Galaxy clusters and cosmic voids in modified gravity scenarios

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Abstract

The so-called ‘cosmic web’, comprising cosmic voids and galaxy clusters, has been proven to be extremely sensitive to deviations from General Relativity. This could be further investigated by future large-scale surveys, such as with the European Space Agency satellite Euclid. In this study, the parameter $|f_{R0}|$ from $f(R)$ gravity is constrained by considering the Euclid survey specifications to predict the observed numbers of voids and clusters in bins of redshift, mass and, only for voids, density contrast. From these values, the Fisher matrix is computed for three values of $|f_{R0}|$, $10^{-4}$, $10^{-6}$ and $10^{-8}$, by assuming a flat Universe with a component that mimics the cosmological constant. The probability density functions are obtained for $|f_{R0}|$ and seven other parameters from the fiducial model considered ($n_s$, $h$, $\Omega_b$, $\Omega_m$, $\sigma_8$, $w_0$ and $w_a$).

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1 Introduction

The ultimate goal of cosmology is to find a unified framework that could explain the Universe as a whole: a complete, powerful theory characterized by elegance and simplicity. Today the so-called ΛCDM model is considered the best available fit to the observational data [21], as it describes the main features of the Universe, such as its accelerated expansion and the formation of structure. However, in contrast with the sought simplicity, this model predicts that the largest contribution to the cosmic density comes from some mysterious ‘dark’ components which are thought to be at most weakly interacting with the ordinary baryonic matter: dark energy in the form of a cosmological constant (Λ) and cold (non-relativistic) dark matter (CDM). As their name suggests, their nature is still an open issue and their properties unknown. Some potential dark matter candidates have been proposed, including supersymmetric particles and axions [4], while dark energy has been interpreted as a type of energy density of the vacuum with a constant equation of state [7] being responsible for the observed cosmic expansion. Nevertheless, they both are the object of an ongoing debate in the scientific community, as the lack of a theoretical motivation for their existence has not yet been solved.

According to the latest data release of the Plank Collaboration [1], dark matter would account for approximately 26% of the total cosmic density, while dark energy would contribute with about 68%. It is thus legitimate to question the validity of the ΛCDM model and raise an argument originally proposed by L. Guzzo in 2002: “Our wonderful ‘standard’ cosmological model seems [...] to be so far essentially based on a) a Dark Matter we do not detect; b) a Dark Energy we do not understand; [...] ¹. Yet everything seems to work: isn’t this reminiscent of epicycles?” [10].

This has been the starting point for the development of several modified gravity theories that aim to provide an alternative to the ΛCDM model. Their common underlying assumption is the validity of Einstein’s equations as a description of the interaction between matter and geometry. However, instead of acting on the right

¹In the original version of the quote, Guzzo was also referring to “a fraction of baryons that we cannot completely find”. This problem has however been recently solved through the identification of the missing baryons in the so-called warm-hot intergalactic medium [8].
side of the equation by adding a cosmological constant to the so-called stress-energy tensor, they contain possible modifications to the Einstein tensor on the left side [29].

One of the most important modified gravity theories is the $f(R)$ gravity. It is a family of theories which propose to add a scalar function to the Einstein-Hilbert action underlying Einstein’s equations. A possible way to verify the validity of this approach is to try to constrain the parameter $|f_{R0}|$ that they introduce, having different definitions according to the specific theory. The aim of this work is actually to achieve this by using the abundance of galaxy clusters and cosmic voids as a probe, according to the specifications of the future survey Euclid [13]. Clusters and voids, in fact, are extreme elements of the co-called ‘cosmic web’, which constitutes a useful sample of the matter distribution in the Universe. Its sensibility to cosmological parameters has been proven in earlier papers [22, 23, 24] and could probably lead to broader applications in constraining other cosmological parameters. The method itself which has been followed will be the main focus of this work, as it might provide useful indications about whether future large-scale surveys like Euclid could result in a redefinition of modern cosmology.

Here follows an overview of the procedure that will later be described in detail. The excursion set formalism will be applied by taking into account the Euclid specifications for voids and clusters. By implementing the properties of $f(R)$ gravity in a private Fortran number count code (already used in [23]), this will allow to calculate the predicted number density of voids and clusters per redshift, density contrast (for voids), radius and mass bin in $f(R)$ gravity. From these number densities the Fisher matrix for the distribution of voids and clusters will be computed (i.e. the inverse of the covariance matrix). In this case, nine parameters with their fiducial values will be considered, including $|f_{R0}|$. In the end, the publicly available CosmicFish code [19, 20] will be used to forecast the constraints on the parameters.

In the following section we will briefly explain the theory behind $f(R)$ gravity and the properties of the cosmic web. In section 3 the process of structure formation will be reviewed in the context of $f(R)$ gravity and the method implemented will be analyzed. The fiducial model and the survey specifications considered will be presented in section 4, while section 5 contains the results obtained and section 6 is dedicated to the conclusions of this work. Some further explanations and additional plots are included in the Appendices.
2 Theory

2.1 $f(R)$ gravity

The original formulation of Einstein’s equations without the cosmological constant $\Lambda$ is [18]

$$G_{\mu\nu} = 8\pi G T_{\mu\nu},$$

where $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the stress-energy tensor, $G$ is the gravitational constant and the units are such that the speed of light $c$ is equal to 1. These equations express how energy (correlated to mass) curves the four-dimensional spacetime considered in General Relativity. They are yielded through the principle of stationary action by the so-called Einstein-Hilbert action [5, 18]

$$S_{GR} = \frac{1}{16\pi G} \int d^4x R \sqrt{-g},$$

where $g$ is the determinant of the metric tensor $g_{\mu\nu}$ and $R$ is the Ricci scalar, related to the curvature properties and defined as $R := g^{\mu\nu} R_{\mu\nu}$ ($R_{\mu\nu}$ is the Ricci tensor).

