International large-scale assessments and mathematics textbooks in a curriculum reform process


KRISTINA PALM KAPLAN

urn:nbn:se:uu:diva-395267
Reforming a curriculum entails more than simply launching new steering documents: it includes the development of new materials, evaluation of reform and identification of needs for further action. Drawing from algebra material surrounding the 2011 Swedish curriculum reform, this thesis contributes to understanding the process of curriculum reform concerning mathematics textbooks and international large-scale assessments.

Two analytical frameworks are applied to algebra tasks in six mathematics textbooks from three series, published in 1995–2015, and algebra items from four consecutive TIMSS tests in 2003, 2007, 2011 and 2015. These frameworks are a Systemic Functional Linguistics-framework based on a social-semiotic perspective and a framework adapted from different views on school algebra. They are used in order to understand algebra characteristics in terms of school algebra discourses and algebraic activities. Additionally, the relationship between Swedish students’ results on TIMSS’ algebra-items and algebra characteristics of the test items is explored statistically. The findings are viewed against the test structure over time.

The results show that five school algebra discourses are identified in both materials: the symbolic, arithmetical, geometrical, (un)realistic and scientific discourse. However, in the TIMSS tests, a relational discourse was further identified. The materials comprise the same algebraic activities, although in somewhat different proportions. The major change in textbooks is conservative since it is an increase of the (un)realistic discourse, which concerns features that may have been prevalent in the Swedish syllabus already a hundred years ago. This discourse may also be interpreted as a transformation of a reform idea in the syllabus, on mathematical literacy, into quite superficial everyday connections.

A comparison of the materials show that the algebra characteristic which increases most in textbooks, decreases in TIMSS. Further, students with high achievements on TIMSS perform significantly better on test items with algebra characteristics that decrease in proportion, compared with those which increase in proportion. Relatively seen, the difficulty of the tests for these students thus increases over time. This implies that the validity of interpreting TIMSS results as a knowledge trend is lessened. It is therefore less relevant to use TIMSS results for evaluating and identifying needs for further action. It is suggested that conclusions about student achievements in algebra should not be drawn based on the validity of interpreting and using the aggregate of TIMSS results in mathematics. Also, policy makers should pay more attention to how TIMSS results are used as a base for curricular reform and evaluation.

**Keywords:** curriculum reform, algebra, textbooks, international assessment, TIMSS, secondary school, test validity, student achievement.

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urn:nbn:se:uu:diva-395267 (http://urn.kb.se/resolve?urn=nbn:se:uu:diva-395267)
This thesis is based on the following papers, which are referred to in the text by their Roman numerals.


III Palm Kaplan, Kristina and Lönnstedt, Ingrid (manuscript). The test structure of TIMSS’ algebra varies between years and may influence the interpretability of Swedish students’ test scores over time.

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Contents

1 Introduction .......................................................................................................................... 15

2 Background.......................................................................................................................... 19
  2.1 Developments in school algebra ..................................................................................... 19
  2.2 Algebra in the intended curriculum, 1994 and 2011 ..................................................... 21
  2.3 Trends in International Mathematics and Science Study .............................................. 24
    2.3.1 About the validity of TIMSS ................................................................................ 26

3 The aim of the thesis............................................................................................................. 28
  3.1 The sub studies of the thesis ....................................................................................... 29
  3.2 Outline of the thesis ..................................................................................................... 30

4 Previous research............................................................................................................... 32
  4.1 Curriculum theory and research on curricular reform in mathematics.......................... 32
  4.2 Research on mathematics textbooks .......................................................................... 34
    4.2.1 Content analysis of mathematics textbooks ....................................................... 34
    4.2.2 Linguistic and discourse analysis of textbooks in mathematics .......................... 36
    4.2.3 The use of mathematics textbooks ..................................................................... 37
  4.3 Studies of school algebra ............................................................................................ 39
  4.4 Research on large-scale assessments ........................................................................... 40
  4.5 Summary of previous research ................................................................................... 44

5 Theoretical considerations.................................................................................................. 45
  5.1 Conceptions of the curriculum ..................................................................................... 45
  5.2 A model for understanding a curriculum reform ......................................................... 47
  5.3 Using results from ILSAs: aspects of validity ............................................................... 49
  5.4 Discourse and language ............................................................................................... 52
    5.4.1 Discerning school algebra discourses in tasks .................................................... 53
    5.4.2 School algebra discourses vs mathematical discourse ........................................ 55
  5.5 Views of school algebra .............................................................................................. 56
    5.5.1 The task as algebraic activities for the student .................................................... 58

6 Analysis and methods....................................................................................................... 59
  6.1 Materials ....................................................................................................................... 59
    6.1.1 The mathematics textbooks ................................................................................. 60
6.1.2 Selection of TIMSS’ test items for comparison with
textbook tasks ................................................................. 61
6.1.3 Selecting groups of students for analysing student
achievements ................................................................. 62
6.2 Systemic Functional Analysis .................................. 63
  6.2.1 The ideational function .................................... 63
  6.2.2 The interpersonal function ............................ 65
  6.2.3 Discerning and identifying discourses .............. 66
6.3 Algebraic activities ............................................. 67
  6.3.1 Categorizing algebraic activities .................... 68
6.4 Statistical analysis of student achievements .......... 69
  6.4.1 Descriptive statistics and prerequisites for inferential
analysis ................................................................. 69
  6.4.2 Inferential analysis .......................................... 73
  6.4.3 Considering the choice of test for inferential analysis... 74
6.5 Comparing algebra characteristics ................... 74
6.6 Methodological considerations .......................... 76
  6.6.1 Validity in research ........................................ 76
  6.6.2 Some comments on rigor and precision ........... 77
6.7 Ethical considerations .......................................... 78

7 Results. Summary of the papers .......................... 80
  7.1 Discourses in school algebra: the textbooks’ different views on
algebra and the positioning of students. Paper I ............. 80
  7.2 Conservative and transformative changes in algebra in Swedish
  7.3 The test structure of TIMSS’ algebra varies between years and
may influence the interpretability of Swedish students’ test scores over
time. Paper III .......................................................... 90

8 Discussion and conclusions ................................. 96
  8.1 Comparing algebra characteristics in textbooks and TIMSS’ test
items ................................................................. 96
  8.2 Transformative and conservative textbooks in the 2011 school
algebra reform .......................................................... 100
    8.2.1 The transformation of a reform idea ................. 100
    8.2.2 Resisting competences while changing content ... 101
    8.2.3 Implications for curricular reform ................. 102
  8.3 A lessened validity of TIMSS results in algebra .......... 103
    8.3.1 Reduced interpretability and relevance over time ... 103
    8.3.2 Different results – in mathematics or in algebra – produce
different relevance ................................................. 105
    8.3.3 More influence from OECD than IEA? .............. 106
    8.3.4 Implications for curricular reform ................. 107
8.4 Drawing conclusions from ILSAs’ results: implications for the
teaching and learning of algebra ............................................................ 108
  8.4.1 Excluding students with low achievements from learning
  algebra? .................................................................................................. 108
  8.4.2 Meaning-building not inherent in tasks with Manipulation. 109
8.5 A brief discussion on missing values ............................................. 111
8.6 Contributions ................................................................................... 111
8.7 Further research ............................................................................. 113
8.8 Summary ....................................................................................... 114

9 Sammanfattning på svenska .............................................................. 117

10 References .......................................................................................... 128

11 Appendix A. Missing values in the TIMSS material ...................... 139
12 Appendix B. Tables and box plots of percent right values .......... 140
13 Appendix C. Table over the diachronic interplay of school algebra
discourses and algebraic activities in the TIMSS tests ....................... 144
14 Appendix D. Box plots of percent right values for missing values in
the inferential analysis ........................................................................... 145
15 Appendix E. Permission to use test items ........................................ 147

Tables

Table 1. *Comparison of the syllabi* .................................................... 22
Table 2. *The textbook material* .......................................................... 60
Table 3. *TIMSS’ test items in the qualitative analysis of algebra
characteristics* ................................................................................. 61
Table 4. *Number of test items in algebra selected for inferential analysis and
average number of responses for Swedish students per selected test
item* ..................................................................................................... 62
Table 5. *The analytic framework of SFL* .......................................... 63
Table 6. *Distribution of the test items possible for inferential analysis* .... 70
Table 7. *Actions that school algebra discourses invite to* ................. 84
Table 8. *Distribution of algebraic activities per textbook* .................. 87
Table 9. *Distribution of tasks in school algebra discourse per textbook* .... 88
Table 10. *The interplay of the school algebra discourses and algebraic
activities in the textbooks* .............................................................. 88
Table 11. *The algebra characteristics in the structure of TIMSS* .......... 93
Table 12. *Comparison of the materials’ 2nd most common characteristics
over time* .......................................................................................... 99
Table 13. Test items which are hybrids and test items with more than one algebraic activity. ................................................................. 139
Table 14. Distribution of percent right values for the school algebra discourses in TIMSS. ........................................................................ 140
Table 15. Distribution of percent right values for the algebraic activities in TIMSS. ........................................................................... 140

Examples

Example 1. A relational process. ................................................................. 64
Example 2. Passive verb form and a material process. ................................. 64
Example 3. Implicit requests to the student with material processes........... 65
Example 4. The symbolic discourse. ............................................................ 81
Example 5. The geometrical discourse. ....................................................... 81
Example 6. The arithmetical discourse. ..................................................... 82
Example 7. The (un)realistic discourse. ..................................................... 83
Example 8. The scientific discourse. .......................................................... 83
Example 9. Generalized arithmetic. ............................................................ 86
Example 10. Qualitative and proportional thinking .................................... 86
Example 11. Functional thinking ................................................................ 87
Example 12. EEEI in the symbolic discourse ............................................ 89
Example 13. The relational discourse ........................................................ 92

