Physics Letters B 788 (2019) 535–541

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Constraining P and CP violation in the main decay of the neutral Sigma hyperon

Shankar Sunil Nair, Elisabetta Perotti, Stefan Leupold *

Institutionen för fysik och astronomi, Uppsala Universitet, Box 516, S-75120 Uppsala, Sweden

ARTICLE INFO

Article history:
Received 13 February 2018
Received in revised form 30 August 2018
Accepted 9 September 2018
Available online 28 November 2018
Editor: V. Metag

Keywords:
Radiative decays of hyperons
Electric dipole moment
CP violation

ABSTRACT

On general grounds based on quantum field theory the decay amplitude for $\Sigma^0 \to \Lambda\gamma$ consists of a parity conserving magnetic and a parity violating electric dipole transition moment. Because of the subsequent self-analyzing weak decay of the Λ hyperon the interference between magnetic and electric dipole transition moment leads to an asymmetry in the angular distribution. Comparing the decay distributions for the Σ^0 hyperon and its antiparticle gives access to possible C and CP violation. Based on flavor SU(3) symmetry the present upper limit on the neutron electric dipole moment can be translated to an upper limit for the angular asymmetry. It turns out to be far below any experimental resolution that one can expect in the foreseeable future. Thus any true observation of a CP violating angular asymmetry would constitute physics beyond the standard model, even if extended by a CP violating QCD theta-vacuum-angle term.

1. Introduction

There is much more matter than antimatter in the universe. If this is not just by chance but has a dynamical origin, then an explanation of this baryon asymmetry should come from the realm of particle physics. Based on Sakharov's conditions [1] this has spurred the search for baryon decays and reactions that show CP violation [2]. (Here C denotes charge conjugation symmetry and P parity symmetry.) Two directions of such searches are weak decays of baryons [3–5] and electric dipole moments (EDMs) [6,7].

In the present work we propose to study a reaction that is in between these two types of reactions, the decay of the neutral ground-state Sigma hyperon to a photon and a Lambda hyperon, $\Sigma^0 \to \gamma\Lambda$. This electromagnetic baryon decay could in principle show an interference between a parity conserving and a parity violating amplitude. The latter would come from an electric dipole transition moment (the former from a magnetic transition moment). Note that the Λ hyperon decays further into pion and proton on account of the weak interaction [8]. The possible interference in the first decay can then be observed as an angular asymmetry in the decay products of the, in total, three-body decay $\Sigma^0 \to \gamma\pi^- p$. Comparing the asymmetry parameters for the particle decay, $\Sigma^0 \to \gamma\Lambda$, and for the corresponding antiparticle decay, $\bar{\Sigma}^0 \to \gamma\bar{\Lambda}$, one can search for C and CP violation.

Let us add right away that the conservative expectation is that one would not find an effect of P, C or CP violation in this decay. This will be substantiated by our explicit estimates given below. Yet, if theory predicts that something is very small, then it might be worth checking this experimentally. Even if one "only" establishes an upper limit, this can help to constrain beyond-standard-model developments. Needless to add that if one found an effect not predicted by established theory, then this would be sensational.

Actually already in 1962 the interplay of magnetic and electric dipole transition moment for the Σ^0 decay has been addressed in [9], though under the implicit assumption of CP conservation. In the present work we will be more general. In addition, to the best of our knowledge, the search for such interference effects, albeit suggested so early, has never been conducted. With recent and ongoing experiments on hyperon production (e.g. at ELSA [10], J-LAB [11,12], GSI [13,14], BEPC II [15,16], KEKB [17], CESR [18]) and in particular with the upcoming PANDA experiment [19,20] as a hyperon–antihyperon factory, it is absolutely timely to establish at least upper limits for the angular asymmetry of the Σ^0 decay.

The process $\Sigma^0 \to \gamma\Lambda$ constitutes the main decay of the ground state Σ^0. Its lifetime is governed by this decay [8]. Therefore the Σ^0 lives much longer than hadronic resonances that decay

* Corresponding author.
E-mail address: stefan.leupold@physics.uu.se (S. Leupold).

https://doi.org/10.1016/j.physletb.2018.09.065

on account of the strong interaction, but much shorter than weakly decaying particles. This makes the analysis of Σ^0 decays challenging, as already pointed out in [9]. Hopefully these problems can be diminished by the increasing production rate of Σ^0 particles and detector quality.

Let us relate and compare the process $\Sigma^0 \to \gamma\Lambda$ in more detail to the mentioned searches for CP violation in weak baryon decays and for EDMs. In our case there is an interference between two amplitudes, a large and a small one. The first one is related to the magnetic transition moment. This amplitude is parity conserving and compatible with Quantum Electrodynamics (QED) and Quantum Chromodynamics (QCD). The second one is related to the electric dipole transition moment and is parity violating. Thus one expects small effects right away. This is in contrast to similar radiative decays where both amplitudes are caused by the weak interaction, for instance $\Xi^0 \to \Lambda\gamma$ [21]; see also the note on "Radiative Hyperon Decays" from the Particle Data Group [8]. For the weak radiative decays the angular asymmetry is appreciably large, but the difference between particle and antiparticle decay is in most cases so far unmeasurably small. Only recently first evidence for a CP violating decay of the Λ_b baryon has been reported [5]. For our case we expect very small effects for the angular asymmetry and for the particle–antiparticle differences.