$f(R)$ gravity theories aim to explain the observed cosmic accelerating expansion by introducing a scalar function of the Ricci scalar $f(R)$ [27] in the Einstein-Hilbert action:

$$S_{f(R)} = \int d^4x R \sqrt{-g} \left[ \frac{1}{16\pi G} R + f(R) \right] + S_m [g_{\mu\nu}, \psi_i],$$

where $S_m$ is the action for other matter fields $\psi_i$ coupled to the metric tensor. Of course, the modifications introduced by $f(R)$ gravity have to match some specific requirements in order to account for observations [28]:

- They include a form of ‘screening’ effect which allows to reduce the equations to the standard case in contexts, such as the Solar System, where General Relativity has largely been tested. [30]

- They contain a term which mimics a cosmological constant in order to justify the accelerated expansion of the Universe.

In this work, following what was proposed in [27], we will consider the formulation of $f(R)$ by Hu & Sawicki [12] and expand it in powers of $R^{-1}$ in a large curvature context:
\[ f(R) \approx -16\pi G \rho_f - \frac{f_{R0} R_0^{n+1}}{n R^n}. \] (4)

Here \( f_{R0} \) is defined as \( f_{R0} := \frac{df}{dR} \big|_{z=0} \) and represents the current value of the scalar field \( \frac{df}{dR} \) describing the propagation of a ‘fifth force’ of nature. \( n \) is a free parameter, while \( \rho_f \) is a constant whose value is chosen to account for the accelerated cosmic expansion. Here a clarification is needed: despite the fact \( \rho_f \) plays the same role as a cosmological constant, it has an intrinsically different origin, as it is not a constant inserted in Einstein’s equation, but rather a term influencing the formulation of the Einstein-Hilbert action.

### 2.2 Structure in the Universe

It has been proven that modified gravity theories such as \( f(R) \) gravity can be tested through the abundance of galaxy clusters and cosmic voids [24]. However, before giving a detailed account of the method which can be followed in order to attain constraints on modified gravity parameters, it is useful to review the fundamental properties of clusters and voids.

Galaxy clusters are the largest known gravitationally bound objects in the Universe whose main component is believed to be a dark matter halo (accounting for approximately 90% of the total mass) in which up to thousands of galaxies are located [2]. On the other hand, cosmic voids are huge underdense regions normally round in shape and almost completely devoid of galaxies [26]. Together with ‘filaments’ and ‘sheets’ of galaxies connecting separate clusters, voids and clusters are the main components of the so-called ‘cosmic web’, which comprises all structures bigger than individual galaxies.

The cosmic web is typically identified at scales of 50-100 Mpc in diameter. The cosmological principle, in fact, which prescribes a homogeneous and isotropic Universe, holds true only above the limit of a few hundreds of Mpc, while, below this value, structure can be found: from atoms to stars, to galaxies, to the largest clusters, matter and energy interact and form bound systems at different scales which are ruled by the laws of Physics.

\[ ^2 \text{This basically refers to the effects of modified gravity in low-density environments.} \]
3 Method

We will now underline the link between the cosmic web and the constraints that can be drawn upon cosmological parameters such as $|f_m|$ from $f(R)$ gravity. The formation of the large-scale structure will be described in the context of $f(R)$ gravity and the mass function from the excursion set formalism calibrated to $N$-body simulations will be used to make predictions for the abundance of voids and clusters. These predictions will be the starting point for the computation of the constraints.

3.1 The formation of structure: the linear growth

First of all, in order to describe the variations of the mean matter density in the formation of structure, it is useful to define the dimensionless density contrast $\delta(\vec{r},t)$ [21]

$$\delta(\vec{r},t) := \frac{\epsilon_m(\vec{r},t) - \bar{\epsilon}_m(t)}{\bar{\epsilon}_m(t)},$$  \hspace{1cm} (5)$$

where $\epsilon_m(\vec{r},t)$ is the matter energy density as a function of the comoving spatial coordinate $\vec{r}$ and time $t$ and $\bar{\epsilon}_m(t)$ is the spatially averaged energy density defined as $\bar{\epsilon}_m(t) := \frac{1}{V} \int_V \epsilon_m(\vec{r},t) d^3\vec{r}$. It is important to underline that the volume $V$ in this definition must be large compared to the biggest structures in the Universe in order to obtain a value which is not dependent on the area of the sky considered. In fact, beyond scales of a few hundreds of Mpc, the already mentioned cosmological principle holds true and the Universe can be considered homogeneous and isotropic. Given this assumption, the density contrast will have a negative value in underdense regions and a positive value in overdense regions.

As stated in [21], the mechanism at the basis of the evolution of $\delta$ and the growth of large-scale structures is gravitational instability. The seeds for this process are believed to be the quantum fluctuations in the density field of the early Universe, which were blown up to classical scales by the extremely rapid cosmic expansion of inflation at approximately $t = 10^{-36}$ s. Since then, gravity has been the force governing the formation of the cosmic web. At this stage, two different phases can be identified in the process of structure formation:

1. A linear growth, as long as $|\delta| < < 1$

2. A non-linear regime, which subsequently follows and requires a more complex mathematical description.
The linear growth of density fluctuations is commonly treated as a spherical isotropic evolution. Starting from the linearly perturbed fluid equations (continuity equation, Euler equation and Poisson equation), it is possible to compute a linearized evolution equation for the density contrast as a function of the scale factor $a$ (see [17] for the complete derivation). The solution to this differential equation, for adequate initial conditions, gives the linear value of the density contrast at any time $t$. In our calculations we have considered the formulation of the evolution equation given in [27], which incorporates the effects of $f(R)$ gravity in the factor $\mu(k,a)$:

$$\delta'' + \left(\frac{3}{a} + \frac{E'}{E}\right)\delta' = \frac{3}{2a^3E^2}\mu(k,a)\delta,$$

where $k$ is the wave number of the fluctuations in Fourier space, primes denote derivatives with respect to the scale factor $a$, $\Omega_{m,0}$ is the dimensionless matter density parameter at present time and $\mu(k,a)$ is given by

$$\mu(k,a) = \frac{(1 + 2\beta^2)k^2 + m^2a^2}{k^2 + m^2a^2},$$

with

$$\beta = \frac{1}{\sqrt{6}}$$

$$m_0 = \frac{H_0}{c}\sqrt{\Omega_{m,0} + 4\Omega_{f,0}}/(n + 1)fR_0,$$

$$m(a) = m_0 \left(\frac{\Omega_{m,0}a^3 + 4\Omega_{f,0}}{\Omega_{m,0} + 4\Omega_{f,0}}\right)^{(n+2)/2},$$

