then be written as,

$$\sqrt{-g_{10}}\,\mathcal{R}^{(10)}\, e^{-2\Phi} = \sqrt{-g_4}\left[\tau\rho^3 e^{-2\Phi}\,\mathcal{R}^{(4)} + \tau^2\rho^2 e^{-2\Phi}\,\mathcal{R}^{(6)}\right]. \tag{2.69}$$

As in the Kaluza-Klein truncation, we want to get rid of the Φ-dependence multiplying the four dimensional Ricci scalar. This imposes,

$$\tau\rho^3 e^{-2\Phi} = 1. \tag{2.70}$$

Using this gives us,

$$\sqrt{-g_{10}}\,\mathcal{R}^{(10)}\,e^{-2\Phi} = \sqrt{-g_4}\left[\mathcal{R}^{(4)} + \tau\rho^{-1}\,\mathcal{R}^{(6)}\right]. \tag{2.71}$$

Since we assumed $\mathcal{M}_{(6)}$ to be curved, $\mathcal{R}^{(6)} \neq 0$, giving us a curvature contribution to the effective potential as,

$$V_{\text{eff}}(\tau,\rho) \equiv \mathcal{R}^{(6)}\,\tau\rho^{-1}. \tag{2.72}$$

Example 2.6 (The Contribution from the Branes to the Effective Potential)**.** As we mentioned, the definition of the field strength can be modified as,

$$F_{(p+1)} = dC_{(p)} + \mathring{F}_{(p+1)}, \tag{2.73}$$

where $\mathring{F}_{(p+1)}$ is a closed, but not exact, form. Varying the Chern-Simons term in the action with respect to the $C_{(p)}$ gives,

$$dF_{(p+1)} = \text{tadpoles quadratic in } F's \overset{!}{=} 0. \tag{2.74}$$

In type IIB string theory, we have,

$$dF_{(5)} = F_{(3)} \wedge H_{(3)} \overset{!}{=} 0. \tag{2.75}$$

Branes can act as sources to the Bianchi identities. Adding them into our system, changes the Bianchi identities as,

$$dF_{(5)} = F_{(3)} \wedge H_{(3)} + j_{(6)}. \tag{2.76}$$

The Chern-Simons Lagrangian will contain operators of the form,

$$\mathcal{L}_{C-S} \supset F_{(3)} \wedge H_{(3)} \wedge C_{(4)}, \tag{2.77}$$

and the action for brane will be,

$$S_{(\text{brane})} = S_{(\text{DBI})} + S_{(\text{WZ})}, \tag{2.78}$$

with,

$$S_{(\text{DBI})} = -T_p \int d^{p+1}\xi\,\sqrt{\tilde{g}_{p+1}}, \tag{2.79}$$

$$S_{(\text{WZ})} = Q_p \int_{WV(p+1)} C_{(p+1)}, \tag{2.80}$$

where $\tilde{g}$ is the metric induced on the worldvolume of the brane. The contribution to the effective potential will be, in general,

$$V_{\text{eff}} = Q_p \text{vol}(WV_{(p+1)}), \tag{2.81}$$

where again, the volume element is moduli dependent.

We can rewrite the Wess-Zumino part of the action as (specializing in IIB),

$$S_{(\mathrm{WZ})} = Q_3 \int_{WV4} C_{(4)} = \int_{\mathbb{R}^{1,9}} j_{(6)} \wedge C_{(4)}, \tag{2.82}$$

identifying

$$Q_3 \equiv \int_{T_{(6)}} j_{(6)}, \tag{2.83}$$

where $T_{(6)}$ is the transverse space of the worldvolume. In order to have $dF_{(5)} = 0$, we have the so called tadpole cancellation condition. The dual of the lagrangians will be of the form,

$$\star_{10} \left(\mathcal{L}_{(\mathrm{DBI})} + \mathcal{L}_{(\mathrm{WZ})}\right) \sim F_{(3)} \wedge H_{(3)} \wedge C_{(4)} + j_{(6)} \wedge C_{(4)}, \tag{2.84}$$

giving us,

$$F_{(3)} \wedge H_{(3)} + j_{(6)} \stackrel{!}{=} 0, \tag{2.85}$$

which can be thought of as some kind of generalized Gauss Law constraint.

2.3.4 Warped Geometries

In a more general context, it will eventually be useful to see how the compactness of the six dimensional internal space appear in the mathematics, and how it affects the four dimensional spacetime. The ten dimensional action for type IIB string theory is,

$$S = \frac{1}{2\kappa_{10}^2} \left\{ \int d^{10}X \sqrt{-g} \left[\mathcal{R}^{(10)} - \frac{|\partial\tau|^2}{2\Im(\tau)^2} - \frac{|G_{(3)}|^2}{2 \cdot 3! \Im(\tau)^2} - \frac{|\tilde{F}_{(5)}|^2}{4 \cdot 5!} \right] + \frac{1}{4i} \int \frac{C_{(4)} \wedge G_{(3)} \wedge \bar{G}_{(3)}}{\Im(\tau)} \right\} + S_{loc}, \tag{2.86}$$

with the identifications,

$$\begin{aligned} \tilde{F}_{(5)} &= F_{(5)} - \frac{1}{2} C_{(2)} \wedge H_{(3)} + \frac{1}{2} B_{(2)} \wedge F_{(3)}, & (2.87) \\ F_{(p+1)} &= dC_{(p)}, & (2.88) \\ H_{(3)} &= dB_{(2)}, & (2.89) \\ G_{(3)} &= F_{(3)} - \tau H_{(3)}, & (2.90) \\ \tau &= C_{(0)} - ie^{-\Phi}, & (2.91) \end{aligned}$$

with Φ the dilaton. Note that we are using a modified definition of the five form flux $\tilde{F}_5$, as we discussed in the last section. S_{loc} is the contribution from local sources. We are interested in warped solution, that is, solutions with a metric on the form,

$$ds_{10}^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn} dy^m dy^n, \tag{2.92}$$

where we use $\tilde{g}_{mn}$ for the metric on the unwarped internal space $\tilde{Y}_6$, following the notation in [8]. The warped space thus have the metric $g_{mn} = \mathcal{H}(y)^{\frac{1}{2}}\tilde{g}_{mn}$, with $\mathcal{H}(y) = e^{-4A(y)}$. We take the following ansatz for the self dual five form flux,

$$\tilde{F}_{(5)} = (1 + \star_{10})d\alpha(y) \wedge \sqrt{-\det g_{\mu\nu}}dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3. \tag{2.93}$$

Varying the action will give us the Bianchi identities, as in (2.76),

$$d\tilde{F}_{(5)} = H_{(3)} \wedge F_{(3)} + 2\kappa_{10}^2 T_3 \rho_3^{loc}, \tag{2.94}$$

with ρ_3^{loc} the D3-brane charge density due to local sources. Comparing with our notation above, we have $2\kappa_{10}^2 T_3 \rho_3^{loc} = j_{(6)}$. Combining (2.93) and (2.94) gives,

$$\tilde{\nabla}^2\alpha = ie^{2A(y)}\frac{G_{mnp} \star_6 \bar{G}^{mnp}}{12\Im(\tau)} + 2e^{-6A(y)}\partial_m\alpha\partial^m e^{4A(y)} + 2\kappa_{10}^2 e^{2A(y)} T_3\rho_3^{loc}, \tag{2.95}$$

where $\tilde{\nabla}^2$ is the Laplace operator on the unwarped internal space.The trace of the Einstein equations yield

$$\tilde{\nabla}^2 e^{4A(y)} = \frac{e^{8A(y)}}{12 \cdot 3!\Im(\tau)}|G_{(3)}|^2 + e^{-4A(y)}\left(|\partial\alpha|^2 + |\partial e^{4A(y)}|^2\right) + 2\kappa_{10}^2 e^{2A(y)}\mathcal{J}_{loc}, \tag{2.96}$$

which results in,

$$\tilde{\nabla}^2\Phi_- = \mathcal{R}^{(4)} + \frac{e^{8A(y)}}{6 \cdot 3!\Im(\tau)}|G_-|^2 + e^{-4A(y)}|\partial\Phi_-|^2 + 2\kappa_{10}^2 e^{2A(y)}\left(\mathcal{J}^{loc} - T_3\rho_3^{loc}\right), \tag{2.97}$$

with the identifications,

$$\Phi_- = e^{4A(y)} - \alpha(y), \tag{2.98}$$
$$G_- = \star_6 G_{(3)} - iG_{(3)}. \tag{2.99}$$

In the noncompact limit, $\text{vol}(Y_6) \to \infty$, giving us an infinite Planck mass. The Friedmann equations, $H^2 = \frac{V}{3M_{\text{pl}}^2} \to 0$, which gives us a vanishing Ricci scalar in quasi de Sitter space ($\mathcal{R}^{(4)} \sim H^2$). Furthermore, D3-brane saturates the tadpole cancellation condition we derived above, which can be written as $\mathcal{J}^{loc} \geq T_3\rho_3^{loc}$. Integrating the left hand side of (2.97), we see that both sides has to vanish (since the left hand side is a total derivative, and we assume no surface terms). The solution then has to satisfy,

$$\Phi_- = G_- = 0. \tag{2.100}$$

This is called an *imaginary self dual solution*, since $iG_{(3)} = \star_6 G_{(3)}$.

But how do we deal with the decoupling of the four dimensional gravity? We will treat this issue, using some different types of warped internal spaces, when considering specific examples of D3-brane inflation.

2.4 The η-Problem

One of the bigger problems we have to deal with is that the compactifications and moduli stabilizations themselves do not decouple from infationary dynamics. If we compactify with D-branes though, we might get rid of some of these problems.The sectors then have to serve as non-interacting modules for the purpose of computing some four-dimensional observable. But we don't want complete decoupling, which will become clear when we study the particular model in more detail.

If we consider two sectors, A and B, they are (sometimes) said to decouple if the geometrical separation is sufficiently large.[3] The assumption can be checked by integrating out the massive fields coupled to A and B respectively [1]. This gives operators of the form

$$\Delta\,\mathcal{L} \supset \frac{1}{M_{AB}^{\delta_A+\delta_B-4}}\,\mathcal{O}_A^{\delta_A}\,\mathcal{O}_B^{\delta_B}, \tag{2.101}$$

where $\mathcal{O}_A^{\delta_A}$ is an operator of dimension δ_A of fields in A (similar for B). M_{AB} is the mass of the open string stretched from A to B. To illustrate this, we use an example. Take $\mathcal{O}_B^{(4)} = V_0$ and $\mathcal{O}_A^{(2)} = \phi^2$, giving us an operator on the form,

$$\Delta\,\mathcal{L} \supset \frac{V_0}{M_{AB}^2}\phi^2. \tag{2.102}$$

The mass $M_{AB} \sim \frac{d}{\alpha'}$, where d is the separation of the sectors A and B, supports decoupling intuitively. Note though that d is bounded by the compactification diameter L. If the compactification is isotropic, we have $\mathcal{V} \sim L^6$ and $M_{\rm pl} \propto \frac{L^3}{g_s\alpha'^2}$. From this we can derive an expression for the fraction of the string mass and the Planck mass,

$$\frac{M_{AB}}{M_{\rm pl}} \lesssim g_s \left(\frac{l_s}{L}\right)^2. \tag{2.103}$$

The coupling between D-brane sectors will be at least gravitational in strength, with operators suppressed by no more than the Planck mass.

Do we have coupling in anistropic compactifications? Assume that we have p large directions of size L and $6-p$ small directions of size S. The fraction will then be,

$$\frac{M_{AB}}{M_{\rm pl}} \lesssim g_s \left(\frac{l_s}{L}\right)^{\frac{1}{2}p-1} \left(\frac{l_s}{S}\right)^{\frac{1}{2}(6-p)}. \tag{2.104}$$

For $p > 1$, we have at least gravitational coupling at large volume.

One significant example, which we will have reason to come back to later, is a warped throat geometry. We consider a warped cone over an angular manifold X_5. For any such X_5, the mass of the stretched string is smaller than the Planck mass. But the compactness prevents decoupling. Considering a D3-brane-anti-brane pair, we have the following potential,

$$V_C(r) = 2T_3\left[1 - \frac{1}{2\pi^3}\frac{T_3 g_s^s \kappa^2}{r^4}\right], \tag{2.105}$$

[3] The principle is the same for separations along warped directions.

with T_3 the tension of the D3-brane, κ the gravitational parameter. This gives us,

$$\eta \approx -\frac{10}{\pi^3}\frac{\mathcal{V}}{r^6}. \tag{2.106}$$

V_C is too steep to allow inflation if X_6 is not highly anisotropic.

So, if the background is warped, the η-problem seems to disappear. Unfortunately, this is not quite the case. The backreaction of D3-branes on the compact geometry leads to certain instabilities and recurrence of the η-problem. Beyond the probe approximation, a D3-brane at y_b acts on a point source, resulting in a perturbation [1],

$$\nabla_y^2\left(\delta e^{-4A(y_b;y)}\right) = -\mathcal{C}\left[\frac{\delta(y_b-y)}{\sqrt{g(y)}} - \bar{\rho}(y)\right], \tag{2.107}$$

with $\mathcal{C} := 2g_s^2\kappa^2 T_3$. $\bar{\rho}(y)$ corresponds to a negative tension source. The solution is,

$$\delta e^{-4A(y_b;y)} = \mathcal{C}\left[\mathcal{G}(y_b;y) - \int d^6y\sqrt{g}\,\mathcal{G}(y;y')\bar{\rho}(y')\right], \tag{2.108}$$

where $\mathcal{G}$ satisfies,

$$\nabla_{y'}^2\,\mathcal{G}(y;y') = \nabla_y^2\,\mathcal{G}(y;y') = -\frac{\delta(y-y')}{\sqrt{g}} + \frac{1}{\mathcal{V}}. \tag{2.109}$$

If we act with $\nabla_{y_b}^2$ on the solution, we get,

$$\nabla_{y_b}^2 \delta e^{-4A(y_b;y)} = -\mathcal{C}\left[\frac{\delta(y_b-y)}{\sqrt{g(y_b)}} - \frac{1}{\mathcal{V}}\right], \tag{2.110}$$

which is not depending on $\bar{\rho}$. The leading terms in the scalar potential will then be,

$$V(y_b) = 2T_3 e^{4A(y_b)} \approx 2T_3\left[1 - \delta e^{-4A(y_b)}\right]. \tag{2.111}$$

The trace of the Hessian matrix gives,

$$\mathrm{Tr}(\eta) \approx \frac{M_{\mathrm{pl}}^2}{T_3}\nabla_{y_b}^2\left[\delta^{-4A(y_b;y)}\right] = -2. \tag{2.112}$$

Ultimately, we can conclude that the D3-brane potential in the presence of an anti-D3-brane, with only sources required for tadpole cancellation, has a steep unstable direction, preventing sustained inflation.

Usually, string inflation preserves supersymmetry down to $H < M_{KK}$. Thus it can be described as a $\mathcal{N}=1$ four dimensional supergravity. Positive vacuum energy spontaneously breaks supersymmetry, which gives us a particular form of the η-problem. The inflaton, denoted φ, is a complex scalar in a Chiral multiplet, with φ itself being a gauge singlet. Hence, its interactions are determined by the Kähler potential $K(\varphi,\bar{\varphi})$ and the superpotential $W(\varphi)$. Considering only one moduli (with straight forward generalization to an arbitrary number), the Lagrangian is, *IS THIS CORRECT INDEX-WISE?*

$$\mathcal{L} = -K_{\varphi\bar{\varphi}}\partial_\mu\varphi\partial^\mu\bar{\varphi} - e^{-\frac{K}{M_{\rm pl}^2}}\left[K^{\varphi\bar{\varphi}}D_\varphi W D_{\bar{\varphi}}W - \frac{3}{M_{\rm pl}^2}|W|^2\right]. \tag{2.113}$$

We can expand this around a reference point $\varphi \equiv 0$,

$$K = K(0) + K_{\varphi\bar{\varphi}}(0)\varphi\bar{\varphi} + \dots, \tag{2.114}$$

giving us the Lagrangian in leading terms,

$$\mathcal{L} \approx -\partial_\mu\phi\partial^\mu\bar{\phi} - V(0)\left[1 + \frac{\phi\bar{\phi}}{M_{\rm pl}^2} + \dots\right]. \tag{2.115}$$

where $\phi := K_{\varphi\bar{\varphi}}(0)\varphi\bar{\varphi}$ is the canonically normalized field. Without fine-tuning, we get an inflaton mass,

$$m_\phi^2 = \frac{V(0)}{M_{\rm pl}^2} + \dots = 3H^2 + \dots, \tag{2.116}$$

resulting in the η-problem, since $\eta = 1 + ...$

2.5 Reheating [1]

Complete inflation models must explain how the energy stored in the inflaton reaches the visible sector, initiating the hot Big Bang. Also, the process cannot produce too many relic particles, and a sufficient fraction of the energy must heat up Standard Model degrees of freedom to allow Baryogenesis. These are the two main questions/problems to address in a string theoretical treatment of reheating.

