Type IIB Compactifications and string dualities

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ABSTRACT: In the present thesis, we offer an introduction to type IIB string compactifications on $\mathbb{T}^d/\Gamma$ toroidal orbifolds. We first describe the technical method to construct these spaces and reduce the string background on it. We will have (non)-geometrical fluxes arising from these spaces which decorate with discrete deformations our four $\mathcal{N} = 1$ dimensional supergravity theory. Solving its equations of motion, we find several families of supersymmetric AdS vacua with fixed moduli, which can be related through a set of $SL(2, \mathbb{Z})$ symmetries.

Contents

English: We live in a universe where we can see height, width and depth. Furthermore, we perceive how time goes by. These four dimensions are always in our daily life. But physics has something to say, and in this case, String theory starts complaining. This theory has been studied for long time, as it seems to be the strongest candidate to describe all the possible interactions of our universe under the same equations. But one of its drawbacks is that it requieres up to 10 dimensions! In this thesis we are going to compactify (to reduce to tiny size) those extra 6 dimensions we do not see in our life. Depending how we hide these, we can get up to 10^{500} different space-times with alternative properties, which are ruled by the moduli. These objects (moduli) live in those six extra dimensions and govern the three big spatial ones through the action of the fluxes. Fluxes are elements which live on specific regions of tiny dimensions and coil around them while changing the properties of the four uncompact dimensions. On top of that, we can find relations between the different moduli. These relations are called dualities, which will allow us to find connections between those 10^{500} different space-times.

Svenska: Vi lever i ett universum där vi kan observera höjd, bredd och djup. Vi kan också uppfatta tid. Dessa fyra rymdtidsdimensioner märker vi av dagligen. Men i fysiken kan det vara annorlunda, och i vårat fall kommer strängteori ha något att säga till om antal dimensioner. Denna teori har nu studerats under en lång tid, och det verkar vara den bästa kandidaten för att beskriva alla möjliga interaktioner i vårt universum under samma ekvationer. En av dess nackdelar är dock att strängteori kräver tio dimensioner! I denna uppsats förklarar vi hur vi kan kompaktifiera (reducera till liten skala) de sex extra dimensionerna vi inte uppfattar i våra dagliga liv. Beroende på hur vi gömmer dessa dimensioner, kan vi få upp till 10^{500} olika rymdtider med olika egenskaper, vilka bestäms av moduli. Dessa objekt (moduli) lever i de sex extra dimensionerna, och beskriver de tre stora rymddimensionerna via verkan av flöden. Flöden är element som lever i en specifik region av de små dimensionerna och slinger sig runt dem samtidigt som de ändrar egenskaperna av de fyra okompaktifierade dimensionerna. Utöver det här, hittar vi relationer som beskriver de olika moduli. Dessa relationer kallas dualiteter, vilket relaterar de olika 10^{500} rymdtiderna.

Castellano: Vivimos en un universo donde las dimensiones de ancho, largo, alto y el paso del tiempo, son aquellas que experimentamos en nuestra vida cotidiana. Pero como siempre, la física tiene algo que decir, y en este caso es la teoría de cuerdas. Dicha teoría ha sido durante largo tiempo la candidata mas fuerte para representar una teoría que pueda describir todas las posibles interacciones físicas de nuestro universo. Una de sus principales características , ¡es que require de hasta 10 dimensiones! En esta tesis compactificaremos dimensiones, que no es más que estudiar como reducir a tamaños muy pequeños las seis restantes que no vemos. Dependiendo de como las escondamos, podemos obtener unos 10^{500} espacio-tiempos diferentes con distintas propiedades que están reguladas por los Moduli. Estos objetos viven en las seis dimensiones diminutas y rigen las propiedades de las grandes a través de flujos. Estos elementos viven en regiones de las dimensiones pequeñas y "enrollandose" a ellas, cambian las propiedades mas fundamentales de las cuatro grandes. Además, los moduli pueden relacionarse con una serie de transformaciones llamadas dualidades, lo que nos permite encontrar puentes entre vacios (espacio-tiempos) aparentemente diferentes.

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1 Introduction

The XX century represented a mighty revolt in our understanding of the universe we live in. During that period, two main theories have taken a stand as imperative tools for this knowledge. General Relativity and Quantum Field Theory. While a vast amount of experiments have tested and ensured the consistency of Einstein's theory, Particle physics phenomenology has found a great candidate to describe its rules, the Standard Model of Physics, as well, extendedly tested in laboratories and colliders . One remarkable goal attempted in all these decades, and still to be achieved, has been the idea of merging these previous frameworks in a quantum theory that can incorporate gravity as well as the other interactions. Among all possible candidates, String Theory is a strong one to provide us with a complete description of the fundamental forces. Besides its initial target for a description of strong interaction between quarks in 1960's, it has evolved until nowadays as a powerful framework for describing high energy physics aspects, based on the assumption of fundamental one dimensional objects (strings) propagating in time.

This theory was found to contain a graviton vibrating mode of the strings and to reproduce supersymmetric versions of GR in a ten dimensional low energy limit background (Supegravity). Furthermore, it includes a web of dualities, i.e. relations between different theories which allow us to see them as limits of the same theory.

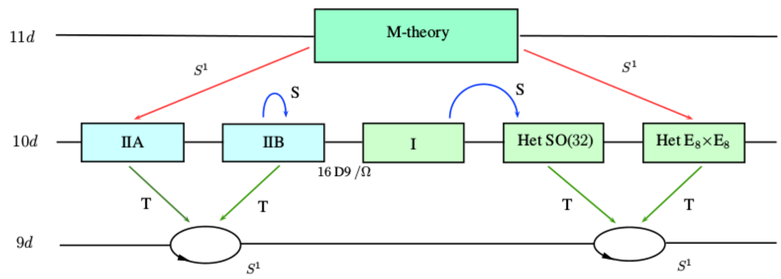


Figure 1. Duality connections between different frameworks in string theory.

As we mentioned above, these theories require at least ten dimensions, due to constraints imposed by quantum consistency [1]. Obviously, our daily life observations show a four dimensional space-time, thus inquiring a reduction of those extra six dimensions somewhere. Theorists found the compactification mechanism as a way to reduce down to four dimensions in order to make contact with low-energy observations. First attempts of compactifications were Kaluza-Klein ones, where a five dimensional space has Einstein's gravity theory coupled to a U(1) gauge theory [2]. But, as low energy string theories require ten dimensions, these compactifications started being studied in internal Calabi-Yau manifolds. By formulating our theory in those spaces, we decorate our four dimensional description with some preserved supersymmetries in a Minkowski space ($\Lambda = 0$) and a generous amount of scalar fields controlling size and shape deformations of six extra dimensions, called moduli, arising from these compactifications in the manifold [3].

But this space (Minkowski), has been proven to not represent the universe we live in. In the last two decades, several measures of the Cosmic Microwave Background (CMB) and supernovae have shown that we live in a universe with a positive cosmological constant ($\Lambda > 0$). This cosmological constant represents the contribution of an unknown energy in Einstein equations that drives our universe through an accelerated expansion [4], called dark energy. This source satisfies a specific equation of state derived from FLRW equations of the so-called $\Lambda - CDM$- model. This model states that around 70% of our universe is dominated by an energy which generates an accelerated expansion in the visible regions of our universe. Furthermore, in order to make contact between the current $\Lambda - CDM$- model that describes our universe and those observations, it was necessary to include a sudden expansion of the universe right after the so called Big Bang. This period is known as inflation, and it was proposed in order to explain certain issues about the homogeneity and isotropy of the observable cosmos, which have never been in thermal contact [5].

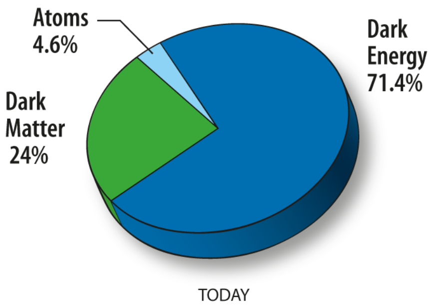


Figure 2. Estimated composition of the observable universe.

These new observational conditions should also be UV-completed within. While the accelerated expansion needs a search for de Sitter vacua without tachyonic modes, the latter demands quasi-de Sitter states with very flat potentials for the inflaton[1]. This means that there should be a vacuum with similar properties in our compactified theories. Based on a statistical analysis of string compactifications, the amount of possible solutions (vacua) is around 10^{500} [6]. This is well known as the string landscape. However, to find a realistic de Sitter solution with this configuration seems hard, as very few of them have consistency and stability, because of moduli stabilisation issues or the presence of tachyonic modes [7].

The study of type II superstring compactifications during these last two decades has motivated the research of de Sitter vacua. Type IIB theory reduced on Calabi-Yau manifolds, when only taking into account NS-NS and R-R fields strengths associated to gauge potentials, offers a four dimensional description which lacks a scale parameters describing the size of compact dimensions and a not minimal supersymmetry [8]. Furthermore, this implies the presence of flat directions in the potential that rules the expected mass value of the scalar fields of the compactification. This can be solved introducing quantum non-perturbative corrections to the superpotential of the moduli giving an anti de Sitter solution. This solution can be uplifted to de Sitter minima including specific branes breaking the supersymmetry [9].

This issue can be reconsidered if we interpret somehow deformations of supergravity models as lower dimensional effective description of something called non-geometrical fluxes [10], as they do not have a totally defined interpretation in higher dimensions. On the other hand, to switch them on, allows us to find even more vacua with fixed moduli minima positions. Among all these solutions, we find the so-called STU-models. These vacua enjoy several inner symmetries based on the dualities beneath string theory [11, 12]. These "new" fluxes are constrained by the algebra arising from the supergravity compactification imposed on the singular limit of our Calabi-Yau manifold, better known as toroidal orbifolds. Even if the different minima obtained from the equations of motion in our theory do not look similar, they can be related using chains of the aforementioned

[1]around 60 e-folds.

dualities.

The aim of this thesis is to give a proper introduction into effective descriptions arising from string compactifications of type IIB string theory. These require to introduce non-geometrical fluxes, while they do not have a defined higher dimensional interpretation, they ensure us a moduli fixation which end up in a vast set of solutions (vacua) of certain dynamical equations in our compactified space. The final goal is to relate a chosen set of these solutions through different dualities equipped in our theory. We hope to offer a better understanding of the physical relations between different solutions in the landscape.

This thesis is organized in this specific way. In chapter 2 we will summarize as an introduction those key topics we need for succesful compactifications of IIB theory. A quick reminder in specific features of Supersymmetry and Supergravity as low energy limit of string theory are mentioned. As well, a mathematical approach of the main properties of the compactified space, Calabi-Yau manifolds, are present. In its last part, Orbifolds are introduced as the best tool to reduce our theories in the requiered way. In chapter 3 we describe how to build up all the Fluxes description, their inner dualities and constraints that arise from the Algebra and dualities. At last, in chapter 4, we will use everything constructed in previous chapters to come up with several of those landscape solutions that we will relate through the inner dualities of our theory. Finally, some more material, as compendium of equations or methodology and programming code used in the thesis are included in the appendices.

2 Compact space construction Mathematical toolkit

In this Chapter we will offer an extended description of all the required tools to describe flux compactifications and potentials based on them. These will lead us to descriptions of Anti de Sitter vacua in section 4. First, we will introduce type II supergravity theories in ten dimensions, mainly focusing on IIB theory, as it will be the main background of this thesis. Its internal symmetries and parities will be studied, as they are completely necessary for building a $\mathcal{N}=1$ supersymmetric theory in four dimensions.

Next section will describe the mathematical properties of complex geometry and Calabi-Yau Manifolds. These objects are powerful tools for encoding the information of the compactified dimensions in our $\mathcal{M}_{10} = \mathcal{M}_{1,3} \times \mathcal{M}_6$ model. Unfortunately, they have several limitations, e.g. to construct smooth metrics of Calabi-Yau Manifolds is still an open problem in Mathematics. In that sense, we can build up a similar kind of identification. These are Orbifolds, which offer more accurate description of this space, fulfilling almost all the requirements for a $\mathcal{N}=1$ supersymmetric compactification. The methods for its construction and similarities to CY-manifolds will be explained. We will end this chapter with the specific compactification of a type IIB theory with this technology.

2.1 Type IIB Supergravity action and its symmetries

2.1.1 Superstrings in Worldsheet and Multiplets

In the Green-Schwarz formalism the action of a superstring of type II propagating in a flat background reads [1]:

$$S_{IIB}^{Light\ cone} = -\frac{1}{4\pi\alpha'}\int_{\sum} d^2\sigma \left(\partial\left(X_L^i + X_R^i\right)\right)^2 + \frac{i}{\pi}\left(\psi^a \partial\, \psi^a + \tilde{\psi}^a \partial\, \tilde{\psi}^a\right), \tag{2.1}$$

Where α is the Regge slope, with inverse relation to the string tension.This action describes the behaviour of left $(X_L, \tilde{\psi}^a)$ and right (X_R, ψ^a) excitations of the string on the worldsheet for the bosonic and fermionic fields along the eight remaining orthogonal dimensions in the light cone description. The index 'a' corresponds to the spinorial representation 8_s of the group $SO(8)$[2] while $\dot{a}$ corresponds to its conjugate one.

After quantising this theory and imposing the adequate boundary conditions, we can obtain a description of the levels for left and right modes, such that:

$$N = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + n\psi_{-n}^a \psi_n^a, \qquad \tilde{N} = \sum_{n=1}^{\infty} \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i + n\tilde{\psi}_{-n}^a \tilde{\psi}_n^a. \tag{2.2}$$

Then, the massless states in the spectrum of this theory will be created by the tensor product of right and left fermionic zero modes acting on a vacuum state. Rewriting those 8 annihilation and creation operators under 4 new ones such that $\sqrt{2}b_m = \left(\psi_0^{2m-1} + i\,\psi_0^{2m}\right)$, we get a set of anticommutation relations satisfying:

$$\{b_m, b_n^\dagger\} = \delta_{mn} \quad , \quad \{b_m, b_n\} = \{b_m^\dagger, b_n^\dagger\} = 0 \qquad\qquad m,n = 1...4. \tag{2.3}$$

This is like reducing the symmetry from $SO(8)$ to $SU(4)\times U(1)$. In this symmetry the annihilation

[2]This representation corresponds to the little group in the light cone gauge

operators are the fundamental representation of $SU(4)$ with charge $\frac{1}{2}$ while the creation operators transform in the conjugate representation. Now, we can apply this operators to the vacuum state to generate a 16-dimensional representation of $SU(4) \times U(1)$ that encodes the bosonic and fermionic states, with an even or odd number of creation operators, respectively.

STATE	$SU(4) \times U(1)$	representation
$\|0\rangle$	$1\ (q = 1)$	
$b^\dagger_m b^\dagger_n \|0\rangle$	$6\ (q = 0)$	$8_v\ \ NS$
$b^\dagger_m b^\dagger_n b^\dagger_i b^\dagger_j \|0\rangle$	$1\ (q = -1)$	
$b^{(\dagger)}_m \|0\rangle$	$4\ (q = \pm\frac{1}{2})$	
$b^{(\dagger)}_m b^{(\dagger)}_n b^{(\dagger)}_i \|0\rangle$	$\bar{4}\ (q = \pm\frac{1}{2})$	$8_s(8_c)\ \ R$

Table 1. Fermionic and bosonic modes in light-cone representation

Table (1) states that NS-sector transforms as a vector, while R transforms as a spinorial representation. This builds up a $(8_v \oplus 8_c)$ representation for the right-moving part of both sectors. As we are working in IIB theory, and the chirality for both sectors is the same, to describe the left-moving sector we need to duplicate the aforementioned representation. The combination of Left-Right sectors will end up with a 256 states set of massless ground states, such that:

Sector	Decomposition	Fields
NS-NS	$(8_v \otimes 8_v) = 35_v \oplus 28_v \oplus 1_v$	$g \oplus B_2 \oplus \phi$
R-R	$(8_c \otimes 8_c) = 35_c \oplus 28_c \oplus 1_v$	$C_4 \oplus C_2 \oplus C_0$
NS-R	$(8_v \otimes 8_c) = 8_s \oplus 56_s$	$\chi_1 \oplus \psi_1$
R-NS	$(8_c \otimes 8_v) = 8_s \oplus 56_s$	$\chi_2 \oplus \psi_2$

Table 2. Decomposition of massless fields representations

As we can see in table (2), the different sectors combine into a set of fields that can be embedded in the ten dimensional low-energy description.

2.1.2 Supersymmetric action at large distance

Setting the Regge slope string parameter $\alpha' \to 0$ gives a description of type II theories in the large distance limit $(L \gg l_s)$[3]. These theories are equipped with a $\mathcal{N} = 2$ supersymmetry in ten dimensions. Even if this model lacks a description of massive modes, it offers a rich background in the low-energy limit with the most fundamental features of string theory.

This theory contains a set of masless fields, composed by [11]:

- A symmetric graviton field G_{MN}.[1]
- An antisymmetric tensor field B_{MN}.
- A scalar field, the dilaton, ϕ.

[3] This means our strings are studied on the limit of pointlike objects.
[1] We will use capital latin letters for our 10 dimensional space

- A set of p-forms, describing gauge fields C_p, where p must be an even number for our IIB theory.
- Fermionic superpartners of previous fields. We are not taking them into account as they break Lorentz invariance of the action.

In our IIB case, the most general action we can write for massless fields reads like:

$$S_{IIB} = S_{NS} + S_{RR} + S_{CHS}. \tag{2.4}$$

These three different terms correspond to sectors controlling the dynamics of various fields. The first one, the Neveu-Schwarz term, denotes the universal sector consisting of G,B, ϕ and their derivatives. This term is given by:

$$S_{NS} = \frac{1}{2\kappa^2}\int_{\mathcal{M}_{10}} d^{10}x\,\sqrt{-G}\left(R - \frac{1}{2}\partial^M\phi\partial_M\phi - \frac{1}{2}e^{-\phi}|dB_2|^2\right). \tag{2.5}$$

We consider the factor κ^2 related to a function depending on ten dimensional Newton's constant G_{10} and the dilaton field ϕ. The derivative term of the antisymmetric tensor denotes its field strength. The second term in equation (2.4) accounts for the Ramond-Ramond fields. These fields are described by the p-forms C_0, C_2 and C_4. This term reads like:

$$S_{RR} = -\frac{1}{4\kappa^2}\int_{\mathcal{M}_{10}} d^{10}x\,\sqrt{-G}\left(e^{2\phi}|F_1|^2 + e^{\phi}|\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right). \tag{2.6}$$

Where F strengths are defined as a combination of:

$$\begin{aligned} F_1 &= dC_0, \\ \tilde{F}_3 &= dC_2 - dB_2 \wedge C_0, \\ \tilde{F}_5 &= dC_4 + \frac{1}{2}\left(B_2 \wedge dC_2 - C_2 \wedge dB_2\right). \end{aligned} \tag{2.7}$$

The $\tilde{F}_5$ in eq(2.7) must be constrainted to its selfdual part by hand, as a supersymmetric theory must have the same amount of degrees of freedom for bosonic and fermionic modes. This theory contains as well a topological Chern-Simons term S_{CS} in the action, described by:

$$S_{CS} = -\frac{1}{4\kappa^2}\int_{\mathcal{M}_{10}} d^{10}x\,\sqrt{-G}\left(C_4 \wedge dB_2 \wedge dC_2\right). \tag{2.8}$$

This term is the responsible one of switching on/off the main geometric fluxes to acquire non-zero expectation values for moduli fields, as we will see in section (3).

This type IIB theory enjoys several symmetries that will be used further in the thesis to reduce the amount of degrees of freedom and to break the supersymmetries to the desired $\mathcal{N} = 1$ value in 4 dimensions. Let's study them in detail.