$E$ in equation (6) is defined as $E(a) = \frac{H(a)}{H_0}$, where $H_0$ is the Hubble constant and $H(a)$ is the Hubble parameter at $a$. For a flat universe containing matter and a component with current density parameter $\Omega_{f,0}$ (denoted in this way as it is introduced by $f(R)$ gravity), we have [21]

$$E(a) = \sqrt{\frac{\Omega_{m,0}}{a^3} + \Omega_{f,0}}.$$  

In the following the parameter $n$ from equation (10) has been fixed to 1 while the density parameters $\Omega_{m,0}$ and $\Omega_{f,0}$ are assigned some fiducial values (see subsection 4.1) to account for the observed accelerated expansion of the Universe. $|fR_0|$ is the parameter we want to constrain and has been given three possible fiducial values (see subsection 4.1).
It is sometimes more convenient to refer to the growth suppression factor $g = \frac{\delta}{a}$ instead of $\delta$, in order to highlight the deviations with respect to matter-dominated epochs, the only ones in which the density perturbations grow at a significant rate with $\delta \approx a$ [15]. This is particularly relevant to analyze the effects of the presence of a different component (a cosmological constant, or, in this case, a component with current density parameter $\Omega_{f,0}$). Equation (6) can thus be rewritten as

$$g'' a + \left(5 + \frac{E'}{E} a\right) g' = \left(\frac{3}{2} \frac{\Omega_{m,0}}{a^4 E^2} \mu(k, a) - \frac{3}{a} - \frac{E'}{E}\right) g. \quad (12)$$

We have computed the solutions for $g$ from equation (12) by considering $z = 20$ and $g = 1$ as initial conditions: this seems reasonable for a high value of redshift allowing to assume a purely matter-dominated growth with $\delta \approx a$ and thus $g = 1$.

In order to show the effects of the dependence of $g$ from equation (12) on the wave number $k$, a feature introduced by $f(R)$ gravity, we have plotted $g$ as a function of $\log_{10}(1 + z)$ for four different values of $k$ \(^3\):

![Figure 1: Plots of the growth suppression factor $g$ for four values of $k$ with the data analysis framework ROOT [6].](image)

The value $k = 1.0 \times 10^{-4} \text{ h/} \text{Mpc}$ corresponds to large-scale fluctuations and is very close to the General Relativity case: it can thus be used as a reference. The values $3.3 \times 10^{-3} \text{ h/} \text{Mpc}$ and $k = 2.4 \times 10^{-2} \text{ h/} \text{Mpc}$ correspond to medium scales, while $k = \ldots$

\(^3\)The conversion between the scale factor $a$ and the redshift $z$ is trivial [21]: considering the current value of the scale factor $a(t_0) = 1$, we have $z = \frac{1}{a} - 1$. 

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2.2 $h$/Mpc corresponds to small scales. By observing the plots, it seems reasonable to conclude that the dependence on $k$ is stronger at large scales than at small ones: the difference between the two curves with higher $k$ appears to be much less significant than the one between the plots with $3.3 \times 10^{-3}$ $h$/Mpc and $k = 2.4 \times 10^{-2}$ $h$/Mpc, despite the fact that in both cases there is a step of approximately one order of magnitude in $k$.

### 3.2 The non-linear regime and the mass function

The solutions for $\delta$ in equation (6) or $g$ in equation (12) have a physical meaning only as long as $|\delta| << 1$ [17]. The transition between the linear and the non-linear regime is a gradual process and includes a phase of quasi-linear growth while $|\delta| \approx 1$. As a reference point, however, it is possible to identify a specific threshold for the density parameter ($\delta_v$ for voids of $\delta_c$ for clusters) beyond which the process is treated as completely non-linear and a different mathematical description is needed. A viable approach is to employ the framework of the so-called ‘excursion set theory’, which consists of a procedure to connect linear perturbation theory and non-linear effects. However, before giving a review of the excursion set theory for voids and clusters following [32], it is necessary to introduce some definitions and notations.

First of all, the density contrast $\delta$ needs to be smoothed on a particular scale $R$ of interest. This can be done by introducing a ‘window function’ $W(k, R)$ that weights the density contrast in a useful way [32]: assuming isotropy such that $\delta(\vec{r}) = \delta(r)$, the smoothed density contrast at a given time $t$ is

$$\delta(r, R) := \int \frac{d^{3}k}{(2\pi)^3} \delta(k) W(k, R) e^{-ikr}, \quad (13)$$

where $\delta(k)$ is the Fourier transform of the density contrast and the most reasonable window function is a simple sphere in real space:

$$W(r, R) = \begin{cases} \frac{3}{4\pi R^3} & \text{if } r \leq R \\ 0 & \text{if } r > 0 \end{cases}. \quad (14)$$

Since $\delta(\vec{r}, t)$ is assumed to be a Gaussian field and smoothing is a linear operation, also the smoothed density contrast is Gaussian distributed. Thus, it is possible to calculate its variance

$$\sigma_{\delta}^2 = S(R) = \int \frac{dk}{(2\pi)^2} k^2 P(k) |W(k, R)|^2, \quad (15)$$
where $P(k)$ is the matter power spectrum defined as the mean square amplitude of the Fourier components of the density contrast $P(k) := \langle |\delta(k)|^2 \rangle$ [21]. It is important to underline that the variance $S(R)$ can be used as a parameter for the smoothing scale $R$ with a one-to-one correspondence, as in hierarchical formation models $S(R)$ tends to infinity when $R$ tends to zero and thus incorporates information about the scale.