Though nature violates all discrete symmetries P, C and CP,[1] this happens to different degrees and different interactions behave differently. Therefore in an analysis of the decay $\Sigma^0 \to \gamma\Lambda$ it is useful to distinguish conceptually between a scenario of C violation but CP conservation and a scenario of C conservation but CP violation. We will present observables to test both scenarios for the process of interest. Yet for our concrete estimates we will focus on strong CP violation.

From an experimental point of view, the strong interaction conserves P, C and CP separately. Yet from the theory side it would be natural to expect CP violation in QCD based on the non-trivial topological structure of the non-abelian gauge theory [23]. Such a "theta-vacuum-angle term" is C conserving but P and CP violating. In particular, it gives rise to an EDM of the neutron. So far, no EDM of the neutron has been experimentally established [24], but the very small upper limit raises the question of why CP violation in the strong sector is so unnaturally small (the "strong-CP problem") [25]. Note that the weak CP violation leads to an EDM of the neutron that is many orders of magnitude below the experimental upper limit [6]. Therefore we focus on possible strong CP violation in the following. As we will show below, our decay $\Sigma^0 \to \gamma\Lambda$ is related to the neutron EDM via the approximate SU(3) flavor symmetry [26]. In the framework of chiral perturbation theory, one can take care of the explicit symmetry breaking in a systematic way [27–29]. Based on the experimental upper limit of the neutron EDM we will provide an upper limit for the CP violating effect on the Sigma decay chain.

The rest of the paper is structured in the following way: In the next section we will present a general parametrization of the transition moments for radiative decay amplitudes of baryons, discuss the impact of C and CP symmetry/violation and relate all this to the angular asymmetries of baryon and antibaryon decays. In section 3 we will calculate all relevant decay widths and angular distributions. In section 4 we will determine an upper limit for those effects caused by strong CP violation, which is in turn related to the QCD theta-vacuum-angle term. Here baryon chiral perturbation theory [27–29] can be used to relate the neutron EDM to the Σ^0 decay. Finally a summary and an outlook will be provided in section 5.

2. Transition moments and angular decay asymmetries

For the coupling of baryons to the electromagnetic current J_μ, we follow in principle [28] but adopt the definition of the photon momentum to our decay process:

$$\langle B'(p')|J^\mu|B(p)\rangle = e\,\bar{u}_{B'}(p')\,\Gamma^\mu(q)\,u_B(p) \tag{1}$$

with $q := p - p'$ and

$$\begin{aligned}\Gamma^\mu(q) = &\left(\gamma^\mu + \frac{m_{B'} - m_B}{q^2}\,q^\mu\right) F_1(q^2)\\ &+ i\left(\gamma^\mu q^2 + (m_B + m_{B'})\,q^\mu\right)\gamma_5\,F_A(q^2)\\ &- \frac{i}{m_B + m_{B'}}\,\sigma^{\mu\nu}q_\nu\,F_2(q^2)\\ &- \frac{1}{m_B + m_{B'}}\,\sigma^{\mu\nu}q_\nu\gamma_5\,F_3(q^2)\,.\end{aligned} \tag{2}$$

If B and B' have the same intrinsic parity then the functions $F_1(q^2)$ and $F_2(q^2)$ are the P conserving Dirac and Pauli transition form factors. $F_A(q^2)$ and $F_3(q^2)$ are the P violating Lorentz invariant transition form factors and are termed the anapole form factor and the electric dipole form factor, respectively. We note in passing that ideas how to access the q^2 dependence of F_1 and F_2 for the transition of Σ^0 to Λ have been presented in [30].

For the neutron, the Pauli form factor at the photon point is related to the anomalous magnetic moment by [8]

$$F_{2,n}(0) = \kappa_n \approx -1.91 \tag{3}$$

while the EDM of the neutron is given by

$$d_n = \frac{e}{2m_n}\,F_{3,n}(0)\,. \tag{4}$$

The decay $\Sigma^0 \to \gamma\Lambda$ is only sensitive to the magnetic (dipole) transition moment [8,30]

$$\kappa_M := F_{2,\Sigma\Lambda}(0) \approx 1.98 \tag{5}$$

and the electric dipole transition moment (EDTM)

$$d_{\Sigma\Lambda} := \frac{e}{m_{\Sigma^0} + m_\Lambda}\,F_{3,\Sigma\Lambda}(0)\,. \tag{6}$$

The (transition) charge must vanish, $F_{1,\Sigma\Lambda}(0) = 0$, and the anapole moment $F_{A,\Sigma\Lambda}$ does not contribute for real photons, technically based on $q^2 = 0$ and $q^\mu\epsilon_\mu = 0$ where ϵ_μ denotes the polarization vector of the photon. In section 4 we will relate the electromagnetic properties of the neutron and of the Σ^0–Λ transition.