2.1.3 Symmetries of IIB Superstring theory

In this thesis, we will focus on compactifications from ten to four dimensions. In this sense, we need to understand the different hidden symmetries in our ten dimensional background, in order to account for the properties of our lower dimensional descriptions. These symmetries are principally encoded in the spinorial behaviour on the worldsheet description of eq (2.1). From this action we can detect two different symmetries related to the parity of the fields and the fermionic sector.

Fermionic number

The fermionic part of the action described in eq (2.1) is invariant under the following discretes symmetries :

$$\begin{aligned} (-1)^{F_R} &: \psi^a \to -\psi^a, \\ (-1)^{F_L} &: \tilde{\psi}^a \to -\tilde{\psi}^a. \end{aligned} \tag{2.9}$$

Where $F_{L,R}$ corresponds to the fermionic number of left and right sectors. This occurs for both sectors. Then, we can group all the massless fields of IIB theory, according to parity, as shown in table 3:

	Even	Odd
$(-1)^{F_R}$	g, B_2, ϕ, χ_1, ψ_1	C_0, C_2, C_4, χ_2, ψ_2
$(-1)^{F_L}$	g, B_2, ϕ, χ_2, ψ_2	C_0, C_2, C_4, χ_1, ψ_1

Table 3. Action of the Fermionic Number.

Worldsheet parity Ω_p

Type IIB superstring theory contains spinors ψ^a with the same space-time chirality in action (2.1). This means that interchanging the left and right sectors will result in the same description, but our bosonic fields from the action (2.5) will be affected; Half of them will acquire an even parity, whereas the rest will get an odd one. This can be seen in next table:

	Even	Odd
Ω_p	g, C_2, ϕ	C_0, B_2, C_4

Table 4. Action of parity Ω_p.

SL(2,$\mathbb{Z}$) Self-Duality

The type IIB action has another interesting symmetry that relates different regions of our theory [13]. To notice this symmetry, we need to define new scalar and forms, based on our previous ones in eq(2.4):

$$\begin{aligned} S &= C_0 + \imath e^{-\phi}, \\ G_3 &= dC_2 - i\, e^{-\phi/2}\, B_2. \end{aligned} \tag{2.10}$$

Where S is known as the axiodilaton field, which will be used for the rest of this thesis. With these definitions, our action looks like:

$$
\begin{aligned}
S_{IIB} =& \frac{1}{2\kappa^2}\int_{\mathcal{M}_{10}} d^{10}x\,\sqrt{-G^E}\left(R - \frac{\partial_M S\,\partial^M \bar{S}}{2(Im\,S)^2} - \frac{|G_3|^2}{2Im\,S} - \frac{|F_5|^2}{4}\right) + \\
&+ \frac{1}{8\imath\kappa^2}\int_{\mathcal{M}_{10}} \frac{1}{Im\,S}\,C_4\wedge G_3\wedge \bar{G}_3.
\end{aligned}
\tag{2.11}
$$

The previous action is expressed in the Einstein frame (A rescaling of the metric such that $G^E_{MN} = e^{\frac{\phi}{2}}G_{MN}$ and take us to the "string frame"). $\tilde{F}_5$ self-duality must be imposed again by hand. This action reflects a symmetry that affects the axiodilaton field and B_2 and C_2 as well. The first one is transformed in a non-linear way, such that:

$$
S \to \frac{aS+b}{cS+d}, \qquad \text{with} \;\; ad-bc=1. \tag{2.12}
$$

This means that the imaginary part of our S field, the dilaton, can relate weak to strong coupling regions in the moduli space of the theory, so perturbative states at one point of our moduli space are described by perturbative and non-perturbative ones after the transformation. This is why this self-duality cannot be checked order by order due to its non-perturbative nature.

The action of this duality on the fields B_2 and C_2 can be denoted by the matrix transformation:

$$
\begin{bmatrix} C_2 \\ B_2 \end{bmatrix} \to \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} C_2 \\ B_2 \end{bmatrix}. \tag{2.13}
$$

This group can be described by its three generators, such that:

$$
\mathbf{T} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \to S' = S+1, \quad \mathbf{S} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \to S' = \frac{-1}{S}, \quad \mathbf{R} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \to S' = S. \tag{2.14}
$$

As we can see, the R-generator acts as a reflections on the pair (B_2, C_2), which can be identified as an orientifold of the type $\mathbf{R} = (-1)^{F_L}\Omega_p$. This self-duality is usually called S-duality.

2.2 Calabi- Yau Manifolds

Before introducing the Calabi-Yau manifold's properties we want to make use of, we give a brief summary of the elements living in a manifold, so we can introduce further more complicated definitions for the elements and properties of this space.

A p-form A_p is an antisymmetric tensor whose rank is p. We can describe its components as:

$$
A_p = \frac{1}{p!}\,A_{\mu_1\ldots\mu_p}\,dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_p}. \tag{2.15}
$$

Where $\wedge$ stands for an antisymmetric tensor product. We can map this form to a $p+1$-form space, using the exterior derivative, such that:

$$
d\,A_p = \frac{1}{p!}\,\partial_{\mu_1}\,A_{\mu_2\ldots\mu_{p+1}}\,dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_{p+1}}. \tag{2.16}
$$

The exterior derivative (2.16) allows us to define two important characteristics of p-forms. They are called closed if:

$$d\,A_p = 0. \tag{2.17}$$

Or exact if there is a specific $p-1$-form we can derive them from, as:

$$A_p = d\,B_{p-1}. \tag{2.18}$$

These two previous definitions help us to define a group based on the quotient space,

$$H^p(M) = \frac{C^p(M)}{Z^p(M)}, \tag{2.19}$$

Where $C^p(M)$ stands for the group of closed p-forms in the manifold and $Z^p(M)$ denotes the group of exact p-forms on M. The quotient group $H^p(M)$ is called de Rham Cohomology group, and it contains only unique closed p-forms, as it considers equivalent those ones that can be related through an exact form. The dimension of this space is called Betti number. It is an important feature of the manifold, as its invariance characterizes it.

Next, we introduce the space where these forms live. A complex manifold of dimension n is a manifold whose transition function are requiered to be biholomorphic[4]. In spaces like these, the p-forms (2.15) split their indices into p holomorphic and q antiholomorphic ones as:

$$A_{p,q} = \frac{1}{p!\,q!}\,A_{a1\ldots a_p\bar{b}_1\ldots\bar{b}_q}\,dz^{a_1}\wedge\ldots\wedge dz^{a_p}dz^{\bar{b}_1}\wedge\ldots\wedge dz^{\bar{b}_q}. \tag{2.20}$$

These complex manifold can host a (1,1)-tensor J, called complex structure[5]. This tensor has a set of components defined by:

$$J^a_b = i\,\delta^a_b \qquad J^{\bar{a}}_{\bar{b}} = -i\,\delta^{\bar{a}}_{\bar{b}} \qquad J^{\bar{a}}_b = J^a_{\bar{b}} = 0. \tag{2.21}$$

We can equip this manifold with a complex Riemmanian metric such that:

$$ds^2 = g_{ab}dz^a dz^b + g_{a\bar{b}}dz^a dz^{\bar{b}} + g_{\bar{a}b}dz^{\bar{a}}dz^b + g_{\bar{a}\bar{b}}dz^{\bar{a}}dz^{\bar{b}}. \tag{2.22}$$

As a special case, we can remove the diagonal conjugated terms $g_{\bar{a}\bar{b}} = \bar{g_{ab}}$, which leaves us with an Hermitian manifold. The complex structure (2.21) can be defined as:

$$J = i\,g_{a\bar{b}}\,dz^a\wedge dz^{\bar{b}}. \tag{2.23}$$

This describes a Kähler manifold if $dJ = 0$. This exterior derivative splits into Dolbeault operators,

$$d = \partial + \bar{\partial} = dz^a\frac{\partial}{\partial\,z^a} + d\bar{z}^{\bar{a}}\frac{\partial}{\partial\,\bar{z}^{\bar{a}}}. \tag{2.24}$$

[4]Holomorphic means totally complex differentiable in a neighbourhood of every point of the function domain.
[5]We have assume its Nijenhuis tensor is 0 for the present work. For deeper understanding, check [14].

Which maps (p,q)-forms to $(p+1,q)$-forms and $(p,q+1)$-forms, respectively. This exterior derivatives are nilpotent and anticonmute. Using these properties in $dJ = 0$, we can deduce these manifolds satisfy $\partial_a\, g_{b\bar{c}} = \partial_b\, g_{a\bar{c}}$, and therefore the metric can be locally written as:

$$g_{a\bar{b}} = \frac{\partial}{\partial\, z^a}\frac{\partial}{\partial\, \bar{z}^{\bar{b}}}\, \mathcal{K}(z,\bar{z}). \tag{2.25}$$

Where $\mathcal{K}$ is the Kähler potential. This is an holomorphic function defined up to the addition of arbitrary holomorphic functions. This function, together with a closed Ricci form,[6] are representatives of Kähler manifolds. The latter can be used as a representative of the cohomology first Chern class,

$$c_1 = \frac{1}{2\pi}[\mathcal{R}]. \tag{2.26}$$

Now we are ready to introduce the description of a Calabi-Yau manifold. This is a Kähler manifold with vanishing first Chern class (2.26). A fundamental theorem says that a compact Kähler manifold has vanishing first Chern class if and only if it can be equipped with a nowhere vanishing holomorphic top form Ω [14]. In local coordinates, this form is:

$$\Omega(z_1...z_n) = f(z_1...z_n)\, dz^1 \wedge dz^2 ... \wedge dz^n. \tag{2.27}$$

An important ingredient for characterizing a Calabi-Yau manifold is to specify its Betti numbers. These b_p are the dimension of the p-th de Rham cohomology of the manifold M. If the space can be equipped with a metric, the Betti numbers can be decomposed in terms of Hodge numbers $h^{p,q}$, which counts the number of harmonic[7] (p,q)-forms on the manifold as,

$$b_k = \sum_{p=0}^{k} h^{p,k-p}. \tag{2.28}$$

Hodge numbers give an important description of the Calabi-Yau manifold we study, although it is not a complete characterization since different Calabi-Yau can have the same Hodge numbers. Besides that, there are really important symmetries and dualities among these numbers that help us to display these in the so called Hodge diamond.

$$\begin{array}{ccccccc} & & & h_{3,3} & & & \\ & & h_{3,2} & & h_{2,3} & & \\ & h_{3,1} & & h_{2,2} & & h_{1,3} & \\ h_{3,0} & & h_{2,1} & & h_{1,2} & & h_{0,3} \\ & h_{2,0} & & h_{1,1} & & h_{0,2} & \\ & & h_{1,0} & & h_{0,1} & & \\ & & & h_{0,0} & & & \end{array} \tag{2.29}$$

The first symmetry we find is the one given by complex conjugation, which interchanges the holomorpic and antiholomorphic indices of the forms living in the manifold, making a switch in the cohomology groups those forms belong to. Their Hodge numbers are related by:

[6] The torsion vanishes in a Kähler manifold.

[7] A_p is an harmonic form if and only if $\Delta_p A = (d + d^\dagger)^2\, A_p = 0$

$$h^{p,q} = \tilde{h}^{q,p}. \tag{2.30}$$

As well, a Poincaré duality gives the relation:

$$h^{p,q} = h^{n-q,n-p}. \tag{2.31}$$

This follows from the isomorphism we can stablish between $H^p(M)$ and $H^{d-p}(M)$. This means that for every closed p-form integrated over the manifold M, we can find $(d-p)$-cycle N such that:

$$\int_M A \wedge B = \int_N B. \tag{2.32}$$

Last symmetry is as well related with the isomorphism above. If we contract a $(p,0)$-form with the complex conjugate of $(n,0)$-form and contract with the metric to make a $(0,n-p)$-form. This makes Hodge number satisfy the relation:

$$h^{p,0} = h^{0,n-p} \tag{2.33}$$

We will work with simply-connected Kähler complex manifolds, which means $h^{0,0} = 1$ (constant functions). Furthermore, they have a vanishing first homology group. Then:

$$h^{1,0} = h^{0,1} = 0. \tag{2.34}$$

If we consider the case $n = 3$ of a Calabi-Yau manifold[8] and we apply the three previous symmetries, we can get rid of describing $h_{p,q}$ spaces in (2.29) as they are related to each other through dualities. In fact, the only spaces we have to study are $h_{1,1}$ and $h_{2,1}$. The full set of Hodge numbers is displayed as follows:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h_{1,1} & & 0 & \\ 1 & & h_{2,1} & & h_{1,2} & & 1 \\ & 0 & & h_{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{2.35}$$

So far we have presented the decomposition of the Calabi-Yau space for (p,q)-forms, but we have not introduced a description of these forms apart from J and Ω. Furthermore, as we mentioned above, a main drawback is the fact that different Calabi-Yau spaces can have the same Hodge numbers. Some of them can be related by deformations of specific elements describing their shapes and sizes. These elements are the moduli, who physically represent massless scalars in our theory. We can deform them in order to connect Calabi-Yau with same Hodge numbers [9]. These moduli live in a Moduli Space defined by complex structures and Kähler structures.

[8] 6 real dimensions if we consider holomorphic and antiholomorphic indices.

[9] We will use a similar characteristics of a Orientifold space in 4 to relate different vacuas through their moduli

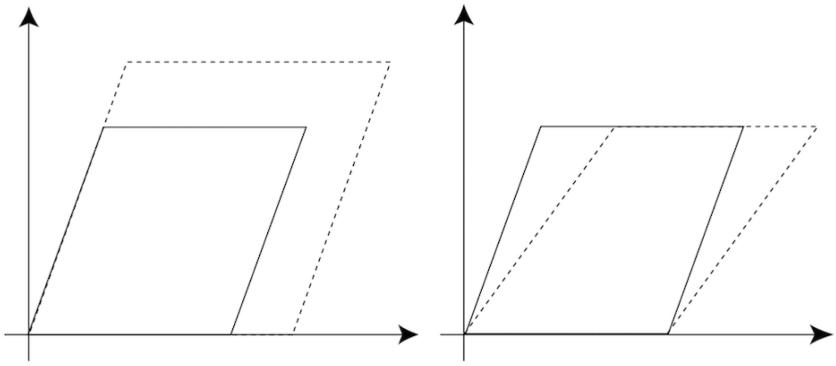


Figure 3. Deformations of structures in a $\mathbb{T}^2$. Kähler structure left and complex one right.

As an introductory example for these moduli, we can identify shape deformations of the thorus as the complex structure τ and those deformations related to the size with the Kähler moduli ρ. These moduli are related with the radii of the thorus as:

$$\tau = i\,\frac{R_2}{R_1}, \qquad \rho = i\,R_2\,R_1. \tag{2.36}$$

These deformations corresponds to metric deformations of our torus. So we need to raise this description to a 3 dimensional Calabi-Yau Space, as we are compactifying our ten dimensional string background in perfectly separable pieces of $\mathcal{M}_4 \times \mathcal{M}_6$. In fact, the contributions of our background in a Calabi-Yau space will come from the metric g_{MN} and the antisymmetric two form B_{MN}. This ten dimensional form can give rise to four dimensional fields such that two forms, one forms and scalars. This scalars b_i are moduli originating from the B field decomposition in both spaces. We will offer a deeper explanation in the next sections.

We can get some more information about the space regarding the contribution of our ten dimensional g_{MN} metric in our Calabi-Yau Manifold. We can ask ourselves the question: What happens to the Ricci tensor of our manifold if we vary the metric a little bit? This is:

$$R_{\mu\nu}(g + \delta\,g) = 0. \tag{2.37}$$

As g is a Ricci-flat metric, any diffeomorphism[10] will leave it flat [13]. If we expand our Ricci tensor using the correct gauge fixing to remove contributions of change of coordinates, we arrive to the expression:

$$\nabla^\rho \nabla_\rho \delta\,g_{\mu\nu} + 2\,R^{\rho\sigma}_{\mu\nu}\,\delta\,g_{\mu\nu} = 0. \tag{2.38}$$

Due to the properties of the index structure in Kähler manifolds, the components of the metric $g_{i\bar{j}}$ and g_{ij} decouple and we can study them separately. For $\delta\,g_{i\bar{j}}$ we would have an expression of the form:

$$\nabla^\mu \nabla_\mu \delta\,g_{i\bar{j}} + 2\,R^{i\bar{j}}_{k\bar{l}}\,\delta\,g_{k\bar{l}} = 0. \tag{2.39}$$

[10]coordinate change in the metric

Where μ sums over coordinates $(k, \bar{k})$. In this sense, we can see $\delta\, g_{i\bar{j}}$ as a (1,1)-form that represents pure variations of the metric which can be expanded as:

$$\delta\, g_{i\bar{j}} = \sum_{a=1}^{h^{1,1}} \tilde{t}^a\, b^a_{i\bar{j}}. \tag{2.40}$$

We have chosen $\tilde{t}^a$ to represent a real Kähler moduli coupled to a base of $(1,1)$-forms in the space $h^{1,1}$. This moduli will get an addition of contributions from the antisymmetric two form B, as we mentioned above, we can expand it in a base of $(1,1)$-form.This is known as the complexification of the Kähler form and results into a complexification of the moduli $\tilde{t}^a$ as:

$$\left(i\, \delta g_{i\bar{j}} + \delta B_{i\bar{j}}\right)\, dz^i \wedge d\bar{z}^{\bar{j}} = \sum_{a=1}^{h^{1,1}} t^a\, b^a. \tag{2.41}$$

On the other hand, we can inspect the contribution of $\delta\, g_{\bar{i}\bar{j}}$ in equation (2.38). This generates a $(0,1)$-form living in the holomorphic tangent bundle which corresponds to deformations of the complex structure [13]. In order to 'project' this form living in the tangent bundle to one of the homology group of our CY-manifold we can use our unique Ω holomorphic $(3,0)$-form to define an isomorphism between the tangent bundle space and $H^{2,1}(X)$ by defining a mixed form,

$$\Omega_{ijk}\delta g^k_{\bar{l}}\, dz^i \wedge dz^j \wedge dz^{\bar{l}}. \tag{2.42}$$

We can expand this form in the basis of harmonic $(2,1)$-forms on our CY-manifold as:

$$\Omega_{ijk}\delta g^k_{\bar{l}} = \sum_{a=1}^{h^{2,1}} t^a\, c^a_{ij\bar{l}}. \tag{2.43}$$

Where, in the same spirit as in equation (2.40), we define t^a as the complex structure moduli. These two different contributions of deformations of the metric g and the variation of the antisymmetric two form B define a natural metric of the moduli space. This reads:

$$ds^2 = \frac{1}{2\,\mathrm{Vol}_6} \int g^{i\bar{j}}\, g^{k\bar{l}} \left(\delta g_{ik}\, \delta g_{\bar{j}\bar{l}} + \left(\delta g_{i\bar{l}}\, \delta g_{k\bar{j}} - \delta B_{i\bar{l}}\, \delta B_{k\bar{j}}\right)\right) \sqrt{g}\, d^6x \tag{2.44}$$

The former part inside the parenthesis corresponds to deformations of the complex structure while the latter represents those ones associated with the complexified Kähler form. These completely separable metrics come from the product between two different moduli spaces $\mathcal{M}_{(1,1)} \times \mathcal{M}_{(2,1)}$. From the first part of the metric, we can derive the form of the Kähler potential in this space. We are going to define a set of $(2,1)$-forms such that:

$$\chi_a = \frac{1}{2}(\chi_a)_{ij\bar{k}}\, dz^i \wedge dz^j \wedge dz^{\bar{k}}. \tag{2.45}$$

Where we identify the components of the form as:

$$(\chi_a)_{ij\bar{k}} = -\frac{1}{2}\Omega^{\bar{l}}_{ij}\partial_{t^a}\, g_{\bar{k}\bar{l}}. \tag{2.46}$$

We can invert expression (2.46) to find how the metric components change. Plugging that into the first part of (2.44) we arrive to the formula:

$$ds^2 = 2\, G_{a\bar{b}}\, \delta t^a \delta t^{\bar{b}} = -2 \left(\frac{i \int \chi_a \wedge \bar{\chi}_{\bar{b}}}{\int \Omega \wedge \bar{\Omega}} \right) \delta t^a \delta t^{\bar{b}}. \tag{2.47}$$

We can now ask ourselves how the expression of the forms change depending on the coordinate system of t^a moduli we use. In fact, the variation of our Ω $(3,0)$-form with respect to this moduli can be understood as a linear combination of our χ_a form (which is based on (2.42)) and the $\Omega_{3,0}$-form times an element K_a depending on our moduli t^a. This reads:

$$\partial_a \Omega = K_a\, \Omega + \chi_a. \tag{2.48}$$

If we remember from equation (2.25), the metric of a moduli space can be derived from a Kähler potential. If we insert equation (2.48) in (2.47), while we consider the condition (2.25) one can see that the metric of this space is Kähler whose potential is described by:

$$\mathcal{K} = -\log \left(i \int \Omega \wedge \bar{\Omega} \right). \tag{2.49}$$

A next logical step would be to expand the ten dimensional forms and scalars $\{C_i, B_{MN}, g_{MN}, \phi\}$ in a perfect decomposition of a $\mathcal{M}_4 \otimes \mathcal{M}_6$. We do exactly this in next section, where we realise that all the masless scalar fields can be arranged in a set of $\mathcal{N} = 2$ multiplets. Of course, this is a nice approach, but still a set of rigid algebra conditions[11] not enough to describe a minimal supersymmetry (supergravity) in a four dimensional space-time.

