Coleman–Weinberg Potential

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Task

retype the project plan

Problem (a)

Assume that $m^2 = -\mu^2 < 0$, so that the symmetry $\phi(x) \rightarrow e^{-i\alpha}\phi(x)$ is spontaneously broken. Write out the expression for $\mathcal{L}$, expanded around the broken-symmetry state by introducing

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\sigma(x) + i\pi(x)),$$  \hfill (1)

where $\phi_0$, $\sigma(x)$, and $\pi(x)$ are real-valued. Show that the $A_\mu$ field acquires a mass. This mechanism of mass generation for vector fields is called the Higgs mechanism.

The Lagrangian density $\mathcal{L}$ is as given ([1], p. 469):

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu\phi)^\dagger D^\mu\phi - m^2\phi^\dagger\phi - \frac{\lambda}{6}(\phi^\dagger\phi)^2,$$  \hfill (2)

where $\phi(x)$ is a complex-valued scalar field and $D_\mu = (\partial_\mu + ieA_\mu)$.

Instead of $m^2$, we write $-\mu^2$. Let us define the potential:

$$V(\phi,\phi^\dagger) = -\mu^2\phi^\dagger\phi + \frac{\lambda}{3}(\phi^\dagger\phi)^2.$$

Let us find $\phi_0$ – the vacuum expectation value, which is non-zero is symmetry is broken. For this, we should find minima of the potential $V(\phi,\phi^\dagger)$:

$$\frac{\partial V}{\partial \phi}(\phi_0) = 0,$$

$$\frac{\partial V}{\partial \phi^\dagger}(\phi_0) = -\mu^2\phi + \frac{\lambda}{3}\phi^\dagger\phi = \frac{3\mu^2}{\lambda}.$$  \hfill (3)

To expand the Lagrangian density around the broken symmetry state, we insert

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\sigma(x) + i\pi(x))$$

into the Lagrangian density. $\phi = \phi_0 + \frac{1}{\sqrt{2}}(\sigma + i\pi)$ and $\phi^\dagger = \phi_0 + \frac{1}{\sqrt{2}}(\sigma - i\pi)$.

1) $(D_\mu\phi)^\dagger D^\mu\phi \rightarrow ((\partial_\mu + ieA_\mu)(\phi_0 + \frac{1}{\sqrt{2}}(\sigma + i\pi)))^\dagger(\partial^\mu + ieA^\mu)(\phi_0 + \frac{1}{\sqrt{2}}(\sigma + i\pi)) = (\frac{1}{\sqrt{2}}(\partial_\mu\sigma - i\partial_\mu\pi) - ieA_\mu\phi_0 + \frac{1}{\sqrt{2}}eA_\mu(-i\sigma - \pi))(\frac{1}{\sqrt{2}}(\partial^\mu\sigma + i\partial^\mu\pi) + \frac{1}{\sqrt{2}}(\partial^\mu\pi - i\partial^\mu\sigma))$.
The one-loop correction comes from a constant field that can be chosen real by proper Weyl transformation. The following expressions can be used to simplify the Lagrangian density:

\[ L = \frac{1}{2} (\partial_{\mu} \phi)^2 + \frac{1}{2} (\partial_{\mu} \pi)^2 + e^2 A_{\mu} A^\mu \phi_0^2 + \frac{1}{2} e^2 A_{\mu} A^\mu \pi^2 \]

\[ + \frac{1}{2} e^2 A_{\mu} A^\mu \sigma^2 + \frac{2}{\sqrt{2}} e A_{\mu} \phi_0 \partial_{\mu} \pi + \frac{2}{\sqrt{2}} e A_{\mu} A_{0} \phi_0 \sigma + e A_{\mu} \sigma \partial_{\mu} \pi - e A_{\mu} \pi \partial_{\mu} \sigma \]

2) \[ \mu^2 \phi^4 \phi \rightarrow \mu^2 (\phi_0 + \frac{1}{\sqrt{2}} (\sigma + i\pi)) (\phi_0 + \frac{1}{\sqrt{2}} (\sigma - i\pi)) = \mu^2 (\sigma^2 + \frac{\pi^2}{2} + \frac{2}{\sqrt{2}} \phi_0 \sigma) \]

