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Teachers' Roles in Preschool Children's Collective Mathematical Reasoning

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ABSTRACT

The aim of this paper is to study teachers' roles in preschool children's mathematical reasoning using analysis of epistemological moves. Three moves were identified: instructional moves, confirming move and a new move, concluding move. There were no generative moves encouraging the children to produce different arguments for choice of strategy or conclusion. Both the confirming move and the concluding move functioned as an end of the reasoning and thereby limited the opportunities for the children to learn creative mathematical reasoning. This although that several encounters were created by the preschool teachers, both as formal planned situations and unformal such as free play.

Keywords: epistemological moves, mathematical reasoning, preschool education

INTRODUCTION

Young children's mathematical reasoning is receiving increasing attention (Sumpter and Hedefalk, 2015), such as studies that indicate that young children are more competent to develop and demonstrate processes regarding mathematical thinking than previously reported (e.g. Mulligan and Vergnaud, 2006). The focus on mathematical reasoning is indeed relevant for younger children as well: mathematical reasoning is, alongside problem solving, the core of what it means to think mathematically (Niss, 2003). They create what is called mathematical proficiency (Abrantes, 2001; Kilpatrick et al., 2001), which is here understood to mean the actions adopted by an individual who identifies a situation as problematic and is favourably disposed to solve the problem and proceeds to do so. The solution is done by deploying a strategy in a series of not necessarily linear steps. This deployment and creation of steps is what characterises mathematical reasoning, where the non-linearity is related to creativity. These are competencies relevant for all and not just a mathematical elite (OECD, 2013).

Recent studies show that children use different competencies in their reasoning (Sumpter and Hedefalk, 2015). However, when looking at the development of mathematical thinking, there is evidence that these competencies are not developed without the provision of a learning opportunity (Bergqvist and Lithner, 2012; Bobis et al, 2005). Also, it has been indicated that if children have access to a guide, they are more likely to go further in their mathematical thinking especially if that person asks key questions (Laine et al., 2017; van Oers, 1996). The importance of the teacher has been stressed in mathematical reasoning teaching (e.g., Bergqvist and Lithner, 2012; Kilpatrick et al, 2001; Shimizu, 1999). This is in line with previous studies on active learning environment (e.g. Derry and Lesgold, 1996; Marks, 2000). This implies that to avoid rote thinking, teachers need to know what to do, when to do it and the implications of their actions (Laine et al., 2017; Lester and Cai, 2016; Liljedahl, 2016).

However, when analysing Swedish secondary school teachers' presentations, we see that most task solutions that are presented to the students are based on algorithms with only rare opportunities to see aspects of creative mathematical reasoning (Bergqvist and Lithner, 2012). We don't know how such results are translated to the preschool level, especially with informal settings as an important learning opportunity: although the competence mathematical reasoning is the same and education in Swedish preschools should strive for children to "develop their mathematical skill in putting forward and following reasoning" (School Agency, 2011, p. 10), teaching should be play-based (Sumpter and Hedefalk, 2015). The way the education differs at preschool invites a different set of problems:

Finding ways to educate such a teacher for young children, with a deep understanding of the psychological characteristics of young children's playful learning, with valid mathematical understandings, with abilities to demonstrate the relevance of mathematical creativity and the attitude to improvise in her pedagogical practice within a strongly structured field (see, for example, Sawyer, 2011) may be the biggest challenge that we face in our attempts to improve mathematics education for young children (van Oers, 2013, p. 271-272).

In order to understand what it means to understand mathematical reasoning, in an informal setting as well, we need to look at preschool teachers' roles in collective mathematical reasoning. This is the aim of this paper: to study the opportunities for participation in mathematical reasoning that are presented to children at the preschool level. The research questions posed are: (1) what different arguments can be found in collective mathematical reasoning in a preschool setting?; and, (2) what are the different roles of the teacher connected to these arguments?

