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Phenomenology of new Neutral Vector Bosons and Parton Distributions from Hadronic Fluctuations

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Abstract
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The Higgs particle was first predicted in 1964, and was discovered in the summer of 2012 at the Large Hadron Collider (LHC). This discovery was the latest in a long list of successful Standard Model predictions spanning the last fifty years. However, some of the Standard Models predictions, such as massless neutrinos, are not in agreement with experiment. Thus, extensions of the Standard Model should be considered. Furthermore, some issues, such as how quarks are bound within the proton, are difficult to study from first principles.

In paper I and II of this thesis, a class of models that contains a new TeV scale neutral vector boson is studied. The parameter space of this class of models is constrained using electroweak precision constraints and 13 TeV LHC data. Gauge anomalies are cancelled both by choosing appropriate fermion charges, and by adding Green-Schwarz terms.

The Higgs mechanism is often studied at leading order, but there are also important radiative corrections. These radiative corrections, which change the ground state energy, can both be IR divergent and gauge dependent. In paper III it is shown how to solve both of these problems. In particular, IR divergences are shown to be spurious.

In paper IV of this thesis, rapidity gaps at the LHC are explained by using a colour singlet two-gluon ladder exchange (BFKL). These exchanges, together with a soft-gluon model, are implemented in a complete Monte Carlo simulation, and reproduce observed rapidity gaps at the LHC.

The momentum distributions of bound partons, quarks and gluons, are described by parton distribution functions (PDFs). In paper V and VI of this thesis, a physically motivated model for PDFs is presented. This model can reproduce proton structure function data, and gives a possible solution to the proton spin puzzle.

Keywords: QCD, Higgs, Gauge Symmetry, Standard Model, BFKL, PDF, DIS, Beyond the Standard Model, Colliders, Phenomenology, Effective Potential, Anomaly

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List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

I Andreas Ekstedt, Rikard Enberg, Gunnar Ingelman, Johan Löfgren, and Tanumoy Mandal. Constraining minimal anomaly free U(1) extensions of the Standard Model. JHEP, 11:071, 2016.

II Andreas Ekstedt, Rikard Enberg, Gunnar Ingelman, Johan Löfgren, and Tanumoy Mandal. Minimal anomalous U(1) theories and collider phenomenology. JHEP, 02:152, 2018.

III Andreas Ekstedt and Johan Löfgren. On the relationship between gauge dependence and IR divergences in the $\hbar$-expansion of the effective potential. JHEP, 01:226, 2019.

IV Andreas Ekstedt, Rikard Enberg, and Gunnar Ingelman. Hard color singlet BFKL exchange and gaps between jets at the LHC. 2017. [arXiv:1703.10919][hep-ph]

V Andreas Ekstedt, Hazhar Ghaderi, Gunnar Ingelman, and Stefan Leupold. Nucleon parton distributions from hadronic quantum fluctuations. 2018.[arXiv:1807.06589][hep-ph]

VI Andreas Ekstedt, Hazhar Ghaderi, Gunnar Ingelman, and Stefan Leupold. Towards solving the proton spin puzzle. 2018. [arXiv:1808.06631][hep-ph]

Contents

Preface

This thesis is concerned with unanswered questions within elementary particle physics. Our best theory of particle physics, the Standard Model, describes a wide range of observed phenomena. In particular, how matter particles, quarks and leptons, interact through forces. The Standard Model is a quantum field theory, in which particles are described by fields. Although gravity is not part of the Standard Model, it is possible to describe gravity with a quantum field theory at low energies ($\ll 10^{19}$ GeV). However, gravity is much weaker than the other forces and can be neglected in most circumstances.

There are however unsolved problems in particle physics. Some problems, such as how quarks are bound within the proton, are expected to be solvable within the Standard Model. While others, such as why neutrinos are massive, or why there is much more matter than anti-matter in the universe, might require an extension of the Standard Model.

This summary is written with the intent to provide a theoretical background for the included papers, and the text assumes that the reader is familiar with quantum field theory. Chapter one concisely introduces the Standard Model and collider physics, and chapter two discuss the role of symmetries in quantum field theories. Section 2.2.2 is considerably more technical than the rest of the thesis, and summarizes gauge anomalies. The final chapter briefly summarizes the included papers.

1. Introduction to Particle Physics

> "Mathematics catalogues everything that is not self-contradictory; within that vast inventory, physics is an island of structures rich enough to contain their own beholders"
>
> — Greg Egan, Oceanic

Particle physics studies how matter behaves at short distances ($\lesssim 10^{-15}$m). The quantum theory of matter originally assumed that particles, except for the photon, could not be created or destroyed. However, processes such as radioactive decays showed that particles could both be created and destroyed. These, and other, considerations led to the combination of quantum mechanics and special relativity into quantum field theory, in which particles are described by quantum fields.

The Standard Model is a quantum field theory and is our best description of nature at the fundamental level. However, the Standard Model is not complete. Indeed, experiments conducted during the 21th and 20th century indicate problems with the Standard Model. Extensions of the Standard Model try to solve these problems.

1.1 The Standard Model

The Standard Model of particle physics is our most accurate description of nature. For example, measurements of the electron's magnetic moment agree with theoretical predictions to an accuracy greater than one in 10 billion [1]. The Standard Model describes how matter interact via forces. These forces are mediated by particles: photons, $W^{\pm}$ and Z, and gluons, mediate the electromagnetic, weak, and strong force respectively. The gravitational force is extremely weak at low energies and is not part of the Standard Model.

There are nineteen free parameters in the Standard Model, twelve which describe particle masses, and seven that describe interactions. Massive particles—such as quarks, charged leptons, Z, and $W^{\pm}$—get a mass through the

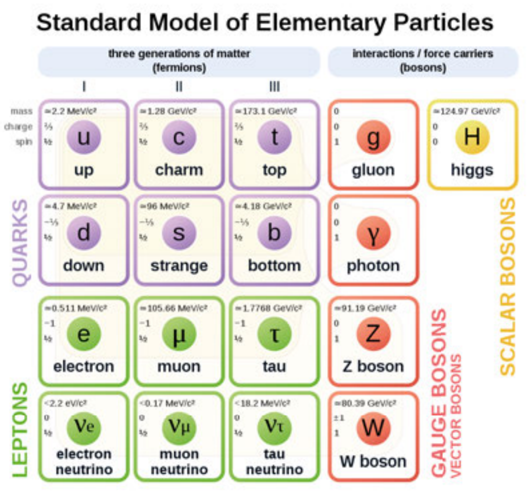


Figure 1.1. The particle content of the Standard Model [3,4]. There are three generations of matter. Each generation consists of an up- and a down-type quark; a charged lepton, and a neutrino. Up-type quarks have an electric charge of $+2/3$, while down-type quarks have an electric charge of $-1/3$. The electromagnetic, weak, and strong force are mediated by the gauge bosons. Massive particles directly interact with the Higgs particle.

Higgs mechanism (see section 2.3.1). Massless particles (photons, gluons, and neutrinos) do not directly interact with the Higgs particle.

Matter particles are divided into quarks and leptons. Quarks interact with the electromagnetic, strong and weak force. There are three families of quarks, and each family consists of an up- and a down-type quark. Up-type quarks (up, charm, and top) have electric charge $q = +\frac{2}{3}$, and down type quarks (down, strange, bottom) have electric charge $q = -\frac{1}{3}$. Quarks have never been observed as free particles, but are instead bound together by the strong force. Hadrons, bound states of quarks, are divided into baryons, mesons, and exotic hadrons. Three quarks bind to form a baryon; a quark and an anti-quark pair form a meson; exotic hadron consists of more than three quarks.

Leptons interact through the electromagnetic and the weak force. There are three families of leptons; each family consists of a charged lepton (electron, muon, and tau) and an electrically neutral neutrino (ν_e, ν_μ, ν_τ). These neutrinos are extremely light, with various bounds indicating a mass smaller than one eV [2].

The number of leptons and quarks is conserved in Standard Model processes. These conserved numbers—defined as the number of leptons minus the number of anti-leptons $L = N_l - N_{\bar{l}}$, and the number of quarks minus the number of anti-quarks (up to a factor of three) $B = \frac{1}{3}\left(N_q - N_{\bar{q}}\right)$—are known as lepton and baryon number.

Gluons mediate the strong force (described by Quantum Chromodynamics), and interact both with themselves, and with quarks. Gluons have not been observed as free particles; instead, gluons bind quarks together to form hadrons. The strength of the strong force decreases at large energies, which is known as asymptotic freedom [5,6]. A consequence of asymptotic freedom is that bound quarks and gluons interact as free particles at large energies ($E \gg 1$GeV).

The weak interaction is mediated by the $W^{\pm}$ and Z particles. Both of these particles are massive. Therefore, low-energy processes can not create Z or W particles; hence suppressing the weak force at low energies. Fermi's theory of beta decay describes this low-energy limit of the weak force [7,8]. The Z particle only interacts with the same type of particles, and can not change flavours; whereas, $W^{\pm}$ can change lepton and quark flavours.

1.2 Beyond the Standard Model

Rarely has there been a theory more successful than the Standard Model. Nevertheless, the Standard Model is not the whole story. For example, neutrinos are not massless, as assumed by the Standard Model, but are massive [9,10]. Furthermore, the universe consists mostly of matter; while the Standard Model predicts a symmetry between matter and antimatter. There are in addition some theoretical curiosities, such as why gravity is roughly 10^{21} times weaker than the other forces. Extensions of the Standard Model, motivated by experiments and theory, try to solve these problems.

1.2.1 Massive neutrinos

Neutrinos do not have masses according to the Standard Model. However, experiments [9,10] have shown that neutrinos have finite masses. The discovery of neutrino oscillations showed that neutrinos have non-zero masses, and that lepton number is not conserved.

Fermions, unlike scalars and vectors, can have two types of masses: a Dirac mass and a Majorana mass. And yet, there are no right-handed neutrinos in the Standard Model, which are required to form a Dirac mass. Neutrinos could get a mass from the Higgs mechanism if right-handed neutrinos were added to the Standard Model. Then again, masses from the Higgs mechanism are proportional to the weak scale (~ 100 GeV) and the strength of the neutrino Higgs interaction Y_ν. A neutrino mass of 1 eV implies that the Yukawa coupling is tiny $Y_\nu \sim 10^{-11}$, which is one million times smaller than the electron's Yukawa coupling. Thus, a Dirac mass does not explain why the neutrinos are light, it only rephrases the problem to ask why Y_ν is small.

The second alternative, a Majorana mass, is more attractive. A small Majorana mass can be generated from the seesaw mechanism [11,12]. The idea (type-I seesaw) is to add right-handed neutrinos that both have Majorana and Dirac masses. The Dirac mass mixes the left- and right-handed neutrinos. The mixing can be removed by rotating the fields, which gives two Majorana mass eigenstates, or particles.

One particle is light, while the other is heavy (see section 3.1). The light neutrino is identified with the Standard Model neutrino. As one particle gets heavier, the other particle becomes lighter. For example, light neutrino masses of the order $\sim$ 1eV are possible if the original Dirac and Majorana masses are of the order $\sim$ 1 MeV, and $\sim$ 1 TeV respectively. Moreover, the seesaw mechanism is natural in models with new forces, such as models with a new neutral vector boson, the Z'. Majorana masses for the right-handed neutrinos can be generated from a heavy Higgs particle—which also gives a mass to the Z' boson. This possibility is investigated in paper I.

1.2.2 Matter-antimatter asymmetry

Why is there more matter than antimatter in the Universe? This question is known as the baryon asymmetry problem [13,14]. A tiny bit of this asymmetry can be explained by the Standard Model, but not enough.

Sakharov formulated three conditions required to explain the asymmetry [15]: baryon number violation, CP violation, and a loss of thermal equilibrium. The first two conditions are necessary for processes to both violate baryon number, and for baryons to interact differently from anti-baryons. The third condition is necessary because an asymmetry generated by a process in thermal equilibrium can be washed out by the inverse process. A loss of thermal equilibrium allows for an asymmetry to be generated, and maintained, after equilibrium is lost.

Electroweak Baryogenesis is one possible explanation of the baryon asymmetry, and is based on the idea is that while the electroweak symmetry is exact at high temperatures, the electroweak symmetry is broken at temperatures close to the weak scale $T \sim 100$ GeV. This is the electroweak phase transition. Thermal equilibrium is lost if the transition from the symmetric to the broken phase occurs through a first-order phase transition. Bubbles would nucleate, and expand, in such a phase transition (similar to boiling water); an asymmetry can be generated in walls of the bubbles.

The effective potential incorporates both thermal and radiative corrections to the classical potential energy (see section 2.3.3), and can be used to describe the electroweak phase transition. However, there are subtle issues with the effective potential even at zero temperature. Reliable predictions require that the radiative corrections are understood and treated consistently. Thermal corrections are much more complicated than zero temperature corrections, and it is important to first understand the zero temperature potential before moving on to the finite temperature potential. Paper III of this thesis studied the effective potential at zero temperature. This paper showed how to perform consistent perturbative calculations for observables.

