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Constructions in higher-dimensional Auslander-Reiten theory

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Abstract

Pasquali, A. 2019. Constructions in higher-dimensional Auslander-Reiten theory. *Uppsala Dissertations in Mathematics* 114. 42 pp. Uppsala: Acta Universitatis Upsaliensis. ISBN 978-91-506-2754-1.

This thesis consists of an introduction and five research articles about representation theory of algebras.

Papers I and II focus on the tensor product of algebras from the point of view of higher-dimensional Auslander-Reiten theory. In Paper I we consider the tensor product Λ of two algebras which are n- respectively m-representation finite. In the case when Λ itself is $(n+m)$-representation finite, we construct its $(n+m)$-almost split sequences explicitly in function of the n- and m-almost split sequences of the factors. In Paper II we use the constructions of Paper I to prove the following result: the tensor product of an n- and an m-complete acyclic algebras (in the sense of Iyama) is $(n+m)$-complete and acyclic.

Papers III and IV deal with the combinatorics of Postnikov diagrams, or equivalently of the Grassmannian cluster category. This is motivated by 2-dimensional Auslander-Reiten theory: we are interested in constructing self-injective Jacobian algebras as they are the 3-preprojective algebras of 2-representation finite algebras. In Paper III we investigate when the stable Jacobian algebra associated to a (k,n)-Postnikov diagram is self-injective. We prove that this happens if and only if the Postnikov diagram is invariant under rotation by $2\pi k/n$. In Paper IV (joint with Thörnblad and Zimmermann) we determine a necessary and sufficient condition on (k,n) for such a symmetric Postnikov diagram to exist, namely $k \equiv -1$, 0 or 1 modulo $n/GCD(k,n)$. As a corollary, we prove that there exist self-injective planar quivers with potential with Nakayama automorphism of any prescribed order, answering a question by Herschend and Iyama.

Paper V (joint with Giovannini) is about skew group algebras. Let G be a finite group acting on a quiver with potential (Q, W), such that certain assumptions hold. We construct a quiver with potential (Q_G, W_G) such that the skew group algebra of the Jacobian algebra of (Q, W) is Morita equivalent to the Jacobian algebra of (Q_G, W_G). Moreover, we show that this construction is a duality if G is abelian. We also apply our results to quivers with potential associated to Postnikov diagrams.

Keywords: Representation theory, higher-dimensional Auslander-Reiten theory, Postnikov diagram, 2-representation finite algebra, self-injective algebra, quiver with potential, skew group algebra

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né dolcezza di figlio, né la pieta
del vecchio padre, né 'l debito amore
lo qual dovea Penelope far lieta,
vincer potero dentro a me l'ardore
ch'i' ebbi a divenir del mondo esperto
e de li vizi umani e del valore;
ma misi me per l'alto mare aperto
sol con un legno e con quella compagna
picciola da la qual non fui diserto.

—Dante, Inferno XXVI

List of papers

This thesis is based on the following papers, which are referred to in the text by their roman numerals.

I A. Pasquali, Tensor products of higher almost split sequences. *J. Pure Appl. Algebra* **221** (2017), 645–665.

II A. Pasquali, Tensor products of n-complete algebras. *J. Pure Appl. Algebra* **223** (2019), 3537–3553.

III A. Pasquali, Self-injective Jacobian algebras from Postnikov diagrams. *Algebr. Represent. Theory* (2019), https://doi.org/10.1007/s10468-019-09882-8.

IV A. Pasquali, E. Thörnblad, and J. Zimmermann. Existence of symmetric maximal noncrossing collections of k-element subsets. arXiv:1808.03556, submitted for publication.

V S. Giovannini and A. Pasquali. Skew group algebras of Jacobian algebras. *J. Algebra* **526** (2019), 112–165.

Contents

1. Foreword

Fatti non foste a viver come bruti, ma per seguir virtute e canoscenza.

—Dante, Inferno XXVI

Mathematics is not an empirical science. There are hypotheses and predictions and educated guesses, but there are no experiments. The real world is not a concern of mathematical research, which instead focuses on the world of ideas. The analogue of an experiment in this world is indeed testing a theory against an example, but this takes place in one's mind (and partially on coffee-stained paper) and not in a laboratory.

The process of designing an experiment in the sciences requires a deep understanding of the theory, so that one can exclude all non-relevant factors and expect one result if the theory is correct and another result if the theory is wrong. There are limitations in the form of available resources and technology.

The process of choosing examples in mathematics is similar. They should be simple enough that computations are possible, yet deep and structured enough that one expects them to either falsify a hypothesis or confirm it in a non-trivial way. There are limitations in the form of computabiliy, checkability of properties, and availability of examples in the first place.

How are examples created (or found, depending on one's personal Weltanschauung)? One can often try to construct "the" prototypical example of a certain object or phenomenon, by finding something that has the properties in question "and no more". One can also, often, look for examples in adjacent areas of research, in hope to shed light not only on the topic but also on hidden connections that might be of interest themselves.

The latter is what this thesis is in essence about. It is a collection of theorems and constructions which in the end leaves us with more examples than we had before. The original motivation of all the papers comprised in this thesis comes from higher-dimensional Auslander-Reiten theory, a very specific subspecialty of representation theory of algebras. However, the constructions involved are freely borrowed from other areas of algebra (broadly interpreted). On the one hand, this allows for novelty, finding expected structure in unexpected places. On the other hand, maybe this structure is there for a reason. And by understanding these reasons one can hope to reveal hidden links and connections between different subjects in algebra. From my point of view, this thesis is thus a worthy contribution to an exciting and still to a large extent unexplored part of human knowledge.

The structure is as follows. The core of this thesis consists of five research papers that I wrote, with coauthors, during my time as a doctoral student. These articles are attached at the end. The next chapters are devoted to presenting and recalling some of the necessary background, and then stating the main results of the papers. Then the reader will find a short perspective on possible further research directions, as well as a summary in Swedish. Last but not least, in Chapter 6 I express my gratitude to everyone who accompanied me in this journey.

2. Preliminaries

The purpose of this chapter is twofold. First, it contains a condensed list of definitions and results which are necessary in order to state the results of the papers. Second, it provides a short historical and contextual introduction to some areas of representation theory of algebra (specifically, those relevant for this thesis).

2.1 Finite-dimensional algebras

We will recall some definitions and basic results in representation theory of finite-dimensional algebras. We refer to [17] [4] for a more detailed treatment.

2.1.1 Algebras and modules

Informally, an algebra is a ring with a compatible vector space structure, or a vector space with a nice associative bilinear operation. Formally, let k be a field.

Definition 2.1.1. A *k-algebra* is a ring with identity Λ which is a k-vector space, such that

$$\lambda(ab) = (\lambda a)b = a(\lambda b)$$

for all $a,b \in \Lambda$ and $\lambda \in k$.

In this thesis, the field k is somewhat in the background, so we will often just speak of Λ as an *algebra*. Historically, much of the theory was developed for algebras over an arbitrary artinian ring [5]. For reference, we summarise here the additional assumptions on the field we need to make. In Papers I and II, we need k to be perfect. In Papers III and IV, we work over the field $\mathbb{C}$, and in fact we denote by k a natural number. In Paper V, we need no assumptions for the main results, and we assume $k = \mathbb{C}$ for some of the applications.

Almost all of the algebras appearing in this thesis are finite dimensional (as vector spaces), so we assume that $\dim_k \Lambda < \infty$ from now on. We also assume that Λ is connected, i.e. that it cannot be written as a nontrivial product

$\Lambda \cong \Lambda_1 \times \Lambda_2$. An algebra *morphism* is a linear ring morphism. The *opposite algebra* Λ^{op} of Λ is Λ as a vector space, but with product $\cdot_{op}$ given by $a \cdot_{op} b = ba$.

If M is a vector space, the set $\mathrm{End}(M)$ of linear maps from M to M is an algebra with product given by composition. A *left* Λ*-module* is a vector space M with an algebra morphism $\Lambda \to \mathrm{End}(M)$. A *right* Λ*-module* is a vector space M with an algebra morphism $\Lambda^{op} \to \mathrm{End}(M)$. If M is a left Λ-module, we can see elements of Λ as acting from the left as linear maps on M, and we write $a \cdot m$ or am for the element we obtain if we let $a \in \Lambda$ act on $m \in M$. Similarly, we write $m \cdot a$ or ma if M is a right Λ-module. A *left (respectively right)* Λ*-module morphism* is a linear map $\varphi : M \to N$ such that $\varphi(a \cdot m) = a \cdot \varphi(m)$ for all $a \in \Lambda, m \in M$ (respectively, $\varphi(m \cdot a) = \varphi(m) \cdot a$). The vector space of Λ-module morphisms from M to N is denoted $\mathrm{Hom}_\Lambda(M,N)$. It has a subspace

$$\mathrm{rad}_\Lambda(M,N) = \{f \in \mathrm{Hom}_\Lambda(M,N) \mid \mathrm{id}_M - g \circ f \text{ is invertible} \, \forall g \in \mathrm{Hom}_\Lambda(N,M)\}.$$

Modules over an algebra are also called *representations* of the algebra, hence the name "representation theory" for the study of modules over algebras. Here again, we are mostly interested in finite-dimensional modules. We denote by $\mathrm{mod}\,\Lambda$ and $\Lambda\mathrm{mod}$ the categories of right respectively left finite-dimensional Λ-modules. One could say that the goal of representation theory (of finite-dimensional algebras) is to describe these categories. These are abelian categories, so they admit a notion of direct sum, kernels of morphisms and exactness. Moreover, the Krull-Schmidt theorem ensures that any module can be decomposed completely with respect to direct sums, and that the summands are uniquely determined. Modules which cannot be decomposed further are called *indecomposable*. An algebra Λ is called *representation finite* if $\mathrm{mod}\,\Lambda$ has finitely many indecomposable objects up to isomorphism.

