Evolution of Cavity Tip Vortices in High-Pressure Turbines

Albin Berglund
Abstract

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This degree project in applied physics studies the tip gap flows over the rotor blades of a high-pressure turbine. The rotor blade used in the study has an improved design that utilizes both a cavity tip and an uneven profiling to reduce turbine loss. The designed rotor blade is shown to admit a 21% lower leakage mass flow rate across the tip gap than a reference rotor blade with a flat tip. By studying the designed rotor blade using transient CFD, the flow field of the tip gap region has been studied through one blade passage. The flow field characteristics of particular interest are the leakage mass flow rate across the tip gap region, which is proportional to turbine loss, and the characteristic vortices that reside within the cavity tip. By using post-processing scripts, the leakage mass flow rate has been calculated for every time step across one blade passage, showing a strong time dependence. The characteristic vortices are found using two different vortex detection algorithms, and their respective vorticity magnitude is shown to depend on the leakage mass flow rate. The simulation shows that the vorticity magnitude is increasing above a threshold of leakage mass flow rate, and that it is decreasing under this threshold. This effect is shown to destabilize the leakage mass flow rate, increasing its amplitude over its period of one blade passage.
Evolution av virvelströmmar kring rotorblad med kaviteter inuti högtrycksturbiner


På 70-talet insåg ingenjörer att detta läckageflöde naturligtvis också skedde inuti själva flygplanets motor. En turbin har ju vingar i form av rotorblad, och dessa rotorblad erfar också tryckskillnad. Under 90-talet kopplades detta läckageflödet direkt till förluster inuti turbinen. I modern tid har ingenjörer därför arbetat med att minimera förlusterna associerade med läckageflöde.


Arbetet lyckades minska läckageflödet med hjälp av det förbättrade rotorbladet. Detta medförde också att den simulerade turbinens effekt ökade. Virvelströmmarna visades vara starkt tidsberoende, och de kunde kopplas till läckageflödets massflöde.
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1 Introduction

In an unshrouded turbine, there is a gap between rotor blades and turbine casing that prevents the rotating blade from grazing against its outer boundary. Normally, in commercial turbines, this gap has a height of around 1.5% of rotor blade span. During turbine operation there is a flow in this region driven by the pressure gradient between the suction and pressure sides of the blade. Research on losses in turbomachines in Denton (1993) has shown that the mixing losses arising from this leakage flow accounts for as much as 30% of aerodynamic losses in a stage, and research in Hourmouziadi and Albrecht (1987) has shown that an increase in blade gap by 1% decreases stage efficiency by 2%. The stage, which is one stator row and one rotor row in an axial turbine, is the usual turbine segment used in simulation and experiment. The stage of the axial turbine used in this project, with marked rotor blades and stator blades, can be seen in Figure 2. By using rotor blade geometries with design features such as cavities, winglets and/or squealers it is possible to change the tip gap flow field and increase turbine efficiency. These design features are all depicted in Figure 1. Here, the suction side squealer to the right joins the pressure side squealer to the left, forming an enclosed cavity inside the blade tip. Notice how the pressure side squealer is inclined, forming a winglet that extends out into the turbine main flow.

![Figure 1: Cavity tip with a squealer to the right and a combined winglet-squealer to the left.](image)

The flow field characteristics of the tip gap region most relevant to loss is, according to Denton, tip gap leakage mass flow rate and the angle of the flow exiting the tip gap. Winglet design predominantly reduces loss by decreasing leakage mass flow rate by reducing the driving pressure gradient over the blade, and by changing the exit angle of the leakage flow. Schabowski and
H. Hodson (2013) showed how cavity and squealer designs reduce leakage mass flow rate by separating the flow and so constricting it while it passes the tip gap.

In this research it is the cavity tip design that is studied. The research uses computational fluid dynamics (CFD) to study the vortices residing in the cavity tip region during turbine operation. The vortex dynamics during
rotor blade passage over stator blade is discussed, and then related to the leakage mass flow rate passing through the tip gap region. As a reference, the leakage mass flow rate of the cavity tip design is compared to that of a normal flat tip design.

1.1 A brief history on tip gap research

Early descriptions of the behavior of flow in the tip gap region and its effects on downstream flow field were published in S. Sjolander and Amrud (1987). They discovered sets tip-leakage vortices as the tip leakage flow rolled up into the main flow. Bindon (1989) investigated the loss arising from the tip gap flow and the tip-leakage vortices, and concluded that the majority of losses came either from downstream mixing, as the tip-leakage vortices mixed with the main flow, or from internal gap wall shear. Bindon highlighted that future turbine blade geometries have to work the trade-off between gap losses and downstream losses. This was later validated in Bindon and Morphis (1992) when a contouring of the tip blade managed to keep tip gap flow attached, reducing internal gap loss but increased downstream mixing loss, leading to an overall unchanged loss.

