Study of the decays $D_S^+ \to K_S^0 K^+$ and $K_S^0 K^+$
Using an $e^+e^-$ annihilation data sample corresponding to an integrated luminosity of 3.19 fb$^{-1}$ and collected at a center-of-mass energy $\sqrt{s} = 4.178$ GeV with the BESIII detector, we measure the absolute branching fractions $B(D_s^+ \rightarrow K_S^0 K^+)$ = (1.425 $\pm$ 0.038$^{\text{stat}}$ $\pm$ 0.031$^{\text{syst}}$)% and $B(D_s^+ \rightarrow K_L^0 K^+)$ = (1.485 $\pm$ 0.039$^{\text{stat}}$ $\pm$ 0.046$^{\text{syst}}$)% . The branching fraction of $D_s^+ \rightarrow K_S^0 K^+$ is compatible with the world average and that of $D_s^+ \rightarrow K_L^0 K^+$ is measured for the first time. We present the first measurement of the $K_S^0$-$K_L^0$ asymmetry in the decays $D_s^+ \rightarrow K_S^0 K^+$, and $R(D_s^+ \rightarrow K_L^0 K^+)$ = $\frac{B(D_s^+ \rightarrow K_S^0 K^+)}{B(D_s^+ \rightarrow K_L^0 K^+)}$ = ($-2.1 \pm 1.9^{\text{stat}}$ $\pm$ 1.6$^{\text{syst}}$)% . In addition, we measure the direct $CP$ asymmetries $A_{CP}(D_s^+ \rightarrow K_S^0 K^+)$ = (0.6 $\pm$ 2.8$^{\text{stat}}$ $\pm$ 0.6$^{\text{syst}}$)% and $A_{CP}(D_s^+ \rightarrow K_L^0 K^+)$ = ($-1.1 \pm 2.6^{\text{stat}}$ $\pm$ 0.6$^{\text{syst}}$)% .

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I. INTRODUCTION

Two-body hadronic decays of charmed mesons, $D \rightarrow P_1 P_2$ (where $P_{1,2}$ denotes a pseudoscalar meson), serve as an ideal environment to improve our understanding of the weak and strong interactions because of their relatively simple topology [1,2]. Charmed-meson decays into hadronic final states that contain a neutral kaon are particularly attractive. Bigi and Yamamoto [3] first pointed out that the interference of the decay amplitudes of the Cabibbo-favored (CF) transition $D \rightarrow \bar{K}^0\pi$ and the doubly-Cabibbo-suppressed (DCS) transition $D \rightarrow K^0\pi$ can result in a measurable $K^0_{S} - K^0_{L}$ asymmetry

$$R(D \rightarrow K^0_{S,L}\pi) = \frac{B(D \rightarrow K^0_{S}\pi) - B(D \rightarrow K^0_{L}\pi)}{B(D \rightarrow K^0_{S}\pi) + B(D \rightarrow K^0_{L}\pi)}. \tag{1}$$

A similar asymmetry can be defined in $D^+_{s}$ decays by replacing $\pi$ with $K$. Additionally, as pointed out in Ref. [4], the interference between CF and DCS amplitudes can also lead to a new $CP$ violation effect, which is estimated to be of an order of $10^{-3}$. The measurement of $K^0_{S} - K^0_{L}$ asymmetries and $CP$ asymmetries in charmed-meson decays with a neutral kaon provides insight into the DCS process, as well as information to explore $D^0\bar{D}^0$ mixing, $CP$ violation and SU(3) flavor-symmetry breaking effects in the charm sector [5,6].

On the theory side, there are different phenomenological models which give predictions for the $K^0_{S} - K^0_{L}$ asymmetries, such as: the topological-diagrammatic approach [2] under the SU(3) flavor symmetry (DIAG) or incorporating the SU(3) breaking effects [SU(3)$_{FB}$] [7–9], the QCD factorization approach (QCDF) [10], and the factorization-assisted topological-amplitude (FAT) [11]. The predicted $K^0_{S} - K^0_{L}$ asymmetries in charmed-meson decays from these different approaches, as well as the measured values reported by the CLEO Collaboration [12], are summarized in Table I. Considering the large range of values predicted for the $K^0_{S} - K^0_{L}$ asymmetries, their measurements provide a crucial constraint upon models of the dynamics of charmed meson decays.

![Image](https://example.com/phys_rev_d_99_112005.png)

**FIG. 2.** A schematic diagram illustrating the BESIII detector. The BESIII detector consists of a superconducting solenoid, a multiwire proportional counter (MIP) system, a layer of strip silicon detector (SSD), an internal calorimeter (RIC), and a steel/yair calorimeter (YAI). The full detector is immersed in a magnetic field of $1 T$. (a) Measured $D^+_{s}$ decay branching fractions and $CP$ asymmetries. (b) Predicted $K^0_{S} - K^0_{L}$ asymmetries from different phenomenological models and the CLEO measurements.

**TABLE I.** Predictions for $K^0_{S} - K^0_{L}$ asymmetries in charmed-meson decays from different phenomenological models and the CLEO measurements.

