Search, Bargaining and Employer Discrimination

by

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Abstract
This paper analyses Becker’s (1971) theory of employer discrimination within a search and wage-bargaining setting. Discriminatory firms pay workers who are discriminated against less, and apply stricter hiring-criteria to these workers. It is shown that the highest profits are realized by firms with a positive discrimination coefficient. Moreover, once ownership and control are separated, both highest profits and highest utility may be realized by firms with a positive discrimination coefficient. Thus, market forces, like entry and/or takeovers do not ensure that wage differentials due to employer discrimination will disappear.

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1. Introduction

In his seminal work on discrimination, Becker (1971) assumes that some agents have a "taste" for discrimination. Wages for women and blacks are lower because employers, co-workers, or consumers require a premium to interact with these groups. As pointed out by, e.g., Arrow (1972,1973) and Cain (1986), wage-differentials due to discrimination can, however, only be sustained as a short-run phenomenon in Becker’s model. In the long run they will disappear through segregation. That is, people belonging to a particular group interact exclusively with each other. In the case of employer discrimination, entry by employers without prejudices is an additional market force which will eliminate wage-differentials. As long as wage-differentials exist, non-discriminatory firms will be more profitable, driving discriminatory firms out of the market.

This paper re-examines Becker’s (1971) model of employer discrimination. The two main departures from Becker’s framework are the introduction of search frictions in the labour market and wage-setting by Nash-Bargaining. The resulting predictions differ substantially from those of Becker’s original model. In particular, wage-equalisation will not take place and profits are highest for firms with a positive discrimination coefficient (though more discriminatory employers have lower utility). Furthermore, in an extended setting with separation of ownership and control, the utility as well as profits may be highest for firms with a positive discrimination coefficient. More fundamentally, this paper establishes that wage-differentials caused by employer taste for discrimination may not be eliminated through market forces,
when there are search frictions, both workers and firms have some bargaining power, and ownership and control are separated.

Consider a model with two types of workers. One type is valued equally by all firms, while the valuation of the other type depends on the firm’s taste for discrimination (its discrimination coefficient). For most of the paper, workers belonging to the first group will be labelled male and those of the second group female. As in Pissarides (1984, 1985) matches between workers and firms differ in productivity.

Employers’ taste for discrimination affects profits through wages and hiring decisions. Compared to nondiscriminatory firms, discriminatory firms pay female workers a lower wage and male workers a higher wage, and the total wage bill is lower, provided that the firm is not too discriminatory. The discriminatory firms will, however, take non profit-maximising hiring decisions. Nevertheless, being not too discriminatory yields a higher profit. More precisely, there always exists a positive discrimination coefficient such that the positive effect of lower wages dominates the negative effect of suboptimal hiring policy. This holds because the wage-effect is of first order, while the hiring-effect is of second order.

Although not too discriminatory employers make higher profits, they have a lower utility, compared to nondiscriminatory employers. This suggests that there is scope for takeovers by nondiscriminatory employers. Employer’s utility in discriminatory firms may, however, not be lower once separation of ownership and control is introduced. When the person who conducts the wage-negotiation, say a manager, is not the residual claimant, discriminatory firm’s pay even lower wages for
female workers. The reason is that a discriminatory manager incurs the entire utility loss of employing female workers but receives only part of the profits. Provided that the manager’s share of the profits is sufficiently low, total utility (as well as profits) for the owners and the manager are highest in firms with a (not too) discriminatory manager. This implies that the owner’s profits are higher when employing such a discriminatory manager, even if the manager is compensated for the disutility that he derives from hiring female workers. Hence, market forces will drive out nondiscriminatory managers rather than discriminatory ones.

Becker’s theory of discrimination has been further developed by several authors. As in the present model, Akerlof (1985), Borjas and Bronars (1989), Sattinger (1992), and Sasaki (1997) introduce search friction and show that wage-differentials due to taste for discrimination is sustainable in the long run. These models differ from the present model with respect to the source of discrimination and the mechanism why discrimination may persist. Akerlof and Borjas and Bronars consider a model with discriminatory consumers. Wage-differentials are sustainable in the long run because shops with “wrong” employees will either have to wait longer for the right, (i.e. nonprejudiced), consumer or to sell at a lower price. Sattinger and Sakaki consider discriminatory co-workers. In Sattinger’s model minority worker will earn lower wages because firms that specialise in these workers have higher recruiting costs. Sasaki shows that in the presence of asymmetric co-worker discrimination, nondiscriminatory workers earn lower wages. Closely related to the present paper is Black (1995). He analyses employer taste for discrimination in a search environment but assumes that employers set the wages. In his model it is shown that wages will not
equalise, while both profits and utility are decreasing in the discrimination coefficient. In contrast, the present paper assumes wage-bargaining which is crucial for our main result that employer discrimination can be profit- as well as utility-maximising.¹

In addition to explaining why employer discrimination can be an equilibrium phenomenon, the model yields several predictions about wages and employment of different groups. First, irrespective of whether the firm is discriminatory, female workers earn less than male workers (for a given productivity). Second, female wages are lower and male wages are higher (for a given productivity) in firms which are more discriminatory. Third, more discriminatory firms apply stricter hiring standards for female workers than for male workers. Combining these results allows us to analyse the relationship between the composition of workers and wages. In particular, the present model predicts that male workers will earn less in environments where the proportion of female workers is higher.

The paper is organised as follows. Section 2 presents the basic model and shows that the highest profits are realised by firms with a positive discrimination coefficient. Section 3 introduces separation of ownership and control. Section 4 discusses the model’s implications for the correlation between wages and the composition of workers within firms and across occupations. Section 5 concludes the paper.

¹ Other discrimination papers where search frictions play a crucial role are Verma (1994) and Rosén (1997).
2. The Basic Model

The standard search model with non-cooperative wage-determination is taken as the starting point. Following Becker (1971), the important alternation is that the employer’s utility is a function of both, profits and employees’ characteristics, such as sex and race.

It is a continuous time model with two types of workers, \( i = f \) denotes female and \( i = m \) male. The proportion of each type, \( \alpha_i \), is exogenously given. Employers differ in the disutility that they derive from employing a \( f \)-worker. An employer of type \( c \) derives a disutility of \( c \) for each \( f \)-worker they employ (and zero disutility for each employed \( m \)-worker). \( c \) is distributed among employers according to the density function \( g(c) \), \( c \in [0, c^*] \). The density function is continuous and differentiable. \( G(c) \) is the corresponding distribution function. Each employer has one job. The mass of jobs and workers is unity.