Following [8] the successive decay $\Lambda \to \pi^- p$ is parametrized by the matrix element

$$\mathcal{M}_2 = \bar{u}_p(\mathcal{A} - \mathcal{B}\gamma_5)\,u_\Lambda \tag{7}$$

where $\mathcal{A}$ and $\mathcal{B}$ are complex numbers. To stay in close analogy to this parametrization we write the decay matrix element for the first decay $\Sigma^0 \to \gamma\Lambda$ as

$$\mathcal{M}_1 = \bar{u}_\Lambda\,(a\,\sigma_{\mu\nu} - b\,\sigma_{\mu\nu}\gamma_5)\,u_{\Sigma^0}\,(-i)q^\nu\epsilon^{\mu*}\,. \tag{8}$$

The two decay parameters a and b are related to the transition moments via

$$a = \frac{e}{m_{\Sigma^0} + m_\Lambda}\,\kappa_M\,, \qquad b = i\,d_{\Sigma\Lambda}\,. \tag{9}$$

[1] In quantum field theory the third discrete symmetry, time reversal symmetry T, is intimately tied to CP via the CPT theorem [22]. In our decay, time reversal arguments are not directly applicable because we do not study the inverse formation process $\Lambda\gamma \to \Sigma^0$. Therefore we focus on CP instead of T.

The decay asymmetries that will finally show up in the angular distribution of the three-body decay $\Sigma^0 \to \gamma\pi^- p$ are defined by (see, e.g., [8])

$$\alpha_{\Sigma^0} := \frac{2\mathrm{Re}(a^* b)}{|a|^2 + |b|^2} \tag{10}$$

and

$$\alpha_\Lambda := \frac{2\mathrm{Re}(s^* p)}{|s|^2 + |p|^2}, \tag{11}$$

with $s := \mathcal{A}$ and $p := \eta\mathcal{B}$. We have introduced $\eta := |\vec{p}_p|/(m_p + E_p)$ to compensate for the p-wave phase space relative to the s-wave. Here m_p denotes the mass of the proton and E_p ($\vec{p}_p$) its energy (three-momentum) in the rest frame of the Λ decaying into pion and proton.

To reveal C and/or CP violation for our processes one has to compare particle and antiparticle decays. To this end we introduce the following matrix elements that correspond to (7) and (8):

$$\mathcal{M}_{\bar{2}} = -\bar{v}_\Lambda(\bar{\mathcal{A}} - \bar{\mathcal{B}}\gamma_5)\, v_p\,, \tag{12}$$

$$\mathcal{M}_{\bar{1}} = -\bar{v}_{\Sigma^0}\,(\bar{a}\,\sigma_{\mu\nu} - \bar{b}\,\sigma_{\mu\nu}\gamma_5)\, v_\Lambda\,(-i)q^\nu \epsilon^{\mu *}\,. \tag{13}$$

In analogy to (10) and (11) we also introduce the asymmetries

$$\alpha_{\bar{\Sigma}^0} := \frac{2\mathrm{Re}(\bar{a}^* \bar{b})}{|\bar{a}|^2 + |\bar{b}|^2} \tag{14}$$

and

$$\alpha_{\bar{\Lambda}} := \frac{2\mathrm{Re}(\bar{s}^* \bar{p})}{|\bar{s}|^2 + |\bar{p}|^2}, \tag{15}$$

with $\bar{s} := \bar{\mathcal{A}}$ and $\bar{p} := \eta\bar{\mathcal{B}}$.

C and CP conservation/violation influence the phases of the parameters a and b. In general, the products of a decay show some final-state interaction that leads to an additional phase; see, e.g., [2] and the note on "Baryon Decay Parameters" in [8]. It is useful to distinguish these two effects. To simplify things we note, however, that overall phases can be freely chosen in quantum mechanics. Only phase differences matter. Consequently we decompose

$$b = b_W\, e^{i\delta_F}\,, \qquad a = a_W \tag{16}$$

with the parameters from the direct decay without final-state interactions labeled by an index W. The phase difference δ_F between a and b encodes the final-state interaction. For this final-state interaction between Λ and γ we assume that it is dictated by (P and C conserving) QED, which implies that it is the same for Λ–γ and $\bar{\Lambda}$–γ. In Appendix A we demonstrate that the parity conserving contribution a to the decay amplitude can be chosen real and positive for the particle and the antiparticle decay. CP conservation would imply that the parity violating contribution b_W is real, while C conservation would imply that b_W is purely imaginary; see Appendix A for details.

Since for our parameter α_{Σ^0} the relative size between a and b will matter, it might be illuminating to provide the corresponding information for α_Λ and its building blocks s and p (or $\mathcal{A}$ and $\mathcal{B}$) and also for the antiparticle amplitudes. The overall strength of $\mathcal{A}$ and $\mathcal{B}$ enters the decay width of the Λ, see (22) and (24) below. Here we concentrate on the relative strength and present the ratio $r := |p/s|$ and the relative phase δ_Λ between s and p, i.e. $p/s =: re^{i\delta_\Lambda}$. The Particle Data Group provides the following information [8]:

$$\frac{2r\cos\delta_\Lambda}{1+r^2} = \alpha_\Lambda = 0.642 \pm 0.013 \tag{17}$$

and

$$\frac{2r\sin\delta_\Lambda}{1-r^2} = \tan\phi_\Lambda = \tan[(-6.5 \pm 3.5)^\circ] \tag{18}$$