Figures

Figure 1. The curriculum reform as a process. .............................................. 48
Figure 2. Non-normal data distribution in the geometrical discourse for students with low achievements. .......................................................... 71
Figure 3. Simulated density of Beta (1, 4.2) after multiplication by 0.5 .... 72
Figure 4. Histogram of sample means. .......................................................... 72
Figure 5. Normal Q-Q plot. ...................................................................... 73
Figure 6. Percent right values of school algebra discourses for students with high achievements. ................................................................. 94
Figure 7. Diachronic comparison of textbook tasks and TIMSS’ algebra items. .............................................................................................. 98
Figure 8. The (partly) transformative textbooks in the 2011 school algebra reform. .................................................................................... 101
Figure 9. The validity of TIMSS’ algebra results over time. Interpretation of results and the relevance of using them in a reform process............ 104
Figure 10. Implications of TIMSS results for the teaching and learning of algebra. ...................................................................................... 108
Figure 11. Percent right values in algebraic activities for all students. ....... 141
Figure 12. Percent right values in school algebra discourses for all students ............................................................... 141
Figure 13. Percent right values in algebraic activities for students with low achievements. .................................................. 142
Figure 14. Percent right values in school algebra discourses for students with low achievements. .................................................. 142
Figure 15. Percent right values in algebraic activities for students with high achievements. .................................................. 143
Figure 16. Percent right valued for students with high achievements in the test items not selected for inferential analysis ................ 145
Figure 17. Percent right values for all students in the test items not selected for inferential analysis. ................................. 146
Figure 18. Percent right values for student with low achievements in test items not selected for inferential analysis. ......................... 146
Abbreviations

ILSAs International large-scale assessments
TIMSS Trends in International Mathematics and Science Study
IEA International Association for the Evaluation of Educational Achievement
PISA Programme for International Student Assessment
OECD Organisation for Economic Co-operation and Development
SFL Systematic Functional Linguistics
MD The textbook *Matte Direkt*
Y The textbook *Matematikboken Y*
F The textbook *Formula 8*
Acknowledgements (mostly in Swedish)

Det är på jobbet det händer. En dag för vad som känns som tusen år sedan i ett litet arbetsrum på GUC ställde Maria Asplund frågan vad vi skulle arbeta med om vi inte var gymnasielärare. Och sedan sade hon att det nog skulle passa mig att doktorera. Tänk, om jag inte hade kommit på det själv. Jag prisar din visdom, Maria.


Paul Andrews, thank you so very much for your patience with my questions concerning statistics, for showing interest in my work, and for fika. Kirsti Hemmi, all participants in the Nordic Network for Algebra Learning and Uffe
Thomas Jankvist: I am most grateful for the many wise comments and questions you have contributed with during these years. Tusind tak!


Jag vill även passa på att rikta ett särskilt tack till min skugghandledare: Sara Backman Prytz, som mystiskt nog har dykt upp på jobbet i synnerhet de dagar som Johan har varit hemma och vabbat. Ingen har beordrat mig att ta fikapauser, äta lunch och gå hem i tid som du.


Kristina Palm Kaplan  
Uppsala, oktober 2019
1 Introduction

Several countries have sought to implement algebra beginning in the early school years in their curricula (Britt & Irwin, 2011; National Council of Teachers of Mathematics, 2006; Ponte & Guimarães, 2014). As for the Swedish curriculum reform in 2011, algebra became more emphasized in the new syllabus of mathematics, for earlier school years as well as with more details in the descriptions. Even though this Swedish reform was not specific for algebra or even mathematics, the school algebra content in Sweden is an interesting case for studying curriculum reform: in commentary material to the new syllabus in 2011, better knowledge in algebra was particularly requested. One of the reasons for this request was stated to be the low student achievements reported by international large-scale assessments (Skolverket, 2011a). Indeed, the algebra results of Swedish students in these assessments seem to have been low for a long time.

Low results on international large-scale assessments (henceforth ILSAs) have been reported repeatedly, not only for Swedish students but for students in all Nordic countries (Grønmo, Borge, & Hole, 2014; Olsen & Grønmo, 2006). ILSAs such as Trends in International Mathematics and Science Study, TIMSS, and Programme for International Student Assessment, PISA, have been paid an increasing attention. In Germany the government backed a plan of action following PISA 2000 (de Lange, 2007). In Slovenia, TIMSS was used to monitor and evaluate a curriculum reform (Štraus, 2005). New content standards in the U.S have been warranted by low results on these assessments and ILSAs have generated considerable changes in national educational assessments and supervision in Norway and Sweden (Grek, 2017; Nortvedt, 2018; Turgut, 2013). Policy makers use results on ILSAs to initiate and evaluate different kinds of educational reforms. Johansson and Hansen (2018) argue that some countries may even revise their curricula to make them more like the TIMSS’ framework.

Drawing inferences from ILSAs may be problematic, though, for both policy makers and researchers. According to Messick (1989), the validity of an assessment is a judgment of how fair and appropriate the inferences made and actions taken are, as supported by empirical evidence and theoretical principles. This is not just ‘traditional’ validity, i.e., a judgment of whether the assessment measures what it set out to measure or not. Validity in this version is a judgment of the interpretation of the test results, and of how the test results
are interpreted and used as a basis for further actions. Policy makers and researchers should therefore pay close attention to how they make use of the results of ILSAs. Do ILSAs provide descriptions of knowledge trends in countries in such a way that these can be used as bases for educational reform, or are they just another Holy Grail?

The current strengthened position of ILSAs in a Swedish context of educational reform is in contrast to mathematics-specific reforms in the 20th century. Between 1910 and 1960, changes in school mathematics were often introduced and driven textbook producers (Prytz, 2017). The role of the state was to examine and approve textbooks’ correspondence to the syllabus, which was achieved through a textbook review that was mandatory for all school subjects. The textbook review started in 1938. (Johnsson Harrie, 2009; Prytz, 2017). The state became more active as drivers of reform in the 1960–70s during the New Math project, but mathematics textbooks still played a key role in the implementation of the New Math: textbook materials in mathematics were tested and developed before the 1969 syllabus was launched, and textbooks were submitted to review (Prytz, 2017, 2018). Later, the state review process changed and became more of a guideline for teachers to use when choosing textbooks. It ended completely in 1991 and textbooks were from then on produced on a market under very weak regulation (Johnsson Harrie, 2009). This means that textbook authors have a quite independent position against policy makers now, as compared to earlier. What the selection of content in mathematics textbooks in a curriculum reform looks like today in relation to this weak regulation seems to have gone unexplored.

School governance has thus during the last decades become globalized and evaluative through influences from international assessments, as well as deregulated through the changed position of textbooks. Here, the use of ILSAs results may be understood as a part of governing the school system by objectives and results (Sundberg & Wahlström, 2012). Forsberg et al. (2017, p. 373) write:

> In the past the rationale of educational reform in Sweden was grounded in centralism, universality, social engineering, and consensus. This was followed by reforms based on ideas about decentralization, professionalization, and democracy. At present the rationale is based on recentralization, management, quality, and evaluation. These different rationales relate to the power and control that different actors have over education. As a consequence, questions about how the system is governed, how curriculum goals are established, and how content is selected and evaluated are addressed.

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1A joint committee from the Nordic countries was formed in 1960 through political initiative, to develop and try out a new curriculum (both documents and textbooks). The main idea was to base the new curriculum in modern mathematics e.g. set theory, algebra and logic, as well as psychology. It also involved teaching methods and the use of concrete materials (Prytz & Karlberg, 2016).
This means that different rationales for educational reform are given depending on what actors are in power. Concerning the Swedish 2011 curriculum reform with focus on mathematics, both textbooks and ILSAs are important to consider as products of such actors. They may affect the selection of content. Yet, Forsberg et al. (2017) do not mention textbooks, textbook authors, or any subject-specific content.

In relation to the 2011 reform in Sweden and mathematics, it is more appropriate to look at TIMSS than PISA. TIMSS and PISA have different aims, so different inferences can be drawn from the assessments. OECD (2013) claims that PISA assesses students’ mathematical literacy and that the results may be used for cross-country comparisons and for countries’ progressions over time. Mathematical literacy is defined by OECD as the ability to use mathematics in different contexts, and for making judgments and taking decisions as an engaged citizen. IEA (Pierre, 2017) on the other hand, claims that TIMSS assesses students’ mathematics skills and knowledge. The results may be used to measure curricular implementation, to look at knowledge trends within a country over time, and for cross-country comparisons. Mathematical literacy is not mentioned in the TIMSS framework. Mathematics is defined by IEA as a combination of cognitive skills in different content domains, which are, broadly speaking, in common for the curricula in the participating countries. If the aim is to measure changes in students’ knowledge in algebra, to evaluate the effectiveness of a curriculum, or to initiate curriculum reform, one could expect that TIMSS should serve for this purpose. However, to interpret TIMSS results in terms of, for instance, students’ mathematical literacy may be difficult since this is not a part of the TIMSS’ framework. To understand in detail the interpretations and conclusions which can be made from TIMSS results, TIMSS’ test items should also be considered. The importance of understanding ILSAs’ test items has previously been pointed out by Säljö and Radišić (2018).

According to Howson, Keitel and Kilpatrick (1981) there are several stages in a curriculum reform. First, the need for development must be identified. A plan for action must be formed and accepted, for example by creating and deciding on a new syllabus. It is essential that new materials be developed, in the sense that new tasks, textbooks, teacher guides or other resources may be needed in order to implement the reform. Disseminating the new ideas and materials to teachers, and implementing the ideas in classrooms are also vital for the adoption of a new curriculum. Finally, an evaluation, e.g., of students’ results over time, may lead to further development (Howson et al., 1981).