We remark that $\mathrm{mod}\,\Lambda$ and $\Lambda^{op}\mathrm{mod}$ are isomorphic categories. There is moreover a duality between $\mathrm{mod}\,\Lambda$ and $\Lambda\mathrm{mod}$ given by $D = \mathrm{Hom}_k(-,k)$. If ϕ is an automorphism of Λ and $M \in \mathrm{mod}\,\Lambda$, we can define a "twisted" module structure M_ϕ on M by $m \cdot_\phi a = m \cdot \phi(a)$.

An algebra Λ is always a module over itself from both sides, with action given by multiplication. It is called *basic* if it does not have isomorphic summands as a module over itself. Non-basic algebras play an important role in Paper V, but for any algebra Λ there always exists a basic algebra Λ_b such that $\mathrm{mod}\,\Lambda$ is equivalent to $\mathrm{mod}\,\Lambda_b$. Two algebras with equivalent module categories are called *Morita equivalent*.

2.1.2 Quivers with relations

An important tool for studying modules over algebras is the language of quivers and quiver representations. This was introduced in [13] [14] to address the problem of classifying representation finite algebras, and has since become standard.

A *quiver* is a finite directed graph (loops and multiple edges are allowed). For a quiver Q, we denote by Q_0 its set of vertices and by Q_1 its set of arrows (i.e. oriented edges). Given a quiver Q, we can define an algebra kQ in the following way. A basis of kQ is the set of oriented paths in Q (where we declare that there is a path e_i of length 0 at every vertex i). In particular, kQ is finite dimensional if and only if Q has no oriented cycles. Multiplication of paths is given by concatenation if possible, and multiplying two non-composable paths yields 0. This multiplication rule is extended by linearity to elements of kQ which are not paths. The resulting algebra kQ is called the *path algebra* of Q. This is defined even for quivers with oriented cycles, but then it is an infinite dimensional algebra. Since Q_0 is finite, kQ has a unit given by the sum of all paths of length 0.

A representation of a quiver Q is the assignment of a vector space V_i to every vertex $i \in Q_0$ and of a linear map $V_i \to V_j$ to every arrow $i \to j$ in Q_1. The category of representations of Q is equivalent to the category of left kQ-modules. However, not all algebras are isomorphic (or even Morita equivalent) to a path algebra, a fact which motivates the next construction.

Any path algebra kQ has a two-sided ideal J generated by arrows. An ideal I of kQ is called *admissible* if there exists $n \geq 2$ such that $J^n \subseteq I \subseteq J^2$. By factoring out ideals of this form (even if kQ is infinite dimensional) we can in fact construct all basic algebras.

Theorem 2.1.2 ([4, Theorem II.3.7])**.** *Let Q be a (connected) quiver and I an admissible ideal of kQ. Then the algebra kQ/I is a (connected) basic finite-dimensional algebra. If the base field k is algebraically closed, then every (connected) basic finite-dimensional algebra is isomorphic to an algebra of this form.*

From the point of view of representation theory, we only care about algebras up to Morita equivalence, so by this result it is enough to look at path algebras quotiented by admissible ideals. Usually, one speaks of *relations* for a (nicely) chosen set of generators of an admissible ideal, so that Q becomes a *quiver with relations*.

2.1.3 Homological algebra

In this section we recall some homological properties and constructions of the category $\operatorname{mod}\Lambda$. We will only mention the ones we are going to need, but the interested reader is referred to [9] for a deeper treatment of homological algebra.

As we mentioned earlier, $\operatorname{mod}\Lambda$ is an abelian category, which allows us to talk about direct sums, kernels, complexes, and exactness. If $M \in \operatorname{mod}\Lambda$, we denote by $\operatorname{add}M$ the full subcategory of $\operatorname{mod}\Lambda$ whose objects are all direct sums of direct summands of M. Recall that Λ is naturally a right Λ-module, so we can define *projective Λ-modules* to be the objects of $\operatorname{add}\Lambda$. Since Λ is also a left Λ-module, we can dually define *injective Λ-modules* to be the objects of $\operatorname{add}D\Lambda$. If Λ itself is an injective Λ-module, then Λ is called a *self-injective algebra*. Such algebras play an important role in Papers III and V. If Λ is basic and self-injective, then there always exists an automorphism ϕ of Λ and a map $\Lambda \to D\Lambda$ which is simultaneously an isomorphism $\Lambda \cong D\Lambda$ of left Λ-modules and an isomorphism $\Lambda_\phi \cong D\Lambda$ of right Λ-modules. This automorphism is called a *Nakayama automorphism* and is unique as an outer automorphism of Λ.

We need to introduce projective resolutions, see [9, Chapter V]. If $M \in \operatorname{mod}\Lambda$, a *projective resolution* of M is an exact complex

$$\cdots \longrightarrow P_2 \longrightarrow P_1 \longrightarrow P_0 \longrightarrow M \longrightarrow 0$$

such that P_i is projective for all $i \geq 0$. This is not uniquely determined, but there always exists one. The minimal length (in $\mathbb{Z}_{\geq 0} \cup \{\infty\}$) of a projective resolution is an invariant of M called its *projective dimension* and denoted $\operatorname{proj.dim} M$. The supremum of all projective dimensions of all finite-dimensional Λ-modules is an invariant of Λ, called its *global dimension* and denoted $\operatorname{gl.dim}\Lambda$. So $\operatorname{gl.dim}\Lambda = 0$ means that all Λ-modules are projective, i.e. Λ is Morita equivalent to the path algebra of a quiver with no arrows (if $k = \bar{k}$). Similarly, if $k = \bar{k}$, $\operatorname{gl.dim}\Lambda \leq 1$ means that Λ is Morita equivalent to the path algebra of a quiver (necessarily a quiver with no oriented cycles).

If $N \in \operatorname{mod}\Lambda$, we can apply the functor $\operatorname{Hom}_\Lambda(-,N)$ to a projective resolution of M, to get a complex of the form

$$0 \longrightarrow \operatorname{Hom}_\Lambda(P_0,N) \longrightarrow \operatorname{Hom}_\Lambda(P_1,N) \longrightarrow \operatorname{Hom}_\Lambda(P_2,N) \longrightarrow \cdots$$

The cohomology of this complex in position i does not depend on the choice of projective resolution, and it is denoted by $\operatorname{Ext}^i_\Lambda(M,N)$. This construction is in fact functorial in both M and N.

Certain quotient categories of $\operatorname{mod}\Lambda$ will play an important role. The construction we will now explain works for any exact category, a fact which is used in Paper III. Given an exact category $\mathscr{E}$, we can consider the ideal $\mathscr{I}$ consisting of all morphisms factoring through a projective object. The *stable category* $\underline{\mathscr{E}}$ of $\mathscr{E}$ is the quotient category $\mathscr{E}/\mathscr{I}$. Its objects are the same as those of $\mathscr{E}$, and its morphism spaces $\underline{\operatorname{Hom}}(X,Y)$ are the quotients of those of $\mathscr{E}$ by the subspaces of maps $X \to Y$ that factor through a projective object. Dually one can define the *costable category* $\overline{\mathscr{E}}$ of $\mathscr{E}$, by quotienting out morphisms that factor through an injective object.

2.2 Higher-dimensional Auslander-Reiten theory

We will briefly recall some of the rich theory developed by Auslander and Reiten in the end of the 20th century (most of it is collected in [5]). This was then reframed as the "$d = 1$ case" of a more general theory by the school of Iyama [23] [22] [24] [26] [27]. One can interpret the parameter d both as the global dimension of the algebras involved, and in some cases as the dimension of a space in which some object naturally lives. For a suggestive example, see [25].

This higher-dimensional Auslander-Reiten theory has been found to have strong connections to higher homological algebra [28] [15] [30], and has found applications outside representation theory in algebraic geometry [2] [21] [20].

One of the issues of the theory has from the beginning been the scarcity of examples. An important motivation for all the papers included in this thesis was to look for, construct, and study new examples coming from various constructions.

2.2.1 Almost split sequences

In representation theory of finite-dimensional algebras, one of the most important theorems is the existence of almost split sequences. We will now explain the definitions needed to state this if $\operatorname{gl.dim}\Lambda \leq 1$, but we remark that the results are true for any global dimension. We choose not to work in the full generality in order to better show where the definitions in dimension d come from. The interested reader is advised to consult [4] and [5] for a much deeper and broader treatment.