One possibility to break more symmetries and to get effective theories in 4d is to compactify our Backgrounds on 6 dimensional toroidal orbifolds. Merging three $\mathbb{T}^2$ thorus as a 6d manifold, we can identify with finite symmetries points of it with a group G acting on the target space. These points can be identified among them, but several singularities will appear associated to these points. Far from these points, toroidal orbifolds seem like smooth manifolds whose structure is inherited from our $\mathbb{T}^6$ manifold. It is in this sense that we can consider a $\mathbb{T}^6$ as a singular limit of a Calabi-Yau manifold. In the next section, we offer a camly introduction and methodology about the structure and composition of these singular limits.

[11] We are looking for the richest supersymmetric solutions of the landscape.

2.3 Construction of $\mathbb{T}^6/\mathbb{Z}^2 \times \mathbb{Z}^2$ Orbifolds and Orientifolds

In this section we will construct all the geometric tools we need to define $\mathbb{T}^6/\mathbb{Z}^2 \times \mathbb{Z}^2$ Orbifolds and the action of Orientifolds. As we mentioned at the end of last section, this model is one of the easiest methods to compactify a type IIB theory in order to get an $\mathcal{N} = 1$ effective field theory in four dimensions. To achieve this, we need to include O-3 and O-7 planes, and at least, generalised background fluxes to give a proper description in the low dimensional theory (The so called STU models).

2.3.1 The $\mathbb{T}^6$ Torus

We briefly describe the factorization of a $\mathbb{T}^2$ torus on a Complex plane. As we can see in picture 2.3.1, the basic torus has been identified through the orientation of it different sides. As well, the parameter τ describes the deformation of the torus. This is totally related with the complex structure of the Calabi-Yau manifold in previous section. The element invariant can be written as,

$$ds^2 = |dx + \tau dy|^2 = dx^2 + |\tau|^2\, dy^2 + 2\, Re\tau\, dxdy. \tag{2.50}$$

$ds^2 = |dx + \tau dy|^2$ in the Complex plane. Demanding $det\, g = 1$ for the metric, we get:

$$\mathbf{g} = \frac{1}{Im\,\tau}\begin{bmatrix} |\tau|^2 & Re\,\tau \\ Re\,\tau & 1 \end{bmatrix}. \tag{2.51}$$

If we want to reproduce a limit description of a six-dimensional Calabi-Yau space, we can reproduce it with three different copies of the aforementioned torus. A part from that, instead of encoding the torus into a a complex space $\mathbb{C}$, we can identify every torus with a two manifold and a set of three different complex structures. The basis of forms which conform this space will be $\{\eta_i\}$ $i = 1...6$. As in [11] and [12] we will use Latin indices for horizontal directions and Greek ones for vertical.

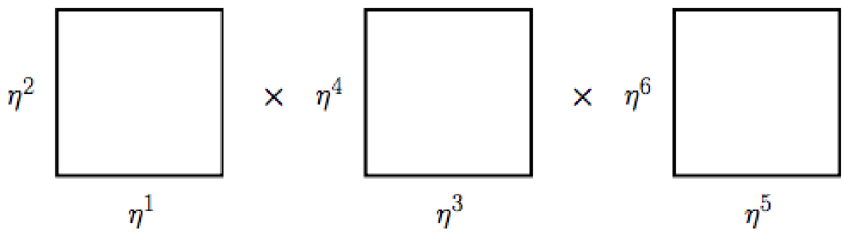


Figure 4. Torus identification

If we recall the general properties of Calabi-Yau manifolds in previous section, we had a hodge diamond of forms in our manifold. With this new description, we can reproduce set of forms that have the same properties of those ones living in the groups described with the Betti numbers. Indeed, we can reproduce set 2,3,4 and 6 [12] forms combining the 1-forms of the base through the

[12] 5-forms are not allowed. Continue reading for more information.

wedge product. Then:

$$\begin{aligned} \omega_1 &= \eta_1 \wedge \eta_2, \\ \omega_2 &= \eta_3 \wedge \eta_4, \\ \omega_3 &= \eta_5 \wedge \eta_6. \end{aligned} \tag{2.52}$$

The set of 4-forms can be obtained if we use the Hodge duality in this space, such that:

$$\begin{aligned} *_6\omega_1 &= \eta_3 \wedge \eta_4 \wedge \eta_5 \wedge \eta_6, \\ *_6\omega_2 &= \eta_1 \wedge \eta_2 \wedge \eta_5 \wedge \eta_6, \\ *_6\omega_3 &= \eta_1 \wedge \eta_2 \wedge \eta_3 \wedge \eta_4. \end{aligned} \tag{2.53}$$

For the 3-forms, we should not make the mistake of building up a product of the form $\alpha = \eta_1 \wedge \eta_2 \wedge \eta_3$ as the two first one-forms generate the ω_1 form, so we would not get forms that belong to the cohomological space. Then, we should try to build them with a "leg" in each torus (A 1-form). The possible combinations and their Hodge-dual ones read as follows:

$\alpha_0 = \eta_1 \wedge \eta_3 \wedge \eta_5$	$\beta^0 = *_6\alpha_1 = \eta_2 \wedge \eta_4 \wedge \eta_6$
$\alpha_1 = \eta_2 \wedge \eta_3 \wedge \eta_5$	$\beta^1 = *_6\alpha_1 = \eta_1 \wedge \eta_4 \wedge \eta_6$
$\alpha_2 = \eta_1 \wedge \eta_4 \wedge \eta_5$	$\beta^2 = *_6\alpha_2 = \eta_2 \wedge \eta_3 \wedge \eta_6$
$\alpha_3 = \eta_1 \wedge \eta_3 \wedge \eta_5$	$\beta^3 = *_6\alpha_3 = \eta_2 \wedge \eta_4 \wedge \eta_5$

Table 5. Set of allowed three-forms.

The volume in this 6 dimensional space can be obtained through the orientation:

$$\int_{\mathcal{M}_6} \eta_1 \wedge \eta_2 \wedge \eta_3 \wedge \eta_4 \wedge \eta_5 \wedge \eta_6 = \mathcal{V}_6. \tag{2.54}$$

It is useful to notice that the basis of cohomological forms satify:

$$\int_{\mathcal{M}_6} \alpha_0 \wedge \beta^0 = -\mathcal{V}_6, \qquad \int_{\mathcal{M}_6} \alpha_I \wedge \beta^J = \mathcal{V}_6 \delta_I^J, \qquad I, J = 1...3. \tag{2.55}$$

As a description of a limit of a Calabi-Yau Manifold, we should construct the two forms that determine the geometry of this space. The Kähler form $\mathbb{J}$ that belongs to $h_{1,1}$ cohomology group and the volume form $\Omega_{3,0}$ and its conjugate. The volume form can be built up from the one-forms describing the structure of the torus $dT^i = \eta_a + \tau_i \eta_b$ such that:

$$\Omega_{3,0} = dT^1 \wedge dT^2 \wedge dT^3 = \alpha_0 + \sum \tau_I \alpha_I + \sum \tau_1 \tau_2 \tau_3 \frac{\beta^I}{\tau_I} + \tau_1 \tau_2 \tau_3 \beta^0. \tag{2.56}$$

The Kähler form can be described in the basis of 2-forms ω_i and reads:

$$\mathbb{J} = \sum x_I \omega_I. \tag{2.57}$$

Where x_I is for the moduli that specifies the size of the torus I. If we come back to eq (2.50), the generalization of this metric for the compact space of three torus looks like:

$$dS_{T^6} = \sum_{I=1}^{3} \frac{A_I}{Im\, \tau_I} \left(|\tau_I|^2 (\eta_{2I-1})^2 + (\eta_{2I})^2 + 2\, Re\, \tau_I\, \eta_{2I-1} \otimes \eta_{2I} \right). \tag{2.58}$$

With all the forms that can be produced in this toroidal space, we can represent the Hodge Diamond introduced in section 2.2 for a $\mathbb{T}^6$ Torus such that:

$$\begin{array}{ccccccc} & & & \int \mathcal{V}_6 & & & \\ & & 0 & & 0 & & \\ & 0 & & \omega_i & & 0 & \\ \Omega_{3,0} & & \{\alpha_n\} & & \{\beta^n\} & & \Omega_{0,3} \\ & 0 & & *_6\omega_i & & 0 & \\ & & 0 & & 0 & & \\ & & & \Omega \wedge \bar{\Omega} & & & \end{array} \tag{2.59}$$

2.3.2 The $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold and Orientifold description

We can describe the action of the Orbifold group $\mathbf{G} = \mathbb{Z}_2 \times \mathbb{Z}_2$ through its generators acting on the 1-form basis of the $\mathbb{T}^6$ as:

$$\begin{aligned} \theta_1 &: (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6) \to (\eta_1, \eta_2, -\eta_3, -\eta_4, -\eta_5, -\eta_6), \\ \theta_2 &: (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6) \to (-\eta_1, -\eta_2, \eta_3, \eta_4, -\eta_5, -\eta_6). \end{aligned} \tag{2.60}$$

We can include as well in this generator's group the action of the operator $\mathbb{I}$ and $\theta_3 = \theta_1\theta_2$. Now we can check that all the forms that we have described in previous section are invariant under the action of this group, so the Orbifold description is consistent. But, if we want to compactify a type IIB theory on this geometry, we fail our goal of obtaining a $\mathcal{N} = 1$ in four dimensions. Instead, we still get a $\mathcal{N} = 2$, non-chiral 4-dimensional Supergravity.

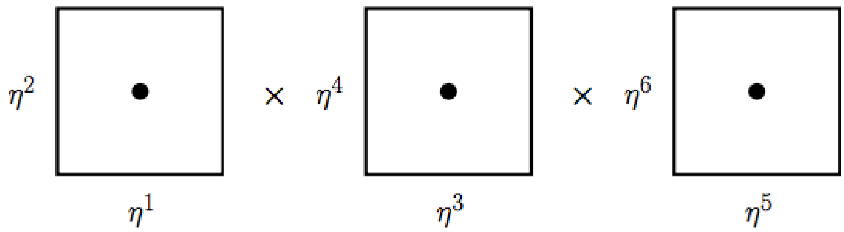


Figure 5. O3-plane located and fixed under the action of σ in $\mathbb{T}^6$

To achieve our desired target, we can introduce an extra reflection $\mathbb{Z}_2$ that will help us to fold down to $\mathcal{N} = 1$ the symmetry of our description. This so called "orientifold involution" acts on the space of 1-forms such that:

$$\sigma : (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6) \to -(\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6). \tag{2.61}$$

This operation keeps forms as ω_i and $*_6\omega_i$ even under the action and project out those ones composed by 3 different 1-forms which remain odd . If we include this operation in the group of generators of the $\mathbb{Z}_2^2$ we get a $\mathbb{Z}_2^3$ symmetry. The action of the object σ with the different elements of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ will create plane-mirror-symmetries which leave invariant the $\mathbb{T}^6$ space. These are:

$$\sigma\mathbb{I}(\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6) = -(\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6), \tag{2.62}$$

$$\begin{aligned}\sigma\theta_I(\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6) &= (+,+,-,-,+,+), \\ &= (-,-,+,+,+,+), \\ &= (+,+,+,+,-,-).\end{aligned} \tag{2.63}$$

The action of $\sigma\mathbb{I}$ in equation (2.62) creates a reflection of all the coordinates of the three torus in our space. This involves the creation of 4^3 O3-planes in the same number of determined fixed points, as we can see in figure 5.

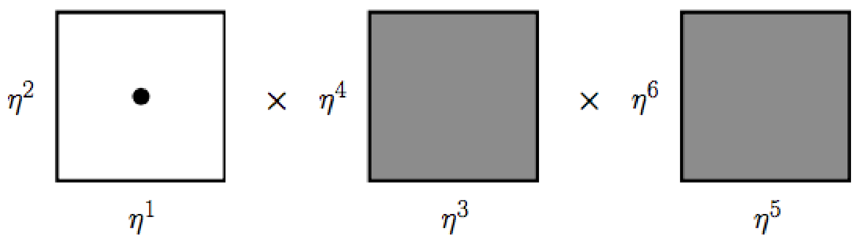


Figure 6. O7-plane wrapping in our manifold, fixed under the action of the $\sigma\,\theta_1$ group element

On the other hand, the action described in equation (2.63) reflects only one of the three torus, creating four different O7-planes in four fixed points of the internal space, as we can appreciate in figure 6. The number n in On-planes comes from the number of dimensions that remain invariant in our 9 spacial dimensions (compact and uncompact ones).

2.4 Compactifications of IIB theory beneath $\mathbb{T}^6/\mathbb{Z}^2 \times \mathbb{Z}^2 \bigcup (-1)^{F_L}\Omega_p\sigma$

With the previous sections above, we are fully equiped for embedding our ten dimensional description in 6 compact dimensions. In Calabi-Yau compactifications, we can choose an easy 10-dimensional block diagonal metric [15], whose element of line looks like:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{i\bar{j}}dy^i dy^{\bar{j}}. \tag{2.64}$$

Where $g_{\mu\nu}$ is the metric of the 4-dimensional Minkowski Space and $g_{i\bar{j}}$ belongs to the Calabi-Yau Manifold Y. This means, we can describe our 10-dimensional fields as:

$$A_{MNJK...}(x,y) = A_{\mu\nu\gamma\alpha...}(x) + F_{ijkl...}(y). \tag{2.65}$$

Where F represent whatever combination of forms up to same number of indices in ten dimensional description. Then, we can expand the forms appearing in equation (2.4) as:

$$G_{MN}(x,y) = g_{\mu\nu}(x) + g_{i\bar{j}}(y), \tag{2.66}$$

$$B_{MN}(x,y) = B_{\mu\nu}(x) + b^A(x)\omega_A(y), \tag{2.67}$$

$$C_{MN}(x,y) = C_{\mu\nu}(x) + c^A(x)\omega_A(y). \tag{2.68}$$

For C_4 we need to take several things into account . The first one is that there will be no dinamical part of this tensor in four dimensions. The second one is the fact that we still need to apply the self-dual constraint imposed by hand. That means from all the possible combination of forms, only half of them will have relevance in our description. The form looks like:

$$C_{MNPQ}(x,y) = D_2^A(x) \wedge \omega_A(y) + \rho_A(x) *_6 \omega_A(y) + V^k(x) \wedge \alpha_k(y) + U_k(x) \wedge \beta^k. \tag{2.69}$$

Where, we will have to choose one form from each package (D_2^A, ρ_A) and (V^k, U_k). All of this 4 dimensional forms must be grouped in multiplets given by a $\mathcal{N} = 2$ supersymmetry [15] such that:

Multiplet	Copies	Fields
Gravity Multiplet	1	$g_{\mu\nu}, V^0$
Vector Multiplet	$h_{2,1}$	V^k, z^k
hypermultiplet	$h_{1,1}$	ρ_A, b^A, c^A, x^A
double-tensor Multiplet	1	B_2, C_2, ϕ, C_0

Table 6. Multiplets in $\mathcal{N} = 2$.

Where we got rid of two forms of the pairs mentioned before. As we introduced the orientifold, we need to specify how this works over our fields. If we apply together the Fermionic number and the Worldsheet parity introduced in section 2.1.3, we see that our bosonic fields (From this point on, we will not care about fermionic fields) behave as:

	Even	Odd
$(-1)^{F_L}\Omega_p$	g, ϕ, C_0, C_4	B_2, C_2

Table 7. Action of $(-1)^{F_L}\Omega_p$.

This means, in order to keep all the bosonic fields invariant, the four dimensional fields B_2 and C_2 do not appear in our $4 \oplus 6$ dimensional description. As well, the action of the orientifold σ in the internal space will split the cohomology groups into even and odd forms, and we will have to choose those elements of the expansion that make the fields remain invariant under the orientifold action. We will use latin indices α, β for even forms. Otherwise, we will use a, b for Odd forms. So the forms will look like:

$$G_{MN}(x,y) = g_{\mu\nu}(x) + g_{i\bar{j}}(y), \tag{2.70}$$

$$B_{MN}(x,y) = b^a(x)\omega_a(y), \tag{2.71}$$

$$C_{MN}(x,y) = c^a(x)\omega_a(y), \tag{2.72}$$

$$C_{MNPQ}(x,y) = \rho_\alpha(x) *_6 \omega_\alpha(y) + V^k(x) \wedge \alpha_k(y). \tag{2.73}$$

With these new expansions, the multiplets in table 6 decouple into even and odd ones, such that:

Multiplet	Copies	Even Fields	Odd Fields
Gravity Multiplet	1/0	$g_{\mu\nu}$	
Vector Multiplet	$h_{2,1+}/h_{2,1-}$	V^k_{even}	z^k_{odd}
hypermultiplet	$h_{1,1+}/h_{1,1-}$	ρ_α, x_α	b^a, c^a
double-tensor Multiplet	1/0	ϕ, C_0	

Table 8. Multiplets in $\mathcal{N} = 1$ after Orientifold.