1.2.3 The hierarchy problem

Gravity is much weaker than the other forces. The difference in strength is related to that gravity is associated with the Planck scale ($\sim 10^{19}$GeV), and the other forces are associated with the weak scale (~ 100 GeV). The Higgs boson should naively have a mass close to the Planck scale, or any other scale of new physics. Parameters must be tuned for the Higgs particle, and by extension other Standard Model particles, to be light.

This tuning can be understood from a model with two interacting scalar particles [16]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 + \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2}M^2\Phi^2 - \kappa\frac{1}{4}\phi^2\Phi^2 - \frac{1}{4!}\eta\phi^4.$$

The "light" scalar, ϕ, has mass m, and the "heavy" scalar, Φ, has mass M. We will assume that Φ decouples from the theory at low energies $k^2 \ll M^2$; similar to how the Z can not be excited for low energies $k^2 \ll M_Z^2$. The light scalar ϕ would also decouple unless the mass is of the same order, or smaller, than the low-energy scale k^2. Hence, we will assume that the light scalar does not decouple.

The light scalar plays the role of the Higgs boson, and the heavy scalar is a heavy particle that talks with the Higgs. The theory does not contain the heavy scalar at low energies, and the low-energy theory can be described by an effective Lagrangian

$$\mathcal{L}_{EFT} = \frac{1}{2}Z_\phi(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m_L^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \dots.$$

This low-energy Lagrangian should describe the same physics as the full theory (in low-energy domain); thus, coefficients of the effective theory must be chosen to give the same result as the full theory. For example, leading order scattering processes imply $\lambda = \eta$, $m_L^2 = m^2$ and $Z_\phi = 1$. Higher order corrections come from loops. Matching the light scalar propagator in both energy regimes gives (up to sub-leading terms) [16]

$$m_L^2 = m^2 - \kappa\frac{1}{32\pi^2}M^2. \tag{1.1}$$

By assumption $m_L \ll M$—which means that $m \sim M$. The behaviour shows that the light scalar "wants" to be heavy; that is, the light scalar can only have a light mass by tuning the original masses and couplings.

This example shows a generic feature of theories with scalar particles—it is unnatural for the Higgs particle to be light if it interacts with heavy particles. However, the Higgs particle can be light without tuning. One of the main motivations for beyond the Standard Model extensions is to explain why the Higgs boson is light.

1.3 Quantum Chromodynamics

Quantum Chromodynamics (QCD) is a quantum field theory that describes how quarks and gluons interact via the strong force. Whereas partons, quarks and gluons, are fundamental particles—hadrons are not. Nevertheless, partons have not been observed as free particles; instead, produced partons form jets. These jets are created by a combination of high- and low-energy processes. Low-<energy processes can not be reliably calculated in QCD by perturbative methods. Despite this, much of the the structure of jets can be understood from perturbative processes, involving quarks and gluons.

Collisions in hadron colliders are different from collisions in lepton colliders. Thus, different kinematic variables are needed. Jets are defined by their size and their mass. Using solid angles to measure jet sizes is problematic because jets closer to beam axis are more contaminated from background processes; whereas, jet sizes defined by rapidity are less sensitive to contamination.

Hadrons consists of bound partons, but high-energy processes occur on short enough time-scales for partons to behave as free particles. Bound partons can, at high energies, be described as free partons sharing the hadron's momentum. The distributions of parton momenta, parton distribution functions, can not be calculated perturbatively, but must be modelled.

1.3.1 Jet formation

High-energy processes can be described by perturbative methods. Partons produced in these collisions end up forming jets. Partons can, before a jet is formed, radiate and split into gluons and quarks. The probability, in the soft-collinear limit, for a parton to emit a gluon is [17]

$$dP_{i\to ig} = \frac{2\alpha_s}{\pi} C_i \frac{dz}{z} \frac{d\theta}{\theta}. \tag{1.2}$$

The strong coupling constant, α_s, times the colour factors C_i ($C_F = \frac{4}{3}$ for quarks and $C_A = 3$ for gluons) determine the likelihood for the splitting. The fraction of the parton's momenta, z, taken by the quark, determines how "soft" the emitted gluon is, while the relative angle θ determines how wide the eventual jet is. Gluons, on average, have more emissions than quarks because $C_A \sim 3C_F$.

A high-energy parton produces, through multiple emissions, a set of final state particles. However, perturbative QCD becomes unreliable for low energies (< 1GeV), and these low energies are described by non-perturbative models. These models describe the processes that create hadrons from quarks and gluons. Hadrons are created between partons that have exchanged colour. For instance, partons that have radiated gluons at wider angles produce wider jets.

1.3.2 Jet Kinematics

Collisions of protons result in many soft processes that cannot be described by perturbative QCD. However, some collisions involve large momentum transfers where partons interact directly. These colliding partons share the hadron's momentum, which means that the center of momentum frame is different for the partons and the hadrons. In particular, the partons center of momentum frame is longitudinal boosted with respect to the hadrons center of momentum frame. It is therefore useful to work with observables that are insensitive to the absolute longitudinal momentum.

Each momentum can be separated into one longitudinal component parallel to the beam axis ($\hat{z}$ direction) and two components transverse to the beam axis ($\hat{x}$, $\hat{y}$ direction)—conveniently done with light-cone coordinates. Light-cone coordinates, for momentum $(p) = (p^0, \vec{p})$, are defined as

$$
\begin{aligned}
p^{\pm} &= p^0 \pm p^3, \\
p \cdot q &= \frac{1}{2}\left(p^+ q^- + p^- q^+\right) - \vec{p}_\perp \cdot \vec{q}_\perp.
\end{aligned}
$$

Momentum components in the transverse direction can be described by an angle and the absolute value of the transverse momentum

$$\vec{p}_\perp = p_T \cos\phi + p_T \sin\phi.$$

Consider a longitudinal boost

$$
\begin{aligned}
p^0 &\to p^0 \cosh\xi + p^3 \sinh\xi, \\
p^3 &\to p^0 \sinh\xi - p^3 \cosh\xi,
\end{aligned}
$$

in which the light-cone components transform as $p^{\pm} \to e^{\pm\xi} p^{\pm}$. This transformation can be rephrased by defining rapidity: $y = \frac{1}{2} \log \frac{p^+}{p^-}$,

$$p^{\pm} = E_T e^{\pm y}, \tag{1.3}$$

$$E_T^2 \equiv (p^0)^2 - (p^3)^2. \tag{1.4}$$

Rapidity transforms linearly for longitudinal boosts $y \to y + \xi$; hence rapidity differences are frame invariant. The related quantity of pseudorapidity is defined as $\eta = -\log\left[\tan \frac{\theta}{2}\right]$ [18], where θ is the angle between the beam axis and $\vec{p}$ ($\vec{p} \cdot \hat{z} = |\vec{p}| \cos\theta$). Rapidity and pseudorapidity are equal for massless particles and will be used interchangeably. For example, a rapidity of $y = 0$ is perpendicular to the beam axis, and a rapidity of $y \sim 7$ is almost parallel with the beam axis ($\theta \approx 0.1^o$).

Defining ΔR, a boost invariant distance, as

$$\Delta R \equiv \sqrt{(\Delta y)^2 + (\Delta\phi)^2}, \tag{1.5}$$

gives a natural size measure. Jets would naively be defined by the opening angle $\Delta\Omega = \sqrt{(\Delta\theta)^2 + \sin^2\theta(\Delta\phi)^2}$, which is not boost invariant. However, opening angles are related to ΔR as

$$(\Delta\Omega)^2 = \frac{1}{\cosh^2 y}(\Delta R)^2. \tag{1.6}$$

At the central part of the detector ($y = 0$), solid angles are equivalent to ΔR. Conversely, close to the beam axis the opening angle gets squeezed. That is, jets that are circular in the central directions are still circular—but have a smaller opening angle—close to the beam axis. Using normal opening angles to define jet sizes disproportionally contaminates (for a given jet size) jets close to the beam axis. In contrast, the QCD background is roughly uniformly distributed in rapidity [19,20]. As a consequence, jets with a given size ΔR are evenly contaminated. Moreover, ΔR is longitudinally boost invariant, the opening angle $\Delta\Omega$ is not. The size of a jet is therefore defined by ΔR.

1.3.3 Parton Distribution Functions

Say that we are interested in a collision of n particles, produced by colliding two (massless) partons $q_i + q_j \to 1 + 2 + \ldots + n$; the cross section is [21]

$$d\sigma_{ij} = \frac{|\mathcal{M}(ij \to 1, \ldots, n)|^2}{2s_{ij}}$$
$$\times (2\pi)^4 \delta^4 \left[q_i + q_j - (p_1 + \ldots + p_n) \right] \Pi_{a=1}^{n} \int \frac{d^3 p_a}{(2\pi)^3 E_{p_a}}.$$

This is the familiar cross section formula; everything is weighted by the probability amplitude $|\mathcal{M}|^2$, and there is a phase space integral for each particle—with a delta function enforcing momentum conservation.

Partons are not free particles, but are bound within hadrons. Therefore, each parton only takes a fraction of the hadron's momentum. The distribution of partons' momenta is described by Parton Distribution Functions (PDFs). The PDFs, $f_i(x,Q)dx$, are defined as the probability for a parton i to carry a fraction $[x, x+dx]$ of the hadron's momentum, and having transverse momentum less than Q. In addition, quarks do not carry all of a hadron's momentum. Indeed, gluons roughly carry half of the protons momentum.

Whereas parton collisions are weakly coupled at high energies, PDFs come from strongly coupled systems. Nevertheless, the non-perturbative physics that describes the PDFs factorize from the perturbative physics describing parton scattering. The cross-section for two colliding hadrons P_1, P_2 can be written as an integral of the partonic cross sections weighted by the PDFs [21]

$$d\sigma = \int_0^1 dx_i dx_j f_i(x_i, Q) f_j(x_j, Q) d\sigma_{ij}. \tag{1.7}$$

The strategy is to measure the PDFs at a scale Q_0^2, and to use this measurement to calculate other processes. This strategy would be cumbersome if a new measurement was required for each Q^2. Instead, PDFs are measured at a starting scale Q_0^2 and then evolved to a larger Q^2. This evolution is described by perturbative QCD through the Dokshitzer–Gribov–Lipatov–Altarelli –Parisi (DGLAP) equations [22–24]:

$$\frac{d}{d\log Q} f_i(x,Q) = \frac{\alpha_s(Q^2)}{\pi} \int_0^1 dz dy \delta(zy - x) \sum_j P_{ij}(z) f_j(y,Q). \tag{1.8}$$

Protons consist of two up quarks and one down quark, which are expected to each carry a third of the proton's momentum. For example, figure 1.2 shows the Q^2 evolution of proton PDFs, and the PDFs for up- and down-type quarks are, as expected, peaked at $x \sim \frac{1}{3}$ at the low $Q^2 = 0.77$ GeV scale.

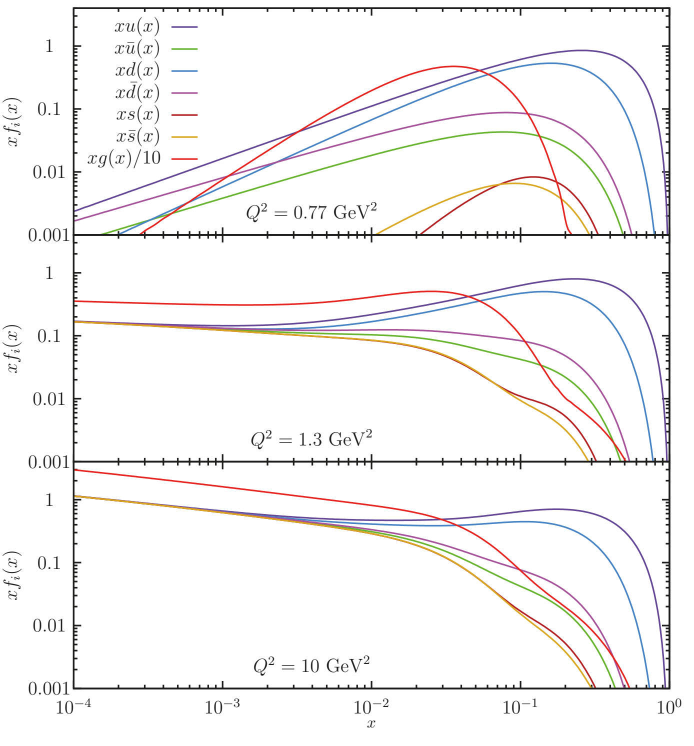


Figure 1.2. Proton PDFs for three different Q^2 values. Distributions are weighted with x. The figure is taken from paper V.