Let Λ be an algebra of global dimension at most one. Then for every $X \in \operatorname{mod}\Lambda$, the space $\operatorname{Ext}^1_\Lambda(X,\Lambda)$ is a (left) Λ-module in a natural way. This in fact makes $\tau = D\operatorname{Ext}^1_\Lambda(-,\Lambda)$ into a functor on $\operatorname{mod}\Lambda$. Similarly, $\tau^- = \operatorname{Ext}^1_{\Lambda^{op}}(D-,\Lambda)$ is also a functor on $\operatorname{mod}\Lambda$. These functors are called *Auslander-Reiten translations* and will play a crucial role in describing $\operatorname{mod}\Lambda$. In particular, they appear in the so-called almost split sequences:

Theorem 2.2.1 ([4, Theorem IV.3.1]). *Let $L \in \operatorname{mod}\Lambda$ be indecomposable and non-injective. Then there exists a unique (up to isomorphism) exact sequence*

$$0 \longrightarrow L \longrightarrow M \longrightarrow N \longrightarrow 0$$

such that N is indecomposable and the induced sequence

$$0 \longrightarrow \operatorname{Hom}_\Lambda(-,L) \longrightarrow \operatorname{Hom}_\Lambda(-,M) \longrightarrow \operatorname{rad}_\Lambda(-,N) \longrightarrow 0$$

is exact on $\operatorname{mod}\Lambda$. *Moreover, in this case* $N \cong \tau^- L$ *and* $L \cong \tau N$. *Such a sequence is also determined by the choice of a non-projective indecomposable N, and the induced sequence*

$$0 \longrightarrow \operatorname{Hom}_\Lambda(N,-) \longrightarrow \operatorname{Hom}_\Lambda(M,-) \longrightarrow \operatorname{rad}_\Lambda(L,-) \longrightarrow 0$$

is also exact on $\operatorname{mod}\Lambda$.

Such sequences are called *almost split sequences*. The name comes from the following observation: if we replaced rad with Hom in the definition, the condition that $\operatorname{Hom}_\Lambda(-,M) \to \operatorname{Hom}_\Lambda(-,N)$ is surjective would imply that the sequence is split. In fact, $\operatorname{rad}_\Lambda(-,N)$ is the largest possible subspace of $\operatorname{Hom}_\Lambda(-,N)$ such that we can have surjectivity onto it but the sequence does not split. We remark that from the definition it follows that the maps $L \to M$ and $M \to N$ are radical.

If Λ has global dimension (at most) one, this allows us to recursively construct the module category of Λ by starting with the projective indecomposables and constructing almost split sequences (this process is often referred to as "knitting"). In particular, if Λ is representation finite, we can obtain essentially full information about $\operatorname{mod}\Lambda$ in this way.

Theorem 2.2.2. *Let* Λ *be representation finite, with* $\operatorname{gl.dim}\Lambda \leq 1$. *Let* $P_1, \dots, P_n$ *and* $I_1, \dots, I_n$ *be non-isomorphic representatives of the indecomposable projective respectively injective* Λ*-modules. Then:*

1. *There is a permutation* σ *and positive integers* $l_1, \dots, l_n$ *such that* $P_j \cong \tau^{l_j - 1} I_{\sigma(j)}$ *for all j.*
2. *We have*

$$\operatorname{mod}\Lambda = \operatorname{add}\left(\bigoplus_{j=1}^{n} \bigoplus_{p=0}^{l_j-1} \tau^p I_j \right) = \operatorname{add}\left(\bigoplus_{j=1}^{n} \bigoplus_{p=0}^{l_j-1} \tau^{-p} P_j \right).$$

3. *The Auslander-Reiten translations induce quasi-inverse equivalences*

$$\underline{\operatorname{mod}}\Lambda \underset{\tau}{\overset{\tau^-}{\rightleftarrows}} \overline{\operatorname{mod}}\Lambda.$$

2.2.2 d-cluster tilting

In recent years, Iyama and collaborators have developed a version of Auslander-Reiten theory for algebras of higher global dimension. In this setting one looks at a suitable subcategory of $\operatorname{mod}\Lambda$ which has similar homological properties to the whole module category in the classical case, but with all the $\operatorname{Ext}^1_\Lambda$ replaced by $\operatorname{Ext}^d_\Lambda$, where $d = \operatorname{gl.dim}\Lambda$. This subcategory is called a d-cluster tilting subcategory, and if it is generated by a single module then it has finitely many indecomposables up to isomorphism, which means that we can hope to describe it completely using analogous techniques as in the dimension one case.

Precisely, a d-cluster tilting Λ-module is a module M such that

$$\begin{aligned}\operatorname{add} M &= \{X \in \operatorname{mod}\Lambda \mid \operatorname{Ext}^i_\Lambda(X,M) = 0\,\forall i = 1,\dots,d-1\} = \\ &= \{X \in \operatorname{mod}\Lambda \mid \operatorname{Ext}^i_\Lambda(M,X) = 0\,\forall i = 1,\dots,d-1\}.\end{aligned}$$

An algebra Λ is called *d-representation finite* if $\operatorname{gl.dim}\Lambda \leq d$ and there exists a d-cluster tilting Λ-module. The category $\operatorname{add} M$ is called a *d-cluster tilting subcategory* of $\operatorname{mod}\Lambda$.

We remark that a 1-cluster tilting module M is such that $\operatorname{add} M = \operatorname{mod}\Lambda$. In particular, Λ is 1-representation finite if and only if Λ is representation finite and $\operatorname{gl.dim}\Lambda \leq 1$. A 2-cluster tilting module is usually just called a cluster tilting module (and in fact this is the origin of the name "d-cluster tilting"). Observe that a d-cluster tilting module must have all indecomposable injectives and all indecomposable projectives as summands.

Let Λ be d-representation finite. Then we can define the *higher-dimensional Auslander-Reiten translations* by $\tau_d = D\operatorname{Ext}^d_\Lambda(-,\Lambda)$ and $\tau_d^- = \operatorname{Ext}^d_{\Lambda^{op}}(D-,\Lambda)$. We have an analogue of Theorem 2.2.2:

Theorem 2.2.3 ([25, Proposition 1.3])**.** *Let Λ be d-representation finite. Let $P_1,\dots,P_n$ and $I_1,\dots,I_n$ be non-isomorphic representatives of the indecomposable projective respectively injective Λ-modules. Then:*

1. *There is a permutation σ and positive integers $l_1,\dots,l_n$ such that $P_j \cong \tau_d^{l_j-1} I_{\sigma(j)}$ for all j.*
2. *There exists a unique (up to isomorphism) basic d-cluster tilting Λ-module M, given by*

$$M = \bigoplus_{j=1}^{n} \bigoplus_{p=0}^{l_j-1} \tau_d^p I_j = \bigoplus_{j=1}^{n} \bigoplus_{p=0}^{l_j-1} \tau_d^{-p} P_j.$$

3. *The higher Auslander-Reiten translations induce quasi-inverse equivalences*

$$\operatorname{add}(M/P) \underset{\tau_d}{\overset{\tau_d^-}{\rightleftarrows}} \operatorname{add}(M/I),$$

where $P = \bigoplus_{j=1}^n P_j$ *and* $I = \bigoplus_{j=1}^n I_j$.

2.2.3 d-almost split sequences

One key result by Iyama is the existence of a higher-dimensional analogue of almost split sequences.

Theorem 2.2.4 ([24]). *Let Λ be d-representation finite with d-cluster tilting module M, and let $L \in \operatorname{add} M$ be indecomposable and non-injective. Then there exists a unique (up to isomorphism) exact sequence*

$$0 \longrightarrow L \longrightarrow M_d \longrightarrow \cdots \longrightarrow M_1 \longrightarrow N \longrightarrow 0$$

such that

1. *N is indecomposable and all the maps are radical.*
2. *The induced sequence of functors*

$$0 \longrightarrow \operatorname{Hom}_\Lambda(-,L) \longrightarrow \operatorname{Hom}_\Lambda(-,M_d) \longrightarrow \cdots$$

$$\cdots \longrightarrow \operatorname{Hom}_\Lambda(-,M_1) \longrightarrow \operatorname{rad}_\Lambda(-,N) \longrightarrow 0$$

   is exact on $\operatorname{add} M$.

Moreover, in this case $N \cong \tau_d^- L$ and $L \cong \tau_d N$. Such a sequence is also determined by the choice of a non-projective indecomposable N, and the induced sequence

$$0 \longrightarrow \operatorname{Hom}_\Lambda(N,-) \longrightarrow \operatorname{Hom}_\Lambda(M_1,-) \longrightarrow \cdots$$

$$\cdots \longrightarrow \operatorname{Hom}_\Lambda(M_d,-) \longrightarrow \operatorname{rad}_\Lambda(L,-) \longrightarrow 0$$

is also exact on $\operatorname{add} M$.

Such sequences are called *d-almost split sequences*, and as one could expect they play a central role in the study of d-representation finite algebras. Their properties and combinatorial characterisation for tensor products of algebras are the main subject of Papers I and II.

2.3 Preprojective algebras and quivers with potential

In this section we focus on the case $d = 2$ of d-dimensional Auslander-Reiten theory. There is one more construction “in dimension one” which generalises neatly to dimension two, namely the preprojective algebra.

2.3.1 Quivers with potential

For later convenience, before we discuss preprojective algebras we should present the definition of a quiver with potential and its Jacobian algebra [11]. Let Q be a quiver. A potential W on Q is a k-linear combination of cyclic paths in Q, up to cyclic permutations. The pair (Q,W) is called a quiver with potential or a QP for short. Given a potential, one can define an ideal of relations on Q and thus a quotient algebra, called the Jacobian algebra, as follows. For an arrow a in Q, we define the cyclic derivative with respect to a as a linear map $\partial_a : kQ \to kQ$ by setting $\partial_a(p) = \sum_{p=uav} vu$ for every path $p \in kQ$. Observe that $\partial_a(p)$ is 0 unless p is a cycle, in which case it is invariant under cyclic permutation. In particular, the element $\partial_a(W) \in kQ$ is defined. The Jacobian algebra $\mathscr{J}(Q,W)$ is defined by

$$\mathscr{J}(Q,W) = \frac{kQ}{\langle \partial_a(W) \,|\, a \in Q_1 \rangle}.$$

This is not a priori a finite-dimensional algebra, and in the generality we defined it the ideal we quotient by is not necessarily admissible. In particular, a necessary condition is that all cycles appearing in W have length at least three. It is often more convenient to consider the completed version of the Jacobian algebra, where the difference is that we allow for infinite sums in the path algebra and for potentials to be infinite linear combinations of cycles.