<table>
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<tbody>
<tr>
<td>$R(D^0 \rightarrow K^0_{S,L}\pi^0)$ (%)</td>
<td>10.7</td>
<td>10.7</td>
<td>10.6</td>
<td>9.3$^{+4}_{-2}$</td>
<td>11.3 ± 0.1</td>
</tr>
<tr>
<td>$R(D^+ \rightarrow K^0_{S,L}\pi^+)$ (%)</td>
<td>−0.5 ± 1.3</td>
<td>−1.9 ± 1.6</td>
<td>−1.0 ± 2.6</td>
<td>⋯</td>
<td>2.5 ± 0.8</td>
</tr>
<tr>
<td>$R(D^+<em>s \rightarrow K^0</em>{S,L}K^+)$ (%)</td>
<td>−0.22 ± 0.87</td>
<td>−0.8 ± 0.7</td>
<td>−0.8 ± 0.7</td>
<td>11$^{+4}_{-14}$</td>
<td>1.2 ± 0.6</td>
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In this paper, measurements of the absolute branching fractions for the decays $D^+_s \rightarrow K^0_{S}K^+$ and $D^+_s \rightarrow K^0_{L}K^+$, the $K^0_{S}K^0_{L}$ asymmetry, and the corresponding $CP$ asymmetries are performed using a sample of $e^+e^-$ annihilation data collected at $\sqrt{s} = 4.17$ GeV with the BESIII detector at the BEPCII. The data sample corresponds to an integrated luminosity of 3.19 fb$^{-1}$. Throughout the paper, charge conjugation modes are implicitly implied, unless otherwise noted.

II. BESIII DETECTOR AND MONTE CARLO SIMULATION

The BESIII detector is a magnetic spectrometer that operates at the BEPCII $e^+e^-$ collider [17]. The detector has a cylindrical geometry that covers 93% of the $4\pi$ solid angle and consists of several subdetectors. A main drift chamber (MDC) with 43 layers surrounding the beam pipe measures momenta and specific ionization of charged particles. Plastic scintillator time of flight counters (TOF), located outside of the MDC, provide charged-particle identification information, and an electromagnetic calorimeter (EMC), consisting of 6240 CsI(Tl) crystals, detects electromagnetic showers. These subdetectors are immersed in a magnetic field of 1 T, produced by a superconducting solenoid, and are surrounded by a multi-layered resistive-plate chamber (RPC) system interleaved in the steel flux return of the solenoid, providing muon identification. In 2015, BESIII was upgraded by replacing the two end-cap TOF systems with multigap RPCs, which achieve a time resolution of 60 ps [18]. A detailed description of the BESIII detector is presented in Ref. [19].
TABLE II. Summary of the $D_s^-$ ST yields, along with the ST and DT detection efficiencies for that decay mode. The uncertainty is statistical only. The decay branching fractions of subsequent decays in the ST side are not included in the efficiencies. The decay branching fraction of $K_S^0 \rightarrow \pi^+\pi^-$ in the signal side is included in $\epsilon_{ST}$.

<table>
<thead>
<tr>
<th>Tag mode</th>
<th>$N_{ST}$</th>
<th>$\epsilon_{ST}$ (%)</th>
<th>$\epsilon_{DT}$ (%)</th>
<th>$\epsilon_{MM}$ (%)</th>
</tr>
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<tbody>
<tr>
<td>$K^+ K^- \pi^-$</td>
<td>$141285 \pm 631$</td>
<td>$42.15 \pm 0.03$</td>
<td>$13.58 \pm 0.07$</td>
<td>$16.33 \pm 0.10$</td>
</tr>
<tr>
<td>$K^- \pi^+ \pi^-$</td>
<td>$18051 \pm 575$</td>
<td>$48.84 \pm 0.26$</td>
<td>$16.35 \pm 0.08$</td>
<td>$19.73 \pm 0.12$</td>
</tr>
<tr>
<td>$\pi^+ \pi^- \pi^-$</td>
<td>$40573 \pm 964$</td>
<td>$56.05 \pm 0.18$</td>
<td>$18.47 \pm 0.08$</td>
<td>$22.55 \pm 0.12$</td>
</tr>
<tr>
<td>$K^+ K^- \pi^-\pi^0$</td>
<td>$41001 \pm 840$</td>
<td>$10.61 \pm 0.03$</td>
<td>$3.86 \pm 0.04$</td>
<td>$5.02 \pm 0.06$</td>
</tr>
<tr>
<td>$\pi^0 \eta_{\rho}$</td>
<td>$26360 \pm 833$</td>
<td>$35.33 \pm 0.16$</td>
<td>$12.41 \pm 0.07$</td>
<td>$15.59 \pm 0.10$</td>
</tr>
<tr>
<td>$\rho^- \eta$</td>
<td>$32922 \pm 878$</td>
<td>$16.65 \pm 0.06$</td>
<td>$5.99 \pm 0.06$</td>
<td>$8.84 \pm 0.09$</td>
</tr>
<tr>
<td>$K_S^0 K^- \pi^+ \pi^-$</td>
<td>$8081 \pm 283$</td>
<td>$18.47 \pm 0.11$</td>
<td>$6.16 \pm 0.05$</td>
<td>$7.72 \pm 0.07$</td>
</tr>
<tr>
<td>$K_S^0 K^+ \pi^- \pi^-$</td>
<td>$15331 \pm 249$</td>
<td>$21.44 \pm 0.06$</td>
<td>$6.82 \pm 0.05$</td>
<td>$8.21 \pm 0.07$</td>
</tr>
<tr>
<td>$K_S^0 K^0 \pi^0$</td>
<td>$11380 \pm 385$</td>
<td>$16.97 \pm 0.12$</td>
<td>$5.94 \pm 0.05$</td>
<td>$7.82 \pm 0.07$</td>
</tr>
<tr>
<td>$K_S^0 K_S^0 \pi^-$</td>
<td>$5015 \pm 164$</td>
<td>$22.86 \pm 0.11$</td>
<td>$6.95 \pm 0.05$</td>
<td>$8.98 \pm 0.07$</td>
</tr>
<tr>
<td>$\pi^0 \eta$</td>
<td>$19050 \pm 512$</td>
<td>$46.60 \pm 0.19$</td>
<td>$16.06 \pm 0.07$</td>
<td>$21.99 \pm 0.13$</td>
</tr>
<tr>
<td>$\pi^0 \eta_{\epsilon' \eta}$</td>
<td>$7694 \pm 137$</td>
<td>$18.80 \pm 0.05$</td>
<td>$6.16 \pm 0.05$</td>
<td>$8.45 \pm 0.08$</td>
</tr>
<tr>
<td>$\pi^0 \eta_{\epsilon' \epsilon'}$</td>
<td>$5448 \pm 169$</td>
<td>$22.30 \pm 0.11$</td>
<td>$7.47 \pm 0.06$</td>
<td>$9.70 \pm 0.08$</td>
</tr>
</tbody>
</table>