3) \[ \frac{\lambda}{6} (\phi^\dagger \phi)^2 \rightarrow -\frac{\lambda}{6} (\phi_0^2 + \frac{\sigma^2}{2} + \frac{\pi^2}{2} + \frac{2}{\sqrt{2}} \phi_0 \sigma)^2 = -\frac{\lambda}{6} (2 \phi_0^2 \sigma^2 + \frac{\sigma^4}{4} + \frac{\pi^4}{4} + \frac{\pi^2}{2} + \frac{4\phi_0^2 \sigma^2}{\sqrt{2}} + \phi_0^2 \sigma^2 + \phi_0^2 \pi^2 + \frac{2}{\sqrt{2}} \phi_0 \sigma^3 + \frac{2}{\sqrt{2}} \phi_0 \sigma \pi^2) \]

All constants were omitted because the energy level can always be shifted. The following expressions can be used to simplify the Lagrangian density:

\[ \mu^2 \frac{\pi^2}{2} - \frac{\lambda}{6} \frac{3 \phi^2}{\lambda} \pi^2 = 0 \text{ and } \mu^2 \frac{2}{\sqrt{2}} \phi_0 \sigma - \frac{\lambda}{6} \frac{4 \phi^2}{\sqrt{2}} \phi_0 \sigma = 0. \]

Thus, the Lagrangian density is as follows:

\[ \mathcal{L}(\sigma, \pi, A_{\mu}) = -\frac{1}{4} (F_{\mu\nu})^2 + \frac{1}{2} (\partial_{\mu} \sigma)^2 + \frac{1}{2} (\partial_{\mu} \pi)^2 - \mu^2 \sigma^2 + e^2 \phi_0^2 A_{\mu} A^\mu + \frac{1}{2} e^2 A_{\mu} A^\mu (\sigma^2 + \frac{\pi^2}{2} \phi_0 \sigma) + \frac{2}{\sqrt{2}} e A_{\mu} \phi_0 \partial_{\mu} \pi + e A_{\mu} (\sigma \partial_{\mu} \pi - \pi \partial_{\mu} \sigma) - \frac{\lambda}{6} (\frac{\sigma^4}{4} + \frac{\pi^4}{4} + \frac{\pi^2 \sigma^2}{2} + \frac{2}{\sqrt{2}} \phi_0 \sigma^3 + \frac{2}{\sqrt{2}} \phi_0 \sigma \pi^2). \]

We can see that \( A_{\mu} \) acquired the mass \( m_{A_{\mu}} = 2 e^2 \phi_0^2 \) from the term \( e^2 \phi_0^2 A_{\mu} A^\mu \).

**Problem (b)**

Working in Landau gauge \( (\partial_{\mu} A_{\mu} = 0) \), compute the one-loop correction to the effective potential \( V(\phi_{cl}) \). Show that it is renormalized by counterterms for \( m^2 \) and \( \lambda \). Renormalize by minimal subtraction, introducing a renormalization scale \( M \).

From Eq. 11.63 [1], the one-loop correction is \( \frac{1}{2} \text{logdet}(-\frac{\partial^2 \mathcal{L}}{\partial \phi_{cl} \partial \phi_{cl}}) \), where \( \mathcal{L}_1 \) is the term depending only on the renormalized parameters, namely \( \phi = \phi_{cl} + \eta_1 + i\eta_2, \eta_1 \) and \( \eta_2 \) are real-valued. Here, \( \phi_i = \eta_1, \eta_2, A_{\mu} \). \( \phi_{cl} \) is a constant field that can be chosen real by proper Weyl transformation. The one-loop correction comes from \( \frac{1}{2} \int d^4 x d^4 y t_1(x) \frac{\partial^2 \mathcal{L}_1}{\partial \eta_1(x) \partial \eta_2(y)} t_2(y) \); thus, we can consider only terms of the Lagrangian density that depend on the second-order fluctuating fields \( \eta_1, \eta_2, A_{\mu} \).