Jacobian algebras were introduced (to algebraists) in [11] to connect representation theory and cluster theory (i.e. the study of phenomena related to the cluster algebras of [12]). In fact, if T is a (2-)cluster tilting object in a suitable category, it often happens that $\mathrm{End}(T)$ is a Jacobian algebra. It is therefore not surprising that quivers with potential and their Jacobian algebras have been widely and successfully used to categorify cluster algebras in various contexts [1] [8] [32]. Jacobian algebras also make an appearance in 2-dimensional Auslander-Reiten theory as preprojective algebras, as we will now explain.

2.3.2 The (classical) preprojective algebra

If Δ is a simply laced Dynkin diagram and Q is a quiver with underlying graph Δ, there is a way to associate a finite-dimensional algebra $\Pi(Q)$ to Q that, up to isomorphism, depends only on Δ. Moreover, Q can be recovered from $\Pi(Q)$ with the datum of a certain grading corresponding to the choice of an orientation of Δ. The construction is as follows: let us define the double quiver $\overline{Q}$ of Q by $\overline{Q}_0 = Q_0$ and $\overline{Q}_1 = Q_1 \cup \{\bar{a} : j \to i \,|\, a : i \to j \in Q_1\}$. Observe that, as a quiver, $\overline{Q}$ only depends on Δ and not on the orientation Q. We define the preprojective algebra $\Pi(Q)$ by

$$\Pi(Q) = \frac{k\overline{Q}}{\left\langle e_i \left(\sum_{a \in Q_1} a\bar{a} - \bar{a}a\right) e_i \,|\, i \in Q_0 \right\rangle}.$$

This definition was introduced in [16], and the algebra $\Pi(Q)$ turns out to be finite-dimensional and self-injective. Moreover, if Q' is another quiver with underlying graph Δ, then there is an isomorphism $\Pi(Q) \cong \Pi(Q')$. Another definition was later given in [6]: one can define $\Pi(Q)$ to be the tensor algebra

$$\Pi(Q) = \bigoplus_{i \geq 0} \operatorname{Ext}^1_{kQ}(D(kQ), kQ)^{\otimes i}.$$

One can recover the path algebra kQ in the following way. We can define a grading on $k\overline{Q}$ by setting all arrows $a \in Q_1$ to have degree zero, and all arrows $\bar{a}$ to have degree one. Then the relations defining $\Pi(Q)$ are homogeneous, so we obtain an induced grading on $\Pi(Q)$. One can check that we can then recover kQ as the degree zero part $kQ \cong \Pi(Q)_0$. Note that all orientations of Δ appear in this way for a suitable choice of grading.

2.3.3 The 3-preprojective algebra

This construction generalises neatly, mutatis mutandis, to global dimension two. A suitable 2-dimensional analogue of representation-finite path algebras (i.e. path algebras of Dynkin quivers) is given by 2-representation finite algebras. If Λ is 2-representation finite, one can define [27] the (3-)preprojective algebra $\Pi(\Lambda)$ to be

$$\Pi(\Lambda) = \bigoplus_{i \geq 0} \operatorname{Ext}^2_{\Lambda}(D\Lambda, \Lambda)^{\otimes i}.$$

This is again finite-dimensional and self-injective, and moreover it was shown by Keller [31] that, if k is algebraically closed, there exists a QP (Q, W) such that $\Pi(\Lambda) \cong \mathscr{J}(Q, W)$.

Like in the 1-dimensional case, there are many 2-representation finite algebras sharing the same preprojective algebra, and one can recover them all by a suitable choice of grading. Specifically, a *cut* C on a QP (Q, W) is a set of arrows of Q such that every cycle of W has exactly one arrow in C. One can then define a grading on kQ by setting arrows in C to have degree one and all the other arrows to have degree zero. Since the potential is homogeneous by definition, one gets a grading on the Jacobian algebra, and one can consider the degree zero part $\mathscr{J}(Q, W)_C$, called a truncated Jacobian algebra.

Theorem 2.3.1 ([19]). *Let $\mathscr{J}(Q,W)$ be a self-injective Jacobian algebra, such that the ideal $\langle \partial_a(W) \,|\, a \in Q_1 \rangle$ defining it is admissible. If C is a cut, then $\mathscr{J}(Q,W)_C$ is 2-representation finite. In this case,*

$$\mathscr{J}(Q,W) \cong \Pi(\mathscr{J}(Q,W)_C).$$

Moreover, if k is algebraically closed, every basic 2-representation finite algebra can be described in this way for some cut C on the QP (Q,W) of its preprojective algebra.

This result motivates the investigation of self-injective Jacobian algebras (and of their cuts). We carry out such an investigation in Papers III, IV and V. We are able to draw from different areas of algebra to construct many new examples of self-injective Jacobian algebras, and in some cases to construct cuts on them. In the following, we say that a QP is self-injective if its Jacobian algebra is self-injective.

3. Summary of papers

In the core of this thesis we address various problems and constructions related to higher-dimensional Auslander-Reiten theory. In this chapter, we summarise the main results contained in the papers of which this core consists. The themes covered are tensor products (Papers I and II), Postnikov diagrams and quivers with potential (Papers III, IV and V) and skew group algebras (Paper V). Paper I deals with the question of describing $(n+m)$-almost split sequences over a tensor product of an n- and an m-representation finite algebra, when this is known to be $(n+m)$-representation finite. In Paper II we extend the results and constructions of Paper I to the weaker setting of d-complete algebras. We prove that if A and B are acyclic n- respectively m-complete algebras, then $A \otimes B$ is acyclic $(n+m)$-complete. In Paper III we study QPs constructed combinatorially from Postnikov diagrams, and prove that they are self-injective if and only if the diagram is rotation invariant. Motivated by this, we investigate in Paper IV for which parameters there exist rotation-invariant Postnikov diagrams, and find a necessary and sufficient condition. Paper V is dedicated to the study of skew group algebras of Jacobian algebras, and of how one can translate into combinatorial operations on the QP level the algebraic construction of taking skew group algebras. We obtain various results, which we apply in particular to self-injective QPs coming from Postnikov diagrams.

3.1 Paper I

The first construction we address in this thesis is that of tensor product. If A and B are k-algebras, one can take their tensor product (as vector spaces) $\Lambda = A \otimes_k B$, and define multiplication componentwise to get a k-algebra. To ensure that homological algebra behaves well, we assume that k is perfect (in particular, this guarantees that $\operatorname{gl.dim}(\Lambda) = \operatorname{gl.dim}(A) + \operatorname{gl.dim}(B)$). Let A and B be an n- and an m-representation finite algebra. In general it is not true that Λ is $(n+m)$-representation finite, but there is a necessary and sufficient condition for when this happens, found in [18]. The question we investigate in Paper I is:

Question 3.1.1. *Suppose that A, B and Λ are n-, m- and $(n+m)$-representation finite respectively. What is the connection between the higher almost split sequences in* $\operatorname{mod} A$, $\operatorname{mod} B$ *and* $\operatorname{mod} \Lambda$*?*

One naive guess would be to say that higher almost split sequences over Λ are total tensor products of higher almost split sequences over A and B. This cannot be true, however, since the total tensor product of complexes of length $n+2$ and $m+2$ has length $n+m+3$, while $(n+m)$-almost split sequences have length $n+m+2$.

A description of the $(n+m)$-cluster tilting subcategory $\mathscr{C}_\Lambda$ of Λ, as well as of the $(n+m)$-Auslander-Reiten translations, was obtained in [18]. It turns out that

$$\mathscr{C}_\Lambda = \operatorname{add}\left(\bigoplus_{i\geq 0} \tau_n^{-i} A \otimes \tau_m^{-i} B\right).$$

Moreover, for every indecomposable $N \otimes M \in \mathscr{C}_\Lambda$ we have $\tau_{n+m}^{\pm}(N \otimes M) \cong \tau_n^{\pm}(N) \otimes \tau_m^{\pm}(M)$. In particular, we have that $\tau_n^-(N) \neq 0$ and $\tau_m^-(M) \neq 0$ precisely if $\tau_{n+m}^-(N \otimes M) \neq 0$. We define *slices*: slice i of $\operatorname{mod}\Lambda$ is the full subcategory of $\operatorname{mod}\Lambda$ given by $\mathscr{S}_\Lambda(i) = \operatorname{add}(\tau_{n+m}^{-i}\Lambda)$. In a similar way we define slices of $\operatorname{mod}A$ and $\operatorname{mod}B$, so that $\otimes$ gives a map $\mathscr{S}_A(i) \times \mathscr{S}_B(i) \to \mathscr{S}_\Lambda(i)$.

Assume that $N \otimes M \in \mathscr{C}_\Lambda$ is the starting point of an $(n+m)$-almost split sequence. Then there is i such that $N \in \mathscr{S}_A(i)$, $M \in \mathscr{S}_B(i)$ and $N \otimes M \in \mathscr{S}_\Lambda(i)$. Moreover, N is the starting point of an n-almost split sequence and M of an m-almost split sequence. Describing the $(n+m)$-almost split sequences in $\operatorname{mod}\Lambda$ can be seen as completing the information we have about the category $\mathscr{C}_\Lambda$: given the n-almost split sequences starting in N and the m-almost split sequence starting in M, can we describe the $(n+m)$-almost split sequence starting in $N \otimes M$ (when $N \otimes M \in \mathscr{C}_\Lambda$)?