Further research showed, however, that endwall and blade design could indeed decrease overall losses. Yang et al. used squealers to decrease tip heat transfer coefficients and leakage flow in Yang et al. (2002), and concluded in further research in Acharya et al. (2003) that blades with suction side squealers had the lowest heat transfer coefficients and tip-leakage flows, and better efficiency than normal flat tip blades. The research in Yang et al. (2002) showed that the range of heat transfer coefficient for flat tip blades at 700-1300 W/m²K is decreased to 500-650 W/m²K using squealer designs. The heat transfer coefficient relates heat flux between a fluid and a solid to their temperature difference, meaning a tip with squealers experiences less heat transfer from a passing fluid than a flat tip. The squealer tip design superiority over the flat tip design was further cemented in Schabowski and H. Hodson (2013), where they discussed three ways of decreasing tip leakage loss: reduction of driving pressure with winglets, load splitting with multiple squealers and discharge coefficient reduction by preventing flow reattachment using thin squealers. By using a combined squealer-winglet design, the slope of the pressure loss coefficient, the proportionality constant between inlet relative tip gap stagnation pressure and inlet relative downstream total pressure, plotted at different tip gap widths was 22% lower than for the flat tip
design. In practice, this means that rotor blades with improved designs are more resilient to widening tip gaps. The effect of thin squealers discussed in Schabowski and H. Hodson (2013) is similar to that of Denton (1993), where flow over thick blades and thin blades were differentiated by whether or not the flow reattached after separating at pressure side tip gap. The cavity tip vortices in Schabowski and H. Hodson (2013) enhanced internal gap mixing, reducing the mixing losses of the tip leakage flow. This shows the correlation between tip gap vortical structures and turbine efficiency. The research of Schabowski and Hodson included both CFD and experiment, and highlighted a good coherence of numerical and experimental results.

In more recent research, the effects of shroud motion on the tip leakage flow in cavity tips have been increasingly included in simulation. During turbine operation, the wall shear arising from the relative motion between shroud and rotor blade changes the tip leakage flow. Chao (2015) studied the thermal performance of blades with cavity tips with shroud motion. The study found that the shroud motion creates a scraping vortex in addition to the cavity vortex also found in experiments with stationary shrouds, and a corner separation vortex at the near leading edge suction side. It is the evolution of these three cavity tip vortices that this paper covers. Earlier research in Bindon and Morphis (1988) that also studied the effects of shroud relative motion showed a pressure increase near the blade suction side, while pressure side portion of the blade remained unchanged. Bindon and Morphis (1988) argued that the shroud motion was an important consideration in tip gap loss formation.

1.2 The rotor tip design

The cavity tip design used in the simulations of this research can be seen in Figure 3. This design increases turbine efficiency by decreasing downstream mixing loss and by increasing internal gap wall shear and loss.

Since turbine rotor blades have thicknesses four times greater than the tip gap, the flow will, according to S. A. Sjolander and Cao (1995), reattach before exiting at the blade suction side. By using a cavity tip, the locations at which the flow reattaches after separation changes. When the cavity tip rims are thin, the reattachment of the flow is delayed, increasing the contraction of the tip gap jet. It is, according to Schabowski and H. Hodson (2013), this increased contraction that creates a double seal that decreases the leakage mass flow rate and downstream mixing loss. The cavity tip design used in
this research also utilizes the benefit of the winglet tips. It has a radially inclined pressure side rim, that reduces the driving pressure across the tip gap, a design feature used in Schabowski, H. Hodson, et al. (2014) which combined squealers and winglets in one single design. The radial inclination is set at 30 degrees, a number that was arrived upon after simulating with test geometries with inclinations of 15, 30, 45 and 60 degrees.

Figure 3: Turbine rotor blade with cavity tip.
There are also several non-aerodynamic aspects to designing a good rotor blade that are omitted in this simulation. There are cooling features, such as the cooling holes that are used in blade film cooling. Cooling helps to control widening blade gaps, which is a common problem in turbomachines since the hot tip-leakage flow increases erosion and thermal oxidation. It has been shown that the efficiency gain of improved geometries increases with increasing blade gap. This finding in Bunker (2004) further amplifies the efficiency benefits of improved blade designs. Cavity tip geometries increase thermal loading, and in some designs, also blade stress. In fact, both Chao (2015) and Schabowski, H. Hodson, et al. (2014) found that most aerodynamically improved blade tip geometries put further requirements on cooling and stress optimization. Despite this, cooling and stress optimization are not included in the present design, as the current research does not concern heat transfer or structural load.

1.3 The simulation

The present research studies the cavity geometry rotor blade using a transient simulation. In a transient simulation, the evolution of tip gap and main flow vortices can be studied. The simulation displays vortex interaction and the effects of rotor-stator interaction and rotor-stator offset on these vortices as the simulation steps forward in time.

Vortex interaction is complicated, but also key in understanding tip gap flow. By understanding the vortex dynamics in the tip gap, the research aims to find out how vortices affect and depend upon the leakage mass flow rate across the tip gap region.

Stator-rotor position is often held fixed at no offset in CFD. In this simulation, the effects of this offset on the leakage mass flow rate and cavity tip vortices are discussed.

2 Theory

2.1 Solver

All simulations are solved using ANSYS CFX. It is a vertex-centered and fully-coupled solver that uses finite elements as its discretization method. As a fully-coupled solver, it solves for all governing equation in any given
cell simultaneously. In contrast, a segregated solver, which is the more common of the two, solves the momentum equation before solving the continuity equation. A fully-coupled solver is preferable at high Mach numbers with compressible flow, such as the simulation in this project.

The CFX solver accounts for transient interaction at the sliding interface between the stator and the rotor domain. This is useful in this study where the tip gap vortices are compared to transient behavior in the stator domain, such as the stator wake.

The experimental validity of the CFX solver for tip leakage flows was shown in Coull, Atkins, and H. P. Hodson (2013), where it accurately captured the correct flow physics in the tip gap region.

2.2 Turbulence modeling of tip gap flows

In a study on the effects on tip gap flow from different exit Mach numbers in Wheeler, Atkins, and Li (2011) it was found that transonic tip gap flow has low turbulent dissipation. The research argued that the resulting flow field in the tip gap region is not sensitive to the choice of turbulence model. Therefore, the turbulence model of this simulation, which is presented in the methodology, is chosen on its merits in robustness and capturing of flow separation.