This process becomes even clearer if the curriculum is regarded through the curriculum model put forward by the TIMSS’ textbook study, which depicts the curriculum in four layers (Valverde, Bianchi, Wolfe, Schmidt, &

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2 The content domains for Year 8 are number, algebra, geometry, and data and chance. The cognitive domains are knowing, applying, and reasoning (Mullis, Martin, & Arora, 2012).
Houang, 2002). In this model, the intended curriculum refers to educational system goals and formal steering documents, e.g., course plans which are formulated in the syllabus; the potentially implemented curriculum refers to the textbook as a mediator, which translates the syllabus into more concrete formulations which may be used in the classroom; the implemented curriculum refers to strategies, practices and activities which are used and negotiated by teachers and students in the classroom; and the attained curriculum refers to students’ achievements and the assessment of them. A curriculum reform may thus be interpreted as concerning changes in any of these layers, or in a broader sense, all of the layers.

The focal point for this thesis is the part of the Swedish 2011 curriculum reform that concerns lower secondary school algebra. It will be referred to in this thesis as the 2011 school algebra reform. It involves mathematics textbooks and TIMSS’ test items as the potentially implemented curriculum and TIMSS results as an aspect of the attained curriculum. In accordance with Howson et al. (1981), a curriculum reform is here understood as a process and not just the launching of a new syllabus. It involves textbooks in order to understand the development of new materials in curriculum reform, and TIMSS’ test items in order to understand in what ways TIMSS results may be interpreted and used in curriculum reform. In the thesis, I will describe and compare changes during the period 1995–2015 in characteristics of lower secondary school algebra in mathematics textbooks and TIMSS tests, and explore TIMSS results in order to understand the process of the 2011 school algebra reform.
2 Background

This chapter provides a brief overview of developments in school algebra, of changes in algebra in the Swedish syllabus 1994–2011, and of TIMSS. Different kinds of algebra have been stressed in different times and for different students. In the first section, a short historical account of this dynamic is presented. It involves both a Swedish perspective and an international outlook. In the second section, the Swedish syllabi of 1994 and 2011 are described to outline their similarities and differences with respect to algebra, as well as some other central features. Finally, an introduction to the TIMSS’ assessment framework is given for those who are not acquainted with the IEA studies. This section includes how TIMSS’ validity has been addressed in reports from IEA and, in a Swedish context, by the Swedish National Agency of Education.

2.1 Developments in school algebra

In Sweden, institutionalized teaching for all children evolved during the 19th century but not everyone studied algebra. Reckoning and geometry were taught as two different subjects in Folkskolan, a school for Year 1–6 founded in 1842. Many students only went through this primary education. Geometry was introduced in Year 5. In tandem, mathematics was taught as one subject with arithmetic, geometry and algebra as topics in Realskolan (Year 4–9), the lower secondary education mainly recruiting students from the upper classes. Algebra was taught in Year 6–9 (Prytz, 2015). This did not really change until 1962 when Grundskolan replaced both Folkskolan and Realskolan, and created one school for all in order to put an end to an uneven distribution of children from different social classes.

Internationally, algebra during the 20th century mainly revolves around three aspects (Ponte & Guimarães, 2014). First, algebra was understood as a theory for equation solving and preparation for infinitesimal calculus. The manipulation of algebraic expressions and equations was stressed. Second, the

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3 Realskolan was actually a part of Läroverket, a secondary education which emanated from the medieval cathedral schools. The education in Läroverket was split up in two parts in the early 20th century. These were named Realskolan (year 4–9) and Gymnasiet (year 9–12), respectively. The students who attended Gymnasiet in general quit Realskolan already after Year 8 (Prytz, 2015).
German mathematics professor Felix Klein (1849–1925) and others introduced the idea of connecting algebra to functions and modelling, thus emphasizing applications of algebra. During the first half of the 20th century, functions gradually became central in the syllabi of upper secondary school. Third, by the late 1960s, the New Math movement raised the view of algebra as a structure. According to Prytz (2017), the New math movement was also very influential in the Swedish curriculum reform of 1969. The mathematics syllabus changed, and algebra and set theory came to be introduced in Year 1.

After the brief New Math era, many countries reduced the amount of algebra in the curriculum and different countries stressed different aspects of algebra. Generally, countries stressing realistic problem-solving and word problems put less weight on manipulating algebraic expressions and equations (Kieran, 2007; Ponte & Guimarães, 2014). In Sweden, problem-solving in everyday situations was introduced as a new main area in the syllabus from 1980 and algebra was downplayed (Skolverstyrelsen, 1982). In the 1994 syllabus, this changed and problem-solving was understood as a competence, while algebra was seen as being a tool for problem-solving (Skolverket, 1994). This implies that both the 1980 and the 1994 syllabus may have attempted to connect algebra with functions, modelling and applications. However, these syllabi do not particularly stress functions.

International research trends in the late 1980s gave rise to a new field – early algebra. This field has shifted focus from characterizing algebra in terms of different topic areas, to enhancing processes such as reasoning and representing, and to different activities which can promote the development of algebraic thinking (Kieran, Pang, Schifter, & Ng, 2016). The field has also carried thoughts and evidence of the child as someone who can learn algebra and develop algebraic thinking beginning in the early years. As mentioned in the Introduction, early algebra has become included in different national curricula for primary education since the late 1980s and early 1990s.

Hence, it is possible to speak of three main perspectives on algebra: the ‘traditional curriculum’ where algebra is viewed as having to do with algebraic expressions and equations; the functional perspective where algebra concerns modelling; and the structural perspective where elements of set theory is the focus (Ponte & Guimarães, 2014). In the trend of early algebra, these perspectives may be combined since the enhancement of processes does not exclude specific topics in the same manner as a more content-based curriculum.

It is also possible to speak of three main types of algebra tasks, which have evolved over the years:

From a certain time, algebra began to constitute, in itself, a wealth of problems, giving rise to two large didactic movements – one following a purer mathematics line using problems formulated solely in mathematical terms and the other profusely using word problems within the scope of extramathematical contexts. Upon the consolidation of the concept of function as the central idea
of algebra teaching, a third kind of task has progressively gained pre-dominance – tasks involving modelling of real situations. (Ponte & Guimarães, 2014, p. 471)

The difference between word problems and tasks modelling real situations according to Ponte and Guimarães (2014) thus appears to be connected to the kind of algebra which is targeted. They also write that word problems may turn artificial, because they do not capture algebra in a realistic way, particularly not more advanced algebra.

2.2 Algebra in the intended curriculum, 1994 and 2011

A syllabus is a part of the intended curriculum that is established by the parliament. It regulates the teaching of a school subject. The Swedish word is ‘kursplan’. Besides this, the Swedish National Agency of Education also publishes commentary materials adjoining the syllabus, to explain the syllabus further. Both syllabus and commentary material are regarded as parts of the intended curriculum in this thesis. The commentary material in 2011 can be seen as a guideline for both teachers and textbook authors concerning how to interpret the 2011 mathematics syllabus, since it was published the same year as the new syllabus was launched. It has been argued that the Swedish National tests can be understood as a part of the intended curriculum (M. Johansson, 2003). Still, National tests are not developed because of, nor disseminated before, a new syllabus, and all National test items for Year 9 were classified between 2011 and 2013.4 Below are described those parts in the mathematics syllabi and commentary materials which are relevant for the 2011 school algebra reform.5

The 1994 and 2011 syllabi are both documents of a quite abstract character. As opposed to earlier syllabi, neither of these syllabi involves guidelines on how to teach (Prytz, 2015). Nor are there any concrete example tasks or materials. Moreover, the 2011 commentary material does not include concrete examples. This is a difference as compared with the commentary materials of the 1994 syllabi and its predecessor from 1980, as well as to syllabi from some

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4 It could also be argued that the state-initiated in-service education for teachers, Boost for mathematics, is a part of the intended curriculum. Yet, the Boost for mathematics started with a trial period in 2012 and was subsequently conducted in 2013-2016, i.e., after the 2011 syllabus was launched (Skolverket, 2016). Furthermore, its overarching goal was to scaffold teachers’ reflection on their classroom decisions and afford them a broader set of teaching methods rather than to implement the new syllabus.

5 The comparison does not include the 1994 Grading and assessment or the 2011 Knowledge criteria. These parts of the syllabi do not really include algebra-specific information, so they are left aside. According to Sundberg and Wahlström (2012), this part dominates the 2011 syllabus, focuses on results, and has similarities with standards-based curricula.
other countries. For example, guidelines for textbook writers and concrete examples of tasks are included in Chinese steering documents, and in Germany, concrete examples may be found in the syllabi of different Länder (Jablonka, 2019). The level of details in steering documents has been discussed as a question of the teacher’s discretionary space: the more details, the less possibility “for making micro-curricular decisions” (Jablonka, 2019, p. 7). This suggests that the 2011 syllabus and its commentary material aimed toward more teacher autonomy than earlier syllabi.

The main differences concerning algebra content in the Swedish syllabi of 1994 and 2011 (Skolverket, 1994, 2011b) are summarized in Table 1 and outlined below.

Table 1. Comparison of the syllabi.

<table>
<thead>
<tr>
<th>Syllabus</th>
<th>1994</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structure</strong></td>
<td>General introduction. Goals to strive for (GTS). Structure and character of the subject. Goals to achieve (GTA), specified for Year 1–5 and Year 6–9. No further specification on what year a certain core content should be in focus. No guidelines on how to teach.</td>
<td>General Aim. Core content specified for Year 1–3, 4–6 and 7–9. No further specification on what year a certain core content should be in focus. No guidelines on how to teach.</td>
</tr>
</tbody>
</table>
In the 1994 syllabus, the content is described as goals to strive for (GTS) and goals to achieve (GTA). GTS includes, e.g., concepts, formulae, transformations, equations and inequalities. GTA specifies number patterns and unknown numbers in simple formulae for Year 1–5, and formulae, equations and graphs for Year 6–9. GTA also concerns interpreting and using graphs of real world situations. Problem-solving in real life is emphasized as a competence both in syllabus and commentary material (Skolverket, 1994, 1997).