It turns out that the tool we need to describe the operation of "tensoring" an n- and an m-almost split sequence is given by the mapping cone. Given a chain map $\varphi : (A_\bullet, d^A) \to (B_\bullet, d^B)$, its *mapping cone* is the complex $\operatorname{Cone}(\varphi) = A[-1]_\bullet \oplus B_\bullet$ with differential given by

$$d^{\operatorname{Cone}(f)} = \begin{pmatrix} d^A[-1] & 0 \\ \varphi[-1] & d^B \end{pmatrix}.$$

In Paper I we prove:

Theorem 3.1.2. *Let Λ be a d-representation finite algebra. Every d-almost split sequence in* $\operatorname{mod}\Lambda$ *is isomorphic as a complex to the mapping cone of a chain map.*

The chain maps giving d-almost split sequences are of the form $\varphi : C_1 \to C_2$, where C_1 and C_2 are complexes consisting of modules in slice i and $i+1$ respectively, for some i. There is a natural notion of tensor product of chain maps, so one could ask whether $(n+m)$-almost split sequences can be realised as cones of tensor products of maps. The main result of Paper I is that this is indeed the case, thus giving a complete description of higher almost split sequences over a tensor product in terms of the ones over the factors.

Theorem 3.1.3. *Let A, B and $\Lambda = A \otimes B$ be n-, m- and $(n+m)$-representation finite respectively. Let φ be a chain map from $\mathscr{S}_A(i)$ to $\mathscr{S}_A(i+1)$ such that* $\operatorname{Cone}(\varphi)$ *is n-almost split, and let ψ be a chain map from $\mathscr{S}_B(i)$ to $\mathscr{S}_B(i+1)$ such that* $\operatorname{Cone}(\psi)$ *is m-almost split. Then* $\operatorname{Cone}(\varphi \otimes \psi)$ *is $(n+m)$-almost split.*

Since every $(n+m)$-almost split sequence starts in $\mathscr{S}_\Lambda(i) = \mathscr{S}_A(i) \otimes \mathscr{S}_B(i)$ for some i, all $(n+m)$-almost split sequences in $\operatorname{mod}\Lambda$ are accounted for by this theorem.

3.2 Paper II

The idea behind Paper II is to take the constructions of Paper I and perform them in greater generality. In particular, we consider a setting in which slices can be defined and we have $\mathscr{S}_{A\otimes B}(i) = \mathscr{S}_A(i) \otimes \mathscr{S}_B(i)$, but we do not have d-representation finiteness.

Question 3.2.1. *We know that* $\operatorname{Cone}(\varphi \otimes \psi)$ *will have good homolgical properties whenever* $\operatorname{Cone}(\varphi)$ *and* $\operatorname{Cone}(\psi)$ *are n- and m-almost split. This does not depend on the fact that $A \otimes B$ is $(n+m)$-representation finite. Can we still say something about nice subcategories of* $\operatorname{mod} A \otimes B$ *based on the existence of such sequences?*

We consider, instead of d-representation finite algebras, the weaker notion of d-complete algebras. Let Λ be d-representation finite. Recall that, by Theorem 2.2.3, for every indecomposable injective Λ-module I there is $l_I \geq 1$ such that $\tau_d^{l_I-1}(I)$ is projective. Moreover, the d-cluster tilting subcategory is given by

$$\mathscr{C}_\Lambda = \operatorname{add}\left(\bigoplus_{I \text{ ind. injective}} \bigoplus_{j=0}^{l_I-1} \tau_d^j(I) \right).$$

Observe in particular that $\tau_d^{l_I}(I) = 0$. We want to generalise this setup, but without the condition of $\tau_d^{l_I-1}(I)$ being projective.

Definition 3.2.2. A Λ-module T is a *tilting module* if:

1. $\operatorname{proj.dim} T \leq 1$.

2. $\mathrm{Ext}^i_\Lambda(T,T) = 0$ for all $i > 0$.

3. There is an exact sequence

$$0 \longrightarrow \Lambda \longrightarrow T_0 \longrightarrow \cdots \longrightarrow T_m \longrightarrow 0$$

for some m, with $T_i \in \operatorname{add} T$ for all i.

Intuitively, Λ is d-complete if, instead of reaching the indecomposable projectives as the last nonzero translates of the indecomposable injectives, we reach a suitable tilting module. In this case the category $\mathscr{C}_\Lambda$ defined above will not be d-cluster tilting in $\operatorname{mod}\Lambda$, but only in the exact subcategory $T^\perp = \ker \mathrm{Ext}^{>0}(T,-)$. More precisely, we call $\mathscr{P} = \{X \in \mathscr{C}_\Lambda \mid \tau_d X = 0\}$, $\mathscr{C}_P = \{X \in \mathscr{C}_\Lambda \mid X \text{ has no nonzero summands in } \mathscr{P}\}$, and T a basic module such that $\operatorname{add} T = \mathscr{P}$. We define:

Definition 3.2.3. An algebra Λ of global dimension at most d is *d-complete* if the following conditions hold:

1. T is a tilting module.

2. $\mathscr{C}_\Lambda$ is a d-cluster tilting subcategory of $T^\perp$,

3. $\mathrm{Ext}^i(\mathscr{C}_P, \Lambda) = 0$ for every $0 < i < d$.

So d-representation finite is the same as d-complete with $T = \Lambda$. The notion of d-completeness was originally introduced in [25] to deal with higher Auslander algebras. If Λ is d-representation finite, one can define its *d-Auslander algebra* as the endomorphism algebra of the basic d-cluster tilting Λ-module. For $d = 1$ this coincides with the classical notion of the Auslander algebra of a representation finite algebra. One of Iyama's main motivations for introducing higher-dimensional Auslander-Reiten theory was to obtain a generalisation of the Auslander correspondence, which describes a necessary and sufficient homological condition for an algebra to be an Auslander algebra.

If Λ is d-representation finite, its d-Auslander algebra has global dimension at most $d+1$, but it is not usually $(d+1)$-representation finite. However, Iyama proved that if Λ is d-complete, then its d-Auslander algebra is always $(d+1)$-complete. As we saw, if A and B are n- respectively m-representation finite, the tensor product $A \otimes B$ is not $(n+m)$-representation finite in general. In Paper II we prove:

Theorem 3.2.4. *Let A and B be n- respectively m-complete acyclic algebras. Then $A \otimes B$ is $(n+m)$-complete and acyclic.*

This theorem can be seen as a parallel to Iyama's result about higher Auslander algebras, in that we find one more setting in which d-representation finiteness is too strong a property to be preserved, but d-completeness is not.

The assumption of acyclicity is a technical condition explained in Definition 2.5 and §4.4 of Paper II. If $\Lambda = kQ/I$ for an admissible ideal I and k is algebraically closed, then Λ is acyclic if and only if Q is acyclic. We need the assumption that A and B be acyclic in order to make proofs work, but we remark that there are no known examples of d-complete algebras which are not acyclic.

In particular, we get back the characterisation, originally found in [18], of when the tensor product of an n- and an m-representation finite algebra is $(n+m)$-representation finite (in the acyclic case). We say that an algebra Λ of global dimension d is *l-homogeneous* if $\tau_d^{l-1}(D\Lambda) = T$. If Λ is d-complete, this is the same as saying that $l_I = l$ for every indecomposable injective I. We get:

Corollary 3.2.5. *Let A and B be n- respectively m-representation finite acyclic algebras. Then the following are equivalent:*

1. *$A \otimes B$ is $(n+m)$-representation finite;*
2. *A and B are l-homogeneous for some common l.*

Moreover, in this case $A \otimes B$ is also l-homogeneous.

To prove that $A \otimes B$ is $(n+m)$-complete, we first prove that $\operatorname{mod} A \otimes B$ has $(n+m)$-almost split sequences using the same method as in Paper I. Namely, we realise n- and m-almost split sequences over A and B as cones of chain maps, and then verify that the cone of the tensor product is indeed $(n+m)$-almost split. Then, by a recursive construction using these sequences (and making crucial use of acyclicity) we can prove that T is a tilting module in $\operatorname{mod}\Lambda$, which then implies d-completeness using a theorem by Iyama. The difficult part is precisely showing that T is tilting, since the "generating" property needs in principle to be checked for arbitrary $A \otimes B$-modules, and not only for modules of the form $N \otimes M$. Instead, we use an argument involving an alternative generating property of tilting modules, namely that they generate the bounded derived category.

3.3 Paper III

The motivation behind Paper III is: there is a certain combinatorial way of generating planar QPs in the sense of [19]. Can we use it to produce self-injective QPs? Recall that we are particularly interested in self-injective QPs because their Jacobian algebras are the 3-preprojective algebras of 2-representation finite algebras.

The combinatorics is that of Postnikov diagrams [36] [7]. A (k,n)-Postnikov diagram (in this paper k is a number and not a field) is a collection of n oriented curves in a disk with n marked points on the boundary. The curves connect vertex i to vertex $i+k \pmod{n}$, and the key property is that following one curve one sees the others crossing it alternatingly from the left and from the right. To such a diagram one can associate a quiver by putting vertices in the regions whose boundary is alternating, and connecting them with arrows through the crossings. This is illustrated in Figure 3.1.