The visible sector in compactifications of type II string theory is localized on branes, or on intersections of branes. If the energy is localized on branes as well, the inflationary sector, the visible sector and their interactions can be computed, or at least parametrized. In the case of D3-brane-anti-brane inflation in warped throat geometry, this is exactly the case. The warped throat where the inflation occurs is one module, and the Standard Model on D-branes in a different region, which can be warped or an unwarped bulk region. Note though that this choice will affect the result! Here, we choose to consider the case where the inflationary sector and the Standard Model sector are both warped throats.

The actual inflation occurs when the D3-brane passes through an inflection point of the potential, and the accelerated expansion end at regions with steeper potential. When the D3-brane and the anti-D3-brane are sufficiently close to each other, i.e. when their separation is small enough, a tachyon develops. The instability created by this tachyon causes the D3-brane to fragment and decay into highly excited, non-relativistic string modes. They then decay into massive Kaluza-Klein excitations of supergravity (such as massless string states) when reaching the inflationary throat. These modes have mutual interactions, which are suppressed by the IR-scale, $m_{IR} \sim e^{A_{IR}} M_{\rm pl} \ll M_{\rm pl}$, where $e^{A_{IR}}$ is the warp factor at the tip of the throat. The coupling to Kaluza-Klein zero modes (e.g. the graviton) is suppressed by the Planck mass, so the warping creates a gravitational potential, keeping massive particles in the throat. They have to tunnel in order to escape.

In order for all this to work, we need to have the following timescale,

$$\tau_{\rm therm} \ll \tau_{\rm graviton} \ll \tau_{\rm tunnel}, \tag{2.117}$$

where $\tau_{\rm therm}$ is the thermalization time for Kaluza-Klein modes in the throat, $\tau_{\rm graviton}$ is the timescale for decay into gravitons and $\tau_{\rm tunnel}$ is the timescale of tunneling. There is still a lot of energy in the throat shortly after decay from excited strings to Kaluza-Klein modes though. It is essential that a sufficiently large fraction of this channel to Standard Model degrees of freedom. Otherwise, we will have various problems. We might get an overproduction of gravitons, leading to dominating gravitational radiation, which ruins the Nucleosynthesis. To avoid this, $\tau_{KK} \gtrsim \tau_{\rm tunnel}$. We might also have excitations of other sectors, if the Kaluza-Klein modes tunnel to intermediate throats. Also, if we have reheating above the local string scale $e^{A_{IR}}/\sqrt{\alpha'}$, we could have an extreme production of excited string states in strongly warped throats. The effects of this still has to be analyzed (and it remains challenging).

2.6 Inflating with D-branes in Warped Geometries

In string compactifications, the position of localized sources corresponds to scalar fields in the four dimensional effective theory. The relative position between a D3-brane and an anti-D3-brane can serve as an inflaton candidate. The sources, i.e. the branes, then attract each other both gravitationally and through "Coulomb forces".[4] At very small separations, a tachyon appears, which indicate a natural end of inflation. There is one problem with the Coulomb force though. The strength of this force suffers from size problem of the compact internal space. We can suppress the Coulomb force by warping the extra dimensions, but we can also find other contributions to the potential, e.g. from moduli stabilization, which has to be dealt with as well. How to determine, and ultimately deal with, such contributions is sort of the million dollar question.

2.6.1 Introduction

This section will introduce the model of warped D-brane inflation dealing with a bunch of different examples, starting with the more elementary ones.

Example 2.7 (Introducing Warped D-Brane Solutions [1])**.** Consider N D3-branes in ten dimensional Minkowski spacetime. The source contribute with a non-trivial background for massless fields. In string frame, the metric will then look like,

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(r)} \left(dr^2 + r^2 d\Omega^2_{S_5} \right), \tag{2.118}$$

[4]Note that the objects carry opposite charges with respect to the $C_{(4)}$-flux.

with the *warp factor* defined as,

$$e^{4A(r)} := 1 + \frac{L^4}{r^4}, \quad \frac{L^4}{(\alpha')^2} = 4\pi g_s N. \tag{2.119}$$

This is a warped solution. It has a constant diliaton and a non-trivial four-form potential,

$$\alpha(r) := (C_{(4)})_{tx^i} = e^{4A(r)}. \tag{2.120}$$

The self dual five-form flux is,

$$\tilde{F}_5 = (1 + \star_{10}) dC_4. \tag{2.121}$$

Before moving on, we just remind ourselves about what a line element in AdS_5 looks like,

$$ds^2_{AdS_5} = \frac{L^2}{r^2} dr^2 + \frac{r^2}{L^2} \eta_{\mu\nu} dx^\mu dx^\nu, \tag{2.122}$$

and we see that our metric solution in (2.118) reduces to exactly this when $r \ll L$.

The D3-brane action in Einstein frame will be,

$$S_{D_3} = -T_3 \int d^4\sigma \sqrt{-\det G^E_{ab}} + \mu_3 \int C_4. \tag{2.123}$$

The coordinates on the worldvolume must coincide with the spacetime coordinates in order to keep Poincaré invariance. Using the metric (2.118) and assuming that the angular coordinates are fixed, the lagrangian will be,

$$\mathcal{L} = -T_3 e^{4A(r)} \sqrt{1 + e^{-4A(r)} g^{\mu\nu} \partial_\mu r \partial_\nu r} + T_3 \alpha(r), \tag{2.124}$$

which we can expand for small velocities, $\dot{r} \ll e^{4A(r)}$,

$$\mathcal{L} \approx -\frac{1}{2} (\partial\phi)^2 - T_3 \left(e^{4A(r)} - \alpha(\phi) \right), \tag{2.125}$$

where $\phi^2 := T_3 r^2$ is the conically normalized field. From the definition of $\alpha(\phi)$ we see that the D3-brane experiences no force in the AdS-background, since the term multiplying T_3 in (2.125) vanishes.

If we want to be realistic, the AdS-background is bad, since then we have a non-compact spaectime, $0 \leq r \leq \infty$. Furthermore, the metric is singular for $r = 0$. Instead, we try to place D3-branes in finite warped throat regions of a flux compactification. We will review some geometry before going to the actual physics. We use the same convention as in [1].

A *singular conifold* is a Calabi-Yau cone X_6, represented as the locus over $\mathbb{C}^4$,

$$\sum_{A=1}^{4} = z_A^2 = 0. \tag{2.126}$$

This results in a rather special cone base. By setting $z^A = x^A + iy^A$ (2.126) will result in,

$$x \cdot x = \frac{1}{2}\rho^2; \quad y \cdot y = \frac{1}{2}\rho^2; \quad x \cdot y = 0. \tag{2.127}$$

This, at least topologically, looks like a three sphere and a two sphere, fibered over a three sphere. In fact, this is a very particular Einstein manifold, $T^{1,1} = (SU(2) \times SU(2))/U(1)$. This has an isometry group $SU(2) \times SU(2) \times U(1)$ and the metric is,

$$d\Omega^2_{T^{1,1}} := \frac{1}{9}\left(d\psi + \sum_{i=1}^{2}\cos\theta_i d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right), \tag{2.128}$$

and the metric on the conifold will be,

$$ds^2 = dr^2 + r^2 d\Omega^2_{T^{1,1}}, \tag{2.129}$$

with $r := \sqrt{\frac{3}{2}\rho^2}$. This metric can be written as a Kähler metric by introducing complex coordinates z^α with $\alpha \in \{1,2,3\}$. The Ricci-flat metric will look like,

$$ds^2 = k_{\alpha\bar{\beta}} dz^\alpha d\bar{z}^\beta, \tag{2.130}$$

with Kähler potential

$$k(z_\alpha, \bar{z}_\beta) = \frac{3}{2}\left(\sum_{A=1}^{4}|z_A|^2\right)^{\frac{2}{3}}, \quad k_{\alpha\bar{\beta}} = \partial_\alpha\partial_{\bar{\beta}}k. \tag{2.131}$$

One problem still remains. We have a singularity for $z_A = 0$. To avoid this, we can deform the conifold slightly by changing the definition and instead consider,

$$\sum_{A=1}^{4} z_A^2 = \epsilon, \tag{2.132}$$

which changes the three equations in x and y we had before to,

$$x \cdot x - y \cdot y = \epsilon^2, \tag{2.133}$$
$$x \cdot x + y \cdot y = \rho^2. \tag{2.134}$$

At the tip of the conifold, $\rho^2 = \epsilon^2$, we still have a copy of S^3, while the S^2 shrinks to zero size. Far from the tip, we can approximate the deformed conifold by a singular conifold.

Now, moving on to the physics, we illustrate this with two examples.

Example 2.8 (D-Branes on Confiold Singularities [1])**.** Stack N D-branes at $z_A = 0$, resulting in the following line element after back-reacting,

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(r)}\left(dr^2 + r^2 d\Omega^2_{T^{1,1}}\right), \tag{2.135}$$

with

$$e^{-4A(r)} = 1 + \frac{L^4}{r^4}; \quad L^4 = \frac{27\pi}{4}g_s N(\alpha')^2. \tag{2.136}$$

In the limit $r \ll L$, this gives us a $AdS_5 \times T^{1,1}$-geometry.

Example 2.9 (Stacking Different D-Branes on a Deformed Conifold [1])**.** Another alternative is to stack N D3-branes at the tip, and M D5-branes where the S^2 collapses. This can be realized by replacing the branes by their associated flux charges. The fluxes are quantized as,

$$\frac{1}{(2\pi)^2\alpha'}\int_A F_{(3)} = M, \tag{2.137}$$

$$\frac{1}{(2\pi)^2\alpha'}\int_B H_{(3)} = K, \tag{2.138}$$

with $M, K \in \mathbb{Z}$, $M, K \gg 1$. Here, A and B are independent three cycles. This results in a non trivial warping as well, and the line element is written as,

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(r)}d\tilde{s}^2, \tag{2.139}$$

where $d\tilde{s}^2$ is the deformed conifold geometry. Far from the tip, this is well approximated by the singular conifold, with the warp factor given as,

$$e^{-4A(r)} = \frac{L^4}{r^4}\left[1 + \frac{3g_sM}{8\pi K} + \frac{3g_sM}{2\pi K}\ln\left(\frac{r}{r_{\text{UV}}}\right)\right], \tag{2.140}$$

with $L^4 := \frac{27\pi}{4}g_sN(\alpha')^2$ and $N := MK$. The warp factor reaches a minimum at the tip,

$$e^{A_{\text{IR}}} = \exp\left(-\frac{2\pi K}{3g_sM}\right). \tag{2.141}$$

This example provides the basis for studies of warped D-brane inflation, it's the canonical example of warped throat geometry. Note though that we have to consider a finite portion of this example, usually from the tip $r = r_{\text{IR}}$ to $r = r_{\text{UV}}$. Otherwise we have no gravity, leading to an infinite compactification volume and Planck mass. Furthermore, the approximation using a finite portion of the total noncompact space is only valid in the mid-throat region ($r_{\text{IR}} \ll r \ll r_{\text{UV}}$) [8]. In the next example, we treat another type of conifold structure, making the approximation valid in the whole throat.

Example 2.10 (Gluing the Resolved Conifold to a Calabi-Yau Bulk)**.** The limitations of the deformed conifold to the mid-throat region motivates the use of something else. We will here, as in [8], use the so called *resolved conifold*, and glue it onto a Calabi-Yau bulk space, and use this as our six dimensional internal space. The resolved conifold has the metric,

$$\begin{aligned} ds^2_{RC} = \tilde{g}_{mn}dy^m dy^n = \kappa^{-1}(r)dr^2 + \frac{1}{9}\kappa(r)r^2\left(d\psi + cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2\right)^2 \\ + \frac{1}{6}r^2\left(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2\right) + \frac{1}{6}\left(r^2 + 6u^2\right)\left(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2\right), \end{aligned} \tag{2.142}$$

with $\kappa(r) = \frac{r^2+9u^2}{r^2+6u^2}$, and u is called the resolution parameter and has dimension of length.

Placing a stack of N D3-branes extended in the four noncompact dimensions, at the northpole of the S^2, i.e. at the tip of the conifold, gives us the ten dimensional geometry,

$$ds^2 = \mathcal{H}^{-\frac{1}{2}}(\rho, \theta_2) ds^2_{FRW} + \mathcal{H}^{\frac{1}{2}}(\rho, \theta_2) ds^2_{RC}, \tag{2.143}$$

where the four dimensional spacetime has been chosen to be FRW. ρ is a dimensionless radial coordinte, $\rho = \frac{r}{3u}$. The warp factor gets rather complicated. It is a solution of the Green's functions equation for the Laplace operator on the resolved conifold, which can be written as [8],

$$\mathcal{H}(\rho, \theta_2) = \left(\frac{L_{T^{1,1}}}{3u}\right)^4 \sum_{l=0}^{\infty} (2l+1)\, \mathcal{H}_l^A(\rho) P_l \left[\cos(\theta_2)\right], \tag{2.144}$$

where $L_{T^{1,1}} = \frac{27\pi}{4} N g_s \alpha'^2$ is the length scale of the $T^{1,1}$. P_l are Legendre polynomials, and $H_l^A(\rho)$ can be given in terms of the hypergeometric function ${}_2F_1(a, b, c; z)$,

$$H_l^A(\rho) = \frac{2\tilde{C}_\beta}{\rho^{2+2\beta}} {}_2F_1\left(\beta, 1+\beta, 1+2\beta; -\frac{1}{\rho^2}\right), \tag{2.145}$$

$$\tilde{C}_\beta = \frac{\Gamma(1+\beta)^2}{\Gamma(1+2\beta)}, \tag{2.146}$$

$$\beta = \sqrt{1 + \frac{3}{2} l(l+1)}. \tag{2.147}$$

The difference from the deformed conifold is that we now have angular dependence in the warp factor as well.

As we did for the other conifold structures, we have to consider a large but finite part of the throat. Then we want to glue this on a Calabi-Yau space, which we call a *bulk space*. This will result in perturbations for the $\Phi_- = e^{A(y)} - \alpha(y)$. The noncompact limit gives us $\tilde{\nabla}^2 \Phi_- = 0$, but cutting the throat off at $r = r_{\text{UV}}$ gives us perturbations of the form,

$$\tilde{\nabla}^2 \Phi_- = \mathcal{R}^{(4)} + \frac{e^{8A(y)}}{6\Im(\tau)} |G_-|^2 + e^{-4A(y)} |\partial \Phi_-|^2 + 2\kappa_{10}^2 e^{2A(y)} \left(\mathcal{J}^{loc} - T_3 \rho_3^{loc}\right). \tag{2.148}$$

Making certain assumptions (which are discussed in [8]), e.g. that the corrections to Φ_- and G_- are small of order $\mathcal{O}(\delta)$, and since we are inflating with D3-branes, the perturbations will to leading order in the large volume limit be,

$$\tilde{\nabla}^2 \Phi_h = 0, \tag{2.149}$$

and if we have a non-negligible curvature,

$$\tilde{\nabla}^2 \Phi_- = \mathcal{R}^{(4)}. \tag{2.150}$$

This reasoning is also valid for the deformed conifold - as we will see later, when discussing the potential, we will get solutions of the same form. Unfortunately, we don't know that exact solutions of the Laplace equation for the deformed conifold - we only know them in the mid throat region where we can approximate the space as $AdS_5 \times T^{1,1}$. For the resolved conifold, we can again take advantage of the hypergeometric function ${}_2F_1$. Note that since we probe the tip of the warped resolved conifold, we want solutions independent of (θ_1, ϕ_1) and ψ. According to [8] there are two such independent solutions. One is the $H_l^A(\rho)$ we discussed above, and the other is,

$$H_l^B(\rho) = {}_2F_1(1-\beta, 1+\beta, 2; -\rho^2), \tag{2.151}$$

and thus the most general solution will be given by,

$$\Phi_h(\rho, \theta_2, \phi_2) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left[a_l H_l^A(\rho) + b_l H_l^B(\rho) \right] Y_{lm}(\theta_2, \phi_2). \tag{2.152}$$

Again we stress that this solution is valid *anywhere* in the throat, in particularly near the tip. The asymptotic behaviour of the radial functions are,

$$\frac{2}{\rho^2} + 4\beta^2 \ln(\rho) + \mathcal{O}(1) \xleftarrow{0 \leftarrow \rho} H_l^A(\rho) \xrightarrow{\rho \to \infty} \frac{2\tilde{C}_\beta}{\rho^{2+2\beta}}, \tag{2.153}$$

$$\mathcal{O}(1) \xleftarrow{0 \leftarrow \rho} H_l^B(\rho) \xrightarrow{\rho \to \infty} \mathcal{O}(\rho^- 2 + 2\beta). \tag{2.154}$$

In [8], they also treat solutions to the Poisson equation, $\tilde{\nabla}^2 \Phi_- = \mathcal{R}_4$, but we leave that for now.