This yields (we do not go through the exercise of carrying along the (correlated) errors of α_Λ and ϕ_Λ)

$$r \approx 0.37\,, \qquad \delta_\Lambda \approx -0.13\,. \tag{19}$$

For the antiparticle amplitudes $\bar{s}$ and $\bar{p}$ one must obtain the same ratio r as for the particle case: $r = |\bar{p}/\bar{s}|$. But the relative phase $\delta_{\bar{\Lambda}}$ between $\bar{s}$ and $\bar{p}$ will be different from δ_Λ. The decays $\Lambda \to \pi^- p$ and $\bar{\Lambda} \to \pi^+ \bar{p}$ are governed by the weak interaction [31]. Here the CP violation is very small. The pion–baryon system shows a strong final-state interaction. Therefore the phase between the amplitudes $\bar{s}$ and $\bar{p}$ is not small. If one ignores CP violation one obtains $\delta_\Lambda = \delta_{\bar{\Lambda}} + \pi$, which leads to $\alpha_\Lambda = -\alpha_{\bar{\Lambda}}$. So far the experimental results are compatible with this assumption of (approximate) CP conservation. According to [8] the current experimental status is

$$\alpha_{\bar{\Lambda}} = -0.71 \pm 0.08\,, \tag{20}$$

which should be compared with (17).

3. Integrated and differential decay widths and angular distributions

The matrix element (8) gives rise to the decay width

$$\Gamma_{\Sigma^0 \to \gamma\Lambda} = (|a|^2 + |b|^2)\,\frac{(m_{\Sigma^0}^2 - m_\Lambda^2)^3}{8\pi m_{\Sigma^0}^3}\,. \tag{21}$$

For later use we also provide the partial decay width of the Λ hyperon:

$$\Gamma_{\Lambda \to \pi^- p} = \frac{\lambda^{1/2}(m_\Lambda^2, m_p^2, m_\pi^2)}{16\pi m_\Lambda^3}\, R_\Lambda \tag{22}$$

with the Källén function

$$\lambda(a,b,c) := a^2 + b^2 + c^2 - 2(ab + bc + ac) \tag{23}$$

and [32]

$$R_\Lambda := |\mathcal{A}|^2((m_\Lambda + m_p)^2 - m_\pi^2) + |\mathcal{B}|^2((m_\Lambda - m_p)^2 - m_\pi^2)\,. \tag{24}$$

In general, the corresponding decay widths for the antiparticle decays are obtained by replacing the parameters a, b, $\mathcal{A}$, $\mathcal{B}$ by the corresponding parameters for the antiparticles.

For the full three-body decay $\Sigma^0 \to \gamma p\pi^-$, shown on the left-hand side of Fig. 1, we start with the double-differential decay width [8]

$$\frac{d^2\Gamma_{\Sigma^0 \to \gamma p\pi^-}}{dm_{23}^2\, dm_{12}^2} = \frac{1}{(2\pi)^3}\,\frac{1}{32\, m_{\Sigma^0}^3}\,\left\langle |\mathcal{M}_3|^2 \right\rangle, \tag{25}$$

where we have introduced $m_{23}^2 := (p_\gamma + p_p)^2$ and $m_{12}^2 := (p_p + p_\pi)^2$. Our decay proceeds via the very narrow Λ hyperon as an intermediate state. Consequently we will integrate over m_{12}^2 and focus on a single-differential decay width. In addition, we rewrite the m_{23}^2 dependence into a dependence on the angle θ between proton and photon defined in the rest frame of Λ. Note that in this frame we have $\vec{p}_{\Sigma^0} = \vec{p}_\gamma$ and $\vec{p}_p = -\vec{p}_\pi$ which defines a plane. This plane is depicted on the right-hand side of Fig. 1. One finds for the single-differential decay rate:

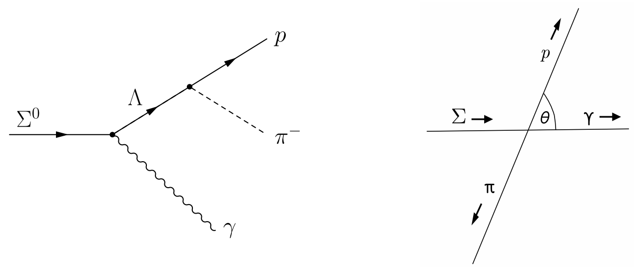

Fig. 1. *Left-hand side:* Feynman diagram for the decay process $\Sigma^0 \to \gamma p \pi^-$. *Right-hand side:* The decay process as seen in the Λ rest frame.

$$\frac{d\Gamma_{\Sigma^0\to\gamma p\pi^-}}{d\cos\theta} = \frac{1}{(2\pi)^3}\,\frac{1}{64\,m_{\Sigma^0}^3}\int dm_{12}^2\,\left\langle|\mathcal{M}_3|^2\right\rangle\,\frac{m_{\Sigma^0}^2-m_{12}^2}{m_{12}^2} \times \lambda^{1/2}(m_{12}^2,m_p^2,m_\pi^2)\,. \tag{26}$$

In (25), (26) the matrix element $\mathcal{M}_3$ for the three-body decay is given by

$$\begin{aligned}\mathcal{M}_3 &= \bar{u}_p\,(\mathcal{A}-\mathcal{B}\gamma_5)\,(\not{p}_\Lambda+m_\Lambda)\,(a\,\sigma_{\mu\nu}-b\,\sigma_{\mu\nu}\gamma_5)\\ &\quad\times u_{\Sigma^0}\,(-i)p_\gamma^\nu\,\epsilon^{\mu*}\,D_\Lambda(m_{12}^2)\\ &=: \mathcal{M}_{\rm red}\,D_\Lambda(m_{12}^2)\end{aligned} \tag{27}$$

with a Λ resonance propagator

$$D_\Lambda(s) := (s-m_\Lambda^2+im_\Lambda\Gamma_\Lambda)^{-1} \tag{28}$$

and the total width $\Gamma_\Lambda = 1/\tau_\Lambda$ where τ_Λ denotes the lifetime of the Λ.