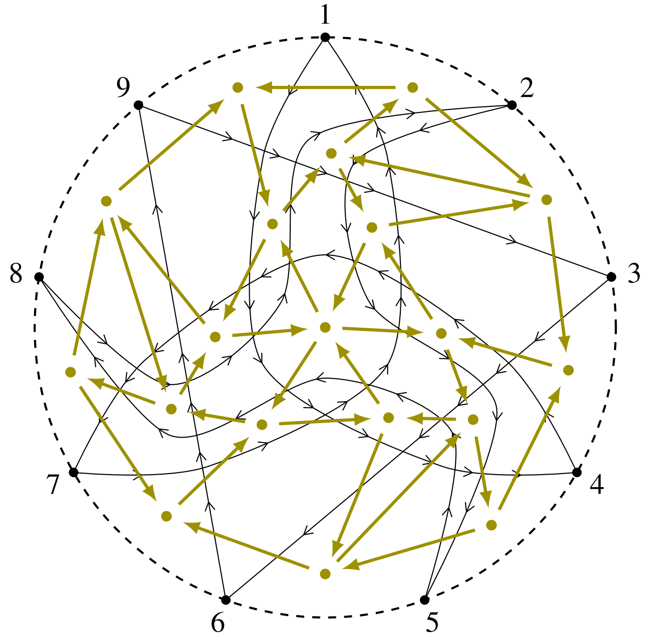

Figure 3.1. A rotation-invariant (3,9)-Postnikov diagram and the corresponding quiver.

One obtains a planar QP (by setting the potential to be the sum of the boundaries of the faces) whose Jacobian algebra is infinite dimensional. Factoring out the idempotent corresponding to the boundary vertices, one gets on the other hand a finite-dimensional Jacobian algebra, and it makes sense to ask whether this is self-injective. In practice, we just remove the boundary vertices, so from the quiver of Figure 3.1 we obtain the planar QP of Figure 3.2.

Question 3.3.1. *What kind of Jacobian algebras arise in this way? Can we characterise their self-injectivity?*

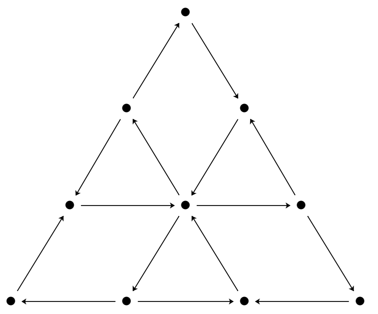

Figure 3.2. The planar self-injective QP corresponding to the Postnikov diagram of Figure 3.1.

In all the examples of self-injective planar QPs shown in [19], a Nakayama automorphism acts by a rotation, and it turns out that the same is true here. The main result of Paper III is:

Theorem 3.3.2. *The Jacobian algebra constructed from a (k,n)-Postnikov diagram is self-injective if and only if the diagram is invariant under rotation by $\frac{2\pi k}{n}$. In this case, a Nakayama automorphism acts by this rotation.*

Observe that one can interpret the action of a Nakayama automorphism as subtracting $k \pmod{n}$ from all labels, a fact which plays a role in the proof (as well as in Paper IV).

The proof uses a categorification of Postnikov diagrams constructed in [29]. There is an algebra $B = B(k,n)$ such that the combinatorics of (k,n)-Postnikov diagrams governs the vanishing of Ext^1 between certain Cohen-Macaulay B-modules. One can define a B-module L_I for each vertex I of the quiver associated to the diagram, and it turns out that $T = \bigoplus_I L_I$ is a cluster tilting object in the stable category $\underline{\mathrm{CM}}(B)$. By a result of [7], the stable endomorphism algebra $\underline{\mathrm{End}}_B(T)$ is in fact isomorphic to the Jacobian algebra associated to the Postnikov diagram. Moreover, $\underline{\mathrm{CM}}(B)$ is 2-Calabi-Yau, which implies that $\underline{\mathrm{End}}_B(T)$ is a self-injective algebra if and only if $T \cong T[2]$. We compute the action of the functor $[2]$ on the modules L_I, and prove that $T \cong T[2]$ precisely when the Postnikov diagram is rotation invariant.

We also consider cuts on QPs coming from Postnikov diagrams. Using an isoradial embedding constructed in [35], we show:

Proposition 3.3.3. *If (Q,W) is a QP constructed from a Postnikov diagram, then every arrow of Q is contained in a cut.*

For planar QPs, the condition that every arrow be contained in a cut was introduced and studied in [19]. In particular, Herschend and Iyama prove that if it holds, then all the truncated Jacobian algebras $\mathscr{J}(Q,W)_C$ are derived equivalent.

Paper III also contains some new examples of self-injective QPs, constructed from Postnikov diagrams. This answers in the negative a question asked in [19]. In particular, a family of planar self-injective QPs with Nakayama automorphism of arbitrarily large order is constructed.

3.4 Paper IV

A natural question to ask in view of the results of Paper III is:

Question 3.4.1. *For which pairs (k,n) do there exist rotation-invariant (k,n)-Postnikov diagrams?*

If we do not ask for rotation invariance, the answer is for all $n \geq k \geq 1$. If we do, however, the situation is more complicated. In Paper IV we answer this question:

Theorem 3.4.2. *There exists a rotation-invariant (k,n)-Postnikov diagram if and only if k is congruent to $0, 1$ or -1 modulo $n/\operatorname{GCD}(k,n)$.*

In this paper we use the language of *maximal noncrossing collections* instead of that of Postnikov diagrams. Two subsets I and J of $\{1,\ldots,n\}$ are said to be *noncrossing* if there do not exist cyclically ordered a,b,c,d such that $a,c \in I \setminus J$ and $b,d \in J \setminus I$. To a Postnikov diagram one can associate a collection of mutually noncrossing k-element subsets of $\{1,\ldots,n\}$ by assigning to every region with alternating boundary the set of starting points of curves that have the region to their left (see Figure 3.1).

By results in [36] and [35], the resulting collection of k-element sets is maximal among the collections of mutually noncrossing k-element sets, and we call it a maximal noncrossing collection. Moreover, all maximal noncrossing collections arise in this way [35], so Postnikov diagrams and maximal noncrossing collections are interchangeable as combinatorial objects. We choose here to work with collections because they are easier to construct explicitly. The rotation invariance can be phrased in this language by demanding that the collection be invariant under adding $k \pmod{n}$ to all labels (we call such collections *symmetric*).

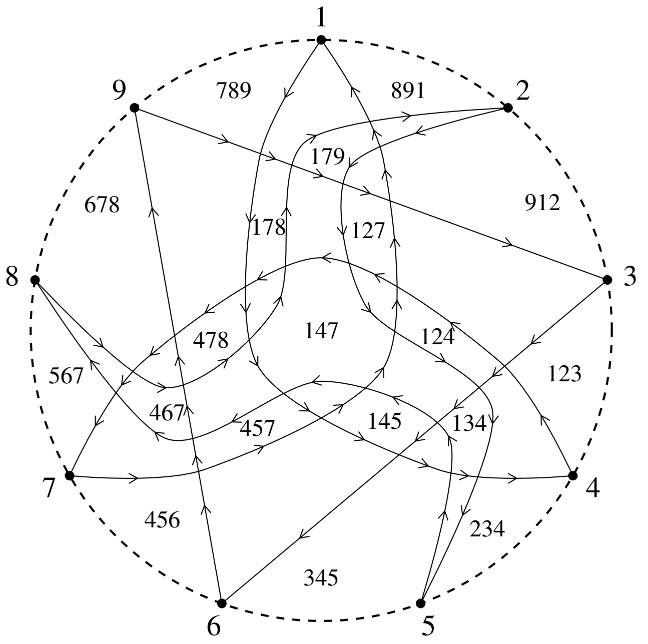

Figure 3.3. The maximal noncrossing collection associated to the Postnikov diagram of Figure 3.1.

The two implications in the theorem are proved with different techniques. Necessity of the numerical condition follows from analysing the isoradial embedding (see [35]) of the quiver associated to a Postnikov diagram. This quiver must have a central vertex or a central cycle in order to be rotation invariant, and the condition follows from considerations on its associated k-element set(s).

Sufficiency of the numerical condition is proved by explicitly constructing a symmetric maximal noncrossing collection when the condition holds. This is the core of the paper, and it makes crucial use of the fact that all maximal noncrossing collections of k-element subsets of $\{1,\ldots,n\}$ have exactly $k(n-k)+1$ elements [35]. It is worth noting that we do not get all the possible symmetric maximal noncrossing collections this way, so a complete classification is still open.