Whereas at higher Q^2, distributions grow larger at smaller x. This growth is due to quarks emitting gluons, and gluons splitting into quark anti-quark pairs; the splitting gives each parton a lower energy, and distributions are therefore pushed to smaller x.

To understand the DGLAP equation, consider a quark radiating a gluon. Assume that the radiated gluon ends up having a fraction x of the hadron's momentum, by taking a fraction z of the initial quark's momentum. Thus, the initial quark (j) must have started with a fraction $y = \frac{x}{z}$ of the hadron's momentum. Increasing the transverse momentum scale from Q to $Q + \Delta Q$ allows the quark to radiate a gluon with probability

$$\frac{\Delta Q}{Q} \frac{\alpha_s(Q^2)}{\pi} \int_x^1 \frac{dz}{z} P_{qg}(z) f_q\left(\frac{x}{z}\right),$$

which gives a new gluon distribution

$$f_g(x, Q) + \frac{\Delta Q}{Q} \frac{\alpha_s(Q^2)}{\pi} \int_x^1 \frac{dz}{z} P_{qg}(z) f_q\left(\frac{x}{z}\right).$$

Evolving to higher Q^2 pushes all distributions to lower x—the emitted gluon always takes a fraction of the quarks momentum. The splitting functions $P_{ij}(z)$ have been calculated in perturbative QCD, and are currently known to NNLO (next-to-next-to-leading order) accuracy [25,26].

2. Symmetries

> "You take another step forward and here I am again, like your own reflection in a hall of mirrors."
>
> — Walton Simons

Particles are described by quantum field theories—which combines quantum mechanics and special relativity. Each quantum field theory is defined by an action. This action describes classical dynamics and is the starting point for the quantum theory. Symmetries of the quantum theory are described by the effective potential. A classical symmetry is not guaranteed to be a symmetry of the quantum theory; indeed, symmetries can be broken by quantum effects, known as anomalies. Anomalies serve as consistency checks, and are useful to describe low-energy dynamics.

2.1 Quantum Field Theory

Particles can be thought of as excitations of a field, in the same way as sound is a wave in air. Fields have a value at every point in space-time. Analogously, quantum fields have quantum operators at every point in space-time. A quantum field theory is defined by an action. Observables are calculated from the action with the help of the path integral. Particles such as vectors (spin-1 particles) can only be described by a gauge invariant quantum field theory.

2.1.1 Fields

Particle creation is natural in a field description; a field can be converted to another field, similar to how a temperature gradient can create a wind. Particles are classified by their spins and masses. Particles with different spin behave differently under Poincaré transformations and are described by different types of fields. Fields are classified by how they transform under Lorentz transformations $x \to \Lambda x$. For example, a scalar field $\Phi(x)$, describing a spin-0 particle, transforms as $\Phi(x) \to \Phi'(x) = \Phi(\Lambda^{-1}x)$. That is, the transformed scalar field, in the new coordinates, is the same as the original field evaluated in the original space-time point. Other fields include spinor fields $\psi^a(x)$ and vector fields $A^\mu(x)$, describing spin-$\frac{1}{2}$ and spin-1 particles respectively.

Particle physics is a theory within the quantum field theoretical framework. Theories are defined by a Lagrangian $\mathcal{L}[\Psi]$, which is built out of fields; together with free parameters describing particle masses and interaction strengths. The action, for a collection of fields $\Psi(x)$, is defined as the Lagrangian integrated over space-time

$$S[\Psi] = \int d^4x \mathcal{L}[\Psi]. \tag{2.1}$$

The classical equations of motion are found by minimizing the action, also known as the principle of least action $\delta S[\phi, \Psi] = 0$. The Euler-Lagrange equations follow from the principle of least action [21,27]

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi(x))} - \frac{\partial \mathcal{L}}{\partial \Psi(x)} = 0. \tag{2.2}$$

2.1.2 Global Symmetries

Theories are easier to work with if they have a high degree of symmetry. Symmetries allow us to understand why electric charge is conserved, and why photons can not interact with themselves. Classical symmetries are not always symmetries of the quantum theory, and vice versa.

Classical symmetries leave the action and the equations of motion invariant. A field transformations $\Psi(x) \to \Psi'(x)$ is a symmetry if $S[\Psi'] = S[\Psi]$.

Continuous symmetries of the form

$$\Psi(x) \to \Psi'(x) = \Psi(x) + \epsilon F[\Psi] \tag{2.3}$$

define conserved charges if the equations of motions are obeyed. The conserved current j^μ—that defines the conserved charge Q—depends on the form of $F[\Psi]$, and can be found from Noether's theorem [27]

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)} F[\Psi], \tag{2.4}$$

$$\partial_\mu j^\mu = 0, \tag{2.5}$$

$$\frac{d}{dt} \int d^3x j^0 = \frac{d}{dt} Q = 0. \tag{2.6}$$

On the other hand, quantum symmetries depend on the action through the path integral. Classical particles follow the path that minimizes the action. Quantum mechanics extends this notion and allows particles to follow any path—not only the path minimizing the action. Moreover, all paths are not equally probable, and each path is weighted by the phase $e^{iS[\Psi]}$. The path that minimizes the action is the most probable path, and the path integral sums all possible paths.

The generating functional is defined as the vacuum-to-vacuum correlator in the presence of an external source J_Ψ,

$$\left\langle 0|e^{-iHT}|0\right\rangle_{J_\Psi} \equiv Z[J_\Psi] = \int D\Psi e^{iS[\Psi]+i\int d^4x J_\Psi(x)\Psi(x)}. \tag{2.7}$$

The generating functional is important for several reasons. First, correlation functions can be calculated from the generating functional:

$$\langle 0|T\Psi(x_1)\Psi(x_2)\ldots|0\rangle = \frac{\delta}{i\delta J_\Psi(x_1)}\frac{\delta}{i\delta J_\Psi(x_2)}\ldots Z[J_\Psi]|_{J_\Psi=0}.$$

Second, symmetries of the classical theory are related, through the path integral, to symmetries of the full quantum theory. These symmetries are not symmetries of the classical action, but of the effective action.

The effective action, a quantum version of the classical action, depends on classical fields, not quantum fields; symmetries of the quantum theory can be understood from the effective action. First, define $W[J]$ as the logarithm of the generating functional,

$$Z[J_\Psi] = e^{iW[J_\Psi]} = \int D\Psi e^{iS[\Psi]+i\int d^4x J_\Psi(x)\Psi(x)}. \tag{2.8}$$

Second, actions depend on fields, not currents; currents can be swapped in favour of fields by a Legendre transformation. That is, define $\psi(x)$ as the expectation value of the quantum field $\Psi(x)$ in the presence of a source $J_\Psi(x)$

$$\psi(x) = \frac{\delta W[J_\Psi]}{\delta J_\Psi} \equiv \langle 0|\Psi(x)|0\rangle_{J_\Psi}. \tag{2.9}$$

The effective action, $\Gamma[\psi]$, is defined as

$$\Gamma[\psi] = W[J_\Psi] - \int d^4x \psi(x) J_\Psi(x), \tag{2.10}$$
$$\frac{\delta\Gamma[\psi]}{\delta\psi(x)} = -J_\Psi(x).$$

By setting the source to zero ($J_\Psi = 0$), the effective action becomes identical to $W[0]$. And we find the quantum version of the principle of least action

$$\frac{\delta\Gamma[\psi]}{\delta\psi(x)} = 0. \tag{2.11}$$

Consider now a symmetry transformation $\Psi(x) \to \Psi'(x) = \Psi(x) + \epsilon F[\Psi]$ which leave the classical action invariant. The generating functional transforms as

$$\int \mathcal{D}\Psi' e^{iS[\Psi'] + \int d^4x \Psi'(x) J_\Psi(x)} = \int D\Psi e^{iS[\Psi] + \int d^4x \Psi(x) J_\Psi(x) + \epsilon \int d^4x F[\Psi] J_\Psi(x)},$$

where it was assumed that the integration measure is invariant under the transformation $\mathcal{D}\Psi' = \mathcal{D}\Psi$. Granted, the integration measure is not always invariant; indeed, chiral fermion measures are not always invariant, as discussed in section 2.2.2. The path integral covers all fields; using Ψ or Ψ' does not matter because all fields are included in the integration. Hence, both the integration over Ψ and the transformed variables Ψ' lead to the same generating functional. That is,

$$\int \mathcal{D}\Psi e^{iS[\Psi] + \int d^4x \Psi(x) J_\Psi(x)} = \int D\Psi e^{iS[\Psi] + \int d^4x \Psi(x) J_\Psi(x) + \epsilon \int d^4x F[\Psi] J_\Psi(x)}.$$

This implies, to first order in the infinitesimal parameter ϵ, that

$$\int d^4x \left(J_\Psi(x) \langle F[\Psi] \rangle_{J_\Psi} \right) = 0. \tag{2.12}$$

Or terms of the effective action

$$\int d^4x \left(\frac{\delta\Gamma[\psi]}{\delta\psi} \langle F[\Psi] \rangle_{J_\Psi} \right) = 0 \tag{2.13}$$

The situations is simpler for linear symmetries ($F[\Psi] = \alpha\Psi$) because $\langle F[\Psi] \rangle = F[\langle \Psi \rangle]$; showing that the effective action is invariant under the transformation

$$\delta_\epsilon \Gamma[\psi] = \int d^4x \frac{\delta\Gamma[\psi]}{\delta\psi(x)} \delta_\epsilon \psi(x) = 0, \qquad (2.14)$$
$$\delta_\epsilon \psi \equiv \epsilon F[\psi].$$

That is, linear symmetries of the classical action are also symmetries of the full quantum theory.

2.1.3 Gauge Symmetries

Particles are the physical degrees of freedom. Massless vector particles, such as the photon, have two degrees of freedom; the spin (projection) can either be parallel or opposite to the momentum. Yet we describe photons with a vector field $A^\mu(x)$, which naively has four degrees of freedom; a Lorentz index can take four values (0,1,2,3). Therefore, a consistent field description of photons requires that some degrees of freedom cancel.

Actually, a vector field that satisfies the equation of motion only has three degrees of freedom. Hence, for on-shell photons, only one degree of freedom needs to be accounted for. This third degree of freedom must be "irrelevant". That is, the extra degree of freedom must cancel in all calculations. To understand why cancellations are needed, consider how a photon's polarization vector (with polarization $\lambda = \pm 1$) changes under a Lorentz transformation [27]

$$\epsilon^\mu_\lambda(p) \to \Lambda^\mu{}_\nu \epsilon^\nu_\lambda(\Lambda p) + \alpha(p, \Lambda) p^\mu. \qquad (2.15)$$

The first term, $\Lambda^\mu{}_\nu \epsilon^\nu_\lambda$, is the usual Lorentz transformation for a vector field. The second term, $\alpha(p, \Lambda) p^\mu$, is an inhomogeneous piece proportional to the photon's momentum p^μ. Scattering amplitudes involving photons, $\epsilon^\mu \mathcal{M}_\mu$, are only Lorentz invariant if the inhomogeneous piece is "irrelevant". That is $p^\mu \mathcal{M}_\mu = 0$, which is known as the Ward–Takahashi identity [28,29]. The requirement $p^\mu \mathcal{M}_\mu = 0$ implies that the photon only couples to conserved currents $p^\mu j_\mu = 0 \longleftrightarrow \partial^\mu j_\mu = 0$.

Similarly, vector fields describing photons must transform in the same way as the polarization vectors, that is

$$A^\mu(x) \to \Lambda^\mu{}_\nu A^\nu(\Lambda^{-1} x) + \partial^\mu \alpha(x).$$

Thus, the Lagrangian must be invariant under the transformation

$$A^\mu(x) \to A^\mu(x) + \partial^\mu \alpha(x).$$

This symmetry is a gauge symmetry, a local symmetry.

To understand how the degrees of freedom cancel, consider how this works in Quantum Electrodynamics (QED). The gauge transformation is

$$A^{\mu}(x) \to A'^{\mu}(x) = A^{\mu}(x) + \partial^{\mu}\alpha(x). \tag{2.16}$$

The gauge symmetry removes one degree of freedom from $A^{\mu}(x)$, for example by setting $\vec{\nabla}\cdot\vec{A}(x) = 0$. All fields are still off-shell, virtual, at this point, and gauge transformations satisfying $\nabla^2\alpha(x) = 0$ maintain $\vec{\nabla}\cdot\vec{A}(x) = 0$. Real on-shell photons obey Maxwell's equations

$$\begin{aligned} &\nabla^2 A^0(x) = 0, \\ &\nabla^2\vec{A}(x) - \partial_t^2\vec{A}(x) = 0. \end{aligned}$$

Another degree of freedom can be removed for on-shell photons. A gauge transformation, $A^0 \to A^0 + \partial^0\alpha(x) = 0$ is possible if $\nabla^2 A^0(x) = 0$—because $\nabla^2\alpha = 0$. Hence, virtual photons have three degrees of freedom, and real photons have two degrees of freedom.