2.6.2 The Potential

The compactification volume is the sum of the throat volume,

$$\mathcal{V}_T := \int d\Omega^2_{T^{1,1}} \int_{r_{\rm IR}}^{r_{\rm UV}} r^5 dr e^{-4A(r)} = 2\pi^4 g_s N (\alpha')^2 r_{\rm UV}^2, \tag{2.155}$$

and the volume of the bulk space $\mathcal{V}_B$. We can bound the Planck mass from below by neglecting the bulk volume,

$$M_{\rm pl}^2 > \frac{N}{4} \frac{r_{\rm UV}^2}{(2\pi)^3 g_s (\alpha')^2}. \tag{2.156}$$

Furthermore, the field range available for a D3-brane is,

$$\Delta\phi^2 < T_3 r_{\rm UV}^2 = \frac{r_{\rm UV}^2}{(2\pi)^3 g_s (\alpha')^2}. \tag{2.157}$$

Combining these two equations yields,

$$\frac{\Delta\phi}{M_{\rm pl}} \leq \frac{2}{\sqrt{N}}. \tag{2.158}$$

Supergravity demands $N \gg 1$ and thus precludes super Planckian field ranges. The kinematic reasoning we have used above does not allow for observation of gravitational waves [1], so let us now consider the potential as well. We have,

$$V(\phi) = T_3 \left(e^{4A(\phi)} - \alpha(\phi) \right), \tag{2.159}$$

which vanishes if we have imaginary self dual fluxes. Certain compactification can break this condition. Adding an anti-D-brane to the compactification perturbs the background supergravity and makes the D3-brane experience a force.

$$V(\phi) = D_0 \left(1 - \frac{27}{64\pi^2} \frac{D_0}{\phi^4} \right); \quad D_0 \ll 2T^3 \tag{2.160}$$

where $D_0 := 2T^3 e^{4A(r_{\text{IR}})}$. Unfortunately, we have further contributions to the potential, making it less flat. The contribution from the curvature coupling is described by,

$$V_{\mathcal{R}}(\phi) = \frac{1}{12} \mathcal{R} \phi^2, \tag{2.161}$$

In de Sitter space, we get,

$$V(\phi) = V_C(\phi) + V_{\mathcal{R}}(\phi) \approx V_0 H^2 \phi^2 + \cdots \Rightarrow \eta \approx \frac{2}{3} + \ldots, \tag{2.162}$$

and we see that this gives a mass to the inflaton. We need to consider further contributions in order to see wether D-brane inflation can occur or not. Then we have to allow the D3-brane to backreact on the geometry, giving a position dependence to the compactification volume.

2.6.3 Moduli Stabilization and Back-Reaction

The Kähler moduli stabilization involves non-perturbative effects on D7-branes wrapping certain four cycles, where volume will depend on the D3-brane position, which affects the gauge coupling on the wrapped D7-branes. This leads to important corrections to the D3-brane potential.

Example 2.11 (Back Reaction and its Contribution to F-Term Potential in $\mathcal{N} = 1$ SUSY [1])**.** The four dimensional $\mathcal{N} = 1$ supersymmetric effective theory has the F-term potential,

$$V_F = e^K \left[K^{I\bar{J}} D_I W D_{\bar{J}} W - 3|W|^2 \right], \tag{2.163}$$

where I, J runs over all moduli. Stabilized complex structure moduli and the dilaton are assumed to be constant at sufficiently high energies. The remaining are Kähler moduli and brane position moduli. We can assume that we only have one Kähler moduli T, since the generalization is straight-forward. We define $Z^I := \{T, z^\alpha\}$. At tree level, the Kähler potential is given by,

$$K = -2 \ln \mathcal{V}, \tag{2.164}$$

and we neglect other corrections to K in this reasoning. The backreaction of the D3-brane on the compactification volume gives,

$$\mathcal{V} = \left[T + \bar{T} - \gamma k(z_\alpha, \bar{z}_\alpha)\right]^{\frac{3}{2}}. \tag{2.165}$$

It can be shown that,

$$\gamma = \frac{T_3}{6}\left(T + \bar{T}\right)_{\text{IR}}, \tag{2.166}$$

where T_{IR} is the Kähler modulus near the tip of the throat. The Kähler potential will be of the form,

$$K(Z_I, \bar{Z}_I) = -3\ln\left(T + \bar{T} - \gamma k(z_\alpha, \bar{z}_\alpha)\right) := -3\ln\left[U(Z^I, \bar{Z}^I)\right]. \tag{2.167}$$

The F-term potential combined with a superpotential will be,

$$\begin{aligned} V_F(T, z_\alpha) = \frac{1}{3U^2}\Big\{&\left[T\bar{T} + \gamma(k_\gamma k^{\gamma\bar{\delta}} k_{\bar{\delta}} - k)\right]|W_{,T}|^2 - 3(\bar{W}W_{,T} + c.c.) \\ &(k^{\alpha\bar{\delta}} k_{\bar{\delta}} \bar{W}_{,T} W_{,\alpha} + c.c.) + \frac{k^{\alpha\bar{\beta}}}{\gamma} W_{,\alpha}\bar{W}_{,\beta}\Big\}. \end{aligned} \tag{2.168}$$

If the potential doesn't depend on the brane coordinate, $W = W(T)$, the second line of the F-term potential vanishes. Then we write the potential as,

$$V_F(r) \approx \frac{V_0}{(1 - \frac{1}{6}\phi)^2} \approx V_0 + \frac{1}{3}\frac{V_0}{M_{\text{pl}}^2}\phi^2. \tag{2.169}$$

The inflaton gets a mass of order Hubble scale,

$$H^2 \approx \frac{V_0}{3M_{\text{pl}}^2}. \tag{2.170}$$

Gaugino condensation on a stack of N_c D7-branes leads to,

$$|\Delta W| \propto \exp\left[-\frac{2\pi}{N_c}\mathcal{V}_4\right]. \tag{2.171}$$

Changing the D3-brane position alters the warp factor, and imposes a ϕ-dependence on $\mathcal{V}_4$ and thus on ΔW.

One can compute the backreaction of the D3-branes on the four cycle wrapped by the D7-brane. For a four cycle defined as a holomorphic embedding,

$$f(z_\alpha) = 0, \tag{2.172}$$

we get [1],

$$W(T, z_\alpha) = W_0 + \mathcal{A}(z_\alpha)e^{-aT}, \quad a := \frac{2\pi}{N_c}, \tag{2.173}$$

with,

$$\mathcal{A}(z_\alpha) = \mathcal{A}_0 \left(\frac{f(z_\alpha)}{f(0)} \right)^{\frac{1}{N_c}}. \tag{2.174}$$

We want to find embeddings $f(z_\alpha)$ leading to forces which can balance the curvature coupling. To-date, only one such embedding is known, namely,

$$f(z_1) = \mu - z_1, \tag{2.175}$$

giving a potential,

$$V_F(\phi) \approx V_0 + ... + \lambda \phi^{\frac{3}{2}} + \frac{V_0}{M_{\text{pl}}^2} \phi^2 + ... \tag{2.176}$$

The last terms contribute with different signs to η. Let ϕ_0 denote the point where $\eta(\phi_0) = 0$. Around ϕ_0, $|\eta| \ll 1$. Fine tuning includes demanding ϕ_0 being in a warped throat, and that the potential is monotonic, with small first derivative, $\epsilon \ll 1$. If this can be arranged, we have inflation near an approximate inflection point.

This has several weaknesses though. We assume that the throat decouples from the bulk, which is rarely the case. Also, the warped throat region is approximated by a finite part of a non-compact warped Calabi-Yau cone. Now, we instead assume that all compactification effects can be expressed as,

$$\delta\Phi(r) = \delta\Phi(r_{\text{UV}}) \left(\frac{r}{r_{\text{UV}}} \right)^{\Delta - 4}, \tag{2.177}$$

where Δ is related to the scaling dimension of the operator AdS/CFT-dual to $\delta\Phi$. The spectrum of perturbation can be expressed as leading order correction to the D3-brane potential.

2.6.4 The Potential - Again

The effective action of D-brane probes is specified by supergravity field. We thus intend to find the most general supergravity solution for a finite warped throat, looking like a K.S.-solution in the infrared limit, by classifying all perturbations $\delta\Phi$. Generally, it is challenging to determine the spectrum of Δ. But if we approximate the warped throat region by a finite portion of $T^{1,1}$, we can use powerful group theory techniques. Using spectroscopy of $T^{1,1}$, we can determine the leading non-renormalizable modes. We will try to illustrate this and work out some details in the below example.

Example 2.12. To determine the D3-brane potential, we will be interested in field solutions on the form,

$$\Phi_- := e^{4A} - \alpha, \tag{2.178}$$

using the metric ansatz,

$$ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} dy^m dy^n, \tag{2.179}$$

with $g_{\mu\nu}$ being a maximaly symmetric four dimensional metric. Field equations of type IIB supergravity yields the master equation,

$$\nabla^2 \Phi_0 = \mathcal{R}^{(4)} + \frac{g_s}{96}|\Lambda|^2 + e^{-4A}|\nabla\Phi_-|^2 + S_{\rm loc}, \tag{2.180}$$

where $S_{\rm loc}$ describes localized sources due to anti-D3-branes, and,

$$\Lambda := \Phi_+ G_- + \Phi_- G_+, \tag{2.181}$$
$$G_\pm := (\star_6 \pm i) G_{(3)}, \tag{2.182}$$

with equations of motion for the three form flux,

$$d\Lambda + \frac{i}{2}\frac{d\tau}{\Im(\tau)} \wedge (\Lambda + \bar{\Lambda}) = 0. \tag{2.183}$$

The potential can be split as,

$$V(x, \Psi) = V_0 + V_C(x) + V_{\mathcal{R}}(x) + V_{\mathcal{B}}(x, \Psi), \tag{2.184}$$

with $x := \frac{r}{r_{\rm UV}}$ and Ψ denoting dependence on all five angular coordinates. V_0 is all contributions from distant sources of supersymmetry breaking that exert negligible forces on the D3-brane, only contributing to inflationary vacuum energy. $V_C(x)$ is the (extremely flat) Coulomb potential sourced by $S_{\rm loc}$,

$$V_C(x) = D_0 \left(1 - \frac{27}{64\pi^2}\frac{D_0}{T_3^2 r_{\rm UV}^4}\frac{1}{x^4}\right). \tag{2.185}$$

The Friedmann equation related $\mathcal{R}^{(4)} = 12H^2$ to the energy density, $V \approx V_0 + D_0$. The curvature potential induces a mass term,

$$V_{\mathcal{R}}(x) = \frac{1}{3}\mu^4 x^2 + ..., \quad \mu^4 := (V_0 + D_0)\frac{T_3 r_{\rm UV}^2}{M_{\rm pl}^2}. \tag{2.186}$$

This is sort of the curvature aspect of the eta problem. The bulk potential is,

$$V_{\mathcal{B}}(x, \Psi) = \mu^4 \sum_{LM} c_{LM} x^{\Delta(L)} f_{LM}(\Psi), \tag{2.187}$$

with $L := (j_1, j_2, R)$ and $M := (m_1, m_2)$ labelling the $SU(2) \times SU(2) \times U(1)$ quantum numbers under the isometries of $T^{1,1}$. f_{LM} are angular harmonics on $T^{1,1}$ and $\Delta(L)$ is given from spectroscopic analysis of $AdS_5 \times T^{1,1}$.

Let us split the bulk-solution as,

$$\Phi = \Phi_h + \Phi_f, \tag{2.188}$$
$$\nabla^2 \Phi_h = 0, \tag{2.189}$$
$$\nabla^2 \Phi_f = \frac{g_s}{96}|\Lambda|^2. \tag{2.190}$$

The harmonic solution satisfies,

$$\Delta_h(L) := 2\sqrt{H(j_1, j_2, R) + 4}, \tag{2.191}$$

$$H(j_1, j_2, R) := 6\left(j_1(j_1+1) + j_2(j_2+1) - \frac{1}{8}R^2\right), \tag{2.192}$$

while the flux contribution will look like,

$$\Delta_f(L) = \delta_i(L) + \delta_j(L) - 4, \tag{2.193}$$

$$\delta_1(L) := -1 + \sqrt{H(j_1, j_2, R+2) + 4}, \tag{2.194}$$

$$\delta_2(L) := \sqrt{H(j_1, j_2, R) + 4}, \tag{2.195}$$

$$\delta_3(L) := 1 + \sqrt{H(j_1, j_2, R-2) + 4}. \tag{2.196}$$

Using the restriction of the quantum numbers, one can determine the $\Delta' s$.

2.6.5 Masses of Scalar Fields

An effective theory describing inflating D3-branes in a conifold region attached to a stabilized compactification has a natural mass scale. All continuos global symmetries are broken, saying that the six scalar fields parametrizing the D3-brane position have masses of $m \sim \mathcal{O}(H)$. But we want $m \ll H$, so we need certain cancellations. D3-brane inflation give rise to models of multi-field-inflation. We need to solve the equations of motion numerically, without approximations. The spectrum of scalar masses is predicted by a matrix model,

$$\mathcal{M} = \begin{pmatrix} A\bar{A} + B\bar{B} & C \\ \bar{C} & \bar{A}A + \bar{B}B \end{pmatrix}, \tag{2.197}$$

where A, B, C are complex symmetric 3×3 matrices, with entries of random complex numbers picked from a Gaussian distribution.

3 Compactifications of Low-Energy String Theory

In order to fuse inflation and string theory, it is of utmost importance to investigate if there exists ten dimensional string theory models resulting in an effective four dimensional description with an effective de-Sitter (dS) potential.[5] We will investigate two candidates in detail and see if we can have configurations agreeing somewhat with modern observations.

[5]One might rather say that the correct thing to do would be to find models in eleven dimensional M-theory resulting in a four dimensional effective theory with a dS-potential. However, since all the string theory models are obtained upon compactifying M-theory on S^1, our reasoning is fine.

3.1 Type IIB SUGRA on $\mathbb{T}^6$

We start with the action on the form,

$$S = \frac{1}{2\kappa_{10}^2}\left\{\int d^{10}X\sqrt{-g_{10}}\left[e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\right) - \frac{1}{12}|F_{(3)}|^2\right]\right.$$
$$\left. + \frac{1}{2}\int C_{(4)} \wedge F_{(3)} \wedge H_{(3)}\right\} + S_{\text{loc}}, \tag{3.1}$$

where we deal with the contribution from local sources later. We will use two different metric ansatz's for this, one being more general than the other. We split this into different subsections.