In the 2011 syllabus, core content is described in detail and for three stages. For lower secondary school, Year 7–9, the content now includes the meaning of the concept of variable and the use of variables. In the commentary material, general descriptions for calculating areas of geometrical figures or for costs of mobile phone contracts are suggested (Skolverket, 2011a). Just as before, patterns are described in the commentary material as a prerequisite for the introduction of the more formal algebra in Year 7–9 (Skolverket, 1997, 2011a).

The 2011 syllabus introduces the heading Relationships and changes. The topics included here – e.g. functions and linear equations – are thus set apart from algebra. This is a difference against the 1994 syllabus, where content was described under one general heading instead of several topic-specific headings. Moreover, the commentary material explicates that algebra is needed in geometry, for further studies and for the core content in Relationships and change (Skolverket, 2011a).

Both the 1994 and the 2011 syllabi can be said to be oriented towards competences⁶. This is not unique for Sweden but follows an international trend where mathematics is understood as processes instead of products (Boesen et al., 2014; M. Johansson, 2003). The focus on processes can be described as taking a competence perspective on mathematics. It builds on ideas of knowing mathematics as doing mathematics and lifelong learning, similar to what has been put forward by the NCTM Standards in the U.S, the Danish Mathematical Competency Framework, the EU and the OECD (Boesen et al., 2014; Sundberg & Wahlström, 2012).

The competence perspective was first adopted in the 1994 syllabus. In the 2011 syllabus the competence perspective was further clarified: students were supposed to be given opportunities for developing their problem-solving, their mathematical reasoning and argumentation, their understanding of concepts, and their ability to communicate their conclusions (Skolverket, 2011b). In other words, the competence perspective entails a lot more than basic skills and procedures. That transformation (of symbolic expressions) is no longer

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⁶ The National Agency of Education has chosen not to use the word ‘competence’ in the Swedish syllabus in order to avoid introducing too many concepts. Still, the National Agency of Education claim that the formulations in the syllabus are in line with the competence perspective on mathematics (Sundberg & Wahlström, 2012).
mentioned in the 2011 syllabus, can be seen in light of the enhanced competence perspective. However, while the competence perspective dominates the part called Aim in the 2011 syllabus, the part called Core content is more content-focussed (Sundberg & Wahlström, 2012).

Following this development, in the 2011 syllabus, problem-solving can be understood both as a core content and as a competence. This is new compared to the 1994 syllabus. To formulate and solve problems or evaluate problem-solving is highlighted as a competence while problem-solving as a content is highlighted in relation to everyday situations or situations relevant to the students (Skolverket, 2011b). Moreover, for generalized reasoning in problem-solving, students need algebra (Skolverket, 2011a).

Compared with previous syllabi, the 2011 syllabus is also more aligned with the international trend of early algebra, discussed in the previous section. Here, the competence perspective is developed in relation to different topic areas (Skolverket, 2011a). Notably, for algebra, the ability to generalize is enhanced in terms of general ways of expressing calculations, general solving methods, general reasoning in problem-solving, and making general descriptions of different situations.

Finally, the formulations about problem-solving in the 2011 syllabus resonate with what is often conceptualized as mathematical literacy or numeracy. According to Jablonka (2015), mathematical literacy was developed as a critique against mathematics as ‘products’, and it can be understood as competences which cross different mathematical content areas or practices, with a focus on the learners’ problem-solving. It thus establishes links between mathematics and everyday practice, as well as with professional practices. The OECD description of mathematical literacy (OECD, 2013) emphasizes the students’ ability to solve problems in the real world, in contexts such as personal, societal, occupational or scientific. From its first assessment in 2000, we know that PISA has been very influential in Sweden (cf. Grek, 2017). It is thus likely that the formulations about problem-solving in the Swedish syllabus are partly due to the influence of OECD. This influence, and others, from ILSAs on the 2011 syllabus, have been further argued for by Sollerman (2019). The 2011 syllabus can be understood as emphasizing problem-solving in the real world with algebra as a tool, embracing the idea of mathematical literacy.

2.3 Trends in International Mathematics and Science Study

The assessment framework in TIMSS builds on cognitive aspects and content aspects of knowledge. The cognitive aspects are: knowing the facts; procedures and concepts; applying knowledge in order to solve problems; and reasoning in
new and complex situations (cf. Mullis & Martin, 2013). One of the four content domains for Year 8 is algebra. The topic areas included in algebra are:

- expressions and operations,
- equations and inequalities,
- relationships and functions.

Students are assessed based upon their abilities to:

- use symbolic expressions, equations and inequalities to represent problem situations,
- transform these and calculate values of expressions in order to solve the problems,
- generalize patterns in numbers, words or symbolic expressions,
- interpret, relate and represent functions with different semiotic resources,
- identify and understand different properties of functions (Mullis & Martin, 2013).

A TIMSS test entails a large number of test items in order to cover all of its cognitive and content aspects. IEA produces a new test every fourth year. Some items are so called trend items and are reused in up to three consecutive tests. For every TIMSS test, new items are added while others are released into the public domain. According to IEA (Mullis & Martin, 2013), this is how the countries participating in TIMSS can have comparable data on the student achievements over the period of years in which they have participated. Hence, the use of trend items is IEA’s way of preserving a similar test structure and addressing test validity over time.

It is not intended that the students who take the test should answer all test items. That would take too much time. Instead, the items are divided into different booklets and a student is tested on the test items in one booklet. A student’s achievement is first assessed as raw scores on the test items in the booklet. The raw score is the actual score of a student on an actual test item. Based on item response theory, plausible values are then calculated to extrapolate what the results would look like if the student had answered all of the test items. The item response theory is thus a guarantee for generalization from the test items taken to all test items in the assessment.

As comparison across groups is central to TIMSS, the assessment uses a set of benchmarks. These are standardized achievement scales, originally based on the achievements of the students from the participating countries in TIMSS 1995. The average of the benchmarks is set to 500 and the standard deviation to 100 (M. O. Martin, 2005). The students’ plausible values are then interpreted in relation to the benchmarks.
2.3.1 About the validity of TIMSS

As mentioned in the Introduction, a main issue when drawing inferences from ILSAs and using their results as a basis for curricular reform, is the question of test validity. There are several reports on the validity of TIMSS, which together implicate the presence of aspects of validity that are not yet accounted for.

IEA’s own reports on test-curriculum matching analysis (Mullis, Martin, & Arora, 2012; Mullis, Martin, & Foy, 2008; Mullis, Martin, Foy, & Hooper, 2016; Mullis, Martin, Gonzalez, & Chrostowski, 2004) show that the difference between the Swedish curricula and the TIMSS tests is not big enough to have an influence on the students’ results in total. This analysis compares the aggregate of TIMSS results in mathematics. The conclusion is based on TIMSS’ surveys taken by their national research coordinators (NRCs), who report on TIMSS’ content coverage in their respective countries’ syllabi. However, this TIMSS survey in 2003 also shows that 67% of the algebra topics are not included in the Swedish curriculum for Year 8⁷, and the remaining 33% is only included for the more able students (Mullis et al., 2004, p. 184). The survey taken by the NRCs thus implies that there are large differences in algebra content coverage between TIMSS and the Swedish curriculum. Even if these differences do not influence the students’ results in total, it remains an open question as to whether or not they influence the students’ results in algebra.

As for Swedish reports ordered by the National Agency of Education, Lindström (2006) studies content coverage by comparing frameworks and test items from TIMSS 2003, PISA 2003 and a Swedish national evaluation from 2003. The proportion of algebra tasks in TIMSS 2003 is compared with the proportion of the number of pages spent on algebra in a popular Swedish textbook series; where the proportion is calculated in relation to all topics which are represented in TIMSS, and covered by the textbooks for grade 7 and 8 (Lindström, 2006, p. 45ff). These proportions are found to be approximately the same. Still, this comparison hinges on the assumption that the specific chosen textbook series gives a good representation of the syllabus. This assumption is not scrutinized by Lindström (2006).

Another report from the National Agency of Education by Sollerman and Pettersson (2016) compares the framework’s content descriptions and test items of TIMSS 2015 to the Swedish syllabus of 2011 and the test items of the Swedish National test in 2013. The report finds that the TIMSS’ framework is mainly in agreement with the Swedish syllabus of 2011, and the

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⁷ As already stated, the goals in the Swedish syllabus are written for year 7–9. The national research coordinators are asked to estimate which items more than 50 percent of the students in year 8 have met. This estimation includes experiences from researchers, people at the National Agency of Education working with the syllabus and national tests, and experienced teachers (M. Axelsson, personal communication, August 28, 2019).
TIMSS’ test items are concordant with the Swedish national test items. The comparison is not entirely unproblematic, since TIMSS is conducted in year 8, the Swedish syllabus entails goals for Year 7–9, and the National tests are given in Year 9. What is tested in Year 8 by TIMSS may very well be taught in Swedish school Year 9. The comparison by Sollerman and Pettersson (2016) solves this problem by selecting for analysis only those test items which are deemed by TIMSS’ national research coordinators, as being relevant for Swedish students in or before Year 8. However, as described above, TIMSS’ test-curriculum matching analysis involves large discrepancies when it comes to algebra. In fact, of the 37 test items in TIMSS 2015 Year 8 reported by the NRC as not covered in the Swedish curriculum (IEA, 2013), 22 items are categorized as algebra by TIMSS, or 58 percent. The missing values in the comparison by Sollerman and Pettersson (2016) are thus highly skewed in relation to TIMSS’ algebra. This may not be a problem if one aims at drawing inferences solely from the aggregate of TIMSS results in mathematics; but as the commentary material to the 2011 syllabus shows, results from ILSAs are also used to draw inferences about students’ subject-specific knowledge such as algebra (Skolverket, 2011a).