2.3 Vortices

The main flow and the tip gap flow of a rotor row houses different characteristic vortices. These vortices are strong enough to affect the flow field, and are therefore important in explaining how cavity tip geometries increase turbine efficiency. The vortices inside the cavity tip are: the cavity vortex, the scraping vortex and the corner separation vortex. They are illustrated in Figure 4. The main flow vortices are: the tip-leakage vortex and the passage vortex. They will all be covered briefly below. Keep in mind that this research focuses on the three cavity tip region vortices.
Cavity vortex. When the flow separates at the pressure side of the blade it shears off the rim. The sheared and separated fluid heads into the tip gap region with an increased vorticity, and the height drop of the cavity prevents it from reattaching. The detached flow shears further on the passing jet, resulting in a cavity vortex near the pressure side rim.

Scraping vortex. As a viscous fluid passes over the counter rotating shroud, the shroud shears the fluid, with a shear strongest at the rims of the cavity tip, since the flow contracts between the rim and the shroud. The scraping vortex is a result of this shear, and occupies the cavity tip between the cavity vortex and the suction side rim. It is visualized in Chao (2015). In numerical simulation, the scraping vortex appears when a relative motion between the shroud and the rotor blade has been specified.

Research on the effects of relative endwall movement in Bindon and Morphis (1988) found that the presence of a scraping vortex acts to reduce the pressure over the tip gap, reducing leakage mass flow rate.

Corner separation vortex. In the suction side leading edge corner, the static pressure is at a local maximum. This drives a flow ingestion that creates the corner separation vortex. The origin of the static pressure maximum
is described in Coull, Atkins, and H. P. Hodson (2014) as impingement of cross-passage secondary flows.

*Tip leakage vortex.* When the tip leakage jet exits over the suction side rim, it exits at a different velocity and a different angle than the passing main flow, creating a vortex sheet. As this vortex sheet rolls up, a tip leakage vortex is created. In the Denton (1993) loss model, the dynamics of the tip leakage vortex do not affect loss, but rather the flow field at the tip gap suction side at which it is created.

*Passage vortex.* As described in Acharya (2006), the flow passing the leading edge of the rotor creates a vortex at the junction of the hub and the leading edge. The flow splits up into a horseshoe vortex, with one leg entering the passage at the suction side of the rotor, and one leg at the pressure side.

### 2.4 Mesh generation parameters and wall treatment

When creating the mesh used in the simulation, there are a multitude of mesh parameters that need to be considered based on choice of geometry, physics, boundary conditions, and turbulence model.

The expansion factor of the mesh at the wall is the volume difference of neighboring cells as the mesh expands out from the walls. This expansion factor controls the way the mesh grows out of the boundary layers into areas with a coarser grid. When creating the mesh, the distance between the walls and the closest cells determines, together with the expansion factor, how the boundary layer is resolved. The wall distance of the first cell is estimated using Blasius equation

\[
y_{\text{wall}} = 6\left(\frac{V_{\text{ref}}}{\nu}\right)^{-\frac{7}{4}}\left(\frac{L_{\text{ref}}}{2}\right)^{\frac{1}{8}}y^+,\quad (1)
\]

where \(y_{\text{wall}}\) is the distance of the nearest grid point to the wall, \(V_{\text{ref}}\) is the reference velocity of the flow, in this case the inlet velocity, \(\nu\) is the kinematic viscosity and \(L_{\text{ref}}\) is the reference length which is set as blade chord. \(y^+\) is the only variable not intrinsic to the simulation setup, i.e. the wall distance \(y_{\text{wall}}\) is found by finding \(y^+\). This \(y^+\) is the non-dimensional near wall distance, and is given by

\[
y^+ = \frac{yu_{\tau}}{\nu},\quad (2)
\]
where $y$ is the distance from the wall, $u_\tau$ is the friction velocity and $\nu$ is the
kinematic viscosity. $y^+$ relates velocity at some distance from the wall in
a relation known as the law of the wall. In modern CFD, it is common to
alternate between a wall function, such as law of the wall, and low-Reynolds
number turbulence models to more accurately and more efficiently describe
near-wall flow. This means that for finer mesh and low $y^+$ the solver will
integrate to the wall, and for courser mesh and high $y^+$, the solver will instead
default to wall functions. This relaxes the $y^+$ requirements throughout the
mesh; where strict use of low-Re models the $y^+$ throughout the mesh would
have to be less than 2, the $y^+$-values are in this simulation relaxed to be less
than 10. A contour of $y^+$-values at the rotor blade can be seen in Figure 5.

![Figure 5: $y^+$-values at the rotor blade.](image)

### 2.5 Tip leakage mass flow rate

To extract the tip leakage mass flow rate from the converged flow field, 37
polylines along the tip gap at the blade suction side are extracted from the
mesh. These 37 polylines can be seen in Figure 6 as the red lines along the
k-direction. Each polyline crosses 165 mesh cells inside the tip gap, meaning
the leakage mass flow rate is summed over 5735 cells. This is done in a FORTRAN script using the equation

\[ \dot{m}_{\text{leak}} = \sum_{j=1}^{37} \sum_{k=10}^{165} \dot{m}_{k,j}, \]  

where the indices represent all cells at the tip gap exit. Each cell’s exit side face mass flow rate is given by

\[ \dot{m}_c = \rho_c A_c v_c \cos \theta, \]  

where \( \dot{m}_c \) is the cell face mass flow rate, \( \rho_c \) is the cell density, \( A_c \) is the cell face area, \( v_c \) is the flow velocity and \( \theta \) is the angle between the cell face normal and the flow direction. The tip gap definition used in leakage mass flow rate calculation and the mesh orientation can be seen in Figure 6.