The Lagrangian for photons interacting with (fermion) fields, $\Psi(x)$, is [21,30]

$$\begin{aligned} &\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\Psi}\not{D}\Psi, \\ &F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,\ D_\mu = \partial_\mu\Psi + iq_\Psi A_\mu\Psi, \end{aligned} \tag{2.17}$$

which is invariant under the $U(1)$ gauge transformations

$$\begin{aligned} &A_\mu \to A_\mu + \partial_\mu\epsilon, \\ &\Psi \to e^{-iq_\Psi\epsilon}\Psi. \end{aligned} \tag{2.18}$$

Whereas photons can be described by gauge theories, they can not interact with themselves—photons are not electrically charged. Vector boson self-interactions violate Bose-Fermi statistics, and unitarity, unless the theory has a non-abelian gauge symmetry [31]. Non-abelian gauge theories allow for vector particles to self-interact. A non-abelian vector field, A_μ, is a collection of fields defined as $A_\mu = A_\mu^a T^a$. Generators, a set of matrices T^a, span a semi-simple group $\mathcal{G}$. This group is an extension of the $U(1)$ phase transformation $\Psi \to e^{-\epsilon q_\Psi}\Psi$. That is, a non-abelian gauge transformation is $\Psi \to T(\omega)_\Psi\Psi$, and fields can transform in different representations, $T(\omega)_\Psi$. These representations are analogous to different electric charges $\Psi \to e^{-iq_\Psi}\Psi$, but mix different fields instead of only changing the phase.

Particles such as gluons can be described by a non-abelian (with group $\mathcal{G}$) Lagrangian

$$\begin{aligned}
&\mathcal{L} = -\frac{\mathrm{Tr}\left[F^{\mu\nu}F_{\mu\nu}\right]}{4} + \mathrm{Tr}\left[\overline{\Psi}(\not{D}\Psi)\right], \\
&F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu], \\
&\not{D}\Psi = \not{\partial}\Psi + g T_\Psi(\not{A})\Psi.
\end{aligned} \tag{2.19}$$

This Lagrangian is invariant under the gauge transformations

$$\begin{aligned}
&\Psi \to T_\Psi(\omega)\Psi, \\
&A_\mu \to \omega A_\mu \omega^{-1} + \omega\partial_\mu\omega^{-1}, \\
&\omega \in \mathcal{G}.
\end{aligned} \tag{2.20}$$

The coupling constant g is equivalent to the electric charge. Vector interactions are proportional to $[T^a, T^b] = i f^{abc}$. For example, the three gluon interaction is [21,30]

$$\begin{aligned}
\left\langle 0|A^a_\mu(k)A^b_\nu(p)A^c_\rho(q)|0\right\rangle =& g f^{abc}\left[g^{\mu\nu}(k-p)^\rho\right. \\
& \left. g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\right].
\end{aligned}$$

2.2 Anomalies

Not all classical symmetries are symmetries of the quantum theory. It is possible for symmetries to be broken by quantum corrections; these symmetries are anomalous. Massless vectors can only be described by a gauge invariant theory. An anomalous gauge symmetry implies that the theory is inconsistent. Nevertheless, anomalous global symmetries are useful, for example in describing how the neutral pion interacts with photons.

2.2.1 Anomalous Symmetries

Some symmetries are not "true" symmetries of the full theory. Radiative corrections can break a classical symmetry. A symmetry that is broken by quantum corrections is called an *anomaly*. There is no problem if global symmetries are broken by radiative corrections. However, anomalous local symmetries are inconsistent.

To understand how anomalies arise, consider again how the generating functional transforms under a continuous symmetry $\Psi(x) \to \Psi'(x) = \Psi(x) + \epsilon F[\Psi(x)]$. Anomalies arise if the integral measure is not invariant under the symmetry transformation: $\mathcal{D}\Psi' = e^{i\epsilon\int d^4x\mathcal{A}(x)}\mathcal{D}\Psi$. Following the same steps as in section 2.1.2, the effective action transforms as

$$\delta_\epsilon\Gamma[\psi] = \int d^4x \frac{\delta\Gamma[\psi]}{\delta\psi(x)}\delta_\epsilon\psi(x) = \epsilon\int d^4x\mathcal{A}(x), \tag{2.21}$$

where $\mathcal{A}$ is the anomaly. That is, the quantum theory is not invariant under the original symmetry.

In addition, symmetries of the effective action need not be symmetries of the classical action. Indeed, if the classical action is not invariant under the symmetry, $S[\Psi'] = S[\Psi] + \epsilon\int d^4xB(x)$, it is still possible for the full theory to be invariant if $B(x) = -\mathcal{A}(x)$. Only the effective action has to be invariant.

2.2.2 Anomaly calculations

Not all theories have anomalous symmetries, only theories with chiral fermions can be anomalous. A fermion can either be left- or right-handed. The left- and right-handed components do not have to transform in the same way. That is, there can be chiral transformations.

Consider a set of fermions, χ, described by the Lagrangian

$$\mathcal{L}[\Psi,\chi] = i\bar{\chi}_L \not{D}\chi_L + i\bar{\chi}_R\not{D}\chi_R + \dots$$

Performing the path integral only over the fermions gives [30,32,33]

$$\int \mathcal{D}\Psi\mathcal{D}\chi_L\mathcal{D}\chi_R e^{iS[\Psi,\chi]} = \int \mathcal{D}\Psi e^{i\tilde{S}[\Psi]}e^{i(\Phi_R[\Psi]-\Phi_L[\Psi])}$$

The new action, $e^{i\tilde{S}[\Psi]}$, is gauge invariant, but the phase $e^{i(\Phi_R[\Psi]-\Phi_L[\Psi])}$ does not have to be; the phase factor vanish for fermions with vector couplings.

The remaining section shows how to find general gauge anomalies; however, the details are technical and condensed. The results are summarized at the end of this section.

The procedure to find a general anomaly requires some compact notation. It is convenient to work in Euclidean space and to write gauge fields as one-forms $A \equiv A_\mu dx^\mu$:

$$\begin{aligned} &d \equiv \partial_\mu dx^\mu, d^2 = 0,\\ &F = \frac{1}{2}F_{\mu\nu}dx^\mu dx^\nu = dA + A^2,\ F^2 = \frac{1}{4}\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\rho\sigma},\\ &\delta A = dv + [A,v], \end{aligned} \tag{2.22}$$

where the field strength, F, is written as a two-form. Anomalies, coming from the chiral phase, can be found from the descent equations [30,32,33]. Assume that a chiral fermion is charged under a gauge field
$\mathcal{A} = A_1 \oplus A_2 \oplus + \dots$
First, define the Chern Character

$$\mathrm{ch}(F) = \frac{i}{24\pi^2}\mathrm{STr}\left[F^3\right], \tag{2.23}$$

where the supertrace runs over all fermions with a plus sign for right-handed fermions and a minus sign for left-handed fermions. Second, write the Chern Character as a total derivative acting on a *Chern-Simons form*

$$\mathrm{ch}(F) = \mathrm{d}Q_5. \tag{2.24}$$

The Chern-Simons form is not unique because $(Q_5 + \mathrm{d}\alpha)$ is equivalent to $\mathrm{d}Q_5$ ($\mathrm{d}^2 = 0$). Third, perform a gauge transformation (δA) of the Chern-Simons form, and define Q_4 as

$$\delta Q_5 = \mathrm{d}Q_4. \tag{2.25}$$

The anomaly is given by $\epsilon_a \mathcal{A}^a(x) = Q_4$—together with going to Minkowski space $A \to -iA,\ F \to -iF,\ v \to -i\epsilon$.

Consider two abelian gauge groups, call them $U(1)_x$ and $U(1)_y$. Fermions have the charges q_i^x, q_i^y under these gauge groups, and an arbitrary $U(1)$ charge is denoted by q_i.

The Chern Character is given by

$$\mathrm{ch}(F) = \frac{i}{24\pi^2}\mathrm{STr}\left[F_x^3 + 3F_x^2 F_y + 3F_x F_y^2 + F_z^3\right], \tag{2.26}$$

where the middle terms are known as mixed anomalies. Let us start with a purely abelian anomaly ($F^3 = (\mathrm{d}A)^3$). Chern-Simons terms are of the form $\mathrm{ch}(F) = \mathrm{d}Q_5$, and in this case $Q_5 = \frac{i}{24\pi^2}\mathrm{STr}\left[A\mathrm{d}A^2\right]$.An abelian gauge transformation $\delta A = \mathrm{d}v$ gives $\delta Q_5 = \frac{i}{24\pi^2}\left[\mathrm{d}v\mathrm{d}A^2\right] = \mathrm{d}Q_4$. Or in Minkowski space

$$Q_4 = -\epsilon\frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\rho\sigma}\left[\sum_R q_i^3 - \sum_L q_i^3\right]. \tag{2.27}$$

Now consider the mixed anomalies $F_x F_y^2$. Two possible Chern-Simons forms are

$$Q_5 = 3\frac{i}{24\pi^2}\mathrm{STr}\left[A_x F_y^2\right], \qquad (2.28)$$
$$Q_5 = 3\frac{i}{24\pi^2}\mathrm{STr}\left[F_x A_y F_y\right].$$

Both of these Chern-Simons forms are possible. The choice of Q_5 is arbitrary (up to adding a closed form $\mathrm{d}\alpha$); a common choice is the symmetrical scheme where each gauge field is weighted equal. For example, a symmetrical scheme corresponds to

$$Q_5 = \frac{i}{24\pi^2}\mathrm{STr}\left[A^x (F^y)^2\right] + 2\frac{i}{24\pi^2}\mathrm{STr}\left[F^x A^y F^y\right]. \qquad (2.29)$$

Continuing with the descent, the anomaly is given by

$$Q_4 = -\epsilon_x \frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\rho} F^y_{\mu\nu} F^y_{\rho\sigma}\left[\sum_R q_i^x (q_i^y)^2 - \sum_L q_i^x (q_i^y)^2\right] \qquad (2.30)$$
$$-2\epsilon_y \frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\rho} F^x_{\mu\nu} F^y_{\rho\sigma}\left[\sum_R q_i^x (q_i^y)^2 - \sum_L q_i^x (q_i^y)^2\right].$$

There are also anomalies corresponding to non-abelian gauge fields; these also come as pure and as mixed. Mixed non-abelian anomalies involve two non-abelian gauge fields and one abelian, because the non-abelian generators are traceless ($\mathrm{Tr}[t_a]$).

As an example of a theory with mixed non-abelian anomalies we choose the gauge field $\mathcal{A} = B \oplus A$, where A is an abelian field, and $B \equiv B^a t^a$ is a non-abelian field. The Chern Character (with non-abelian field strength G) is

$$\mathrm{ch}(F) = \frac{i}{24\pi^2}\mathrm{STr}\left[G^3 + 3G^2 F + F^3\right]. \qquad (2.31)$$

The pure abelian anomaly is unchanged, while the Chern-Simons form for the purely non-abelian anomaly is [32]

$$Q_5 = \frac{i}{24\pi^2}\mathrm{STr}\left[B\mathrm{d}B^2 + \frac{3}{2}B^3\mathrm{d}B + \frac{3}{5}B^5\right]. \qquad (2.32)$$

Finally, the anomaly is

$$
\begin{aligned}
Q_4 &= \frac{i}{24\pi^2}\mathrm{STr}\; v\mathrm{d}\left[B\mathrm{d}B + \frac{1}{2}B^3\right] \\
&= -\frac{1}{96\pi^2}\epsilon^a\epsilon^{\mu\nu\sigma\rho}\partial_\mu\left[B^b_\nu\partial_\rho B^c_\sigma + \frac{1}{4}B^b_\nu B^e_\rho B^f_\sigma f^{efc}\right] \\
&\times\left(\sum_R \mathrm{Tr}\left[t^a t^b t^c\right] - \sum_L \mathrm{Tr}\left[t^a t^b t^c\right]\right).
\end{aligned} \tag{2.33}
$$

Again, there is an arbitrariness with the mixed anomaly—the symmetric scheme Chern-Simons form is

$$
Q_5 = 2\frac{i}{24\pi^2}\mathrm{STr}\left[(B\mathrm{d}B + \frac{2}{3}B^3)F\right] + \frac{i}{24\pi^2}\mathrm{STr}\left[G^2 A\right], \tag{2.34}
$$

with corresponding anomalies:

$$
\begin{aligned}
Q_4 = &-2\frac{1}{48\pi^2}\epsilon^a\epsilon^{\mu\nu\sigma\rho}\partial_\mu B^b_\nu F_{\rho\sigma}\left[\sum_R \mathrm{Tr}\left[t^a t^b q_i\right] - \sum_L \mathrm{Tr}\left[t^a t^b q_i\right]\right] \\
&-\frac{1}{96\pi^2}\epsilon\epsilon^{\mu\nu\sigma\rho}G^a_{\mu\nu}G^b_{\rho\sigma}\left[\sum_R \mathrm{Tr}\left[t^a t^b q_i\right] - \sum_L \mathrm{Tr}\left[t^a t^b q_i\right]\right]
\end{aligned} \tag{2.35}
$$