\[ \mathcal{L} = -\frac{1}{4} (F_{\mu\nu})^2 + (D_{\mu} \phi)^\dagger D^\mu \phi - m^2 \phi^\dagger \phi - \frac{\lambda}{6} (\phi^\dagger \phi)^2, F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \text{ and } D_{\mu} = \partial_{\mu} + ie A_{\mu}. \]
Let us compute $\mathcal{L}_1$.
1) $-\frac{1}{2}(F_{\mu\nu})^2 = \frac{1}{2}((\partial_{\mu}A_{\nu})^2 - \partial_{\mu}A_{\nu}\partial^{\nu}A^{\mu})$
2) $(\partial_{\mu} - ieA_{\mu}(\phi_{cd} + \eta_1 - i\eta_2)(\partial^{\mu} + ieA^{\mu})(\phi_{cd} + \eta_1 + i\eta_2) = (\partial_{\mu}\eta_1 - i\partial_{\mu}\eta_2 - ieA_{\mu}\phi_{cd} - ieA_{\mu}\eta_1 - eA_{\mu}\eta_2)(\partial^{\mu}\eta_1 + i\partial^{\mu}\eta_2 + ieA^{\mu}\phi_{cd} + ieA^{\mu}\eta_1 - eA^{\mu}\eta_2) 
\rightarrow (\partial_{\mu}\eta_1)^2 + (\partial_{\mu}\eta_2)^2 + e^2\phi_{cd}^2A_{\mu}A^{\mu} + 2e\phi_{cd}\partial_{\mu}\eta_2A^{\mu}$
3) $-m^2(\phi_{cd} + \eta_1 - i\eta_2)(\phi_{cd} + \eta_1 + i\eta_2) \rightarrow -m^2(\eta_1^2 + \eta_2^2)$
4) $-\frac{1}{6}(\phi_{cd}^2 + \phi_{cl}^2\eta_1 + \eta_2^2) \rightarrow \frac{1}{6}(4\phi_{cl}^2\eta_1 + 2\phi_{cd}^2\eta_2^2 - \frac{1}{3}(3\phi_{cd}^2\eta_1 + \phi_{cd}^2\eta_2^2)$
Thus,

$$\mathcal{L}_1 = -\frac{1}{2}((\partial_{\mu}A_{\nu})^2 - \partial_{\mu}A_{\nu}\partial^{\nu}A^{\mu}) + e^2\phi_{cl}^2A_{\mu}A^{\mu} + 2e\phi_{cd}\partial_{\mu}\eta_2A^{\mu} + \eta_1(\partial^2 - m^2 - \lambda\phi_{cd}^2)\eta_1 + \eta_2(\partial^2 - m^2 - \frac{\lambda}{3}\phi_{cd}^2)\eta_2$$ (5)

The next step is to compute the second variational derivative.
1) $\frac{\delta^2S}{\delta\eta_1\delta\eta_2} = \frac{\delta^2S}{\delta\eta_1}\int d^4x\eta_1(x)(\partial_\mu - 2 - m^2 - \lambda\phi_{cd}^2)\eta_1(x) = 2(-\partial^2 - m^2 - \lambda\phi_{cd}^2)$
The minus sign before the partial derivative squared comes from integrating by parts and omitting the surface integral because fields are zero at infinity.
2) $\frac{\delta^2S}{\delta\eta_1\delta\eta_2} = 0 = \frac{\delta^2S}{\delta\eta_1\delta\eta_2}$
3) $\frac{\delta^2S}{\delta\mu\delta\eta_2} = 2(-\partial^2 - m^2 - \frac{\lambda}{3}\phi_{cd}^2)$
4) $\frac{\delta^2S}{\delta\mu\delta\eta_2} = 2e\phi_{cd}\delta\mu\partial_{\mu}A^{\mu} = 0$ because of the Landau gauge.
5) $\frac{\delta^2S}{\delta\mu\delta\eta_2} = 2e^2\phi_{cd}^2g^{\mu\nu}g^{\nu\xi} - \frac{1}{2}\delta\mu\delta\eta_2 A^{\mu} A^{\xi} A^{\nu} = 2e^2\phi_{cd}^2g^{\mu\nu} - \frac{1}{2}\delta\mu\delta\eta_2 A^{\mu} A^{\xi} A^{\nu} = 2e^2\phi_{cd}^2g^{\mu\nu} - \partial_\mu g^{\mu\nu}$

Thus, we get

$$\frac{i}{2}\log\det[-\frac{\delta^2S}{\delta\phi_i\delta\phi_j}] = \frac{i}{2}\log[\det(\partial^2 + m^2 + \lambda\phi_{cd}^2)\det(\partial^2 + m^2 + \frac{\lambda}{3}\phi_{cd}^2)].$$

$$\det(\partial^2 g^{\mu\nu} - 2e^2\phi_{cd}^2 g^{\mu\nu} - \partial_\mu g^{\mu\nu})$$ (6)

The fields $\eta_1$ and $\eta_2$ were rescaled using a Weyl transformation, so factors of 2 were omitted in the first two determinants.