Both Swedish reports build their comparisons on categories that are partly based on the aspects of content and cognition in the TIMSS’ framework. All of the reports above discuss to what extent TIMSS’ test items make a good representation of the national curriculum, i.e. the relevance of using TIMSS as a measure of Swedish students’ knowledge in mathematics. None of them investigates other aspects of test validity such as value implications of using ILSAs’ results, the interpretability of TIMSS results over time as trends in knowledge, or the relevance of using TIMSS results to understand student achievements in subject-specific areas such as algebra. In the chapter Theoretical considerations, aspects of test validity are further discussed. Research on ILSAs is described in the chapter Previous research.
3 The aim of the thesis

The aim of this thesis is to understand the process of a curriculum reform. Textbooks can be seen as a potentially implemented curriculum, which functions as a link between the syllabus and the classroom. Textbooks can also be considered to fulfil a function in curriculum reform through concretizing reform ideas of a new syllabus for use in classrooms. ILSAs can be seen as instruments for measuring students’ knowledge, as an attained curriculum. Results in ILSAs can also be considered to fulfil a function for evaluating curriculum reform and for identifying needs for further action. In this thesis is investigated in what ways textbooks and ILSAs may constitute prerequisites for planning and acting out curriculum reform. Specifically, the algebra part of the 2011 curriculum reform is studied since knowledge in algebra was identified as an area which needed improvement.

Two different frameworks are used to describe algebra characteristics in algebra tasks. A social-semiotic framework is developed to identify school algebra discourses, and a framework of algebraic activities is adapted to categorize what sort of algebra the student is supposed to engage in.

To address how mathematics textbooks change in connection to a curriculum reform, algebra characteristics in textbooks’ algebra tasks during the period 1995–2015 are compared. Changes in the algebra characteristics are then linked to reform ideas on algebra between 1994 and 2011 in the mathematics syllabus.

To address the use of ILSAs in curriculum reform in mathematics, aspects of TIMSS’ validity are investigated. First, algebra characteristics of TIMSS’ algebra items from four consecutive assessments, 2003, 2007, 2011 and 2015, are described. This is done in order to capture the structure of TIMSS and changes in this structure over time. The same two frameworks are used for this comparison, as for the textbook tasks.

In relation to the algebra characteristics in TIMSS, Swedish students’ achievements on the algebra items from the four TIMSS tests are then explored. This is done to identify potential differences in results related to different algebra characteristics. If the structure of TIMSS changes over time,

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8 In this thesis, the structure of TIMSS is understood as the school algebra discourses and algebraic activities which are identified in a TIMSS-test, and the distribution of these algebra characteristics in a TIMSS test. The structure of TIMSS over time may hence show potential differences in the algebra of the TIMSS-tests.
such differences imply that the relative difficulty of TIMSS may not stay the same. If so, both how TIMSS results are interpreted as trends over time and the relevance of using TIMSS results in curriculum reform, may be affected.

Finally, the diachronic interplay of different algebra characteristics in TIMSS is compared to the algebra characteristics of the textbooks and their changes. This comparison is done in order to address the relevance of using TIMSS results to evaluate Swedish students’ knowledge in algebra, during the time period before and after the 2011 school algebra reform. This comparison may also address potential consequences of using TIMSS results in curriculum reform. The research questions are:

1. What characterizes algebra tasks in Swedish textbooks for Year 8?
2. In what ways and to what extent do algebra characteristics in Swedish textbooks’ algebra tasks change over time?
3. What characterizes algebra items in TIMSS?
4. In what ways and to what extent do algebra characteristics in TIMSS’ algebra items change over time?
5. To what extent do students’ results significantly differ on TIMSS’ algebra items in relation to the different algebra characteristics of TIMSS’ algebra items?
6. In what ways do algebra characteristics in the textbook tasks and the TIMSS’ test items differ?

The questions are answered through investigating algebra tasks in six Swedish mathematics textbooks in three series for Year 8 from the curricula of 1994 and 2011; algebra items for Year 8 from TIMSS 2003, 2007, 2011 and 2015; and student achievements on the algebra items in these TIMSS tests.

Moreover, the answers to the questions are put in a wider context of a curriculum reform. In the Theoretical consideration, a model for this context is developed from Valverde et al. (2002) and Howson et al. (1981). Through this model, textbook tasks and TIMSS’ test items are related to each other as parts of stages in the process of the 2011 school algebra reform.

3.1 The sub studies of the thesis

In the first study, a method for identifying school algebra discourses in textbook tasks in algebra is developed, with a social-semiotic framework. Focus is on how the view of algebra and the student is constructed in text through linguistic resources. The paper answers to the first research question.

The second study focuses on how mathematic textbooks change in relation to the syllabus in 2011 school algebra reform. Adding algebraic activities as a second framework of algebra characteristics, the two types of algebra characteristics in the textbooks are compared. The comparison shows in what ways
and to what extent algebraic activities and school algebra discourses change. These changes are then viewed in relation to reform ideas on algebra introduced in the 2011 syllabus. The paper answers to the first and second research question.

In the third study, algebra characteristics – i.e., algebraic activities and school algebra discourses – are analysed with the same frameworks as before, but in four consecutive TIMSS tests. The proportions of the algebra characteristics in these four tests are then compared to understand differences in the structure of TIMSS over time. Student achievements on the TIMSS’ algebra items are explored with statistical inferential analysis in relation to the algebra characteristics. The findings are then viewed against the structure of TIMSS over time. The paper provides answers to the third, fourth and fifth research question.

The final research question is answered in the first part of the chapter Discussion and conclusions through a comparison of the diachronic interplay of algebra characteristics in the textbooks and the TIMSS tests.

3.2 Outline of the thesis

In the Introduction chapter of this thesis, it was argued that mathematics textbooks and ILSAs are relevant in order to understand a curriculum reform in mathematics as a process. The Background chapter gave a short historical account of school algebra and compared the syllabi of 1994 and 2011. TIMSS and some issues of validity concerning TIMSS were also described. This chapter has developed the aim and research questions of the thesis.

The following chapter, Previous research, provides an insight into research fields that are relevant for the thesis: curriculum theory and research on curricular reform in mathematics; research on textbooks in mathematics; studies of school algebra; and research on (mostly international) large-scale assessments. The fifth chapter discusses theoretical considerations involving a model for interpreting the results, a conceptualization of test validity, and two analytical frameworks.

The sixth chapter, Analysis and methods, is comprised of several parts. First, the material is characterized. Then, two qualitative analyses of the school algebra discourses and algebraic activities are outlined. This is followed by an account of the statistical analysis. After a description of how the algebra characteristics are compared over time, the Analysis and method chapter ends with methodological and ethical considerations.

The seventh chapter presents the results of the thesis. It consists of a summary of the three papers in the thesis. It is followed by the chapter Discussion and conclusions.

As stated above, the first part of the chapter Discussion and conclusions starts with a comparison of the didactic interplay of the algebra characteristics
in the two materials. How changes in the textbooks are related to ideas in the 2011 syllabus are described. This is followed by a discussion on aspects of TIMSS’ validity with respect to the 2011 school algebra reform. Some implications for the teaching and learning of algebra are raised. The discussion is closed by three short sections: the reflection upon the selection of test items for the statistical analysis, the overall contributions of the thesis, and suggestions for further research. Finally, in the ninth and last chapter, a summary in Swedish of the entire thesis is given.
4 Previous research

Understanding mathematics textbooks and ILSAs in curricular reform on school algebra is a complex problem. Curriculum theory and research on curriculum reform can give a picture of the ways in which the selection of content has been addressed. This overview constitutes the first part of the chapter.

Three aspects of research on textbooks in mathematics are examined in the second part of the chapter. Content analysis is described considering school algebra. Linguistic and discourse analysis of textbooks in mathematics is presented in order to give the reader an idea of the affordances of such perspectives on mathematics. Research on the use of textbooks in mathematics is then highlighted to understand what conclusions can be made from research on textbooks.

In the third part of the chapter, studies of school algebra are presented. Finally, studies concerning large-scale assessments, their validity, and their items in relation to student achievements are discussed.

4.1 Curriculum theory and research on curricular reform in mathematics

Curriculum theory has a long tradition in Sweden. As mentioned in the Introduction, Forsberg et al. (2017) remarked upon how governance is a question of power and control not only of the school system, but also of curriculum goals and selection and evaluation of content. Forsberg et al. (2017) differentiated between system governance such as formal power-coercive rules, and management processes where (groups of) individuals exert influence. These individuals were, e.g., politicians, professionals, school principals and teachers. The empirical material in curriculum theory has often consisted of different kinds of policy documents, and in some cases interviews (cf. Helgøy, 2006; Ofstedal Telhaug, Asbjørn Mediås, & Aasen, 2006; Segerholm, 2009).

As an example, Sundberg and Wahlström (2012) investigated shifts in educational reform through discourse on what knowledge is legitimized, between the 1994 and 2011 Swedish syllabi. They found that the curriculum reform in 2011 supported a denationalized and instrumental conception of education, with influences from EU and the OECD. The 2011 syllabus was described as a standards-based curriculum in the sense that the focus was on its Knowledge
demands, which appeared to stress results and specified outcomes. While Sundberg and Wahlström (2012) investigated subject-specific parts of the syllabus, their reasoning and conclusions were general. In addition, their focus was upon policy documents, that is to say, the intended curriculum, not the potentially implemented.