The performance of the BESIII detector is evaluated using a GEANT4-based [20] Monte Carlo (MC) program that includes a description of the detector geometry and simulates its response. In the MC simulation, the production of open charm processes directly produced via $e^+e^-$ annihilation are modeled with the generator CONEXC [21], which includes the effects of the beam energy spread and initial-state radiation (ISR). The ISR production of vector charmonium states [$\psi(3770)$, $\psi(3686)$ and $J/\psi$] and the continuum processes ($q\bar{q}$, $q = u, d, s$) are incorporated in KKM [22]. The known decay modes are generated using EVTGEN [23], which assumes the branching fractions reported in Ref. [13]; the fraction of unmeasured decays of charmonium states is generated with LUNDCHARM [24]. The final-state radiation (FSR) from charged tracks is simulated by the PHOTOS package [25]. A generic MC sample with equivalent luminosity 35 times that of data is generated to study the background. It contains open charm processes, the ISR return to charmonium states at lower mass, and continuum processes (quantum electrodynamics and $q\bar{q}$). The signal MC samples of 5.2 million $e^+e^- \rightarrow D_s^{\pm} \overline{D}_s^{\mp}$ events are produced; in these samples the $D_s^{\pm}$ decays into $\gamma/\rho^0/e^+e^-D_s^{\mp}$, while one $D_s$ decays into a specific mode in Table II and the other into the final states of interest $K_S^0 K^{\mp}$ or $K_S^0 K^\mp$. The signal MC samples are used to determine the distributions of kinematic variables and estimate the detection efficiencies.

III. DATA ANALYSIS

The cross section to produce $e^+e^- \rightarrow D_s^{\pm} \overline{D}_s^{\mp}$ events at $\sqrt{s} = 4.178 \text{ GeV}$ is $(889 \pm 59_{\text{stat.}} \pm 47_{\text{syst.}}) \text{ pb}$, which is one order of magnitude larger than that to produce $e^+e^- \rightarrow D_s^+ D_s^-$ events [14]. Furthermore, the decay branching fraction $B(D_s^{\pm} \rightarrow \gamma D_s^\mp)$ is $(93.5 \pm 0.7)\%$ [13]. Therefore, in the data sample used, $D_s^+$ candidates arise mainly from the process $e^+e^- \rightarrow D_s^{\pm} \overline{D}_s^{\mp} \rightarrow \gamma D_s^+ D_s^-$, along with small fractions from the processes $e^+e^- \rightarrow D_s^{\pm} \overline{D}_s^{\mp} \rightarrow \rho^0 D_s^+D_s^-$ and $e^+e^- \rightarrow D_s^0 D_s^-$. The outline of the reconstruction is described first, with all details given later in this section.

In this analysis, a sample of $D_s^-$ mesons is reconstructed first, which are referred to as “single tag (ST)” candidates. The ST candidates are reconstructed in 13 hadronic decay modes that are listed in Table II. The $D_s^- \rightarrow K_S^0 K^-$ decay mode is not included to avoid double counting in $D_s^0 \rightarrow K_S^0 K^*$ measurement. Here, $\pi^0$ and $\eta$ candidates are reconstructed from a pair of photon candidates, $K_S^0$ candidates are formed from $\pi^+\pi^-$ pairs, and $\rho^{\pm(0)}$ candidates are reconstructed from $\pi^\pm\pi^0(\mp)$ pairs, unless otherwise indicated by a subscript.

In the sample of events with ST candidates, the process $D_s^+ \rightarrow K_S^0 K^*$ is reconstructed by selecting a charged kaon and a $K_S^0$ candidate from those not used to reconstruct the ST candidates, which is referred to as “double tag (DT)”. To reconstruct the $D_s^+ \rightarrow K_S^0 K^*$ decay, the photon from the decay $D_s^\pm \rightarrow \gamma D_s^{\mp}$ and the charged kaon from $D_s^+$ decay are selected to determine the missing-mass-squared

$$MM^2 = (P_{\gamma} - P_{D_s} - P_{K} - P_{K^*})^2.$$  \hspace{1cm} (2)

where $P_{\gamma}$ is the four-momentum of the $e^+e^-$ initial state and $P_i (i = D_s^\mp, \gamma, K^*)$ is the four-momentum of the corresponding particle.

