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Pressure in dark source flux cosmology

Jan-Åke Schweitz
Dept. of Engineering Sciences, Uppsala University, Sweden
jan-ake.schweitz@angstrom.uu.se

Abstract

A cosmological pressure concept is defined from virial equations for open systems that undergo Hubble expansion. A uniform dark source flux model is assumed, comprising non-relativistic baryonic and dark matter ('dust'); relativistic radiation; and dark energy. The pressures thus derived are compared with corresponding pressures derived from the Friedmann equation in standard cosmology. The pressures are found to agree for non-relativistic matter and relativistic radiation, but differ for dark energy. The 'exotic' negative pressure of dark energy in standard cosmology is replaced by more down-to-earth positive pressure in the present theory. The reason for this deviation is that the local energy conservation criterion, which is built into the field equation in general relativity, is replaced by a compelling consequence of Hubble's law: the criterion for balanced expansion. In the choice between questionable energy conservation in an accelerating system on one hand, and the experimentally verified Hubble's law on the other, the latter is chosen in the present analysis.

1. Background

Dark source flux (DSF) cosmology was first presented in 2014 by Schweitz, modelling an expanding universe featuring a singular Big Bang beginning [1]. In 2015 this theory was extended to a model of a universe featuring a non-singular Big Bang beginning, thus revealing an interesting kinship with the Planck unit system [2]. In 2017 both theories were included in a more comprehensive article on DSF cosmology, which also included discussions on several other Big Bang related issues [3]. The most important feature of DSF cosmology is a uniformly distributed, non-conservative source flux of dark energy, driving flat-metric expansion. It is shown that this theory explains all observed expansion features of the universe, qualitatively as well as quantitatively. The effect of DSF is vividly illustrated by the 'Hubble Puddle metaphor' in [3].

Pressure is an implicit property in DSF cosmology, so an explicit definition of pressure is not really necessary in this theory. Neither positive nor negative pressures appear in explicit form. Hence precarious conjectures about pressures, or about equations of state that explicitly relate pressure to energy density in different cosmic fluids, can be avoided.

In standard cosmology (SC) and General Relativity (GR) the pressure concept is ill defined and therefore confusing – it stands in conflict with the Occam's razor philosophy implemented in articles [1] – [3]. However, since DSF theory unavoidably

will be compared with SC, we shall here discuss the cosmological pressure concept anyway. It must be emphasized, though, that DSF cosmology is completely independent of SC and GR. The pressure issue only arises when DSF cosmology is compared with SC-GR, and the problem lies entirely on the SC-GR side.

2. Pressure in General Relativity and Standard Cosmology

The cosmological pressure concept, which is of central importance in SC, see e.g. [4] and [5], is ambiguous and not compatible with pressures as defined in thermodynamics or fluid mechanics. In these disciplines, pressures are equivalent to kinetic energy densities – making negative pressures meaningless. Nonetheless, in SC negative pressure is a property assigned to dark energy (often described by the forgiving term 'exotic').

This property stems from the fact that a local energy conservation postulate (to be more exact: vanishing covariant divergence) is built into Einstein's field equation. When a cosmological constant term – equivalent to a *positive* vacuum energy density – is added, the field equation responds the way it is designed to respond; it adds a *negative* energy density of the same magnitude (in the form of negative pressure), in order to maintain energy conservation.

In SC formalism, a pressure P is defined by a Friedmann equation derived from the vanishing covariant divergence of the energy-momentum tensor. The latter is taken to be a diagonal tensor of a perfect fluid (which in itself is a dubious constraint on a system undergoing accelerating expansion). The same equation can easily be deduced using the classical thermodynamics equation of state: $dU = -PdV$. Both derivations are based on similar initial constraints (local energy conservation and an adiabatic perfect fluid assumption) and suffer from the same fundamental problems. For convenience, we shall here discuss the simpler thermodynamics version, but the conclusion we arrive at is valid in both cases.

