This is the accepted version of a paper presented at *XIIIth International conference on Electrical Machines, September 3-6 2018, Alexandroupoli, Greece*.

Citation for the original published paper:

Winding Design Independent Calculation Method for Short Circuit Currents in Permanent Magnet Synchronous Machines  
In: *2018 XIII International Conference on Electrical Machines (ICEM)* (pp. 1021-1027).  
https://doi.org/10.1109/ICELMACH.2018.8506920

N.B. When citing this work, cite the original published paper.

Permanent link to this version:  
http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-363290
Winding Design Independent Calculation Method for Short Circuit Currents in Permanent Magnet Synchronous Machines

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Abstract—When designing permanent magnet (PM) synchronous machines the demagnetizing effect of armature short circuit currents on the PMs needs to be considered. In some cases there can be a need to estimate the demagnetizing field from the winding without knowing the winding scheme. To do this, a lumped parameter model of the dynamics of the magnetic field and armature current distribution is proposed. Validation of the model using two different machines shows acceptable agreement. The proposed model is found to be useful for its particular purpose of determining the approximate short circuit current distribution in the armature without knowing the winding design.

Index Terms—Lumped Parameter Model, Machine modeling and general design, Permanent Magnet Machines, Rotating Electrical Machines, Short Circuit Current Distribution, Winding Scheme

I. INTRODUCTION

When designing a permanent magnet (PM) machine short circuit currents in the armature needs to be considered as they have a significant risk of demagnetizing the PMs. To be able to determine these is therefore a necessary part of PM machine design [1]–[4]. This is usually done by either transient finite element (FE) simulation coupled with a circuit model, as in [5], or some combination of static FE calculation for parameter estimation and a parametric transient model. In [6] it is discussed how the theory commonly used for short circuit calculations in wound rotors can be adapted for use with PM machines. With some approximations there exists analytical solutions for the transient short circuit currents, as discussed in [7]. In [8] an analytical expression for the steady state short circuit current taking magnetic saturation into account is presented. All of these methods depend on either the winding scheme is known so that resistance and inductance can be calculated, or on measurements of the winding inductance and resistance. Depending on the design procedure there might be a need to get a reasonable estimate of the short circuit current distribution before the winding scheme has been determined. Also, it can be beneficial to be able to compare different rotor topologies, as done in [2], and different PM materials without putting too much consideration into the electrical design of the stator.

In this paper, an approach not depending on the winding scheme but only on the space occupied by the windings and their resistivity is developed. Fields of application for such a model include the study and comparison of different PM rotor topologies, early stage rotor design, and design of the magnetic circuit of machines with a large or varying number of phases.

The approach adopted here is to modify Park’s classic theory [9] by computing the lumped parameters directly from the fields in the machine instead of calculating phase quantities and then transforming them. The new method is then verified by comparing its predictions of a symmetric short circuit to the result of transient FE simulations, coupled with circuit models, of two different machines.

II. THE CIRCUIT PARAMETER MODEL

The commonly used circuit parameter model has the form

\[ v_d = R_s i_d + \frac{d\psi_d}{dt} - \omega_{el} \psi_q \]  

(1)

\[ v_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_{el} \psi_d, \]

(2)

where \( v \) is a terminal voltage, \( i \) a current, \( \psi \) a flux linkage, \( R_s \) the resistance in one phase of the stator winding, \( \omega_{el} \) the electrical angular frequency, and \( d \) and \( q \) subscripts denote direct and quadrature axis, respectively. This model can be found in many places, such as [10], and is used in various application e.g. [11], but differs from [9] in notation and that motor reference is used instead of generator reference.

To be able to use this model structure without knowing the number of turns or number of phases of the winding, winding scheme independent lumped parameters need to be defined. In the next section it is shown how this can be done with only knowledge of the space available for the windings, and the resistivity of the winding. The former is given by allowed stator size and allowed magnetic flux density. The latter is typically that of copper at the machine operating temperature with some correction for the slot area fraction not occupied by copper.
where $\hat{J}_{z,d}$ is the peak current density on the direct axis, and similarly for the quadrature axis

$$ J_z = -\hat{J}_{z,q} \sin \theta_{el} $$

where $\hat{J}_{z,q}$ is the peak current density on the quadrature axis. Inserting (5) into (3) and similarly (6) into (4) allows us to relate $\hat{J}_{z,d}$ and $F_d$, and $\hat{J}_{z,q}$ and $F_q$, respectively, as

$$ F_d = \hat{J}_{z,d} A_{eff} $$

and

$$ F_q = -\hat{J}_{z,q} A_{eff} $$

where

$$ A_{eff} = \int_S \cos^2 \theta_{el} \; dS $$

is an effective winding cross section.