Ignoring the small contribution from the process $e^+e^- \rightarrow D_s^0 D_s^-$, the numbers of ST ($N_{ST}$) and DT ($N_{DT}$) events, for a specific tag mode $i$, are
\[ N_{\text{ST}}^i = 2 \times N_{D^0; D^+} \times B_{\text{tag}}^i \times c_{\text{ST}}^i, \quad (3) \]
\[ N_{\text{DT}}^i = 2 \times N_{D^0; D^+} \times B_{\text{tag}}^i \times B_{\text{sig}} \times c_{\text{DT}}^i, \quad (4) \]

respectively. Here, \( N_{D^0; D^+} \) is the total number of \( e^+ e^- \rightarrow D^0_D; D^+ \) events in the data sample, \( B_{\text{tag}}^i \) is the branching fraction for the \( i \)th ST decay mode, and \( B_{\text{sig}} \) is the branching fraction of the signal decay; \( c_{\text{ST}}^i \) and \( c_{\text{DT}}^i \) are the ST and DT detection efficiencies, respectively, which are evaluated from the signal MC samples corresponding to the \( i \)th tag mode. The value of \( c_{\text{DT}}^i \) includes the branching fraction \( B(K_S^0 \rightarrow \pi^+ \pi^-) \) of the signal side in the analysis of \( D^+_s \rightarrow K_S^0 K^+ \). The factors of 2 in Eqs. (3) and (4) are the result of including charge-conjugated modes in the analysis. We combine Eqs. (3) and (4) for each of the 13 tag modes to obtain
\[ B_{\text{sig}} = \frac{N_{\text{DT}}^{\text{tot}}}{\sum_i N_{\text{ST}}^i \times c_{\text{DT}}^i / c_{\text{ST}}^i}, \quad (5) \]

where \( N_{\text{DT}}^{\text{tot}} = \sum_i N_{\text{DT}}^i \) is the total number of DT events.

### A. Selection of ST events

Good charged tracks, except for the daughter tracks of \( K_S^0 \) candidates, are selected by requiring the track trajectory to approach the interaction point (IP) within \( \pm 10 \) cm along the beam direction and within 1 cm in the plane perpendicular to the beam direction. In addition, the polar angle \( \theta \) between the direction of the charged track and the beam direction must be within the detector acceptance by requiring \( |\cos \theta| < 0.93 \). Charged particle identification (PID) is performed by combining the ionization-energy loss \( (dE/dx) \) measured by the MDC and the time-of-flight measured by the TOF system. Each charged track is characterized by the PID likelihood for the pion and kaon hypotheses, which are \( L(\pi) \) and \( L(K) \), respectively. A pion (kaon) candidate is identified if it satisfies the condition \( L(\pi) > L(K) \) \( [L(K) > L(\pi)] \).

Good photon candidates are selected from isolated electromagnetic showers which have a minimum energy of 25 MeV in the EMC barrel region \((|\cos \theta| < 0.8)\) or 50 MeV in the EMC end-cap region \((0.86 < |\cos \theta| < 0.92)\). To reduce the number of photon candidates that result from noise and beam backgrounds, the time of the shower measured by the EMC is required to be less than 700 ns after the beam collision. The opening angle between a photon and the closest charged track is required to be greater than \( 10^\circ \), which is used to remove electrons, hadronic showers, and photons from FSR. \( \pi^0 \) and \( \eta \rightarrow \gamma \gamma \) candidates are reconstructed from pairs of photon candidates that have an invariant mass within the intervals \((0.115, 0.150)\) and \((0.50, 0.57)\) GeV/c\(^2\), respectively. To improve the momentum resolution, a kinematic fit is performed, constraining the \( \gamma \gamma \) invariant mass to its nominal value \([13] \); the \( \chi^2 \) of the fit is required to be less than 20 to reject the combinatorial background. \( \eta \rightarrow \pi^+ \pi^- \pi^0 \) candidates are selected by requiring the corresponding invariant mass to be within the interval \((0.534, 0.560)\) GeV/c\(^2\).

In order to improve the efficiency of the \( K_S^0 \) selection, \( K_S^0 \) candidates are reconstructed from tracks assumed to be pions without PID, and the daughter tracks are required to have a trajectory that approaches the IP to within \( \pm 20 \) cm along the beam direction and \(|\cos \theta| < 0.93 \). The \( K_S^0 \) candidates are formed by performing a vertex-constrained fit to all oppositely charged track pairs. To suppress combinatorial background, the \( \chi^2 \) of the vertex fit is required to be less than 200 and a secondary vertex fit is performed to ensure that the \( K_S^0 \) candidate originates from the IP. The flight length \( L \), defined as the distance between the common vertex of the \( \pi^+ \pi^- \) pair and the IP in the plane perpendicular to the beam direction, is obtained in the secondary vertex fit, and is required to satisfy \( L > 2\sigma_L \), where \( \sigma_L \) is the estimated uncertainty on \( L \); this criterion removes the combinatorial background formed from tracks originating from the IP. The four-momenta after the secondary vertex fit are used in the subsequent analysis. The \( K_S^0 \) candidate is required to have a mass within the interval \((0.487, 0.511)\) GeV/c\(^2\).