THEORETICAL BACKGROUND

Reasoning in mathematics education is often denoted as some type of 'high-quality thinking' but with little explanation of what it could encompass (Lithner, 2008). This is true for research focusing on the preschool level as well where few studies define what it means to perform mathematical reasoning independent of mathematical area (Sumpter, 2016). The starting point is to separate creative mathematical reasoning and imitative reasoning, where the former is based on mathematics and the latter on other arguments often non-mathematical, and reasoning is defined as the line of thought adopted to generate assertions and conclusions when solving mathematical tasks (Lithner, 2008). This is a product and we see it as a sequence or several sequences that begin with the tasks and ends with an answer, where the answer could be no conclusion at all. Reasoning is a chain of arguments where arguments can be defined as a collection of statements where some are offered in support of another (Toulmin, 2003). It should be stressed that creative mathematical thinking is not restricted to people with an exceptional ability in mathematics since it includes rather elementary reasoning (Lithner, 2008). However, it could be difficult to produce without sufficient key competencies and a supporting environment. Also, teaching mathematical problem solving doesn't automatically mean that students become mathematically proficient, meaning that there is something more that is required in order to teach 'mathematical thinking' (Lester and Cai, 2016). Previous research has shown that creative mathematical reasoning can be difficult for students, both at the secondary level (e.g., Bergqvist, Lithner and Sumpter, 2008) and at the primary level (Pehkonen, 2000). Research also indicates that it is not the implementation of a strategy, the deployment of a strategy that provides the main difficulty for students, but more finding the right arguments for the choice of strategy (e.g. Kester et al., 2004).

Following the idea of learning opportunity and the importance of the teachers when creating them with regard to mathematical reasoning, teachers need to be able to pick up children's mathematical ideas (Bergqvist and Lithner, 2012; van Oers, 1996; Shimizu, 1999). Also, since the key component of Swedish preschool education (children age 1-5) is the emphasis on play and whereby education should not be formal schooling, this should happen independently of whether the activity is planned or informal (School Agency, 2011). One could argue that, at least in such a system as the Swedish one, the role of the teacher at the preschool level is particularly important since the format and context is much more varied and less predictable. Such an argument is supported by Sawyer (2011) when discussing the balance between structure and improvisations regarding what it is that makes good teachers great: that relying on improvisation requires skills. However, these skills are on a general level and when giving examples that are mathematics-specific, the analysis doesn't focus on the teachers' roles (Martin and Towers, 2011).

Continuing to focus on mathematics, van Oers (1996) concluded that teachers at preschool level can promote mathematical reasoning although in his study there is no definition of reasoning and there is no further suggestion of how to do it besides asking the question 'are you sure?' There is also no explanation of why this question works beyond the notion that it should be related to the solutions suggested by the children and not when the problem is initiated. Time then seems to be a factor but no further discussion why is provided. Another study raising the importance of asking the right questions at the right time, although focusing on teacher education, is Shimizu (1999). There is a separation between the initiation of the task and other key questions, but also the instructions a teacher should give at the students' desk. One of the things required is the ability to scan and assess students' problem solving progression, but there are no explanations of what or of how this scanning should be done, of what makes a key question or of what to assess when deciding what to ask, meaning it is hard to distinguish what is generating and promoting mathematical reasoning and what is not. The teachers' role is stressed by Liljedahl (2016), who focuses more on how questions are answered when students are working on a task. His conclusion was that the questions of the types 'proximity questions' and 'stop thinking questions' should not be answered, although acknowledged, in order to create a 'building thinking classroom'. The teacher should only answer 'keep thinking questions' to avoid leading questions and scaffolding, but there are no details of how to answer them and when.

Another study takes the opposite perspective and instead looks at the importance of specific leading questions, with the aim to promote mathematical reasoning (Davydov and Tsvetkovich, 1999). A clear structure is presented about what to say and what mathematical properties to emphasise, but no justifications are presented and there is no analysis of why the structure works. The idea of certain questions being constructive is supported by Laine et al. (2017). In the conclusion, they state that a "teacher who is accustomed to guiding problem-solving knows certain types of good questions that will help the pupils to move forward" (Laine et al., 2017, p.16). Considering the close relationships between problem solving and mathematical reasoning (Niss, 2003) both being part of mathematical proficiency (Abrantes, 2001; Kilpatrick et al, 2001), it is plausible to make the inference that the same questions could function for mathematical reasoning. Nonetheless, no examples are given illustrating what is a 'good question' or whether the quality of the question is dependent on the time and situation.