![Figure 6: The definition of the tip gap exit in red at blade suction side. The calculation of leakage mass flow rate sums over indices k and j.](image)

2.6 Tip gap leakage mass flow rate relation to turbine loss

Turbine loss is defined as creation of entropy throughout the machine. This loss primarily arises from the mixing of the tip gap leakage jet and the main
flow. Denton (1993) presented the equation in an appendix of a paper on loss mechanisms in turbine machines that relates changes in tip gap leakage mass flow rate to change in entropy production. The equation disregards the vortex dynamics of the tip leakage vortex and simply relates the entropy creation, $\Delta \hat{s}_{\text{mix}}$, to the difference in velocity and angle of the leakage jet and the main flow and to the mixing Mach number

$$\Delta \hat{s}_{\text{mix}} = C_p(\gamma - 1)M_{ss}^2 \left(1 - \frac{V_{\text{leak}} \cos \theta}{V_{ss}}\right) \frac{\Delta \hat{m}_{\text{leak}}}{\hat{m}_{\text{passage}}},$$

where $M_{ss}$ is the suction side Mach number, $\gamma$ is the heat capacity ratio, $C_p$ is the specific heat capacity, $\hat{m}_{\text{passage}}$ is the passage leakage mass flow rate, $\hat{m}_{\text{leak}}$ is the leakage mass flow rate, $\theta$ is the angle between main and leakage flow, $V_{\text{leak}}$ is the leakage mass flow velocity and $V_{ss}$ is the suction side velocity, where the mixing takes place. The equation shows the proportionality of leakage mass flow rate, $\hat{m}_{\text{leak}}$, to turbine loss expressed as entropy production.

### 2.7 Vortex core identification methods

To visualize the vortices within a the tip gap, they must first be identified from the computed flow field. There is, however, no single mathematical definition of a vortex. Work in this field has instead produced several methods that identifies a vortex core from the data sets of simulation. These methods are listed and discussed in Jiang, Machiraju, and Thompson (2005). Given the turbulent flow of the tip gap region, these methods are the best way to track coherent structures throughout the simulation. In this research, two different methods have been tested to see how applicable they are when working with tip gap flows. They are both very popular within CFD, with the first one being the $\lambda_2$-criterion, and the second one being the Q-criterion. Both methods are visualized in post-processing as isosurfaces within the tip gap region.

#### 2.7.1 The $\lambda_2$-criterion

The $\lambda_2$-criterion identifies vortex cores by finding the symmetric part, $S$, and anti-symmetric part, $\Omega$, of the velocity gradient tensor. The method defines a vortex core as a region where
\[
\frac{S^2 + \Omega^2}{2}
\] (6)

has two negative eigenvalues. By arranging the three eigenvalues as \(\lambda_1\), \(\lambda_2\) and \(\lambda_3\), a vortex core occurs at negative \(\lambda_2\) since Equation 6 is symmetrical. The vortices are visualized by an isosurface at negative \(\lambda_2\), which also gives the method its name. The method was first presented in Jeong and Hussain (1995).

### 2.7.2 The Q-criterion

The Q-criterion identifies vortex cores by finding a connected region where the second invariant of the velocity gradient tensor is positive. The characteristic equation of a velocity tensor gradient is

\[
\lambda_i^3 + P\lambda_i^2 + Q\lambda_i + R = 0,
\] (7)

where \(\lambda_i\) are the eigenvalues. The second invariant \(Q\) is

\[
Q = \frac{1}{4}(\Omega_i\Omega_i - 2S_{ij}S_{ij}).
\] (8)

The Q-criterion presented in Hunt, Wray, and Moin (1988) defines a vortex by the region where the rate-of-rotation tensor, \(\frac{1}{3}\Omega_i\Omega_i\) in Equation 8, is greater than the rate-of-strain tensor, \(\frac{1}{2}S_{ij}S_{ij}\) in Equation 8. Visualization is made with isosurfaces, just as for the \(\lambda_2\)-method.

### 2.7.3 Streamwise vorticity

Given the turbulent flow within the tip gap region, the post-processors need to separate the cavity vortex, the scraping vortex and the corner separation vortex from non-coherent structures and from each other. A method to filter vortices based on the streamwise orientation was created. This was done by first calculating vorticity vectors along \(x\), \(y\) and \(z\) from the velocity field \(V\). The velocity field gives vorticity vectors \(\omega_x\), \(\omega_y\) and \(\omega_z\)

\[
\omega_x = \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}, \quad \omega_y = \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \quad \text{and} \quad \omega_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}
\] (9)
in each cell, which then gives streamwise vorticity by projection of these vectors onto the streamwise direction. The streamwise direction is defined as being tangential to the turbine blade camber line. Using the mesh data, the camber line was represented mathematically by a third degree polynomial by interpolation of the mesh. By mapping each cell to its closest point on this polynomial and then differentiating, the streamwise direction of each cell is found.

The streamwise vorticity $\omega_s$ is also used outside of vortex filtering. On contour maps, it can distinguish the scraping vortex from the cavity vortex, since these two cavity tip vortices have different orientation. Using these contour maps, the vorticity generation can be measured through one blade passage. By averaging the streamwise vorticity inside the tip gap at each time step, the net generation or dissipation of vortices inside the tip gap can be measured and related to leakage mass flow rate.

2.8 Turbine power

The turbine power is found by calculating instantaneous power at every time step. To find the power gained by using cavity tips, a turbine using cavity tip rotor blades is compared to one using flat tip rotor blades. The instantaneous power $P$ is given by

$$P = \tau \omega,$$

(10)

where $\tau$ is the torque along the turbine shaft axis and $\omega$ is the angular velocity. A comparison between the cavity tip and the flat tip power using this method is accurate, since the angular velocity and the pressure inlet and pressure outlet boundary conditions of the two cases are identical.