The \( \eta' \) candidates are reconstructed via the decay modes \( \gamma \rho^0 \) and \( \pi^+ \pi^- \eta \) by requiring the corresponding invariant masses to be within the intervals \((0.936, 0.976)\) and \((0.944, 0.971)\) GeV/c\(^2\), respectively. The \( \rho^0 \) candidates are reconstructed from \( \pi^+ \pi^- \) pairs that have a mass greater than 0.52 GeV/c\(^2\). The \( \rho^\pm \) candidates are reconstructed from \( \pi^\pm \pi^0 \) combinations that have an invariant mass within the interval \((0.62, 0.92)\) GeV/c\(^2\).

To suppress the background with \( D^s \) decay \( D^s \rightarrow \pi D \), the momentum of charged and neutral pions is required to be greater than 100 MeV/c. For \( K^- \pi^+ \pi^- \) ST candidates, the invariant mass of the \( \pi^+ \pi^- \) pair is required to be outside the interval \((0.478, 0.518)\) GeV/c\(^2\) to remove \( D_s^- \rightarrow K_S^0 K^- \) decays. The ST \( D_s^- \) candidates are reconstructed via all the possible selected particle combinations.

The invariant mass of the system recoiling against the selected \( D_s^- \) is defined as
\[ M_{\text{rec}} = \sqrt{(\sqrt{s} - \sqrt{p^2 + M_{D_s}^2})^2 - p^2}, \quad (6) \]

where \( p \) is the momentum of the ST \( D_s^- \) candidate in \( e^+ e^- \) CM frame, and \( M_{D_s} \) is the nominal mass of the \( D_s \) meson \([13] \). \( M_{\text{rec}} \) is required to be within the interval \((2.05, 2.18)\) GeV/c\(^2\). For a specific ST mode, if there are multiple combinations satisfying the selection criteria, only the candidate with the minimum value of \(|M_{\text{rec}} - M_{D_s}|\) is retained for further analysis. These requirements also accept the events in which the ST \( D_s \) comes from the decay of the primary \( D_s^+ \).
To determine the ST yield, a binned maximum likelihood fit to the distribution of the $D_s^+$ invariant mass $M_{\text{tag}}$ is performed for each tag mode; the distributions and fit results are shown in Fig. 1. In the fit, the probability density function (PDF) that describes the signal is the shape of the signal MC distribution, taken as a smoothed histogram and convolved with a Gaussian function to account for any resolution difference between data and MC simulation. The background is described by a second- or third-order Chebyshev polynomial function. The ST yields determined by the fits, along with the corresponding $\epsilon_{\text{ST}}$, estimated from the generic MC sample, are summarized in Table II.

**B. Branching fraction measurement of $D_s^+ \to K_S^0K^+$**

The signal decay $D_s^+ \to K_S^0K^+$ is reconstructed recoiling against the selected ST $D_s^-$ candidate. We select a $D_s^+ \to K_S^0K^+$ candidate if there is only one $K_S^0$ candidate and one good track, which is identified as a kaon and has positive charge, recoiling against the ST $D_s^-$ candidate; $K^+$ and $K_S^0$ candidates are selected by applying the selection criteria described in Sec. III A. In addition, to suppress combinatorial backgrounds, we reject events in which there are additional charged tracks that satisfy $|\cos \theta| < 0.93$ and approach the IP along the beam direction within ±20 cm.

To determine the DT signal yield, a two-dimensional (2D) unbinned maximum likelihood fit is performed on the invariant mass of the $K_S^0$ and $K^+$ ($M_{K_S^0K^+}$) vs $M_{\text{tag}}$ distribution of selected events, which is summed over the 13 ST modes, as shown in Fig. 2. In the fit, the total PDF is described by summing over the individual PDFs for the following signal and background components, where $x$ represents $M_{K_S^0K^+}$, and $y$ stands for $M_{\text{tag}}$.

(i) **Signal:** $F_{\text{sig}}(x, y) \otimes G(x; \mu_x, \sigma_x) \otimes G(y; \mu_y, \sigma_y)$

$F_{\text{sig}}(x, y)$ is a 2D function derived from the signal MC distribution by using a smoothed 2D histogram; $G(x; \mu_x, \sigma_x)$ and $G(y; \mu_y, \sigma_y)$ are Gaussian functions that compensate for any resolution difference between data and MC simulation for the variables $M_{K_S^0K^+}$ and $M_{\text{tag}}$, respectively. In the 2D fit, the parameters of $G(x; \mu_x, \sigma_x)$ and $G(y; \mu_y, \sigma_y)$ are fixed to the values determined by fitting the corresponding one-dimensional (1D) distributions.

(ii) **BKGI:** $F_{\text{BKGI}}(x, y) \otimes G(y; \mu_y, \sigma_y)$

This PDF describes the background composed of a correctly reconstructed ST $D_s^-$ recoiling against a combinatorial background, which are distributed in the horizontal band in Fig. 2. $F_{\text{BKGI}}(x, y)$ is derived from the distribution of this type of background in
Using Eq. (5), the branching fraction is determined to be identified as a kaon and have opposite charge candidates in data, summed over the 13 tag modes.