Therefore, we argue that, with respect to mathematical reasoning, it is not clear what makes a question from a teacher function as a key question and if and how this differs dependent on content or time. Also, we anticipate that the ability to pick up ideas and ask the right question, hence providing the right stimuli, is a challenge (c.f. Bergqvist and Lithner, 2012) especially for preschool teachers since they often do not know about the mathematical topic beforehand and what mathematical competencies that could be involved due to the centrality of free play (c.f. Laine et al., 2017; Sawyer, 2011). Such a study has been done in research looking at preschool teachers in play-based situations who have different educational content in focus (Hedefalk, 2014; Klaar and Öhman, 2014). As theoretical tools to study teachers' roles, Practical Epistemological Analysis (PEA) and Epistemological Move Analysis (EMA) were used. Practical epistemology is used as a tool for describing the route that meaning making takes, and the meaning-making processes involved (Wickman and Östman, 2002). It is in this meaning-making process that the teacher takes on different roles in order to direct children/students in different directions suggesting that learning and teaching are understood as "ongoing changes in human conduct" (Caiman and Lundegård, 2017, p. 6). Previous research in science and technology education in primary school and secondary school (e.g., Lidar et al., 2006; Lundqvist and Östman, 2009) and in preschool in general education (Hedefalk, 2014; Klaar and Öhman, 2014) has identified the following moves: confirming, reconstructing, instructional, generative, and reorienting moves. In the confirming move, the teacher confirms that the children are recognizing the correct phenomenon, or confirms that the children are undertaking a valid process, by agreeing with what the children say or do. The reconstructing move makes the children pay attention to the 'facts' they have already noticed but have not yet perceived as valid. The instructional move gives the child a direct and concrete instruction for how to act, to discover what is worth noticing. In the generative move, the teacher enables the children to generate explanations by summarizing the important facts in the context of the activity. This is similar to the Lithner and Bergqvist (2012) theoretical framework regarding stimulation argumentation, both predictive and verifying, including reflections. In their study of such argumentation, creative reflection, was sparse.

Finally, the reorienting move indicates that other properties may be worth investigating and encourages the children to take another, alternative direction (Hedefalk, 2014; Lidar et al., 2006; Klaar and Öhman, 2014). Hence, some comments or questions from the teacher will generate different moves depending on the context and interaction with the students. Therefore, van Oers' (1996) suggestion "Are you sure?" could be a key question and create meaning, but it could also not generate an epistemological move. By studying the interaction between the content, here with a focus on different arguments in mathematical reasoning, and the participants and their interplay, there is a possibility to offer such explanations.

METHODS

Method of Data Collection

The data was collected via video filming at a preschool in Sweden during 17 visits over a period of two months. The researcher (second author) had access to three groups of children, the groups are here named Dundret, Kebnekaise, and Skanderna with the ages 1-2, 3-4 and 5 respectively (more information about the data collection can be found in paper masked). The names of the children and teachers have been altered in the transcripts: the children that belong to Dundret have names beginning with D, to Kebnekaise with K, and to Skanderna with S. Out of 24 hours and 10 minutes of film, 13 hours and 30 minutes were transcribed since we were interested in interactions between actors. Hence, episodes where children sat quietly, not interacting with peers or teachers, were not transcribed. The interactions we choose to analyse were restricted to mathematical reasoning both planned and informal. A first analysis resulted in 21 episodes consisting of collective mathematical reasoning in different mathematical areas such as measurement (e.g., height, weight and time) using both informal and formal units, counting and basic arithmetic (e.g., adding) and geometry (e.g., shapes).

Method of Analysis

In order to answer the two research questions, we perform two different analyses. The first analysis is a modification of Lithner's (2008) framework to focus on collective mathematical reasoning. It focuses on the content in the mathematical reasoning, the different arguments (Sumpter and Hedefalk, 2015). The second analysis focuses instead on the different roles of the teachers. What these two analyses share is the structuring of data, a step-by-step transcription stating actors and actions and utterances. In addition to the data structure, the first analysis requires an organization of the data, a four-step structure (Lithner, 2008): (1) A task situation is met (TS); (2) A strategy choice is made (SC); (3) The strategy is implemented (SI); and, (4) A conclusion is obtained (C). In Lithner's (2008) framework, there are two types of arguments attached to two of these steps. The strategy choice can be supported with predictive arguments that aim to answer the question 'Why will the strategy solve the task?'. The implementation of the strategy can be supported with verifying arguments. Their function to answer the question 'Why did the strategy solve the task?'. This categorisation of arguments was used in Bergqvist and Lithner (2012) when studying upper secondary school teachers and mathematical reasoning. However, a first test of the present analysis showed that this categorisation did not cover arguments focusing on the conclusion and the evaluation of it. Therefore, the choice was, just as Sumpter and Hedefalk (2017), to add evaluative arguments. Such arguments focus on the conclusion and how and in what way the conclusion is an answer to the initial question.