During the 20th century, the governance of school in Sweden was centralized with nationally specified and managed conditions. The 1970s and the early 1980s lead to a decentralization with more local autonomy, while the period after 1985 can be characterized as becoming increasingly governed by objectives and results and influenced by ILSAs and neo-liberalism. According to Segerholm (2009), quality assurance, evaluation policy, and the activities which emerged from them was a new means for governing schools. This picture has been complemented by descriptions of how local authorities in Sweden gained influence during the 1990s at the same time as the choice of schools, privatization, and school vouchers contributed to marketization and competition (Börjesson, 2016; Quennerstedt, 2006). This research thus acknowledged that ILSAs have gained influence. Within recent curriculum theory research, linear top-down models of curricular change have been questioned (Nordin & Sundberg, 2018; Sundberg, 2005). Curricular change may instead be considered as processes of recontextualization or translation (Nordin & Sundberg, 2018). Nevertheless, the curriculum theory research mentioned above have not looked further into how the influence of ILSAs on the policy level relates to the selection and evaluation of subject-specific content, such as mathematics. Nor has it looked at recontextualizations or translations related to textbooks: for example, textbook authors and publishing companies were not mentioned as exerting influence when management processes were discussed by Forsberg et al. (2017), or as actors in curriculum change by Nordin and Sundberg (2018).

Textbooks have been highlighted in studies on mathematic-specific curricular reform. Using textbooks, syllabi, teacher journals and reports from development projects, Prytz (2017, 2018) showed how textbook producers were important actors in reforms of the Swedish mathematics curriculum during a large part of the 20th century. The implementation of the 1994 curriculum reform has been scrutinized with classroom observations, teacher interviews and teacher surveys (Boesen et al., 2014). One of the conclusions was that National tests and a new syllabus were not sufficient means of governance for implementing the new competence perspective that was mentioned in the Background chapter. For 35% of the time spent on a lesson, the students worked only with procedures. However, neither Prytz (2017, 2018) nor Boesen et al. (2014) studied curricular reform with a focus on algebra or in relation to ILSAs.

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9 The studies in question focus on the period 1910-1980.
This thesis draws attention to the selection of content in a curriculum reform, namely algebra, and on how this selection relates to the increased influence of ILSAs as well as to the more traditional influence of mathematics textbooks.

4.2 Research on mathematics textbooks

The field of mathematics textbook research has been dominated by content analysis, both in a Nordic and an international (mostly English-speaking) perspective (Fan, Zhu, & Miao, 2013; Rezat & Strässer, 2015). Many studies have seen gaps between syllabi and mathematics textbooks. Why these gaps occur and how they may be closed is “a challenge for researchers, textbook developers, policy makers and teachers” (Fan et al., 2013, p. 638). Textbook studies also have concerned the use of mathematics textbooks and their impacts in the classroom; while less common topics have regarded, e.g., methodological questions or issues of gender, equality, and value (Fan et al., 2013). In relation to research on textbooks in mathematics, the present thesis relates changes in textbooks to curricular reform through linking these changes to ideas on algebra which are expressed in the 2011 syllabus.

Research on content in mathematics textbooks is described first in this section. It is followed by linguistic and discourse studies of mathematics textbooks. The section ends with a short discussion on the use of mathematics textbooks. For the sake of brevity, ‘textbooks’ in the following always refers to ‘mathematics textbooks’. 

4.2.1 Content analysis of mathematics textbooks

Content analysis of textbooks mostly has involved comparisons, limited topics, problem-solving, cognitive demands in tasks, and didactical aspects such as progression (Fan et al., 2013; Rezat & Strässer, 2015). This is the case for Nordic textbook studies of school algebra as well: Häggström (2008) compared algebra content in Chinese textbook tasks with Swedish textbook tasks, for Year 8. Partanen and Tolvanen (2017) studied algebra content in textbooks. Their work was a part of a development project aiming at designing early algebra teaching materials in Finland, and was directed towards primary school. Lundberg (2011) investigated the concept of proportionality, and Kongelf (2015) the introduction of symbolic algebra, with its conceptions and possible misconceptions. Both used qualitative content analysis. Kongelf (2015) found that generalized arithmetic – which he described as the gradual transition from an operational to a structural understanding of algebra – was only prevalent in Norwegian textbooks to a small extent. Manipulations dominated the textbook content despite the fact that generalized arithmetic was a part of the Norwegian syllabus for lower secondary school. Lundberg (2011) on the other hand compared textbook tasks with national tests in Sweden.
While gaps were observed in these studies between the textbooks on one hand and the syllabus and respectively the national tests as a part of the intended curriculum on the other, Kongelf (2015) and Lundberg (2011) have not addressed these gaps in relation to curricular reform nor the changes of these gaps over time. Therefore, they did not answer questions about how the algebra content changes in curricular reform.

A historic comparison of school algebra was given by Jakobsson-Åhl (2006) in a phenomenographic content analysis of upper secondary school algebra in Swedish textbooks from 1960–2000. School algebra was emphasized as a skill to manipulate expressions; as structures; and as applications in geometry and everyday life. More specifically, Jakobsson-Åhl (2006) found that in the early 1960s, manipulations of literal expressions dominated and in word problems students were required to construct models related to physics and finance. During the New Math era, in the late 1960s and early 70s, the skill to manipulate was downplayed. Instead graphs and tables were stressed, and number structures formed the basis for all mathematics. Word problems also became connected to geometrical applications and the students were required either to construct or to use given models. After the New math era, between 1978 and 1994, algebra was embodied as equations and expressions. Tables and graphs, and word problems were still a part of the algebraic content and new mathematics might be introduced through a problem-solving approach. After 1994, alongside equations, expressions, tables and graphs, the textbooks of upper secondary school related to word problems with societal and everyday applications. Mostly, models were given and students were required to use them. Furthermore, tasks about relationships and number patterns appeared (Jakobsson-Åhl, 2006). These results are much in concordance with the international trends in the algebra curriculum, as discussed in the Background chapter. The study did not relate the development of algebra in textbooks to curricular reform or address gaps between steering documents and textbooks.

The implementation of the syllabus in textbooks was addressed by Johansson (2003) in a content analysis of three textbooks for Year 7 in one series, following different syllabi. She found that the content did not change appreciably, though new sections on, e.g., problem-solving and thematic work were added after the syllabus reforms of 1980 and 1994. Moreover, content in the textbooks did not mirror the syllabus reforms. New formulations such as the status of authentic problem-solving in the 1980 reform and the importance and role of mathematics in the 1994 reform, were not much addressed by the textbooks. Most of the word problems throughout the books concerned private economy, while word problems relating to other school subjects were halved in the last of the textbooks. Concerning the role of mathematics, only narrative text blocks and word problems were analysed. This implies that other kinds of tasks were not considered as a means to convey any role of mathematics. As is described in the following section though, this may be an unwarranted assumption.
4.2.2 Linguistic and discourse analysis of textbooks in mathematics

There are reasons to believe that linguistic methods and studies of discourse can add to the research on textbooks in mathematics. According to Herbel-Eisenmann (2007), the NCTM Standards proposed a shift away from teacher and textbooks as authorities and towards students’ own reasoning and participation, a clear reform idea. In her study of one textbook section on mathematical modelling designed to follow the NCTM Standards, the ideas she identified regarding authority were mixed. While humans were presented in the text as doing mathematics in different contexts, the linguistic choices in the text presented mathematics as absolutist, and the relation between the author and reader as unequal and authoritative: the latter may not even be possible to change in textbooks. Herbel-Eisenmann’s (2007) study can thus be seen as an implication of consequences for all studies on textbooks, of how the syllabus is transformed through the textbooks: linguistic choices in the textbooks affect and may even undermine the intentions of the policymakers. Findings similar to Herbel-Eisenmann’s (2007) were made by Le Roux (2008). She found that a mathematics course could be reformed with the aim of helping marginalized students, and the tasks used in the course might still hinder these students’ access to the mathematics through their discursive features. Le Roux’s (2008) analysis of one typical problem in a reformed mathematics course in South Africa showed how the discourse has transformed from being procedural to conceptual, at the same time as a traditional word problem was reproduced. This means that in order to solve the problem, the mathematics was more important to understand than the context. The students were positioned as already knowledgeable of mathematics instead of as apprentices.

Alshwaikh (2016) performed a multimodal analysis of the mathematical discourse concerning geometry in Palestinian textbooks10. His results showed how mathematics was constructed as specialized, in the sense that mathematical objects were presented rather than mathematical activity which produced objects. The student was mostly positioned as a ‘scribbler’ through engaging in material processes, “e.g. use, cut, write, measure, calculate” (Alshwaikh, 2016, p. 173). Dowling (1996) stressed textual features in a study of two different textbooks from the UK. He found that the textbooks distributed different kinds of messages to students. On one hand, mathematics was “presented as having potentially universal descriptive power” (Dowling, 1996, p. 407). On the other hand, mathematics was presented as a prerequisite “for optimum participation in the public domain” (Dowling, 1996, p. 406). Students were thereby offered to learn either about mathematics or about themselves, depending on what textbook they worked with. Generally speaking, Dowling

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10 In Palestine, there is one textbook for each subject. They are supervised by the Ministry of Education (Alshwaikh & Morgan, 2014).
(1996) claimed that it is not possible to maintain practices, e.g. in domestic and manual situations, when the practices are textualized into mathematics, nor to maintain mathematics in a text about any other practice.

In a Nordic perspective, Ebbelind and Segerby (2015) have examined, among other things, how Year 4 students were invited to participate in mathematics by working on their own in silence, through an analysis of linguistic choices in textbooks. The studies mentioned in this section point out how mathematics is linguistically constructed in different ways in relation to the student: as something one is to scribble; as something which can help the student to describe almost anything or to cope with daily matters; or as something which is to be carried out on one’s own.

According to Österholm and Bergqvist (2013), many claims about mathematical texts are not sufficiently founded empirically. They surveyed such claims about linguistic features in research literature. Their aim was also to test the validity of the claims empirically, as found in Swedish mathematics and history textbooks. Compared with history textbooks, mathematics textbooks included fewer nominalizations, less complex sentences and had an inclination towards more human agents in the text. The study was thus restricted to the linguistic features themselves.