In summary, abelian mixed and pure anomalies—corresponding to $U(1)$ gauge fields x,y—are given by

$$
\mathcal{A}^{\text{abelian,pure}}_{x,y} = -\frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\rho\sigma}\left[\sum_R (q_i^{x,y})^3 - \sum_L (q_i^{x,y})^3\right],
$$

$$
\begin{aligned}
\mathcal{A}^{\text{abelian,mixed}}_{x} = &-\frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\rho}F^y_{\mu\nu}F^y_{\rho\sigma}\left[\sum_R q_i^x(q_i^y)^2 - \sum_L q_i^x(q_i^y)^2\right] \\
&-2\frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\rho}F^x_{\mu\nu}F^y_{\rho\sigma}\left[\sum_R q_i^y(q_i^x)^2 - \sum_L q_i^y(q_i^x)^2\right],
\end{aligned}
$$

and equivalently for the other gauge field y. Pure non-abelian anomalies are given by

$$
\begin{aligned}
\mathcal{A}^{a,\text{non-abelian,pure}} = &-\frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\rho}\partial_\mu\left[B^b_\nu\partial_\rho B^c_\sigma + \frac{1}{4}A^b_\nu B^e_\rho B^f_\sigma f^{efc}\right] \\
&\times\left(\sum_R \mathrm{Tr}\left[t^a t^b t^c\right] - \sum_L \mathrm{Tr}\left[t^a t^b t^c\right]\right).
\end{aligned}
$$

Mixed non-abelian anomalies are given by

$$\mathcal{A}^{a,\text{non-abelian,mixed}} = -2\frac{1}{48\pi^2}\epsilon^{\mu\nu\sigma\rho}\partial_\mu B^b_\nu F_{\rho\sigma}\left[\sum_R \mathrm{Tr}\left[t^a t^b q_i\right] - \sum_L \mathrm{Tr}\left[t^a t^b q_i\right]\right]$$

$$\mathcal{A}^{\text{abelian,mixed}} = -\frac{1}{96\pi^2}\epsilon^{\mu\nu\sigma\rho}G^a_{\mu\nu}G^b_{\rho\sigma}\left[\sum_R \mathrm{Tr}\left[t^a t^b q_i\right] - \sum_L \mathrm{Tr}\left[t^a t^b q_i\right]\right].$$

All the anomalies are presented in the symmetric scheme, and the trace is taken over the appropriate representations of each fermions.

2.2.3 Anomaly matching

Gauge anomalies are proportional to a trace over all chiral fermions. These anomalies, if finite, would result in an inconsistent theory. This is because massless vectors can only be consistently described by fields if the theory is gauge invariant. There are two ways to remove anomalies. First, fermion charges can be chosen such that the supertrace vanishes. Second, anomalies can sometimes be cancelled by using an action that is not gauge invariant. Focusing on the latter option, the idea is that only the effective action needs to be gauge invariant, not the classical action. For example, consider chiral anomalies in QCD. The Lagrangian for (two flavour) QCD is

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + i\bar{q}\not{D}q, \; q = (u,d)^T. \tag{2.36}$$

The *axial* transformation $q \to e^{i\epsilon\gamma^5}q$ is anomalous. Left- and right-handed fermions transform as

$$\delta_\epsilon q_L \to -i\epsilon q_L, \; \delta_\epsilon q_R \to i\epsilon q_R. \tag{2.37}$$

That is, the symmetry is generated by $t_L = -1, \; t_R = 1$. Quarks are in the fundamental representation of $SU(3)_c$, and generators are normalized as $\mathrm{Tr}\left[t^a t^b\right] = \frac{1}{2}\delta^{ab}$. The anomaly is, according to section 2.2.2, given by

$$\mathcal{A}(x)^{\text{axial}} = -\frac{N_c N_f}{32\pi^2}\epsilon^{\mu\nu\sigma\rho}F^a_{\mu\nu}F^a_{\rho\sigma}, \tag{2.38}$$

where N_f is the number of coloured fermions.

Other QCD symmetries are anomaly free; however, there are additional anomalies in the presence of electromagnetism. A chiral symmetry—under which the quarks transformed as $q_L \to U_L q_L, \; q_R \to U_R q_R$—breaks down to the diagonal subgroup $SU(2)_V$ when quarks are confined into hadrons.

That is, the vacuum condensate [30]

$$\Lambda^3 \equiv \langle 0|\bar{q_L} q_R + \bar{q_R} q_L|0\rangle \to \left\langle 0|\bar{q_L} U_L^\dagger U_R q_R + \bar{q_R} U_R^\dagger U_L q_L|0\right\rangle, \tag{2.39}$$

is only invariant if $U_L = U_R$. Pions are the Goldstone bosons of this symmetry breaking. The neutral pion is related to the chiral symmetry ($\delta_\epsilon \pi^0 = \epsilon f_\pi$)

$$\delta_\epsilon u_L \to -i\epsilon u_L,\ \delta_\epsilon u_R = i\epsilon u_R, \tag{2.40}$$

$$\delta_\epsilon d_L \to i\epsilon d_L,\ \delta_\epsilon d_R = -i\epsilon d_R. \tag{2.41}$$

Call this symmetry $U(1)_{\pi^0}$. This symmetry is not anomalous with respect to QCD because contributions from u and d have opposite signs, and cancel in the super-trace $\mathrm{STr}\left[t^a t^b t_{\pi^0}\right]$. Nevertheless, there is a mixed anomaly between $U(1)_{\pi^0}$ and electromagnetism [21,30]

$$S\mathrm{Tr}\left[q_{em}^2 t_{\pi^0}\right] = -2N_c\left[\left(\frac{2e}{3}\right)^2 - \left(\frac{-1e}{3}\right)^2\right] = -\frac{2N_c e^2}{3}, \tag{2.42}$$

which corresponds to the anomaly

$$\mathcal{A}(x)^{\pi^0} = -\frac{N_c e^2}{48\pi^2}\epsilon^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\mu\nu}. \tag{2.43}$$

Note, this anomaly is not given in the symmetric scheme, meaning that there is only an $U(1)_{\pi^0}$ anomaly, not an electromagnetic anomaly

In the end, electromagnetism breaks the chiral symmetry anyway, why does it matter that the symmetry is anomalous? There is a lovely trick due to t'Hooft [34] that can be used to derive how the neutral pion couples to photons. The idea is to do a thought experiment and pretend that the $U(1)_\pi^0$ symmetry is a real gauge symmetry. If this was the case, the theory would be inconsistent—because unphysical degrees of freedom would no longer cancel. One solution to this problem is to add "heavy" fermions with quantum numbers chosen to cancel the anomaly. There would be no anomaly and everything would be consistent.

After the quarks are confined, during the strong phase transition, they are no longer part of the spectrum, and can no longer contribute to the anomaly. Yet, the heavy fermions are still sticking around and are responsible for an anomaly $\mathcal{A}^{H,\pi^0} = -\mathcal{A}(x)^{\pi^0}$. It appears that the theory is inconsistent in the confined phase. However, a theory can not become inconsistent due to a phase transition. Thus, the anomaly in the confined theory must be compensated by one of the new mesons. Recall that π^0 transforms as $\delta_\epsilon \pi^0(x) = \epsilon f_\pi$ under the $U(1)_{\pi^0}$ symmetry. Contributions from the heavy fermions can be compensated by a term

$$\mathcal{L} \supset -\frac{N_c e^2 \pi^0(x)}{48\pi^2 f_\pi} \epsilon^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\mu\nu}. \tag{2.44}$$

Remarkably, this term gives the correct pion decay rate [30] $\Gamma(\pi \to \gamma\gamma) = \frac{N_c^2 \alpha^2 m_\pi^3}{144\pi^3 f_\pi^2}$, in agreement with experiment.

Similarly, some gauge anomalies can be cancelled if there is a field that transforms non-linearly ($\delta_\epsilon \Psi = \Psi + \epsilon f \Psi$) under the symmetry. This idea was used in the eighties, by Michael Green and John Schwartz, to cancel anomalies in string theory [35]. Also, this method is used in paper II to cancel anomalies in a Standard Model extension.

2.3 Spontaneous breaking of symmetries

> "Perhaps you were expecting some surprise, for me to reveal a secret that had eluded you, something that would change your perspective of events, shatter you to your core. There is no great revelation, no great secret. There is only you."
>
> — Kreia

We saw in section 2.1.3 that photons had to be described by a gauge invariant theory. Indeed, photons have two degrees of freedom, while (on-shell) vector fields have three. A gauge symmetric theory ensures that the leftover degrees of freedom cancel. Massive vectors, for example Z bosons, on the other hand, already have three degrees of freedom—the degrees of freedom already match up. Hence, it might be surprising that theories with massive vector particles face difficulties. These difficulties occur at high energies—high enough for the Z mass to be negligible.

Massive vectors have three possible polarizations, or degrees of freedom: two transverse, and one longitudinal. The two transverse polarizations are, at high energies, equivalent to a massless vector particle. The longitudinal polarization, on the other hand, has nowhere to go. There is, therefore, a mismatch between low- and high-energy degrees of freedom. Furthermore, the amplitude for longitudinal Z scattering blows up for energies much large than the Z mass [27]

$$\epsilon_L^\mu(p)\mathcal{M}_\mu \underset{\frac{M_Z}{E}\to\infty}{\longrightarrow} \frac{p^\mu}{M_Z}\mathcal{M}_\mu + \mathcal{O}\left(\frac{M^2}{E^2}\right). \tag{2.45}$$

However, this problem disappears if Z couples to a conserved current, $p^\mu \mathcal{M}_\mu = 0$. That is, if the theory is gauge invariant at high energies.

Actually, the problem is even worse. The Z interacts with the $W^\pm$ particles; at high energies these particles behave as massless vectors. As mentioned in section 2.2.2, massless vectors can only interact with themselves if the theory has a non-abelian gauge symmetry. A field theory that describes Z and $W^\pm$ particles must have a non-abelian gauge symmetry at high energies.

Nevertheless, massive vectors break the gauge symmetry. Thus, massive vectors can only be described by a field theory if the gauge symmetry is softly broken, meaning that the gauge symmetry is exact at high energies. The Higgs mechanism is one such soft breaking. Additionally, the Higgs mechanism replaces the longitudinal components of massive vectors by scalar particles at high energies; thus, correctly matching low and high-energy degree of freedom. Massive vectors have three degrees of freedom at low energies—which separate into a massless vector and a scalar particle at high energies.

2.3.1 The Higgs mechanism

The ground state does not have to share the symmetries of the action. As an example consider a rock, in this case a large irregular rock; the laws governing the motion of a falling rock are rotationally symmetric, but the rock itself is not rotationally symmetric. The rock breaks the rotational symmetry of the system. The symmetry is spontaneously broken by the initial conditions. Similarly, the ground state energy does not have to be gauge invariant—only the action must be gauge invariant.

Particles are represented by fields; specifically by excitations of fields. These excitations are created by disturbing the vacuum. The vacuum energy must be a local minimum—excitations around a local maximum are unstable. Hence, all fields should be expanded around their vacuum expectation values. Which fields are allowed to have a finite vacuum expectation value? Spinor and vector fields have a non-trivial transformation under the Lorentz group; a finite vacuum breaks Lorentz symmetry. In particular, the vacuum would have a preferred direction in space-time.

Scalars, on the other hand, can have finite vacuum expectation without breaking Lorentz invariance, and are therefore used to break gauge symmetries, without breaking the Lorentz symmetry. The idea is that particles get a mass from the finite vacuum expectation value, that is from their interactions with the scalars.

As an example, consider scalar QED with a complex scalar and a massless vector, described by the Lagrangian

$$\begin{aligned} \mathcal{L} &= (\mathcal{D}_\mu \phi)^\dagger \mathcal{D}^\mu \phi - V(\phi, \phi^\dagger), \\ \mathcal{D}_\mu \phi &= \partial_\mu \phi(x) - ieA_\mu(x)\phi(x), \\ V(\phi, \phi^\dagger) &= \frac{\lambda}{4}(|\phi|^2 - v^2)^2,\ v^2 = \frac{4\mu^2}{\lambda}. \end{aligned} \tag{2.46}$$

This Lagrangian is gauge invariant under the transformations

$$\phi(x) \to \phi(x)e^{-i\alpha(x)},\ A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x). \tag{2.47}$$

The ground state, or vacuum, is a minimum of the energy—which at tree-level is [36]

$$\begin{aligned} E = \int d^3x \Big[&\frac{1}{4}F_{ij}F_{ij} + \frac{1}{2}F_{0i}F_{0i} + \frac{1}{2}(\mathcal{D}_0\phi)^\dagger (\mathcal{D}_0\phi) \\ &+ \frac{1}{2}(\mathcal{D}_i\phi)^\dagger (\mathcal{D}_i\phi) + V(\phi, \phi^\dagger) \Big]. \end{aligned} \tag{2.48}$$

Kinetic terms are minimized when $A_\mu = 0,\ \partial_\mu \phi = 0$, or when the gauge field is pure gauge $A_\mu = \partial_\mu \epsilon$. The vacuum is determined by the minimum of the potential, $V(\Phi)$. Hence, the scalar vacuum expectation value, $\langle \phi\phi^\dagger \rangle = v^2$, is a circle of radius v (in ϕ space).