Now, the objective of the excursion set theory is to relate the initial density field to specific structures in the evolved field [32]. This can be done starting from three elements:

1. The characterization of the primordial density fluctuations during the inflationary epoch

2. The evolution of the growth suppression factor $g(a)$ according to linear perturbation theory

3. The identification of the thresholds for the collapse into a virialized cluster ($\delta_c$) and the formation of a cosmic void ($\delta_v$).

Given these premises, the problem consists in identifying the smoothing scale $R$ (or the variance $S(R)$) at which the density contrast field $\delta$ crosses the thresholds. This can be done by evaluating the field at various values of the smoothing scale $R$ and a fixed spatial coordinate $r$, creating a series of $\delta$-trajectories normally referred to as ‘random walks’. The probability distribution $\Pi(\delta, S)$ to find the value $\delta$ at a certain value of $S$ is given by the Fokker-Planck equation with adequate initial conditions:

$$\frac{\partial \Pi}{\partial S} = \frac{1}{2} \frac{\partial^2 \Pi}{\partial \delta^2}. \quad (16)$$

From the exact formulation of $\Pi$ it is possible compute the multiplicity function $f(S)$, corresponding to the fraction of random walks which have surpassed the threshold at some given $S$. The interesting aspect of the multiplicity function is that it contains $S$, which is a linear quantity computed from equations (6) and (15), as a parameter to describe the non-linear evolution of a void or a cluster. $f(S)$ then enters the so-called ‘mass function’, which gives a prediction for the number of objects (voids or clusters) per unit of volume: in the context of this study, this function is the objective of the whole analysis of structure formation, as it has been proven to be extremely sensitive to the value of cosmological parameters such as $|f_{R0}|$ [24]. Its precise formulation slightly differs for clusters and voids and will be shortly presented for both.
3.2.1 Cosmic voids

In the case of voids, it is first of all necessary to identify the non-linear threshold \( \delta_v < 0 \) \[32\]. This corresponds to the transition to a non-linear regime, characterized by the phenomenon of ‘shell-crossing’: since underdense areas expand more rapidly than the average expansion rate, the inner, less dense shells of a void accelerate outward faster than the denser outer shells, eventually overtaking them. Another aspect which must be taken into account is the fact that an underdense patch might be contained in a larger overdense patch which could collapse and eliminate the void: this is known as the ‘void in cloud’ effect and can be treated by introducing another threshold \( \delta^c_v \) which the density contrast should not surpass at any scale. The objective of applying the excursion set formalism is thus to compute the largest scale at which \( \delta_v \) is crossed provided that \( \delta^c_v \) is not crossed at larger scales.

Another issue which must be solved is the so-called ‘galaxy bias’ \[32\], which refers to the discrepancy between the clustering of ordinary matter and the underlying dark matter. Moreover, the mass function must be evaluated by considering the extrapolated linear radius of a void, which leads to the computation of a linear density threshold \( \delta^\text{lin}_v \) actually entering the mass function. More details about the corrections implemented to model such effects according to \[22\] can be found in Appendix A.

In this context, the computation of the mass function can be done by slightly modifying the Fokker-Planck equation to include the effects of \( f(R) \) gravity. The result for voids has been taken from \[27\], where all the calculations can be found. The multiplicity function is

\[
f(S) = \frac{|\delta^\text{lin}_v|}{\sqrt{S(1 + D_v)}} \sqrt{\frac{2}{\pi}} e^{-\left(\frac{(|\delta^\text{lin}_v| + \beta_v S)^2}{2S(1 + D_v)}\right)},
\]

(17)

where the dependence on the parameter \( f_{R0} \) is contained in the parameters \( D_v \) and \( \beta_v \).

Voivodic et al. \[27\] present some mean values and 1\( \sigma \) errors for the two parameters fitted from \( N \)-body simulations \[4\] for four steps of \( |f_{R0}| \): \( 10^{-4}, 10^{-5}, 10^{-6} \) for \( f(R) \) gravity and \( 10^{-8} \), assumed to correspond to General Relativity. In the context of this study, these values have been fitted using the data-analysis framework ROOT \[6\] with simple functions monotonically depending on \( \log_{10} |f_{R0}| \) in order to find the precise expression for \( D_v \) and \( \beta_v \) (see appendix A). The multiplicity function can then be inserted in the mass function, i.e. the the differential number density of voids per unit \( \ln R \):

\[4\]The values considered are labeled as 1LDB (one linear diffusive barrier) in \[27\].
\[
\frac{dn}{d\ln R} = \frac{f(S) d\ln \sigma^{-1} d\ln R_L}{V(R) d\ln R_L} \bigg|_{R_L(R)} \frac{V(R)}{V(R_L)},
\]
(18)
where the subscript \(L\) denotes linear quantities, \(V(R)\) is the volume, \(\frac{V(R)}{V(R_L)}\) is a normalizing factor and \(\sigma\) is derived from equation (15).

3.2.2 Galaxy clusters

In the case of galaxy clusters the transition to a non-linear growth is conventionally identified when the density contrast surpasses a given threshold \(\delta_c\) and the halo undergoes a ‘spherical collapse’ [17]. One feature of the non-linear regime is the possibility that a region might have not crossed the threshold at a certain scale \(R\), but might have crossed it at a larger scale [32]. This is known as the ‘cloud in cloud’ effect and can be addressed through the excursion set formalism in an analogous way to the ‘void in cloud’ effect: the objective is in this case to compute the largest scale at which \(\delta_c\) is crossed.