3 Methodology

3.1 The mesh

The setup used in the simulation was a 1:1 stator-rotor stage with 36 stators and 36 rotors. It can be seen in Figure 2. With a 1:1 stage, there is no pitch change across the stator-rotor interface. The choice of a 1:1 stator-rotor ratio
is because it relaxes computation times since the domain interface handler in the solver is sensitive changes in pitch. It also allows for simulation of one stator blade with one rotor blade using rotationally periodic mesh.

The stator mesh was designed in ANSYS TurboGrid and the rotor mesh in NUMECA AutoGrid. The stator mesh was scaled to align pitch at a 1:1 ratio with the rotor mesh. Both meshes can be seen together resting on the hub in Figure 7, with the rotor in the front and the stator in the back. Notice that because the area of interest for this research is the tip gap region of the rotor blade, the grid density is much higher.

Figure 7: Isometric view of rotor and stator mesh. The rotor is in the front.

The starting point of the rotor mesh was profile data from a turbine blade designed for a test facility at the Institute of Thermal Turbomachinery and Machine Dynamics at the Technical University of Graz as presented in Erhard (2000). This blade data was used in AutoGrid together with a basin geometry by NUMECA. The periodicity of the stator and rotor around the Z-axis was created by making the mesh Z-axis periodical. The topology of the mesh was edited manually around the cavity and the outlet was highly staggered to ensure good orthogonality. The mesh blocks were edited manually using IGG by NUMECA.
A grid independency study found the grid densities at which a finer grid do
not affect the flow features of interest. This independency study is especially
critical in a transient simulation, where any redundant mesh greatly affects
computation time. The blocks studied were those in proximity to the cavity
tip, since that is the region of interest. The number of lines in the mesh that
are parallel to the shroud was varied from 21 to 41. After finding the ideal
number, the number of lines between the cavity rims was varied within the
same range. The variable of interest when homing on grid independency was
the leakage mass flow rate across the tip gap. The leakage mass flow rate
was dependent on variations along the tip gap up until 37 mesh lines, and
showed little dependency on variations between the suction and the pressure
side.

Using Equation (1) and (2), the wall distance was set to $2.5 \mu m$. The
expansion factor of the mesh, which determines the growth of the mesh from
the highly resolved boundary layers and outwards into less dense mesh, was
kept below 1.3.

The dimensions of the mesh at the tip gap were set following the grid
independency study. The resulting rotor mesh can be seen in Figure 8 and
Figure 9. Notice that the rotor blade uses an O-grid around the blade, making
up the blade skin block, and inside the cavity. The inlet block, outlet block,
upper block and lower block are all H-grids. The rotor mesh contains 4.11
million cells and the stator mesh 0.43 million cells.
Figure 8: Side view of the rotor mesh with cavity.
3.2 Numerical simulation

Initial values. The transient simulation needs a flow field as initial value before solving. This flow field was found by setting up a steady state simulation using a stage rotor-stator interface, which circumferentially averages flow data at the stator-rotor interface. The steady state simulation used boundary conditions from the same test facility that provided the rotor blade data used in meshing. The stator inlet used a total pressure at 3.439 bar at 454.4 K, and the rotor outlet used a static pressure at 1.102 bar. All boundary conditions are displayed in 1.

Boundary conditions. The shroud, the hub and the blade were all as-
signed a no-slip condition with adiabatic heat transfer. The inlet and outlet boundary conditions were given by the converged steady state simulation. The stator domain is stationary, and the rotor domain rotates at 11000 rpm around the Z-axis. All components rotate within the rotor domain except for the shroud which is set as a counter-rotating wall.

**Stator-rotor domain interface.** The interface at the rotor and stator domains have different grid densities and sliding mesh. It is handled by the CFX transient rotor-stator interface, which accounts for all stator-rotor interaction.

**Turbulence model.** The choice of turbulence model is the SST-model presented in Menter (1994). The SST-model uses the k-ω model in the viscous sublayer, and transitions to a k-ε model in the free stream. Both the k-ω model and the k-ε model use Reynolds-averaged Navier-Stokes. SST is chosen for this simulation because of its robustness and its good representation of flow separation, which is the characteristic flow feature of cavity tips that alter the tip leakage jet.

**Time step size.** The time step used in the transient simulation was found by running a test simulation using a method for adaptive time stepping in the CFX solver homing in on 3-5 coefficient loops. This time step was rounded down to give 30 steps per blade passage and a total simulation time of 0.01212 seconds. Moving to the main simulation, the benefit of having a constant time step over an adaptive is that a constant time step is useful when writing post-processing scripts, when for example creating a rotor stationary frame of reference.

**Monitor points.** Monitor points are placed out in the domain at points of particular interest. These monitor points are used to monitor transient flow features, such as vortices and shock, and to assure that the blade passages are reaching some periodicity.
Table 1: Boundary conditions of the reference turbine.

<table>
<thead>
<tr>
<th>Inlet pressure</th>
<th>Inlet temperature</th>
<th>Outlet pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.439 bar (total)</td>
<td>454.4 K</td>
<td>1.102 bar (static)</td>
</tr>
</tbody>
</table>

Table 2: Output of the reference turbine.

<table>
<thead>
<tr>
<th>Angular velocity</th>
<th>Mass flow rate</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>11000 RPM</td>
<td>18.1 kg/s</td>
<td>1.94 MW</td>
</tr>
</tbody>
</table>

Table 3: Parameters of the reference turbine.