Following [15] we need to define a set of complex fields to describe the Kähler Metric of our space, as we did in section 2.2. With all the fields described in table 8 we can build up a set of complex fields that describe the complex structure, the kähler structure and the features inherited from the ten dimensional background. Actually, we saw this last complex field in section 2.1.3. This is the axio-dilaton,

$$S = C_0 + \imath e^{-\phi}. \tag{2.74}$$

If we recall the complex parameter τ_I in our torus description, was the one which describes the inner structure of the manifold. We can build up a complex field from scalars that belong to the Kaluza-Klein expansion of the metric. Those are ρ, x and z. The field z does not have the same dimension as the other two, so it is excluded. Then,

$$U_I = \tau_I = \rho_\alpha + i\, x_\alpha, \qquad I = 1...3. \tag{2.75}$$

To obtain the fields controlling the Kähler structure, it is necessary to derive them from the so called Kähler 4-form,

$$\mathcal{J} = C_4 + \frac{\imath e^{-\phi}}{2} (\mathbb{J} \wedge \mathbb{J}) \simeq \sum_{I=1}^{3} T_I *_6 \omega_I. \tag{2.76}$$

In previous equation, we composed the most general 4-form to obtain the remaining T_I fields. We included two copies of Kähler form times the complex part of the axio-dilaton, as a Weyl scaling. The moduli T_I are obtained as:

$$\begin{aligned} C_4 + \frac{\imath e^{-\phi}}{2} (\mathbb{J} \wedge \mathbb{J}) &= \sum_{I=1}^{3} T_I *_6 \omega_I, \\ \rightarrow \left(C_4 + \frac{\imath e^{-\phi}}{2} (x_I \omega_I \wedge x_J \omega_J) \right) \wedge \omega_K &= \sum_{I=1}^{3} T_I *_6 \omega_I \wedge \omega_K, \\ \int_{\mathcal{M}_6} C_4 \wedge \omega_K + \frac{\imath e^{-\phi}}{2} \int_{\mathcal{M}_6} (x_I \omega_I \wedge x_J \omega_J) \wedge \omega_K &= \sum_{I=1}^{3} T_I \int_{\mathcal{M}_6} *_6 \omega_I \wedge \omega_K, \\ \frac{1}{\mathcal{V}_6} \int_{\mathcal{M}_6} \left(C_4 \wedge \omega_K + \imath e^{-\phi} (x_J x_K) \right) &= T_I. \end{aligned} \tag{2.77}$$

With all the moduli defined in equations (2.74),(2.75) and (2.77) we can described the dynamics of the space of this fields with a Kähler potential, as we did in section 2.2, such that:

$$K(S,T,U) = -\left(\log(-\imath(S-\bar{S})) + \sum_{I=1}^{3}\log(-\imath(U_I-\bar{U_I})) + \sum_{I=1}^{3}\log(-\imath(T_I-\bar{T}_I))\right). \qquad (2.78)$$

This potential requieres an explanation. The term depending on U_I fields is the analogue one to the Kähler potential in equation (2.49). This part of the equation (2.78) takes into account the complex-structure moduli space, but, due to the mirror symmetries in the inner space, it is neccesary to include those Kähler-structure moduli T_I that would behave as complex-structure moduli from a IIA theory compactification [16]. As well, the contribution of the dilaton must appear, as the field expands $\phi(x,y) = \phi(x)\phi(y)$ and affects our six dimensional space in a natural way. This set of seven fields Φ_I will span a group $\left(\frac{SL(2,\mathbb{R})}{SO(2)}\right)^7$.

The dynamics of these seven moduli fields Φ_I are described in terms of the Lagrangian density,

$$\mathcal{L}_{moduli} = \sqrt{-g}K_{I\bar{J}}\partial_\mu\Phi^I\partial^\mu\bar{\Phi}^J - V(\Phi). \qquad (2.79)$$

These fields are couple to gravity and the Kähler Metric of the moduli field space is:

$$K_{I\bar{J}} = \frac{\partial^2 K}{\partial\Phi_I\partial\Phi_{\bar{J}}}. \qquad (2.80)$$

Where the potential part in equation (2.79) is 0. In that sense, our seven moduli will not acquire a vacuum expected value (vev) so they can not be stabilized at a minimum. This can be partially solved in presence of background fluxes, switching them on in the Chern-Simons term in equation (2.4). Acting like this, we will induce a non trivial $V(\Phi) \neq 0$ through interacting fields, which will allow us to determine their minimal value. This is the content of the next chapter.

3 Generalized compactifications

As we mentioned in previous chapter, unless we do not activate background fluxes, the potential in the Langrangian (2.79) will remain 0. But we did not define the form of this potential. In this chapter, based on the results extracted from the previous one, we will build up all the remaining p-forms and functions to come up with a description of an effective four dimensional potential for our moduli fields. To preserver supersymmetry, we need a holomorphic function depending on our fields, that we call superpotential W. This will require the presence of geometric and non-geometric fluxes, in order to keep the inner dualities of the moduli unbroken. After that, we will raise up our six dimensional compactified description to the effective four dimensional one. The last part of this chapter defines the constraints imposed in the inner algebra of our fluxes.

3.1 Creation of Superpotential W

We will start describing the form of the superpotential W. As it is stated in [17], in a $\mathcal{N} = 1$ supergravity, The superpotential must be an object depending on the moduli fields present in the theory, describing the dynamics of those fields in a moduli space. This object must be covariant under the action of the symmetries present in the moduli fields. In that spirit, we can build up a holomorphic 'function' such that:

$$W = \alpha + S\,\beta. \tag{3.1}$$

Where α, β are holomorphic objects and we have decided to start with S-modulus for simplicity. The previous equation represents a sketch of a holomorphic 'function' which must be covariant[13] under $SL(2,\mathbb{R})_S$. Again, for simplicity in our concepts, we choose the subgroup $SO(2,\mathbb{R})_S$. If so, previous holomorphic 'function', after an inversion of the S-moduli looks like:

$$W' = \frac{\alpha\, S - \beta}{S}. \tag{3.2}$$

We can see, that in order to preserve the initial form of the function in the numerator[14], the objects α, β should transform as well under the action of the inversion, one into each other. If we remember, in section 2.1.3, we saw that B_2 and C_2 belong to an extended $SL(2,\mathbb{Z})$ group. As we saw in equations (2.71, 2.72), these background forms have an easy description in the our $\mathbb{T}^6$ manifold. So, we could think these forms are good candidates to substitute the objects α and β in equation (3.2). To prove our first attemp wrong, we need to notice that even if we integrated on the whole $\mathbb{T}^6$ space through the (0,3) Ω volume form, we would not get a function[15], but a 1-form. In that sense, the exterior derivative of these forms, dC_2 and dB_2 (Better known by F_3 and H_3 respectively) are good candidates to close the space and get a holomorphic function result. In fact, if we check their behaviour under a $SO(2,\mathbb{R})$ group:

$$\begin{bmatrix} F_3 \\ H_3 \end{bmatrix}' \rightarrow \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} F_3 \\ H_3 \end{bmatrix} = \begin{bmatrix} H_3 \\ -F_3 \end{bmatrix}. \tag{3.3}$$

Taking α as F_3 and β as $-H_3$ and integrating over the whole $\mathbb{T}^6$ manifold using the (0,3) volume-form Ω, we have a holomorphic function of the form:

[13] The invariance of the superpotential under discrete displacements is shown at AppendixB

[14] we will see later in the chapter what happens with the denominator.

[15] This is the reason why we used ' ' before. It was not really a function.

$$W = \int_{\mathbb{T}^6} (F_3 - S\, H_3) \wedge \Omega. \tag{3.4}$$

Which is the well-known superpotential for $\mathcal{N} = 1$ in four dimensional space-time derived by Gukov, Vafa and Witten in [17].

Now, we need to develop the structure of the three-forms H, F in our $\mathbb{T}^6$ manifold. If we remember, we developed a set of (2,1) and (3,0)-forms in table 2.3.1. We can expand our three forms in the most general basis of these forms which are invariant under the action of our orientifold. The most general expressions for both of them are:

$$\bar{H} = \alpha_0\, b_3 + \alpha^I\, b_{2,I} + \beta^I\, b_{1,I} + \beta^0\, b_0, \tag{3.5}$$

$$\bar{F} = \alpha_0\, a_3 + \alpha^I\, a_{2,I} + \beta^I\, a_{1,I} + \beta^0\, a_0. \tag{3.6}$$

Where we have choose $a_{i,I}, b_{i,I}$ as parameters of the fluxes. These parameters are the responsible ones for giving a non-zero value when integrating each flux on a 3-cycle section of the $\mathbb{T}^6$. If we compute the superpotential (3.4), using the expansion (2.56) for the volume-form, we will obtain a more specific expression of this, such as:

$$W = \int_{\mathbb{T}^6} (F_3 - S\, H_3) \wedge \Omega = P_1(U_I) + S\, P_2(U_I). \tag{3.7}$$

Where P_i Represents a polynomial of third order depending on the complex structure $\tau_I = U_I$, such that:

$$P_1(U_I) = a_0 - \sum_{K=1}^{3} a_1^K U_K + \sum_{K=1}^{3} a_2^K \frac{U_1 U_2 U_3}{U_K} - a_3 U_1 U_3 U_3, \tag{3.8}$$

$$P_2(U_I) = -b_0 + \sum_{K=1}^{3} b_1^K U_K - \sum_{K=1}^{3} b_2^K \frac{U_1 U_2 U_3}{U_K} + b_3 U_1 U_3 U_3. \tag{3.9}$$

This result was obtained by substituting τ_I in the volume-form for U_I as we saw in (2.75) and using the integration rules derived in (2.55). As we stated at the beginning of the section, the superpotential must depend on the interacting moduli of the theory. But, as we check from equation (3.7) we still lack the presence of the Kähler T-moduli, which makes our superpotential gain a scale structure. At this point, it is recommended to take a step back and reconsider our description.

This Master thesis has been developed in the IIB framework but, What is the IIA interpretation of all of these? As we stated in the introduction, both frameworks are related via T-duality in some specific dimension. If we reproduced the same steps we have done until this moment in that framework, we would arrive to a superpotential of the form:

$$W_A = P_1(T_I) + S\, P_2(T_I) + U\, P_3(T_I). \tag{3.10}$$

We can appreciate the presence of our seven moduli, where T_I and U_I have interchanged roles (U-moduli in the IIA description again has to do with the Kähler structure of the manifold [16]). As both theories are related via duality, they must offer a superpotential describing the same physics. At this point, we need to extend our fluxes in order to find a superpotential with a scale structure in both theories. This extension is called generalized fluxes and is explained in next section.

3.1.1 Generalized Fluxes

As it is stated in [10, 11] we can perform a set of T-dualities in our NS-NS $\bar{H}_3$ fluxes that lead to generalized fluxes. This transformation would have a conceptual look as:

$$\bar{H}_{123} \xrightarrow{T_a} \omega^1_{23} \xrightarrow{T_b} Q^{12}_3 \xrightarrow{T_c} R^{123}. \tag{3.11}$$

Each T_i-duality switches between a IIB and IIA flux description. This set of NS-NS fluxes would define a supergravity algebra invariant under T-duality [11]. We know that ω^1_{23} and R^{123} are fluxes that belong to a IIA theory [10, 16]. In fact, both are even under the action of the orientifold (2.61) in a type IIB string theory compactification, and they are projected out. But this is not the case for Q^{12}_3, although their components are limited by the action of the orbifold (2.60). Specifically, up to 24 components, as we can see in next table:

Flux directions	Components	Flux parameter
Q_a^{bc}	$Q_1^{35}, Q_3^{51}, Q_5^{13}$	$\tilde{c}_1^1, \tilde{c}_1^2, \tilde{c}_1^3$
$Q_a^{b\gamma}$	$Q_1^{36}, Q_3^{52}, Q_5^{14}$	$\hat{c}_2^1, \hat{c}_2^2, \hat{c}_2^3$
$Q_a^{\beta c}$	$Q_1^{45}, Q_3^{61}, Q_5^{23}$	$\check{c}_2^1, \check{c}_2^2, \check{c}_2^3$
$Q_a^{\beta\gamma}$	$Q_1^{46}, Q_3^{62}, Q_5^{24}$	c_3^1, c_3^2, c_3^3
Q_α^{bc}	$Q_2^{35}, Q_4^{51}, Q_6^{13}$	c_0^1, c_0^2, c_0^3
$Q_\alpha^{\beta c}$	$Q_2^{45}, Q_4^{61}, Q_6^{23}$	$\hat{c}_1^1, \hat{c}_1^2, \hat{c}_1^3$
$Q_\alpha^{b\gamma}$	$Q_2^{36}, Q_4^{52}, Q_6^{14}$	$\check{c}_1^1, \check{c}_1^2, \check{c}_1^3$
$Q_\alpha^{\beta\gamma}$	$Q_2^{46}, Q_4^{62}, Q_6^{24}$	$\tilde{c}_2^1, \tilde{c}_2^2, \tilde{c}_2^3$

Table 9. Allowed Q-fluxes in our Orbifold description

The main issue with this flux is its lack of global geometrical interpretation. After one T-duality, the fluxes ω^1_{23} can be interpreted as a form factor in the algebra of a twisted $\mathbb{T}^6$ geometry. But after a second T-duality from our H_3 NS-NS flux, not a global, but a local geometrical interpretation could be achieved. This local interpretation requires the addition of T-maps to the usual transition ones between patches in a manifold description [10]. The next issue we face is to include this flux in our sperpotential W. As we are looking for a T_i explicit dependence of our W, we can contract our Q-flux with the Kähler 4-form (2.76) such that:

$$(Q.\mathcal{J})_{ijk} = \frac{1}{2} Q^{mn}_{[i} (\mathcal{J})_{jk]mn} \tag{3.12}$$

This will ensure us that we obtain a proper (3,0)-form for the flux. This can be expanded in a basis of (3,0) and (2,1)- forms, as we previously did. In fact, (2.76) carries the scalars T_i, but the base of forms $*_6 w_I$ has been contracted with the Q^{jk}_i, which must be completed with the base (2.3.1):

$$Q.\mathcal{J} = T_K \left(\square^K_3 \, \alpha_0 + \square^{??}_2 \, \alpha_j + \square^{??}_1 \, \beta_j + \square^K_0 \, \beta_0 \right). \tag{3.13}$$

Where the boxes represents objects whose dimension and indices we do not know yet. K runs from 0 to 3 for the Kähler moduli. As α_0, β_0 are unique $(3,0)$- forms, it is clear that $\Box_0$ and $\Box_3$ must be scalars that we will call c_3 and c_0, respectively. For boxes one and two, we can realise that there must be two indices running. One for each of the $(2,1)$-forms α_j, β_j and another one for the moduli T. Then, the aspect of this contracted object is:

$$Q.\mathcal{J} = T_K \left(c_3^K\, \alpha_0 - c_2^{jK}\, \alpha_j + c_1^{jK}\, \beta_j + c_0^K\, \beta_0 \right). \tag{3.14}$$

Where the matrices have the form:

$$c_i^{jK} = \begin{bmatrix} -\tilde{c}_i^1 & \check{c}_i^3 & \hat{c}_i^2 \\ \hat{c}_i^3 & -\tilde{c}_i^2 & \check{c}_i^1 \\ \check{c}_i^3 & \hat{c}_i^1 & -\tilde{c}_i^3 \end{bmatrix}, \qquad i = 1, 2. \tag{3.15}$$

The upper index belongs to the correspondant moduli T_i while the different 'hat' of c_i runs for the different components of the basis α, β. The contribution of this Q-flux in the superpotential W will take the form of:

$$\int_{\mathbb{T}^6} (Q.\, \mathcal{J}) \wedge \Omega = P_3^K(U_I) T_K. \tag{3.16}$$

Where we obtained P as a polynomial of third order calculating with the same integration rules as for (3.4). This polynomial is:

$$P_3(U_I) = c_0^k + \sum_i^{i=3} c_1^{iK} U_i - \sum_i^{i=3} c_2^{iK} \frac{U_1 U_2 U_3}{U_i} - c_3^K U_1 U_2 U_3. \tag{3.17}$$

This polynomial completes the presence of all the moduli S, T_i, U_i in our superpotential. It makes it T-duality invariant, but breaks its S-duality. In fact, if we sketched a S-duality, the new superpotential would like:

$$W' = \frac{\alpha\, S + \beta + S\, T\, \gamma}{S}. \tag{3.18}$$

As we did in (3.2), we can not related γ to another partner, as in the case for α, β. In order to keep everything invariant, we can create a new flux P in the same spirit as the Q one. This new P flux will be the S-dual partner of Q, as:

$$\begin{bmatrix} Q \\ P \end{bmatrix}' \to \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} Q \\ -P \end{bmatrix}. \tag{3.19}$$

This way, we ensure ourselves that S-duality will affect our superpotential by pairs, and each term will have a S-dual partner. This P flux can be described exactly the same way we did for the Q-flux. To contract it with the Kähler four-form and identifying the terms of the matrices. The form of the superpotential, with all the terms included, is:

$$W = \int_{\mathbb{T}^6} ((F_3 - S\, H_3) + (Q - S\, P).\mathcal{J}) \wedge \Omega =$$
$$= P_1(U_I) + S\, P_2(U_I) + T_K\, P_3^K(U_I) + S\, T_K\, P_4^K(U_I). \tag{3.20}$$

P-flux has the same structure and parameters as the Q-flux polynomial (3.17) with reverse sign. This complete superpotential is invariant under $SL(2, \mathbb{R})$ S and T-dualities. It is described by 64 parameters, such that $8 + 8$ come from the geometrical H_3, F_3 fluxes and $24 + 24$ come from the generical allowed Q_i^{jk}, P_i^{jk} parameters.

3.2 Isotropic Supergravity flux models

So far, we have described a four dimensional $\mathcal{N} = 1$ supergravity model with seven different moduli whose dynamics depend on the inner structure of our $\mathbb{T}^6$ manifold. But, as we commented in last section, this description depends on up to 64 different parameters, what could make our calculations a completely chaos. In order to keep things under control, we can introduce an extra feature that will simplify our calculations generously. We can add a symmetry to our $\mathbb{T}^6$, that relates the 3 different $\mathbb{T}^2$ to each other. This will reduce the two triads of T_K and U_K to a single pair of Kähler and Complex structure forms, such that:

$$U_1 = U_2 = U_3 = U, \qquad T_1 = T_2 = T_3 = T. \tag{3.21}$$

This will have an immediate effect in our flux parameters, such that the amount will be reduce from 64 to 20[16]. This means the summation on the index K will drop and be substituted by a three factor when needed. After this Isotropic symmetry, the different monomomials that belong to the polynomials P_i in equation(3.20) can be identify with specific parameters as we can see in next table:

[16]The effect of the symmetry will reduce it to 24. The remaining four parameters can be reduced by constraints, as we will see in 3.49

Coupling	Type IIB Fluxes	Flux parameter
1	F_{abc}	a_0
U	$F_{ab\gamma}$	a_1
U^2	$F_{a\beta\gamma}$	a_2
U^3	$F_{\alpha\beta\gamma}$	a_3
S	H_{abc}	$-b_0$
SU	$H_{ab\gamma}$	$-b_1$
SU^2	$H_{a\beta\gamma}$	$-b_2$
SU^3	$H_{\alpha\beta\gamma}$	$-b_3$
T	$Q_c^{\alpha\beta}$	c_0
TU	$Q_c^{\alpha b} = Q_c^{a\beta}$, $Q_\alpha^{\beta\gamma}$	$c_1, \bar{c}_1$
TU^2	$Q_\gamma^{a\beta} = Q_\gamma^{\alpha b}$, Q_c^{ab}	$c_2, \bar{c}_2$
TU^3	Q_γ^{ab}	c_3
ST	$P_c^{\alpha\beta}$	$-d_0$
STU	$P_c^{\alpha b} = P_c^{a\beta}$, $P_\alpha^{\beta\gamma}$	$-d_1, -\bar{d}_1$
STU^2	$P_\gamma^{a\beta} = P_\gamma^{\alpha b}$, P_c^{ab}	$-d_2, -\bar{d}_2$
STU^3	P_γ^{ab}	$-d_3$

Table 10. Relation between Superpotential monomials and flux parameters.

One last effect of this extra $\mathbb{Z}^3$ symmetry is that the potential of the moduli-space (2.78) will look like:

$$K(S,T,U) = -\left(\log(-i\,(S-\bar{S})) + 3\log(-i\,(U-\bar{U})) + 3\log(-i\,(T-\bar{T}))\right). \tag{3.22}$$

And a superpotential W reduced to the form:

$$W(S,T,U) = P_1(U) + S\,P_2(U) + T\,P_3(U) + S\,T\,P_4(U). \tag{3.23}$$

Where the polynomials $P_i(U)$ can be found in equations (A.2-A.5).