Workers and firms are risk-neutral, infinitely lived, and have a common discount rate \( r \). Workers are either unemployed or employed, and jobs are vacant or occupied. Only unemployed workers and vacant jobs engage in search. An unemployed worker is matched with a vacancy and a vacancy with an unemployed worker at the same constant rate \( \phi \). A match will result in employment if and only if

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\(^3\) When the mass of workers equals the mass of jobs, the mass of unemployed will equal the mass of vacancies. Assuming a matching technology with constant returns to scale implies that the matching rate is constant and the same for workers and firms. Constant matching rates for workers and firms simplify the existence proof of equilibrium.
both firm and worker prefers employment to continuing search. Let $z_i$ and $y(c)$ denote the probability of employment for a type $i$ worker and for a type $c$ firm, given that a match has occurred. Employed workers separate from jobs at an exogenously given rate $s$.

A match has productivity $x$, where $x$ is a random drawing from the density function $f(x)$, with $x \in [0, \bar{x}]$. $f(x)$ is continuous, differentiable and the same for all workers and firms. $F(x)$ is the corresponding distribution function. For simplicity, it is assumed that $\bar{x} < c$.

Denote by $U_i$ the present discounted utility of an unemployed worker and by $J_i(c,x)$ the present discounted utility of an employed worker holding a job with match productivity $x$ and discrimination coefficient $c$. Without loss of generality, the income flow while unemployed is set equal to zero. In steady state, $U_i$ satisfies

$$rU_i = \phi z_i (E[J_i(c,x) | i] - U_i). \quad (1)$$

At the rate $\phi z_i$ the worker finds employment, in which case the expected increase in utility is $E[J_i(c,x) | i] - U_i$. Analogously, the present discounted utility for an employed worker of type $i$ and productivity $x$ in a job of type $c$ satisfies

$$rJ_i(c,x) = w_i(c,x) + s(U_i - J_i(c,x)), \quad (2)$$

where $w_i(c,x)$ is the wage. Using (1) and (2) gives
\[ rU_i = \frac{\phi_i E[w_i(c, x) | i]}{r + s + \phi_i} \]  

(3)

and

\[ J_i(c, x) = \frac{w_i(c, x) + sU_i}{r + s}. \]  

(4)

Denote by \( V(c) \) the present discounted value of a vacancy of type \( c \) and by \( H_i(c, x) \) the present discounted value of a type \( c \) job occupied by a type \( i \) worker of productivity \( x \). In steady state \( V(c) \) satisfies

\[ rV(c) = \phi y(c) (E[H_i(c, x) | c] - V(c)). \]  

(5)

At a rate \( \phi y(c) \) the vacancy is filled, in which case the expected increase in the value of the job is \( E[H_i(c, x) | c] - V(c) \). \( H_i(c, x) \) satisfies

\[ rH_i(c, x) = x - w_i(c, x) - c_i + s(V(c) - H_i(c, x)), \]  

(6)

where \( c_f = c \) and \( c_m = 0 \). The utility flow to the employer is \( x - w_i(c, x) - c_i \). Using (5) and (6) gives
\[ rV(c) = \frac{\phi(c)E[x - w_i(c,x) - c_i | c]}{r + s + \phi(c)}, \tag{7} \]

and

\[ H_i(c,x) = \frac{x - w_i(c,x) - c_i + sV(c)}{r + s}. \tag{8} \]

We now turn to the wage-determination and the employment decision. The wage is assumed to be determined by the Nash-bargaining solution. The wage of a type \( i \) worker who works in a type \( c \) firm and has a match productivity \( x \) is determined by

\[ \text{Max} \Omega_{w_i(c,x)} = \left( J_i(c,x) - U_i \right)^{\beta} \left( H_i(c,x) - V(c) \right)^{1-\beta}. \tag{9} \]

The wage-equation is obtained by solving (9) and using (4) and (8),

\[ w_i(c,x) = \beta(x - c_i - rV(c)) + (1-\beta)r U_i. \tag{10} \]

The wage is increasing in the productivity \( x \) and the worker’s outside option \( U_i \), but decreasing in the discrimination coefficient \( c_i \) and the firm’s outside option \( V(c) \).

A match between a worker and a firm results in employment if and only if both, the firm and the worker, gain from employment. That is, if \( J_i(c,x) > U_i \) and \( H_i(c,x) > \)
Given that the wage is determined by (10), these conditions can be rewritten as
\[ J_i(c) + H_i(c, x) > U_i + V(c), \]
which in flow terms is equivalent to
\[ x - c_i > rU_i + rV(c). \]
This latter inequality translates into a cut-off productivity \( \mu_i(c) \), where firms hire a worker if and only if \( x > \mu_i(c) \). The optimal cut-off level is given by

\[
\mu_i(c) = \begin{cases} 
  rU_i + rV(c) + c_i, & \text{if } rU_i + rV(c) + c_i \leq \bar{x} \\
  \bar{x}, & \text{otherwise.}
\end{cases}
\]

Denote the proportion of type \( i \) workers among the unemployed \( \lambda_i \), the distribution of vacancies \( H(c) \), and its corresponding density function \( h(c) \). It follows that the employment probability is
\[ z_i = \int_0^{\bar{x}} [1 - F(\mu_i(c))] h(c) dc \]
and the hiring probability is
\[ y(c) = \lambda_f \left[ 1 - F(\mu_f(c)) \right] + \lambda_m \left[ 1 - F(\mu_m(c)) \right]. \]
Using these equalities and equations (3), (7), and (10) we obtain expressions for \( rU_i \) and \( rV(c) \).

\[
rU_i = \frac{\beta \phi \int_0^{\bar{x}} \int_0^{\mu_f(c)} (x - c - rV(c)) f(x) dx h(c) dc}{r + s + \beta \phi c}, \quad (12)
\]

\[
rV(c) = \frac{(1 - \beta) \phi \left\{ \lambda_f \int_0^{\bar{x}} (x - c - rU_f) f(x) dx + \lambda_m \int_0^{\bar{x}} (x - rU_m) f(x) dx \right\}}{r + s + (1 - \beta) \phi y(c)}, \quad (13)
\]

\(^4\) When the wage is determined by (9) \( J_i(c, x) = U_i \Leftrightarrow H_i(c, x) = V(c) \).
Where the equilibrium cut-off levels are determined by (11). Notice that the cut-off levels given by (11) maximise (12) and (13) respectively. (Differentiating (12) and (13) with respect to \( \mu_i(c) \) gives
\[
\frac{\partial rU_i}{\partial \mu_i(c)} = 0 \iff \frac{\partial rV(c)}{\partial \mu_i(c)} = 0 \iff \mu_i(c) = rU_i + rV(c) + c_i.
\]

To complete the model we need expressions for the proportions of unemployed, of each type, \( \lambda_i \), and the distribution of vacancies, \( H(c) \). These are derived in appendix 1. In Appendix 2, the existence of equilibrium is proved.