Recall that in Paper III we established that the Jacobian algebras constructed from rotation-invariant Postnikov diagrams are self-injective, with a Nakayama automorphism induced by rotation. A corollary of our result is therefore:

Corollary 3.4.3. *There exist infinitely many self-injective Jacobian algebras with Nakayama automorphism of any prescribed order.*

3.5 Paper V

Paper V deals with yet another construction, namely that of skew group algebra. If Λ is a k-algebra and G is a finite group acting on Λ by algebra automorphisms, one can define the skew group algebra ΛG to be $\Lambda \otimes_k kG$ as a vector space, with the "twisted" multiplication induced by

$$(\lambda \otimes g)(\mu \otimes h) = \lambda g(\mu) \otimes gh.$$

The algebra ΛG is not basic in general, so it is not the quotient of a path algebra. However, it is interesting to study ΛG up to Morita equivalence, in particular the connections between the quiver of Λ and that of ΛG. This was done extensively by Reiten and Riedtmann [37], who proved that many properties of $\operatorname{mod} \Lambda G$ are inherited from $\operatorname{mod} \Lambda$. One interesting question is to describe the quiver Q_G of a basic version Λ_G of ΛG explicitly as a function of the quiver of Λ. This was done in [37] if G is cyclic, and in [10] for any G if Λ is hereditary or a preprojective algebra. However, describing the relations one needs to impose on Q_G to obtain Λ_G is difficult in general. In this paper we address the following question:

Question 3.5.1. *Let (Q,W) be a QP with a group G acting on it in a nice way, and let $\Lambda = \mathscr{J}(Q,W)$. Is the skew group algebra ΛG Morita equivalent to another Jacobian algebra $\mathscr{J}(Q_G,W_G)$? If it is, can we explicitly describe the potential W_G?*

A positive answer to the first part of this question comes from work by Le Meur [33]. He proves that the skew group dg algebra of the *Ginzburg dg algebra* of (Q,W) is Morita equivalent to the Ginzburg dg algebra of another QP. Then one can get the statement for Jacobian algebras by taking zeroth cohomology. The potential one obtains on Q_G is the image of W via a natural map, but it is not expressed as a linear combination of cycles. By a similar approach, Amiot and Plamondon [3] manage to describe W_G explicitly if $G \cong \mathbb{Z}/2\mathbb{Z}$.

In the first part of Paper V, we explicitly describe the potential W_G under some assumptions on the action of G on (Q,W). We work in the case where G is cyclic, and we impose some combinatorial conditions on the length of the orbits of vertices in function of W. The precise statement we prove is:

Theorem 3.5.2. *Let (Q,W) be a QP and Λ its Jacobian algebra. Let G be a finite cyclic group acting on (Q,W) as per the assumptions (A1)–(A7) of §3.1 of Paper V. Let $\eta \in \Lambda G$, Q_G and W_G be as in Section 3 of Paper V. Then*

$$\mathscr{J}(Q_G,W_G) \cong \eta(\Lambda G)\eta.$$

These conditions (A1)–(A7) are satisfied in many examples, namely whenever (Q,W) is a planar QP on which G acts by rotations. Thus we get many examples coming from self-injective Jacobian algebras constructed in Papers III and IV (one can take G generated by a power of the Nakayama automorphism).

Reiten and Riedtmann observe that, in all the examples they compute, there is a natural action of the dual group $\hat{G}$ of G on Q_G. They prove that in fact there is always a natural action of $\hat{G}$ on ΛG, and that if G is abelian then the algebra $(\Lambda G)\hat{G}$ is Morita equivalent to Λ. Thus taking skew group algebras is in some sense a duality in the abelian case. In the second part of the paper, we prove that in our setting the action of $\hat{G}$ on ΛG restricts to the basic version by an action on (Q_G, W_G). This action still satisfies our assumptions, so we can explicitly construct a QP $((Q_G)_{\hat{G}}, (W_G)_{\hat{G}})$ whose Jacobian algebra is isomorphic to Λ. We prove that, as one could expect,

Theorem 3.5.3. *There is an isomorphism of QPs*

$$((Q_G)_{\hat{G}}, (W_G)_{\hat{G}}) \cong (Q,W)$$

which induces an isomorphism of algebras

$$\theta \left((\eta \, (\Lambda G) \, \eta) \, \hat{G} \right) \theta \cong \Lambda,$$

where θ is the idempotent defined in Section 5 of Paper V.

The third part of Paper V is dedicated to studying planar QPs with G acting by rotations and their skew group algebras. We construct cuts on Q_G from G-invariant cuts on Q, and show some conditional results about the truncated Jacobian algebras of ΛG.

4. Afterword

Strenuousness is the immortal path, sloth is the way of death.

—H. W. Tilman, When Men & Mountains Meet

Only by getting to the top does one realise that there are so many mountains around. The results in this thesis are the product of a long process, during which many questions were answered. Fortunately, many were left unanswered, and many more new questions were asked. The aim of this chapter is to outline some possible directions for future (and present) research.

4.1 Generalising Postnikov diagrams

In the scope of the main result of Paper III, there is still a lot to be done. Suppose we construct a Postnikov diagram such that the permutation afforded by the strands is not $i \mapsto i+k \pmod{n}$, but which is symmetric. This corresponds to a collection of noncrossing subsets which is maximal in a suitable submatroid of the matroid of all k-element subsets of $\{1,\dots,n\}$ [35]. The QP one gets is still rotation-invariant, but not always self-injective. Yet, all examples of planar self-injective QPs that I know which have no interior vertices of valency 2 can be realised in this way. Are there others? Exactly what conditions do we need for self-injectivity?

Here is a proposed strategy for looking at these questions. Let $\mathbb{J}$ be a noncrossing collection which is maximal in some suitable matroid. One should associate an algebra B' to $\mathbb{J}$ in a similar way as how B is constructed, and there should exist a map $B \to B'$ (corresponding to the fact that $\mathbb{J}$ can be embedded in a collection which is actually maximal). By studying properties of this map and the corresponding functors $\operatorname{mod} B' \to \operatorname{mod} B$, one can hope to obtain information about self-injectivity of the endomorphism algebra of the (conjectural) cluster tilting B'-module associated to $\mathbb{J}$. In particular, I hope that the restriction of scalars functors $\mathrm{CM}(B') \to \mathrm{CM}(B)$ behave reasonably with respect to the triangulated structures of the stable categories.

4.2 More on skew group algebras

One problem with the main result of Paper V is the dependence on many assumptions on both the QP and the group action. These assumptions are satisfied in the cases we were interested in, but are indeed quite strong. One could try to generalise the result to the case of an arbitrary group acting reasonably on an arbitrary QP.

This was done in [34], using techniques from monoidal categories. However, the formulas Le Meur obtains depend on solving a linear system which can be quite big. In an ongoing joint project with Giovannini and Plamondon, we are trying to give simple formulas for the potential W_G for any abelian group G. In fact, we hope to obtain a slightly stronger result, namely an isomorphism not only on the level of Jacobian algebras but also on the level of dg algebras (this is the generality of [3] [33] [34], but not of Paper V).

4.3 Postnikov diagrams on orbifolds

We observed in Paper V how one can always apply our skew group QP construction to the self-injective QPs coming from Postnikov diagrams. On the one hand, one could try to find a combinatorial model in the spirit of Postnikov diagrams for the resulting QPs. This should be axiomatised as some kind of strand diagram on the disk with an orbifold point. On the other hand, one can construct the skew group category of $\mathrm{CM}(B)$, and this will be a Frobenius, stably 2-Calabi-Yau category. One could imagine it being equivalent to $\mathrm{CM}(B_G)$, where B_G is the skew group algebra of B. Furthermore, there is hope to describe cluster tilting modules in $\mathrm{CM}(B_G)$ combinatorially, and maybe one can recover a "skew" version of [7, Theorem 10.3]. It is an ongoing project with Baur and Velasco to investigate these questions.

5. Sammanfattning på svenska (Summary in Swedish)

Den här avhandlingen består av fem artiklar om representationsteori av algebror. I det här kapitlet kommer vi att återge en del av den algebraiska bakgrunden till avhandlingens resultat, och sedan att sammanfatta själva resultaten. För fördjupningar inom algebra och speciellt representationsteori hänvisar vi till [17] och [4].

5.1 Bakgrund

En algebra är en mängd där man kan addera och multiplicera ihop element, samt multiplicera element med skalärer (till exempel reella tal). Ett sätt att beskriva en algebra är att presentera den som vägalgebran av ett koger, modulo några relationer. Ett koger är en uppsättning punkter med en uppsättning pilar mellan dem. Vägalgebran är mängden av alla summor av alla skalärmultiplar av riktade vägar i ett koger, med multiplikation given av sammansättning av konsekutiva vägar. Vägalgebran kan modifieras genom att kvota bort en del relationer, vilket innebär att vissa linjära kombinatoner av vägar blir lika med noll. Alla basala ändligdimensionella algebror över en algebraiskt sluten kropp kan skrivas som kvot av en vägalgebra på det sättet.

Representationsteori handlar om att beskriva moduler över algebror. Dessa är vektorrum där algebran agerar som en mängd av endomorfier. Problemet att beskriva modulkategorin av en algebra är generellt olösbart, men man kan byta till enklare, mer specifika problem. En idé är att beakta endast en delkategori om vilken man kan säga någonting. Det är motivationen bakom högdimensionell Auslander-Reitenteori, som uppfanns under de senaste 15 år av Iyama och hans medarbetare [24].

På ett koger kan man ange en potential, det vill säga en linjärkombination cykler. En potential ger upphov till vissa relationer och på det sättet fås en så kallad Jacobialgebra [11].

5.2 Avhandlingens resultat

I Artikel I och II betraktar vi högredimensionell Auslander-Reitenteori för tensorprodukten $\Lambda = A \otimes B$ av en n- och en m-representationsändlig algebra, se också [18]. Resultatet är en algebra som vanligtvis inte är $(n+m)$-representationsändlig, men vi antar att den ar det i Artikel I. Här undersöker vi formen på de så kallade d-nästan kluvna följderna i $\operatorname{mod}\Lambda$. Vi bevisar att beroendet på A och B kan beskrivas med hjälp av total tensorprodukt, en klassisk homologisk konstruktion. De n- respektive m-nästan kluvna följderna hos A respektive B kan realiseras som koner av vissa avbildningar φ respektive ψ. Med hjälp av total tensorprodukt fås en avbildning $\varphi \otimes \psi$ vars kon ger den sökta $(n+m)$-nästan kluvna följden.