3.1.1 Ideal Torus

We want to compactify the above action on an ideal $\mathbb{T}^6$, giving us the metric,

$$ds_{10}^2 = g_{\mu\nu}dx^\mu dx^\nu + \delta_{mn}dy^m dy^n, \tag{3.2}$$

where we have taken the six dimensional torus to be $\mathbb{T}^6 = S^1 \times ... \times S^1$, i.e. as the product of six circles. We also assume the radii of the circles to be of unit length. The ten dimensional Ricci scalar equals the four dimensional Ricci scalar, since $\mathbb{T}^6$ is flat. The determinant of the ten dimensional metric equals the determinant of the four dimensional metric, and these two observations result in that the gravitational parameter in ten dimensions κ_{10} equals the four dimensional gravitational parameter κ_4. Thus we get no contribution to the effective potential from the curvature part of the action.

Now we consider the flux terms. We assume that we have two independent 3-cycles of $\mathbb{T}^6$, the A-cycle (wrapped by $F_{(3)}$) and the B-cycle (wrapped by $H_{(3)}$).The quantization conditions are,

$$\int_{\mathcal{C}_A^{(3)}} F_{(3)} = 4\pi^2\alpha' M, \tag{3.3}$$

$$\int_{\mathcal{C}_B^{(3)}} H_{(3)} = 4\pi^2\alpha' K, \tag{3.4}$$

with $M, K \in \mathbb{Z}$. This gives us,

$$\int_{\mathbb{T}^6} |F_{(3)}|^2 = 3!(4\pi^2\alpha')^2 M^2, \tag{3.5}$$

$$\int_{\mathbb{T}^6} |H_{(3)}|^2 = 3!(4\pi^2\alpha')^2 K^2. \tag{3.6}$$

To simplify this, we use units such that $\alpha' = (4\pi^2)^{-1}$, resulting in

$$\int_{\mathbb{T}^6} |H_{(3)}|^2 = 3!K^2, \tag{3.7}$$

$$\int_{\mathbb{T}^6} |F_{(3)}|^2 = 3!M^2, \tag{3.8}$$

$$\int_{\mathbb{T}^6} F_{(3)} \wedge H_{(3)} = MK. \tag{3.9}$$

Since we want the dilaton dependence multiplying the Ricci scalar to vanish in the four dimensional effective action, $e^{-2\Phi}$ has to be constant, which we normalize to 1. Therefore, the kinetic term in the dilaton field vanishes. The action then becomes,

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g_4} \left[\mathcal{R}^{(4)} - \frac{1}{2}(M^2 + K^2) \right] + \frac{1}{2} MK \int C_{(4)} + S_{\text{loc}}. \tag{3.10}$$

We now add sources, e.g. n D3-branes and m O3-planes, each with the contribution,

$$S_{\text{loc}}^{D3} = -T_3^{D3} \int d^4\xi \sqrt{-g} + \int j_{(6)}^{D3} \wedge C_{(4)}, \tag{3.11}$$

$$S_{\text{loc}}^{O3} = -T_3^{O3} \int d^4\xi \sqrt{-g} + \int j_{(6)}^{O3} \wedge C_{(4)}, \tag{3.12}$$

with a total contribution,

$$S_{\text{loc}} = nS_{\text{loc}}^{D3} + mS_{\text{loc}}^{D3}. \tag{3.13}$$

Furthermore, for positive tension objects we know that $|Q_3| = T_3$, with $Q_3 = \int_{\mathbb{T}^6} j_{(6)}$ in this case, and for negative tension objects the equality is $|Q_3| = |T_3|$. We also know that both sources have positive charge. Moreover, we have to take into consideration that $C_{(4)}$ is a dyonic field, and the self duality of its flux ensures that objects carrying electric charge with respect to $C_{(4)}$ will also carry magnetic charge of equal size. The tadpole cancellation condition will hence look like,

$$\frac{1}{2\kappa_4^2} MK + nQ_3^{D3} + mQ_3^{O3} = 0. \tag{3.14}$$

Plugging this into the action yields,

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g_4} \left[\mathcal{R}^{(4)} - \frac{1}{2}(M^2 + K^2) \right] - nT_3^{D3} \left(2 \int d^4\xi \sqrt{-g} \right) - \frac{MK}{2\kappa_4^2} \int d^4\xi \sqrt{-g}. \tag{3.15}$$

Since the sources are assumed to be spacetime filling, all the integrals in the expression above are taken over the same space. This results in,

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g_4} \left[\mathcal{R}^{(4)} - \frac{1}{2}(M^2 + K^2) - MK - 4n\kappa_4^2 T_3^{D3} \right], \tag{3.16}$$

which can be written on the form,

$$S = \int d^4x\sqrt{-g_4}\frac{1}{2}\left(\frac{\mathcal{R}^{(4)}}{\kappa_4^2} - 2V_{\text{eff}}\right), \tag{3.17}$$

with,

$$V_{\text{eff}} = \frac{1}{4\kappa_4^2}\left(M^2 + K^2\right) + \frac{MK}{2\kappa_4^2} + 2nT_3^{D3}. \tag{3.18}$$

Using the statement regarding the tension for D3-branes in [9], and combining it with our use of units, the tension is,

$$T_3^{D3} = 2\pi. \tag{3.19}$$

Using [9] again, we have the following for the gravitational parameter,

$$2\kappa_4^2 = (2\pi)^{-1}, \tag{3.20}$$

and thus, the potential can be written as,

$$V_{\text{eff}} = 2\pi\left[\frac{1}{2}(M+K)^2 + 2n\right]. \tag{3.21}$$

Choosing to eliminate the D3-brane contribution instead, the potential is written as,

$$V_{\text{eff}} = 2\pi\left[\frac{1}{2}(M-K)^2 + 2m\right]. \tag{3.22}$$

To be sure which one to choose, we need to include moduli fields in the metric ansatz.

3.1.2 Including Moduli

A slightly more general metric ansatz than the one used in the previous section would be,

$$ds^2 = \tau g_{\mu\nu}dx^\mu dx^\nu + \rho\tilde{g}_{mn}dy^m dy^n, \tag{3.23}$$

where τ and ρ are four dimensional scalars. The metric $\tilde{g}$ has unit determinant, so ρ is basically the volume of the $\mathbb{T}^6$. The action will be the same as before,

$$\begin{aligned} S = \frac{1}{2\kappa_{10}^2}\Bigg\{&\int d^{10}X\sqrt{-g_{10}}\left[e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\right) - \frac{1}{12}|F_{(3)}|^2\right] \\ &+ \frac{1}{2}\int C_{(4)}\wedge F_{(3)}\wedge H_{(3)}\Bigg\} + S_{\text{loc}}, \end{aligned} \tag{3.24}$$

but this time the metric determinant and the Ricci scalar will look different, namely,

$$\sqrt{-g_{10}} = \tau^2\rho^3\sqrt{-g_4}, \tag{3.25}$$

$$\mathcal{R}^{(10)} = \tau^{-1}\mathcal{R}^{(4)}. \tag{3.26}$$

Furthermore, we have to investigate the behaviour of the fluxes. We investigate the H-flux as an example, but the reasoning is analogous for the F-flux. The absolute value is (up to permutation factors, which for reasons that will become clear will appear on the most left hand side and the most right hand side),

$$|H_{(3)}|^2 \sim H_{MNP}H^{MNP} = H_{MNP}H_{QRS}g^{MQ}g^{NR}g^{PS} = H_{mnp}H_{qrs}\tilde{g}^{mq}\tilde{g}^{nr}\tilde{g}^{ps}\rho^{-3} \sim |H_{mnp}|^2\rho^{-3}, \tag{3.27}$$

and the conclusion is,

$$|H_{(3)}|^2 = |H_{mnp}|^2\rho^{-3}, \tag{3.28}$$
$$|F_{(3)}|^2 = |F_{mnp}|^2\rho^{-3}. \tag{3.29}$$

Plugging all this into the action yields,

$$S = \frac{1}{2\kappa_{10}^2}\left\{\int d^{10}X\sqrt{-g_4}\tau^2\rho^3\left[e^{-2\Phi}\left(\tau^{-1}\mathcal{R}^{(4)} + 4(\partial\Phi)^2 - \frac{1}{12}\rho^{-3}|H_{mnp}|^2\right)\right.\right.$$
$$\left.\left. -\frac{1}{12}\rho^{-3}|F_{mnp}|^2\right] + \frac{1}{2}\int C_{(4)}\wedge H_{(3)}\wedge F_{(3)}\right\} + S_{\text{loc}}. \tag{3.30}$$

Again we want the dilaton dependence multiplying the Ricci scalar to vanish (i.e., we want to work in Einstein frame) and we conclude that

$$e^{2\Phi} = \tau\rho^3. \tag{3.31}$$

The quantization conditions are the same as before, but they are valid for the six dimensional absolute value. All this yields,

$$S = \frac{1}{2\kappa_{10}^2}\left\{\int d^{10}X\sqrt{-g_4}\left(\mathcal{R}^{(4)} + 4\tau(\partial\Phi)^2 - \frac{1}{12}\tau\rho^{-3}K^2 - \frac{1}{12}\tau^2M^2\right)\right.$$
$$\left. +\frac{1}{2}MK\times\text{vol}(\mathbb{T}^6)\int C_{(4)}\right\} + S_{\text{loc}}. \tag{3.32}$$

The kinetic term for the dilaton looks like,

$$\frac{1}{2\kappa_{10}^2}\int d^{10}X\sqrt{-g_4}4\tau\partial_M\Phi\partial_N\phi g^{MN}. \tag{3.33}$$

Both the dilaton and the metric depend on all our coordinates, in particular the compact ones. Thus, we can Fourier expand them, resulting in,

$$\frac{1}{2\kappa_{10}^2}\int d^{10}X\sqrt{-g_4}4\tau\partial_M\left[\prod_{j=4}^{9}\sum_{k_j=0}^{\infty}\Phi^{(k_j)}(x^\mu)\exp\left(\frac{ik_jy^j}{R_j}\right)\right]$$
$$\times\,\partial_N\left[\prod_{l=4}^{9}\sum_{k_l=0}^{\infty}\Phi^{(k_l)}(x^\mu)\exp\left(\frac{ik_ly^l}{R_l}\right)\right]\prod_{m=4}^{9}\sum_{k_m=0}^{\infty}g^{MN}_{(k_m)}(x^\mu)\exp\left(\frac{ik_my^m}{R_m}\right). \tag{3.34}$$

As for this term, we will have a bunch of different cases. If we first consider when M or N attains values corresponding to coordinates in the internal space, we see that we will get at least one factor of k_i, $i = j, l$ in front of everything, making the terms including the zero modes vanish. Since the exponentials are periodic functions, and the integrals over the coordinates of internal space is taken over one period, they vanish after integration. Thus, the only contribution will come when m and n attains values corresponding to coordinates in the four dimensional spacetime. As before, for all the modes (except the zero modes) we will have an integral of a periodic function over a whole period, making them vanish. Thus, the only surviving terms are those including the zero modes, and (3.34) can be written as,

$$\frac{\mathrm{vol}(\mathbb{T}^6)}{2\kappa_{10}^2}\int d^4x\sqrt{-g_4}4\tau\partial_\mu\Phi\partial_\nu\Phi g_{10}^{\mu\nu} = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g_4}4(\partial\Phi)^2, \tag{3.35}$$

where the square is now with respect to $g_4^{\mu\nu}$, hence the vanishing of the moduli dependence. The action can now be written as,

$$S = \frac{1}{2\kappa_4^2}\left\{\int d^4x\sqrt{-g_4}\left[\mathcal{R}^{(4)} + 4(\partial\Phi)^2 - \frac{1}{12}\left(\tau\rho^{-3}K^2 + \tau^2M^2\right)\right] + \frac{1}{2}MK\int C_{(4)}\right\} + S_{\mathrm{loc}}. \tag{3.36}$$

We add sources in the same way as we did for the ideal torus. The contribution from those will be,

$$S_{\mathrm{loc}} = -nT_3^{D3}\int d^4\xi\sqrt{-g}e^{-\Phi} + n\int j_{(6)}^{D3}\wedge C_{(4)} - mT_3^{O3}\int d^4\xi\sqrt{-g}e^{-\Phi} + m\int j_{(6)}^{O3}\wedge C_{(4)}. \tag{3.37}$$

The tadpole cancellation condition will be,

$$\frac{1}{2\kappa_4^2}MK + nQ_3^{D3} + mQ_3^{O3} = 0, \tag{3.38}$$

with the charges defined as the integral of the corresponding current $j_{(6)}$ over the torus. Since we are in Einstein frame, we know that $T_3^{D3} = Q_3^{D3}$ and $T_3^{O3} = -Q_3^{O3}$ with the charges being positive. Thus (3.38) can be written as,

$$\frac{1}{2\kappa_4^2}MK + nT_3^{D3} - mT_3^{O3} = 0. \tag{3.39}$$

The induced metric is, as usual,

$$g_{\mu\nu} = \partial_\mu X^M\partial_\nu X^N g_{MN}. \tag{3.40}$$

Choosing the gauge in which the X^Ms coincide with x^μ, the determinant will be,

$$\sqrt{-g} = \tau^2\sqrt{-g_4}, \tag{3.41}$$

resulting in a total action of the form,

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g_4}\left[\mathcal{R}^{(4)} + 4(\partial\Phi)^2 - \frac{1}{2}\left(\tau\rho^{-3}K^2 + \tau^2M^2\right) - 4nT_3^{D3}\kappa_4^2e^{-\Phi}\tau^2 - MKe^{-\Phi}\tau^2\right]. \tag{3.42}$$

In our units, we have, as before,

$$T_3^{D3} = 2\pi, \tag{3.43}$$

$$2\kappa_4^2 = \frac{1}{2\pi \text{vol}(\mathbb{T}^6)}, \tag{3.44}$$

giving us,

$$\begin{aligned} S &= \int d^4x\sqrt{-g_4}\frac{1}{2}\left\{\frac{\mathcal{R}^{(4)} + 4(\partial\Phi)^2}{\kappa_4^2} - 2\left[\frac{1}{24\kappa_4^2}\left(\tau\rho^{-3}K^2 + \tau^2M^2\right) + 2nT_3^{D3}\tau^{\frac{3}{2}}\rho^{-\frac{3}{2}} + \frac{MK\tau^{\frac{3}{2}}\rho^{-\frac{3}{2}}}{4\kappa_4^2}\right]\right\} \\ &= \int d^4x\sqrt{-g_4}\frac{1}{2}\left[\frac{\mathcal{R}_{(4)}}{\kappa_4^2} - K_{ij}\partial\Phi^i\partial\Phi^j - 2V_{\text{eff}}\right], \end{aligned} \tag{3.45}$$

with K_{ij} a Kähler metric[6] and,

$$V_{\text{eff}} = \frac{1}{4\kappa_4^2}\left(\tau\rho^{-3}K^2 + \tau^2M^2\right) + 2nT_3^{D3}\tau^{\frac{3}{2}}\rho^{-\frac{3}{2}} + \frac{MK}{2\kappa_4^2}\tau^{\frac{3}{2}}\rho^{-\frac{3}{2}}, \tag{3.46}$$

which is,

$$V_{\text{eff}} = 2\pi\left[\text{vol}(\mathbb{T}^6)\frac{\left(\tau^{\frac{1}{2}}\rho^{-\frac{3}{2}}K + \tau M\right)^2}{2} + 2n\tau^{\frac{3}{2}}\rho^{-\frac{3}{2}}\right]. \tag{3.47}$$

In terms of the number of O3-planes, we have,

$$V_{\text{eff}} = 2\pi\left[\text{vol}(\mathbb{T}^6)\frac{\left(\tau^{\frac{1}{2}}\rho^{-\frac{3}{2}}K - \tau M\right)^2}{2}\right] - 2mT_3^{O3}\tau^{\frac{3}{2}}\rho^{-\frac{3}{2}} \tag{3.48}$$

Considering the form (3.47), we know that we have an extremal point when the gradient of the potential is zero, which gives,

$$\frac{\rho^{-3}K^2 + 2\tau M^2}{2}\text{vol}(\mathbb{T}^6) + 3n\tau^{\frac{1}{2}}\rho^{-\frac{3}{2}} + \frac{3}{2}MK\tau^{\frac{1}{2}}\rho^{-\frac{3}{2}}\text{vol}(\mathbb{T}^6) = 0 \tag{3.49}$$

$$-\frac{3\rho^{-4}\tau K^2}{2}\text{vol}(\mathbb{T}^6) - 3n\tau^{\frac{3}{2}}\rho^{-\frac{5}{2}} - \frac{3}{2}MK\tau^{\frac{3}{2}}\rho^{-\frac{5}{2}}\text{vol}(\mathbb{T}^6) = 0 \tag{3.50}$$

Multiplying (3.50) by $\tau^{-1}\rho$ and adding it to (3.49) yields,

$$-\rho^{-3}K^2 + \tau M^2 = 0 \tag{3.51}$$

which gives us,

$$K^2 = M^2e^{2\Phi} \tag{3.52}$$

[6]From here on, we freeze out the dilaton, making the kinetic dilaton term vanish.