As mentioned, scalar fields should be expanded around the vacuum expectation value $\langle \phi\phi^\dagger \rangle = v^2$, not around $\langle \phi \rangle = 0$. For example, by redefining the scalar field: $\phi = \frac{1}{\sqrt{2}}(h(x) + v + i\chi(x))$. Interactions with the scalar field, ϕ, generate a mass for one scalar, the Higgs $h(x)$, and for the "photon" $A_\mu(x)$.

$$\mathcal{L} \supset \frac{e^2 v^2}{2} A_\mu(x) A^\mu(x) + \frac{\lambda v^2}{4} h^2(x). \tag{2.49}$$

Although the "photon" is massive, the action is gauge invariant—but the vacuum is not. Indeed, the Lagrangian is invariant under the transformations

$$h(x)+v \to \alpha(x)\chi(x),\ \ \chi(x) \to -\alpha(x)(h(x)+v), \tag{2.50}$$
$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x).$$

This form of the gauge symmetry is hidden at low energies, but becomes apparent at large momenta $k^2 \gg v^2$. Furthermore, high-energy scattering of the "photons" longitudinal component is equivalent to scattering of χ; formalized by the Goldstone equivalence theorem [21].

2.3.2 The Standard Model

The Standard Model describes self-interacting $W^\pm$ and Z bosons, and must be based on a gauge symmetry. This gauge group is

$$\mathcal{G}_{\text{SM}} = \underbrace{SU(3)_c}_{\text{QCD}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Electroweak}}. \tag{2.51}$$

There are two electroweak fields, $W_\mu \equiv W_\mu^a \frac{\tau^a}{2}$ for $SU(2)_L$, and B_μ for $U(1)_Y$. The gauge group $SU(2)_L$ is the group of two-by-two unitary matrices that only rotate left-handed fields. For example, the gauge fields can be written in matrix form as

$$W_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}. \tag{2.52}$$

And B^μ is the gauge field for the hypercharge transformation: $U(1)_Y$. Left- and right-handed fermions have a different charge under this group. The standard model is a chiral theory.

Massive vectors, $W^\pm$ and Z, get their masses through the Higgs mechanism. The vacuum expectation value of the Higgs field, ϕ, is not invariant under the full electroweak group, but only under the electromagnetic subgroup. That is, the electroweak symmetry is broken down to electromagnetism by the Higgs' vacuum expectation value. This Higgs field is an $SU(2)_L$ doublet with hypercharge $Y_H = 1$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}.$$

The Higgs vacuum expectation value, $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & v \end{pmatrix}^T$, is not symmetric under the full $SU(2)_L \times U(1)_Y$ symmetry. The vacuum expectation value is only invariant under a symmetry transformation $U = \exp(iT^a\alpha(x))$ if

$U\langle\phi\rangle = \langle\phi\rangle$. In particular, unbroken generators annihilate the vacuum expectation value $T^a\langle\phi\rangle = 0$; these are $\tau^3 + \mathbb{1}$. Massive vector bosons get a mass through their interactions with the Higgs field

$$\mathcal{L}_{\text{Kinetic}} \supset \left|\left(\partial_\mu - igW_\mu - i\frac{g' B_\mu}{2}\right)\phi\right|^2 \sim m_W^2 W_\mu^+ W_\mu^- + \frac{m_Z^2}{2} Z_\mu Z^\mu + \ldots \quad (2.53)$$

The Z and $W^\pm$ particles are linear combinations of the original W^μ and B^μ fields, with masses and eigenstates [21,30]

$$\begin{aligned} m_W &= \frac{gv}{2},\ W^\pm \equiv \frac{1}{\sqrt{2}}\left(W^1 \mp iW^2\right), \\ m_Z &= \frac{v\sqrt{g^2+g'^2}}{2},\ Z_\mu = \cos\theta_w W_\mu^3 - \sin\theta_w B_\mu. \end{aligned} \quad (2.54)$$

The photon eigenstate A_μ, and the Weinberg angle are defined as [21,30]

$$\begin{aligned} \tan\theta_w &= \frac{g'}{g}, \\ A_\mu &= \sin\theta_w W^3 + \cos\theta_w B_\mu. \end{aligned} \quad (2.55)$$

Fermions get a mass from the Yukawa terms [21,30]

$$\mathcal{L}_m \supset -\lambda_d^{ij}\bar{Q}_L^i \phi d_R^j - \lambda_u^{ij}\epsilon^{ab}\bar{u}_{La}^i \phi_b^\dagger u_R^j - \lambda_l^{ij}\bar{L}_L^i \phi l_R^j + \text{h.c}, \quad (2.56)$$

where i, j runs over all families. The Yukawa matrices are generally not diagonal, but can be diagonalized by the redefinitions [21]

$$\begin{aligned} u_R^i &\to W_u^{ij} u_R^j,\ d_R^i \to W_d^{ij} d_R^j,\ l_R^i \to W_l^{ij} l_R^j, \\ u_L^i &\to U_u^{ij} u_L^j,\ d_L^i \to U_d^{ij} d_L^j,\ L_L^i \to U_l^{ij} L_L^j. \end{aligned} \quad (2.57)$$

These transformations diagonalize the Yukawa couplings (except the Goldstone boson couplings), but introduce off-diagonal charged current interactions for the quarks. Charged currents ($\mathcal{L} \supset W^{-,\mu}J_\mu^+ + W^{+,\mu}J_\mu^-$) $J_\mu^+ = \frac{1}{\sqrt{2}}\bar{u}_L^i \gamma_\mu d_L^i \to \frac{1}{\sqrt{2}}\bar{u}_L^i \gamma_\mu (U_u^\dagger U_d)^{ij} d_L^j$ are in general not diagonal. The charged current interactions are parametrized by the CKM matrix $V \equiv U_u^\dagger U_d$. Neutral-currents, and charged lepton currents ($W^+ \to l^+ + \nu_l$), interactions are all diagonal at tree level.

The Standard Model is a chiral symmetry, and as we saw in 2.2.2 this means that there can be gauge anomalies. These anomalies are proportional to the sum over all fermion gauge charges. The fermion super traces for these anomalies are (see Table 3.1 and section 2.2.2)

$$\begin{aligned} SU(2)_L^2U(1)_Y : &\sum_L \mathrm{Tr}\left[t^i t^j Y_i\right] - \sum_R \mathrm{Tr}\left[t^a t^b Y_i\right] \\ &= \frac{\delta^{ij}}{2}\left[(-1) + 3 \times \frac{1}{3}\right] = 0 \end{aligned} \tag{2.58}$$

$$\begin{aligned} SU(3)_c^2U(1)_Y : &\sum_L \mathrm{Tr}\left[t^a t^b Y_i\right] - \sum_R \mathrm{Tr}\left[\tau^a \tau^b Y_i\right] \\ &= \frac{\delta^{ab}}{2}\left[2 \times 1 \times (\frac{1}{3}) - (\frac{4}{3}) - (-\frac{2}{3})\right] = 0 \end{aligned} \tag{2.59}$$

$$\begin{aligned} U(1)_Y^3 : &\sum_L \mathrm{Tr}\left[Y_i^3\right] - \sum_R \mathrm{Tr}\left[Y_i^3\right] \\ &= \left[2 \times 3 \times (\frac{1}{3})^3 + 2 \times (-1)^3 - 3 \times (\frac{4}{3})^3 - 3 \times (\frac{-2}{3})^3 - (-2)^3\right] \\ &= 0 \end{aligned} \tag{2.60}$$

Fortunately, all gauge anomalies cancel in the Standard Model.

2.3.3 Radiative corrections to symmetry breaking

We saw in section 2.3.1 how the Higgs field, by breaking the symmetry, gave mass to vectors and fermions. Yet, section 2.3.1 only used the tree level energy. There are also important radiative corrections to the energy. A quantum corrected version of the Higgs mechanism can be studied with the effective potential.

The Higgs field interacts with other particles as they propagate, and these interactions change the energy of the system. There are, therefore, corrections to the vacuum expectation value. To illustrate how to include quantum corrections to the Higgs mechanism, consider a theory with an unspecified symmetry—which contains a collection of scalar fields Φ, and a set of matter fields Ψ. The Higgs mechanism is based on that fields should be expanded around the true ground state. Call this ground state $\phi \equiv \langle 0|\Phi|0\rangle$. Scalar fields can be expanded around this vacuum expectation value by defining $\Phi = \tilde{\Phi} + \phi$—where the vacuum expectation value, ϕ, is a priori unknown.

To find radiative corrections to the energy, consider the generating functional from section 2.1.2

$$e^{-iTE[J]} = e^{iW[J]} = Z[J] = \int \mathcal{D}\Phi\mathcal{D}\Psi e^{iS[\Phi,\Psi]+i\int d^4xJ(x)\Phi(x)}. \tag{2.61}$$

We only consider an external source, $J(x)$, for the scalars Φ. The vacuum expectation value of Φ is

$$\frac{\delta}{\delta J(x)}W[J]=\frac{\int \mathcal{D}\Phi\mathcal{D}\Psi\ \Phi(x)e^{iS[\Phi,\Psi]+i\int d^4xJ(x)\Phi(x)}}{\int \mathcal{D}\phi\mathcal{D}\Psi e^{iS[\Phi,\Psi]+i\int d^4xJ(x)\Phi(x)}} \tag{2.62}$$
$$=\langle 0|\Phi(x)|0\rangle_J \equiv \phi(x)^J,$$

Next, we want to figure out how the energy depends on $\phi^J(x)$, not $J(x)$. The external source can be swapped for $\phi^J(x)$ by using the Legendre transformation

$$\Gamma[\phi^J]\equiv W[J]-\int d^4xJ(x)\phi^J(x), \tag{2.63}$$
$$\frac{\delta}{\delta\phi^J(x)}\Gamma[\phi^J]=-J(x),$$

where the effective action, $\Gamma[\phi]$, is defined in the same way as in section 2.1.2

To find the static energy, set the external source to zero. The vacuum expectation value $\phi(x)$ (removing all reference to the source) extremizes the effective action

$$\frac{\delta}{\delta\phi(x)}\Gamma[\phi]=0. \tag{2.64}$$

Note that formally the effective action is the same as minus the energy times T (for a vanishing source), and is the quantity that we want. Kinetic terms are minimized if the vacuum expectation value is constant. The effective potential (density) is defined as

$$\Gamma[\phi]\equiv-(VT)\mathcal{V}(\phi),$$

where the space-time volume has been factored out because ϕ is constant throughout space-time. The effective potential is the radiatively corrected potential energy, and the vacuum expectation value extremizes the effective action

$$\frac{\partial}{\partial\phi}\mathcal{V}(\phi)|_{\phi=\phi_m}=0. \tag{2.65}$$

We expect that the leading order contribution to the effective potential is the tree level potential

$$\mathcal{V}(\phi)=V_0(\phi). \tag{2.66}$$

Indeed, neglecting all quantum fluctuations, that is $\Phi \to \phi, \ \Psi \to 0$, gives

$$e^{iS[\phi]} = e^{-i(VT)V_0(\phi)}. \tag{2.67}$$

Higher order contributions can be calculated from vacuum diagrams (shown in figure 2.1).

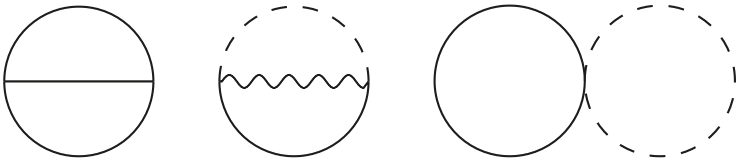


Figure 2.1. Examples of two-loop vacuum diagrams in a theory with scalars and vector bosons.