First, one should remember that classical thermodynamics is valid only for equilibrium states in isolated systems where energy is conserved (the First Law of Thermodynamics). But our expanding universe is *not* in equilibrium; certainly not in the 'explosive' initial phase. Furthermore, the state equation: $dU = -PdV$ defines mechanical energy; work done by the system on the surroundings (hence the minus sign) when its volume is increased by dV under equilibrium pressure P. But our expanding, universal system does not feature equilibrium pressure – it varies dramatically from Big Bang and forward – and the energy change dU in an expanding volume element is not negative in our case; on the contrary it is positive, since the amount of dark energy increases with the expansion. We conclude that the negative pressure of dark energy resulting from this theory is *a model artefact caused by an improper state equation, requiring negative P to achieve positive dU.*

The basic problem here is the fact that energy conservation is erroneously implemented in a system undergoing accelerating flat-metric expansion, in direct conflict with generally accepted physics for accelerating systems in flat space. Another problem is the fact that the equation of state: $dU = -PdV$ implies entropy change $dS = 0$ at all temperatures, which in turn implies that every increment of the expansion process is thermally reversible. This certainly seems like an overly bold

presumption, considering the dramatic thermal development of the universe from Big Bang and forward.

We conclude that $dU = -PdV$ is an improper equation of state for our expanding universe, and so is the Friedmann equation, which is based on similar, too restricted initial constraints.

In DSF cosmology, on the other hand, *no energy conservation, no reversible expansion process, no quasi-equilibrium state, no perfect fluids, no strange pressure concepts are assumed.* Hence, this theory is much less restricted than SC.

3. Virial theorems in open, homogeneous and isotropic systems

Instead of using adiabatic thermodynamics, or an equivalent Friedmann equation, we shall here use virial theorems for open systems, derived by the author some 40 years ago [6] – [8], to make stringent definitions of pressure in DSF cosmology.

The analysis may seem a bit 'messy' in some parts, since we need to distinguish between relativistic contributions and non-relativistic contributions, and *at the same time* distinguish between expanding states and non-expanding states. But in the end, it all boils down to two simple contributions in the cosmological pressure, so please hold on.

We will study the same uniformly distributed cosmic fluids as are defined in SC: non-relativistic baryonic and dark matter; relativistic radiation; and dark energy. In SC, dark energy is seen as a non-relativistic medium, which actually turns out to be supported by the present results.

The general idea in the derivation of the virial theorem is to differentiate a sum of one-particle observables with respect to time, then find the long-term time average of the resulting derivative and finally show that this average must be vanishing for the observed system (see [6]). This reasoning requires stable time averages for the system in question. Apparently, a metrically expanding universe featuring a dramatic Big Bang beginning is not such a system, at least not in time intervals of interest to us.

The situation is saved, however, by the uniform universe assumption, which removes the time average limitation. In this universe, the momentum distribution is assumed to be homogeneous and isotropic everywhere in space at all times. The momentum density level may vary over time, but it is always homogeneous and isotropic. The sum of one-particle observables of interest here is [6]:

$$M(t) = \sum \mathbf{r}_i \cdot \mathbf{p}_i(t). \tag{1}$$

$\mathbf{p}_i(t)$ is the momentum of the ith particle located at $\mathbf{r}_i$ in the observed volume at the time t. In the case of homogeneous and isotropic momentum distribution, this sum can be expressed as a volume integral:

$$M(t) = \int_V \mathbf{r} \cdot \mathbf{p}(\mathbf{r}, t) \, \mathrm{dv}. \tag{2}$$

$\mathbf{p}(\mathbf{r},t)$ is the net momentum density at $\mathbf{r}$ at time t. In the case of isotropic momentum distribution, the net momentum vector is $\mathbf{p}(\mathbf{r},t) \equiv 0$ for all $\mathbf{r}$ and t values. Hence we have $M(t) = 0$ and $\frac{d}{dt}M(t) = 0$ at all times (not only as long-term averages), meaning that the virial expressions derived in [6] are valid at all discrete times from Big Bang to the ultimate state of the uniform universe. Note that this result is not some type of adiabatic approximation; in the uniform universe picture it is an exact result. (Of course, the uniform universe is a simplified picture of the real universe, but that's another question.)