How to deal with second-order tensors? This part of the integral comes from the photon propagator. Let us omit small parameters, and we get: $P^{\mu\nu} = g^{\mu\nu} - \frac{k^{\mu}k^{\nu}}{k^2}$. When we compute the one-loop correction, the following integral appears: $\int [dA]e^{-\frac{1}{2}\int d^4x \eta_i \partial_\mu A_{i\mu}(x)(2e^2\phi_{cd}^2 g^{\mu\nu} + k^2 P^{\mu\nu})A_{i\nu}(y)}$. What is $P^{\mu\nu}$?
\[ P^{\mu\nu}(k) P_{\nu}^{\rho}(k) = g^{\mu\rho} + g^{\mu\nu} \frac{k_{\rho}}{k^2} - \frac{k^{\nu} k^{\rho}}{k^2} g_{\nu}^{\rho} - \frac{k^{\mu} k_{\rho}}{k^2} = P^{\mu\rho}(k). \] Thus, \( P^{\mu\nu} \) is a projection operator, whose eigenvalues can be 0 or 1. Thus, while calculating the volume integral, we will get a constant or the ordinary contribution that includes the mass \( 2e^2 \phi_{cl}^2 \).

Using Eqs. (11.71) and (17.73) [1], the one-loop correction is as follows:

\[
\frac{i}{2} \log \det \left[ \frac{\delta^2 S_1}{\delta \phi_i \delta \phi_j} \right] = \frac{\Gamma(-d/2)}{2(4\pi)^{d/2}} \left( (2e^2 \phi_{cl}^2)^{d/2} + (m^2 + \lambda \phi_{cl}^2)^{d/2} + (m^2 + \frac{\lambda}{3} \phi_{cl}^2)^{d/2} \right) \tag{7}
\]

It is easy to prove that it is renormalized by counter-terms for \( m^2 \) and \( \lambda \):

\[- \delta m \phi_{cl}^2 \text{ and } - \delta \lambda \phi_{cl}^4 \text{ (corrections to bare } m^2 \text{ and } \lambda). \]

In the one-loop correction, we have \( m^2, e^2, \) and \( \lambda \). However, \( e \) is related only to terms with fluctuating fields \( A_{\mu} \) not related to \( \phi \). The term with \( \partial_{\mu} \eta \partial A_{\mu} \) gives no contribution; thus, only the counter-terms for \( m^2 \) and \( \lambda \) contribute.

Using Eq. (11.78) [1], we obtain

\[
\frac{i}{2} \log \det \left[ \frac{\delta^2 S_1}{\delta \phi_i \delta \phi_j} \right] \to \frac{1}{4(4\pi)^2} \left( (m^2 + \lambda \phi_{cl}^2)^2 \left( \log \frac{m^2 + \lambda \phi_{cl}^2}{M^2} - 3/2 \right) +\right.
\]
\[+ \left. (m^2 + \frac{\lambda}{3} \phi_{cl}^2)^2 \left( \log \frac{m^2 + \frac{\lambda}{3} \phi_{cl}^2}{M^2} - 3/2 \right) + 3(2e^2 \phi_{cl}^2)^2 \left( \log \frac{2e^2 \phi_{cl}^2}{M^2} - 3/2 \right) \right) \tag{8}
\]

where \( M \) is the normalization scale.

**Problem (c)**

In the result of part (b), take the limit \( \mu \to 0 \). The result should be an effective potential that is scale-invariant up to logarithms containing \( M \). Analyze this expression for \( \lambda \) very small, of order of \( e^4 \). Show that with this choice of coupling constants, \( V(\phi_{cl}) \) has a symmetry-breaking minimum at a value of \( \phi_{cl} \) for which no logarithm is large, so that a straightforward perturbation theory analysis should be valid. Thus, the \( \mu^2 = 0 \) theory, for this choice of coupling constants, still has spontaneously broken symmetry, due to the influence of quantum corrections.
According to part (b), the effective potential is as follows:

\[ V_{\text{eff}} = -\mu^2 \phi_{\text{cl}}^2 + \frac{\lambda}{6} \phi_{\text{cl}}^4 + \frac{1}{4(4\pi)^2}((-\mu^2 + \lambda \phi_{\text{cl}}^2)^2(\log\frac{-\mu^2 + \lambda \phi_{\text{cl}}^2}{M^2} - 3/2) + \\
(-\mu^2 + \frac{\lambda}{3} \phi_{\text{cl}}^2)(\log\frac{-\mu^2 + \frac{\lambda}{3} \phi_{\text{cl}}^2}{M^2} - 3/2) + 12e^4 \phi_{\text{cl}}^4(\log\frac{2e^2 \phi_{\text{cl}}^2}{M^2} - 3/2)) \quad (9) \]

up to the one-loop correction.