We now characterise the equilibrium outcomes of \( V(c), U_i, W(c,x) \) and \( \mu_i(c) \).

First, we examine how the equilibrium value of a vacancy varies with the discrimination coefficient \( c \). Taking the derivative of (13) with respect to \( c \) gives
\[
\frac{drV(c)}{dc} = \frac{\partial rV(c)}{\partial c} + \frac{\partial rV(c)}{\partial \mu_i(c)} \frac{d\mu_i(c)}{dc} + \frac{\partial rV(c)}{\partial \mu_m(c)} \frac{d\mu_m(c)}{dc}.
\] (14)

Because \( \frac{\partial rV(c)}{\partial \mu_i(c)} = 0 \) for interior cut-off levels and \( \frac{d\mu_i(c)}{dc} = 0 \) for \( \mu_i(c) = \bar{x} \), equation (14) reduces to
\[
\frac{drV(c)}{dc} = \frac{\partial rV(c)}{\partial c} = \frac{-(1-\beta)\phi \lambda_i [1-F(\mu_i(c))]}{r+s+(1-\beta)\phi \gamma(c)}.
\] (15)

It follows directly from (15) that \(-1 < \frac{drV(c)}{dc} < 0\) if \( \mu_i(c) < \bar{x} \) and \( \frac{drV(c)}{dc} = 0 \) if \( \mu_i(c) = \bar{x} \). We can now establish the following Lemma.
Lemma 1: There exists a level \( c = \hat{c} \), \( \hat{c} \in (0, \bar{x}) \), such that \( f \)-workers are employable by, and only by, firms of type \( c < \hat{c} \).

Proof: It follows directly from (11) that Lemma 1 holds if there exists a \( \hat{c} \in (0, \bar{x}) \) such that \( rU_f + rV(c) + c \geq \bar{x} \) for \( c \geq \hat{c} \). To prove this it is sufficient to show that

(i) \( rV(c) + c \) is strictly increasing in \( c \),
(ii) \( rU_f + rV(c) + c < \bar{x} \) for \( c = 0 \), and
(iii) \( rU_f + rV(\bar{c}) + \bar{c} > \bar{x} \).

(i) It follows directly from (14) that \( \frac{d(rV(c) + c)}{dc} > 0 \).

(ii) Assume to the contrary that \( rU_f + rV(c) + c \geq \bar{x} \) for \( c = 0 \). Since \( rV(c) + c \) is increasing inc no female worker is then employable in any firm, and hence \( U_f = 0 \).

But from (13) follows that \( rV(c) < \bar{x} \) and hence \( rU_f + rV(c) + c \geq \bar{x} \) for \( c = 0 \), is a contradiction.

(iii) From \( rU_f + rV(c) \geq 0 \), and \( \bar{c} > \bar{x} \) follows that \( rU_f + rV(\bar{c}) + \bar{c} > \bar{x} \).

Lemma 2: Firms utility: The present discounted value of an vacancy \( V(c) \) is strictly decreasing inc for \( c \in [0, \hat{c}) \), and \( V(c) = V(\hat{c}) \) for \( c \geq \hat{c} \).

Proof: From (15) and Lemma 1 follows that \( \frac{drV(c)}{dc} < 0 \) for \( c < \hat{c} \), and \( \frac{drV(c)}{dc} = 0 \) for \( c \geq \hat{c} \).
The discrimination coefficient affects the value of a vacancy negatively as long as the firm hires some $f$-workers.

**Lemma 3: Workers utility:** The present discounted utility of an unemployed $m$-workers is higher than that of an unemployed $f$-workers, i.e. $U_m > U_f$

**Proof:**

\[
ru_m = \frac{\beta\phi \int_0^c (x - rV(c))f(x)dxh(c)dc}{r + s + \beta\phi \int_0^c f(x)dxh(c)dc} > \frac{\beta\phi \int_0^{\bar{c}} (x - rV(c))f(x)dxh(c)dc}{r + s + \beta\phi \int_0^{\bar{c}} f(x)dxh(c)dc} = ru_f
\]

The first inequality follows from that in equilibrium, $\mu_m(c)$ maximises $ru_m$. $\mu_f(c) < \bar{c}$ for $c \in [0, \bar{c})$ where $\bar{c} > 0$ implies that $\int_0^{\bar{c}} cf(x)dxh(c)dc > 0$ and hence the second inequality holds.  

Because firms value $f$-workers less, these workers have lower utility.
**Lemma 4:** **Wages:** (i) \( w_m(c,x) > w_f(c,x) \). (ii) \( w_m(c,x) \) is strictly increasing in \( c \) for \( c \in [0, \hat{c}) \), and \( w_m(c,x) = w_m(\hat{c},x) \) for \( c = \hat{c} \). (iii) \( w_f(c,x) \) is strictly decreasing inc.

**Proof:** Part (i) follows from the wage-equation (10) and \( rU_m > rU_f \). Part (ii) follows from the wage-equation (10) and Lemma 2. Part (iii) follows from the wage-equation and \( \frac{d(rV(c)+c)}{dc} > 0 \).

For a given \( c > 0 \) and \( x \), wages of \( m \)-workers are higher than those of \( f \)-workers because of two reasons. First, \( m \)-workers have a better bargaining position \( (rU_m > rU_f) \). Second, a discrimination coefficient greater than zero lowers \( f \)-workers wages directly. Moreover, \( m \)-workers earn higher wages also in nondiscriminatory firms \( (c = 0) \) because of their better bargain position. When firms are more discriminatory, while still hiring female workers, \( m \)-workers’ wages increase, because of the weakened bargaining position of those firms. The \( f \)-workers’ wages decrease with the discrimination coefficient. The direct effect of the discrimination coefficient dominates the effect that the firm’s bargaining position weakens with the discrimination coefficient.