4. Virial equations in non-relativistic and relativistic open systems

In the following, we distinguish between relativistic contributions (superscript *(rel)*) and non-relativistic contributions (superscript *(norel)*). When neither *(rel)* nor *(norel)* is indicated, the expression is valid for mixed relativistic and non-relativistic matter.

In DSF and standard cosmology, a *non-relativistic* universe comprises baryonic and dark matter, and probably dark energy (let's leave that question till later), but *not* relativistic radiation (photons and neutrinos). For a non-relativistic open system, the virial equation is expressed [6]:

$$2T^{(norel)} + Z_V^{(norel)} + Z_S^{(norel)} = 0 . \tag{3}$$

$T^{(norel)}$ is the kinetic energy content of the observed volume V; the volume virial $Z_V^{(norel)}$ is the virial of all forces acting on the content of V; and the surface virial $Z_S^{(norel)}$ is the virial of the momentum flux through the (imaginary) limiting surface S of V. (These virial terms will be defined below.)

A nearly fully *relativistic* universe consists of photons and neutrinos, but probably not dark energy. The virial equation then is:

$$T^{(rel)} + Z_V^{(rel)} + Z_S^{(rel)} = 0 . \tag{4}$$

In this equation, $Z_V^{(rel)}$ and $Z_S^{(rel)}$ are defined in the same ways as in the non-relativistic case, but the factor 2 in the kinetic term now is 1. This equation can be derived in the same manner as our Eq. (3) is derived in [6], with the modification that non-relativistic single-particle energies $T_i = p_i^2 / (2m_i)$ are replaced by fully relativistic single-particle energies $T_i = p_i c$.

So far, we have treated non-relativistic and relativistic systems as separate cases. But in the real universe, non-relativistic and relativistic matter exists simultaneously. A combined virial equation still features a kinetic term (two actually), a volume virial, and a surface virial:

$$2T^{(norel)} + T^{(rel)} + Z_V + Z_S = 0 . \tag{5}$$

The virial terms now include both non-relativistic and relativistic matter, but in the kinetic terms the two matter types must be separated. Now, let's define the virial terms.

5. Volume virials, surface virials, and pressure

The basic expression for a volume virial is: $Z_V = \sum \mathbf{r}_i \cdot \mathbf{F}_i$. In a system with forces of interaction of $1/r^2$ type (such as gravitational forces) the volume virial Z_V equals the potential energy integrated over V, i.e.

$$Z_V = U, \tag{6}$$

where U is a negative quantity. Let us look at a spherical volume $V(R)$ of radius R. For uniform matter density, the volume virial $Z_V = U$ then is well defined, since the matter inside $V(R)$ is unaffected by gravitational forces from the outside, according to Newton's shell model.

At this point, it must be emphasized that Eq. (6) is valid only in epochs when gravitational interaction is of $1/r^2$ type and dominates over other types of interaction. In very early epochs, this may not have been the case and the volume virial then is unknown and a virial theorem cannot be defined.