Now, let us take the limit \( \mu^2 \to 0 \)

\[ V_{\text{eff}} \to \frac{\lambda}{6} \phi_{\text{cl}}^4 + \frac{3e^4 \phi_{\text{cl}}^4}{16\pi^2} (\log\frac{2e^2 \phi_{\text{cl}}^2}{M^2} - 3/2). \]

Let us find the potential minimum.

\[ \frac{\partial V_{\text{eff}}}{\partial \phi_{\text{cl}}} = \phi_{\text{cl}}^3 (\frac{2\lambda}{3} + \frac{3e^4}{4\pi^2} (\log\frac{2e^2 \phi_{\text{cl}}^2}{M^2} - 3/2) + \frac{e^4}{4\pi^2}) \]

In addition to en extremum at \( \phi_{\text{cl}} = 0 \), we get the following equation:

\[ \frac{2\lambda}{3} + \frac{3e^4}{4\pi^2} (\log\frac{2e^2 \phi_{\text{cl}}^2}{M^2} - 3/2) + \frac{e^4}{4\pi^2} = 0 \]

\[ \frac{2\lambda}{3} + \frac{e^4}{4\pi^2} (3\log\frac{2e^2 \phi_{\text{cl}}^2}{M^2} - 3.5) = 0 \]

\[ \log\frac{2e^2 \phi_{\text{cl}}^2}{M^2} = -\frac{8\pi^2}{9e^4} + \frac{7}{6} \Rightarrow \phi_{\text{cl}}^2 = \frac{M^2}{2e^2} e^{-\frac{8\pi^2}{9e^4}} - \frac{7}{6} \] other extrema. As \( \Phi_{\text{cl}} \) has the exponential dependence, no logarithm of it will be large. Thus, straightforward perturbation analysis would be valid. Additionally, as \( \Phi_{\text{cl}} \neq 0 \), symmetry is broken at \( \mu^2 = 0 \).

**Problem (d)**

Sketch the behaviour of \( V_{\text{eff}} \) as a function of \( m^2 \), on both sides of \( m^2 = 0 \), for the choice of coupling constants made in part (c).

Let us divide Eq.(9) by \( M^4 \). Also, we will omit terms proportional to \( \lambda^2 \).

We get:

\[ \frac{V_{\text{eff}}}{M^4} = -\frac{\mu^2}{M^2} \phi_{\text{cl}}^2 + \frac{\phi_{\text{cl}}^4}{6M^2} + \frac{1}{4(4\pi)^2}(\lambda(-\frac{\mu^2}{M^2} + \frac{\phi_{\text{cl}}^2}{M^2})^2(\log\frac{-\mu^2 + \lambda \phi_{\text{cl}}^2}{M^2} - 3/2) + \\
\lambda(-\frac{\mu^2}{M^2} + \frac{\phi_{\text{cl}}^2}{3M^2})^2(\log\frac{-\mu^2 + \frac{\lambda}{3} \phi_{\text{cl}}^2}{M^2} - 3/2) + 4\phi_{\text{cl}}^4(\log\frac{2e^2 \phi_{\text{cl}}^2}{M^2} - 3/2)) \]
We will consider the parameter \( \frac{\mu^2}{\lambda M^2} \equiv \mathcal{M}^2 \). Also, \( \frac{\phi^2}{M^2} \equiv \Phi_{cl}^2 \). Thus, we get the following expression:

\[
\frac{V_{eff}}{\lambda M^4} \rightarrow -\mathcal{M}^2 \Phi_{cl}^2 + \frac{\Phi_{cl}^4}{6} + \frac{\Phi_{cl}^4}{16\pi^2} (\log 2 e^2 \Phi_{cl}^2 - 3/2).
\] (10)

To get the right balance between the quadratic and quartic parabolas, we fixed \( e^2 = \frac{10^{-6}}{2} \). Sketching Eq. (10) using the program SciDAVis, we get Figs. 1(a) and 1(b).