In the second step of the analysis of the arguments, we use the notion of anchoring. It is important to note that anchoring does not refer to the logical value of the argument, since it allows us to talk about reasoning that is incorrect (Lithner, 2008). This helps us to look at the foundation and how it is used. Anchoring is seen as the fastening of the relevant mathematical properties, or alternatively, of the components reasoned about. These components are objects, transformations, and concepts (Lithner, 2008). Certain mathematical properties will be surface and other intrinsic depending on the task, e.g., when comparing fractions, the size of the numerator and denominator is a surface property, whereas the quotient is the intrinsic property. In Lithner's (2008) framework, different types of reasoning can be classified. Here, we will only focus on the different types of arguments and their foundation.

For the second analysis, we need analytical tools focusing on the teachers' roles. Here, the choice was Practical Epistemological Analysis (PEA) and Epistemological Move Analysis (EMA). By using PEA, we can understand why a conversation takes a certain path and why it stays on the same path. There are four concepts in focus in PEA: encounter, stand fast, gap and relations (Wickman and Östman, 2002). An encounter is a specific situation in terms of what the participators interact with and here we will focus on encounters between children and teachers where mathematical discussions take place. What stands fast for the participator is identified in their actual use of words within the practice. When the participator uses a word without hesitation or questioning, such words are said to stand fast in the particular situation. Standing fast is a situational description of the meaning that words have in action. When the participator hesitates, when what is happening cannot be taken for granted, there is a gap. When a gap is noticed it can, according to Wickman and Östman (2002), be filled by establishing relations to what stands fast in the encounter. Then it is possible for the participators to proceed in their meaning making again.

PEA is then combined with analysis of Epistemological Move Analysis (EMA). EMA is an analytical method that aims to generates knowledge about the role a teacher plays in children's meaning making (Wickman and Östman, 2002). The focus of the analysis is how the teacher directs children's meaning making in different ways (Lidar et al., 2006). When the children respond, verbally or non-verbally, to the teacher's direction, we call it an epistemological move. The epistemological moves of the teacher show the children both what counts as knowledge and appropriate means of obtaining knowledge.

For each of the selected episodes, the task situations (TS) were identified with an appropriately chosen grain size. Then the central decisions for each TS were identified alongside the argumentation for these decisions. The mathematical content was analysed using the notion of anchoring and mathematical properties. Then, we applied PEA and EMA. First, we analyse how the actors move forward during the mathematical discussions, i.e., an analysis of the children's practical epistemology. Then we analyse what stands fast, i.e., what they all agree on, what is understandable and reasonable for the children. In order to get to the direction of meaning making, we also look for a gap, i.e., where there is doubt in the interaction between the actors. Here we can see that the actors do not agree on what is said to be reasonable, and in particular how the gaps are filled. These are situations that could indicate learning opportunities.

Table 1. Instructional and confirming move

Line	Person	Data	Argument	EMA
2705	Solveig [T]:	Then we should not forget to say how much we have. How much was it? One litre. [Walks to the table with a colander, then pours the content into the colander]	Initiating a TS: How much is the volume? SC: measuring with a standard unit, reading the scale. SI: Straightforward. C: 1l.	Instructional move. Concluding move.
2706	Sandor:	Here is some more. [reaching for the measure putting some bits of rhubarb in it]		
2707	Sam:	One litre.	Repeating the conclusion.	
2708	Solveig [T]:	That was, one litre is what we have now. Uhm. [goes back to the place where she puts the dough into a pie dish].	Repeating the conclusion.	Confirming move.
2709	Sandor	It is special when you take long [ones]. I take these [holding up a stalk of a certain length]	Indirect measure of lengths. Comparing magnitudes.	
2710	Solveig [T]:	Maybe we can have a whole extra litre [of rhubarb].	Beginning of a new TS.	Instructional move.
2711	Sandor and Sam [at the same time]:	Yes.		
[…]				
2783	Solveig [T]:	Hey you, it is time for you to empty your bits into the measure. And how much do we have then? [pointing at the measure, Svante pours the content from the measure into the colander]	TS: How much is the total content?	
2784	Svante:	Two litres. [using the measure]	SC: No explicit arguments is given. First measure using standard unit l, then adding result to previous result: 1+1 = 2. SI is straightforward with no hesitation. C: Two litres.	
2785	Solveig [T]:	Two litres, correctly! These that are a bit long, could you cut them please? You have got the… [referring to the ability to use a sharp knife] remember which is the straight side. Look at the knife. Yes, you must look. This one you have to work a bit more with [picking up a stalk and putting it on Samuel's tray]		Confirming move.