**Lemma 5:** **Cut-off productivity:** (i) The \( m \)-workers cut-off productivity, \( \mu_m(c) \) is decreasing in \( c \) for \( c \in [0, \hat{c}) \), and \( \mu_m(c) = \mu_m(\hat{c}) \) for \( c = \hat{c} \). (ii) \( \mu_m(0) > \mu_m(\hat{c}) \). (iii) The \( f \)-workers’ cut-off productivity, \( \mu_f(c) \), is strictly increasing in \( c \) for \( c \in [0, \hat{c}) \), and \( \mu_f(c) = \bar{x} \) for \( c = \hat{c} \). (iv) \( \mu_m(0) > \mu_f(0) \).
Proof: (i) The cut-off equation (11) and Lemma 2 imply that, $\mu_m(c)$ is decreasing in $c$ for $c \in [0, \hat{c})$, and $\mu_m(c) = \mu_m(\hat{c})$ for $c = \hat{c}$. (ii) Part (i) implies that $\mu_m(0) = \mu_m(\hat{c})$. Since $V(c)$ is strictly decreasing in $c \in [0, \hat{c})$, it follows from (11) that $\mu_m(0) = \mu_m(\hat{c})$ iff $\mu_m(0) = \mu_m(\hat{c}) = \bar{x}$. Now $\mu_m(\hat{c}) = \bar{x}$ implies that $rV(\hat{c}) = 0$ (since $\mu(\hat{c}) = \bar{x}$) which contradicts that $\mu_m(\hat{c}) = \bar{x}$ (since $rU_m < \bar{x}$). Part (iii) follows from the cut-off equation (11) and that $c + V(c)$ is strictly increasing in $c$. Part (iv) follows from the cut-off equation and $rU_m > rU_f$. \\Firms with a high discrimination coefficient are more selective in employing women but less selective in employing men. The reason for the latter is that the option value, $V(c)$, is decreasing in $c$. Thus, the $f$-workers are better (of higher productivity) and the $m$-workers are worse in more discriminatory firms. Moreover, because of different outside opportunities for these two groups, also non-discriminatory firms apply different hiring standards for women and men.

We will now analyse the relationship between the discrimination coefficient and profits. The present discounted value of profits from a vacancy in flow terms, $r\Pi(c)$, is equal to $rV(c)$ net of the disutility $c_i$. Thus, the analogue to (7) in profit terms is

$$r\Pi(c) = \frac{\phi(c)E[x - w_i(c, x) | c]}{r + s + \phi(c)}.$$  

(16)
Inserting the wage-equation (10) into (16) yields

\[ r\Pi(c) = \]

\[ \phi \left\{ \lambda_f \int_{\mu_f(c)}^{\bar{x}} ((1 - \beta)(x - rU_f) + \beta (rV(c) + c))f(x)dx + \lambda_m \int_{\mu_m(c)}^{\bar{x}} ((1 - \beta)(x - rU_m) + \beta rV(c))f(x)dx \right\} \]

\[ r + s + \phi r(c) \]

(17)

Differentiating \( r\Pi(c) \) with respect to \( c \), we obtain

\[ \frac{dr\Pi(c)}{dc} = \frac{\partial \Pi(c)}{\partial c} + \frac{\partial \Pi(c)}{\partial rV(c)} \frac{drV(c)}{dc} + \frac{\partial \Pi(c)}{\partial \mu_f(c)} \frac{d\mu_f(c)}{dc} + \frac{\partial \Pi(c)}{\partial \mu_m(c)} \frac{d\mu_m(c)}{dc}. \]  

(18)

A change in the discrimination coefficient affects profits through wages and hiring standards. The wage-effect covers the direct effect of discrimination on the \( f \)-workers’ wages (first term of (18)) and the indirect effect on the value of a vacancy (second term).

A change in \( c \) affects the hiring-standards applied to \( m \)-workers (third term) and \( f \)-workers (fourth term). In order to sign \( \frac{dr\Pi(c)}{dc} \) for \( c < \hat{c} \), we analyse the terms in (18)
in two steps.\footnote{If $c > \hat{c}$ all the terms in (18) are equal to zero.} First, we derive the wage effect. Using (17) we find that the direct wage effect of $c$ on profits is

$$\frac{\partial \Pi(c)}{\partial c} = \frac{\beta \phi \lambda_c [1 - F(\mu_c(c))]}{r + s + \phi(y(c))} > 0.$$  \hspace{1cm} (19)

The indirect wage effect of $c$ through the value of a vacancy is

$$\frac{\partial \Pi(c)}{\partial V(c)} \frac{drV(c)}{dc} = \frac{\beta \phi y(c) drV(c)}{r + s + \phi(y(c))} < 0.$$  \hspace{1cm} (20)

Using (15), (19) and (20) we find that the sum of the two wage effects is

$$\frac{\partial \Pi(c)}{\partial c} + \frac{\partial \Pi(c)}{\partial V(c)} \frac{drV(c)}{dc} = \frac{(r + s) \beta \phi \lambda_c [1 - F(\mu_c(c))]}{(r + s + \phi(1 - \beta)y(c))(r + s + \phi(y(c)))} > 0.$$  \hspace{1cm} (21)

Hence, for given hiring standards, the wage-bill decreases and profits increase in $c$.

In the second step, we explore the effect of $c$ on profits through its impact on hiring standards. To derive explicit expressions for the last two terms in (18), it is useful to rewrite the profit function as

$$r \Pi(c) = rV(c) + Y(c),$$  \hspace{1cm} (22)
where \( Y(c) = \frac{c \phi \lambda_f [1 - F(\mu_f(c))]}{r + s + \phi y(c)} \).

Using equations (22), (11), that \( \frac{\partial V(c)}{\partial \mu_f(c)} = 0 \), and that \( -1 < \frac{drV(c)}{dc} < 0 \), we find that the effect on profits from changed hiring-standards is negative.

\[
\frac{\partial \Pi(c)}{\partial \mu_f(c)} \frac{d\mu_f(c)}{dc} = \frac{-c \phi \lambda_f \mu_f(c)[r + s + \phi \lambda_m[1 - F(\mu_m(c))]}{(r + s + \phi y(c))^2} \left( \frac{drV(c)}{dc} + 1 \right) < 0. \tag{23}
\]

\[
\frac{\partial \Pi(c)}{\partial \mu_m(c)} \frac{d\mu_m(c)}{dc} = \frac{c \phi \lambda_f[1 - F(\mu_f(c))]}{(r + s + \phi y(c))^2} \frac{drV(c)}{dc} < 0. \tag{24}
\]

Hence, the sum of the effects on wages is positive while the effect on profits from changed hiring-standards is negative. Being discriminatory lowers the wage-bill for a given work force, but leads to non profit-maximising hiring decisions. We are now ready to state the following proposition.

**Proposition 1.** The highest profits are realised by firms with a positive discrimination coefficient.
Proof: At \( c = 0 \), \( r\Pi(c) = r\nu(c) \) and hence \( \frac{\partial r\Pi(c)}{\partial \mu_i(c)} = \frac{\partial r\nu(c)}{\partial \mu_i(c)} = 0 \). Thus, at \( c = 0 \)

\[
\frac{dr\Pi(c)}{dc} = \frac{\partial r\Pi(c)}{\partial c} + \frac{\partial r\Pi(c)}{\partial \nu(c)} \frac{dr\nu(c)}{dc},
\]

which by (21) is positive. 