We will have to investigate both possible roots to this equation, namely,

$$K = \pm M e^{\Phi} \tag{3.53}$$

If we have the "positive" root, we get,

$$3\tau\left(M^2 \mathrm{vol}(\mathbb{T}^6) + n e^{-\Phi}\right) = 0 \tag{3.54}$$

Since we want the contribution from the fluxes to be non-trivial, this must be valid for any M. Also, we want $\tau \neq 0$, and hence we can discard the "positive" root. We conclude,

$$K = -M e^{\Phi} \tag{3.55}$$

which gives us, when plugged into (3.49)

$$n = 0 \tag{3.56}$$

and we see that the solution does not allow D3-branes.

If we now consider (3.48) and plug in the tadpole cancellation condition with the additional constraint that $n = 0$, the potential attains the form,

$$V_{\text{eff}} = 2\pi \left[\mathrm{vol}(\mathbb{T}^6) \frac{\left(\tau^{\frac{1}{2}} \rho^{-\frac{3}{2}} K + \tau M\right)^2}{2} \right]. \tag{3.57}$$

as expected. The extremal point is clearly a minima, since the potential vanishes at the point, and is a total (real) square.

The relevant equations of motion for type IIB string theory (for a detailed derivation, see appendix) are,

$$\mathcal{R}^{(10)} - 4\Delta\Phi + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2 = 0 \tag{3.58}$$

$$e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{24}|H_{(3)}|^2\right) - \frac{1}{24}|F_{(3)}|^2 = 0 \tag{3.59}$$

If we consider a frozen dilaton, they will attain the form,

$$\mathcal{R}^{(10)} - \frac{1}{12}|H_{(3)}|^2 = 0 \tag{3.60}$$

$$e^{-2\Phi}\left(\mathcal{R}^{(10)} - \frac{1}{24}|H_{(3)}|^2\right) - \frac{1}{24}|F_{(3)}|^2 = 0 \tag{3.61}$$

Combining these two clearly yields,

$$|F_{(3)}|^2 = e^{-2\Phi}|H_{(3)}|^2 \tag{3.62}$$

which is exactly what the minimizing of the potential has given us, in terms of K and M, see (3.52). Our compactification agrees with the ten dimensional equations of motion, making it consistent.

3.1.3 No-Scale Structures and Potential Uplifting

It is clear that the effective potential we arrived at has 0 as a global and only minimum. This implies that our effective cosmological constant vanishes, meaning that our potential results in a Minkowski vacuum and the model falls under the category *no-scale structures*. Compactifications of the simplest type, e.g. the one we performed above, will have a potential which is semi-positive definite, which thus can be put on the form

$$V = e^K \left(g^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3|W|^2 \right) \tag{3.63}$$

where W is a superpotential (whose form can be determined) and K is a Kähler potential. We use the same conventions as in [9], i.e.,

$$g^{a\bar{b}} = \partial^a \partial^{\bar{b}} K, \tag{3.64}$$

$$D_a W = \partial_a W + W \partial_a K. \tag{3.65}$$

One way to avoid this type of minima is to consider further corrections to the potential, if such exist.

In [10], two specific corrections to the potential are mentioned, the details of which we will not discuss here. One of them supposes that the corrections come from Euclidean D3-branes, and the other suggests that it comes from stacks of D7-branes wrapping 4-cycles. However, these effects will contribute in similar ways, and we will have corrections to the superpotential of the form,

$$\delta W = A e^{ia\sigma} \tag{3.66}$$

Such corrections can change our Minkowski minimum to an AdS minimum. Let us now consider such a correction, and let us do it at the tree-level. Then, the Kähler potential and the superpotential can be written as

$$K = -3 \ln \left[-i \left(\sigma - \bar{\sigma} \right) \right], \tag{3.67}$$

$$W = W_0 + A e^{ia\sigma} \tag{3.68}$$

For a supersymmetric minimum, we have,

$$D_\sigma = 0 \tag{3.69}$$

Following [10], we introduce $\sigma = i\rho$, with $\rho \in \mathbb{R}$ (in fact, this ρ coincides with our radial modulus). Furthermore, we let a, A and W_0 be real as well. This gives us,

$$\begin{aligned} D_\rho W = 0 \iff & \partial_\sigma \left(W_0 + A e^{-a\rho} \right) + \left(W_0 + A e^{-a\rho} \right) \partial_\rho \left(-3 \ln(2\rho) \right) = 0 \\ \iff & -aAe^{a\rho} - \frac{3}{2\rho} \left(W_0 + A e^{-a\rho} \right) = 0 \\ \iff & W_0 = -Ae^{a\rho} \left(1 + \frac{2}{3} \rho a \right) \end{aligned} \tag{3.70}$$

The minimum value of the potential is thus,

$$V_{\min} = -3e^K |W|^2 = -\frac{a^2 A^2 e^{-2a\rho_{\rm cr}}}{6\rho_{\rm cr}} \tag{3.71}$$

where we have assumed that $\sigma \gg 1$, and thus discarding all terms of $\mathcal{O}(\sigma^{-2})$ and lower.

This method changes the potential so that we get an AdS minimum instead. But, we want the minimum to be of dS-type. This can be achieved by introducing a few $\bar{D}3$-branes. Again, we refer to [10] for details. The potential will be perturbed as,

$$\delta V = \frac{D}{\rho^3} \tag{3.72}$$

where D depends on the number of anti-branes, the tension etc. The total expression for the potential then becomes,

$$V = \frac{aAe^{-a\rho}}{2\rho^2}\left(\frac{1}{3}\rho a A e^{-a\rho} + W_0 + Ae^{-a\sigma}\right) + \frac{D}{\sigma^3} \tag{3.73}$$

For wisely chosen parameters, this uplifts the minimum to one of a dS-type. For further discussion of this process, see [10].

3.2 Type IIA SUGRA on a Twisted Torus

To make our model more realistic, we want to consider a six dimensional compact space with curvature. Our starting point will be an action of the form

$$\begin{aligned} S = \frac{1}{2\kappa_{10}^2}\Bigg\{\int d^{10}X\sqrt{-g_{10}}\Bigg[e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\right) + \\ -\frac{1}{2}\left(|F_{(0)}|^2 + \frac{1}{2!}|F_{(2)}|^2 + \frac{1}{4!}|F_{(4)}|^2 + \frac{1}{6!}|F_{(6)}|^2\right)\Bigg] \\ + \int \left(F_{(2)}\wedge dC_{(7)} + F_{(0)}B_{(2)}\wedge dC_{(7)}\right)\Bigg\} + S_{\rm loc} \end{aligned} \tag{3.74}$$

where we are inspired by [11]. Note that the exterior derivative is now slightly changed, $d \to d_\omega = d + \omega$, where ω denotes a constant spin connection. Our general metric ansatz will be of the form

$$ds_{10}^2 = \tau g_{\mu\nu}dx^\mu dx^\nu + \rho M_{mn} e^m \otimes e^n \tag{3.75}$$

where M_{mn} is an object of unit determinant with inverse M^{mn} and $e^m \equiv e^m_\mu dx^\mu$ are vielbeins. The six dimensional part might contain additional moduli dependence. To make the treatment as clear as possible, we split the calculations into pieces.

3.2.1 Curvature Contribution - Metric Flux

The main difference from the IIB-case described in Section 3.1 is that we now assume that we have a curved internal space. The ten dimensional Ricci scalar is written as,

$$\mathcal{R}^{(10)} = \tau^{-1}\,\mathcal{R}^{(4)} + \rho^{-1}\,\mathcal{R}^{(6)} \tag{3.76}$$

and our goal is to find $\mathcal{R}^{(6)}$. It can be written as,

$$\mathcal{R}^{(6)} = \mathcal{R}_{\mu\nu ab}\, e^{\mu a} e^{\nu b} = \left(\partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega_\mu{}^c{}_a \omega_\nu{}^c{}_b - \omega_\nu{}^c{}_a \omega_\mu{}^c{}_b\right) e^{\mu a} e^{\nu b} \tag{3.77}$$

where $\omega_{\mu ab}$ is a spin connection. It contains the components of a connection one-form, $\omega_{\mu ab} dx^\mu = \omega_{ab}$. Also, $\omega^c{}_{ab} = \omega^\mu{}_{ab} e^c_\mu$. Our goal is to write this completely in the vielbein basis. What we mean when we say that the spin connection is to be constant, is that $\omega^\mu{}_{\nu\rho}$ is constant. Furthermore, it can be shown that ω is traceless, and the third term in (3.77) vanishes.[7] Following the reasoning of [4], the terms including derivatives can be integrated by parts inside the action, and the contribution can be written as,

$$\mathcal{R}^{(6)} = \omega^r{}_{[st]} \omega^{st}{}_r = \omega^r{}_{st} \omega^s{}_{t'r} M^{tt'} \tag{3.78}$$

Introducing U^γ_b, where $U^\beta_a U^{\beta'}_a = M^{\beta\beta'}$, we can write the spin connection as [4],

$$\omega_{c[ab]} = \frac{1}{2}\omega^\alpha{}_{\beta\gamma}\left(U^\beta_a U^\gamma_b U_{\alpha c} + 2U^\beta_{[c} U^\gamma_{|c|} U_{\alpha b]}\right) \tag{3.79}$$

Plugging this into (3.77) results in,

$$\begin{aligned}
\mathcal{R}^{(6)} &= \omega_{cab}\omega^{abc} \\
&= \frac{1}{4}\omega^\alpha{}_{\beta\gamma}\omega^{\alpha'}{}_{\beta'\gamma'}\left(U^\beta_a U^\gamma_b U_{\alpha c} + 2U^\beta_{[a} U^\gamma_{|c|} U_{\alpha b]}\right)\left(U^{\beta' b} U^{\gamma' c} U^a_{\alpha'} + 2U^{\beta'[b} U^{\gamma|a|} U^{c]}_{\alpha'}\right) \\
&= \frac{1}{4}\omega^\alpha{}_{\beta\gamma}\omega^{\alpha'}{}_{\beta'\gamma'}\Big(M^{\gamma\beta'}\delta^{\alpha'}_\alpha \delta^\beta_{\alpha'} + M^{\beta\gamma'} M^{\gamma\beta'} M_{\alpha\alpha'} - M^{\beta\gamma'}\delta^\gamma_{\alpha'}\delta^{\beta'}_\alpha + M^{\gamma\gamma'}\delta^{\beta'}_\alpha \delta^\beta_\alpha + \\
&\quad - M^{\beta\beta'} M^{\gamma\gamma'} M_{\alpha\alpha'} + M^{\beta\gamma'}\delta^\gamma_{\alpha'}\delta^{\beta'}_\alpha - M^{\beta\gamma'} M^{\gamma\beta'} M_{\alpha\alpha'} - M^{\beta\beta'}\delta^\gamma_{\alpha'}\delta^{\gamma'}_\alpha + M^{\gamma\beta'}\delta^\beta_{\alpha'}\delta^{\gamma'}_\alpha\Big) \\
&= -\frac{1}{4}\left(2\omega^a{}_{bc}\omega^b{}_{ac} M^{cd} + \omega^a{}_{bc}\omega^d{}_{ef} M_{ad} M^{be} M^{cf}\right)
\end{aligned} \tag{3.80}$$

3.2.2 Contribution from Local Sources and Fluxes

We will consider D6-branes and O6-planes as local sources. They will fill the four non-compact dimensions, and three of the compact dimensions. Exactly which will be dealt

[7] We impose that ω is traceless since it is a one form flux. Such a contribution would mean that we would be supposed to assume that our internal space has non-trivial one cycles, wrapped by such a one-form flux. Existence of one-cycles would impose that our manifold has a boundary, which can be the case, but it leads to nasty calculations. Thus, it makes sense to demand $\omega$to be traceless as a first step.

with later. The DBI- and WZ-action for these object will be,

$$S_{\text{loc}} = -\int_{\text{WV}} d^7\xi T_6^{D6}\sqrt{-\tilde{g}}e^{-\Phi} + \int Q^{D6}C_{(7)}\wedge j_{(3)}^{D6} + \\ -\int_{\text{WV}} d^7\xi T_6^{O6}\sqrt{-\tilde{g}}e^{-\Phi} + \int Q^{O6}C_{(7)}\wedge j_{(3)}^{O6} \tag{3.81}$$

where j is basically just some volume element, considered as a tadpole when writing the WZ-part such that it matches with the bulk action. Varying the whole action w.r.t. $C_{(7)}$ results in a tadpole cancellation condition, which can be thought of as,

$$d_\omega F_{(2)} + F_{(0)}H_{(3)} = -Q_{D6}j_{(3)}^{D6} - Q_{O6}j_{(3)}^{O6} \tag{3.82}$$

which can be written as,

$$\omega\cdot F_{(2)} + F_{(0)}H_{(3)} = (T_{O6} - T_{D6})j_{(3)} \tag{3.83}$$

where $\omega\cdot F_{(2)}$ is a 3-form with components $\omega^p{}_{[mn}F_{p]q}$. We will carry out more explicit calculations when we specify the metric.

3.2.3 Dimensional Reduction

In order to carry out any specific calculations, we need to specify the metric a bit more. Let's take the ansatz (in agreement with [12]),

$$ds_{10}^2 = \tau g_{\mu\nu}dx^\mu dx^\nu + \rho\left(\sigma M_{ab}dy^a dy^b + \sigma^{-1}M_{ij}dy^i dy^j\right) \tag{3.84}$$

where $a = 1, 2, 3$ and $i = 4, 5, 6$.

Source	a			i		
$D6^{\parallel}/O6^{\parallel}$	×	×	×	–	–	–
$D6^{\perp}/O6^{\perp}$	–	–	×	×	×	–
	–	×	–	×	–	×
	×	–	–	–	×	×

Table 1: A schematic view over what compact dimensions are wrapped by the local sources. All sources furthermore wrap all four spacetime dimensions.

As for the fluxes, we have,

$$|H_{(3)}|^2 = \rho^{-3}\sigma^3|H_{ijk}|^2 + \rho^{-3}\sigma^{-1}|H_{abk}|^2 \tag{3.85}$$
$$|F_{(0)}|^2 = |F_{(0)}|^2 \tag{3.86}$$
$$|F_{(2)}|^2 = 3|F_{ai}|^2\rho^{-2} \tag{3.87}$$
$$|F_{(4)}|^2 = 3|F_{aibj}|^2\rho^{-4} \tag{3.88}$$
$$|F_{(6)}|^2 = |F_{aibjck}|^2\rho^{-6} \tag{3.89}$$

As before, we will have quantization conditions for all the fluxes. Again, we use units where $4\pi^2\alpha' = 1$, and a volume factor is understood in all cases.

IIA-Fluxes	Parameter
F_{aibjck}	α_0
F_{aibj}	-α_1
F_{ai}	α_2
$F_{(0)}$	-α_3
H_{ijk}	β_0
$\omega^a{}_{ij}$	β_1
H_{abk}	γ_0
$\omega^j{}_{ka} = \omega^i{}_{bk}, \omega^a{}_{bc}$	γ_1

Table 2: A summary of the quantization conditions explaining which parameters are related to what fluxes upon integration over the internal space.