Using the fact that the lifetime of the Λ is extremely long [8] one can carry out the integral that appears in (26):

$$\begin{aligned}&\int dm_{12}^2\,\left\langle|\mathcal{M}_3|^2\right\rangle\,\frac{m_{\Sigma^0}^2-m_{12}^2}{m_{12}^2}\,\lambda^{1/2}(m_{12}^2,m_p^2,m_\pi^2)\\ &=\int dm_{12}^2\,|D_\Lambda(m_{12}^2)|^2\,\left\langle|\mathcal{M}_{\rm red}|^2\right\rangle\,\frac{m_{\Sigma^0}^2-m_{12}^2}{m_{12}^2}\,\lambda^{1/2}(m_{12}^2,m_p^2,m_\pi^2)\\ &=\frac{1}{m_\Lambda\Gamma_\Lambda}\int dm_{12}^2\,\frac{m_\Lambda\Gamma_\Lambda}{(m_{12}^2-m_\Lambda^2)^2+(m_\Lambda\Gamma_\Lambda)^2}\,\left\langle|\mathcal{M}_{\rm red}|^2\right\rangle\\ &\quad\times\frac{m_{\Sigma^0}^2-m_{12}^2}{m_{12}^2}\,\lambda^{1/2}(m_{12}^2,m_p^2,m_\pi^2)\\ &\approx\frac{\pi}{m_\Lambda\Gamma_\Lambda}\,\left\langle|\mathcal{M}_{\rm red}|^2\right\rangle\,\frac{m_{\Sigma^0}^2-m_\Lambda^2}{m_\Lambda^2}\,\lambda^{1/2}(m_\Lambda^2,m_p^2,m_\pi^2)\end{aligned} \tag{29}$$

where we have used

$$\frac{m_\Lambda\Gamma_\Lambda}{(m_{12}^2-m_\Lambda^2)^2+(m_\Lambda\Gamma_\Lambda)^2} \approx \pi\,\delta(m_{12}^2-m_\Lambda^2)\,. \tag{30}$$

This replacement is completely appropriate when comparing to the experimental procedure where the intermediate Λ is actually measured by a displaced vertex.

Note that the reduced matrix element $\mathcal{M}_{\rm red}$ is now evaluated for onshell momenta of the Λ. Then the average $\langle|\mathcal{M}_{\rm red}|^2\rangle$ depends only on m_{23}^2 or, in other words, on the angle θ between proton and photon. Finally one obtains for the single-differential decay rate:

$$\frac{d\Gamma_{\Sigma^0\to\gamma p\pi^-}}{d\cos\theta} = \frac{1}{2}\,\Gamma_{\Sigma^0\to\gamma\Lambda}\,\mathrm{Br}_{\Lambda\to\pi^-p}\,(1-\alpha_\Lambda\alpha_{\Sigma^0}\cos\theta) \tag{31}$$

with the branching ratio $\mathrm{Br}_{\Lambda\to\pi^-p} := \Gamma_{\Lambda\to\pi^-p}/\Gamma_\Lambda$ and the asymmetries defined in (10), (11). In terms of the number of events, N, this reads

$$\frac{dN}{d\cos\theta} = \frac{N}{2}\,(1-\alpha_\Lambda\alpha_{\Sigma^0}\cos\theta)\,. \tag{32}$$

For the corresponding antiparticle decay chain one finds

$$\frac{d\bar{N}}{d\cos\theta} = \frac{\bar{N}}{2}\,(1-\alpha_{\bar{\Lambda}}\alpha_{\bar{\Sigma}^0}\cos\theta)\,, \tag{33}$$

with the number of events $\bar{N}$.

If one found an angular asymmetry in the particle and the antiparticle decays, then one could determine the parameters α_{Σ^0} and $\alpha_{\bar{\Sigma}^0}$ since the asymmetries α_Λ and $\alpha_{\bar{\Lambda}}$ have been measured [8]. According to the definitions (10), (14) and the relations given in the appendix an observable to test CP symmetry is given by

$$\mathcal{O}_{\rm CP} := \alpha_{\Sigma^0}+\alpha_{\bar{\Sigma}^0}\,, \tag{34}$$

i.e. this quantity vanishes if CP is conserved. Thus a non-vanishing value signals CP violation. A corresponding observable to test C symmetry is

$$\mathcal{O}_{\rm C} := \alpha_{\Sigma^0}-\alpha_{\bar{\Sigma}^0}\,. \tag{35}$$

Before we move on to an estimate of these parameters, it should be stressed that a measurement of the single-differential decay rate provides only the product of the two asymmetry parameters for the Sigma and the Lambda decay. To understand the implications of a non-vanishing experimental result for this product, we discuss first the particle case alone and then the comparison of particle and antiparticle case. The present situation is that the asymmetry parameter of Λ has been determined and is non-vanishing, see (17), while the asymmetry parameter of the Σ^0 has never been measured and is probably extremely small (see next section). An experimental finding of a non-vanishing value for the product would therefore be extremely interesting, irrespective of the result for the antiparticle case. If this product is different for particle and antiparticle decays one should, of course, analyze whether the origin lies in the first or the second decay or in both. Since the asymmetry parameters for Λ and $\bar{\Lambda}$ can also be obtained from other production/decay chains [8], one needs a combined analysis of all these decays. But at present the situation is such that the Lambda decays are compatible with the absence (smallness) of CP violation. Therefore the observation of CP violation via non-trivial results for the angular distributions (32) and (33) would be sensational, no matter which of the successive decays is influenced by CP violation.