All fields are described as a collection of harmonic oscillators, and contribute to the energy. If a field interacts with the scalars Φ, the frequency depends on the background field ϕ. The next-to-leading order contribution to the effective potential comes from summing the energies of all these harmonic oscillators [37]

$$V_1(\phi) = \sum_\Psi (-1)^{2s_\Psi}(2s_\Psi + 1)\int d^3k \frac{1}{2}\omega_\Psi(\phi, k), \ \omega_\Psi^2(\phi, k) = \vec{k}^2 + M_\Psi^2(\phi).$$

Note that any particle that does not interact with the Higgs field only contributes a constant piece to the energy—these particles do not change the vacuum expectation value. After the integration is performed, the next-to leading order contribution to the effective potential is

$$V_1(\phi) = \sum_\Psi (-1)^{2s_\Psi}(2s_\Psi + 1)M_\Psi^4(\phi)\left(\log\left(\frac{M_\Psi^2(\phi)}{\mu^2}\right) + \text{const}\right), \tag{2.68}$$

where the constant depends on the particle type and the renormalization scheme.

3. Papers

> "There is no real ending. It's just the place where you stop the story."
>
> — Frank Herbert

Previous chapters have provided a background for the papers making up this thesis; whereas this chapter gives a brief summary of every paper. Reprints of these papers were made with the permission from the publishers, under the Creative Commons Attribution 4.0 International License [38].

3.1 Papers I and II

Extensions of the Standard Model often modify the gauge group. Common examples include grand unified theories and composite Higgs models. These extensions—by their very nature—include new vector particles. Also, it is natural to include right-handed neutrinos in these models. In particular, theories with a (single) neutral vector, Z', are based on the gauge group (see section 2.3.2)

$$\mathcal{G}_{\mathrm{SM+z}} = SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_z. \tag{3.1}$$

This theory contains a new neutral vector particle, the Z' boson. We have not observed a massless vector boson, which means that the Z' particle must be massive. That is, the $U(1)_z$ symmetry must be broken (for instance by a new Higgs field).

3.1.1 A new neutral vector

Similarly to the Z particle, the Z' gets a mass from the Higgs mechanism. This new Z' particle could be accessible at the Large Hadron Collider if the symmetry breaking happens at the TeV scale, that is, if the Z' has a mass around a couple of TeV.

Fermions are, analogously to the hypercharge group $U(1)_Y$, charged under the new gauge group—with charges given in Table 3.1.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_z$
q_L	3	2	1/3	z_q
u_R	3	1	4/3	$z_H + z_q$
d_R	3	1	−2/3	$z_q - z_H$
l_L	1	2	−1	z_l
e_R	1	1	−2	$z_l - z_H$
H	1	2	+1	z_H

Table 3.1. *The charge assignments for the fermions and scalars of the model. Charges are chosen to allow for Standard Model Yukawa couplings.*

Anomaly	Trace	Expression
$[U(1)_z]^3$	$\mathrm{Tr}\left[z^3\right]$	$-z_H^3 - 3z_H z_\ell^2 - z_\ell^3 + 3z_H^2(z_\ell + 6z_q)$
$[U(1)_z]^2\,[U(1)_Y]$	$\mathrm{Tr}\left[Yz^2\right]$	$4z_H(z_\ell + 3z_q)$
$[U(1)_z]\,[U(1)_Y]^2$	$\mathrm{Tr}\left[Y^2 z\right]$	$4(z_\ell + 3z_q)$
$[SU(2)_L]^2\,[U(1)_z]$	$\mathrm{Tr}\left[\left\{t^i, t^j\right\} z\right]$	$(6z_q + 2z_\ell)$
$[SU(3)_C]^2\,[U(1)_z]$	$\mathrm{Tr}\left[\left\{t^a, t^b\right\} z\right]$	0

Table 3.2. *Possible gauge anomalies, together with the corresponding fermion traces. The table is expressed in terms of the generators t^i of $SU(2)_L$, t^a of $SU(3)_c$, Y of $U(1)_Y$, and z of $U(1)_z$. Fermion charges are taken from Table 3.1.*

Moreover, the fermions contribute to a new class of anomalies involving the $U(1)_z$ group. These anomalies can either be removed if the contributions from all fermions cancel (these constraints are given in Table 3.2), or by a method similar to section 2.2.3.

Taking the first approach, and demanding that all anomalies in table 3.2 cancel, severely limits the theory. For example, the only non-trivial solution is where all $U(1)_z$ fermion charges are proportional to the corresponding hypercharge values. It is, therefore, interesting to consider other options. One such option is to introduce right-handed neutrinos. These neutrinos are uncharged under all Standard Model interactions; however, the right-handed neutrinos can be charged under the $U(1)_z$ gauge group. Furthermore, left-handed neutrinos (ν_e, ν_μ, ν_τ) have tiny masses($m_\nu \leq 1\text{eV}$) [2]—which can be explained by the right-handed neutrinos through the seesaw mechanism [11,12].

For example, assume that the Z' particle gets a mass from the same Higgs field that give right-handed neutrinos a Majorana mass, that is $\langle\varphi\rangle = M$. In addition, assume that left- and right-handed neutrinos mix through a Dirac mass, generated by the Standard Model Higgs field (ν_L and ν_R are Weyl spinors):

$$\mathcal{L} \supset -Y_\nu \bar{L}^i_a \epsilon^{ab}_L \phi^\dagger_b \nu_R - \nu_R \nu_R \varphi + \text{h.c} \tag{3.2}$$

This Lagrangian only describes one family of neutrinos, but it is straightforward to include all three families. The left- and right-handed neutrino mix after the electroweak symmetry breaking. Eigenstate masses, for the Weyl spinors, can be found from the mass matrix ($m_D = \frac{Y_\nu v}{\sqrt{2}}$)

$$\mathcal{L} \supset \frac{1}{2}(\nu_L, \nu_R)\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}. \tag{3.3}$$

The Majorana mass, M, is assumed to be much larger than the Dirac mass. This is natural because we have assumed that $M_{Z'} \sim M \gg v$. The mass matrix can be diagonalized, giving two mass eigenstates

$$|m_{\text{light}}| \approx \frac{m_D^2}{M}, \qquad |m_{\text{heavy}}| \approx M. \tag{3.4}$$

The Dirac mass should be of the same order as the other fermion masses. Thus, if we take the Dirac mass to be equal to the electron mass, and assume that $M \approx 1$ TeV, we find $m_{\text{light}} \sim 1$ eV. The seesaw mechanism naturally explains why neutrinos are light, without tuning the parameters.

3.1.2 Anomalous gauge symmetry

Consider now, contrary to the previous section, that there are gauge anomalies. That is, assume that the traces in Table 3.2 are finite. Yet, anomalies can be removed by the same method used in section 2.2.3. For this to work, it is necessary to include a field that transforms non-linearly under the $U(1)_z$ symmetry. Consider the (Stueckelberg) Lagrangian describing a charged pseudoscalar

$$\mathcal{L} \supset \frac{1}{2}\left(\partial^\mu a + M g_z B_z^\mu\right)^2, \tag{3.5}$$

which is invariant under the gauge transformations

$$a(x) \to a(x) + M g_z \epsilon_z(x), \qquad B_z^\mu \to B_z^\mu - \partial^\mu \epsilon_z(x). \tag{3.6}$$

Anomalies can be redefined so that only the $U(1)_z$ symmetry is anomalous (see section 2.2.2). Next, all $U(1)_z$ anomalies can be removed by adding the terms

$$\mathcal{L}_{\text{anomaly}} \supset \frac{a}{96\pi^2 M}\epsilon^{\mu\nu\sigma\rho}\Big(A_{zz}g_z^2 F_z^{\mu\nu}F_z^{\mu\nu} + g_z g' A_{zy} F_z^{\mu\nu}F_y^{\mu\nu} + g'^2 A_{yy}F_y^{\mu\nu}F_y^{\mu\nu}$$
$$+A_{zL}g^2\mathrm{Tr}\left[F_W^{\mu\nu}F_W^{\mu\nu}\right]\Big), \tag{3.7}$$

where the coefficients depend on the $U(1)_z$ fermion charges, and are given in paper II. This kind of anomaly cancellation could come from integrating out heavy fermions, or from other mechanism. However, the origin of the terms is not important. This type of cancellation parametrizes any high-energy anomaly cancellation.

3.2 Paper III

The effective potential, introduced in section 2.3.3, incorporates radiative corrections to the classical potential. The effective potential is important for studying phase transitions, and is an observable if the system is weakly coupled to gravity. Yet, is the effective potential gauge invariant? In gauge theories, it is necessary to choose a gauge for the path integral to converge [21,27]. For example, one common gauge choice is the Fermi gauge

$$\mathcal{L}_{g.f} = -\frac{1}{2\xi}(\partial_\mu A^\mu(x))^2. \tag{3.8}$$

Although this gauge term explicitly breaks gauge invariance, observables do not depend on the particular gauge fixing. Thus, a quantity explicitly depending on ξ is not gauge invariant, and hence not a physical quantity. In fact, the one-loop effective potential in scalar QED is not gauge invariant, as can be seen in figure 3.1.

But the ground state energy (the effective potential evaluated at its minimum) is gauge invariant. This can be seen from the Nielsen identity, which describes how the effective potential depends on the gauge parameter [39]

$$\left(\xi\frac{\partial}{\partial\xi} + C(\phi,\xi)\frac{\partial}{\partial\phi}\right)\mathcal{V}(\phi,\xi) = 0. \tag{3.9}$$

The Nielsen coefficient, $C(\phi,\xi)$, can be calculated order-by-order in perturbation theory, and is in general non-zero. Note, that while the effective potential is gauge independent in its minima, the vacuum expectation value

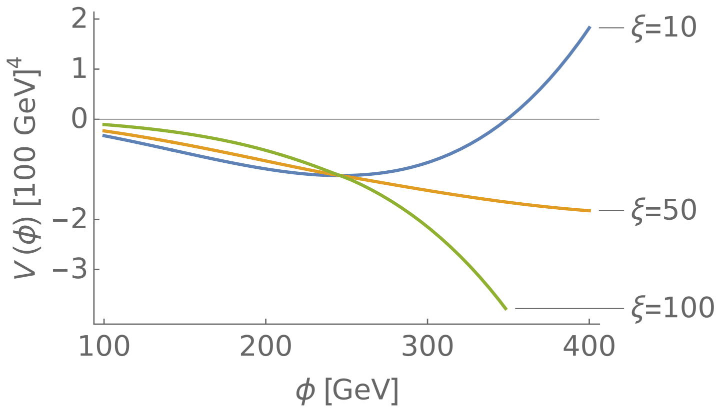
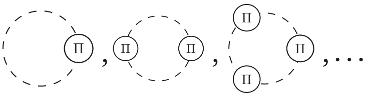


Figure 3.1. Dependence of the 1-loop effective potential on the gauge fixing parameter. The tree level minimum is fixed to $\phi = 246$ GeV.

ϕ that extremizes the effective potential is not [39]

$$\partial_\xi \mathcal{V}(\phi)|_{\phi=\phi_m} = 0,$$
$$\xi \partial_\xi \phi_m = C(\phi_m, \xi).$$

Furthermore, the effective potential is IR divergent when calculated perturbatively. One class of divergent diagrams are the daisy diagrams (as explain in paper III)



In particular, if $G(\phi)$ is a Goldstone mass depending on the background field, ϕ; contributions from the L:th loop behave, in Landau gauge $\xi = 0$, as

$$V_L \sim G^{3-L}. \tag{3.10}$$

The Goldstone mass, G, vanish in the tree level minima, which means that there are IR divergences at higher loop orders. Moreover, the 1-loop potential in figure 3.1 does not appear to be gauge invariant, even at its minimum. Indeed, the minimum is lower for $\xi = 100$ than for $\xi = 10$.

Both IR divergences, and the gauge invariance problem, are spurious. To elaborate, both ϕ_m *and* $V(\phi)$ are calculated perturbatively. Both quantities depend on a power counting parameter, denoted as $\hbar$. This $\hbar$ denotes the perturbative order, and higher orders in $\hbar$ are more suppressed. Explicitly, both the minimum and the potential are calculated as a power series in $\hbar$

$$\begin{aligned}\phi_m &= \phi_0 + \hbar\phi_1 + \hbar^2\phi_2 + \ldots, \\ \mathcal{V}(\phi) &= V_0(\phi) + \hbar V_1(\phi) + \hbar^2 V_2(\phi) + \ldots,\end{aligned} \tag{3.11}$$

showing that the ground state energy, $\mathcal{V}(\phi_m)$, is given by

$$\mathcal{V}(\phi_m) = \left[V_0(\phi) + \hbar\left(\partial_\phi V_0(\phi)\phi_1 + V_1(\phi)\right) + \ldots\right]_{\phi=\phi_0}. \tag{3.12}$$

All the terms in $\mathcal{V}(\phi_m)$ are, order-by-order in $\hbar$, free of IR divergences and gauge invariant. If we try to minimize the effective potential numerically we neglect some higher order terms, but include other. We should, therefore, be extra careful when calculating the effective potential perturbatively.