The surface virial is [6]:

$$Z_S = \oint_S \mathbf{r} \cdot (\bar{\vec{p}}(\mathbf{r}) - \vec{p}(\mathbf{r}))\, dS, \tag{7}$$

where $\bar{\mathbf{p}}$ is the mean *influx* density of momentum through the surface element dS at the point $\mathbf{r}$, and $\vec{p}$ is the corresponding *outflux* density. The dimension of $\vec{p}$ is momentum per time unit and area unit, or energy per volume unit, or pressure. For *any* fluid with uniform and isotropic distribution of momentum, we have:

$$Z_S = -3PV, \tag{8}$$

where $P = |\bar{\mathbf{p}} - \vec{p}| = 2|\vec{p}|$ is the pressure. The virial equation (5) now is:

$$2T^{(norel)} + T^{(rel)} + U - 3PV = 0, \tag{9}$$

i.e. the pressure is:

$$P = (2T^{(norel)} + T^{(rel)} + U)/(3V) = (2\varepsilon_T^{(norel)} + \varepsilon_T^{(rel)} + \varepsilon_U)/3. \tag{10}$$

The ε terms are energy densities. We can now make a simplifying assumption, corresponding to the zero pressure of 'dust' assumed in SC. We assume $\varepsilon_T^{(norel)} = 0$, so Eq.(10) now reduces to:

$$P = (\varepsilon_T^{(rel)} + \varepsilon_U)/3 . \tag{11}$$

In the special case of a fully relativistic *photon+neutrino gas* the *U*-term can be neglected, and we are left with the well known state equation for a photon gas:

$$P = \varepsilon_T^{(rel)}/3 . \tag{12}$$

6. Metric expansion

All expressions derived so far can, in fact, include metric expansion. In the expanding state, an isotropic Hubble flow superposes all other motion of the particles in the system. Naturally, this will change the numerical values of the kinetic T-terms in the virial equation (5), but not the value of the volume virial Z_V, since this is velocity independent. The surface virial Z_S still is valid, since the momentum distribution of the metrically expanding system still is isotropic at every point in space, but its numerical value will change due to the superposed Hubble expansion.

In DSF theory, the dynamics of the universe features two opposing energy densities: one expansive kinetic energy density and one contractive gravitational energy density. The DSF theory states that these two energy components locally are in exact dynamic balance at all times, i.e. the positive kinetic term and the negative gravitational term always add up to zero:

$$\varepsilon_{\text{kin}}(\mathbf{r},t) + \varepsilon_{\text{grav}}(\mathbf{r},t) = 0 . \tag{13}$$

This is not only in complete accord with Hubble's law; it actually is a compelling consequence of it (see [3], sec. 5.7). Thus, in DSF theory the expansion of the universe is *not* due to the expansive kinetic energy of matter overcoming the contractive gravitational energy – they are always in balance. Instead, the expansion is due to the ever ongoing dark source flux that generates a resistance-less expansion of all matter embedded in the flat expanding metric, as illustrated by the resistance-less drift of the leaves in the Hubble Puddle metaphor [3]. The DSF theory shows that the source flux in the dynamically balanced universe drives a metric expansion *exactly* as described by Hubble's law and relates the expansion rate to the source flux rate.

Hubble's law always is defined in a local frame and the balance criterion (13) is defined in the same frame. But we know that Hubble's law is valid for every choice of local frame, and so is the balance criterion. This makes the balance criterion a global feature, i.e. also in a global perspective the kinetic expansion energy always is outbalanced by gravitational energy. Thus there exists a global balance equation corresponding to Eq. (13), which is time dependent but not space dependent. For simplicity, we keep the same notation in the global case as was defined in the local equation.

We wish to relate this balance equation to the ε terms in the pressure expression of Eq. (10). For the gravitational term, seemingly this is easy; we have $\varepsilon_{\text{grav}} = \varepsilon_U$. However, in an unlimited universe, ε_U is notoriously difficult to express in an

unambiguous way. But – as we shall see – this is no problem for us, since ε_U will be outbalanced anyway in the final pressure expression.

In the following, we will use superscript *(X)* to indicate expanding state, and *(noX)* for non-expanding state. When none of these superscripts are indicated, the expression is valid in both states.