The 2D fit gives a signal yield of $1.92 \pm 0.10$, and the uncertainty is statistical. The residual $\chi^2$ of the fit is 100, with 100 degrees of freedom. The fit is performed to the data, with the signal MC samples, and the other decaying into any of ST modes, with the projection of the fit result superimposed. The data are shown as the black dots with error bars, the blue solid line is the total fit projection, the red short-dashed line is the projection of the BKGII component, the green long-dashed line is the projection of the BKGIII component, and the magenta dotted-dashed line is the projection of the signal component. The residual $\chi^2$ between the data and the total fit result, normalized by the uncertainty, is shown beneath the figures.

**C. Branching fraction measurement of $D_s^- \rightarrow K_{L}^0 K^+$**

The $D_s^- \rightarrow K_{L}^0 K^+$ candidates are reconstructed by requiring the event to have only one good track recoiling against the ST $D_s^-$ candidate; the charged track is required to be identified as a kaon and have opposite charge candidates in data, summed over the 13 tag modes. The detection efficiencies for the individual ST mode, obtained with the signal MC samples, are summarized in Table II. Using Eq. (5), the branching fraction is determined to be $\mathcal{B}(D_s^- \rightarrow K_{L}^0 K^+) = (1.425 \pm 0.038_{\text{stat}} \%)$.

**FIG. 2.** Distribution of $m_{tag}$ vs $m_{K_{S}^0 K^+}$ for $D_s^+ \rightarrow K_{S}^0 K_+^+$ candidates in data, summed over the 13 tag modes.

**FIG. 3.** (a) Distributions of $m_{K_{L}^0 K^+}$ and (b) $m_{tag}$, summed over the 13 tag modes, with the projection of the fit result superimposed. The data are shown as the black dots with error bars, the blue solid line is the total fit projection, the red short-dashed line is the projection of the BKGII component, the green long-dashed line is the projection of the BKGIII component, the magenta dotted-dashed line is the projection of the signal component. The residual $\chi^2$ between the data and the total fit result, normalized by the uncertainty, is shown beneath the figures.

compared with ST $D_s^-$. The $K^+$ is selected with the criteria described in Sec. III A. We further suppress combinatorial backgrounds by requiring no additional charged tracks that satisfy the requirements described in Sec. III B.

In this analysis, the ST and signal candidates are assumed to originate from the decay chain $\gamma e^+ e^- \rightarrow D_{s}^{\pm} D_{s}^- \rightarrow g D_{s}^{\mp} D_{s}^- \rightarrow g D_{s}^{\pm} K_{L}^0 K^+$, with one $D_{s}^- \rightarrow K_{L}^0 K^+$ decaying into any of ST modes, and the other decaying into $K_{L}^0 K^+$. We reconstruct the $K_{L}^0$ candidate using a kinematic fit that applies constraints arising from the masses of the ST $D_s^-$ candidate, the signal $D_s^-$ candidate, the intermediate state $D_s^{\pm}$, and the initial four-momenta of the event. In the kinematic fit, the $K_{L}^0$ signal candidate is treated as a missing particle whose four-momentum is determined by the fit. The fit is performed to
select the $\chi$ candidate from the decay $D_1^{\pm} \to \gamma D_1^{\pm}$ under two different hypotheses that constrain either the invariant mass of the selected $\gamma$ and signal $D_1^{\pm}$ or the selected $\gamma$ and the ST $D_1^{-}$ to the nominal mass of the $D_1^{-}$ meson; the hypothesis that results in the minimum value of $\chi^2$ is assumed to be the correct topology. If there are multiple photon candidates, which are not used to reconstruct the ST candidate, the fit is repeated for each candidate and the photon that results in the minimum value of the $\chi^2$ is retained for further analysis. For each event, the four-momentum of the missing particle assumed in the kinematic fit is used to determine the $MM^2$ of the $K_0^0$ candidate. In order to reduce combinatorial background, $\chi^2 < 40$ is required. To further suppress background with multiple photons, we reject those events with additional photons which have an energy larger than 250 MeV and an opening angle with respect to the direction of the missing particle greater than 15°.

To determine the signal yield, an unbinned maximum likelihood fit is performed on the $MM^2$ distribution of selected events from all 13 ST modes combined, as shown in Fig. 4. In the fit, three components are included: signal, peaking, and nonpeaking backgrounds. The PDFs of these components are described below, where $x$ represents $MM^2$.

(i) Signal: $F_{\text{sig}}(x) \otimes G(x; \mu^\prime, \sigma^\prime)$

(ii) Peaking background: $F_{\text{bkg}}^{K_0^0}(x) \otimes G(x; \mu^\prime, \sigma^\prime)$