The next step in the analysis involves EMA, the teacher's role in meaning making. The teacher's epistemological positions are examined using moves identified in earlier research, but also allowing for new moves to be identified. Finally, the different moves were compared to the different arguments (supportive, verifying, or evaluative) using the structuring of the data, thus enabling us to focus on the teachers' roles in collective mathematical reasoning.

RESULTS

The analysis showed that three different types of epistemological moves reoccurred and that these had three different roles in mathematical reasoning. These three were instructional, confirming and a new move, a concluding move. We will here present three episodes to illustrate these moves and their different functions in the reasoning. The numbered lines indicate position in transcriptions, *T* stands for 'teacher' and explanations and further information is provided in square brackets. Pauses not relevant to the task situation are marked with […].

Repeating instructional and confirming epistemological move

The most common situation in the data was teachers initiating a task situation (TS) using an instructional move and then, with a confirming move, establish a conclusion with no further arguments. In this episode, four boys (Sandor, Samuel, Sam and Svante) from Skanderna are going to bake a rhubarb pie with their teacher Solveig. The transcript begins when they have been cutting rhubarb for approximately 30 minutes, see Table 1.

The teacher instructs the children to measure the amount of rhubarb using a measure with standard unit litre (row 2705). However, since she herself provides the strategy choice including implementation of the strategy and conclusion without any arguments, this means that the children are not participating in this particular part of the sequence. However, there are some indications of what stands fast, such as what the unit is and how to operate the measure as nobody shows any hesitation when asked about it or using it. Also, the confirmation of the conclusion ("one litre") is repeated by Sam (line 2707), thus making it an epistemological move, a concluding move. The teacher makes the children focus on the measure and how much rhubarb they have cut up by repeatedly commenting on the amount of rhubarb in the liter measure (lines 2710 and 2783). The TS is initiated (line 2783), and a conclusion is given when Svante states "Two litres" (line 2784). There are no arguments provided besides the film showing Svante operating with the measurement. The conclusion is confirmed by the teacher as a

Table 2. Confirming and concluding move as part of conclusion

Line	Person	Data	Argument	EMA
2501	Kasper:	Yes, but the house is bigger than the rock.	Final C. New TS and C. Argument not provided. House's height > rock's height.	
2502	Kristina [T]:	Where?		
2503	Kasper:	The house is bigger than the rock.	House's height > rock's height.	
2504	Kristina [T]:	The house? Yes, definitely. Because the house, I can step in [the house], right?	Agreeing to C: provides argument using transitivity: Since House > Teacher, and Teacher > Rock, therefore House > Rock.	Confirming move. Concluding move.
2505	Kasper:	Uhm.	Confirming C.	

reasonable conclusion (line 2785), but there is no encouragement to produce any supporting, verifying, or evaluating arguments.

Confirming Move and Concluding Move

Another situation that occurred in the data was the combination of a confirming move and a new move, here called a concluding move. It appeared in the conclusion in relation to evaluative arguments: why is this conclusion an answer to the TS? It is here illustrated with this encounter where two children from Kebnekaise playing in the woods and who have found a rock. When they start trying to climb the rocks, a teacher sees this and interacts with the children. The main TS for this encounter is: what is rock's height in relation to other objects/people? In this part of the reasoning, the teacher (Kristina) and one of the children (Kasper) have disagreed about the height of the rock in relation to the teacher creating a gap (see **Table 2**).

There is one incident where a gap occurs. It starts when Kasper says that the house is bigger than the rock (line 2501). This is a statement that is confirmed by the teacher as a valid statement (line 2502). However, she doesn't encourage Kasper to give any arguments for this conclusion but provides them herself. The arguments are related to the TS and not to the SC or the SI meaning that they are evaluative arguments functioning as control. The gap is not filled in this chain of interactions. What stands fast is that the rock is smaller than the house, which turns out to be the final C to the TS. The encounter ends with Kasper's recognition of the conclusion, (line 2505). According to PEA and EMA. Kasper's 'uhm' is seen as recognition, an affirmative statement, and not an objection or contradiction. Therefore, the conclusion is an epistemological move.