The intuition for proposition 1 is as follows. In the absence of discrimination \((c = 0)\), the chosen cut-off level is profit maximising. Therefore, a change in \( c \) leads to a hiring effect which is only of second order, while the effect on wages is of first order. Proposition 1 contrasts with the implications of Becker’s (1971) original work and Black (1995). In their models, a firm with a zero discrimination coefficient always make higher profits, given that there are wage-differentials.\(^6\)

Profits are, however, not increasing in \( c \) for all values of \( c \). For some values of \( c \), the negative effect from suboptimal hiring outweighs the lower wage bill. This is most easily seen in the case where \( c = \hat{c} \). If \( c \) is so high that no females are hired, the discriminatory firm makes lower profits. It pays \( m \)-workers a higher wage than nondiscriminatory firms and forgo profits by not employing \( f \)-workers.\(^7\)

The result that being discriminatory is profit-maximising depends on two assumptions; wage determination through bargaining and search frictions. Wage bargaining implies that discriminatory firms pay \( f \)-workers less than nondiscriminatory

\(^6\)There are few tests of the relationship between profits and discrimination. One exception is Reich (1981). In a cross section of 48 areas he finds a negative correlation between the ratios of black workers’ wages to white workers’ wages and profits. This study is extensively discussed in Cain (1986).

\(^7\)Formally, at \( c = \hat{c} \), no females are hired and \( r\Pi(\hat{c}) = r\nu(\hat{c}) \). Since \( r\nu(\hat{c}) < r\nu(0) \) and \( r\nu(c) = r\Pi(c) \) for \( c = 0 \) we have \( r\Pi(\hat{c}) = r\nu(\hat{c}) < r\nu(0) = r\Pi(0) \).
firms. Search frictions imply that firms with (not too) positive discrimination coefficients also hire female workers. With regard to other modelling assumptions the result is fairly robust.\footnote{For example, the results still hold if we would assume that all workers are equally productive or that general productivity levels differ among workers. However, match-specific productivity differences yield the most tractable model. For the same reason the discrimination coefficient is assumed to be continuously distributed without mass points. In the bargaining it is assumed that the outside opportunity is the threat point. Alternatively, one may assume that the outside option is only a constraint in the bargaining. This would have affected some of the Lemmata but not Proposition 1.}

In Becker’s model wage-differentials due to employer taste for discrimination are unstable in the long run when ownership can be transferred at a low cost (see e.g. Arrow 1972). Nondiscriminatory entrepreneurs profit by buying out discriminatory entrepreneurs. In the present model, a similar long-run stability problem arises. Although discriminatory owners may make higher expected profits, they will have lower expected utility, compared to the nondiscriminatory owner. (Recall that $V(c)$ is decreasing in $c$.) Consequently, there is scope for takeovers by nondiscriminatory owners. The next section will show that this long run stability problem may not arise, once ownership and control are separated.

So far we have implicitly assumed that employers are manager-owners. This section considers separation of ownership and control. That is, the person who takes the hiring decision and bargains with the worker is not the sole owner, or more precisely, is not the residual claimant. One can think of managers and shareholders, an owner and the personnel director, or a partnership where one partner is in charge of hiring and wage-bargain. Subsequently, manager refers to the person who takes hiring decisions and conducts the wage-bargaining, and owners to the party who’s utility is affected by profits, but who are not in charge of these tasks.

It is assumed that the managers may or may not have a taste for discrimination and maximises his own utility, whereas the owners only care about profits. It is crucial that the manager’s utility depends on profits. Profits may either enter directly into the manager’s remuneration package through e.g. stock options, or indirectly through e.g. the likelihood of keeping the job. For simplicity, we assume that the manager’s wage consists of a fixed payment \( a \) and a proportion \( \iota \) of the profits. When the job is vacant the manager’s instantaneous utility is \( a \) and owners return is \(-a\). When the job is occupied by a worker of type \( i \) and the manager is of type \( c \), the manager gets a flow utility of \( a + \iota(x - w_i(c,x)) - e_i \), and the owners’ return is: \(-a + (1 - \iota)(x - w_i(c,x))\)

Solving for the value of a vacancy and the value of an occupied job in the same way as in section 2 yields that the present discounted value of a vacancy to a manager of type \( c \) \( VM(c) \) satisfies
\[ rVM(c) = a + \frac{\phi \left[ \lambda_f \int_{\beta_f(c)}^\tau (t(x - w_f(c,x)) - c)f(x)dx + \lambda_m \int_{\beta_m(c)}^\tau (t(x - w_m(c,x))f(x)dx \right]}{r + s + \phi y(c)} \]  

(25)

The present discounted value of a job occupied by a worker of type \( i \) and productivity \( x \) to a manager of type \( c \) \( HM_i(c) \) satisfies

\[ HM_i(c) = \frac{a + t(x - w_i(c,x)) - c_i + sVM(c)}{r + s}, \]  

(26)

and the present discounted expected profit for the owners with a manager of type \( c \), \( VO(c) \), satisfies

\[ rVO(c) = -a + \frac{\phi \left[ \lambda_f \int_{\beta_f(c)}^\tau (1-t)(x - w_f(c,x))f(x)dx + \lambda_m \int_{\beta_m(c)}^\tau (1-t)(x - w_m(c,x))f(x)dx \right]}{r + s + \phi y(c)} \]  

(27)

The wage will now be determined by the following maximisation problem, (where \( U_i \) and \( J(c) \) are defined by (3) and (4)):

\[ \max_{w_i(c,x)} \Omega(c,x) = (J(c,x) - U_i)^\beta (HM_i(c,x) - VM(c))^{1-\beta}. \]  

(28)

A wage-equation is obtained by solving (28) and using (4) and (26),
$w_{c,x} = \beta \left( x + \frac{a}{t} - c_i - \frac{rVM(c)}{t} \right) + (1 - \beta) rU_i.$ \hspace{1cm} (29)

Comparison of (10) and (26) shows that the direct effect of the discrimination coefficient is now magnified by $\frac{1}{t}$.

In accordance with the last section, one can show that, from the owners point of view, the optimal discrimination coefficient differs from zero if the manager is not compensated for his disutility of hiring $f$-workers. There is, however, an even stronger result.

**Proposition 2:** The discrimination coefficient that maximises the sum of the owners’ profit and the manager’s utility is positive if

$$t < \frac{\beta (r+s)}{r+s + (1 - \beta) \phi y(c)}.$$ \hspace{1cm} (30)

*Proof:* See appendix 3.