The mass function is then computed and the formulation we have adopted in this study is given by Lombriser et al. in [16]. Instead of using the parameter \(S\) it is in this context more useful to define the ‘peak threshold’ \(\nu := \delta_c/\sqrt{S}\). Thus, the mass function takes a slightly different (but equivalent) form:

\[
\frac{dn(M)}{dM} dM = \bar{\rho}_m M \phi(\nu) d\nu dM,
\]
(19)
where \(\bar{\rho}_m\) is the mean matter density and \(\phi(\nu)\) is the fraction of mass of collapsed objects per logarithmic interval in \(\nu\). In this case,

\[
\nu \phi(\nu) = A \sqrt{\frac{2}{\pi} a \nu^2} \left[ 1 + \left( a \nu^2 \right)^{-p} \right] e^{-a \nu^2/2},
\]
(20)
where \(a = 0.707\) and \(p = 0.3\) and \(A\) is a normalization constant such that \(\int_0^\infty \phi(\nu) d\nu = 1\), thus \(A = 0.32\). An important aspect of the ‘Parameterized Post-Friedmann’ approach followed by Lombriser et al. is that the density threshold \(\delta_c\) is computed by including a statistical average over different environmental densities. This is incorporated in the way in which the variance \(S\) is parameterized: its expression interpolates between the \(S\) computed for \(\Lambda\)CDM and the one computed for \(f(R)\) gravity,

\[
S^{1/2}(M) = \frac{S_{f(R)}^{1/2}(M) + (M/M_{th})^\mu S_{\Lambda CDM}^{1/2}(M)}{1 + (M/M_{th})^\mu}.
\]
(21)
M is the mass of the cluster, while the values for the free parameters $\mu$ and $M_{th}$ are computed from $N$-body simulations: $\mu \approx 1.415$ and

\[
M_{th} = \bar{M}_{th} \left( \frac{10^6 |f_{R0}|}{3/2} \right)^{3/2} M_\odot / h,
\]

with $\bar{M}_{th} \approx 2.172$ and $h = H_0 \ 100^{-1} \ \text{km}^{-1} \ \text{s} \ \text{Mpc}$.

### 3.3 The Fisher matrix method

Once introduced the mass functions, it is possible to define a model to predict the abundance of voids and clusters in given bins in redshift and radius (for voids) or mass (for clusters). This is very relevant in order to try to constrain $f(R)$ gravity, since these number counts are sensitive to the theory of gravity considered.

The investigated area of the sky as well as the information about the bins, i.e. the width of the bins and the intervals considered for each observable, can be fixed according to the specifications of future large-scale surveys. In this way, the result attained might be an indicator about whether these surveys could provide good constraints on the parameters considered. In the context of this study, we have chosen to focus on the specifications of the future European Space Agency survey Euclid [13], which will be presented in the next section.

The model for the number counts is analogous to the one used in [22, 23, 24]:

\[
N_{\text{obs}} = \int \int \int p(O|O_t) n \left[ M(O_t), z \right] \frac{dM}{dO_t} \frac{dV}{dz} dO_t dO dV
\]

where $O$ is the size observable (radius for voids and mass for clusters), $O_t$ is the true value of $O$, $M(O_t)$ is the mass of the object, $z$ is the redshift, $V$ is the volume and $p(O|O_t)$ corresponds to the probability of measuring $O$ given $O_t$. The differential number density $n \left[ M(O_t), z \right]$ is computed from the mass function. The integrals are performed for each bin according to the survey specifications considered and the result is then used to compute the so-called ‘Fisher matrix’.

The Fisher matrix is an extremely powerful tool to predict the results of an experiment [31] given two elements:

1. A fiducial model which must be tested, in this case $f(R)$ gravity and a flat Universe containing cold dark matter, whose parameters are assigned some values considered as valid guesses for the real ones

2. Some observables depending on the model parameters, in this case the number counts for each bin.
For a model with $N$ parameters, the Fisher matrix will be a $N \times N$ matrix. The expression we used for its elements is based upon the Poisson log likelihood function [22, 23]:

$$L = \sum_i N_i \ln \bar{N}_i - \bar{N}_i,$$  

(24)

where $N_i$ is the observed number of objects in the $i$-bin and $\bar{N}_i$ is the model prediction. Thus, the $mn$ element of the Fisher matrix is

$$F_{mn} = \sum_i \frac{1}{N_i} \frac{\partial \bar{N}_i}{\partial \theta_m} \frac{\partial \bar{N}_i}{\partial \theta_n},$$  

(25)

where $\theta_m$ are the model parameters, assumed to be Gaussian-distributed. In this rather standard formulation, the Fisher matrix is the inverse of the covariance matrix for the model parameters: it contains the uncertainties on the model parameters and this information, together with the fiducial values, can be used to compute a Gaussian probability distribution for the parameters,

$$g(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2},$$  

(26)

where $\sigma$ is the standard deviation on the parameters and $\mu$ is the fiducial value considered. The probability distribution for $|f_{R0}|$ will ultimately show if it is possible to constrain $f(R)$ gravity by using the large-scale structure of the Universe as a probe.

A last important aspect to underline is that two Fisher matrices computed with different survey specifications for the same fiducial model can be summed in order to strengthen the constraints on the model parameters. This is particularly useful to combine the results for voids and clusters and verify if they provide complementary information about $f(R)$ gravity, as previous studies [22, 23, 24] seem to suggest.