3.2.1 Inner Dualities of the Superpotential

One of the main features of the superpotential (3.23) is the set of inner symmetries that it has. This is driven by the $SL(2,\mathbb{Z})$ group that rules the transformation of each of the moduli fields S,T,U. As we saw in 2.1.3, this is mainly driven by modular transformation, inversion of the components of the group and reflections. In this section we add a dilation to the moduli transformation, as this is allowed by a $SL(2,\mathbb{Z})$ group. We first talk about the different inversion of the moduli and how this affects the (super)potential.
For S-modulus, an inversion will behave as:

$$W\left(S \to \frac{-1}{S}\right) = \frac{-S\,P_1 - P_2 - S\,T\,P_3 - T\,P_4}{S}. \tag{3.24}$$

This implies that the flux parameters of polynomials P_1 and P_2 are interchanged as:

$$a_0 \to -b_0, \tag{3.25}$$
$$a_1 \to -b_1, \tag{3.26}$$
$$a_2 \to -b_2, \tag{3.27}$$
$$a_3 \to -b_3. \tag{3.28}$$

In the same way, $c_i, \bar{c}_i$ are interchanged with $d_i, \bar{d}_i$. On the other hand, if we perform an inversion in the T-modulus, the superpotential would transform as:

$$W\left(T \to \frac{-1}{T}\right) = \frac{-T\,P_1 - S\,T\,P_2 - P_3 - S\,P_4}{T}. \tag{3.29}$$

Which indicates that there must exist are relation between P_1, P_2 and P_3, P_4 in order to preserve the symmetries. The flux parameters are related as:

$$a_0 \to -3\,c_0, \tag{3.30}$$
$$a_1 \to -2\,c_1 + \bar{c}_1, \tag{3.31}$$
$$a_2 \to 2\,c_2 - \bar{c}_2, \tag{3.32}$$
$$a_3 \to -3\,c_3. \tag{3.33}$$

The same relations exist for polynomial elements of P_2 and P_4. Not so complicated is the inversion of the U-modulus $U \to -1/U$. A superpotential of the form (3.10) will transform as:

$$W \sim 1 + U + U^2 + U^3 \to \frac{-1 - U - U^2 - U^3}{U^3} \sim \frac{-W}{U^3}. \tag{3.34}$$

This enforces a symmetry among the flux parameters to keep the superpotential invariant, such that:

$$a_0 \to a_3, \tag{3.35}$$
$$a_1 \to -a_2, \tag{3.36}$$
$$a_2 \to a_1, \tag{3.37}$$
$$a_3 \to -a_0. \tag{3.38}$$

And the same applies for b_i parameters, so the geometrical flux parameters behave in an identical way. For our $P_i(U)$ non- geometrical parameters, we only need to add the transformation set for tilde parameters, so:

$$\tilde{c}_1 \to -\tilde{c}_2, \tag{3.39}$$
$$\tilde{c}_2 \to \tilde{c}_1. \tag{3.40}$$

This leaves us with a $-1/U^3$ factor from (3.34). This factor is removed with the action of the Kähler moduli and metric in equation (3.10), as the inner term factor $-1/U^3$ will be countered by the action of the Kähler potential in the exponential dropping a $-U^3$ factor. This cancellation takes place as well with inversions of the S and T-moduli, where the denominator created during the inversion is compensated with a term arising from the Kähler metric in the exponential of the potential we will study in section 3.4.

The next set of transformations affects the 3 different moduli in the same way, but the behaviour of their inner parameters after these shifts is different. The main idea is to perform a general shift with dilation of the form:

$$\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} \quad S \to a\,S + b. \tag{3.41}$$

If we set $a = 1$ for a pure shift and plug this into (3.10) we get:

$$\begin{aligned} W(S,T,U) &= P_1(U) + (S+n)\,P_2(U) + T\,P_3(U) + (S+n)\,T\,P_4(U) = \\ &= (P_1(U) + n\,P_2(U)) + S\,P_2(U) + T\,(P_3(U) + n\,P_4(U)) + S\,T\,P_4(U). \end{aligned} \tag{3.42}$$

Which means a change in the contribution for the flux parameters in H_3 and Q_3. This linear transformations are displayed in Appendix B. Similarly, we can perform a modular transformation in the T-modulus. In this case, the change of the superpotential will be:

$$\begin{aligned} W(S,T,U) &= P_1(U) + S\,P_2(U) + (T+m)\,P_3(U) + S\,(T+m)\,P_4(U) = \\ &= (P_1(U) + m\,P_3(U)) + S\,\,(P_2(U) + m\,P_4(U)) + T\,P_3(U) + S\,T\,P_4(U). \end{aligned} \tag{3.43}$$

Which relates the polynomials in a different way as (3.42). Rearranging the terms in those polynomials under the same monomials U_i, SU_i we can get the new parameters (B.11-B.15) for a_i and similarly for b_i, leaving c_i, d_i invariant.

The last possible shift we can perform is in our complex structure modulus U. This will generate a geometric transformation with a long set of monomials to rearrange, as we will have to expand values of the form $(U + r)^p$ where p goes from 0 to 3. This means we will have third order factors with zeroth order monomials U and viceversa. After the rearrangement of terms, the parameters look like (B.16-B.19) for the geometric parameters a_i,b_i and a little more tricky ones (B.20-B.25) for non-geometric parameters.

3.3 Flux Algebra and Tadpole Cancellations

In the previous section we have stated the existence of different fluxes further than the geometrical background $\bar{H}_3$, $\bar{F}_3$. As in (3.11), performing a set of string-dualities we can complete a whole picture of fluxes that included in the superpotential (3.23) will give the same physics description in IIA and IIB theories. The set of dualities in (3.11) suggests a possible interpretation of different layers of algebra on a string background [10]. The role of the flux parameters can be interpreted as structure constants of those algebras. On the other hand, we can interpret our (non) geomeometrical fluxes as "twist" factors that affect the metric in the $\mathbb{T}^6$ description, which must fulfil a set of Bianchi identities, as:

$$\bar{F}_{[w}\bar{H}_{xyz]} = 0. \tag{3.44}$$

Which arise from the integration over a four-cycle of the differential form whose "torsion" parameter is the flux. For further reading, we recommend [10, 16]. In this work, we will follow [11] and [12] interpretation. It results that the duality chain (3.11) include a whole set of fluxes that fill out a $GL(6)$ representation for a $\mathcal{N} = 4$ supergravity group invariant under T-transformation. The set of fluxes behaves as structure constants of 6 Kaluza-Klein generators Z_a and 6 B-field gauge generators X^a. The commutators of the supergravity algebra $GL(6)$ are:

$$[Z_a, Z_b] = \bar{H}_{abc}X^c + \omega^c_{ab}Z_c, \tag{3.45}$$
$$[Z_a, X^b] = -\omega^b_{ac}X^c + Q^{bc}_a Z_c, \tag{3.46}$$
$$[X^a, X^b] = Q^{ab}_c X^c + R^{abc}Z_c. \tag{3.47}$$

As we mentioned above, ω^a_{bc} and R^{abc} fluxes are even under the orientifold (2.61) and are projected out, so the algebra calculations reduce generously. This offers a T-fold description of this supergravity in our IIB orientifold. The different gauge/isometric subgroups will be determined by the Jacobi identities of the algebra (3.45). If we force the Q-flux to satisfy the $[X, [X, X]]$ Jacobi identity, we can determine the constraints in the gauge group as:

$$Q^{[ab}_d\, Q^{c]d}_e = 0. \tag{3.48}$$

This will result in 8 conditions paired in identical copies from all the possible index choices in (3.48). An important issue that must be solved from these 8 conditions is the complexity of $\bar{c}_i$ if we do not consider a symmetry between $Q^{a\beta}_c$ and $Q^{\alpha b}_c$ fluxes. Imposing that symmetry, it follows [11]:

$$\check{c}_1 = \hat{c}_i = c_1, \qquad\qquad i = 1, 2. \tag{3.49}$$

Setting this result into the 8 paired conditions from the Jacobi constraints, we obtain a set of 3 equations as:

$$c_0(c_2 - \bar{c_2}) + c_1(c_1 - \bar{c_1}) = 0, \tag{3.50}$$
$$c_2(c_2 - \bar{c_2}) + c_3(c_1 - \bar{c_1}) = 0, \tag{3.51}$$
$$c_0c_3 - c_1c_2 = 0. \tag{3.52}$$

With the same goal, we can compute the Jacobi identity for the isometries Z_a. This Jacobi constraint will result into a mixed gauge-isometry condition that relates geometrical and non geometrical fluxes such that:

$$[Z_a, [Z_b, Z_c]] + \text{ perm.} = \bar{H}_{bcd}[Z_a, X^d] + \text{ perm.} = \bar{H}_{b[cd}Q^{ad}_{e]} = 0. \tag{3.53}$$

Where this set of constraints XXX and ZZZ represent the coset space of the supergravity algebra and the gauge subalgebra of X^a. These two algebras are sufficient and necessary to describe an effective supergravity theory. If we plug in expression (3.53) the flux parameters displayed on table (10) we obtain a set of conditions not invariant under S-transformation of the form:

$$c_0 b_2 - c_2 b_0 + (c_1 - \bar{c_1}) b_1 = 0, \tag{3.54}$$
$$c_0 b_3 - c_3 b_0 + (c_1 - \bar{c_1}) b_2 = 0, \tag{3.55}$$
$$c_1 b_2 - c_3 b_0 - (c_2 - \bar{c_2}) b_1 = 0, \tag{3.56}$$
$$c_1 b_3 - c_3 b_1 - (c_2 - \bar{c_2}) b_2 = 0. \tag{3.57}$$

We can act with S-duality on the conditions (3.12,3.53), as we want our constraint S-invariant. If we act systematically with this, we find a triplet of pure non geometrical constraints, such that:

$$Q_d^{[ab}\, Q_e^{c]d} = 0 \;\longleftrightarrow\; Q_d^{[ab}\, P_e^{c]d} + P_d^{[ab}\, Q_e^{c]d} = 0 \;\longleftrightarrow\; P_d^{[ab}\, P_e^{c]d} = 0. \tag{3.58}$$

This first condition is exactly the same as in (3.48) and the second one is a copy of this one in the $SL(2,\mathbb{Z})_S$ dual P flux of Q. The third one rules out the conditions that both flux must follow with each other. The equation constraints for this algebra conditions can be found in (A.13 , A.18).

The last constraint to be S-dualised is (3.53). It results in a $SL(2,\mathbb{Z})$singlet mix of the four fluxes developed for this IIB theory, read as:

$$\bar{H}_{b[cd} Q_{e]}^{ad} - \bar{F}_{b[cd} P_{e]}^{ad} = 0. \tag{3.59}$$

Which now is a completely S-dual invariant constraint and, after plug in the flux parameters in table (10) it yields a set of equations(A.19 , A.22).

3.3.1 Tadpole Cancellations

Not only the supergravity algebra in a $\mathbb{T}^6$ manifold will constrain our flux parameters. With no background fluxes, the R-R fields in (2.6) are limited by the action of a set of Bianchi identities with sources, such that:

$$d\, F_1 = 0, \tag{3.60}$$
$$d\, \tilde{F}_3 = *_6\, j_6, \tag{3.61}$$
$$d\, \tilde{F}_5 = *_6\, j_4. \tag{3.62}$$

As we switch on the non-geometrical fluxes, these identities must be promoted to more general ones as the sources can be spread on higher p-branes. We only have one geometrical background flux that belongs to the R-R sector, $\bar{F}_3$. This one can be combined with other fluxes and understood as tadpole cancellations conditions of the different C_i forms present in our description. In the IIB theory, the first tadpole condition observed is that one from eq (2.8). The total charge of the orientifold is -16[17] and it is totally distributed on the 64 O3-planes located at the singular points of our orientifold (2.61). This charge can be compensate with the addition of D3-branes [11, 16].

[17] The total charge of these O-planes can be calculated through the expression $Q_{D_p} = -2^{p-5} \times$ #fixed p. in σ.

If we include fluxes (3.5,3.6) in equation (2.8) and the value of the sources, after calculations, the cancellation condition is:

$$a_0 b_3 - 3a_1 b_2 + 3a_2 b_1 - a_3 b_0 = 16 - N_{D3}. \tag{3.63}$$

Normally, we will eliminate the presence of extra D3 branes ($N_{D3} = 0$) in order to keep our calculations easier.

For the next set of tadpole conditions, we must remember that a p-brane is an extended object that couples to a set of $p+1$-forms. In a IIB theory, we have a set of C_{2p}-forms. Those ones allowed by our orientifold (2.61) are electrically coupled to D3 and D7-branes. A $SL(2,\mathbb{Z})$ triplet of C_8 is coupled to D7-brane with electrical charges p^2, r, q^2 respectively. [11]. As this forms are 8-forms, in order to contract them in the whole $\mathcal{M}_{10}$ supergravity space, we need to contract them with a set of 2-forms related with the fluxes. This set of two-forms are related with our $\bar{F}_3$ and $\bar{H}_3$ (because of the RR-RR and NS-NS dependence) and the non geometrical fluxes Q and P. We can contract our non-geometrical fluxes with geometrical ones, as we did in (3.12), so they can be reduce to 2-forms and contracted with C_8^i as:

$$\int_{\mathcal{M}_{10}} C_8 \wedge (Q \cdot \bar{F}_3), \qquad \int_{\mathcal{M}_{10}} C_8' \wedge (Q \cdot \bar{H}_3 + P \cdot \bar{F}_3), \qquad \int_{\mathcal{M}_{10}} \tilde{C}_8 \wedge (P \cdot \bar{H}_3). \tag{3.64}$$

The first contribution of $C_8 \wedge (Q \cdot \bar{F}_3)$ yields:

$$a_0 c_3 + a_1(2c_2 - \bar{c}_2) - a_2(2c_1 - \bar{c}_1) - a_3 c_0 = N_7. \tag{3.65}$$

Where we will normally set $N_7 = 0$ for easy. For the second and third tadpole cancellation conditions in (3.64) for the R-R potentials we have:

$$\begin{aligned} &b_0 c_3 + b_1(2c_2 - \bar{c}_2) - b_2(2c_1 - \bar{c}_1) - b_3 c_0 + \\ &a_0 d_3 + a_1(2d_2 - \bar{d}_2) + a_2(2d_1 - \bar{d}_1) - a_3 d_0 = N_7', \end{aligned} \tag{3.66}$$

$$b_0 d_3 + b_1(2d_2 - \bar{d}_2) - b_2(2d_1 - \bar{d}_1) - b_3 d_0 = \bar{N}_7. \tag{3.67}$$

The values $N_7', \bar{N}_7$ are related to the number of bound states of D7-branes, but they will be set to 0 during this article [18].

3.4 Generalized potential V and effective description

The general form of a four dimensional $\mathcal{N} = 1$ scalar potential is [19]:

$$V = e^{\frac{K}{M_{pl}}} \left(K^{ij} D_i\, W\, D_j \bar{W} - \frac{3}{M_{pl}^2} |W|^2 \right) \tag{3.68}$$

Where the Latin indices runs for all our moduli. The D derivatives corresponds to covariant ones, of the form:

$$D_i\, W = \partial_i W + (\partial_i K)\ W \tag{3.69}$$

This potential corresponds to the Lagrangian that governs the motion of chiral fields in a $\mathcal{N} = 1$ four dimensional supersymmetric theory. In this case, we have included the pair graviton-gravitino, thus we have included gravity interaction. That is the reason for covariant derivative D. The square superpotential term W belongs exclusively to supergravity. When the potential V reaches its minimum, that term represents $m^{3/2}$ of the gravitino mass. The Planckanian mass is included due to the action of gravity. If it goes to ∞ the gravity decouples and we recover the result from supersymmetric chiral Lagrangian.

This potential is invariant under the action of the different $SL(2,\mathbb{R})$ group for the moduli, as the action of them in our superpotential W gets a counter in the Kähler metric and potential. This invariance makes it a good candidate to describe the physics of the vacua we obtain after solving the equations of motion, as we will see in next chapter. Several features of this potential that are remarkable to be mentioned, such that:

- If we decouple gravity, the only present terms in equation (3.68) are the derivatives, such that the second term in equation (3.69) is removed. This implies, that if we do not break supersymmetry ($D_i W = 0$), the result will end up with $V = 0$ describing a Minkowski $\mathcal{N} = 1$ four dimensional space[18]. This type of solutions requiere a non-scale superpotential. Several examples will be offered in next chapter.

- If we do not decouple gravity, we still have the contribution of the Superpotential and a warped moduli space metric. A vanishing $D_i W$ still ensures supersymmetry, so the final value of the potential will be smaller than 0. This corresponds to an AdS vacuum solution, with negative cosmological constant. This is the set of solutions we are going to work with in next chapter, as they offer analytical solutions with dualities.

- We can improve our results by breaking supersymmetry in previous point with $D_\phi = A_\phi + iB_\phi$. The solutions can run into the de Sitter zone, with a positive cosmological constant and a semi-realistic approximation. Its drawback it is that a vast majority of solutions are tachyonic, though fully stable examples can be found [7].

In the next chapter we will offer a fast introduction to solutions in a Minkowski space, and a deeper explanation of their main lack of non-scale dependence. Following, a set of family solutions in Ads Space will be analysed and related to each other with their inner dualities.

[18] This result can be improved with an addition of a non-perturbative potential, uplifting the solutions to meta-stable de Sitter ones.

4 Families of Vacua and Moduli Fixing

Equipped with the mathematical knowledge acquired in the previous sections, we are now ready to solve the equations of motion for our potentials V in order to get their different minima and cosmological constant, as we saw at the end of section 3. The methodology for these will be as follows:

- Determine which fluxes parameters are switched on and off in our superpotential description. In the next section we will demonstrate that we need to switch on a considerable amount of parameters to fix all the moduli.
- Solve the constraints (A.6 - A.22) imposed by the inner algebra of the fluxes, to determine the value of the different parameters. At this point, we will use the same restriction as in [11, 16]. We will limit our values to even integers, in order to avoid exotic orientifold planes.
- Solve the F-flatness conditions for our potential (3.20):
$$D_{\phi_i}W = \partial_{\phi_i}W + K_{\phi_i}W = 0. \tag{4.1}$$
Where i runs for our three moduli in our isotropic prescription and K_{ϕ_i} is the moduli space potential derived with respect to one of those fields. Equation (4.1) will be modified depending on whether we are looking for a Minkowski or Anti de Sitter vacuum solution. As we said in the previous point, if we have enough parameters switched on, we will fix all the moduli.
- Get the minimum value of the vacuum (Λ) and the coupling constant of this configuration. As we want a physical solution, all the imaginary parts of the moduli should be greater than 0, as these carry important information of the manifold's geometry, such as the coupling constant g_s and the size of the cycles wrapped by H, F.
- Obtain the canonical invariant mass matrix and its eigenvalues.

Subsequently, we will present remarkable solutions of Minkowski and anti de Sitter spaces. We will start with $\mathcal{N} = 1$ Minkowski one, as a warm up for more complex, but precise descriptions in a $\mathcal{N} = 1$ Anti de Sitter space.