4. Parameter estimates

We will now use three-flavor chiral perturbation theory in the presence of a theta vacuum angle [27–29] to relate the neutron EDM to the EDTM for the transition of the Σ^0 to the Λ. The experimental upper limit for the former will provide an upper limit for the latter and in this way an upper limit for the decay asymmetry based on the possible strong CP violation.

EDMs for nucleons and hyperons have been calculated in [27–29] but the EDTM for the Sigma-to-Lambda transition has not been addressed. In the present work we provide the result for this EDTM from a complete calculation at the first non-trivial order in the chiral expansion. This calculation encompasses tree-level contributions from the next-to-leading-order (NLO) baryon Lagrangian and one-loop diagrams with one vertex from the NLO baryon Lagrangian but the rest from the leading-order (LO) baryon and LO

Table 1
Coefficients for the loop contributions to the EDTM of Σ^0 to Λ. The first column shows all possible meson–baryon pairs that can contribute in the loop. The second and third columns contain the respective coefficients $C_{\rm ce}$ and $C_{\rm df}$. The indices c–f refer to the loop diagrams depicted in [29]. The parameters F (b_F) and D (b_D) are low-energy constants from the LO (NLO) baryon Lagrangian.

loops	$C_{\rm ce}$	$C_{\rm df}$
$\{\pi^+, \Sigma^-\}$	$-4Db_F$	$4Fb_D$
$\{\pi^-, \Sigma^+\}$	$-4Db_F$	$4Fb_D$
$\{K^+, \Xi^-\}$	$-(3F-D)(b_D+b_F)$	$(D+F)(3b_F-b_D)$
$\{K^-, p\}$	$-(D+3F)(b_D-b_F)$	$(D-F)(3b_F+b_D)$

Table 2
Coefficients for the loop contributions to neutron EDM. See the caption of Table 1 for further details.

loops	$C_{\rm cd}$
$\{\pi^-, p\}$	$2(D+F)(b_D+b_F)$
$\{K^+, \Sigma^-\}$	$-2(D-F)(b_D-b_F)$

meson Lagrangians. Since the mass splitting between the members of the baryon octet emerges only from the NLO baryon Lagrangian, it is a consistent procedure to neglect the mass difference between Σ^0 and Λ for the loop calculation. In practice, this drastically simplifies the loop calculation.[2]

Since we follow exactly the framework of [29], we refer to this reference for further details. In the following we will need the expressions for the neutron EDM determined in [29] and for the Σ^0–Λ EDTM calculated here. Loops with various meson–baryon combinations contribute. They are collected in Table 1 for the Σ^0–Λ transition and in Table 2 for the neutron. Note that the tree-level part shows the SU(3) flavor symmetry while the loops break it due to the different meson masses that appear already at LO in the mesonic chiral Lagrangian. The important point is that chiral perturbation theory provides a systematic expansion around the flavor symmetric chiral limit where the quark masses vanish and the baryon masses of the members of a multiplet are degenerate.

The tree-level results are given by

$$d_{\Sigma\Lambda}^{\rm tree} = -\frac{4}{\sqrt{3}}\, e\bar\theta_0\, (\alpha w_{13} + w'_{13}) \tag{36}$$

and [29]

$$d_n^{\rm tree} = \frac{8}{3}\, e\bar\theta_0\, (\alpha w_{13} + w'_{13})\,, \tag{37}$$

with low-energy constants w_{13} and w'_{13} from the baryon NLO Lagrangian and $\alpha := 144 V_0^{(2)} V_3^{(1)}/(F_0 F_\pi M_{\eta_0})^2$. Here, F_π denotes the pion decay constant while F_0 (M_{η_0}) is the decay constant (mass) of the singlet eta field. This field is intimately tied to the field $\theta(x)$ that encodes the strong CP violation. In the framework of chiral perturbation theory it is useful to introduce a chirally invariant combination $\bar\theta(x)$ of these two fields. The vacuum expectation value of $\bar\theta(x)$ can be linked back to the vacuum expectation value of the original $\theta(x)$ field via

$$\bar\theta_0 = \left[1 + \frac{4V_0^{(2)}}{F_\pi^2}\,\frac{4M_K^2 - M_\pi^2}{M_\pi^2(2M_K^2 - M_\pi^2)}\right]^{-1} \theta_0 \tag{38}$$

with the masses of pion, M_π, and kaon, M_K. Finally, the quantities $V_i^{(j)}$ are the Taylor coefficients of the functions $V_i(\bar\theta)$ expanded in powers of the field $\bar\theta$. These functions appear as coefficients in the chiral expansion of the meson Lagrangian [33,34]. For technical details we refer to [28,29].