When $t$ is small the wage is bargained down by more than the discrimination coefficient. The reason is that the manager carries the full utility loss from hiring a $f$-worker but receives only part of the extra profits. Thus, when the manager’s share of the profits is small, owners can compensate a slightly discriminatory manager for his
disutility (for example through a higher \( a \)) and still earn higher profits. In this case there is no (economic) incentives to replace a discriminatory manager. On the contrary, employing a nondiscriminatory manager does not lead to higher utility (nor higher profits). As a result, market forces will not ensure that only nondiscriminatory firms survive in the long run.

The model implicitly assumes that the manager cannot affect the number of jobs. In principle, one could expand the model allowing the manager to first choose the firm size and then to make the hiring decisions. Assuming decreasing returns to the number of jobs, discriminatory managers will choose smaller sizes. The effect of the discrimination coefficient on firm size is, like the hiring decision of second order. Therefore, the result is likely to hold also in such an extended setting.

4. Wages and the composition of workers

In this section we investigate in more detail the model’s predictions about the relationship between the wages and the composition of the work force. Given the assumption that all jobs are identical but firms differ in their levels of \( c \), the model is applicable to intra-occupational wage-differentials across firms. Another interpretation of the model is that firms are identical and have several occupations, but that the
discrimination coefficient differs between the occupations. This interpretation makes
the model also applicable to inter-occupational wage-differentials⁹.

Section 2 has shown that more discrimination (higher $c$ values) increases the
proportion of $m$-workers and their wages, and decreases $f$-workers wages. Hence,
working with their own type is associated with higher wages. Viewed differently, the
model predicts that male (white) workers earn less when working in female (black)
dominated occupations/firms, while the reverse holds for female (black). The wages
are lower for male worker in a setting dominated by female workers because of the
stronger bargaining position that firms have in these settings. The wages are lower for
female workers in male dominated settings, because firms are more likely to be
discriminatory.

The crowding theory by Bergman (1974) and the theory of employee taste for
discrimination by Becker (1971) also address the issue of wages and the composition
of workers. The crowding theory is based on the assumption that one group (women
or blacks) is excluded from some occupations. They therefore crowd into the
remaining occupations and drive down marginal productivity and wages in these
occupations. As our model (but for a different reason), it predicts that female
dominated occupations pay lower wages to men. The crowding theory also implies
that black workers or women, who for reasons such as specialised education, work in
sectors dominated by men or whites (i.e., sectors where the discrimination is large),

⁹ For simplicity, we have assumed that $c$ is distributed without mass points and that there is a
continuum of jobs. It is possible to derive all the lemmata and propositions under the alternative
assumption of a discrete number of firm, each with a given number of jobs.
will have a lower wage, due to taste for discrimination, compared to those in other sectors (see Bergman (1974)). This effect is essentially the same as in the present model. In the co-worker discrimination theory, workers who have to work in integrated firms demand higher wages for the disutility of working with the other sex or race, contradicting the predictions of this model (and those of the crowding theory).

Several empirical studies examine how wages are affected by the occupational or intra-firm sex-composition.\textsuperscript{10} The majority of the studies find a negative effect on both men’s and women’s wages from working in female dominated environments. The reported lower wages of men in predominantly female environments is consistent with our theory as well as with the crowding theory. But neither of these theories predicts that women will earn more when working with men.

Hirsch and Schumacher (1992) and Sorensen (1989) study the effect of racial occupational composition on wages. Hirsch and Schumacher find that both black and white earn lower wages if they work in occupations with more black workers. The lower wages of white in predominantly black environments is consistent with our theory as well as the crowding theory. The finding that black earn less in black dominated occupations conflicts with both theories. Sorensen reports a negative effect only on the wages of white male workers working in occupations dominated by black workers, more in line with our theory. Ragan and Tremblay (1988) provide empirical studies on the effect on wages of the composition of race within firms. They report that both white and black workers earned more when working in an integrated firm,

\textsuperscript{10}See e.g. Sorensen (1990), Blau (1977), and Ragan and Tremblay (1988).
contradicting the predictions of our theory but supporting the co-worker discrimination theory.

One should, however, be cautious when comparing the theoretical wage-predictions of this model, (as well as others), directly to different empirical findings. First, the theoretical correlation between wages and composition of workers described above holds only if corrected for productivity differences. Without these corrections, the results may be reversed. For example, firms with high $c$ will have higher standards for $f$-workers and, depending on the functional forms, the average wages may be higher for $f$-workers in such firms. Second, the proportion of types of workers in a firm (for a given $c$) is affected by variables which also affects wages, e.g., search frictions.

5. Conclusion

Becker’s original model of employer taste for discrimination predicts that discriminatory firms earn lower profits than non-discriminatory ones, unless there is total segregation, in which case wage-discrimination will disappear. This paper analyses employer taste for discrimination in an extended framework where there are frictions in the labour market and wages are set by Nash-bargaining. The main findings are that discriminatory firms may have higher profits and that, under separation of ownership and control, employing a manager with a taste for discrimination may be profitable for the owners. This holds even if he is compensated for the disutility incurred from
interacting with the disliked group. Because profits and utility may be higher for
discriminatory firms, they will not be driven out of the market.
Appendix 1

Let $u$ denote the proportion of workers that are unemployed, and $v$ the proportion of jobs that are vacant. The steady state unemployment condition implies that the flow out of unemployment equals the flow into unemployment. Hence,

$$\lambda u \phi z = s(\alpha_i - \lambda u).$$  \hspace{1cm} (A1.1)

Rearranging (A1.1) gives

$$\lambda u = \frac{s\alpha_i}{s + \phi z}.$$ \hspace{1cm} (A1.2)

Using (A1.2) and that $\lambda_f + \lambda_m = 1$ gives,

$$\lambda_f = \frac{\alpha_i(s + \phi z_m)}{\alpha_i(s + \phi z_m) + \alpha_m(s + \phi z_f)}, \quad \lambda_m = \frac{\alpha_m(s + \phi z_f)}{\alpha_m(s + \phi z_f) + \alpha_i(s + \phi z_m)}.$$ \hspace{1cm} (A1.3)

Analogously, the flow of vacancies filled in steady state should equal the flow of new vacancies. That is,

$$h(c)v = \frac{sg(c)}{s + \phi y(c)},$$ \hspace{1cm} (A1.4)
where

\[ v = \int_0^s \frac{s}{s + \phi_y(c)} \, dG(c) . \]  (A1.5)
Appendix 2: Proof of existence of equilibrium.