Here we have made a choice regarding the ω, namely that,

$$\omega^c{}_{ij} = \beta_1 \epsilon_{cij} \tag{3.90}$$

$$\omega^c{}_{ab} = \gamma_1 \epsilon_{cab} \tag{3.91}$$

$$\omega^k{}_{aj} = \gamma_1 \epsilon_{kaj} \tag{3.92}$$

This means that ω coincides with the structure constants for $\mathfrak{su}(2)$. In some sense, this is one of the simplest choices we can make, but on the other hand, as long as ω satisfies the Jacobi identity, that's all we want. Clearly, this choice is compatible with that. In terms of this choice, the Ricci scalar looks like,

$$\mathcal{R}^{(6)} = -\left(-\frac{3}{2}\gamma_1^2\sigma^{-1} + \frac{3}{2}\beta_1^2\sigma^3 - 6\beta_1\gamma_1\sigma\right) \tag{3.93}$$

and furthermore, the scalar product $\omega \cdot F_{(2)}$ has the following surviving components,

$$\omega^p{}_{[mn}F_{p]q} \sim \omega^a{}_{[bc}F_{a]i} + \omega^a{}_{[ij}F_{a]k} + \omega^j{}_{[ka}F_{j]b} \sim \gamma_1\alpha_2 + \beta_1\alpha_2 \tag{3.94}$$

where we haven't taken relative factors into account (they will appear in the action).

Now we can start reducing the dimensions of our action. As before, the determinant of the metric goes as,

$$\sqrt{-g_{10}} = \sqrt{-g_4}\tau^2\rho^3 \tag{3.95}$$

giving us the condition

$$e^{2\Phi} = \tau\rho^3 \tag{3.96}$$

in order to arrive in the four dimensional Einstein frame. As for the determinant describing the different world sheets for the sources, we will have,

$$\sqrt{-g_7}e^{-\Phi} = \sqrt{-g_4}\tau^{\frac{3}{2}} \begin{Bmatrix} \sigma^{\frac{3}{2}} \\ \sigma^{-\frac{1}{2}} \end{Bmatrix} \tag{3.97}$$

After integrating over the compact dimensions, the action is,

$$S = \frac{\text{vol}}{2\kappa_{10}^2} \int d^4x \sqrt{-g_4} \left[\mathcal{R}^{(4)} - \tau\rho^{-1} \left(\frac{3}{2}\beta_1^2\sigma^3 - \frac{3}{2}\gamma_1^2\sigma^{-1} - 6\sigma\gamma_1\beta_1 \right) - \frac{\tau}{2} \left(\rho^{-3}\sigma^3\beta_0^2 + 3\rho^{-3}\sigma^{-1}\gamma_0^2 \right) + \right.$$
$$\left. -\frac{1}{2}\tau^2 \left(\rho^3\alpha_3^2 + 3\rho\alpha_2^2 + 3\rho^{-1}\alpha_1^2 + \rho^{-3}\alpha_0^2 \right) - \tau^{\frac{3}{2}}\sigma^{\frac{3}{2}} \left(\alpha_3\beta_0 - 3\beta_1\alpha_2 \right) + \tau^{\frac{3}{2}}\sigma^{-\frac{1}{2}} \left(3\alpha_3\gamma_0 + 3\alpha_2\gamma_1 \right) \right] \tag{3.98}$$

This can be written on the form

$$S = \frac{1}{2} \int d^4x \sqrt{-g_4} \left(\frac{\mathcal{R}^{(4)}}{\kappa_4^2} - 2V_{\text{eff}} \right) \tag{3.99}$$

with,

$$V_{\text{eff}} = \pi \text{vol} \left[\frac{3}{2}\beta_1^2\tau\rho^{-1}\sigma^3 - \frac{3}{2}\gamma_1^2\tau\rho^{-1}\sigma^{-1} - 6\sigma\tau\rho^{-1}\gamma_1\beta_1 + \frac{\tau}{2} \left(\beta_0^2\rho^{-3}\sigma^3 + \gamma_0^2\rho^{-3}\sigma^{-1} \right) + \right.$$
$$\left. +\frac{1}{2}\tau^2 \left(\rho^3\alpha_3^2 + 3\rho\alpha_2^2 + 3\rho^{-1}\alpha_1^2 + \rho^{-3}\alpha_0^2 \right) + \tau^{\frac{3}{2}}\sigma^{\frac{3}{2}} \left(\alpha_3\beta_0 - 3\beta_1\alpha_2 \right) - \tau^{\frac{3}{2}}\sigma^{-\frac{1}{2}} \left(3\alpha_3\gamma_0 + 3\alpha_2\gamma_1 \right) \right] \tag{3.100}$$

where we have used the same expressions for the tension of the D6-brane and the gravitational parameter as in the previous section.

3.2.4 Deriving the Effective Potential from a Superotential

Inspired by [12], we can derive the same potential from a superpotential,

$$W = \left(a_0 - 3a_1U + 3a_2U^2 - a_3U^3 \right) - \left(b_0 - 3b_1U \right) S + \left(c_0 + c_1U \right) T \tag{3.101}$$

and a Kähler potential

$$K = -\log\left[-i\left(S - \bar{S} \right) \right] - 3\log\left[-i\left(T - \bar{T} \right) \right] - 3\log\left[-i\left(U - \bar{U} \right) \right] \tag{3.102}$$

The fluxes has the following correspondence,

Coupling	IIA-Fluxes	Parameter
1	F_{aibjck}	a_0
U	F_{aibj}	-a_1
U^2	F_{ai}	a_2
U^3	$F_{(0)}$	-a_3
S	H_{ijk}	b_0
SU	${\omega^a}_{ij}$	b_1
T	H_{abk}	c_0
TU	${\omega^j}_{ka} = {\omega^i}_{bk}, {\omega^a}_{bc}$	c_1

Table 3: A summary of which parameters are related to which fluxes in the superpotential. Also, the coupling to the moduli fields is included.

Our goal is to relate the complex scalar fields S, T, U with our moduli fields. Doing so, we will automatically determine the factors understood to multiply the terms in the expression for the potential. In terms of W and K, the potential is given by,

$$V = e^K \left(g^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3|W|^2 \right) \tag{3.103}$$

where $g^{a\bar{b}}$ represents the inverse of the Kähler metric $g_{a\bar{b}}$,

$$g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K \tag{3.104}$$

and the covariant derivative is defined as (when acting on W),

$$D_a W = \partial_a W + W \partial_a K \tag{3.105}$$

The indices a, b runs over all moduli fields. The inverse Kähler metric will look like,

$$g^{a\bar{b}} = - \begin{pmatrix} (S-\bar{S})^2 & & \\ & \frac{(T-\bar{T})^2}{3} & \\ & & \frac{(U-\bar{U})^2}{3} \end{pmatrix} \tag{3.106}$$

and the terms contributing to the potential will be,

$$g^{S\bar{S}} D_S W D_{\bar{S}} \bar{W} = -(S-S)^2 \left(-b_0 + 3b_1 U - W \frac{1}{S-\bar{S}} \right) \left(-b_0 + 3b_1 \bar{U} + \bar{W} \frac{1}{S-\bar{S}} \right) \tag{3.107}$$

$$g^{T\bar{T}} D_T W D_{\bar{T}} \bar{W} = -\frac{(T-\bar{T})^2}{3} \left(c_0 + c_1 U - W \frac{3}{T-\bar{T}} \right) \left(c_0 + c_1 \bar{U} + \bar{W} \frac{3}{T-\bar{T}} \right) \tag{3.108}$$

$$\begin{aligned} g^{U\bar{U}} D_U W D_{\bar{U}} \bar{W} = -\frac{(U-\bar{U})^2}{3} & \left(-3a_1 + 6a_2 U - 3a_3 U^2 + 3b_1 S + c_1 T - W \frac{3}{U-\bar{U}} \right) \times \\ & \left(-3a_1 + 6a_2 \bar{U} - 3a_3 \bar{U}^2 + 3b_1 \bar{S} + c_1 \bar{T} + \bar{W} \frac{3}{U-\bar{U}} \right) \end{aligned} \tag{3.109}$$

In the beginning, we neglected the contribution from axionic states in the action. We have to do the same here. This corresponds to setting the real parts of S, U and T to zero (the real part clearly corresponds to axionic contributions, and the imaginary part to dilaton contributions). Doing so will make our calculations way easier, since $U = -\bar{U} = i\Im(U) := iu$ and thus $U - \bar{U} = 2iu$, with trivial extension to T and S. We will spare the reader from the detailed calculations in what follows - they are lengthy, but straight forward and presenting the details do not serve any purpose. Eventually, the potential will be,

$$\begin{aligned} V = \frac{1}{st^3u^3} & \left(\frac{a_0^2}{32} + \frac{3a_1^2}{32}u^2 + \frac{3a_2^2}{32}u^4 + \frac{a_3^2}{32}u^6 + \frac{b_0^2}{32}s^2 + \frac{3b_1^2}{32}s^2u^2 + \frac{c_0^2}{96}t^2 + \right. \\ & \left. -\frac{c_1^2}{96}t^2u^2 + \frac{b_0 a_3}{16}su^3 - \frac{3b_1 a_2}{16}su^3 - \frac{c_0 a_3}{16}tu^3 - \frac{c_1 a_2}{16}tu^3 - \frac{b_1 c_1}{8}stu^2 \right) \end{aligned} \tag{3.110}$$

and we will get the following system of equations to determine the constants and the relations between S, T, U and the moduli fields,

$$
\begin{aligned}
\frac{1}{32}s^{-1}t^{-3}u^{-3}a_0^2 = \frac{\pi}{2}\rho^{-3}\tau^2\alpha_0^2; &\quad \frac{3}{32}s^{-1}t^3u^{-1}a_1^2 = \frac{\pi}{2}\tau^2\rho^{-1}\alpha_1^2 \\
\frac{3}{32}s^{-1}t^{-3}ua_2^2 = \frac{\pi}{2}\tau^2\rho\alpha_2^2; &\quad \frac{1}{32}s^{-1}t^{-3}u^3a_3^2 = \frac{\pi}{2}\tau^2\rho^3\alpha_3^2 \\
\frac{1}{32}b_0^2st^{-3}u^{-3} = \frac{\pi}{2}\rho^{-3}\sigma^3\tau\beta_0^2; &\quad \frac{3}{32}b_1^2su^{-1}t^{-3} = \frac{3}{2}\pi\tau\rho^{-1}\sigma^3\beta_1^2 \\
\frac{1}{96}c_0^2tu^{-3}s^{-1} = \frac{\pi}{2}\rho^{-3}\sigma^{-1}\tau\gamma_0^2; &\quad -\frac{1}{96}c_1^2s^{-1}t^{-1}u^{-1} = -\frac{3}{2}\pi\tau\rho^{-1}\sigma^{-1}\gamma_1^2 \\
\frac{1}{16}t^{-3}b_0a_3 = \pi\tau^{\frac{3}{2}}\sigma^{\frac{3}{2}}\alpha_3\beta_0; &\quad -\frac{3}{16}b_1a_2t^{-3} = -\pi\tau^{\frac{3}{2}}\sigma^{\frac{3}{2}}\alpha_2\beta_1 \\
-\frac{1}{16}s^{-1}t^{-2}c_0a_3 = -\pi\tau^{\frac{3}{2}}\sigma^{-\frac{1}{2}}\alpha_3\gamma_0; &\quad -\frac{1}{16}c_1a_2s^{-1}t^{-2} = -\pi\tau^{\frac{3}{2}}\sigma^{-\frac{1}{2}}\alpha_2\gamma_1 \\
-\frac{1}{8}t^{-2}u^{-1}b_1c_1 = -6\beta_1\gamma_1 &
\end{aligned} \tag{3.111}
$$

and we can conclude the following,

$$\rho = u; \quad \sigma = s^{\frac{1}{2}}t^{-\frac{1}{2}}; \quad \tau = s^{-\frac{1}{2}}t^{-\frac{3}{2}} \tag{3.112}$$

$$\alpha_0 = \frac{1}{4\sqrt{\pi}}a_0; \quad \alpha_1 = \frac{1}{4\sqrt{\pi}}a_1; \quad \alpha_2 = \frac{1}{4\sqrt{\pi}}a_2; \quad \alpha_3 = \frac{1}{4\sqrt{\pi}}a_3 \tag{3.113}$$

$$\beta_0 = \frac{1}{4\sqrt{\pi}}b_0; \quad \beta_1 = \frac{1}{4\sqrt{\pi}}b_1; \quad \gamma_0 = \frac{1}{12\sqrt{\pi}}c_0; \quad \gamma_1 = \frac{1}{12\sqrt{\pi}}c_1 \tag{3.114}$$

where a volume factor is understood to follow the greek letter parameters. We see that this solution gives us the correct factors for the cross terms as well. Thus, we have derived the effective potential we received upon compactifying from a superpotential and a Kähler potential.

3.2.5 Towards "KKLT" in a Type IIA Setting

After all this reasoning about pure compactifications, we now want to relate this to the actual topic, i.e. inflation. To do so, we want to achieve something similar to the KKLT-scenario, only that we are now in a IIA SUGRA setting. To do so, we first want our theory to yield an AdS-vacuum, which we now can achieve perturbatively instead of taking non-perturbative effects into account. We know that our massive IIA SUGRA results in an effective $\mathcal{N} = 1$ four dimensional description. We also know, as we showed above, that the effective potential can be derived from a superpotential W (3.101) and a Kähler potential (3.102) as,

$$V = e^K \left(g^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3|W|^2 \right) \tag{3.115}$$

The spatial part of our four dimensional manifold looks like $\mathcal{M}_{\text{scalar}} = (SL(2\,\mathbb{R})/SO(2))^3$ consisting of complex scalar fields,

$$\Phi^\alpha = (S, T, U) \tag{3.116}$$

which we chose to represent as,

$$S = s_r + is; \quad T = t_r + it; \quad U = u_r + iu \tag{3.117}$$

These scalars also have a matrix representation,

$$M^\alpha = \left\{ \begin{pmatrix} s & s_r s \\ s_r s & s^{-1} + s_r^2 s \end{pmatrix}, \begin{pmatrix} t & t_r t \\ t_r t & t^{-1} + t_r^2 t \end{pmatrix}, \begin{pmatrix} u & u_r u \\ u_r u & u^{-1} + u_r^2 u \end{pmatrix} \right\} \tag{3.118}$$

We see that these are of the form,

$$\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2\,\mathbb{R}); \quad ad - bc = 1 \tag{3.119}$$

We remind ourselves about the generators of the algebra $\mathfrak{sl}(2\,\mathbb{R})$,

$$\left\langle \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right\rangle \equiv \mathfrak{sl}(2,\mathbb{R}) \tag{3.120}$$

i.e., rotations, dilatations and shifts respectively. Leaving out the rotations (since we are modding them out), we conclude that we should enable shifts and dilatations of our scalar fields without changing the Physics. Thus, We should be able to do $S \to S'$, where $S' = e^{2\lambda} S$ or $S' = S + b$. We have to check how this affects our fluxes, and thus the superpotential. Is our superpotential general enough, or do we generate new terms upon doing fractional linear transformations? Obviously, our superpotential is closed under dilatations; everything that is non-zero will stay non-zero and vice versa. But what about the shifts? Let Λ_β be a shift by β, and consider a term $\tilde{W} = a_0 + b_0 S$ - it is sufficient to treat such a term, since it can be generalized straight forward to match our superpotential. Upon acting, we get,

$$\tilde{W} = a_0 + b_0 S \xrightarrow{\Lambda_\beta} a_0 + b_0(S + \beta) = a_0 + b_0\beta + b_0 S = a_0' + b_0 S \tag{3.121}$$

And we see that we do not generate any new terms. Note though that if we where to not include a_0 in our expression for W, we would generate a non-zero constant term when acting with a shift.