The one-loop diagrams that contribute at the same (first non-trivial) order as the tree-level terms (36) and (37) yield the following results:

$$d_{\Sigma\Lambda}^{\rm loop} = -\frac{4e\bar\theta_0 V_0^{(2)}}{\sqrt{3}F_\pi^4} \sum_{\{M,B\}} (C_{\rm ce} - C_{\rm df})\, J_{MM}(0) \tag{39}$$

and [29]

$$d_n^{\rm loop} = -\frac{8e\bar\theta_0 V_0^{(2)}}{F_\pi^4} \sum_{\{M,B\}} C_{\rm cd}\, J_{MM}(0)\,. \tag{40}$$

The sum covers the meson–baryon pairs listed in Table 1 or 2, respectively. The loop function J_{MM} is essentially the bubble diagram with two meson lines of mass M.[3] It is given by

$$\begin{aligned} J_{MM}(q^2) &= i\int \frac{d^dk}{(2\pi)^d}\,\frac{1}{(k^2-M^2+i\epsilon)((k+q)^2-M^2+i\epsilon)} \\ &= 2L + \frac{1}{16\pi^2}\left(\ln\frac{M^2}{\mu^2} - 1 - \sigma\ln\frac{\sigma-1}{\sigma+1}\right), \end{aligned} \tag{41}$$

where $\sigma = \sqrt{1-4M^2/q^2}$ and L contains the divergence for space–time dimension $d=4$ as will be shown below.

Following still [29] the loop divergence for the neutron case can be absorbed into the renormalization of w'_{13}:

$$w'_{13} = w'^{r}_{13}(\mu) + \frac{24V_0^{(2)}}{F_\pi^4}(Db_F + Fb_D)L \tag{42}$$

where $w'^{r}_{13}(\mu)$ is the finite part of w'_{13}. The dependence on the renormalization scale μ comes through the quantity

$$L = \frac{\mu^{d-4}}{(4\pi)^2}\left\{\frac{1}{d-4} - \frac{1}{2}[\ln(4\pi) + \Gamma'(1) + 1]\right\} \tag{43}$$

and cancels with that of the loops, leaving the EDM scale independent. It is a non-trivial check of our results (36) and (39) that the very same combination of loop divergence and counterterm appears.

Finally, we spell out the numerical values of the low-energy constants that we use for our size estimate of the (upper limit of the) Σ^0–Λ EDTM: To a large extent we follow [29] and use $D = 0.804$, $F = 0.463$, $b_D = 0.068$ GeV^{-1}, $b_F = -0.209$ GeV^{-1}, $F_\pi = 92.2$ MeV, and $V_0^{(2)} = -5\times 10^{-4}$ GeV^4.

The numerical values of the low-energy constants $w'^{r}_{13}(\mu)$ and w_{13} are not separately known. However, there is a recent lattice-QCD determination of the counterterm combination [35]

$$\begin{aligned} w_a(\mu) &:= \alpha w_{13} + w'^{r}_{13}(\mu)\,, \\ w_a(1\ \text{GeV}) &= (0.04 \pm 0.01)\ \text{GeV}^{-1}\,, \end{aligned} \tag{44}$$

which is exactly the quantity entering the tree-level expressions (36) and (37).

With these ingredients we obtain

$$\frac{d_{\Sigma\Lambda}}{d_n} = \frac{d_{\Sigma\Lambda}^{\rm tree} + d_{\Sigma\Lambda}^{\rm loop}}{d_n^{\rm tree} + d_n^{\rm loop}} \approx -0.88\,. \tag{45}$$

[2] Note that the more complicated loop structures caused by the different masses of the two external baryons of our transition do not appear at all for the "direct" EDMs addressed in [27–29] where initial and final states have always the same mass.

[3] We have here a slight misuse of M denoting both the meson mass and the meson itself. This should not cause any ambiguities.

Amusingly, this is numerically close to the ratio obtained just from the tree-level parts: $d^{\text{tree}}_{\Sigma\Lambda}/d^{\text{tree}}_n = -\sqrt{3}/2 \approx -0.87$. Of course, one knows this only after performing the loop calculation.

The experimental upper limit for the neutron EDM can be translated to an upper limit for the Σ^0–Λ EDTM:

$$|d_{\Sigma\Lambda}| \leq 2.5 \times 10^{-26} e\,\text{cm} \tag{46}$$

where we have utilized the current experimental upper bound of the neutron EDM [36], $|d^{\text{exp}}_n| \leq 2.9 \times 10^{-26} e\,\text{cm}$.

Now we have all ingredients for an estimate of the asymmetry α_{Σ^0} defined in (10). Using (9) together with (16) and anticipating $|a| \gg |b|$ we obtain

$$\alpha_{\Sigma^0} \approx -\frac{2 d_{\Sigma\Lambda} \sin\delta_F}{a} . \tag{47}$$

We utilize the first relation of (9) and the experimental value of the magnetic transition moment (5). This gives us the upper limit

$$|\alpha_{\Sigma^0}| \leq 3.0 \times 10^{-12} |\sin\delta_F| . \tag{48}$$

For the most conservative limit one might use for the phase caused by final-state interactions the mathematical bound $|\sin\delta_F| \leq 1$. However, given that the final-state interaction between photon and Λ is an electromagnetic effect, one would rather estimate $|\sin\delta_F| \sim \alpha_{\text{QED}}$ with the fine-structure constant α_{QED}. For the electrically neutral Λ the interaction with the photon is dominantly magnetic and therefore further kinematically suppressed by the photon momentum. Therefore it feels justified to assume $|\sin\delta_F| \leq 10^{-2}$, which leads to

$$|\alpha_{\Sigma^0}| \leq 3.0 \times 10^{-14} . \tag{49}$$

For the slope of the angular asymmetry in (32) we deduce from (49) an upper limit of

$$|\alpha_\Lambda \alpha_{\Sigma^0}| \leq 1.9 \times 10^{-14} \tag{50}$$

where the experimental value of the decay asymmetry for $\Lambda \to p\pi^-$ is given in (17).