4.1 A Toy Model. $\mathcal{N} = 1$ Minkowski space

As an introduction exercise, we will offer a symbolical solution of a superpotential only with geometrical fluxes. For solution purposes, we will recover a non-isotropical superpotential expression, with flux parameters of $\bar{H}_3$ and $\bar{F}_3$ as (3.8). In this case, the superpotential looks like:

$$\begin{aligned} W(S, U_i) =& a_0 - \sum_{K=1}^{3} a_1^K U_K + \sum_{K=1}^{3} a_2^K \frac{U_1U_2U_3}{U_K} - a_3U_1U_2U_3 \\ & S\left(-b_0 + \sum_{K=1}^{3} b_1^K U_K - \sum_{K=1}^{3} b_2^K \frac{U_1U_2U_3}{U_K} + b_3U_1U_2U_3\right). \end{aligned} \tag{4.2}$$

Before solving its equation of motion, we first must show its principal issue, as these solutions do not possess a scale dependence, due to the fact that:

$$D_SW = \partial_SW + K_SW = 0, \tag{4.3}$$
$$D_{T_i}W = \partial_{T_i}W + K_{S_i}W = 0, \tag{4.4}$$
$$D_{U_i}W = \partial_{U_i}W + K_{U_i}W = 0. \tag{4.5}$$

From equation (4.4) we can determine that, as the superpotential does not depend on T_i, its derivative is 0. This does not mean that the metric of the moduli space does not depend on T_i, so the only option left is to set $W = 0$. If so, the covariant derivative reduces to a partial one, that for our superpotential (4.2) is:

$$D_S W = Im\,(\partial_S W) = -\sum_{i=1}^{3} b_1^i\, U_i, \tag{4.6}$$
$$D_{U_1} W = Im\,(\partial_{U_1} W) = -a_2^2\, U_3 - a_2^3\, U_2 - S\, b_1^1, \tag{4.7}$$
$$D_{U_2} W = Im\,(\partial_{U_2} W) = -a_2^1\, U_3 - a_2^3\, U_2 - S\, b_1^2, \tag{4.8}$$
$$D_{U_3} W = Im\,(\partial_{U_3} W) = -a_2^2\, U_1 - a_2^1\, U_2 - S\, b_1^3. \tag{4.9}$$

Where we are considering the imaginary part of the moduli, as they are the ones who carry the physical information of the solutions. In order to have imaginary parts of the moduli different from 0, the determinant of this system of equations must be different from 0, as:

$$(b_1 a_1 + b_2 a_2 - b_3 a_3)^2 = 4 b_1 a_1 b_2 a_2. \tag{4.10}$$

Where we have dropped the upper indices as equations (4.7-4.9) have the same behaviour in those indices. This determinant condition can be solved imposing back a set of isotropical conditions:

$$a_2 = a_3 = -\frac{a_1}{2}, \qquad\qquad b_2 = b_3 = -\frac{b_1}{2}. \tag{4.11}$$

This kind of solution of the constraint plus setting the rest of parameters to 0, will yield a Superpotential of the form:

$$W_{S-dual} = -S\, b_1\, U_1 - a_1\, U_2\, U_3 + \frac{1}{2}\left((U_1 + U_2)\,(b_1\, S + a_1\, U_1)\right). \tag{4.12}$$

If we solve equations (4.7-4.9) again, the system of equations to solve will get reduce to two parameters and 4 fields, such:

$$-b_1\, U_1 + \frac{1}{2}\,(U_2 + U_3)\; b_1 = 0, \tag{4.13}$$
$$-b_1\, S + \frac{1}{2}\,(U_2 + U_3)\; a_1 = 0, \tag{4.14}$$
$$-a_1\, U_3 + \frac{1}{2}\,(b_1\, S + U_1\, a_1) = 0, \tag{4.15}$$
$$-a_1\, U_2 + \frac{1}{2}\,(b_1\, S + U_1\, a_1) = 0. \tag{4.16}$$

From equations (4.15,4.16) we can extract the condition $U_2 = U_3$, which plugged into equation (4.13) gives us a identity between the three fields. In the end we recover some isotropy. If we introduce this in one of the four equations above, we get:

$$U_i = \frac{b_1}{a_1}\, S. \tag{4.17}$$

For the three different values of the U-moduli. As we can appreciate, none of above equations include the presence of a T_i-moduli. This issue would have taken place as well in a IIA description of the background. This means that our solution does not depend on a scale of the compact $\mathbb{T}^6$. As well, the solutions we have found for the U-moduli, has a dependence on S, which can take an arbitrary value. As we stated at the end of Chapter 3, this kind of solutions has a 0 cosmological constant. It is necessary to improve our superpotentials to be able to fix all the moduli.

4.2 $\mathcal{N} = 1$ AdS space

In previous section it has been proved that a Minkowski space without non-geometrical fluxes activated cannot fix the vacuum expectation value for all the moduli. In addition, the description leads to null cosmological constant, a result we would like to improve. In the spirit of 'AdS' vacua solution hunting, we must relax the conditions for our gravitino mass ($W \neq 0$), moving the cosmological constant below 0 and fixing the string coupling constant (not always) at small values. This means that the three F-flatness equations for a more general superpotential of the form (3.20) look like:

$$(S - \bar{S})D_S W = P_1(U) + \bar{S}\, P_2(U) + T\, P_3(U) + \bar{S}\, T\, P_4(U) = 0, \tag{4.18}$$

$$(T - \bar{T})D_T W = P_1(U) + S\, P_2(U) + \left(\frac{2}{3}T + \frac{1}{3}\bar{T}\right)\, (P_3(U) + S\, P_4(U)) = 0, \tag{4.19}$$

$$(U - \bar{U})D_U W = (U - \bar{U})\, \partial_U W - 3\, W = 0. \tag{4.20}$$

Where we have considered that:

$$S = Re\, S + i\, Im\, S, \qquad T = Re\, T + i\, Im\, T, \qquad U = Re\, U + i\, Im\, U. \tag{4.21}$$

Solving equations (4.18-4.20) as we will explain, will come up with fixed solutions for the three moduli. These solutions correspond to the minima of our potential (3.68), which reads:

$$\Lambda = -3\, e^{K(S_0, T_0, U_0)} |W(S_0, T_0, U_0)|^2. \tag{4.22}$$

These minima have associated a string coupling constant $g_s = 1/Im\, S$.

As we saw in the previous point, it is necessary to turn on non-geometrical fluxes Q and/or P to fix and stabilise the masses of all moduli. This causes the amount of equations coming from the algebra constraints to increase generously. In the next two sections we will introduce two of the most representative solutions for these constraints, with arbitrary integer values of the parameters a_i, b_i, c_i, d_i which satisfy equations (A.6-A.22). The first section corresponds to a general family of vacua with two free parameters in the T-moduli. The next section increases the amount of free integer parameters by one, which are included in all the moduli values. The canonical invariant mass matrix will be calculated as well as its eigenvalues.

4.2.1 Example with two parameters

In this section we introduce a detailed explanation of the results for arbitrary a_i, b_i initial parameters in our H and F fluxes. Inspired by [20], we force our F flux to be of the form:

$$P_2(U) = b(U-1)^3. \tag{4.23}$$

This ensures us an easier calculation in the future. In fact, the first tadpole cancellation related to our C_4-form is:

$$a_0 - 3a_1 + 3a_2 - a_3 = \frac{16}{b}. \tag{4.24}$$

At this point, it is important to notice that we cannot just activate three a parameters with the same value. If we did so, we would get a non- integer result for our a parameters, violating the primitive constraint of 'All parameter fluxes must be even integers'. In this case, as we are warming up for more difficult solutions, we decided to switch off those parameters with odd index in the H flux, and imposing $a_0 = a_2$. This means we turned on only fluxes with legs along the two even directions of our $\mathbb{T}^6$ while we observe them from the remaining torus. This will give a first constraint for our 'a' parameters, such that:

$$a = \frac{4}{b}. \tag{4.25}$$

This directs us to the next set of constraints. In this case, we decided to switch completely off the P-flux, while setting the Q-flux parameters untouched. Then, we will find constraint contributions two 8-forms from the $SL(2,\mathbb{Z})$ group and the flux algebra ones $[Q,Q]$ and $H, Q - F, P$ such that:

$$c_3 - (2c_1 - \bar{c}_1) = 0, \tag{4.26}$$
$$c_3 + (2c_2 - \bar{c}_2) - (2c_1 - \bar{c}_1) - c_0 = 0, \tag{4.27}$$
$$c_0(c_2 - \bar{c}_2) + c_1(c_1 - \bar{c}_1) = 0, \tag{4.28}$$
$$c_2(c_2 - \bar{c}_2) + c_3(c_1 - \bar{c}_1) = 0, \tag{4.29}$$
$$c_0 c_3 - c_1 c_2 = 0, \tag{4.30}$$
$$(\times 2!)\ \ c_0 - c_2 + (c_1 - \bar{c}_1) = 0, \tag{4.31}$$
$$(\times 2!)\ \ c_1 - c_3 - (c_2 - \bar{c}_2) = 0. \tag{4.32}$$

Where the first two equations correspond to the tadpole cancellation conditions for D-7 brane sources and equations (4.31,4.32) appear twice in our $HQ - PF$ constraints. The freedom we can obtain from equation (4.30) is remarkable. This condition is dominant in our Q,P fluxes and it is a key to simplify that constraint and get results. In fact, if we set $c_0, c_1 = 0$ in this equation, we will get c_2, c_3 to be free different parameters. From condition (4.26) we determine that $\bar{c}_1$ must be $-c_3$, while from (4.31), we find $c_3 = c_2$. In this case, we can say $c_3 = n$. Inserting this result in the remaining equation, we end up with a set of flux parameters such that:

$$H \to \left(\frac{4}{b}, 0, \frac{4}{b}, 0\right), \tag{4.33}$$
$$F \to (b, b, b, b), \tag{4.34}$$
$$Q \to (0, 0, n, -n, -2\,n, -n). \tag{4.35}$$

This choice of parameters leads to a superpotential W of the form:

$$W(S,T,U) = \frac{4}{b}\left(1 + 3\,U^2\right) + b\,S\,(U-1)^3 + 3\,T\,\left(-n\,U + n\,U^3\right). \tag{4.36}$$

Now it is time to solve equations(4.18,4.19,4.20) as we describe in appendix C. Choosing carefully under restriction the values of the real parts of U and S, we arrive at solutions of the form:

$$S = \frac{1}{b^2}\,(1+i)\,, \tag{4.37}$$

$$T = \frac{2}{b\,n}\left(1 \pm \frac{i}{2}\right), \tag{4.38}$$

$$U = (-1 + 2\,i). \tag{4.39}$$

As we can see, the only field carrying the information concerning both parameters is the complex Kähler moduli T. In this case, it is necessary to include a ±[19] sign in the imaginary part of T, to ensure its positive value depending on the one n acquires. (it is important to notice that n can not be 0) Setting $n \to \infty$ we ensure a 0 deformation in our internal $\mathbb{T}^6$ manifold. We can now check the look of the string coupling constant in this configuration :

$$g_s = \frac{1}{Im\,S} = b^2. \tag{4.40}$$

And the cosmological constant Λ:

$$\Lambda = -\left|\frac{3\,n^3 b^3}{2}\right|. \tag{4.41}$$

This configuration leaves us with several problems to deal with. First of all, we can easily check that the coupling constant cannot be small enough to include perturbative corrections to the theory. Moreover, another issue is the cosmological constant. As it does not have any parameter in the denominator we cannot maket arbitrarily close to 0, giving us interesting but not realistic values.

Finally, we should define our canonical mass matrix for the three moduli. This corresponds to solving the second order of a perturbation of the potential V around the v.e.v of our moduli. A scale invariance could be achieved dividing by the value of the potential itself. As well, to get a physical meaning of this matrix, the contribution of the geometry of the moduli space should be taken into account through the action of its metric input, as they are not canonically normalized yet. In fact, the correct description of this matrix reads as:

$$(m^2)_i^j = K^{jk}\frac{\partial_k \partial_i V}{V}|_{\phi_0}. \tag{4.42}$$

In our case, the matrix takes the next form:

[19] In fact, solving both branches ensures this double sign appears in the solution.

$$
(m^2)_i^j = \begin{pmatrix} \frac{4}{3} & 0 & -\frac{8b}{3n} & 0 & -\frac{128b^2}{75} & -\frac{32b^2}{25} \\ 0 & \frac{4}{3} & 0 & -\frac{16b}{3n} & \frac{16b^2}{25} & -\frac{64b^2}{75} \\ -\frac{n}{2b} & 0 & \frac{4}{3} & 0 & 0 & 0 \\ 0 & -\frac{n}{b} & 0 & \frac{8}{3} & \frac{4bn}{25} & -\frac{16bn}{75} \\ -\frac{2}{b^2} & \frac{3}{4b^2} & 0 & \frac{1}{bn} & \frac{478}{75} & \frac{32}{25} \\ -\frac{3}{2b^2} & -\frac{1}{b^2} & 0 & -\frac{4}{3bn} & \frac{32}{25} & \frac{422}{75} \end{pmatrix} \tag{4.43}
$$

Where we have use the basis of fields $\{Re\,S, Im\,S, Re\,T, Im\,T, Re\,U, Im\,U\}$. If we fix one of the parameters, it is easy to check the moduli's mass can be high for some of them, but the higher for specific moduli, the lighter for other ones. This issue is a general feature of all the vacua families in table 11. In that sense, we can try to improve our families up to two different parameters arising from the flux constraints, so we can use them to have light masses of our moduli fields and fixed values of the cosmological and string coupling constants.

4.2.2 Example with three parameters

Here we will improve our results from previous section. Following almost the same principle than in [20], we choose our H-flux to have the form:

$$
P_1(U) = a\,(1-U)^3. \tag{4.44}
$$

This condition will give us a similar equation to (4.24), where a and b have interchanged their roles. In order to avoid harder options in our H-Q algebra constraints, we decide to switch off all the b_i except b_0, which will determined through the Tadpole C_4 condition as:

$$
b = -\frac{16}{a}. \tag{4.45}
$$

With this, we can start counting the contributions of the tadpole cancellations (where again, we decide not to introduce D7 branes) and algebra constraints. The P-flux will be turned off for easier calculations (to include them just requires more effort to solve the constraints and the code does not display solvable branches).

$$
c_3 + (2c_2 - \bar{c}_2) - (2c_1 - \bar{c}_1) - c_0 = 0, \tag{4.46}
$$

$$
b_0 c_3 = 0. \tag{4.47}
$$

From equation(4.47) it is straightfoward to check $c_3 = 0$ to ensure the equation works. To check the different values of c_i in equation (4.46) we need to consider the algebra constraints as follows:

$$
c_0(c_2 - \bar{c}_2) + c_1(c_1 - \bar{c}_1) = 0, \tag{4.48}
$$

$$
c_2(c_2 - \bar{c}_2) = 0, \tag{4.49}
$$

$$
-c_1 c_2 = 0, \tag{4.50}
$$

$$
-c_2 b_0 = 0, \tag{4.51}
$$

$$
(\times\, 2!) - c_3 b_0 = 0, \tag{4.52}
$$

$$
(\times\, 2!) - c_3 b_0 = 0. \tag{4.53}
$$

This set of constraints ensures several things. The first one, as in equation (4.47), we have again the same result from equation (4.52). As well, $c_2 = 0$ from equation (4.51), which reliefs us from setting $c_1 \to 0$ in equation (4.50).Thus, we choose to set $c_1 = n$. Setting $c_2 = 0$ in equation (4.49) we can set $\bar{c}_2 = p$. This will end up in a system of equations (4.46) and (4.48) for c_0 and $\bar{c}_1$ parameters. Solving them, we can plug the values of the parameters inside the superpotential (A.1):

$$W(S,T,U) = a - 3\,a\,U + 3\,a\,U^2 - a\,U^3 + \frac{16}{a}\,S + 3\,T\,(n + (n+p)\,U - p\,U^2) \tag{4.54}$$

As we proceed previously, we have to solve the equations of motion(4.18,4.19,4.20) to obtain the v.e.v's for S,T,U at the minimum. These moduli are:

$$S = \frac{a^2}{32p^3}\left(-(n+p)^3 \pm \frac{(n+p)^3\,i}{4}\right), \tag{4.55}$$

$$T = \frac{a(p+n)}{2\,p^2}\left(1 \pm \frac{i}{2}\right), \tag{4.56}$$

$$U = \frac{1}{2p}\left((p-n) \pm (p+n)i\right). \tag{4.57}$$

These three fields depend on the parameters left free after imposing the algebra constraints (4.46 - 4.52). They configure a family of infinite solutions, where $\pm$ must be chosen in order to keep the imaginary part of the moduli greater than 0. In this case, as we have this choice present in the three moduli, we must choose the same sign for all the three moduli, to ensure the validity of the equations of motion. This family of solutions comes equipped with string coupling constant:

$$g_s = \frac{128\,p^3}{a^2\,(n+p)^3}. \tag{4.58}$$

While the cosmological constant reads:

$$\Lambda = -\left|\frac{96\,p^6}{a^3(p+n)^3}\right|. \tag{4.59}$$

In this family it is easy to see that if we fix the value of p and take n large enough, we can tune the stringy constant and the cosmological one to very small values. Finally, we present the canonical mass matrix of this set of solution now, in the same basis as in the previous section.

$$\begin{pmatrix} \frac{4}{3} & 0 & \pm\frac{64p}{3a(n+p)^2} & 0 & \mp\frac{256p^2}{3a^2(n+p)^2} & 0 \\ 0 & \frac{4}{3} & 0 & \pm\frac{128p}{3a(n+p)^2} & 0 & \mp\frac{128p^2}{3a^2(n+p)^2} \\ \pm\frac{a(n+p)^2}{16p} & 0 & \frac{4}{3} & 0 & 0 & 0 \\ 0 & \pm\frac{a(n+p)^2}{8p} & 0 & \frac{8}{3} & 0 & \pm\frac{4p}{3a} \\ \mp\frac{a^2(n+p)^2}{16p^2} & 0 & 0 & 0 & \frac{22}{3} & 0 \\ 0 & \mp\frac{a^2(n+p)^2}{32p^2} & 0 & \pm\frac{a}{3p} & 0 & \frac{14}{3} \end{pmatrix} \tag{4.60}$$

Almost all the elements of this matrix can be chosen parametrically small, fixing the value of the winding number n and setting p big enough, except those 'interaction' terms between the T and U moduli. In this case, we can limit these values with a correct choice of the flux parameter a. In the next section we will analyse different solutions for a set of superpotentials.

4.2.3 A bunch of "different" solutions

Here we present a set of interesting solutions we consider remarkable to talk about. The procedure to obtain them was exactly the same one we followed for the aforementioned examples. We turned off specific flux-parameters in our H and F fluxes and solved the algebra constraints for them. Thus, we obtained a superpotential with well defined solutions in the moduli for that configuration.