<table>
<thead>
<tr>
<th>Exit Mach number</th>
<th>Reynold number (blade)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4 M</td>
<td>2.4E+06</td>
</tr>
</tbody>
</table>

3.3 Post-processing

To study all the transient data, the post-processing of the flow fields across the time steps was automated by scripting in ANSYS CFX-Post (Perl), in TecPlot (Python) and in FORTRAN. CFX-Post was used to extract the data from the simulation files that was used in mass flow calculations and FORTRAN was used in the mass flow calculations themselves. The remaining and vast majority of the post-processing work, including contours, streamlines, data processing and vortex cores, was done in TecPlot.

3.4 Reference geometry

To fully understand the effects of the cavity geometry, a reference rotor blade with a flat tip is also used in simulation. The simulation of this flat tip blade follows the exact same methodology as the cavity tip blade. With the exception the removed cavity, the flat tip has the same mesh as the cavity tip. By comparing leakage mass flow rates across the cavity tip and across the flat tip, the benefits of the designed rotor blade can be illustrated.
4 Results

4.1 Flow separation and reattachment

As the pressure gradient drives the flow across the tip gap, the flow separates over the edges of the cavity tips. Figure 10a shows an evenly spaced tangential velocity vector map of flow separation at the pressure side of the blade. The top blue region is the counter-rotating shroud.

The separation can also be seen in on the wall shear map in 10b. Regions in red, with positive wall shear, show sections of the blade where the flow is attached, moving in the general pressure to suction side direction. Other regions indicate detached flow.

4.2 Turbine power

By using a cavity tip, the average power of the turbine is increased by 1.01%. The differences in turbine power between the flat tip and cavity at each time step can be seen in Figure 11. The figure shows that the cavity tip geometry gives 1KW power gain near zero stator-rotor offset, and that the power gain decreases to a minimum at maximum stator-rotor offset.
4.3 Leakage mass flow rate

The leakage mass flow rate across the tip gap is proportional to the downstream mixing loss. The results from the code calculating leakage mass flow rate can be seen in Figure 12. The calculations show that the leakage mass flow rate across the tip gap varies greatly along one blade passage. The leakage mass flow rate reaches its greatest levels at $\frac{2}{3}T$ in the blade passage, and minimum levels near $T$. The peak-to-peak amplitude is 0.415 kg/s, which is an increase of 37.9%. The change from minimum to peak happens in half a blade passage.

The red line in Figure 12 shows the mass flow of a converged steady state simulation that uses the same mesh and the same physics. The blue line shows the period averaged leakage mass flow rate. The steady state simulation has a leakage mass flow rate 2.4% higher than the averaged leakage mass flow rate of the transient simulation.
Figure 12: Leakage mass flow rate across tip gap with cavity geometry. The leakage mass flow rate of an identical steady state simulation in red, and the average transient leakage mass flow rate in blue.

The periodicity of the leakage mass flow rate is the same as that of a blade passage, meaning that the leakage mass flow rate across the tip gap is dependent on the stator-rotor offset. Two monitor points that measured pressure during simulation validate this. They are plotted in Figure 13. The purple monitor point in this figure measured pressure at the mid chord suction side of the rotor blade. It has the same periodicity as the leakage mass flow rate, with coinciding extreme values. As the suction side pressure drops, the leakage mass flow rate reaches peak values. When the suction side pressure peaks, the leakage mass flow rate is at minimum.

Upstream unsteady behavior could also lead to an unsteady leakage mass flow rate. The monitor point in green measured the pressure variation after the at the stator wake. It shows a pressure that peaks two times every blade passage. Comparing the stator wake to the leakage mass flow rate, there are no matching perturbations seen on the leakage mass flow rate.
The mass flow rate across a flat tip geometry, as seen in Figure 14, is vastly different than that of the cavity tip geometry. The differences are especially clear in Figure 15 where the mass flow rates are compared. First of all, the magnitude of the mass flow rate across the flat tip is higher. The average mass flow rate across the cavity tip is 1271 g/s over one blade passage, and 1607 g/s for the flat tip. This is a decrease of average mass flow rate by 21% when using the cavity tip geometry. Moreover, the shape of the mass flow rate curve across one blade passage is different. The amplitude is significantly smaller its appearance less sinusoidal. It seems that the characteristics of the flow field arising from the cavity tip design are prevalent enough to shape the
appearance of the mass flow rate, and not only change the magnitude of the mass flow rate. For example, the flat tip mass flow rate flattens out when the simulation approaches time step 240, while the mass flow rates across the cavity tip continues to decrease even further. The effects of the cavity tip vortices on these differences are discussed in the next section.

Figure 14: Leakage mass flow rate across tip gap with flat geometry.
Figure 15: Comparison of leakage mass flow rates across tip gaps of flat tip and cavity tip.

4.4 Vortex dynamics

Tangential streamlines at minimum flow can be seen in Figure 16a, and at peak max flow in Figure 16b. At minimum flow, the cavity vortex and the scraping vortex are snugly fit in the cavity. The tangential streamlines are very condensed at the interface of these two vortices. At maximum leakage mass flow rate, the scraping vortex is pressed towards its wall boundary, decreasing in size, with a greater separation between the two vortices. It appears that the increasing leakage mass flow rate shapes the vortex size.

In a plane perpendicular to the streamwise direction, the vortices of the cavity tip only coexist at regions close the the leading edge. In Figure 17, all three vortices can be seen on streamwise tangential Mach number contours. The effect of these vortices on the cavity tip flow field can be seen in Figure 18.
Figure 16: Tangential streamlines.

(a) Minimum leakage mass flow rate.  
(b) Peak leakage mass flow rate.

Figure 17: All three cavity tip vortices coexisting.
Figure 18: Streamlines passing vortices inside the cavity tip.