It seem as if relatively few studies of discourse in textbooks have been made (Jones, Bokhove, Howson, & Fan, 2014; Ryve, 2011), and especially so within a Nordic perspective. The studies mentioned above exemplify how studies with a focus on language and discourse may fill a lacuna in research on textbooks in mathematics, through emphasizing questions of equity, inclusion, how students are positioned, and of linguistic complexity. Still, they do not investigate school algebra and many of them appear to promote methodological contributions with detailed analyses of fairly small parts of textbooks. In relation to studies of discourse and language, this thesis contributes by exploring the field. It also develops a method for analysing discourse through linguistic patterns.

4.2.3 The use of mathematics textbooks

Research on textbook use entails the problem of defining both what ‘use’ means, and whose use is meant (Rezat & Strässer, 2015). In this section, studies of how teachers and students use textbooks are described. Textbook use is discussed in the sense of the textbooks’ impact on mathematic lessons and their impact on student achievements.

In the US, the use of curriculum materials such as teacher guides, lesson plans and textbooks has been stressed in relation to curriculum reform in mathematics. A series of case studies on how teachers used curriculum materials based on the NCTM Standards reported upon ways in which different kinds of support for classroom enactment by the materials and different mathematical knowledge for teaching, MKT, framed the implemented curriculum (cf.
Charalambous & Hill, 2012; Hill & Charalambous, 2012). They found that teachers with a high MKT could compensate for teaching materials with a low degree of support, while unsupportive teaching materials could lead to problematic instruction for teachers with low MKT. Additionally, Remillard, Harris and Agodini (2014) compared four different US curriculum materials and their learning opportunities, drawing on a large-scale empirical study of student outcomes.

According to a comparative study in France and Norway, similar egalitarian values in the syllabi have rendered very different teacher practices, related to different educational traditions in the countries (Pepin, Gueudet, & Trouche, 2013). Syllabi, textbooks, and teacher practices were investigated. Specifically, the textbooks’ introductions, the introductions to the topic of geometry, and task differentiation were analysed. The results pointed towards the textbook as a crucial link between the syllabus and the implemented curriculum. It influenced the teacher’s activities in the classroom as well as what “views of the subject matter that students might develop” (Pepin et al., 2013, p. 696). Similarly, Swedish primary school teachers experienced different degrees of support from two separate textbook series and their teacher guides. There were differences as to what extent teachers reported that students work at their own pace with the two textbook series (Neumann, Hemmi, Ryve, & Wiberg, 2015). When it comes to students’ use of textbooks, a study by Sidenvall, Lithner and Jäder (2015) indicated that Swedish upper secondary students used textbooks for task-solving. Theory and worked examples were hardly used. Upper secondary students also worked a lot on their own with task-solving (M. Johansson, 2006). The textbook influenced both what tasks students worked with and what examples and concepts teachers chose for discussion.

In relation to the present thesis, the above mentioned results are interesting since they point to the textbook’s influence on both classroom activities and the view of mathematics enacted in the classroom. They also point to the complexity when teachers with different knowledge use textbooks in different ways, and that Swedish students do a lot of task-solving on their own.

Finally, in a methodological contribution, Törnroos (2005) compared opportunities to learn in Finnish textbooks for Year 5–7 with Finnish students’ results on TIMSS 1999. Textbook tasks functioned well as measures of opportunities to learn, especially when long-term data – in this case nine textbooks from three consecutive school years – was analysed. A comparison of tasks with test items also gave a better picture of opportunities to learn than did questionnaire data on what books had been used, or a comparison of textbooks with the TIMSS’ framework (Törnroos, 2005). This implies that comparing TIMSS’ test items with textbook tasks in the present thesis is a good choice. It can be used for discussing the relevance of using TIMSS results to evaluate Swedish students’ knowledge in algebra.
4.3 Studies of school algebra

Internationally, studies of school algebra typically have concerned how students learn and understand algebra or how early algebra can be incorporated in classroom teaching. Many studies have underlined the central role of the teacher for students to learn algebraic thinking, and claimed that students do not learn to think algebraically without the conscious support of a teacher (Cai & Knuth, 2011; Kieran, 2018; Kieran et al., 2016).

Despite these findings, students’ learning of early algebra and algebraic thinking is not merely a question of the implemented curriculum. Britt and Irwin (2011), Cai (2014), and Blanton et al. (2015) focussed on the intended and the attained curriculum. However, they have not distinguished between syllabi and concrete curriculum materials such as textbooks. Thus, only ‘curriculum’ is used in the description below of these three studies.

Britt and Irwin (2011) compared the results of young students following the New Zealand Numeracy Project, a curriculum which stressed algebraic thinking within arithmetic, with results of students who were following a more traditional curriculum. According to them, the curriculum mattered in terms of significant differences in results on post-tests between student groups which were using the different curricula. These differences in results were persistent also when algebra that was more formal was introduced in secondary school. Similar results were reported by Cai (2014) who compared the impact of a NCTM Standards-based curriculum on middle school students’ achievements with that of a more traditional curriculum. The Standards-based curriculum was described as having a functional approach and the more traditional curriculum, a structural approach. As the students proceeded to high school, differences in their achievements from middle school remained. The importance of the curriculum was also underlined by Pang and Kim (2018). The Korean curriculum did not explicitly mention early algebra, but Pang and Kim (2018) found that Korean students achieved in parity with students in a U.S early algebra implementation study by Blanton et al (2015). They credited the results to a generalized arithmetic perspective in the Korean curriculum.

In relation to Nordic research, the present thesis fills a gap by studying the development of lower secondary school algebra in a curriculum reform process. In the Nordic countries, classrooms and the implemented curriculum have been in focus for research on school algebra, though studies about teachers and student teachers have also been conducted. This research concerned engagement and participation, knowledge progression and transposition, and algebraic proficiency. Small groups in classroom situations of early algebra problem-solving and equation solving were studied (Smedlund, 2017; Tuomela, 2017), aiming at understanding Finnish’ students’ participation, interaction, and engagement. Persson (2010) aimed to improve algebra teaching and learning in a longitudinal study in Swedish upper secondary school. Strømskag (2015, 2017) discussed pattern generalization and suggested a model for instruction, based on what
characterized a didactical situation when Norwegian student teachers solved an algebraic generalization task. In Year 6 and 7, Nyman and Kilhamn (2015) studied how Swedish teachers discussed engagement in algebra while Lundberg and Kilhamn (2018) investigated how knowledge is transposed from the textbook to the classroom in a task on proportions in an everyday context. These two studies described teaching where the mathematical content was not stressed. The teachers focussed on the lessons’ organisation and the didactical contract while the “mathematical problem seems taken for granted” (Nyman & Kilhamn, 2015, p. 623). The knowledge transposition was affected by the everyday context of the task on proportions, which limited the possibilities to engage with the mathematical idea of proportionality (Lundberg & Kilhamn, 2018). Naalsund (2012) studied why algebra is difficult for many Norwegian students. She justified the investigation by referring to the low Norwegian ILSA-results in algebra. Using released algebra items from TIMSS 2003 and adapted versions of these items, students’ written solutions, types of errors, and students’ reasoning about their solutions on the test items were analysed.

A more comprehensive view of school algebra has been taken by the project Towards research-based teaching in algebra – diachronic and synchronic analyses of policy documents, teaching materials and teacher interaction with them (cf. Bråting, Hemmi, Madej, & Röj-Lindberg, 2016; Bråting, Madej, & Hemmi, 2019; Hemmi et al., 2017). The project aimed at characterizing school algebra and identifying teaching traditions in Sweden. The findings so far has shown that the learning trajectories in school algebra were not very clear, regardless of whether steering documents or teachers’ talk about algebra were studied. In focus group interviews, teachers mostly discussed algebra in terms of equation solving and the meaning of the equal sign. Moreover, generalized arithmetic was scarcely found in the syllabus, nor in textbooks for Year 1–6. These latter findings are in concordance with Kongelf’s (2015) and they have been hypothesized by the project to be an explanation for low results on TIMSS and PISA. However, the project has not studied ILSAs and therefore cannot make substantial claims about results on ILSAs as related to algebra in textbooks. In relation to that, the present thesis fills a gap by comparing algebra in a Swedish context to algebra in a large-scale assessment.

4.4 Research on large-scale assessments

Because ILSAs have gained a lot of attention, it is not surprising that research on ILSAs has increased as well. This section focusses on policy research on ILSAs, secondary analyses and critical research on ILSAs. Research on test items from large-scale assessments is also discussed.

ILSAs relation to, or impact on, curricular standards in mathematics seems almost unexplored, but PISA has been studied for its impact on policy (cf. Hopfenbeck et al., 2018; Lindblad, Pettersson, & Popkewitz, 2015; Säljö &
Radišić, 2018). For instance, low PISA-results in Norway made way for a new national quality system (Nortvedt, 2018) and the OECD country review led to the formation of the Swedish School Commission (Grek, 2017). A recent study by Johansson and Hansen (2018) however discussed the selection of mathematics specific content in relation to ILSAs. They compared national mathematics curricula from the countries participating in TIMSS to the TIMSS framework over time, by looking at TIMSS’ test-curriculum matching analysis. They found that there was a trend of harmonizing the topics in the mathematics curricula to the TIMSS framework.

Studies which have used ILSAs’ data to perform secondary analyses constitute a large part of the research on ILSAs (Hopfenbeck et al., 2018; Lindblad et al., 2015). This research has aimed at explaining student performances in mathematics by using background factors such as gender, socio-economic situation, or instructional practices. An example is Eriksson, Helenius and Ryve (2018). In a study of the effectiveness of three different instructional practices, they found that a teaching practice relating to daily life was negatively correlated to students’ mathematics achievements in TIMSS tests, while a practice of memorizing formulas and listening to the teacher was positively correlated to the achievements. Applied problems may be related to real-life situations in the TIMSS framework (Mullis & Martin, 2013), but relating to daily life is not at the core of the frameworks’ theoretical principles. Using TIMSS to draw conclusions about how students solve problems that relate to daily life is then not especially relevant: TIMSS’ test items may not capture ‘daily life’. This in turn suggests that Eriksson et al.’s (2018) result is based on circular reasoning. It becomes visible when the look of the test items is accounted for. The study by Eriksson et al. (2018) exemplifies how secondary data analyses may take for granted in what ways ILSAs’ results may be interpreted, and how they may use ILSAs’ results in quite unproblematic ways, i.e. without accounting for the actual test items.