## 4 Data and procedure

For the computation of the Fisher matrices, we have used an already existing Fortran code [22, 23] run with the engine CosmoMC [11, 14]. Some parts of the code have been modified to implement the effects of $f(R)$ gravity, in particular concerning equation (12) for the linear growth and equations (17) and (20) for the multiplicity functions. The code has been run considering three sets of *Euclid* survey specifications [13], two for voids and one for clusters, and three values for the parameter $|f_{R0}|$ (see subsection
4.1), together with seven additional cosmological parameters [22]: \( n_s, h, \Omega_b, \Omega_m, \sigma_8, w_0 \) and \( w_a \). Thus, in total nine Fisher matrices have been computed for the individual sets of specifications and the fiducial values of \( |f_{R0}| \) and six Fisher matrices have later been obtained by combining the results of the two void specifications and the cluster specification. These six Fisher matrices have later been used to calculate the one-dimensional and two-dimensional marginalized probability distribution functions for the eight parameters with the publicly available CosmicFish code [19, 20].

### 4.1 Fiducial values and survey specifications

The parameter \( |f_{R0}| \) has been assigned three fiducial values: \( 10^{-8} \), considered to be corresponding to General Relativity, and \( 10^{-6}, 10^{-4} \), the two extreme values found in [27]. On the other hand, the fiducial values for the seven additional parameters considered have been taken from [22], according to the best fitting Planck results for a flat Universe containing cold dark matter and a dark energy-like component with equation of state \( w(z) = w_0 + w_a (1 - a) \) (where \( a \) is the scale factor). Given the \( f(R) \) gravity formulation we consider in this study, this component is the one with density \( \rho_f \) mentioned in subsection 2.1. It might be pointed out that this equation of state with varying \( w_0 \) and \( w_a \) is crafted to simulate a \( \Lambda \)CDM cosmology. However, in the Hu-Sawicki formulation of \( f(R) \) gravity we considered [12] the \( \rho_f \) component basically consists of a cosmological constant with a \( f(R) \) modification, thus this extrapolation we made for the equation of state seems to be realistic.

Here follows a list of the seven parameters with their assigned values and a short definition:

1. The scalar spectral index, \( n_s = 0.965 \), describing scalar density fluctuations
2. The Hubble parameter, \( h = 0.673 \), indicating the uncertainty in the measurement of the Hubble constant
3. The mean baryonic matter density \( \Omega_b = 0.0492 \)
4. The mean matter density \( \Omega_m = 0.314 \)
5. The matter power spectrum normalization \( \sigma_8 = 0.831 \)
6. The first parameter \( w_0 = -1 \) from the \( f \)-component equation of state
7. The second equation of state parameter \( w_a = 0 \).
The *Euclid* survey specifications implemented in the code are those considered in [22]: EV-A, voids with bins in redshift, EV-B, voids with bins in redshift, radius and density contrast and EC, clusters with bins in redshift and mass. In the following table, the main features of the three specifications are presented: $z$ is the redshift, $T$ refers to the observable for each type of object (i.e. radius for voids and mass for clusters) and $\delta$ is the density contrast. The investigated area of the sky is the same for all cases.

<table>
<thead>
<tr>
<th>Specifications</th>
<th>EV-A</th>
<th>EV-B</th>
<th>EC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{min}$</td>
<td>0.7</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>$z_{max}$</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>bins for $z$</td>
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<td>13</td>
<td>18</td>
</tr>
<tr>
<td>$\log_{10}(T_{min})$</td>
<td>/</td>
<td>1.15</td>
<td>-1.27</td>
</tr>
<tr>
<td>$\log_{10}(T_{max})$</td>
<td>/</td>
<td>1.85</td>
<td>0.8</td>
</tr>
<tr>
<td>bins for $\log_{10}(T)$</td>
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<td>7</td>
<td>10</td>
</tr>
<tr>
<td>$\delta_{min}$</td>
<td>-0.8</td>
<td>-1.0</td>
<td>/</td>
</tr>
<tr>
<td>$\delta_{max}$</td>
<td>/</td>
<td>-0.5</td>
<td>/</td>
</tr>
<tr>
<td>bins for $\delta$</td>
<td>1</td>
<td>3</td>
<td>/</td>
</tr>
<tr>
<td>area (fraction of sky)</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
</tr>
</tbody>
</table>

Table 1: *Euclid* specifications considered for voids and clusters.

## 5 Results

We have computed the Fisher matrices for the different survey specifications considering the three fiducial values for $|f_{R0}|$ for each of them. Then, we have combined the matrices for the EC survey specifications with the EV-A and EV-B specifications and used the resulting Fisher matrices to obtain the probability density functions for the eight parameters considered.

We present below the one-dimensional and two-dimensional probability density functions for the $|f_{R0}| = 10^{-6}$ case as an example. The plots for the two other values of $|f_{R0}|$ can be found in appendix B. In the plots, the darker contours refer to the 68% confidence level bounds, while the lighter ones refer to the 95% confidence level.

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5The units of $T$ are $m/(\text{Mpc}/h)$ when $T$ represents the radius of a void and $\text{kg}/(10^{15} \ M_\odot/h)$ when $T$ represents the mass of a cluster.
Figure 2: Plots of the probability density functions for the EC and EV-A survey specifications, both separately and combined, for the case $\log_{10} |f_{R0}| = -6$. 


Figure 3: Plots of the probability density functions for the EC and EV-B survey specifications, both separately and combined, for the case $\log_{10} |f_{R0}| = -6$. 
Here follows a list of the 68% confidence constraints placed on $\log_{10}|f_{R0}|$ for all the Fisher matrices considered. We underline that instead of $|f_{R0}|$ we referred to $\log_{10}|f_{R0}|$ as this resulted more convenient in the computations.

| Specifications | $\log_{10}|f_{R0}| = -8.0$ | $\log_{10}|f_{R0}| = -6.0$ | $\log_{10}|f_{R0}| = -4.0$ |
|---------------|-----------------|-----------------|-----------------|
| EV-A          | ±4.0            | ±3.0            | ±30.0           |
| EV-B          | ±0.2            | ±0.1            | ±0.3            |
| EC            | ±20.0           | ±0.4            | ±0.2            |
| EC+EV-A       | ±0.1            | ±0.1            | ±0.03           |
| EC+EV-B       | ±0.1            | ±0.1            | ±0.004          |

Table 2: 68% confidence constraints on $\log_{10}|f_{R0}| = -8.0, -6.0, -4.0$ for all the survey specifications.