Down chord, away from the leading edge and the corner separation vortex, the cavity is occupied by two vortices: the scraping and the cavity vortex alone. Contour maps of the streamwise vorticity at planes normal to the streamwise direction show vorticity generation and arrangement. The contour maps at time steps of peak and minimum leakage mass flow rate are shown in Figure 19. The regions of positive streamwise vorticity are the source of cavity vortices, and the regions of negative streamwise vorticity are the source scraping vortices or corner separation vortices.

The contour maps show that the positive streamwise vorticity is primarily generated at the edge of the suction side and pressure side rim. The negative streamwise vorticity in blue is mostly generated at the shroud, and especially
at the shroud above the pressure and suction side rim. Other sources of vorticity include shear between the counter-rotating vortices themselves. The negative streamwise vorticity seen at the suction side leading edge, which is the origin of the corner separation vortex, is disconnected from the shroud and seems to not originate from the passing tip-leakage jet shearing of the walls.

What is clear from the vorticity contour maps is that the scraping vortex and the cavity vortex are both strongly time dependent. To understand the unsteadiness of these vortices, consider how they arose in the first place: from the wall shear at the endwalls. What happens is that as the leakage mass flow rate reaches peak values, the wall shear rate also increases. This primarily creates vorticity at the pressure and suction side rims, where the wall shear is large, but also at internal shear of the fluid and through shear at shroud and cavity tip floor. It is this production of vorticity that feeds the vortices.

The tip gap region also experience a decrease in vorticity through dissipation and downstream flow. The vorticity of the scraping vortex and the cavity vortex becomes a sum of this dissipation and the production through wall shear and internal shear. By introducing a periodic leakage mass flow rate given by the rotor-stator offset, and by assuming that the shear increases for high leakage mass flow rate, the scraping vortex and the cavity vortex will have the same periodicity as stator-rotor offset: the blade passage. This means that there is some leakage mass flow rate threshold across which the vortices are either dissipating or growing. By studying Figure 20, it is clear
that the cell averaged vorticity of the tip gap region is the same at peak and at minimum mass flow rate.

To visualize the periodicity of the scraping vortex and the cavity vortex, the cell averages of the absolute negative and positive streamwise vorticity are plotted in Figure 20. The cell averaging is done in the whole tip gap region, including the blade cavity. The figure shows the relation of negative streamwise vorticity (corner separation vortex and scraping vortex) and positive streamwise vorticity (cavity vortex) to leakage mass flow rate. Both the negative and the positive streamwise vorticity have a delay in phase compared to the leakage mass flow rate.

What happens in the rotor blade and what explains the phase delay in Figure 20 is that as the leakage mass flow rate reaches peak values, so does the generation of streamwise vorticity. Notice how the extreme values of the leakage mass flow rate coincides with maximum derivative of the negative vorticity and positive vorticity graphs. Above some threshold of leakage mass flow rate, the generation of vorticity is larger than vorticity dissipation.

There is a difference in relative amplitude of 4.34 percentage points between negative and positive streamwise vorticity in Figure 20. Since the vorticity peak levels are periodic in each blade passage, the difference in amplitude must be because of a faster dissipation rate of the scraping vortex compared to the cavity vortex.

The effect of these periodic vortices on the leakage mass flow rate is destabilizing, increasing the leakage mass flow rate peak-to-peak amplitude during one blade passage. After the leakage mass flow rate peaks around time step 229, the pressure will drop, and the leakage mass flow rate will start to decrease. The tip cavity vortex cores, however, will continue to grow even after the leakage mass flow rate peaks, because the leakage mass flow rate is still over the threshold of vorticity creation. The growing vortices act to further reduce the pressure, decreasing the leakage mass flow rate even more. When the leakage mass flow rate comes below the threshold of vorticity creation, the effect is reversed.

The destabilizing effect of vortices on the amplitude of the leakage mass flow rate is also seen when comparing cavity tip to flat tip. The amplitude of the mass flow rate over the flat tip across one blade passage is significantly smaller: the cavity tip leakage mass flow rate varies with 37.9% and the flat tip with 20.2%. The large leakage mass flow rate amplitude of the cavity tip is explained by the one transient flow structure it exhibits and the flat tip lacks: the vortex. This shows the significant effect vortical structures has on
the leakage mass flow rate, and how important control and understanding of these vortices is in rotor blade design.

Reconsidering Figure 16 where the tangential streamlines at peak and minimum leakage mass flow rate are plotted. Comparing those results the the results in Figure 20, the net vorticity at times of peak and minimum leakage mass flow rate is the same. This means that the different vortex structures in Figure 16 is not a product of the vorticity in the tip gap region, but a product of the leakage mass flow rate itself. It means that as the leakage mass flow rate reaches peak values, it pushes the vortices towards the walls, making them denser. Not only does the leakage mass flow rate affect the tip gap vortices in the sense of vorticity production, but also the position of their vortex cores.

Figure 20: The cell averaged absolute values of positive and negative streamwise vorticity for the tip gap region compared to a linearly scaled leakage mass flow rate.

The corner separation vortex shows less transient behavior compared to the cavity vortex and the scraping vortex. Throughout one blade passage, the corner separation vortex near the leading edge maintains vortex core
location, and is decreasing in size. The high pressure region that causes the flow ingestion and the corner separation vortex can be seen in Figure 21 through one blade passage. The pressure and the corner separation vortex peak at no stator-rotor offset, and the vortex then proceeds to dissipate during the whole blade passage until the next pressure spike occurs, after which it grows back to its original size.