$F_{\text{bkg}}^{K_0^0}(x)$ is derived from the distribution of $D_s^+ \to K_0^0 K^+ (D_s^+ \to \eta K^+)$ MC simulated events by using a smoothed histogram. These events form a peaking background if the $K_0^0$ or $\eta$ is not reconstructed. Here, $G(x; \mu^\prime, \sigma^\prime)$ is the Gaussian resolution function, whose parameters are the same as those used in the signal PDF. The expected yields of $D_s^+ \to K_0^0 K^+$ and $D_s^+ \to \eta K^+$ are fixed to 263 and 57, respectively. The expected peaking background yields are estimated by using the equation $N_{\text{data}}^{MM^2} = N_{\text{MC}}^{MM^2} \times \epsilon_{\text{DT}} \times \epsilon_{\text{MM}^2}$, where $N_{\text{data}}^{MM^2}$ is the number of expected peaking background events and $N_{\text{MC}}^{MM^2}$ is the yield of $D_s^+ \to K_0^0 K^+$ or $D_s^+ \to \eta K^+$ selected by using the DT method. Here, $\epsilon_{\text{MC}^{MM^2}}$ and $\epsilon_{\text{DT}}$ are the detection efficiencies of the nominal analysis and the DT method for each mode, respectively; these are estimated from MC simulation samples. The uncertainties of estimated event numbers for $D_s^+ \to K_0^0 K^+$ and $D_s^+ \to \eta K^+$ are 19 and 12, which will be used in the systematic uncertainty study.

(iii) Nonpeaking background: $P(x)$

$P(x)$ is a function to describe the combinatorial background, which is not expected to peak in the $MM^2$ distribution. $P(x)$ is a second-order polynomial function whose parameters are determined from the fit to data.

The fit to the $MM^2$ distribution is shown in Fig. 4. The signal yield determined by the fit is 2349 ± 61 events, where the uncertainty is statistical. Using Eq. (5), the branching fraction is calculated to be $B(D_s^+ \to K_0^0 K^+) = 1.485 \pm 0.039_{\text{stat}}\%$, where the DT detection efficiencies $\epsilon_{\text{DT}}^{MM^2}$ used are summarized in Table II; the values of $\epsilon_{\text{DT}}^{MM^2}$ are estimated from signal MC samples.

D. Asymmetry measurement

By using the measured branching fractions and Eq. (1) the $K_0^0-K_0^0$ asymmetry is determined to be

$$R(D_s^+ \to K_0^0 K^+) = (-2.1 \pm 1.9_{\text{stat}})\%.$$ (7)

To determine the direct CP violation, we also measure the branching fractions for the $D_s^+$ and $D_s^-$ decays separately, using the same methodology as the combined branching fraction measurement. The direct CP asymmetry is defined as
\[ A_{CP}(D_s^+ \rightarrow f) = \frac{B(D_s^+ \rightarrow f) - B(D_s^- \rightarrow \bar{f})}{B(D_s^+ \rightarrow f) + B(D_s^- \rightarrow \bar{f})}, \]  

which leads to the measurements

\[ A_{CP}(D_s^+ \rightarrow K_S^0 K^\pm) = (0.6 \pm 2.8_{\text{stat}} \%) \],  

\[ A_{CP}(D_s^+ \rightarrow K_L^0 K^\pm) = (-1.1 \pm 2.6_{\text{stat}} \%) \],

for the two signal modes.

### IV. SYSTEMATIC UNCERTAINTY

For the absolute branching fractions, which are determined according to Eq. (5), the systematic uncertainties are associated with \( N_{\text{DT}} \), \( N_{\text{ST}}^{\text{tot}} \), and the corresponding ratio of detection efficiencies \( (\epsilon_{\text{DT}}/\epsilon_{\text{ST}}) \). One of the advantages of the DT method is that most of the systematic uncertainties associated with selection criteria for the ST side reconstruction cancel. However, there is some residual uncertainty due to the different decay topologies between DT and ST events; this is referred to as “tag-side bias,” and its effect is considered as one of the systematic uncertainties. For the \( R(D_s^+) \) and \( A_{CP} \) measurements, the systematic uncertainties are calculated by propagating corresponding branching fraction uncertainties from different sources taking into account that some of the uncertainties cancel due to the fact that these observables are ratios as defined in Eqs. (1) and (8).

Table III summarizes the relative uncertainties on the absolute branching fraction and the absolute uncertainties for the asymmetries. The total systematic uncertainties are calculated as the sum in quadrature of individual contributions by assuming the sources are independent of one another.

<table>
<thead>
<tr>
<th>Source</th>
<th>( \mathcal{B}(D_s^+ \rightarrow K_S^0 K^+) )</th>
<th>( \mathcal{B}(D_s^+ \rightarrow K_L^0 K^+) )</th>
<th>( R(D_s^+ \rightarrow K_S^0 K^+) )</th>
<th>( A_{CP}(D_s^+ \rightarrow K_S^0 K^+) )</th>
<th>( A_{CP}(D_s^+ \rightarrow K_L^0 K^+) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^+/K^- ) tracking</td>
<td>0.5</td>
<td>0.5</td>
<td>\cdots</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>( K^+/K^- ) PID</td>
<td>0.5</td>
<td>0.5</td>
<td>\cdots</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>( K_S^0 ) reconstruction</td>
<td>1.5</td>
<td>\cdots</td>
<td>0.7</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>Photon selection and kinematic fit</td>
<td>\cdots</td>
<td>2.0</td>
<td>1.0</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>Extra photon energy requirement</td>
<td>\cdots</td>
<td>0.6</td>
<td>0.3</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>Extra charged track requirement</td>
<td>0.6</td>
<td>0.6</td>
<td>\cdots</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>( ST M(D_s) ) fit</td>
<td>0.9</td>
<td>0.9</td>
<td>\cdots</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>DT fit</td>
<td>0.8</td>
<td>\cdots</td>
<td>0.4</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>( MM^2 ) fit</td>
<td>\cdots</td>
<td>1.5</td>
<td>0.7</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>MC statistics</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Effect of ( B(D_s \rightarrow \gamma D_s) )</td>
<td>\cdots</td>
<td>0.7</td>
<td>0.3</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>Effect of ( e^+ e^- \rightarrow D_s^+ D_s^- )</td>
<td>\cdots</td>
<td>0.4</td>
<td>0.2</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>Tag-side bias</td>
<td>0.3</td>
<td>0.5</td>
<td>0.3</td>
<td>\cdots</td>
<td>\cdots</td>
</tr>
<tr>
<td>Total</td>
<td>2.2</td>
<td>3.1</td>
<td>1.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>
between data and MC simulation for these two requirements are both 0.6%, which are assigned as the systematic uncertainties from these sources.