The first package of solutions corresponds to those ones where Q and P fluxes only depend on one single parameter 'n' and the parameter that belongs to H or F fluxes. Here we display the table:

	Flux parameters a,b,c,d	Fields	Λ
I	(a,a,a,a) $(0,0,0,16/a)$ $(0,0,n,0,n,0)$ $(0,0,0,0,0,0)$	$S=\frac{a^2}{32}\left(1+\frac{i}{4}\right);\ T=\frac{a}{2\,n}\left(-1\pm\frac{i}{2}\right);\ U=(1+i)$	$-\left\lvert\frac{96\,n^3}{a^3}\right\rvert$
II	$(4/b,0,4/b,0)$ (b,b,b,b) $(0,0,n,-n,-2n,-n)$ $(0,0,0,0,0,0)$	$S=\frac{1}{b^2}\left(3+\frac{i}{4}\right);T=\frac{2}{b\,n}\left(1\pm\frac{i}{2}\right);U=\frac{1}{5}(-1+2i)$	$-\left\lvert\frac{3\,n^3b^3}{2}\right\rvert$
III	$(4/b,0,4/b,0)$ (b,b,b,b) $(n,0,0,n,n,0)$ $(0,0,0,0,0,0)$	$S=\frac{1}{b^2}\,(1+i);\quad T=\frac{2}{b\,n}\left(1\pm\frac{i}{2}\right);\quad U=(-1+2\,i)$	$-\left\lvert\frac{3\,n^3b^3}{2}\right\rvert$
IV	$(0,a,a,a)$ $(-16/a,0,0,0)$ $(0,0,-n,n,n,0)$ $(0,0,-1,1,1,0)16\,n/a^2$	$S=\frac{a^2}{34}\left(-\frac{15}{8}+i\right);\ T=\frac{a^2}{2\,n}\left(1\pm\frac{i}{2}\right);\ U=(0+i)$	$-\left\lvert\frac{96\,n^3}{a^3}\right\rvert$

Table 11. Vacua solutions with two different parameters

In table 11, the first column corresponds to the value of the flux parameters in the solution. The second one offers a quick view of the value of the moduli vevs, while the third one offers the value of the cosmological constant. The first feature we can observe is that they can be grouped into two different values for the cosmological constant. In addition, for all of these solution, the only parameter which carries information about the non-geometrical flux parameter 'n' is the expected value of the Kähler modulus T. This modulus behaves exactly in the same way for solutions II and III. This issue relies on the fact that both solutions belong to different choices in the values of the non-geometrical algebraic constraints for the same initial geometrical flux parameters. The drawback for these 'twins' is that neither the coupling constant nor the cosmological constant can be set to small values for a suitable choice of the winding numbers, as these ones must be even integers. In that sense, family I, looks like a nice candidate to get small values in these constants. In fact, if we fix n to a small value and we set $a = 16$, as it is the maximum value it can be without violating the C_4 tadpole condition, we can get values for the string coupling and cosmological constants for 0.5 and -0.027, respectively.

Of course, these values do not represent a correct approach to reality, but seem the most reliable ones we can obtain from this set of families. As we will see and explain further, these families have the same mass spectra and some of them can be proved to be related through dualities. In that sense, we should improve our solutions and present those interesting ones we obtained with two different parameters for the non geometrical fluxes.

	Flux parameters a,b,c,d	Fields	Λ
V	$(a,a,a,0)$ $(0,0,0,16/a)$ $(0,0,p,n,p+n,-n)$ $(0,0,0,0,0,0)$	$S=\frac{a^2}{32p^3}\left((n+p)^3-2p^3\pm\frac{(n+p)^3}{4}\,i\right)$ $T=\frac{a(p+n)}{2\,p^2}\left(-1\pm\frac{i}{2}\right)$ $U=\frac{p}{(p^2+n^2)}\,((p-n)\pm(p+n)i)$	$-\left\lvert\frac{96\,p^6}{a^3(p+n)^3}\right\rvert$
VI	(a,a,a,a) $(-16/a,0,0,0)$ $(n,p,p+n,0,-p,0)$ $(0,0,0,0,0,0)$	$S=\frac{a^2}{32p^3}\left(-(n+p)^3\pm\frac{(n+p)^3}{4}\,i\right)$ $T=\frac{a(p+n)}{2\,p^2}\left(1\pm\frac{i}{2}\right)$ $U=\frac{1}{2p}\,((p-n)\pm(p+n)i)$	$-\left\lvert\frac{96\,p^6}{a^3(p+n)^3}\right\rvert$
VII	$(16/b,0,0,0)$ (b,b,b,b) $(n+p,0,n,p,p,0)$ $(0,0,0,0,0,0)$	$S=\frac{8p^3}{b^2(n+2p)^3}\left(1\pm\frac{i}{4}\right)$ $T=\frac{4p}{b(n+2p)^2}\,(-2\pm i)$ $U=\frac{1}{p}\,(-(p+n)\pm(2p+n)i)$	$-\left\lvert\frac{3b^3(n+2p)^6}{128\,p^3}\right\rvert$
VIII	$(16/b,0,0,0)$ (b,b,b,b) $(n+p,-p,n,0,p,0)$ $(0,0,0,0,0,0)$	$S=\frac{8p^3}{b^2(n)^3}\left(1\pm\frac{i}{4}\right)$ $T=\frac{4p}{b(n)^2}\,(-2\pm i)$ $U=\frac{1}{p}\,(-(p+n)\pm n\,i)$	$-\left\lvert\frac{3b^3n^6}{128\,p^3}\right\rvert$

Table 12. Vacuas with three different parameters.

In table 12 we follow the same schema as in table 11. The main difference is that all the moduli have a dependence on two different parameters, n and p. Furthermore, all the imaginary parts are equipped with a $\pm$ sign, such that, depending of the value of the parameters, we can choose it for a positive value solution. One interesting fact is that families V and VI share the same cosmological and string coupling constant. Later in this chapter, we will analyse if they are related through a chain of dualities.

On previous pages, we have talked about their equivalence and relation, but we did not explain how that works. As we introduced the canonical mass matrix (4.42), we knew beforehand the main property of this. Its eigenvalues are invariant under change of basis. This means that different families of solutions will have different mass spectra, so solutions apparently different with same spectra can be related through specific dualities in the compact space. In that sense, we can calculate the eigenvalues of all our solutions, in order to check if they are really different solutions or not. The eigenvalues, for all the solutions above, are:

$$E = \{-0.226769, 2.08668, 8.14009, -0.728685, 4.36482, 5.03054\}. \tag{4.61}$$

This implies that we got a set of results that describe the same physics in the end. In some sense, this is a bad piece of news, as it means all our work during last pages is in fact, the same result. On the other hand, this allows us to relate different solutions through the moduli-dualities in the superpotentials. This will be our aim for next section.

4.3 Dualities

In this last section, we tried to relate several solutions through the present dualities of the moduli in the superpotential. As in the previous section we argued that all the solutions are, in fact, the same effective description, therefore all of them living in the same moduli space with same orientifold. In section 3, the generators of a $SL(2,\mathbb{Z})$ are inversion, translation and reflection. Dilations are allowed as well. We can use these tools to 'dualize' our potentials, and to try to find relations between them as another test of their equity. Unfortunately, not all the solutions above will have a clear dualization

In the next dualities between superpotentials that we have studied, we have always followed almost the same procedure; that is, to invert required moduli in one superpotential to make them look alike and to shift and dilate the other one with general parameters that must be determined through a later identification of the monomials of both. The first two superpotentials we have studied correspond to solutions II and III in table 11. They look like:

$$W_{II}(S,T,U) = \frac{4}{b} + \frac{12}{b}\,U^2 + S\,b\,(U-1)^3 + 3\,T\,(-n\,U + n\,U^3), \tag{4.62}$$

$$W_{III}(S,T,U) = \frac{4}{b} + \frac{12}{b}\,U^2 + S\,b\,(U-1)^3 + 3\,T\,(n - n\,U^2). \tag{4.63}$$

At first sight, if the H-flux of one the superpotentials had odd monomial U-powers, the duality would be a straightforward U-inversion. As this is not the case, we can perform a U-inversion in the superpotential (4.62) and a set of translations and dilations in (4.63) such that:

$$S \to \sigma\,S + x, \tag{4.64}$$

$$T \to \tau\,T + y, \tag{4.65}$$

$$U \to \lambda\,U + z. \tag{4.66}$$

These transformation leave our previous superpotentials with next appearance:

$$W_{II} = \frac{-4}{b}U^3 + \frac{-12}{b}U + S\,b\,(U-1)^3 + 3\,T\,(n\,U^2 - n), \tag{4.67}$$

$$W_{III} = \frac{4}{b} + \frac{12}{b}\,(\lambda\,U + z)^2 + (\sigma\,S + x)\,b\,(\lambda\,U + z - 1)^3 + 3\,(\tau\,T + y)\,(n - n\,(\lambda\,U + z)^2). \tag{4.68}$$

The translations and dilations in equation (4.68) will give us enough terms to determine the value of our shift parameters if we compare one by one the monomials of both superpotentials. In fact, the set of equations is:

Monomial	W'_{III} parameters	W'_{II} parameters
1	$4/b + 12/b\,z^2 - xb + 3xbz - 3xbz^2 + xbz^3 + 3yn - 3ynz^2$	0
U	$24/b\,\lambda z + 3xb\lambda z - 6xbz\lambda + 3xb\lambda z^2 - 6ny\lambda z$	$-\frac{12}{b'}$
U^2	$12/b\lambda^2 - 3xb\lambda^2 + 3xb\lambda^2 z - 3yn\lambda^2$	0
U^3	$xb\lambda^3$	$-\frac{4}{b'}$
S	$-\sigma b + 3\sigma bz - 3\sigma bz^2 + \sigma bz^3$	$-b'$
SU	$3\sigma b\lambda - 6\sigma b\lambda z + 3\sigma b\lambda z^2$	$3b'$
SU^2	$-3\sigma b\lambda^2 + 3b\sigma\lambda^2 z$	$-3b'$
SU^3	$\sigma b\lambda^3$	b'
T	$3\tau n - 3\tau nz^2$	$-3n'$
TU	$-6n\lambda z\tau$	0
TU^2	$-3n\lambda^2\tau$	$3n'$

Table 13. Monomial comparison.

This corresponds to a direct comparison between the monomials of both superpotentials, where we have identified the flux parameters of the target II superpotential dualized with b', n' Solving thes equations we will obtain the corresponding values of the dilation and translation parameters, as well as the relation between the flux parameters b, n for both potentials. In fact, as we perform dilations, the value of these ones must be greater than 0. If not, the duality will not have a physical meaning. The transformation parameters are:

$$\sigma = -1., \qquad x = \frac{4}{b'^2}, : \qquad \tau = -\frac{n'}{n}, \qquad y = 0, \qquad \lambda = 1, \qquad z = 0, \qquad b = -b' \tag{4.69}$$

As we can check the modulation in T and U moduli is 0, a result we could have predicted comparing this minima values of these fields in table 12. The problematic values correspond to σ and τ. Both of them present a negative value, although the dilation in our T moduli can be 'fixed' relating the negative Q-flux parameters of the II Superpotential with those positives ones in the III. For S dilation, the problems are worse. As the dilation has a non-physical meaning, the only option we have to establish a real duality is to conjugate S_{III} as a non-physical symmetry of S moduli. As well, we could do exactly the same to T_{III} so we do not have to relate both Q-flux parameters.The chain of transformations starting from III to II is of the form:

$$W_{III} \xrightarrow{S'\to\sigma S+x \quad T'\to\tau T \quad U'\to\lambda\, U} W_{III\;Dual} \xrightarrow{S\to\bar{S}} W_{III\;Dual\,2} \xrightarrow{U''\to -1/U'} W_{II} \tag{4.70}$$

Where S', T', U' represent previous dilations and translations. So, we have been able to relate two different solutions in table 11 through a chain of mathematical and allowed physical transformation, except, the relation between the physical branches of the S-moduli in both superpotentials has been reversed. As matter of practice, we can try exactly the same with two different family solutions in table 12. We will analyse solutions V and VI. Their superpotentials look like:

$$W_V(S,T,U) = a - 3\,a\,U + 3\,a\,U^2 + \frac{16}{a} S\,U^3 + 3\,T\,(-p\,U - (n-p)\,U^2 + n\,U^3), \tag{4.71}$$

$$W_{VI}(S,T,U) = a - 3\,a\,U + 3\,a\,U^2 - a\,U^3 + \frac{16}{a} S + 3\,T\,(n + (p-n)\,U - p\,U^2). \tag{4.72}$$

If we pay attention to both potentials, we can observe they may be related through an inversion of the U-modulus. As well, a modular transformation of the S-modulus in the superpotential W would translate a monomial U^3 from the modular transformation of SU^3 to the $\bar{H}_3$ flux parameters, which will make both potentials identical in all the monomials they have. Before performing the transformation, we can notice from table 12 that neither the S nor the T moduli would require a dilation and U-moduli can be reproduce in both sides without taking into account a modular displacement, making the set of dualities (4.64-4.66) easier. After performing the transformation, our dual potentials are:

$$\begin{aligned} W'_V(S,T,U) = {} & a - 3\,a\,\lambda\,U + 3\,a\,\lambda^2\,U^2 + \frac{16}{a}\,\sigma\,\lambda^3\,U^3 + \frac{16}{a} S\,\lambda^3\,U^3 \\ & + 3\,T\,(-p\,\lambda\,U - (n-p)\,\lambda^2\,U^2 + n\,\lambda^3\,U^3) + 3\,y\,(-p\,\lambda\,U - (n-p)\,\lambda^2\,U^2 + n\,\lambda^3\,U^3), \end{aligned} \tag{4.73}$$

$$W'_{VI}(S,T,U) = a - 3\,a\,U + 3\,a\,U^2 - a\,U^3 - \frac{16}{a} SU^3 + 3\,T\,(-n\,U^3 - (p-n)\,U^2 + p\,U). \tag{4.74}$$

We can proceed as we did with the previous example, to compare all the monomials one by one, and trying to find solutions for the different values of the dilation and modular transformation. The monomial identifications follow as:

Monomial	W'_V parameters	W'_{VI} parameters
1	a	a'
U	$-3a\lambda - 3p\lambda y$	$-3a'$
U^2	$-3a\lambda^2 - 3(n-p)\lambda y$	$3a'$
U^3	$16/a\, x\lambda^3 + 3n\lambda^3$	$-a'$
SU^3	$16/a\,\lambda^3$	$-16/a'$
TU	$-3p\lambda$	$3p'$
TU^2	$-3(n-p)\lambda^2$	$-3(p'-n')$
TU^3	$3n\lambda^3$	$-3n'$

Table 14. Monomial comparison for V-VI.

If we try to solve the equations, we may notice that there is no acceptable solution for the system of equations. If we compare the monomial of zeroth order and that one related to SU^3, there is a contradiction between both a parameters. But, we can apply again the non-physical relation of the imaginary parts of both S-moduli, so we ensure that the contradiction is gone. Doing so, and forcing the dilation to be greater than 0, the duality parameters are:

$$a = a', \qquad \sigma = -1, \qquad x = \frac{-a^2}{16}, \qquad y = z = 0, \qquad \lambda = \tau = 1, \qquad n = -n', \qquad p = -p'. \tag{4.75}$$

Where we have forced the dilation $\lambda = 1$, so we have a physical one. As well, in this case we have been able to relate the winding non-geometrical parameters directly between each other, while setting to 0 the modulation of the T-field. If we recreate the chain of dualities as in previous example, we have:

$$W_V \xrightarrow{S' \to S + x \; T' \to T + y \; U' \to \lambda\, U} W_{V\;Dual} \xrightarrow{S \to \bar{S}} W_{V\;Dual\;2} \xrightarrow{U'' \to -1/U'} W_{VI} \tag{4.76}$$

Next step we would like to comment about it is a possible duality between solutions with three different parameters and those ones in table 11. If we compare one superpotential with three parameters and another one with two parameters, first thing we can realise is the similarity between the Q-fluxes if one of the parameters holds a specific relation to the other one. As an example, we are going to compare the relation between the superpotential III in table 11 and VIII in table 12. These superpotentials read:

$$W_{III}(S,T,U) = \frac{4}{b} + \frac{12}{b}\, U^2 + S\, b\, (U-1)^3 + 3\, T\, (n - n\, U^2), \tag{4.77}$$

$$W_{VIII}(S,T,U) = \frac{16}{b} + S\, b\, (U-1)^3 + 3\, T\, \left(n + p - (2p+n)\, U + p\, U^2\right). \tag{4.78}$$

As we can see, from the monomial TU we can determine a relation between p and n that will generate a null contribution of this term. This is $p = -n/2$. This makes the superpotential looks like:

$$W_{VIII}(S,T,U) = \frac{16}{b} + S\, b\, (U-1)^3 + 3\, T\, \left(\frac{n}{2} - \frac{n}{2}\, U^2\right). \tag{4.79}$$

This can be considered as a subfamily of solutions of W_{VIII}. We can see that establishing a direct relation between $n_{III} = n_{VIII}/2$ reduce both Q- fluxes to the same description. As well, we can notice that there is no need of shifting any of the moduli, as all the high order monomonials are present in both superpotentials. So the only need we have is to modulate our superpotential W_{VIII} in such a way we can find a monomial U^2 contribution. This could be done with the most general moduli modulation as:

$$S \to \sigma\, S + x, \qquad\qquad T \to \tau\, T + y. \tag{4.80}$$

We did not consider a modulation of U, as we want to avoid a new mononial TU to appear. If we expand the superpotential and compare the monomials as we previously did we obtain:

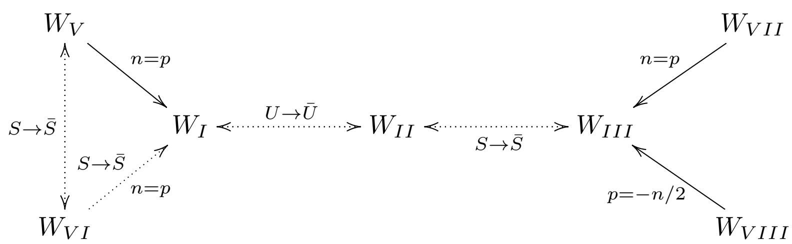
Monomial	W'_{VIII} parameters	W_{III} parameters
1	$16/b - xb + 3yn/2$	$4/b'$
U	$-3xb$	0
U^2	$-3xb - 3yn/2$	$12/b'$
U^3	xb	0
S	$-\sigma b$	$-b'$
SU	$3\sigma b$	$3b'$
SU^2	$-3\sigma b$	$-3b'$
SU^3	σb	b'
T	$3\tau n/2$	$3\,n'$
TU	0	0
TU^2	$-3\tau n/2$	$-3n'$

Table 15. Monomial comparison for VIII-III.

Solving all these equations we can arrive to a similar result than (4.75). This reads:

$$b = b', \qquad \sigma = 1, \qquad x = 0, \qquad \tau = \frac{2\,n'}{n}, \qquad y = -\frac{8}{n\,b'}, \qquad \lambda = 1, \qquad z = 0. \tag{4.81}$$

We can interpret this as a direct dualisation between a sub-family of W_{VIII} and our solutions from W_{III}. In fact, this is not the only duality we have found between solutions with two and three parameters. We have found relations between families V, VI to I and $VII, VIII$ to III. These relations can be represented in a diagram such that:



Where we display all the changes required, as the necessary relations between the parameters for Q-fluxes. Next table shows all the parametrical dualities found among the seven superpotentials we have calculated.

W_i	Modulation and Shifts	Branch	W_f
W_I	$a=-\frac{16}{b};\ \sigma=8;\ x=-\frac{16}{b^2};\ \tau=\frac{4n'}{n};\ \lambda=-\frac{1}{2};\ z=\frac{1}{2}\ ;U\to -1/U$	$U\to\bar{U}$	W_{II}
W_{II}	$b=-b';\ \sigma=-1;\ x=\frac{4}{b^2};\ \tau=-\frac{n'}{n};\ \lambda=1;\ U\to -1/U$	$S\to\bar{S}$	W_{III}
W_V	$a=a';\ \sigma=-1;\ x=-\frac{a^2}{16};\ \tau=1;\ \lambda=1;\ ;U\to -1/U$	$S\to\bar{S}$	W_{VI}
W_V	$a=a';\ n=p=n';\ x=-\frac{a^2}{16}$		W_I
W_{VI}	$a=a';\ n=0;\ p=-n';\ \sigma=-1;\ \tau=1;\ \lambda=1;\ U\to -1/U$	$S\to\bar{S}$	W_I
W_{VII}	$b=-b';\ p=n';\ \sigma=1;\ \tau=1;\ y=-\frac{4}{n'b}\ \lambda=1$		W_{III}
W_{VIII}	$b=b';\ p=-\frac{n'}{2};\ n=n';\ \sigma=1;\ \tau=1;\ y=-\frac{8}{n'b'};\ \lambda=1$		W_{III}

Table 16. All vacua dualities.