I Artikel II används den här konstruktionen i ett mer generellt sammanhang, nämligen för d-fullständiga algebror [25]. Vi bevisar att tensorprodukten $\Lambda = A \otimes B$ av en m- och en n-fullständig algebra är $(n+m)$-fullständig (under ett visst antagande som stämmer i alla kännda exempel). Formlerna i Artikel I används för att visa existensen av $(n+m)$-nästan kluvna följder som sedan leder till $(n+m)$-fullständighet av Λ.

Artikel III adresserar en annan fråga. Den handlar om att konstruera nya exempel på Jacobialgebror av självinjektiva koger med potential. De här algebrorna spelar en viktig roll inom 2-dimensionell Auslander-Reitenteori [19]. Metoden vi använder utnyttjar vissa uppsättning kurvor på enhetscirkelskivan, så kallade Postnikovdiagram. I artikeln bevisar vi att en viss rotationsymmetri hos diagrammet är nödvandig och tillräcklig för algebrans självinjektivitet. Beviset använder en kategorifiering av Postnikovdiagram med moduler över en viss oändligdimensionell algebra. Denna algebra infördes i [29] för att kategorifiera den Grassmannska klusteralgebran. I kategorifieringen motsvarar rotationsymmetri en viss algebraisk invarians hos modulerna, som leder till självinjektivitet.

I Artikel IV undersöker vi symmetriska Postnikovdiagrams existens. Ett sådant diagram beror på två parametrar (k,n), och symmetrivillkoret är inte möjligt för ett godtyckligt val av dem. Vi bevisar att ett nödvandigt och tillräckligt villkor på parametrarna är $k \equiv -1, 0$ eller 1 modulo $n/\operatorname{SGD}(k,n)$. Beviset är konstruktivt, det vill säga vi skapar ett explicit symmetriskt Postnikovdiagram i alla fall där ett finns.

Artikel V undersöker en till konstruktion, nämligen skevgruppsalgebror. De är algebror definierade av en gruppverkan på en algebra. Om den ursprungliga algebran var en Jacobialgebra, vet vi tack vare [33] att dess skevgruppsalgebra också blir (Moritaekvivalent till) en Jacobialgebra. Dessutom kan skevgruppsalgebrans koger beskrivas fullständigt [10] [37]. Å andra sidan, är det i allmänhet svårt att beskriva den nya potentialen på ett explicit sätt. Det är det vi gör i artikeln, under vissa antaganden: vi får formler för potentialen för en Jacobialgebras skevgruppsalgebra. Antagandena stämmer alltid om vi betraktar en Jacobialgebra som kommer från ett Postnikovdiagram. Detta, kombinerat med Artikel III och IV:s resultat, ger oss en rik källa av självinjektiva och symmetriska Jacobialgebror.

Det finns dessutom en viss dualitet hos konstruktionen skevgruppsalgebra. Den duala gruppen verkar på skevgruppsalgebran, och vi bevisar att under våra antaganden blir själva skevgruppsalgebrans skevgruppsalgebra samma som (Moritaekvivalent till) den ursprungliga algebran.

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References

[1] Claire Amiot. Cluster categories for algebras of global dimension 2 and quivers with potential. *Ann. Inst. Fourier (Grenoble)*, 59(6):2525–2590, 2009.

[2] Claire Amiot, Osamu Iyama, and Idun Reiten. Stable categories of Cohen-Macaulay modules and cluster categories. *Amer. J. Math.*, 137(3):813–857, 2015.

[3] Claire Amiot and Pierre-Guy Plamondon. The cluster category of a surface with punctures via group actions. *arXiv:1707.01834v2*, 2017.

[4] Ibrahim Assem, Daniel Simson, and Andrzej Skowroński. *Elements of the representation theory of associative algebras. Vol. 1*, volume 65 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 2006. Techniques of representation theory.

[5] Maurice Auslander, Idun Reiten, and Sverre O. Smalø. *Representation theory of Artin algebras*, volume 36 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1997. Corrected reprint of the 1995 original.

[6] Dagmar Baer, Werner Geigle, and Helmut Lenzing. The preprojective algebra of a tame hereditary Artin algebra. *Comm. Algebra*, 15(1-2):425–457, 1987.

[7] Karin Baur, Alastair D. King, and Robert J. Marsh. Dimer models and cluster categories of Grassmannians. *Proc. Lond. Math. Soc. (3)*, 113(2):213–260, 2016.

[8] Aslak Bakke Buan, Osamu Iyama, Idun Reiten, and David Smith. Mutation of cluster-tilting objects and potentials. *Amer. J. Math.*, 133(4):835–887, 2011.

[9] Henri Cartan and Samuel Eilenberg. *Homological algebra*. Princeton University Press, Princeton, N. J., 1956.

[10] Laurent Demonet. Skew group algebras of path algebras and preprojective algebras. *J. Algebra*, 323(4):1052–1059, 2010.

[11] Harm Derksen, Jerzy Weyman, and Andrei Zelevinsky. Quivers with potentials and their representations. I. Mutations. *Selecta Math. (N.S.)*, 14(1):59–119, 2008.

[12] Sergey Fomin and Andrei Zelevinsky. Cluster algebras. I. Foundations. *J. Amer. Math. Soc.*, 15(2):497–529, 2002.

[13] Peter Gabriel. Unzerlegbare Darstellungen. I. *Manuscripta Math.*, 6:71–103; correction, ibid. 6 (1972), 309, 1972.

[14] Peter Gabriel. Indecomposable representations. II. *Symposia Mat. Inst. Naz. Alta Mat.*, 11:81–104, 1973.

[15] Christof Geiss, Bernhard Keller, and Steffen Oppermann. n-angulated categories. *J. Reine Angew. Math.*, 675:101–120, 2013.

[16] Israel M. Gelfand and Vladimir A. Ponomarev. Model algebras and representations of graphs. *Funktsional. Anal. i Prilozhen.*, 13(3):1–12, 1979.

[17] Pierre Antoine Grillet. *Algebra*. Pure and Applied Mathematics (New York). John Wiley & Sons, Inc., New York, 1999. A Wiley-Interscience Publication.

[18] Martin Herschend and Osamu Iyama. n-representation-finite algebras and twisted fractionally Calabi-Yau algebras. *Bull. Lond. Math. Soc.*, 43(3):449–466, 2011.

[19] Martin Herschend and Osamu Iyama. Selfinjective quivers with potential and 2-representation-finite algebras. *Compos. Math.*, 147(6):1885–1920, 2011.

[20] Martin Herschend, Osamu Iyama, Hiroyuki Minamoto, and Steffen Oppermann. Representation theory of Geigle-Lenzing complete intersections. *arXiv:1409.0668v1*, 2014.

[21] Martin Herschend, Osamu Iyama, and Steffen Oppermann. n-representation infinite algebras. *Adv. Math.*, 252:292–342, 2014.

[22] Osamu Iyama. Auslander correspondence. *Adv. Math.*, 210(1):51–82, 2007.

[23] Osamu Iyama. Higher-dimensional Auslander-Reiten theory on maximal orthogonal subcategories. *Adv. Math.*, 210(1):22–50, 2007.

[24] Osamu Iyama. Auslander-Reiten theory revisited. In *Trends in representation theory of algebras and related topics*, EMS Ser. Congr. Rep., pages 349–397. Eur. Math. Soc., Zürich, 2008.

[25] Osamu Iyama. Cluster tilting for higher Auslander algebras. *Adv. Math.*, 226(1):1–61, 2011.

[26] Osamu Iyama and Steffen Oppermann. n-representation-finite algebras and n-APR tilting. *Trans. Amer. Math. Soc.*, 363(12):6575–6614, 2011.

[27] Osamu Iyama and Steffen Oppermann. Stable categories of higher preprojective algebras. *Adv. Math.*, 244:23–68, 2013.

[28] Gustavo Jasso. n-abelian and n-exact categories. *Math. Z.*, 283(3-4):703–759, 2016.

[29] Bernt Tore Jensen, Alastair D. King, and Xiuping Su. A categorification of Grassmannian cluster algebras. *Proc. Lond. Math. Soc. (3)*, 113(2):185–212, 2016.

[30] Peter Jørgensen. Torsion classes and t-structures in higher homological algebra. *arXiv:1412.0214v3*, 2015.

[31] Bernhard Keller. Deformed Calabi-Yau completions. *J. Reine Angew. Math.*, 654:125–180, 2011. With an appendix by Michel Van den Bergh.

[32] Daniel Labardini-Fragoso. Quivers with potentials associated to triangulated surfaces. *Proc. Lond. Math. Soc. (3)*, 98(3):797–839, 2009.

[33] Patrick Le Meur. Crossed-products of Calabi-Yau algebras by finite groups. *arXiv:1006.1082v2*, 2018.

[34] Patrick Le Meur. On the Morita reduced versions of skew group algebras of path algebras. *arXiv:1810.12612*, 2018.

[35] Suho Oh, Alexander Postnikov, and David E. Speyer. Weak separation and plabic graphs. *Proc. Lond. Math. Soc. (3)*, 110(3):721–754, 2015.

[36] Alexander Postnikov. Total positivity, Grassmannians, and networks. *arXiv:math/0609764v1*, 2006.

[37] Idun Reiten and Christine Riedtmann. Skew group algebras in the representation theory of Artin algebras. *J. Algebra*, 92(1):224–282, 1985.

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