3.3 Paper IV

Hadron colliders produce a large number of jets, together with a background of hadrons. These background particles both come from additional hard interactions, and from hadron remnants. Most hadron scattering processes produce a uniform distribution (in rapidity) of particles. However, there are rare events where two jets are produced with a rapidity "gap" between them. These rapidity gaps are events where two jets are produced with few intermediate particles, and have been seen both at the Tevatron and at the LHC [40–42].

A process where two partons exchange colour, for example a single gluon exchange, is unlikely to produce a rapidity gap. Conversely, a rapidity gap could be produced in processes with no colour exchange. A rapidity gap could be produced by a two gluon exchange because the second gluon can cancel the colour of the first. In particular, two gluons contain a colour singlet

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27. \tag{3.13}$$

The colour singlet (1) corresponds to a configuration with no colour transfer. The cross section for a two gluon exchange is enhanced for jets that are widely separated in rapidity. This enhancement comes from gluon ladder diagrams, in which the two exchanged gluons are connected by an arbitrary number of rungs, as is shown in figure 3.2.

These kind of double gluon exchanges are described by the BFKL [43–45] equation. BFKL exchanges could explain rapidity gaps at the Tevatron [40, 41,46]. In paper IV of this thesis, it is shown how rapidity gaps at LHC also can be explained by BFKL exchanges.

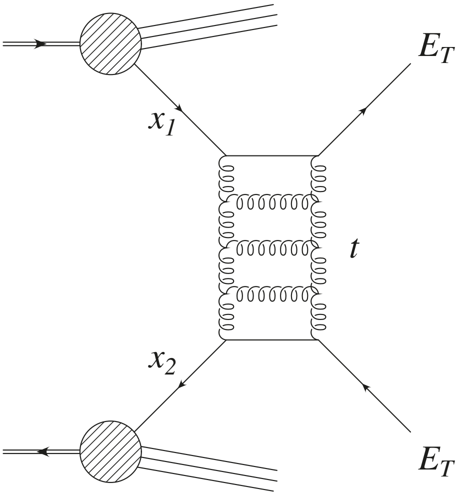


Figure 3.2. Two gluon exchange producing a gap. The two gluons exchange is enhanced by a gluon ladder.

3.4 Papers V and VI

Parton distributions are often parametrized at a starting scale ($\sim$ 1GeV) as [47,48]

$$f_i(x, Q_0) = a_0^i x^{a_1^i} (1-x)^{a_2^i} \exp\left[a_3^i x + a_4^i x^2 + a_5^i \sqrt{x}\right]. \tag{3.14}$$

The free parameters ($a^i_{0,1,2,3,4,5}$) are determined by fitting to experimental data. However, this approach does not explain the PDFs' shapes. Alternatively, PDFs can be modelled by phenomenological models based on physical properties, as is done in paper V and VI. In these papers it was assumed that partons momentum are distributed with a Gaussian distribution. The width σ is expected to be physically given by the uncertainty relation $\Delta x \Delta p \sim \hbar/2$. Taking Δx to be the hadron's size, one expects $\sigma \sim 0.1\,\text{GeV}$.

These distributions, only model "bare" PDFs, that is, only gluons and valence quarks. However, there are also "sea" partons—such as strange and anti-up quarks. Whereas partons are the right high-energy degrees of freedom, hadrons are the right low-energy degrees of freedom. An electron is surrounded by a cloud of electron-positron pairs. Similarly, a hadron, for example a proton, is surrounded by a cloud of hadron fluctuations. That is, the proton state can be written superposition of a bare proton state and a set of hadron states

$$|P\rangle = \alpha_{\text{bare}} |P\rangle_{\text{bare}} + \alpha_{n\pi^+} \left|n\pi^+\right\rangle + \alpha_{K^+\Lambda} \left|K^+\Lambda\right\rangle + \ldots \tag{3.15}$$

A high-energy probe, for example a photon, probes small distances; and can not see the entire hadronic system. Thus, as far as the probing photon is concerned, any quark in the other hadrons belongs to the proton. For example, strange quarks are generated from fluctuations such as $|K^+\Lambda\rangle$.

These kind of models are attractive because the PDFs can be understood from a physical picture. For example, there are more anti-down than anti-up quarks in the proton [48], which is natural from the hadronic point of view because the proton can fluctuate to a $\pi^+ n$ state ($\pi^+ = (\bar{d}u)$). Furthermore, hadronic fluctuations help to explain why quarks only carries a third of the proton's spin—known as the proton spin puzzle [49,50]. Hadronic fluctuations carries orbital angular momentum; a high-energy Q^2 photon can not probe the orbital angular momentum of entire hadronic system. Instead, the photon couples to single quarks, which means that some of the quark's spin gets transferred to orbital angular momentum. Paper VI showed a possible solution to the proton spin puzzle, where hadronic fluctuations and relativistic effects was taken into account.

4. Summary in Swedish—Populärvetenskaplig sammanfattning

Introduktion till Partikelfysik

Världen kan fantastiskt nog helt beskrivas av ett fåtal lagar. I vårt vardagliga liv är vi bekanta med stora föremål, som moln och bilar. Dessa föremål verkar initialt vara strikt annorlunda från varandra. Emellertid är lagarna som beskriver ett moln desamma som de som beskriver en bil. Dessa lagar är förvånansvärt enkla med tanke på hur komplicerad världen tillsynes verkar vara. Komplexiteten kommer dock inte ifrån att naturlagarna är besvärliga; istället är moln komplicerade på grund av att det finns 10^{31} molekyler bara i ett enda moln.

Gravitationskraften är ett exempel på en av dessa fundamentala lagar. Detta är samma kraft som håller jorden i omloppsbana runt solen. Andra krafter, som elektricitet och magnetism, är i själva verket olika aspekter av samma kraft: elektromagnetismen. Gravitationen och elektromagnetismen har under nittonhundratalet fått sällskap av ytterligare två krafter: den starka, och den svaga kraften. Atomkärnor hålls ihop av den starka kraften, medan den svaga kraften orsakar radioaktivt sönderfall.

Fundamentala konstanter

SI enheter—som meter, kilo, och sekund—är inte särskilt praktiska för fundamentala partiklar. Detta beror på att partikelfysiken beskriver hur naturen fungerar på skalor mindre än en proton ($\sim 10^{-15}$m). Enheter som meter och sekunder är inte särskilt “naturliga” på dessa längdskalor.

Enheten GeV används på grund av sammanträffandet att protonens massa är ungefär en GeV. Naturliga enheter ger således ofta extra information. Till exempel är solens massa i naturliga enheter 10^{57} GeV (2×10^{31}kg), vilket innebär att det ungefär finns 10^{57} protoner i solen. Vidare, eftersom längd mäts i GeV^{-1} så är något med en längd 1 GeV^{-1} ungefär lika stort som protonens radie, och en 1 GeV^{-1} är ungefär den tid det tar för ljuset att färdas en protonradie.

Partiklar har en större elektrisk attraktion (repulsion) ju större deras elektriska laddning är. Krafters styrka bestäms av fundamentala konstanter. Dessa konstanter inkluderar partiklars massa och elektriska laddning.

Mycket av den värld som vi ser runt oss kan förklaras av den relativa styrkan av olika krafter. Till exempel kan jordens storklek och massa kan uppskattas som [51]

$$R_\oplus \sim R_e \left(\frac{\alpha}{G_N m_p^2} \right)^{1/2} ,$$

$$M_\oplus \sim m_p \left(\frac{\alpha}{G_N m_p^2} \right)^{3/2} .$$

Jordens radie är stor eftersom den elektromagnetiska kraften mellan protoner och elektroner ($\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$) är 10^{34} gånger större än den gravitationella kraften mellan två protoner ($G_N m_p^2 \approx 10^{-36}$).

Partikelfysik

Naturen beskrivs på en fundamental nivå av partiklar. Ett kraftfullt mikroskop kan se att atomer består av elektroner som kretsar runt en kärna av protoner och neutroner. Ett ännu starkare mikroskop skulle visa att protoner inte är fundamentala, de är uppbyggda av kvarkar. Kvarkar och elektroner är såvitt vi vet fundamentala partiklar, protoner och neutroner är bundna tillstånd av kvarkar.

Partikelfysiken beskriver hur fundamentala partiklar—som elektroner och kvarkar—beter sig. Dessa krafter orsakas av förmedlarpartiklar. Fotoner är elektromagnetismens kraftpartikel. De tre andra krafterna: den svaga kraften, den starka kraften, och gravitationskraften har också förmedlarpartiklar. Den svaga förmedlas av $W^\pm$ och Z partiklar och gluoner förmedlar den starka kraften, medan gravitationskraften tros förmedlas av gravitoner. Den elektromagnetiska och svaga kraften förenas vid höga energier till den elektrosvaga kraften.

Standardmodellen

Standardmodellen är vår nuvarande bästa beskrivning av hur partiklar beter sig. Standardmodellen slutfördes på sjuttiotalet och har gjort förutsägelser som har mäts till en otrolig precision. Standardmodellen beskriver, så vitt vi vet, naturen ner till längdskalor $\sim 10^{-18}$ m—tusen gånger mindre än en proton. För att göra experiment på dessa skalor krävs en enorm mängd energi. Experiment som LHC kolliderar protoner som rör sig en tiondels miljarddel ifrån ljusets hastighet. Således borde protonen ha en enorm energi, men protonerna har faktiskt ungefär samma energi som en flygande mygga, vilket verkar vara väldigt lite. Emellertid är en proton en triljon gånger mindre än en mygga. Således är energi densiteten enorm när två protoner kolliderar.

Standardmodellen inkluderar både kraftpartiklar och materia partiklar. Materia partiklarna kan delas upp i leptoner och kvarkar. Leptoner kan ytligare delas upp i laddade leptoner och neutriner. Elektronen utgör tillsammans med muonen och tauonen de laddade leptonerna. Muoner och tauoner är tyngre versioner av elektronen. Kvarkarna är väldigt små partiklar som bygger upp protoner och neutroner. Vi vet inte exakt hur små kvarkar är men experiment har visat att de åtminstone är tusen gånger mindre än en proton.

Om två protoner kolliderar så kan även kvarkarna kollidera. Kvarkarna delar på protonens energi vilket betyder att en kvark inte tar samma andel av protonens energi i varje kollision. Sannolikheten för att en kvark ska ta en vis andel av protonens energi ges av partonfördelningar. Dessa är tyvärr besvärliga att beräkna och parametriseras ofta utan en fysikalisk insikt. Artikel V och VI använde en fysikaliskt motiverad modell för att beskriva partonfördelningar. Denna modell kunde beskriva data ifrån experiment och gav insikter om hur partonfördelningar uppkommer. Modellen kunde också förklara hur protonens rörelsemängdsmoment är fördelat på kvarkarna.

Higgsmekanismen

Alla fundamentala partiklar vore masslösa utan Higgsfältet. Higgsfältet genomsyrar hela universum och partiklar som interagerar med Higgsfältet utsätts för en friktion och får en massa, vilket kallas för Higgsmekanismen. Kvarkar, elektriskt laddade leptoner, och $W^{\pm}$, Z partiklarna interagerar med Higgsfältet och är massiva. Fotonen är dock masslös och interagerar inte med Higgsfältet. Higgsfältet kräver att det finns en till partikel som kallas Higgspartikeln. Higgspartikeln upptäcktes sommaren 2012 efter att ha förutsagts nästan sextio år tidigare.

Partiklar får en massa genom att interagera med Higgsfältet, vilket också ändrar Higgsfältets energi. Dessa interaktioner är problematiska att beräkna eftersom de naivt ger en oändligt stor energi. Artikel III undersökte detta problemet och visade att oändligheterna försvinner om beräkningarna utförs noggrant.

Utökningar av Standardmodellen

Standardmodellen beskriver en rad fenomen med förvånansvärd precision. Det finns dock skäl att inte vara helt nöjd med Standardmodellen. Till exempel är neutriner (väldigt lätta partiklar som förekommer i radioaktivt sönderfall) masslösa enligt Standardmodellen. Experiment har dock visat att neutriner är massiva. Detta problemet går att lösa genom att utöka Standardmodellen. Dessa utökningar kallas bortom Standardmodellen teorier.

Många utökningar av Standardmodellen inkluderar nya krafter som beter sig som den svaga kraften men är ännu svagare. Dessa nya krafter förmedlas i likhet med de andra av krafterna av partiklar. Nya krafter kan både förklara neutriners massa och är också ett sätt att förena de andra fundamentala krafterna.

Artikel I och II i denna avhandlingen studerade en utökning av Standardmodellen där en ny kraft introducerades. Denna nya kraft verkar genom en förmedlarpartikel, Z', som skulle kunna ses vid LHC. Den nya kraftpartikeln kan påverka hur de andra Standardmodellpartiklarna interagerar. Det är således möjligt att studera effekterna av den nya kraften utan att direkt se förmedlarpartikeln vid LHC.

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