The left column represents the superpotential we are starting from. Then, we specify which kind of inversions, dilation and modulations are done[20]. The column 'Branch' represents those moduli whose positive imaginary part is related with the negative one of the final Superpotential (and viceversa). As we mentioned above, this is a non-physical duality that must be done in order to avoid dilations not allowed by a $SL(2,\mathbb{R})$ group. We have written the not allowed dilations in the table in order to appreciate this fact. Last column is our target Superpotential. It can be noticed that all the families with three different parameters can be related through dualities to those one with just two parameters when there are specific conditions for the inner parameters of the Q-flux. Those solutions are the only ones from those families which can be related to the final desired potentials.

[20]Here we assumed that a no displayed modulation is considered 0.

5 Conclusion and Remarks

Along this thesis we have introduced the vast business of compactifications and the role of cosmology in String Theory. We offered a set of mathematical tools in order to obtain a four dimensional effective description of the ten dimensional background. While this was not a success with strictly mathematical objects, as they did not reproduce our desired $\mathcal{N}=1$ supersymmetric space and the absence of a smooth metric for our moduli spaces, limit behaviour of these managed to achieve a more optimal description of those mechanisms, even though new objects, like O-planes and Orientifolds appear in our description . After creating these spaces, all the data must be transferred from ten to 4 dimensions, affecting the inner rules of the supergravity embedded in our six dimensional compact manifold. These constraints arose from the inner algebra of those spaces, as the representative objects living there follow specific conditions. Aforementioned objects are fluxes, with a globally geometrical, local or absent description. These can be obtained from initial NS-NS $\bar{H}_3$ and R-R $\bar{F}_3$ after applying a chain of T-dualities on them. Finally, we looked for $\mathcal{N}=1$ solutions of Minkowski or Anti de Sitter four dimensional spaces using the developed tools in previous chapters. While non-scales studied Minkowski solutions are not able to fix the vacuum expected value of all the present moduli, creating flat directions in the minima of the Superpotential, anti de Sitter ones offer a fixed description of our solutions to the model with negative cosmological constant. Furthermore, inner dualities embedded in our six dimensional compact spaces allowed us to find a chain of relations among different solutions in our T-fold description.

Although we have found several families of vacua with different cosmological and coupling string constants, we could not find de Sitter solutions with the derived algebra constraints in our six dimensional compact manifold. On the other hand, the inner symmetries encoded in the objects living on our compact manifold have helped us relate apparently different solutions into the same description. As well, a main issue in our IIB compactified theory, is the lack of a geometrical global description in higher dimensions of the quantized fluxes we derived through T-duality. The situation becomes even worse within the IIA compactifications and R-fluxes, which have no meaning using conventional notions of local space-time. But these fluxes must be taken into account to find formulations of IIA and IIB theories which do not depend on backgrounds. Even though we do not have a proper general description of geometry, we have proven the utility of non-geometrical fluxes by switching them on and obtaining families of anti de Sitter solutions.

In order to achieve a desired de Sitter result, we could have uplifted Minkowski solutions with non-perturbative terms in the superpotential as in [21] or to use Super-Symmetry breaking parameters, in order to obtain ratios $K^{i\bar{j}} D_i W\, D_{\bar{j}} \bar{W} > 3\,|W|^2$ so the minima of the Potential V lies in a positive region as in [7]. All in all, these solutions could still[21] inherit a tachyonic dependence in at least one of our 7 no isotropic moduli. This well studied problem leads us to several current questions. Could be that String Theory does not want to host de Sitter spaces? [22] Have we properly understood (Anti) de Sitter properties in String Theory? [23] Shall we look for new types of compactification in order to achieve a desired effective cosmological description? It seems we still have concepts to unravel, descriptions to offer, puzzles to solve. Let's continue working on it.

[21] Over a large number of simulations, very few solutions are tachyonic-free.

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A Guideline of Constraints and Tadpole Cancellations

In this Appendix we offer a compact compendium of all the constraints equation arising from tadpoles and algebra in our six dimensional thorus.

The Isotropic Potential

The most general potential in a IIB theory description looks like:

$$W(S,T,U) = P_1(U) + S\,P_2(U) + T\,P_3(U) + S\,T\,P_4(U). \tag{A.1}$$

With Polynomials:

$$P_1(U) = a_0 - 3a_1U + 3a_2U^2 - a_3U^3, \tag{A.2}$$
$$P_2(U) = -b_0 + 3b_1U - 3b_2U^2 + b_3U^3, \tag{A.3}$$
$$P_3(U) = 3\left(c_0 + (2c_1 - \bar{c}_1)U - (2c_2 - \bar{c}_2)U^2 - c_3U^3\right), \tag{A.4}$$
$$P_4(U) = -3\left(d_0 + (2d_1 - \bar{d}_1)U - (2d_2 - \bar{d}_2)U^2 - d_3U^3\right). \tag{A.5}$$

Tadpole Cancelations

In order to counter the contribution of D3 and D7 branes through the Chern-Simons term in equation (2.4), different fluxes must fulfil a set of equations such that:

$$a_0b_3 - 3a_1b_2 + 3a_2b_1 - a_3b_0 = 16, \tag{A.6}$$
$$a_0c_3 + a_1(2c_2 - \bar{c}_2) - a_2(2c_1 - \bar{c}_1) - a_3c_0 = N_7, \tag{A.7}$$
$$b_0c_3 + b_1(2c_2 - \bar{c}_2) - b_2(2c_1 - \bar{c}_1) - b_3c_0 + a_0d_3 + a_1(2d_2 - \bar{d}_2) + a_2(2d_1 - \bar{d}_1) - a_3d_0 = N_7', \tag{A.8}$$
$$b_0d_3 + b_1(2d_2 - \bar{d}_2) - b_2(2d_1 - \bar{d}_1) - b_3d_0 = \bar{N}_7. \tag{A.9}$$

Algebra Constraints

As we saw in chapter 3, the inner geometry of the fluxes must respect a serie of constraint due to its algebra. Depend on the studied fluxes, these constraints can vary.

[Q,Q]

$$c_0(c_2 - \bar{c_2}) + c_1(c_1 - \bar{c_1}) = 0, \tag{A.10}$$
$$c_2(c_2 - \bar{c_2}) + c_3(c_1 - \bar{c_1}) = 0, \tag{A.11}$$
$$c_0c_3 - c_1c_2 = 0. \tag{A.12}$$

[P,P]

$$d_0(d_2 - \bar{d}_2) + d_1(d_1 - \bar{d}_1) = 0, \tag{A.13}$$
$$d_2(d_2 - \bar{d}_2) + d_3(d_1 - \bar{d}_1) = 0, \tag{A.14}$$
$$d_0d_3 - d_1d_2 = 0. \tag{A.15}$$

[Q,P]

$$c_1(d_1 - \bar{d}_1) + c_0(d_2 - \bar{d}_2) + d_0(c_2 - \bar{c}_2) + d_1(c_1 - \bar{c}_1) = 0, \tag{A.16}$$
$$c_3(d_1 - \bar{d}_1) + c_3(d_2 - \bar{d}_2) + d_2(c_2 - \bar{c}_2) + d_3(c_1 - \bar{c}_1) = 0, \tag{A.17}$$
$$c_3 d_0 - c_2 d_1 - c_1 d_2 + c_0 d_3 = 0. \tag{A.18}$$

HQ-FP

$$c_0 b_2 - c_2 b_0 + (c_1 - \bar{c}_1) b_1 - d_0 a_2 - d_2 a_0 - (d_1 - \bar{d}_1) a_1 = 0, \tag{A.19}$$
$$c_0 b_3 - c_3 b_0 + (c_1 - \bar{c}_1) b_2 - d_0 a_3 - d_3 a_0 - (d_1 - \bar{d}_1) a_2 = 0, \tag{A.20}$$
$$c_1 b_2 - c_3 b_0 - (c_2 - \bar{c}_2) b_1 - d_1 a_2 - d_3 a_0 + (d_2 - \bar{d}_2) a_1 = 0, \tag{A.21}$$
$$c_1 b_3 - c_3 b_1 - (c_2 - \bar{c}_2) b_2 - d_1 a_3 - d_3 a_1 + (d_2 - \bar{d}_2) a_2 = 0. \tag{A.22}$$

B Modular transformation of STU-Moduli

As we saw in chapter 3, performing a set of transformations in our fields will change the aspect of the parameters in the new description. In this appendix we summarize all of those changes:

S→ S + n

$$a'_0 = a_0 + n b_0, \tag{B.1}$$
$$a'_1 = a_1 + n b_1, \tag{B.2}$$
$$a'_2 = a_2 + n b_2, \tag{B.3}$$
$$a'_3 = a_3 + n b_3. \tag{B.4}$$

$$c'_0 = c_0 + n\, d_0, \tag{B.5}$$
$$c'_1 = c_1 + n\, d_1, \tag{B.6}$$
$$\bar{c}_1{}' = \bar{c}_1 + n\, \bar{d}_1, \tag{B.7}$$
$$c'_2 = c_2 + n\, d_2, \tag{B.8}$$
$$\bar{c}_2{}' = \bar{c}_2 + n\, \bar{d}_2, \tag{B.9}$$
$$c'_3 = c_3 + n\, d_3. \tag{B.10}$$

T→ T + m

$$a'_0 = a_0 - 3\, c_0\, m, \tag{B.11}$$
$$a'_1 = a_1 + (2c_1 - \bar{c}_1) m, \tag{B.12}$$
$$a'_2 = a_2 + (2c_2 - \bar{c}_2) m, \tag{B.13}$$
$$a'_2 = a_2 + n b_2, \tag{B.14}$$
$$a'_3 = a_3 - 3\, c_3\, m. \tag{B.15}$$

And similarly for b_i flux-parameters.

U→ U + r

$$a'_0 = a_0 + 3\, a_1 r - 3\, a_2 r^2 + a_3 r^3, \tag{B.16}$$
$$a'_1 = a_2 + 2\, a_2 r - a_3 r^2, \tag{B.17}$$
$$a'_2 = a_2 + a_3 r, \tag{B.18}$$
$$a'_3 = a_3. \tag{B.19}$$

While it looks identical for b_i flux-parameters, for c_i takes another approach:

$$c_0' = c_0 - (2c_1 - \bar{c}_1)r + (2c_2 - \bar{c}_2)r^2 + c_3 r^3, \tag{B.20}$$
$$c_1' = c_1 - c_2 r + \bar{c}_2 r + c_3 r^2, \tag{B.21}$$
$$\bar{c}_1' = c_1 + 2c_2 r + c_3 r^2, \tag{B.22}$$
$$c_2' = c_2 - c_3 r, \tag{B.23}$$
$$\bar{c}_2' = \bar{c}_2 + c_3 r, \tag{B.24}$$
$$c_3' = c_3. \tag{B.25}$$

C Code and methodology in Mathematica

In this chapter we want to explain the procedure we have used to solve the three equations of motion for effective vacua minima search in chapter 4.

The first necessary thing to do is to split the contributions of real and imaginary parts of the three equations of motion. This will result in 6 polynomial equations depending on $\{ReS, ImS, ReT, Im\,T, Re\,U, Im\,U\}$. As all of those parts of the moduli belong to $\mathbb{R}$, we can ensure they can be solved to 0. This system of equation will rarely offer a straightforward solution if solved to 0. In order to obtain specific solutions, we propose to solve four out of six equations in terms of the two remaining values of the fields (Preferably the Real and Imaginary part of the same modulus). If we do so, we will get as a result solutions of the form[22]:

$$\text{res} \to \frac{a^2\left(n\left(3\,\text{imu}^4+\text{imu}^2\left(6(\text{reu}-1)\,\text{reu}+2\right)+\text{reu}\left(3\text{reu}\left((\text{reu}-2)\,\text{reu}+2\right)-2\right)\right)+p\left(2\text{reu}-1\right)\right)}{16\left(\text{imu}^2+\text{reu}^2\right)\left(\text{imu}^2\left(p-n\right)+\text{reu}\left(p\left(\text{reu}-2\right)-n\,\text{reu}\right)\right)}, \tag{C.1}$$

$$\text{ret} \to \frac{a\left(-3\,\text{imu}^4+\text{imu}^2\left(1-6\left(\text{reu}-1\right)\text{reu}\right)-3\left(\text{reu}-1\right)^2\text{reu}^2\right)}{3\left(\text{imu}^2+\text{reu}^2\right)\left(\text{imu}^2\left(p-n\right)+\text{reu}\left(p(\text{reu}-2)-n\,\text{reu}\right)\right)}, \tag{C.2}$$

$$\text{ims} \to 0, \tag{C.3}$$

$$\text{imt} \to 0. \tag{C.4}$$

As we can appreciate, the imaginary parts of moduli S and T are not valid, such that the presence of the dilation $e^{-\phi}$ constraints their value in the real positive and finite branch. In order to get solutions different from 0, we analyze the branches of the solutions offered by the program. If we realise, both solutions display the same denominator. We can analyse this branch when it goes to 0. Of course, for this set of fields that will make the real parts go to infinity, but is in these branches where multidimensional solutions rather than 0 exist. If we impose the denominator to 0 and solve the equation in terms of $Re\,U$, we find:

$$\text{imu} \to \pm i\,\text{reu} \tag{C.5}$$

$$\text{imu} \to \pm\frac{\sqrt{\text{reu}\left(n\,\text{reu}-p\left(\text{reu}-2\right)\right)}}{\sqrt{p-n}}. \tag{C.6}$$

Solutions (C.5) are not valid, as they belong to the imaginary ones. On the other hand, solutions

[22] We use the family of solution V as an example in this Appendix.

(C.6) are totally useful. The $\pm$ sign is an indicative of the future dependence of the value of the integers n, p. As a check measurement, we can use these previous solutions to seize the value of the Real part of U. We can impose a set of conditions in solution (C.6), as $n, p \in \mathbb{Z}$ and $Im\,U > 0$ to constraint the value of the Real part of U. This reads as:

$$\left(0 < \text{reu} < \frac{2p}{p-n} \mid (p \geq 1, n \leq 0) \text{ or } (p > n, n \geq 1)\right). \tag{C.7}$$

Thus, we can use these solutions and evaluate them in (C.1-C.4), so we reduce the amount of variables in one. This will not display solutions for the four of them, but now the values are all different from 0. In fact:

$$\text{ims} \to \frac{a^2\sqrt{p-n}\sqrt{\text{reu}\,(n\,\text{reu} - p\,(\text{reu}-2))}\left(2\,\text{reu}\left(p^2+pn+n^2\right)+p(n-p)\right)}{128p^3\,\text{reu}^2}, \tag{C.8}$$

$$\text{ret} \to \frac{a^2\left(p^3(-(2\,\text{reu}+3)) + 3pn^2 + 2n^3\,\text{reu}\right) - 64p^3\,\text{res}\,\text{reu}}{6ap^3(p+n(2\,\text{reu}-1))}, \tag{C.9}$$

$$\text{imt} \to \frac{a\sqrt{p-n}\sqrt{\text{reu}\,(n\,\text{reu} - p\,(\text{reu}-2))}\left(2\,\text{reu}\left(p^2+pn+n^2\right)+p(n-p)\right)}{4p^3\,\text{reu}\,(p+n(2\,\text{reu}-1))}. \tag{C.10}$$

We can appreciate that the system is not able to find an expression for the real part of S. But now, Imaginary parts carry the desired physical information. As expected, they only depend on $Re\,U$. We can use this advantage to solve the system we left aside at the beginning. If we substitute these previous solutions in the real and imaginary parts of $D_U\,W = 0$, and solve the system of equations for the remaining variables, that, in this case, are the Real parts of U and S, we have:

$$\left\{\text{reu} \to \frac{2m}{m-n}\right\}, \tag{C.11}$$

$$\left\{\text{reu} \to \frac{m(m-n)}{m^2+n^2}, \quad \text{res} \to \frac{a^2\left(-m^3+3m^2n+3mn^2+n^3\right)}{32m^3}\right\}, \tag{C.12}$$

$$\left\{\text{reu} \to \frac{m(n-m)}{2\left(m^2+3mn+n^2\right)}, \quad \text{res} \to \frac{a^2\left(-m^3+3m^2n+3mn^2+n^3\right)}{32m^3}\right\}. \tag{C.13}$$

It is easy to check that solution (C.11) cannot be accepted as valid, due to the lack of $Re\,S$. Among the remaining solutions, we can use the aforementioned constraint (C.7), to decide which pair of solution is the correct one. At last, if we substitute the chosen solution in (C.8-C.10), we will get the moduli displayed in table 12. As a final test, we can prove the validity of these solutions in equation (4.1). If the solution is 0, our moduli are mathematically and physically acceptable with this methodology.

References

[1] J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond*, Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2007, 10.1017/CBO9780511618123.

[2] O. Klein, *Quantentheorie und fünfdimensionale relativitätstheorie*, *Zeitschrift für Physik* **37** (1926) 895.

[3] M. Grana, *Flux compactifications in string theory: A Comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003].

[4] S. Perlmutter, G. Aldering, G. Goldhaber, R. Knop, P. Nugent, P. Castro et al., *Measurements of* Ω *and* Λ *from 42 high-redshift supernovae*, *The Astrophysical Journal* **517** (1999) 565.

[5] A. H. Guth, *Inflationary universe: A possible solution to the horizon and flatness problems*, *Physical Review D* **23** (1981) 347.

[6] M. R. Douglas, *The Statistics of string / M theory vacua*, *JHEP* **05** (2003) 046 [hep-th/0303194].

[7] U. Danielsson and G. Dibitetto, *On the distribution of stable de Sitter vacua*, *JHEP* **03** (2013) 018 [1212.4984].

[8] S. B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D66** (2002) 106006 [hep-th/0105097].

[9] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D68** (2003) 046005 [hep-th/0301240].

[10] J. Shelton, W. Taylor and B. Wecht, *Nongeometric flux compactifications*, *JHEP* **10** (2005) 085 [hep-th/0508133].

[11] A. Guarino, *Generalised Fluxes, Moduli Fixing and Cosmological Implications*, Ph.D. thesis, Madrid, IFT, 2010.

[12] G. Dibitetto, *Gauged Supergravities and the Physics of Extra Dimensions*, Ph.D. thesis, Groningen U., 2012. 1210.2301.

[13] R. Blumenhagen, D. Lüst and S. Theisen, *Basic concepts of string theory*. Springer Science & Business Media, 2012.

[14] K. Becker, M. Becker and J. H. Schwarz, *String theory and M-theory: A modern introduction*. Cambridge University Press, 2006.

[15] T. W. Grimm and J. Louis, *The Effective action of N = 1 Calabi-Yau orientifolds*, *Nucl. Phys.* **B699** (2004) 387 [hep-th/0403067].

[16] G. Aldazabal, P. G. Camara, A. Font and L. E. Ibanez, *More dual fluxes and moduli fixing*, *JHEP* **05** (2006) 070 [hep-th/0602089].

[17] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four folds*, *Nucl. Phys.* **B584** (2000) 69 [hep-th/9906070].

[18] P. Meessen and T. Ortin, *An Sl(2,Z) multiplet of nine-dimensional type II supergravity theories*, *Nucl. Phys.* **B541** (1999) 195 [hep-th/9806120].

[19] D. Z. Freedman and A. Van Proeyen, *Supergravity*. Cambridge University Press, 2012.

[20] J. Shelton, W. Taylor and B. Wecht, *Generalized Flux Vacua*, *JHEP* **02** (2007) 095 [hep-th/0607015].

[21] J. Blåbäck, U. H. Danielsson, G. Dibitetto and S. C. Vargas, *Universal dS vacua in STU-models*, *JHEP* **10** (2015) 069 [1505.04283].

[22] U. H. Danielsson and T. Van Riet, *What if string theory has no de Sitter vacua?*, *Int. J. Mod. Phys.* **D27** (2018) 1830007 [1804.01120].

[23] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, *De Sitter Space and the Swampland*, 1806.08362.