Finally we turn to the antiparticle decay chain. We obtain here the very same estimate as in (47), because the QCD theta-vacuum-angle term conserves C and violates CP; see also the discussion in the appendix. Thus it is useful to provide a corresponding upper limit for the CP-test observable (34):

$$|\mathcal{O}_{\text{CP}}| \leq 6.0 \times 10^{-14} . \tag{51}$$

5. Summary and outlook

We have provided a framework to search for P and CP violation in the decay $\Sigma^0 \to \Lambda\gamma$ utilizing the subsequent weak decay of the Λ to achieve an angular asymmetry. The driving term is an EDTM. For our concrete estimates we have used the CP violating QCD theta-vacuum-angle term. Since this term gives also rise to an EDM of the neutron one can use chiral perturbation theory to relate the two EDMs. Using the experimental upper limit for the neutron EDM provides upper limits for the angular asymmetry in our Σ^0 decay and also for the parameter that tests CP violation by comparing the angular asymmetries for particle and antiparticle. We have found tiny effects which implies that any experimental significance would point to physics beyond QCD, even if extended by a theta-vacuum-angle term.

In principle, also P violation without or with little CP violation can lead to an angular asymmetry. This can be caused for instance by the weak theory. An estimate of this effect is beyond the scope of the present paper. Yet we would like to stress that the CP violation in the weak theory is very small. Thus an observation beyond the tiny effect (51) of a CP violating angular asymmetry as deduced from the Σ^0 and $\bar{\Sigma}^0$ decays would point to physics beyond the standard model.

Our concrete relation between the EDM of the neutron and the EDTM of $\Sigma^0 \to \Lambda$ is based on chiral perturbation theory where the effects from possible strong CP violation can be systematically included. The starting point for establishing this relation is the approximate SU(3) flavor symmetry that assigns the neutron, the Σ^0 and the Λ to the very same multiplet. Chiral perturbation theory takes care of the explicit breaking of this approximate symmetry in a systematic way. But the approximate SU(3) flavor symmetry is a physical effect, not restricted to the standard model extended by a theta-vacuum-angle term. Thus, in principle, it should be possible to relate the EDM of the neutron and the EDTM of $\Sigma^0 \to \Lambda$ also in other beyond-standard-model frameworks that propose appreciably large EDMs. Yet, in practice, a quantitative relation between the two quantities requires an effective field theory that incorporates (a) the electroweak theory, (b) additional terms that encode the beyond-standard-model physics, and (c) chiral perturbation theory to account for the flavor symmetry breaking on the hadron level. At present, not even a marriage of the electroweak theory and chiral perturbation theory has been fully achieved. Consequently, the task to establish a completely model-independent relation between the EDM of the neutron and the EDTM of $\Sigma^0 \to \Lambda$ remains as a future endeavor.

Acknowledgements

We thank the reviewer for pointing out the importance of loop diagrams for our calculation.

Appendix A. Phases and discrete symmetries

An effective Lagrangian for the Σ^0–Λ transition takes the form

$$\begin{aligned}\mathcal{L}_{\Sigma^0\Lambda} = &\frac{1}{2} a_W\, \bar{\Lambda}\sigma_{\mu\nu}\Sigma^0 F^{\mu\nu} + \frac{1}{2}\bar{a}_W\, \bar{\Sigma}^0\sigma_{\mu\nu}\Lambda F^{\mu\nu} \\ &- \frac{1}{2} b_W\, \bar{\Lambda}\sigma_{\mu\nu}\gamma_5\Sigma^0 F^{\mu\nu} - \frac{1}{2}\bar{b}_W\, \bar{\Sigma}^0\sigma_{\mu\nu}\gamma_5\Lambda F^{\mu\nu} .\end{aligned} \tag{A.1}$$

Hermiticity requires $\bar{b}_W = -b^*_W$ and $\bar{a}_W = a^*_W$. We still have the freedom to redefine the field Σ^0 by an arbitrary phase. We choose this phase such that $a_W = \bar{a}_W$ is positive and real.

Using the transformation properties of fermion bilinears [37] it is easy to show that the a terms in our interaction Lagrangian (A.1) conserve P and C symmetry while the b terms break P. In addition, C symmetry implies $b_W = \bar{b}_W$ (i.e. b_W is purely imaginary). Note that conserved C and broken P implies that CP is broken. This is the breaking pattern caused by the QCD theta-vacuum-angle term. What about the case when CP is conserved? This is fulfilled when $b_W = -\bar{b}_W$ (i.e. b_W is purely real) [9]. For a P violating decay, conservation of CP implies that C is violated. Note that for the case when P, C and CP are violated b_W is neither purely real nor purely imaginary.

Finally we recall that electromagnetic final-state interactions induce a phase shift δ_F that is the same for particle and antiparticle. Thus

$$b = b_W\, e^{i\delta_F} , \qquad \bar{b} = \bar{b}_W\, e^{i\delta_F} . \tag{A.2}$$

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