The magnetic flux through an arbitrary non-closed surface can, by Stokes Theorem and the definition of the magnetic vector potential $\vec{A}$, be computed as

$$ \phi = \oint_C \vec{A} \cdot d\vec{l} $$

where $C$ is the right-hand oriented contour bounding the surface and $d\vec{l}$ is a directed segment of infinitesimal length. In the 2D case (10) simplifies since only the $z$-component of $\vec{A}$ is non-zero. The magnetic flux per unit length can then be computed as

$$ \Phi = A_z(\vec{r}_1) - A_z(\vec{r}_2) $$

where $A_z$ is the $z$ component of $\vec{A}$, and $\vec{r}$ the position vector in points 1 and 2.

To compute the flux linking to a winding in 2D one can integrate $A_z$ over the entire plane weighted by the current density distribution resulting from a unit current flowing in the winding. In terms of the above definitions this gives the direct axis linked magnetic flux as

$$ \Phi_d = \int_S A_z \frac{J_{z,d} \cos \theta_{el}}{\hat{J}_{z,d} A_{eff}} \; dS = \frac{1}{A_{eff}} \int_S A_z \cos \theta_{el} \; dS $$

and

$$ \Phi_q = \int_S A_z \frac{-\hat{J}_{z,q} \sin \theta_{el}}{\hat{J}_{z,q} A_{eff}} \; dS = -\frac{1}{A_{eff}} \int_S A_z \sin \theta_{el} \; dS $$

for the the quadrature axis linked flux. The electric field on each winding axis can be similarly integrated to obtain

$$ V_d = \frac{1}{A_{eff}} \int S E_z \cos \theta_{el} \; dS $$
for the direct axis, where \( E_z \) is the out of plane component of the electric field, and

\[
V_q = -\frac{1}{A_{eff}} \int_S E_z \sin \theta_d \, dS \tag{15}
\]

for the quadrature axis. The quantities \( V_d \) and \( V_q \) represent terminal voltage per turn and unit length.

When current flows in a conductor with non-zero resistivity, \( \rho \), there must be an electric field driving the flow of charge according to Ohm’s law. This electric field is given by

\[
E_z = \rho J_z \tag{16}
\]

which can be integrated over the winding domain to give \( V_d \) and \( V_q \). Dividing \( V_d \) and \( V_q \) by the MMF causing it (\( F_d \) and \( F_q \), respectively), gives the resistance of the winding as seen from the magnetic circuit, \( \varrho_s \). Using the direct axis, i.e. \( V_d \) and \( F_d \), one can obtain

\[
\varrho_s = \frac{\int \rho J_{z,d} \cos \theta_d \cos \theta_d \, dS}{J_{z,d} A_{eff}} = \frac{\rho}{A_{eff}} \tag{17}
\]

and the resulting \( \varrho_s \) becomes the same if the quadrature axis is used.

IV. SHORT CIRCUIT CURRENT DISTRIBUTION
CALCULATION METHOD

By replacing the quantities in the original model with their lumped field parameter equivalents the following system of differential equations can be obtained

\[
V_d = \varrho_s F_d + \frac{d\Phi_d}{dt} - \omega_d \Phi_q \tag{18}
\]

\[
V_q = \varrho_s F_q + \frac{d\Phi_q}{dt} + \omega_d \Phi_d \tag{19}
\]

which are not dependent on the winding scheme. A diagram of the relationship between the parameters is shown in Fig. 2.

The simplest short circuit scenario to model using (18) and (19) is the symmetrical which corresponds to \( V_d = V_q = 0 \). The equations can be linearized by the introduction of permeances \( \Lambda_d \) and \( \Lambda_q \) and a constant magnetic flux due to the PMs, \( \Phi_{PM} \), such that

\[
\Phi_d = \Lambda_d F_d + \Phi_{PM} \tag{20}
\]

\[
\Phi_q = \Lambda_q F_q \tag{21}
\]

which can be estimated from two static FE solutions of the magnetic field, one with \( F_d = F_q = 0 \), and one with \( F_d \) and \( F_q \) in the expected order of magnitude of their values during a short circuit. The parameter estimation is further described in Section IV-A.