![Figure 1: Effective potential for different mass parameters](image)

We see that the symmetry is broken until \( \mathcal{M}^2 = 0 \). When \( \mathcal{M}^2 > 0 \), we get symmetry breaking like from \( \phi^4 \) theory. When \( \mathcal{M}^2 < 0 \), we get 2 local extrema above \( \Phi_{cl} = 0 \), which prevent symmetry breaking; then the quadratic parabola dominates and we get only one local minimum at a zero field.

**Problem (e)**

The Callan-Symanzik \( \beta \) functions are \( \beta_e = \frac{e^3}{48\pi^2} \) and \( \beta_\lambda = \frac{1}{24\pi^2} (5\lambda^2 - 18e^2\lambda + 54e^4) \). Sketch the renormalization group flows in the \( (\lambda, e^2) \) plane. Show that every renormalization group (RG) trajectory passes through the region of the coupling constants considered in part (c).
\[ \beta_e = \frac{de}{dT} \text{ and } \beta_\lambda = \frac{d\lambda}{dT}, \text{ where } T - \text{ is a slowly changing parameter. Thus,} \]
\[ d\lambda = 2de(5\lambda^2 - 18e^2\lambda + 54e^4)/e^3. \]

As \( \lambda \sim e^4 \), we can consider the main contribution: \( \lambda \approx 54e^2 + C \), where \( C \) is constant. We see that the flow of the coupling constant \( \lambda \) approaches the critical point. Thus, we have the following RG flow:

![Figure 2: RG flow in the plane \((e^2, \lambda)\)](image)

As can be seen in Fig. 2, every RG trajectory passes through the region \( \lambda \sim e^4 \).

**Problem (f)**

Construct the RG-improved effective potential at \( \mu^2 = 0 \) by applying the results of part (e) to the calculation of part (c). Compute \( \langle \phi \rangle \).

From (c), \( V_{\text{eff}} \rightarrow \frac{\lambda_{\phi^4}}{6} \phi_{cl} + \frac{3e^4\phi_{cl}^4}{16\pi^2} (\log \frac{2e^2\phi_{cl}^2}{M^2} - 3/2). \) To get the RG-improved potential, we should solve the Callan-Symanzik equation: \( (M \frac{\partial}{\partial M} + \beta_\lambda \frac{\partial}{\partial e} + \gamma_{\phi\phi} \frac{\partial}{\partial \phi_{cl}})V_{\text{eff}}(\phi_{cl}, e, \lambda, M) = 0 \) (Eq. (13.24) [1]). However, in our case, we can construct the RG-improved effective potential without solving it. We know that the result should be independent on the scale parameter \( M \). In addition, the coupling constants should be running. It is very easy to construct such a potential, as the term \( \lambda \phi_{cl}^4/6 \) should be the same. We remember that \( \lambda \sim e^4 \), and it is obvious that the term \( \lambda \phi_{cl}^4 \) should be the
same. Thus, only the term $log(\frac{2e^{2}\phi_{cl}^{2}}{m^{2}})$ should be revised. The equation for $\phi_{cl}$ is as follows: $M\frac{\partial \phi'_{cl}}{\partial M} = -\gamma_{\phi} \phi'_{cl}$, where $\gamma_{\phi}$ counts the number of powers of $\phi_{cl}$ in each term of the Taylor expansion and $\phi'_{cl}$ is the running parameter. Thus, $M$ is linear in $\phi_{cl}$. By omitting irrelevant constants, we get

$$V_{eff} \to \frac{\lambda'}{6} \phi'_{cl}^{4} + \frac{3e'_{4}\phi'_{cl}^{4}}{16\pi^{2}}(log2e'^{2} - 3/2), \quad (11)$$

where $e'$, $\phi'_{cl}$, and $\lambda'$ are the running parameters.

From here, we redefine $e' = e$, $\lambda' = \lambda$, and $\phi'_{cl} = \phi_{cl}$.

Let us calculate $\langle \phi \rangle$. The procedure is the same as in (c): Let us find the potential minimum: $\frac{\partial V_{eff}}{\partial \phi_{cl}} = \frac{2\lambda^{3}}{3} + \frac{3e_{4}^{4}}{4\pi^{2}}(log2e'^{2} - 3/2) \phi_{cl}^{3} = 0 \Rightarrow \langle \phi_{cl} \rangle = 0$.

However, the full derivative of the effective potential with respect to the field $\phi_{cl}$ includes the partial derivates over $\lambda$ and $e$ because the coupling constants are running now and depend on the chosen normalization scale, which is chosen to be linear with the field. Thus, symmetry breaking still occurs.
Bibliography