A supersymmetric vacuum is defined as $D_\alpha W = 0$, but thanks to our reasoning, we see that it is sufficient to check $D_\alpha W|_{(S,T,U)=(i,i,i)} = 0$, since we can transform our scalar fields accordingly. We will have,

$$D_SW = \frac{1}{2}b_0 - \frac{1}{2}c_0ts^{-1} + \frac{3}{2}a_1us^{-1} - \frac{1}{2}a_3u^3s^{-1} + \\ + i\left(\frac{3}{2}b_1u - \frac{1}{2}c_1tus^{-1} + \frac{1}{2}a_0s^{-1} - \frac{3}{2}a_2u^2s^{-1}\right) = 0 \quad (3.122)$$

$$D_TW = -\frac{1}{2}c_0 + \frac{9}{2}a_1ut^{-1} - \frac{3}{2}a_3u^3t^{-1} + \frac{3}{2}b_0st^{-1} + \\ + i\left(-\frac{1}{2}c_1u + \frac{3}{2}a_0t^{-1} - \frac{9}{2}a_2u^2t^{-1} - \frac{9}{2}b_1sut^{-1}\right) = 0 \quad (3.123)$$

$$D_UW = \frac{3}{2}a_1 + \frac{3}{2}a_3u^2 + + \frac{3}{2}b_0su^{-1} - \frac{3}{2}c_0tu^{-1} + \\ + i\left(\frac{3}{2}a_0u^{-1} + \frac{3}{2}a_2u - \frac{3}{2}b_1s - \frac{1}{2}c_1t\right) = 0 \quad (3.124)$$

and we get the following relations between the fluxes,

$$a_3 = \frac{5}{3}a_1 \quad (3.125)$$

$$c_0 = 2a_1 \quad (3.126)$$

$$b_0 = -\frac{2}{3}a_1 \quad (3.127)$$

$$a_2 = -\frac{1}{9}a_0 \quad (3.128)$$

$$b_1 = \frac{2}{9}a_0 \quad (3.129)$$

$$c_1 = 2a_0 \quad (3.130)$$

Now one might ask whether this supersymmetric vacua implies extremal values of our scalar potential. The gradient look like $\nabla V = (\partial_S V, \partial_T V, \partial_U V)$, and evaluating this when $(S, T, U) = (i, i, i)$, we get,

$$\partial_S V = \frac{i}{192}\left(3a_0^2 + 9a_1^2 + 9a_2^2 + 3a_3^2 - 3b_0^2 - b_1^2 - 6a_3c_0 + c_0^2 - 6a_2c_1 - c_1^2\right) + \\ + \frac{1}{32}\left(a_0b_0 + a_1b_1\right) \quad (3.131)$$

$$\partial_T V = \frac{i}{192}\left(9a_0^2 + 27a_1^2 + 27a_2^2 + 9a_3^2 + 18a_3b_0 + 9b_0^2 - 54a_2b_1 + 27b_1^2 + \\ -12a_3c_0 + c_0^2 - 12a_2c_1 - 24b_1c_1 - c_1^2\right) + \frac{1}{32}\left(a_0c_0 - a_1c_1\right) \quad (3.132)$$

$$\partial_U V = \frac{i}{192}\left(9a_0^2 + 9a_1^2 - 9a_2^2 - 9a_3^2 + 9b_0^2 + 9b_1^2 + 3c_0^2 - 12b_1c_1 - c_1^2\right) + \\ + \frac{1}{96}\left(-9a_0a_1 - 18a_1a_2 - 9a_2a_3 - 9b_0b_1 + c_0c_1\right) \quad (3.133)$$

By plugging (3.122)-(3.124), we see that all the components vanish! Thus, one could say that our reasoning in fact is self consistent. Since $D_a W = 0$ for every a, it is clear that the potential is negative, and thus being of AdS-type.

Despite all this effort, we still do not arrive at a realistic setup, since we end up with an AdS-vacuum. However, we can use similar techniques as in Section 3.1.3, inspired by [12], in order to, hopefully, end up with a dS-vacuum. We investigate what happens if we take non-perturbative effects into account.

3.2.6 Adding Non-Perturbative Effects

As in the case in the type IIB-setting, we will consider contribution from gaugino condensation. We will also try to explain how this comes about, and why it is reasonable to consider. Since we are treating a type IIA setting including sources, both parallel and orthogonal D6-branes and O6-planes, we have to consider their contributing physics in more detail. In particular, Dp-branes has open-string degrees of freedom, since they can be used to determine non-trivial boundary conditions for open strings. Also, we know that orthogonal to a Dp-brane, there exists a (p+1)-dimensional Super Yang Mills Theory. This can be used to motivate why one should consider gaugino condensation.

To project this to our specific case, i.e. including D6-branes, we will have a 7-dimensional $\mathcal{N} = 1$ SYM theory living orthogonal to the brane. This gives rise to $2^{\lfloor 7/2 \rfloor} = 8$ complex scalars, i.e. 16 real, and thus 8 on-shell real scalars, χ - gauginos. Moreover, we will have five A_μ and three Φ^I. In this case, we might have that the classical expectation value $\langle \chi \rangle_{\text{cl}} = 0$, and the actual gaugino condesnation effect contributing to our superpotential will be to assume $\langle \bar{\chi} \chi \rangle \neq 0$, even though $\langle \chi \rangle_{\text{cl}} = 0$. The contribution will be of the form,

$$W_{(\text{non.pert})} \sim e^{-\frac{\alpha}{g_{\text{YM}}}} \tag{3.134}$$

where g_{YM} is our gauge coupling, and α is related to the rank of the gauge group of the YM-theory. We will have two different possible contributions, coming from theories on the parallel and orthogonal branes respectively. Assuming that the sources wrap a cycle $\mathcal{C}^\perp$ or $\mathcal{C}^\parallel$ respectively, the gauge coupling can be written as,

$$\frac{1}{\left(g_{\text{YM}}^\perp\right)^2} = \frac{\text{vol}(\mathcal{C}^\perp)}{g_s} \tag{3.135}$$

$$\frac{1}{\left(g_{\text{YM}}^\parallel\right)^2} = \frac{\text{vol}(\mathcal{C}^\parallel)}{g_s} \tag{3.136}$$

Using that we have frozen out the dilaton, we have $g_s = e^\Phi = \tau^{\frac{1}{2}} \rho^{\frac{3}{2}}$, and the volumes will be,

$$\int_{\mathcal{C}} \sqrt{\tilde{g}} \tag{3.137}$$

where $\tilde{g}$ is the induced metric on the relevant D6-brane. Using this, we conclude,

$$\frac{1}{\left(g_{\mathrm{YM}}^{\perp}\right)^2} = \frac{\mathrm{vol}(\mathcal{C}^{\perp})}{g_s} = \tau^{-\frac{1}{2}}\sigma^{\frac{3}{2}} = s \tag{3.138}$$

$$\frac{1}{\left(g_{\mathrm{YM}}^{\parallel}\right)^2} = \frac{\mathrm{vol}(\mathcal{C}^{\parallel})}{g_s} = \tau^{-\frac{1}{2}}\sigma^{-\frac{1}{2}} = t \tag{3.139}$$

Thus, we can write the total superpotential, including the non-perturbative contribution as [12],

$$\begin{aligned} W = W_{(\mathrm{pert})} + W_{(\mathrm{non\text{-}pert})} = \left(a_0 - 3a_1 U + 3a_2 U^2 - a_3 U^3\right) - \left(b_0 - 3b_1 U\right) S + \left(c_0 + c_1 U\right) T \\ + (z_1 + iz_2)e^{i\alpha S} + (z_3 + iz_4)e^{i\beta T} \end{aligned} \tag{3.140}$$

here, α and β are related to the rank of the respective gauge groups ($SU(N)$) of the YM-theory, thus they are real. Also, $z_1, ..., z_4$ are real constants.

3.2.7 Searching for dS-Minima

We will now systematically start searching for a configuration that can lead to a dS-minimum of the potential. We will do this in steps, increasing the generality as we go. We will use the relation between the fluxes derived when finding the AdS-vacuum, thus we only have to vary two of the fluxes. Also, we will start considering orthogonal OR parallel gaugino condensation contruibution, to see if this is enough. This might also give us hints around what points we can expect finding a minimum value of the potential. To get even more hints where such points can occur, we will start to consider cases where we set two of the complex moduli S, T, U to be fixed, only letting the moduli appearing in the non-perturbative part vary. Beware though that such a case might not solve $\nabla V = 0$ entirely.

We start with setting $z_3 = z_4 = 0$, and $U = T = i$ and $S = is$, where $s \in \mathbb{R}$. For the following choice of parameters, a positive minimum was found,

$$\alpha = \frac{1}{2} \tag{3.141}$$

$$a_0 = \frac{1}{5} \tag{3.142}$$

$$a_1 = -\frac{1}{5} \tag{3.143}$$

$$z_1 = 2 \tag{3.144}$$

$$z_2 = -2 \tag{3.145}$$

Then, the potential has the following form,

$$V = -\frac{13}{2700} + \frac{e^{-s}}{8} + \frac{e^{-\frac{s}{2}}}{120} + \frac{1}{1350s} + \frac{e^{-s}}{4s} - \frac{e^{-\frac{s}{2}}}{120s} + \frac{s}{1350} + \frac{e^{-s}s}{16} \tag{3.146}$$

which graphically looks like,

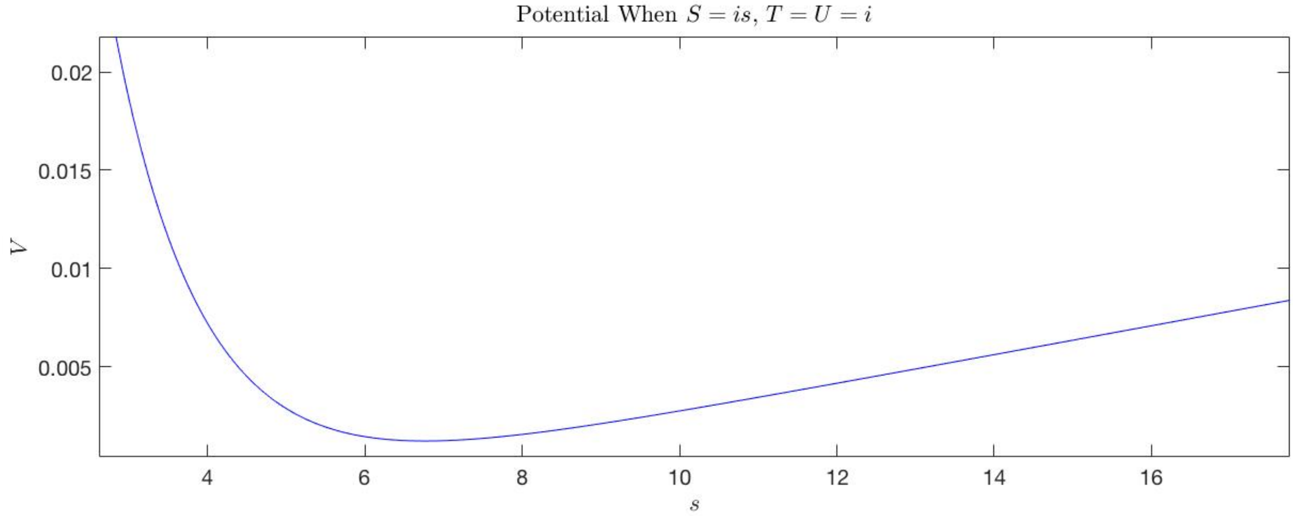

Figure 2: The potential plotted in the direction of s.

The minimum occurs at $s = 6.76261$ and is $V_{min} = 0.00122147$. Unfortunately, there is only one vanishing component of the gradient, motivating us to keep looking. However, when moving out of the this point, i.e., the point $(S, T, U) = (is, i, i)$, it is extremely hard to receive a dS-minimum solving $\nabla V = 0$. To match the contribution from six real scalars, is a task one could work with for a very long time. Usually, one or more directions are not stabilized, as one can see in Figure 3.

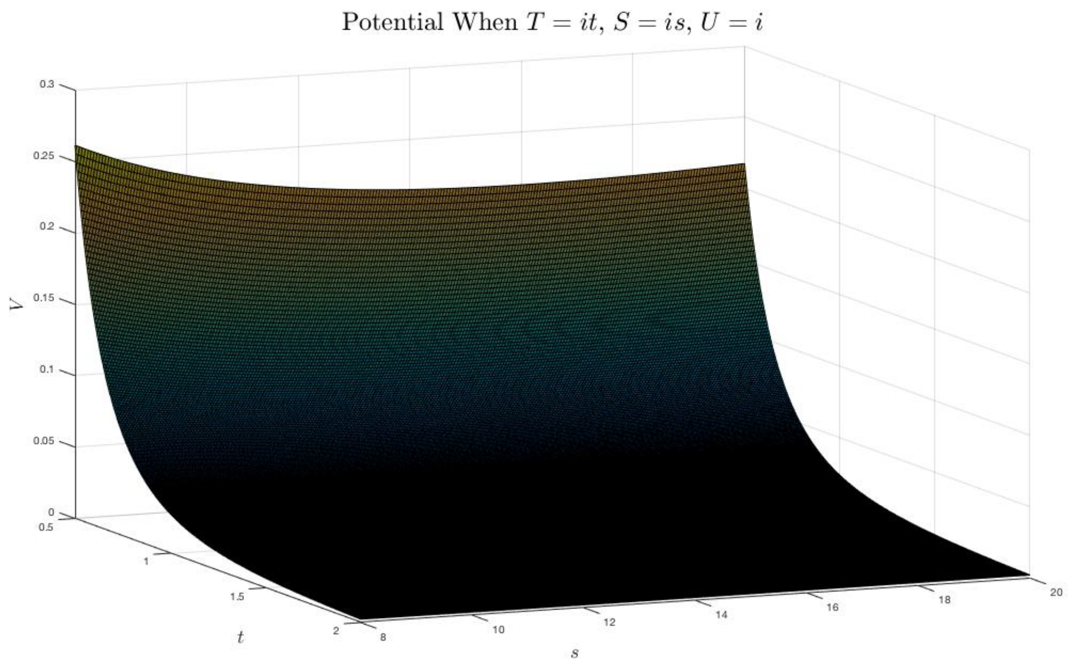

Figure 3: An example for how the searching could look like. Here, the potential is plotted in the imaginary S and T direction, and we can see that we have a minimal behaviour in the s-direction, but not in the t-direction.

Anyhow, inspired by [12], we can use a trick in order to break supersymmetry and thus, hopefully, receiving the desired form of the potential.

To use the trick, we need to consider the whole part of the non-perturbative superpotential in (3.140). In total, we will have 8 perturbative parameters and 4 non-perturbative. Let's denote them $\{F_I\}$, $I = 1, ..., 12$. Also, we can set $\alpha = \beta = 1$. Consider $\alpha S = \alpha(s_r + is) = \alpha s_r + i\alpha s$. Clearly, αs_r can be normalized to s'_r through a re-scaling of s_r (according to the reasoning in Section 3.2.5). The imaginary part is a phase, which we neglect. The same goes for βT, resulting in only 12 parameters in total.

Now, going to the origin and demanding $\nabla V = 0$ gives us a set of six real equations quadratic in the parameters of the form,

$$M^{IJ}_{\alpha,\bar{\alpha}} F_I F_J = 0 \tag{3.147}$$

with $\alpha = 1, 2, 3$. Furthermore, going to the origin and knowing that supersymmetry is supposed to be broken, we impose a SUSY-breaking condition,

$$D_\alpha W_{|(S,T,U)=(i,i,i)} = A_\alpha + iB_\alpha \tag{3.148}$$

which, of course, will be a constant at the origin. This will allow us to rewrite six of the F_I in terms of the SUSY-breaking parameters instead. Our family of parameters can now be thought of as $\{F_a, F_i(A,B)\}$, with $a = 1, .., 6$ and $i = 7, ..., 12$. Generally, the vanishing of the gradient of the potential is,

$$F_a F_b + F_a(A,B)_b + (A,B)_a(A,B)_b = 0 \tag{3.149}$$

But it is known that supersymmetry implies the equations of motion, thus $D_\alpha W = 0$ should give us $\nabla V = 0$. Having $D_\alpha W = 0$, means that $A_\alpha = B_\alpha = 0$, and we will end up with,

$$\nabla V = 0 \Rightarrow F_a F_b = 0 \tag{3.150}$$

which is a contradiction. Thus, the terms quadratic in the F_a cannot appear in equation (3.149), which thus looks like,

$$F_a(A,B)_b + (A,B)_a(A,B)_b = 0 \tag{3.151}$$

This allows us to solve for F_a in terms of A, B without having to deal with terms quadratic in the F_a, and we get $F_I(A,B)$ - a six parameter family of solutions breaking supersymmetry (spontaneously). The task would now be to choose (A,B) such that $V_{\text{min}} > 0$, and the masses of the scalar fields S, T, U are semi-positive definite. However, such a task has its limitations. In order to perform the scanning of the six dimensional parameter space, one first needs to choose values for four of the parameters and then make a surface plot of the remaining two directions, hoping to find a promising point.