Substituting (20) and (21) into (18) and (19) and rearranging to separate the time derivatives give

\[
\Lambda_d \frac{dF_d}{dt} = -\varrho_s F_d + \omega_d \Lambda_q F_q \tag{22}
\]

\[
\Lambda_q \frac{dF_q}{dt} = -\omega_d \Lambda_d F_d - \varrho_s F_q - \omega_d \Phi_{PM} \tag{23}
\]

which by introduction of

\[
\mathbf{F} = \begin{pmatrix} F_d \\ F_q \end{pmatrix}, \tag{24}
\]

\[
\mathbf{\Omega} = \begin{pmatrix} -\varrho_s & \frac{\omega_d \Lambda_q}{\Lambda_d} \\ \frac{\omega_d \Lambda_d}{\Lambda_q} & -\varrho_s \end{pmatrix} \tag{25}
\]

and

\[
\Phi_{PM} = \begin{pmatrix} 0 \\ \frac{\Phi_{PM}}{\Lambda_q} \end{pmatrix} \tag{26}
\]

can be written as a matrix differential equation

\[
\frac{d\mathbf{F}}{dt} = \mathbf{\Omega} \mathbf{F} + \omega_d \mathbf{\Phi}_{PM} \tag{27}
\]

that has a closed form solution if \( \omega_d \) is assumed to be constant. If \( \mathbf{F}_0 = \mathbf{F}(t = 0) \) is the column vector of the MMFs preceding the short circuit, the solution has the form

\[
\mathbf{F}(t) = \exp \left( \mathbf{\Omega} t \right) \left[ \mathbf{F}_0 - \mathbf{F}_{ss} \right] + \mathbf{F}_{ss} \tag{28}
\]

where \( \exp \) is the matrix exponential and \( \mathbf{F}_{ss} = \mathbf{\Omega}^{-1} \omega_d \mathbf{\Phi}_{PM} \) is the steady state solution. While the matrix form is not necessary to solve a system of differential equations of this form, as demonstrated in [7], [9], it makes the solution more compact.

A. Parameter Estimation

To estimate \( \Lambda_d \) and \( \Lambda_q \), two static field solutions are needed, one where \( F_d = F_q = 0 \); quantities from this are denoted by superscript NC (for no current) and one where they are large, denoted superscript HC (for high current). The field solutions can be obtained by FE or some other calculation method where the out of plane component of the vector potential is computed. To perform such calculations a
cross sectional geometry of the machine, a suitable material model for the iron parts of the machine, and an effective resistivity of the winding domain $S$ are needed. The geometry of the stator can be simplified and either have explicit slots and teeth, or have the both represented by an annulus where the magnetic properties of the iron and nonmagnetic material are weighted together by the fractions of space occupied by each. In the first case the slots would constitute $S_s$, in the latter case the entire annulus would constitute $S$. The effective resistivity should be that of the conductor material at operating temperature divided by the area fraction of $S$ occupied by the conductors. For the current purpose large values of $F_{cd}$ and $F_{cq}$ can be considered to be of the same order of magnitude as the calculated short circuit current densities. Estimation with too low $F_{cd}$ and $F_{cq}$ tends to overestimate the maximum magnitude of the short circuit current distribution. Once the field has been computed in the two cases, the permeances are obtained as

$$\Lambda_d = \frac{\Phi_{d}^{HC} - \Phi_{d}^{NC}}{F_{cd}^{HC}}$$

and

$$\Lambda_q = \frac{\Phi_{q}^{HC}}{F_{cq}^{HC}}$$

and the flux from the PMs is $\Phi_{PM} = \Phi_{q}^{NC}$. 

V. VALIDATION

The new model has been verified against transient FE simulations of two different machines. One is a 32-pole experimental wind power generator with a spoke type, ferrite PM rotor that is described in [12]. The other is a rough two-pole machine design with a buried PM rotor using neodymium-iron-boron PMs, similar to the one described in [13]. The winding resistance is in both cases only considering the resistance in the coil sides and calculated using a constant resistivity, i.e. neglecting the impact of thermal effects and skin effect on the resistivity. The short-circuit simulated is a zero impedance symmetrical short-circuit at constant rotational speed. For the 32-pole machine the short circuit happens at no-load conditions. For the two-pole machine the short circuit is simulated both occurring at no-load, and at resistive load corresponding to rated current.

The geometries of the simulated machines are shown in Fig. 3 and 4, and some machine parameters are listed in Table I. Both machines are simulated in the Rotating Machinery, Magnetic module of COMSOL Multiphysics®, with coupled circuit models. The conductors are approximated to have the same current density through the whole conductor turn, the conductor resistance is handled in the circuit model, and eddy currents are neglected. On the curved outer boundaries of the geometries the normal component of the magnetic flux density is set to zero. In the air gap a sliding mesh boundary is used to avoid re-meshing when rotating the rotor. In the 32-pole machine, depicted in Fig. 3, periodic boundary conditions are used on the straight outer boundaries as indicated. The periodicity is also used on the sliding mesh boundary to connect the non-overlapping parts as if there was a repeated copy of the simulation geometry rotated to mate with the non-overlapping part.