The uncertainty related to the limited sizes of MC samples is 0.3% for both \( D_s^+ \rightarrow K_S^0 K^+ \) and \( D_s^+ \rightarrow K_S^0 K^+ \).

The uncertainties associated with ST, DT, and \( M M^2 \) fits are studied by changing the signal and background PDFs, as well as the fit interval; each change is applied separately. Furthermore, in the \( M M^2 \) fit, the effect of the assumed peaking background yields is estimated by changing the fixed numbers of events by \( \pm 1 \sigma \). The systematic uncertainties related to the ST, DT, and \( M M^2 \) fit procedure are 0.9%, 0.8% and 1.5%, respectively; these are the sums in quadrature of the relative changes of signal yield that result from each individual change to the fit procedure.

As discussed previously, the selected ST \( D_s^+ \) sample is dominated by the process \( e^+ e^- \rightarrow D_s^+ D_s^- \rightarrow \gamma D_s^+ D_s^- \), but there is a small contribution from the processes \( e^+ e^- \rightarrow D_s^+ D_s^- \rightarrow \pi^0 D_s^+ D_s^- \) and \( e^+ e^- \rightarrow D_s^+ D_s^- \). In the analysis of \( D_s^+ \rightarrow K_S^0 K^+ \), detailed MC studies indicate that \( e^+ e^- \rightarrow D_s^+ D_s^- \rightarrow \pi^0 D_s^+ D_s^- \) and \( e^+ e^- \rightarrow D_s^+ D_s^- \) processes is negligible in the absolute branching fraction measurement. In the analysis of \( D_s^+ \rightarrow K_S^0 K^+ \), the kinematic fit is performed under the hypothesis that the event is \( e^+ e^- \rightarrow D_s^+ D_s^- \rightarrow \gamma D_s^+ D_s^- \), and the MC studies indicate that the contribution of \( e^+ e^- \rightarrow D_s^+ D_s^- \rightarrow \pi^0 D_s^+ D_s^- \) and \( e^+ e^- \rightarrow D_s^+ D_s^- \) in signal events can be neglected. Thus, the uncertainty of the branching fraction \( B(D_s^+ \rightarrow \gamma D_s^+) \) [13] used in the signal MC simulation must be taken as a source of systematic uncertainty. The systematic uncertainty from excluding the process \( e^+ e^- \rightarrow D_s^+ D_s^- \) is 0.4%, which is the fraction of the ST yields that comes from the process \( e^+ e^- \rightarrow D_s^+ D_s^- \); this fraction is estimated from the MC simulation.

The tag-side bias uncertainty is defined as the canceled uncertainty in the tag side due to different track multiplicities in generic and signal MC samples. By studying the differences of tracking and PID efficiencies between data and MC in different multiplicities, the tag-side bias systematic uncertainties are estimated to be 0.3% for \( D_s^+ \rightarrow K_S^0 K^+ \) and 0.5% for \( D_s^+ \rightarrow K_S^0 K^+ \).

V. SUMMARY AND DISCUSSION

In summary, by using an \( e^+ e^- \) collision data sample at \( \sqrt{s} = 4.178 \) GeV, corresponding to an integrated luminosity of 3.19 fb\(^{-1}\), the absolute branching fractions are measured to be \( B(D_s^+ \rightarrow K_S^0 K^+) = (1.425 \pm 0.038_{\text{stat.}} \pm 0.031_{\text{syst.}})\% \) and \( B(D_s^+ \rightarrow K_S^0 K^+) = (1.485 \pm 0.039_{\text{stat.}} \pm 0.046_{\text{syst.}})\% \); the former is one standard deviation lower than the world average value [13], and the latter is measured for the first time. The \( K_S^0-K_L^0 \) asymmetry in \( D_s^+ \) decay is measured for the first time as \( R(D_s^+ \rightarrow K_S^0 K^+) = (-2.1 \pm 1.9_{\text{stat.}} \pm 1.6_{\text{syst.}})\% \). This measurement is compatible with theoretical predictions listed in Table I. Direct CP asymmetries of the two processes are obtained to be \( A_{\text{CP}}(D_s^+ \rightarrow K_S^0 K^+) = (0.6 \pm 2.8_{\text{stat.}} \pm 0.6_{\text{syst.}})\% \) and \( A_{\text{CP}}(D_s^+ \rightarrow K_L^0 K^+) = (-1.1 \pm 2.6_{\text{stat.}} \pm 0.6_{\text{syst.}})\% \). No significant asymmetries are observed and the uncertainties are statistically dominant.

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