The equilibrium conditions are

\[
\lambda_f = \frac{\alpha_f(s + \phi z_m)}{\alpha_f(s + \phi z_m) + \alpha_m(s + \phi f)}.
\]  

(E1)

\[
\lambda_m = 1 - \lambda_f.
\]  

(E2)

\[
z_i = \int_0^\tau (1 - F(\mu_i(c))) dH(c), \quad i = f,m.
\]  

(E3)

\[
h(c)v = \frac{sg(c)}{s + \phi y(c)}, \quad c \in [0, \bar{c}],
\]  

(E4)

where \( v = \int_0^\tau \frac{s}{s + \phi y(c)} dG(c). \)

\[
y(c) = \lambda_f[1 - F(\mu_f(c))] + \lambda_m[1 - F(\mu_m(c))], \quad c \in [0, \bar{c}].
\]  

(E5)

\[
rU_i = \frac{\beta \phi \int_0^\tau \int_{\min[rU_i + rV(c) + c, \bar{c}]}^\tau (x - c_i - rV(c))f(x)dxH(c)}{r + s + \beta \phi z_i}, \quad i = f,m.
\]  

(E6)

\[
rV(c) = \frac{(1 - \beta) \phi \left\{ \lambda_f \int_{\min[rU_f + rV(c) + c, \bar{c}]}^\tau (x - rU_f - c)f(x)dx + \lambda_m \int_{\min[rU_m + rV(c), \bar{c}]}^\tau (x - rU_m)f(x)dx \right\}}{r + s + (1 - \beta) \phi y(c)},
\]  

\( c \in [0, \bar{c}]. \)

(E7)

\[
\mu_i(c) = \begin{cases} 
  rU_i + rV(c) + c_i & \text{if } rU_i + rV(c) + c_i \leq \bar{x} \\
  \bar{x} & \text{otherwise}
\end{cases}, \quad i = f,m, \quad c \in [0, \bar{c}].
\]  

(E8)
Exogenously given are the variables $\alpha_i$, $\alpha_m$ $s$, $\phi$, $\bar{c}$, $\bar{x}$, $r$, the distribution functions $G(c)$ and $F(x)$, and its corresponding density functions $g(c)$ and $f(x)$. $f(x)$ is continuos and $x \in [0, \bar{x}]$.

The claim is that the system $(E1) - (E8)$ has a solution for any probability distribution $G$ with its mass on $[0,\bar{c}]$.

We first show that there exists a solution if $c$ attains only finitely many values (= $n$). Let $g(c)$ and $h(c)$, $c = c^1, c^2, \ldots, c^n$, be the corresponding probability functions for the (discrete) distributions $G(c)$ and $H(c)$. The interpretation of $g(c)$ and $h(c)$ are the proportions of jobs and vacancies respectively with discrimination coefficient $c$, $c = c^1, c^2, \ldots, c^n$. Then $(E1) - (E8)$ is a system with $6 + 5n$ endogenous variables ($z_i$, $rU_j$, $rV(c)$, $\mu_i(c)$, $\lambda_i$, $h(c)$, $y(c)$, $i = f,m$ and $c = c^1, c^2, \ldots, c^n$) and $6 + 5n$ equations. We will prove that there exists a solution to this system by using Brouwer's fixed point theorem.

Let $Z$ be a vector; $Z = (rU_j, rU_m, \mu_i(c^1), \mu_i(c^2), \ldots, \mu_i(c^n), \mu_m(c^1), \mu_m(c^2), \ldots, \mu_m(c^n), \lambda_j, \lambda_m, y(c^1), y(c^2), \ldots, y(c^n))$, where the components $rU_j$, $rU_m$, $\mu_i(c^1)$, $\mu_i(c^2)$, $\ldots, \mu_i(c^n)$, $\mu_m(c^1)$, $\mu_m(c^2)$, $\ldots, \mu_m(c^n)$ take arbitrary values between 0 and $\bar{x}$, the components $\lambda_j, \lambda_m, y(c^1), y(c^2), \ldots, y(c^n)$ take arbitrary values between 0 and 1, and $\lambda_j + \lambda_m = 1$. Let $A$ denote the range for $Z$, i.e. $A = [0, \bar{x}]^{2n+2} \times [0, 1]^{n+2}$, and $\lambda_j + \lambda_m = 1$.

Let $X$ be a vector; $X = (rV(c^1), rV(c^2), \ldots, rV(c^n), z_j, z_m, h(c^1), h(c^2), \ldots, h(c^n))$, where the components $rV(c^1)$, $rV(c^2)$, $\ldots, rV(c^n)$ take arbitrary values between 0 and $\bar{x}$, the components $z_j$, $z_m$, $h(c^1)$, $h(c^2)$, $\ldots$, $h(c^n)$ take arbitrary values between 0 and 1, and $h(c^1) + h(c^2) + \ldots + h(c^n) = 1$. Let $B$ denote the range of $X$, i.e. $B = [0, \bar{x}]^n \times [0, 1]^{n+2}$, and $h(c^1) + h(c^2) + \ldots + h(c^n) = 1$. 
Lemma A2.1: The equations (E3), (E4) and (E7) specify a continuous (single valued) mapping from $A$ into $B$, $X = \psi (Z)$, $Z \in A$ and $X \in B$.

Proof: (i) Let $L(rV(c')) = (1 - \beta)\phi \left\{ \lambda_j \int_{\text{Min}[rU_j + rV(c'), x]}^{x} (x - rU_j - c')f(x)dx + \lambda_m \int_{\text{Min}[rU_m + rV(c'), x]}^{x} (x - rU_m)f(x)dx \right\}$

$r + s + (1 - \beta)\phi \psi (c')$

, $j = 1, 2, \ldots, n$. Since $L$ is continuous, $L(0) \equiv 0$, $L(\overline{x}) = \overline{x}$ and $\frac{dL}{drV(c')} > 0$ there exist, for each $rV(c')$, a unique solution $rV(c') \in [0, \overline{x}]$ to the equation $L(rV(c')) = 0$. Hence, for a given $Z \in A$ it follows from equation (E7) that $rV(c')$ is uniquely determined, $rV(c') \in [0, \overline{x}]$ and that $rV(c')$ is continuous in $Z, j = 1, 2, \ldots, n$. (ii) Given $y(c') \in [0, 1]$ it follows from equation (E4) that $h(c')$ is uniquely given, that $h(c'^1) + h(c'^2) + \ldots + h(c'^n) = 1$ and that $h(c')$ is and continuous in $Z, j = 1, 2, \ldots, n$. (iii) For given $\mu(c') \in [0, \overline{x}]$, $h(c') \in [0, 1]$ and $h(c'^1) + h(c'^2) + \ldots + h(c'^n) = 1$ it follows from equation (E3) that $z_i$ is uniquely determined, $z_i \in [0, 1]$ and that $z_i$ is continuos in $Z, i = f, m, j = 1, 2, \ldots, n$.