We must be careful with the ε_{kin} term. This term only includes kinetic *expansion* energy (superscript *(X)*). Thus, the balance criterion is:

$$\varepsilon_{\text{kin}} + \varepsilon_U = \varepsilon_T^{(norel/X)} + \varepsilon_T^{(rel/X)} + \varepsilon_U = 0 \ . \tag{14}$$

Any kinetic energy density $\varepsilon_T^{(norel/noX)}$ or $\varepsilon_T^{(rel/noX)}$ that is not directly related to the Hubble expansion is not included in Eq. (14). In the pressure of Eq. (10), on the other hand, *all* kinetic energy terms are included:

$$
\begin{aligned}
P &= [2\varepsilon_T^{(norel)} + \varepsilon_T^{(rel)} + \varepsilon_U]/3 = \\
&= [2(\varepsilon_T^{(norel/X)} + \varepsilon_T^{(norel/noX)}) + (\varepsilon_T^{(rel/X)} + \varepsilon_T^{(rel/noX)}) + \varepsilon_U]/3
\end{aligned} \tag{15}
$$

Using Eq. (14), Eq. (15) reduces to:

$$P = (\varepsilon_T^{(norel/X)} + 2\varepsilon_T^{(norel/noX)} + \varepsilon_T^{(rel/noX)})/3 . \tag{16}$$

We see that the 'problematic' ε_U term now has been eliminated by the balance criterion. Implementing the zero-pressure dust assumption, discussed earlier, we have $\varepsilon_T^{(norel/noX)} = 0$, and Eq. (16) is further reduced to:

$$P = (\varepsilon_T^{(norel/X)} + \varepsilon_T^{(rel/noX)})/3 . \tag{17}$$

In DSF theory the dark energy density $\varepsilon_g = \rho_g c^2$ is identified as the kinetic energy density of the Hubble expansion and we see that dark energy turns out non-relativistic, in agreement with the SC notion. The radiation pressure is given by Eq. (12). Eq. (17) now can be written:

$$P = (\varepsilon_g + \varepsilon_T^{(rel)})/3 = (\rho_g + \rho_{\text{rad}})c^2/3 . \tag{18}$$

Dark energy contributes in the total pressure with the constant value $+\varepsilon_g/3$ at all times. This is a negligible part of the pressure at early times when the total pressure was extreme, but dominant today when total pressure is very low. Hence the dark energy pressure in DSF cosmology deviates from standard cosmology, where it is $-\varepsilon_g$. The error behind this 'exotic' negative pressure is explained in detail in sec. 2. In Eq. (18) ρ_{rad} is fully relativistic, and therefore lacks rest mass. The dark energy density ρ_g is non-relativistic but still lacks rest mass, since kinetic expansion energy does not exist in the *noX* state.

7. Discussion and conclusion

We conclude that dark energy is non-relativistic in agreement with SC. However, its pressure is $+\varepsilon_{\mathrm{g}}/3$ and deviates from standard cosmology, where it is $-\varepsilon_{\mathrm{g}}$. This deviation is due to the fact that the local energy conservation criterion, which is built into the field equation in GR, is replaced by a compelling consequence of Hubble's law: the criterion for balanced expansion. In the choice between questionable energy conservation in a system accelerating in flat space on one hand, and the experimentally verified Hubble's law on the other, we choose the latter.

We note a couple of other things in the final pressure expressions (17) and (18). There is no influence from non-relativistic baryonic or dark matter. This, of course, is a natural consequence of our zero-pressure assumption for 'dust'. Also, there is no *direct* influence on the pressure from the expansion of relativistic radiation ($\varepsilon_T^{(rel/X)}$), since this term is eliminated by the balance criterion. All influence from the expansion of radiation is *indirect* and is due to the dilution of ρ_{rad} in the 'static' term $\varepsilon_T^{(rel/noX)}$. (This is also the case in SC).

In summary, pressures defined here from virial equations agree with those assumed in standard cosmology in two cases: non-relativistic baryonic or dark matter ('dust') and relativistic radiation. In the third case – dark energy – the 'exotic' negative pressure in SC is replaced by less exotic but more down-to-earth positive pressure.

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