Parameters are estimated in a two step process where a first estimation from a FE solution is made in generator mode for rated resistive load. This is then used in the lumped parameter to calculate the $F_{cd}^{HC}$ and $F_{cq}^{HC}$ to be used in a second FE solution of the field, from which the final values
Fig. 5. Comparison of $F_{\text{HC}}$ adjusted to match the transient FE case for easier comparison.

of $F_{\text{HC}}$ and $F_{\text{HC}}$ for parameter estimation.

Comparisons of $F$ computed with the lumped parameter model and transient FE for the three cases are plotted in Fig. 5, 6, and 7. The agreement is reasonable. The maximum error for the lowest value of $F_d$ is 12.5% and happens for the 32-pole machine. The maximum error in maximum magnitude of $F_q$ is 25% and happens for the two-pole when starting the short circuit from no-load conditions.

Fig. 8 shows the field lines in the two-pole machine at minimum $F_d$ for armature currents computed by the lumped parameter model and by the transient FE with a coupled circuit model. The lumped parameter case is obtained by calculating the minimum $F_d$ using the lumped parameter model and inserting the resulting stator current distribution into a static FE model. The transient FE case is taken directly from the FE solver at the instant of minimum $F_d$. Both set of field lines are computed with 31 equally spaced level curves of $A_z$ and the rotor angle in the lumped parameter case is adjusted to match the transient FE case for easier comparison. The same mesh is used for both simulations.

VI. DISCUSSION

A few assumptions have been made when deriving the lumped parameter model. The most prominent among these is the 2D approximation and thereby neglecting the end effects, as is commonly done. While some extra end reactance and end resistance terms to correct for the end effects could be added, determining the terms is not straightforward. This is both because of the general problem of determining the flux in the end regions of the machine due to their inherently three-dimensional (3D) nature but also because of the matter of extending the domain $S$ into 3D in a consistent way. In 2D the out of plane component is the trivial current direction, and inside the stator core this is also the case in 3D, but in the coil ends this does not hold. It could possibly be solved by modeling the coil ends as a ring of conductive material and introducing a vector field for positive direction of the direct or quadrature axis current. A useful definition of such
a field is, however, beyond the scope of the current paper.

As mentioned in the introduction the developed method is mainly useful when the winding scheme is not known, has yet to be determined, or is of minor importance. This can be the case when evaluating or comparing different rotor topologies or PM materials, or in early stages of magnetic circuit design.

The two machines used in the validation are quite different both in terms of magnetic and electric circuit. The two-pole machine is a rather low-voltage–high-current design with very large conductor cross section. The 32-pole machine has very low current both measured as current density in the conductors and in linear current density on the stator.

Neither is a well-optimized machine. The two-pole machine is a quick design only intended to be used for validation of the model. The 32-pole machine is intended as a research prototype where for instance an air gap large enough to easily perform measurements, was a design requirement. They should, however, both serve to validate the lumped parameter model.

The validation shows some discrepancy between the transient FE and the lumped parameter model. Considering that the latter does not model saturation at all, the agreement is acceptable. The only measure taken to take saturation into account is to attempt to get a representative level of saturation when estimating the linearizion. The discrepancies should not pose a problem when using the model for demagnetization estimation since the model tends to overestimate the extreme values of the currents, which would give a conservative estimate when considering the demagnetization of the PMs. Also, the magnitude of the extreme currents is more important than the timing for demagnetization calculations. In practice a factor of safety should be used anyway to account for model inaccuracies both in the short circuit current distribution and the PM material model.

VII. CONCLUSIONS

A commonly used circuit model of a synchronous PM machine is reworked into a lumped parameter model of the magnetic circuit of a synchronous PM machine where integrated field quantities are used. This reworked model allows short circuit currents to be predicted without access to the winding scheme used. Validation in a total of three cases, using two different machines shows reasonable agreement. The proposed model serves its purpose of allowing the short circuit current distribution in an armature winding to be estimated without knowledge of the winding design.

REFERENCES


VIII. BIOGRAPHIES

Petter Eklund was born in Uppsala, Sweden in 1989. He received the M.Sc degree in energy systems engineering from Uppsala University in 2013 and is currently a Ph.D student at the Division of Electricity, Department of Engineering Sciences, of Uppsala University. His main research interest is design of low speed permanent magnet generators that does not use rare-earth permanent magnets.

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