Lemma A2.2: The equations (E1), (E2), (E5), (E6) and (E8) specify a continuous (single valued) mapping from $B$ into $A$, $Z = \varphi (X)$, $Z \in A$ and $X \in B$. ||
Proof: (i) Let $T(rU_i) = rU_i - \beta \phi \int_{j=1}^{j=n} \int_{\min(rU_i + rV(c^j), \lambda_1)}^{\gamma} (x - c^j_i - rV(c^j)) f(x) dx dH(c^j)$, $i = f, m$. Since $T$ is continuous, $T(0) \geq 0$, $T(\bar{x}) = \bar{x}$ and $\frac{dT}{drU_i} > 0$, there exists, for each $rU_i$, a unique solution $rU_i \in [0, \bar{x}]$ to the equation $T(rU_i) = 0$. Hence, for a given $X \in B$ it follows from equation (E6) that $rU_i$ is uniquely determined, $rU_i \in [0, \bar{x}]$ and that $rU_i$ is continuous in $X, i = f, m$. (ii) Given $rV(c^j) \in [0, \bar{x}]$ and $rU_i \in [0, \bar{x}]$ it follows from equation (E8) that $\mu_i(c^j)$ is uniquely determined, $\mu_i(c^j) \in [0, \bar{x}]$ and that $\mu_i(c^j)$ are continuous in $X, i = f, m, j = 1, 2, ..., n$. (iii) Given $z_i \in [0, 1]$ it follows from equations (E1) and (E2) that $\lambda_i$ is uniquely determined, $\lambda_i \in [0, 1], \lambda_f + \lambda_m = 1$ and that $\lambda_i$ is continuous in $Z, i = f, m$. (iv) Given $\lambda_i \in [0, 1], \lambda_f + \lambda_m = 1, \mu_i(c^j) \in [0, \bar{x}]$ it follows from equation (E5) that $y(c^j)$ is uniquely determined, $y(c^j) \in [0, 1]$, and that $y(c^j)$ is continuous in $X, i = f, m, j = 1, 2, ..., n$.

Now $\phi(\psi(Z)) = k(Z)$ is a continuous mapping from $A$ into $A$. An equilibrium of the model is a fixed point of the mapping $k$, that is a value which satisfies $Z_0 = k(Z_0)$. Since $k$ is continuous and the domain $A$ of $k$ is compact and convex we know from Brouwer’s fixed point theorem there exists at least one solution to $Z = k(Z)$.

Since all involved integrands are continuous functions of $c$, the existence of a solution in the case when $G(c)$ is a continuous distribution on a finite interval $[0, \bar{c}]$ can
be obtained by approximating $G$ by the discrete distribution $G^T$, with mass

$$
\int_{\frac{k-\frac{1}{n}}{n}}^{\frac{k}{n}} dG \quad \text{in}
$$

$$\tilde{c} \frac{k}{n}, \ k = 1, 2, ..., n \text{ and then let } n \text{ tend to } \_ \_ \_.$$

Appendix 3: Proof of Proposition 2.

Using (25) and (27) gives

\[ rVM(c) + rVO(c) = \frac{\phi\left(\lambda_f \int_{\mu_f(c)}^c (x - w_f(c,x) - c)f(x)dx + \lambda_m \int_{\mu_m(c)}^c (x - w_m(c,x))f(x)dx\right)}{r + s + \phi_y(c)}. \]  

(A3.1)

Using that \( \frac{\partial rVM(c)}{\partial \mu_i(c)} = \frac{\partial rVO(c)}{\partial \mu_i(c)} = 0 \) at \( c = 0 \), it follows that \( rVM(c) + rVO(c) \) is increasing in \( c \) at \( c = 0 \) if

\[ \lambda_f \int_{\mu_f(c)}^c \left( -\frac{dw_f(c,x)}{dc} - 1 \right)f(x)dx + \lambda_m \int_{\mu_m(c)}^c \frac{dw_m(c,x)}{dc}f(x)dx > 0. \]  

(A3.2)

From the wage-equation (29), we get

\[ \frac{dw_f(c,x)}{dc} = -\beta \left( \frac{1}{t} + \frac{t}{t} \frac{drVM(c)}{dc} \right), \]  

(A3.3)

and

\[ \frac{dw_m(c,x)}{dc} = -\beta \frac{1}{t} \frac{drVM(c)}{dc}. \]  

(A3.4)
Inserting the wage-equation (29) into (25) gives

\[
rVM(c) = \frac{(1 - \beta)\phi \left\{ \lambda_f \int_{\mu_f(c)}^{\bar{c}} (t(x - rU_f) - c)f(x)dx + \lambda_m \int_{\mu_m(c)}^{\bar{c}} (t(x - rU_m)f(x)dx \right\}}{r + s + (1 - \beta)\phi(c)} + a ,
\]

(A3.5)

and at \( c = 0 \)

\[
\frac{drVM(c)}{dc} = \frac{-(1 - \beta)\phi \lambda_f[1 - F(\mu_f(c))]}{r + s + (1 - \beta)\phi(y(c))} .
\]

(A3.6)

Inserting (A3.3), (A3.4) and (A3.6) into (A3.2) gives

\[
\lambda_f \int_{\mu_f(c)}^{\bar{c}} \left( -\frac{dw_f(c, x)}{dc} - 1 \right)f(x)dx + \lambda_m \int_{\mu_m(c)}^{\bar{c}} \left( -\frac{dw_m(c, x)}{dc} \right)f(x)dx > 0
\]

\(\Leftrightarrow\)

\[
\lambda_f[1 - F(\mu_f(c))]( - \frac{dw_f(c, x)}{dc} - 1 ) + \lambda_m[1 - F(\mu_m(c))]( - \frac{dw_m(c, x)}{dc} ) > 0
\]

\(\Leftrightarrow\)

\[
\lambda_f[1 - F(\mu_f(c))](\beta\frac{1}{t} + \frac{1}{t} \frac{drVM(c)}{dc} - 1 ) + \lambda_m[1 - F(\mu_m(c))](\beta\frac{1}{t} drVM(c)) > 0
\]

\(\Leftrightarrow\)

\[
y(c)\beta\frac{1}{t} \frac{drVM(c)}{dc} + \lambda_f[1 - F(\mu_f(c))](\beta - 1 ) > 0
\]

\(\Leftrightarrow\)

\[
-\beta(1 - \beta)\phi(y(c)) + (\beta - t) > 0
\]
\[
\implies t < \frac{\beta(r + s)}{r + s + (1 - \beta)\phi_y(c)}. \tag{A3.7}
\]

Hence, \( VM(c) + VO(c) \) is increasing in \( c \) at \( c = 0 \) if the last inequality holds. ||
References