However, since the conclusion is not a confirmation of something the children have discussed or evaluated, but instead it is the teacher who provides the conclusion, the move is a concluding move. This is a new move that has not has been previously described. Due to its content, we call it a concluding move. It is the teacher who makes the conclusion which is recognised by Kasper. If the conclusion was made by the children using evaluative arguments, it would have become a generative move.

Initiating Evaluative Arguments

In this episode, there is also the same repetition of instructional move when initiating a TS and confirming move regarding the conclusion as in the first episode, but in this particular encounter, there is a variation. As part of a first initiation of evaluative arguments in relation to the conclusion to the main TS, the teacher instead breaks down the TS into a sub-task. Saga (teacher) and some children from Kebnekaise are outdoors playing in the sand pit and an activity is initiated: a leopard (played by Kalle) has a birthday and he is five years old. The sand in the bucket is the cake and the sticks are the candles. When the episode starts, three sticks are already in the bucket and now they need to determine how many more candles should be lit (see **Table 3**).

The teacher initiates the main TS by asking about how many candles are left to light, which then becomes an instructional move (line 1256) when the children show in action that they are now focusing on the amount of lights. One child confirms the instructional move, but also give a conclusion to the TS. (line 1257). The teacher responds to the conclusion with a confirming move, but the difference here compared to the second episode is the teacher doesn't end the reasoning with a concluding move. The teacher tries to engage the children with a question that would focus on evaluative arguments (line 1258), but since there is no change in the children's epistemology, it doesn't become an instructional move or a generative move. The teacher asks again, and this time a sub-task is initiated (line 1260). However, since the question doesn't encourage the children to produce their own arguments or further explain their thinking, it becomes an instructional move instead of a generative move. The sub-TS is solved by counting ending in a conclusion (line 1265). During this part of reasoning, we have one gap: when Kalle disagrees with Kasper's statement about the leopard's age (line 1263). The teacher states that the leopard is five (line 1265) and no further discussion takes place. We see by Karolina's actions, when she counts five candles, (line 1265) that the statement from the teacher is an epistemological move. The teacher then confirms Karolina's actions (SC and SI) and conclusion as reasonable (line 1266).

Table 3. Initiation of evaluative arguments

Line	Person	Data	Argument	EMA
1255	Kasper:	Light [it]. Light [it]. [takes a stick and pulls it towards the three other sticks that are already pushed down in the sand in the bucket.]		
1256	Saga [T]:	How many candles are left to light now?	TS: What is the difference between two amounts, a=5 and the amount of sticks in the bucket, b?	Instructional move
1257	Kasper:	We need more candles! [gets up]	C: b<5, no arguments are given.	
1258	Saga [T]:	More candles? Yes, because how many years do we celebrate? [Kalle push one more stick down into the bucket]	Confirms conclusion. Evaluative argument: what is a?	Confirming move.
1259	Kasper:	I bought one more candle. [comes running with a stick]	SC to previous TS: 1 more is needed, $a - b = 1$. SI straight forward. C: 1.	
1260	Saga [T]:	Yes. How many years is it that you are celebrating, Kalle?	Confirms C. Initiates sub-TS as part of main TS: what is a?	Confirming move. Instructional move.
1261	Kasper:	Uhm. Five! [working with the bucket]	Sub-C: a=5. No arguments are given.	
1262	Saga [T]:	Five. And how many candles do you have now?	Confirms Sub-C: a=5. TS: How many candles? Implicit TS: is the amount of candles the same as the amount of years?	Confirming move. Instructional move.
1263	Kalle:	No, I'm turning six years. [Kalle walks away a few steps]		
1264	Saga [T]:	We have agreed upon five now.	Implicit re-establishment of sub-TS.	Instructional move.
1265	Karolina:	1,2,3,4,5 [points to each stick and counts them]	SC: count sticks, one-to-one, cardinal. SI: Straight forward. C: 5 candles.	
1266	Saga [T]:	Five candles. Now birthday child, it is time to sit down.	Repeating conclusion: 5.	Confirming move.