For the other parameters, we present the combined results for the specifications EV-B and EC, since this combination seems to be the most promising one:

| Parameters   | $\log_{10}|f_{R0}| = -8.0$ | $\log_{10}|f_{R0}| = -6.0$ | $\log_{10}|f_{R0}| = -4.0$ |
|--------------|-----------------|-----------------|-----------------|
| $n_s = 0.965$| ±0.02           | ±0.02           | ±0.02           |
| $h = 0.673$  | ±0.02           | ±0.02           | ±0.01           |
| $\Omega_b = 0.0492$ | ±0.008 | ±0.008 | ±0.005 |
| $\Omega_m = 0.314$ | ±0.001 | ±0.001 | ±0.005 |
| $\sigma_8 = 0.831$ | ±0.0006 | ±0.0006 | ±0.001 |
| $w_0 = -1.00$ | ±0.01 | ±0.01 | ±0.08 |
| $w_a = 0.00$ | ±0.009 | ±0.01 | ±0.5 |

Table 3: 68% confidence constraints on $n_s$, $h$, $\Omega_b$, $\Omega_m$, $\sigma_8$, $w_0$ and $w_a$ for the combined survey specifications EV-B and EC.

6 Conclusions

The first evident result of this study is that cosmic voids and galaxy clusters seem to provide complementary information to constrain the cosmological parameters considered: the combined constraints EC+EV-A and in particular EC+EV-B are definitely the most restrictive ones for all the parameters. This is yet another hint in the direction of adopting the cosmic web as a probe for testing other cosmological models in future studies, as previous works [22, 23, 24] already suggested: future large-scale
surveys combining voids and clusters like *Euclid* will definitely provide useful indications about possible alternatives to the currently favoured model. The constraints obtained for \( \log_{10} |f_{R0}| \) are quite small for all the three values considered, which means that, whatever the measured value will be, the *Euclid* survey will be able to constrain it. In particular, the constraints on \( \log_{10} |f_{R0}| = -4 \) are smaller than in the other two cases and we suspect that this is due to a particularly favourable way in which the information coming from the void and cluster surveys is combined. However, by considering the individual surveys, the constraints on \( \log_{10} |f_{R0}| = -4 \) are not better than for the two other cases and the EV-A specifications give a very bad result. When considered individually, the EV-A specifications seems actually to be the less restrictive in general, as could be expected by considering bins only in redshift and not in mass and density contrast as for EV-B. Nevertheless, they surprisingly give the same constraints as EV-B when combined with the EC specifications for \( \log_{10} |f_{R0}| = -8 \) and \( \log_{10} |f_{R0}| = -6 \): this encourages to investigate the possibility that simpler void surveys, when combined with results coming from clusters, could potentially give enough information to test a fiducial model.

There have been several attempts to constrain \( |f_{R0}| \) and the latest results came from gravitational waves in LIGO observations (\( |f_{R0}| < 5 \times 10^{-7} \) at 68% confidence) [25] and the so-called ‘galaxy clustering ratio’ (\( |f_{R0}| < 5 \times 10^{-6} \) at 68% confidence) [3]. These values seem to suggest the validity of the General Relativity scenario and, if this was proven true, our method could definitely contribute to improve the current constraints. A possible further development in this direction would be to consider other future surveys, such as the four-meter multiobject spectroscopic telescope (4MOST) Galaxy Redshift Survey [9].

The constraints on the parameter \( |f_{R0}| \) are the main objective of this study. However, it can be interesting to briefly analyze the results about the other seven parameters considered. The current *Planck* 68% confidence uncertainties are \( \sigma_{\alpha_s} = 0.004, \sigma_h = 0.005, \sigma_{\Omega_b} = 0.0002, \sigma_{\Omega_m} = 0.007, \sigma_{\sigma_8} = 0.006, \sigma_{w_0} = 0.077 \) and \( \sigma_{w_a} = +0.31, -0.27 \) [1]. The results that we obtained provide better constraints for \( \sigma_8, w_0 \) and \( w_a \) and give reasonable indications regarding the other fiducial values as well. Thus, the method we followed is once more proven to be a valid approach to analyze a cosmological model.

The last important aspect to underline concerns the starting point of this whole study: as stated in the Introduction, General Relativity has been largely tested through Solar System tests. However, the dark components incorporated in our current ΛCDM model, dark matter and dark energy, lack of theoretical justifications. It is thus relevant to test viable alternatives such as \( f(R) \) gravity and question the foundations of cosmology, in order not to fall in the trap of epicycles again.
Post Scriptum