We applied this exact procedure to our case, but due to the inconvenient size of certain equations, we will not present the specific figures. Despite the many tries to modify the SUSY-breaking parameters in a consistent way, we were unable to find a stable dS-minimum of the potential. This might not be too surprising. Looking at [12], they claim to have tried $\mathcal{O}(10^5)$ different configurations, resulting in only 2 stable dS-points. However, these two points were verified. For details and discussion regarding these points, we refer the reader to [12].

4 Concluding Remarks

4.1 Conclusions and Further Outlooks

Summarizing this thesis, we set out with the ambitious goal to join the theory of inflation and string theory. We gave a review on the topic of string inflation, motivating our goal and explaining problems with the existing theory. We discussed concepts as flux compactifications, moduli stabilization et. al. and how these come into place when searching for a string theoretical model including inflation. We also briefly discussed time scales and gave a conceptual treatment of reheating to make clear how involved and pretentious this whole business is. We rounded off the review by treating the specific model of D-brane inflation in warped geometries, where we used the earlier treated concepts.

Since it is of vast importance to have some kind of ten dimensional configuration resulting in a somewhat realistic picture upon compactifying[8], we treated two concrete examples in a somewhat detailed way - Type IIB SUGRA on a $\mathbb{T}^6$, and Type IIA SUGRA on a twisted torus - both with the appropriate choice of Dp-branes and O-planes as local sources. By using the techniques and concepts introduced in the first section, we showed that IIB SUGRA on $\mathbb{T}^6$ lead to no-scale structures, meaning that the effective potential was of Minkowski type. By taking non-perturbative effects into account, we briefly discussed how this potential could be modified as desired, and we also commented on its rather controversial arguments.

Inspired by the $\mathbb{T}^6$ compactification, we considered type IIA SUGRA on a twisted torus, including a scalar curvature on our internal space. After compactifying the ten dimensional theory, we showed that the received effective scalar potential could be derived from a superpotential and a Kähler potential. By once again introducing non-perturbative effects, in this case specified to be gaugino condensation, we strived towards up-lifting the AdS-potential to a dS-potential. By introducing supersymmetry breaking parameters, we where able achieve equations linear in the fluxes, thus avoiding the need of solving quadratic equations. Lastly, our search for a stable dS-minimum began, by scanning the achieved six dimensional parameter space for possible solutions. Unfortunately, we where unsuccessful in finding new stable dS-points.

One completely obvious outlook would be to keep searching for stable dS-points. The optimal would of course be to come up with some kind of formalism which solves it exactly, but that might not be too realistic. Also, an improvement on the work in this thesis would be to take axionic contributions into account, and maybe even try to consider a metric flux ω which is not traceless. Furthermore, it might be interesting to perform the compactification using some other internal manifold and see what might come up.

4.2 Compactifications, Inflation and Late-Time Expansion

As expected, the vastly remote goal of including inflation in string theory remains an open question. However, by continuing the studies on constructing somewhat realistic cosmological models from string theoretical configurations, and in particular finding stable dS-solutions, might give us additional hints about what to expect. It is worth mentioning, that we were able to stabilize five out of the six directions in the parameter space, but unable to stabilize all of them simultaneously - our randomly chosen points resulted in at least one tachyon. It might be interesting to see if this business will explain our current phase of accelerated expansion. Can it be the case that string theory eventually will describe our cosmic history, or will it tell us something completely different? What's sure is that there might be some very interesting answers luring beneath the surface, answers that for sure will revolutionize our knowledge about the World and Universe in which we live.

[8]By realistic here, we mean a configurations that results in a dS-potential upon compactifying, thus resulting in a positive cosmological constant.

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A Equations of Motion: Type IIB SUGRA on $\mathbb{T}^6$

Again, we will start with the action of the form,

$$S = \frac{1}{2\kappa_{10}^2}\left\{\int d^{10}X\sqrt{-g_{10}}\left[e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\right) - \frac{1}{12}|F_{(3)}|^2\right]\right.$$
$$\left. + \frac{1}{2}\int C_{(4)} \wedge F_{(3)} \wedge H_{(3)}\right\} \tag{A.1}$$

but this time, we do not treat any sources. Our assumptions are that our compact space $\mathbb{T}^6$ contain two independent three cycles, which are wrapped respectively by the 3-form fluxes. We will use variational principles exclusively, and for all variations (except the one with respect to the metric) we will rewrite the action as,

$$S = \frac{1}{2\kappa_{10}^2}\left\{\int\left[d^{10}X\sqrt{-g_{10}}\left(\mathcal{R}^{(10)}\,e^{-2\Phi}\right)\right] + \int\left[e^{-2\Phi}\left(4d\Phi\wedge\star d\Phi - \frac{1}{2}H_{(3)}\wedge\star H_{(3)}\right) + \right.\right.$$
$$\left.\left. -\frac{1}{2}F_{(3)}\wedge\star F_{(3)} + \frac{1}{2}C_{(4)}\wedge F_{(3)}\wedge H_{(3)}\right]\right\} \tag{A.2}$$

We will, throughout, assume that surface terms vanish, and thus total derivatives do not contribute to the equations of motion. A very important relation, which will be used several times, is

$$\omega\wedge\star\sigma = \sigma\wedge\star\omega \tag{A.3}$$

Also, we note that all our variations (except, again, the one with respect to the metric) commutes with both the exterior derivative and the Hodge star, which is clear from definitions.

We begin with a variation of $C_{(2)}$,

$$\begin{aligned}
\delta S &= \frac{1}{2\kappa_{10}^2}\int\left[-\frac{1}{2}\delta\left(dC_{(2)}\wedge\star dC_{(2)}\right) + \frac{1}{2}\delta\left(C_{(4)}\wedge H_{(3)}\wedge dC_{(2)}\right)\right] \\
&= \frac{1}{4\kappa_{10}^2}\int\left[d\delta C_{(2)}\wedge\star dC_{(2)} - dC_{(2)}\wedge\delta\star dC_{(2)} - \frac{1}{2}C_{(4)}\wedge H_{(3)}\wedge d\delta C_{(2)}\right] \\
&= \frac{1}{4\kappa_{10}^2}\int\left[-\delta C_{(2)}\wedge d\star dC_{(2)} - dC_{(2)}\wedge\star d\delta C_{(2)} + dC_{(4)}\wedge H_{(3)}\wedge\delta C_{(2)} + C_{(4)}\wedge dH_{(3)}\wedge\delta C_{(2)}\right] \\
&= \frac{1}{4\kappa_{10}^2}\int\left[-\delta C_{(2)}\wedge d\star dC_{(2)} + dC_{(4)}\wedge H_{(3)}\wedge\delta C_{(2)} + C_{(4)}\wedge dH_{(3)}\wedge\delta C_{(2)} - d\delta C_{(2)}\wedge\star dC_{(2)}\right] \\
&= \frac{1}{4\kappa_{10}^2}\int\left[-\delta C_{(2)}\wedge d\star dC_{(2)} + dC_{(4)}\wedge H_{(3)}\wedge\delta C_{(2)} + C_{(4)}\wedge dH_{(3)}\wedge\delta C_{(2)} + \delta C_{(2)}\wedge d\star dC_{(2)}\right] \\
&= \frac{1}{4\kappa_{10}^2}\int d\left(C_{(4)}\wedge H_{(3)}\right)\wedge\delta C_{(2)}
\end{aligned} \tag{A.4}$$

Since the variation is to vanish, we can conclude that,

$$d\left(C_{(4)}\wedge H_{(3)}\right) = 0 \tag{A.5}$$

A completely analogous treatment for a variation with respect to $B_{(2)}$ results in,

$$d\left(C_{(4)} \wedge F_{(3)}\right) = 0 \tag{A.6}$$

Next, we vary with respect to the dilaton field Φ.

$$\begin{aligned}
\delta S = &\frac{1}{2\kappa_{10}^2}\left\{\int\left(d^{10}X\sqrt{-g_{10}}\,\mathcal{R}^{(10)}\,\delta e^{-2\Phi}\right) + \int\left[\delta\left(4d\Phi\wedge\star d\Phi e^{-2\Phi}\right) - \frac{1}{2}H_{(3)}\wedge\star H_{(3)}\delta e^{-2\Phi}\right]\right\} \\
= &\frac{1}{2\kappa_{10}^2}\left\{-2\int d^{10}X\sqrt{g_{10}}\,\mathcal{R}^{(10)}\,e^{-2\Phi}\delta\Phi + \right. \\
&\left. + \int e^{-2\Phi}\left[4\left(d\delta\Phi\wedge\star d\Phi - d\Phi\wedge\star d\delta\Phi\right) - 8d\Phi\wedge\star d\Phi\delta\Phi + H_{(3)}\wedge\star H_{(3)}\delta\Phi\right]\right\} \\
= &\frac{1}{2\kappa_{10}^2}\left\{-2\int d^{10}X\sqrt{g_{10}}\,\mathcal{R}^{(10)}\,e^{-2\Phi}\delta\Phi + \right. \\
&\left. + \int e^{-2\Phi}\left[4d\star d\Phi\wedge\delta\Phi + 4d\star d\Phi\wedge\delta\Phi - 8d\Phi\wedge\star d\Phi\delta\Phi + H_{(3)}\wedge\star H_{(3)}\delta\Phi\right]\right\}
\end{aligned} \tag{A.7}$$

Note that we have the following,

$$\int d\star d\Phi = \int \star\Delta\Phi = \int d^{10}X\sqrt{-g_{10}}\Delta\Phi \tag{A.8}$$

Rewriting everything in terms of coordinates yields,

$$\delta S = \frac{1}{2\kappa_{10}^2}\int d^{10}X\sqrt{-g_{10}}e^{-2\Phi}\delta\Phi\left[-2\,\mathcal{R}^{(10)} + 4\Delta\Phi - 4(\partial\Phi)^2 + \frac{1}{12}|H_{(3)}|^2\right] \tag{A.9}$$

and we can conclude that the dilaton has the following equation of motion,

$$\mathcal{R}^{(10)} - 4\Delta\Phi + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2 = 0 \tag{A.10}$$

Lastly, we vary with respect to the (inverse) metric. We get,

$$\begin{aligned}
\delta S = &\frac{1}{2\kappa_{10}^2}\left(\left(\int d^{10}X\delta\sqrt{-g_{10}}\left[e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\right) - \frac{1}{12}|F_{(3)}|^2\right] + \right.\right. \\
&+ \sqrt{-g_{10}}\left\{e^{-2\Phi}\left[\delta\,\mathcal{R}^{(10)} + 4\delta(\partial_M\Phi\partial_N\Phi g^{MN}) - \frac{1}{12}\delta\left(H_{MNP}H_{QRS}g^{MQ}g^{NR}g^{PS}\right)\right] + \right. \\
&\left.\left.\left. - \frac{1}{12}\delta\left(F_{MNP}F_{QRS}g^{MQ}g^{NR}g^{PS}\right)\right\}\right)\right) \\
= &\frac{1}{2\kappa_{10}^2}\left(\left(\int d^{10}X\sqrt{-g_{10}}\frac{1}{2}(-g_{MN})\delta g^{MN}\left[e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\right) - \frac{1}{12}|F_{(3)}|^2\right] + \right.\right. \\
&+ \sqrt{-g_{10}}\left\{e^{-2\Phi}\left[\mathcal{R}_{MN}\,\delta g^{MN} + 4\partial_M\Phi\partial_N\Phi\delta g^{MN} + \right.\right. \\
&\left. - \frac{1}{12}\left(H_{MNP}H_{QRS}g^{MQ}g^{NR}\delta g^{PS} + (\text{perm})\right)\right] + \\
&\left.\left.\left. - \frac{1}{12}\left(F_{MNP}F_{QRS}g^{MQ}g^{NR}\delta g^{PS} + (\text{perm})\right)\right\}\right)\right)
\end{aligned} \tag{A.11}$$

Now, the technique is to re-name the indices on the components of the 3-forms such that we can extract a factor δg^{MN} from them. Doing so, and absorbing the metric factors (i.e., raising the indices we can) yields,

$$\delta S = \frac{1}{2\kappa_{10}^2}\int d^{10}X\sqrt{-g_{10}}\delta g^{MN}\left\{-\frac{1}{2}g_{MN}\left[e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\right) - \frac{1}{12}|F_{(3)}|^2\right] + \right.$$
$$\left. + e^{-2\Phi}\left[\mathcal{R}_{MN} + 4\partial_M\Phi\partial_N\Phi - \frac{1}{4}H_{MQP}H_N{}^{QP}\right] - \frac{1}{4}F_{MQP}F_N{}^{QP}\right\} \tag{A.12}$$

and we can conclude that,

$$-\frac{1}{2}g_{MN}\left[e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2\right) - \frac{1}{12}|F_{(3)}|^2\right] +$$
$$+ e^{-2\Phi}\left[\mathcal{R}_{MN} + 4\partial_M\Phi\partial_N\Phi - \frac{1}{4}H_{MQP}H_N{}^{QP}\right] - \frac{1}{4}F_{MQP}F_N{}^{QP} = 0 \tag{A.13}$$

This equation is more useful when traced. Doing so, using the full ten dimensional metric, yields,

$$e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{6}|H_{(3)}|^2\right) - \frac{1}{6}|F_{(3)}|^2 = 0 \tag{A.14}$$

In all, we have the following equations of motion,

$$d\left(C_{(4)} \wedge H_{(3)}\right) = 0 \tag{A.15}$$
$$d\left(C_{(4)} \wedge F_{(3)}\right) = 0 \tag{A.16}$$
$$\mathcal{R}^{(10)} - 4\Delta\Phi + 4(\partial\Phi)^2 - \frac{1}{12}|H_{(3)}|^2 = 0 \tag{A.17}$$
$$e^{-2\Phi}\left(\mathcal{R}^{(10)} + 4(\partial\Phi)^2 - \frac{1}{24}|H_{(3)}|^2\right) - \frac{1}{24}|F_{(3)}|^2 = 0 \tag{A.18}$$

References

[1] D. Baumann and L. Mcallister, "Inflation and string theory," arXiv:hep-th 1404.2601v1 [hep-th].

[2] K. Becker, M. Becker, and J. Schwarz, *String Theory and M-Theory: A Modern Introduction.* Cambridge University Press, New York, 2007.

[3] P. Horava and E. Witten, "Heterotic and type i string dynamics from eleven dimensions," *Nucl.Phys.* **B460** (1996) 506–524, arXiv:hep-th/9510209 [hep-th].

[4] J. Scherk and J. H. Schwarz, "How to get masses from extra dimensions," *Nucl.Phys.* **B153** (1979) 61–88.

[5] S. M. Carroll, *Spacetime and Geometry.* Pearson Education, 2004.

[6] D. Baumann, "Tasi lectures on inflation," arXiv:hep-th 0907.5424 [hep-th].

[7] M. R. Douglas and S. Kachru, "Flux compactification," *Rev.Mod.Phys.79:733-796,2007* (January, 2007) 66, arXiv:hep-th 0610102v3 [hep-th].

[8] Z. Kenton and S. Thomas, "D-brane potentials in the warped resolved conifold and natural inflation," *JHEP02(2015)127* (February, 2015) 41, arXiv:hep-th 1409.1221v3 [hep-th].

[9] S. B. Giddings, S. Kachru, and J. Polchinski, "Hierarchies from fluxes in string compactifications," *Phys. Rev. D* **66** (May, 2001) , arXiv:hep-th 0105097v2 [hep-th].

[10] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, "de sitter vacua in string theory," *Phys. Rev. D* **68** (2003) no. 046005, , arXiv:hep-th 0301240v2 [hep-th].

[11] E. Bergshoeff, R. Kallosh, T. Ortín, D. Roest, and A. van Proeyen, "New formulations of d=10 supersymmetry and d8-=8 domain walls," *Class.Quant.Grav* **18** (2001) no. 33, 59–82, arXiv:hep-th 0103233v2 [hep-th].

[12] U. Danielsson and G. Dibitetto, "An alternative to anti-branes and o-planes?," *JHEP 1405 (2014) 013* (2014) 13, arXiv:hep-th 1312.5331v2 [hep-th].