4.5 $\lambda_2$ and Q vortex cores

The best way to visualize the cavity tip vortices are the vortex detection methods presented in the theory section. The results of the $\lambda_2$-criterion are vortex cores that are not distinguishable from each other, even with directional filtering. This problem is touched upon in a comparison on vortex core identification algorithms in Jiang, Machiraju, and Thompson (2005), where the $\lambda_2$-criterion has trouble distinguishing individual vortex cores in regions dense with vortices. The results of the $\lambda_2$-criterion will not be discussed further, but serves as a guideline that the Q-criterion is preferable over it when studying tip gap flows in high pressure turbines.

The results of the Q-criterion, however, are clearly distinguishable vortices. Figure 22 shows the vortex configuration at minimum leakage mass flow rate using surfaces at Q. At this time step, the cavity vortex and the scraping vortex are not growing from the wall shear, but are instead dissipating. As they dissipate, they both move downstream along the cavity. As the vortices approach the trailing edge, the cavity grows thinner. Closing in on the trailing edge, the cavity vortex occupies a growing portion of cavity space, and the scraping vortex becomes thin by constriction.
Figure 21: Total pressure contour at suction side leading edge, the origin of the corner separation vortex.
Figure 22: Vortex configuration at minimum leakage mass flow rate. The contour is colored by streamwise vorticity, highlighting the orientation of the vortices.

Figure 23 shows the vortex configuration at peak leakage mass flow rate. At this time step, the vortices from Figure 22 are barely detected by the Q-isosurface set in the algorithm. As the vortices have moved downstream, the scraping vortex has almost retreated from the trailing edge, and the cavity vortex mainly occupies it. The remaining scraping vortex is thin and of high vorticity. It seems that an increasing leakage mass flow rate primarily constrains the scraping vortex, probably because it is counter-rotating to the passing jet. This would explain the tangential streamlines seen in Figure 16, where the increasing leakage mass flow rate acted to constrict the scraping.
vortex alone. The old cavity vortex from Figure 22 is seen at the trailing edge as a vortex separated from the new vortex growing at the pressure side rim.

At the pressure side rim at this peak leakage mass flow rate, the new cavity vortex is growing fast. The large vorticity near the leading edge indicates that this is the region where the new cavity vortex primarily form. The new scraping vortex also form rapidly at peak leakage mass flow rate. At the shroud near the leading edge, the vorticity is noticeably higher. In contrast to the cavity vortex, the new scraping vortex rejoins the older vortex created at the previous blade passage.

In Figure 24 the whole blade passage is illustrated using one frame for every two time steps. It shows the dissipation and downstream exit of vortices at low leakage mass flow rates, and the increasing vortex generation as the leakage mass flow rate peaks. It also shows that at first the vortices are primarily originating from the leading edge. In the following time steps, however, generation of vorticity is increasingly present downstream, displayed as regions of the cavity vortex connected to the pressure side rim. It is probable that the passing pressure spike of the blade passage, as seen in Figure 13, first peaks vorticity generation at the leading edge, and then continues downstream.

The corner separation vortex in Figure 24 is peaking around the blade passage, and then decreases until the next pressure spike as shown in Figure 21. The dissipation rate is very slow, and the corner separation vortex core in the leading edge-suction side junction is always present.

Figure 24 also explains the appearance of the instantaneous power difference plotted Figure 11. It is clear that large vortices coincide with a high power difference, and small vortices with low power difference. This is especially clear in Figure 25 where Figure 24 and Figure 11 are combined. This figure illustrates the large role that cavity tip vortices play in the superior performance of the cavity tip.
Figure 23: Vortex configuration at peak leakage mass flow rate. The contour is colored by streamwise vorticity, highlighting the orientation of the vortices.
Figure 24: Vortex visualization during one blade passage using the Q-Criterion with one image for every two time steps.
Figure 25: The difference in instantaneous power between cavity tip and flat tip, and the vortex configuration at its peak and minimum time steps. The top vortex configuration is at peak power difference, and the bottom is at minimum power difference.
5 Conclusions

- The cavity tip decreases average mass flow rate and changes the appearance of the mass flow rate plotted across one blade passage. Compared to the flat tip geometry, the cavity tip has 21% lower average leakage mass flow rate averaged over one blade passage.

- The leakage mass flow rate across a cavity tip is periodic with a period of one blade passage. The upstream stator wake did not give any fluctuations on the leakage mass flow rate across the tip gap.

- Both the cavity vortex and the scraping vortex are varying with the leakage mass flow rate at a phase delay of \( \frac{1}{6} T \) (the blade passage period). As they are moving downstream along the cavity, with the scraping vortex dissipating faster than the cavity vortex.

- The corner separation vortex increases with the pressure spike happening at blade passage, and dissipates slowly until the next passage.

- The instantaneous power difference between the cavity tip and the flat tip depends on the vortex size found using the Q-criterion identification. During blade design, power is gained by optimizing cavity tip vortex generation.

- The vortices destabilize the leakage mass flow rate, increasing its fluctuation over one blade passage. The mass flow rate across the cavity tip increases with 37.9% from minimum to peak, and the flat tip with just 20.2%. This shows how strongly leakage mass flow rate depends on vortices, since the only transient structure exhibited by the cavity tip and not the flat tip is the vortices.

- There is a mass flow rate threshold that determines whether net vorticity is decreasing or increasing inside the cavity tip.

- As the leakage mass flow rate increases, the cavity vortex and the scraping vortex are pushed towards the endwalls by the passing flow. They decrease in size while the net vorticity maintains the same.

- The Q-criterion can identify vortices in the tip gap region of high-pressure turbines. The \( \lambda_2 \)-criterion could not separate the three main vortices inside the tip gap region.
References