In a standard report this section should be entitled “Acknowledgements” and is normally quite brief. However, this has not just been a simple Bachelor thesis to me and probably deserves a different ending. The reasons are two: first of all, it has been my first, infinitesimal but meaningful contribution to real research. And, moreover, it has accompanied me all the way during my Erasmus exchange at the Uppsala University, from the first to the last day. Thus, I decided to name this last section “Post Scriptum”, ‘After the writing’, as it is the conclusion of both this report and a special chapter of my life spent in Sweden. Before thanking everyone who has been involved in this project, I would like to briefly write about what I am thanking for: as a remainder about what I am bringing back home in my suitcase, here follows what I learnt during these five months in Uppsala while working with voids and clusters. I begin with a fundamental keyword: ‘humility’, a lesson I will not forget. I have modified the code in the wrong way an infinite number of times, I have read a significant number of papers which sounded incomprehensible at first, I have asked the silliest questions and misunderstood the response. However, I learnt something in the end, and I will go back home determined to understand more. And here comes my second word, ‘patience’. Precious things need their time to blossom, and this was true in a broader sense for my time in Uppsala and more specifically regarding the results of this work. Last but not least, I mention ‘meaningfulness’. While working on this project I reflected several times about the deeper meaning of what I was doing. I asked myself why physics and cosmology should ever matter in a world in which there are still so many other urgent problems to solve. I leave this question open for the future, but my answer, for now, is that Science is not a mere intellectual exercise nor should necessarily have an immediate practical outcome. Science is rather a precious element of our identity and should be more accessible to society, as it is a public treasure which has the extraordinary potential of making people work together.
To conclude, this project has been an experience which has brought me thousands of kilometres away from my comfort zone and made me grow as a student and a person. This was definitely not something I could achieve alone. I start by acknowledging someone who has guided me all along this path, with kindness, patience and enthusiasm: my supervisor and cosmology lecturer Martin Sahlén. It has been a pleasure and a honour working with you, and I will forever value what you taught me. Thank you for having given me a perfect balance between challenges and advice: if I ever become a cosmologist, I will definitely remember that I owe something to you. A huge and warm thank goes to Stefano Camera, who believed in this project
since the beginning and supported me from Turin with his precious advice. I will be glad to present this project for my graduation ceremony, and I am determined to polish every single word even more with your help. I want to mention the Erasmus coordinators of Uppsala and Turin, Dimitri Arvanitis, Mario Bertaina, Igor Pesando, together with the Uppsala course coordinator Rabab Elkarib, who helped me to carefully plan my Erasmus exchange. And, of course, all my gratitude to the Erasmus programme, which gave me the material support to make this experience possible. Also, I will not forget that Erik Zackrisson was the first person I met in Uppsala: thank you for having arranged for this project in the beginning.

A tremendous acknowledgement goes to my co-sailor Elias Waagaard, for having made me feel at home throughout these months in Sweden. With you, Ithaca seems behind the corner and my everyday life is brighter and more colourful: a part of me will forever remain in Uppsala by your side. The whole Waagaard and Walter families have welcomed me as a daughter and grand-daughter and deserve my deepest gratitude for the moments we spent together. A special thank goes to the ‘IKEA’ group, Hanna, Nicole, Margot, Louise, Nodari, Sotiris, Hugo for the beautiful memories we share and for having made my Erasmus experience more enriching and fun. I conclude by mentioning the deep and affectionate support that I have always received from mamma, papà, Fulvio, Sergio, my grandparents nonna Titti, nonno Lucio, nonna Livia, nonno Piermario and my whole family, who are close and dear also when we have thousands of kilometres between us. And before saying what I hope is only a temporary goodbye to Uppsala, I will leave a last thought which seems to constantly accompany me in my peregrinations across Europe: no matter how vast a continent might seem, there will forever be a bridge connecting spirits and hearts, which can be overcome with enough courage, affection and curiosity.

Uppsala, January 7th, 2019
Appendices

A The galaxy bias and the linear density contrast

An effect which must be taken into account in applying the excursion set formalism in the case of voids is the so-called ‘galaxy bias’, the discrepancy between the distribution of baryonic matter and the underlying cold dark matter field. This can be modeled by introducing an additional factor $b_g(z) = \sqrt{(1 + z)}$ \footnote{The factor $b_g$ depends upon the survey details considered and the underlying cosmological assumptions. The expression we adopted is an approximation valid in the case of the expected Euclid galaxy survey in a flat Universe containing dark matter and a dark energy-like component \cite{22}.} which re-scales the non-linear threshold $\delta_v^g$ traced by the galaxy field and gives the ‘true’ value computed from the dark matter field \cite{22}:

$$\delta_v = b_g^{-1}(z)\delta_v^g.$$ \hspace{2cm} (27)

From this value it is then necessary to extrapolate the linear radius of the void and the linear density threshold, which will be the parameters actually entering the mass function \cite{22}. Under the assumption of a spherical expansion model, these linear quantities can be understood as the values computed from linear equations at the same point in space and time as the corresponding non-linear ones. The relation between linear and non-linear radius is

$$\frac{R}{R_L} = (1 + \delta_v)^{-\frac{1}{3}},$$ \hspace{2cm} (28)

where $R_L$ is the linear radius of the void. The linear density contrast is calculated as

$$\delta_v^{lin} = c \left[1 - (1 + \delta_v)^{-\frac{1}{3}}\right],$$ \hspace{2cm} (29)

with $c = 1.594$ according to \cite{22}. The values of $\delta_v^{lin}$ and $R_L$ and are those referred to in equations (17) and (18) for the multiplicity function and the mass function respectively.
B Fits for $D_v$ and $\beta_v$ from the cluster mass function

Figure 4: Fit with ROOT [6] of the mean values of the parameter $D_v$ from [27] with a second order polynomial: $D_v = -0.36 - 0.42 \log_{10} |f_{R0}| - 0.01 \log_{10}^2 |f_{R0}|$. The fit has been later rescaled to attain $D_v = 3.38$ for General Relativity in order to match larger N-body simulations.

Figure 5: Fit with ROOT [6] of the mean values of the parameter $\beta_v$ from [27] with a linear function: $\beta_v = 0.07 + 0.006 \log_{10} |f_{R0}|$. 

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Figure 6: Plots of the probability density functions for the EC and EV-A survey specifications, both separately and combined, for the case $\log_{10} |f_{R0}| = -8$. 

C Additional plots
Figure 7: Plots of the probability density functions for the EC and EV-B survey specifications, both separately and combined, for the case $\log_{10} |f_R| = -8$. 

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Figure 8: Plots of the probability density functions for the EC and EV-A survey specifications, both separately and combined, for the case $\log_{10}|f_{R_0}| = -4$. 

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Figure 9: Plots of the probability density functions for the EC and EV-B survey specifications, both separately and combined, for the case $\log_{10} |f_R| = -4$. 
References