DISCUSSION

In the present paper, the aim was to study preschool children's participation in collective mathematical reasoning, and the teachers' roles in the reasoning. The empirics show that the teachers did help the children to pay attention to mathematics in their everyday lives at the preschool, both during planned activities and free play. As a learning environment, it was interactive and in many ways meaningful context often with a specific content in focus (c.f. Derry and Lesgold, 1996; Marks, 2000), but regarding the results concerning the construction of mathematical reasoning and different arguments, a different image emerges.

Regarding mathematical arguments, there were no supportive or verifying arguments concerning the choice of strategy or the implementation of such in the data. It should be noted that the lack of arguments is not because of the preschool teachers not being able to create learning opportunities; 21 episodes were identified with interactions between children and teachers with mathematical content. The conclusion is that preschool teachers are able to initiate different types of task situations in different mathematical areas. However, the lack of engaging children in mathematical reasoning is similar to what has been reported by Bergqvist and Lithner (2012) regarding the 'opportunity to learn' mathematical reasoning at the upper secondary school level. It appears to be difficult to encourage students to express arguments about different choices at different stages in mathematical reasoning (c.f. Kester et al, 2004). Here, initiations of arguments are replaced by the teacher instead confirming the strategy choice or the conclusion, meaning that the role is about confirming moves instead of generative moves.

The most common epistemological moves, the instructional move and the confirming move, could be considered effective as they make the children pay attention to mathematics (c.f. Bobis et al., 2005). However, it is the generative move that creates the condition for reasoning as it generates different arguments that could include reflection (c.f. Bergqvist and Lithner, 2012). Here, the analysis found no generative moves in the data although there were several situations where mathematics was in focus (e.g., second episode, line 2503). Laine et al. (2017) concluded that guidance without preparation requires an open attitude and good mathematical knowledge. The teachers in our study show both mathematical knowledge and an open attitude, and as such as they both encouraged and engaged in mathematical activities although they were not planned, but still there was something more needed to create a generative move. We suggest, as a possible development in this research area, is to further explore the ability to scan mathematical reasoning (see Shimizu, 1999), if this could possibly be what is missing in order to voice good questions (c.f. Laine et al., 2017). This could include an exploration of what scanning could encompass and how it can be used to create key questions based on the results of the understanding of mathematical ideas.

Looking closer at the confirming moves, when combining EMA and the analysis of the reasoning, we see that the latter one means that the teacher guides and draws conclusions for the children, i.e., a kind of scaffolding. This could be related to Liljedahl's (2016) distinction between three types of questions. When confirming a strategy choice or a conclusion, it is related to the 'stop thinking questions', which could be illustrated with "is this right?". Such a question is about external validation removing the need for the student to think for themselves. Here it is represented by the confirming moves which are mainly found in the conclusions. The teachers act as external validators, almost as an answering section, pointing out what is right and what is wrong. As a comparison with the 'are you sure?' question suggested by van Oers (1996), our results cannot confirm such a conclusion: that a specific question would result in a certain epistemological move. This is here illustrated by the third episode, when the teacher poses the same question twice (line 1258 and line 1260) but with two different reactions from the children. Hence, all that a teacher says does not automatically result in epistemological moves which could be contrasted with previous research aiming to describe key questions (e.g., van Oers, 1996; Shimzu, 1999).

Another result is the discovery that the epistemological moves from research in science teaching were insufficient to describe the teacher's influence on the meaning of the mathematical reasoning (c.f. Hedefalk, 2014; Lidar et al., 2006), as we had to add a new move, a concluding epistemological move. The concluding move differs from the confirming move as it establishes a conclusion in relation to the task situation. If arguments were given, i.e., evaluative arguments, they were provided by the teacher. What concluding moves and confirming moves have in common is that both end the reasoning: the children do not spontaneously continue with the reasoning. A possible explanation could be the difference between mathematics education and science education: that in the former there is often *one* right answer, whereas in the latter there could be several. This requires further investigation.

What we can see in the empirics is a need for the teacher to take the discussions further. The implication then is that this should be addressed by teacher education and in-service education since this is difficult. Shimizu (1999) talks about the need for prospective and beginning teachers to learn key roles. The results from the present study indicate a similar conclusion. If teachers need to know what to do and also when to do it (c.f. Lester and Cai, 2016), especially when taking free play into account (van Oers, 2013; Sawyer, 2011), they need the tools and practice to do it, but they should also be given the explanation of why a certain question could become a generative move in one situation but a confirming move or a concluding move in another situation. Such analysis is needed to understand what makes a key question.

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