Secondary data analyses have also been be comparative and cross-country. A compilation of this kind of studies on results on ILSAs from the Nordic countries has been made concerning, e.g., mathematics teaching on classroom and school levels, teacher attitudes, and high and low performing students in Year 4 (Hansen et al., 2014). An interesting contribution in this compilation is the study by Grønmo et al. (2014), who have compared trends in Nordic countries over time, based on ILSAs’ results. The intended curriculum as reported by TIMSS national research coordinators in Nordic countries showed discrepancies compared with what was reported as the implemented curriculum by the teachers. Specifically, in Sweden the national research coordinator reported a larger amount of algebra in Year 8 than did the teachers. Given that the intended curricula in Norway and Sweden did not have goals for each year, and the reliance on textbooks in these countries – also as reported by TIMSS – was above the international mean, Grønmo et al. (2014, p. 130) concluded:
This indicates that it is the publishers of textbooks for schools, who, to a great extent, decide what should be the objectives for mathematics each year in school […] The problem is likely to be a significant factor influencing the low performance of students in algebra […].

Grønmo et al. (2014) thus acknowledged textbook publishers as an important actor for the selection of content, with the power to affect student achievements. In relation to the present thesis, Grønmo et al. (2014) have pointed out the relevance of studying textbooks in a curriculum reform process.

Critical research on ILSAs has asked questions about ILSAs’ reliability and validity. The cognitive test constructs have been questioned. A large number of methodological questions has been addressed, such as the design of student questionnaires, sampling, the use of plausible values\(^\text{11}\) and measurement bias (Hopfenbeck et al., 2018). The possibilities of cross-cultural and cross-lingual comparison have also been addressed. The results of these studies were somewhat ambiguous, but translation problems might result in differing item difficulty in different countries (Hopfenbeck et al., 2018). Given what critical research has been performed, it seems as if the critique has not concerned comparisons over time. Yet, if the difficulty of an ILSA did change over time, it would affect the possibility to interpret ILSAs’ results in terms of achievements trends in countries.

Building on the report by Sollerman and Pettersson (2016) mentioned in the Background, a recent thesis (Sollerman, 2019) investigated the relevance of ILSAs for the Swedish context. The investigation was done in three parts. First, the contents, frameworks, and test items of TIMSS 2015 and PISA 2012 were compared to the Swedish syllabus of 2011 and the Swedish National test-tasks, respectively. Then the implementation of TIMSS and PISA were compared to the Swedish national test situation. Finally, the results of TIMSS and PISA were compared to students’ final grades in Year 9. Sollerman’s (2019) conclusion was that not all aspects of the Swedish mathematics syllabus were tested in TIMSS and PISA. The concordance between TIMSS and PISA and the Swedish context was quite good though, so these ILSAs are relevant for looking at how Swedish results in mathematics develop. However, the study did not involve a comparison over time and, as commented in the Background chapter, more than half of the missing values in Sollerman’s (2019) TIMSS material involved algebra items. This suggests that there are good reasons to investigate algebra over time in TIMSS. It may say something about the possibility of interpreting TIMSS results over time as being a knowledge trend.

The project The Evolution of the Discourse of School Mathematics, EDSM, has studied changes over time in mathematical discourse (Morgan, 2016b; Morgan & Sfard, 2016a; Morgan & Tang, 2016). This project sought

\(^{11}\) As described in the Background, students do not answer all items in TIMSS. Plausible values are used to extrapolate student achievements from the test-items taken in one booklet to the entire body of test-items.
to understand the mathematical discourse in a UK high-stake large-scale assessment. EDSM showed that mathematical discourse in the assessment items has changed between 1980 and 2011, in the manner that manipulation tended to be required for its own sake and not as earlier, in the context of problem-solving. The project saw how discourse in the assessment items constructed mathematics in ways that on one hand might invite the student to participate, but on the other hand might exclude the student from preparing for higher studies in mathematics. Moreover, the possibility for engaging in more independent reasoning and proving has decreased (Morgan & Tang, 2016). According to Morgan and Sfard (2016b), these changes over time tell us how the expectations for students’ participation in mathematical discourse change. Morgan (2016) also claimed that the textual differences identified might influence students’ achievements in mathematics negatively, as demands decreased. Though the project has studied a British large-scale assessment, the EDSM findings suggest that discourse and language need to be considered when students’ results on ILSAs are being interpreted in terms of trends over time. Nevertheless, the project did not look into students’ achievements on the assessment items nor did it discuss test validity.

Test item characteristics in ILSAs have been related to students’ mathematics achievements by Pedersen (2015), Bergvall (2016) and Dyrvold (2016). They studied mathematical competences adapted to the topic of algebra in TIMSS Advanced\(^\text{12}\), linguistic features in the different TIMSS content areas, and semiotic resources in PISA and Swedish national tests, respectively. Norwegian students solved algebra items better when they involved generating symbolic expressions, than when they involved manipulating and transforming expressions (Pedersen, 2015). As for linguistic features in the area of algebra, low achieving Swedish students had low performances in algebra items with a low proportion of symbols compared with the number of words, while high achieving Swedish students had low performances in algebra items with complex symbolic expressions (Bergvall, 2016). In other topic areas, other differences in performance were found. Finally, naturalistic pictures appeared to matter in affecting how well Swedish students solved test items. Test items including naturalistic pictures were more difficult (Dyrvold, 2016). Though all of these studies involved test item characteristics and their relation to student achievements, none of them explored any aspects of the ILSAs’ validity, or to what extent the test item characteristics changed over time. Nor were trends over time an issue in studies of the practice of test taking. PISA items have been analysed with respect to teachers’ reflections, students’ reasoning and literacy skills, and students’ incorrect answers (Ajello, Caponera, & Palmerio, 2018; Baukal, Pavlović Babić, & Jošić, 2018; Krstić, Šoškić, Ković, & Holmqvist, 2018; Radišić & Baukal, 2018; Selleri & Carugati, 2018). According to Krstić et al. (2018), students with low achievements in PISA lacked

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\(^{12}\) TIMSS Advanced is an assessment by IEA directed at students in upper secondary school.
strategies for reading, whereas all students with high achievements used strategies that were similar to one another’s. This suggests an explanation as to why the high and low achieving students in Bergvall’s (2016) study have low performances in different types of test items.

4.5 Summary of previous research

In this chapter, it has been argued that curriculum theory research in Sweden has not studied textbook authors and changes in textbooks. Research on mathematics specific curricular reform in Sweden has not looked into algebra. It was suggested that Nordic research on textbooks with a focus on algebra has not studied the process of curricular reform. Often, algebra content was instead studied to investigate progression, misconceptions, or for gaps between the intended and potentially implemented curriculum. Additionally, there were few studies with a focus on language and discourse in textbooks of mathematics. Despite the gap, this kind of studies, as Herbel-Eisenmann’s (2007), can capture how reform ideas of a syllabus may transform into something quite different than the reform ideas in textbook tasks.

It was further argued that research on school algebra has emphasized the study of the implemented curriculum and student achievements. Even when research on school algebra did study the impact of a certain curriculum, or of a curriculum reform, as Cai (2014) did, the curriculum does not appear to have been considered as a reform process where both the potentially implemented and attained curriculum are related to the intended.

Finally, it was proposed in this chapter that the validity of ILSAs has not been studied over time. Research on ILSAs has rather concentrated on secondary analysis and comparative cross-country studies. Critical research on ILSAs has addressed test validity in terms of whether the ILSAs can measure the knowledge constructs they claim, whether the results can be used for comparisons between different contexts, and whether the results are useful in specific contexts. Yet, textual differences among mathematics items in large-scale assessments may affect student achievements, and they may change over time. Therefore, textual differences and comparisons over time are important to account for when using the results of ILSAs, as in curricular reform.
5 Theoretical considerations

This chapter develops how textbooks and ILSAs can be understood in relation to the 2011 school algebra reform. First, different conceptions of *curriculum* are discussed in order to clarify what may fit the thesis. In the second part of the chapter, relations between different layers and actors in a curriculum reform are clarified in an elaboration of Valverde et al.'s (2002) curriculum model combined with Howson et al.'s (1981) view of a curriculum reform as a process in several stages. This elaborated model is used in this thesis for understanding and interpreting the results of the papers.

How results on ILSAs may be interpreted and used in curricular reform is addressed through the introduction of Messick's (1989) concept of test validity. This conceptualization is described in the third part of the chapter.

The chapter further describes analytical assumptions concerning how algebra characteristics in the material are understood. Algebra tasks from textbooks and algebra items from TIMSS tests are analysed by means of two different frameworks: a social-semiotic perspective for identifying school algebra discourses, and a theoretical lens of algebraic activities. These analytical assumptions are developed in the fourth and fifth parts of the chapter. In the fourth part, they involve discourse and functional language, a social-semiotic perspective for identifying school algebra discourses, and offered meanings through school algebra discourses. The analytical assumptions in the fifth part of the chapter involve views on algebra in school and how tasks are understood as being algebraic activities for the student.

Finally, to explore the validity of TIMSS, changes in the structure of TIMSS tests are related to student achievements on the test items. This is accomplished through a quantitative view on change and statistical methods. These are explained in the chapter on Analysis and methods.

5.1 Conceptions of the curriculum

As remarked in the Introduction, different actors in different contexts can be of importance for curricular reform. For example, Venezky (1992) claims there is a tradition in the US of not having national educational goals and values, e.g., a national syllabus. This leaves more influence to actors other than national educational authorities and makes, for instance, curriculum material interesting to study. In Sweden, the first official Swedish national curriculum
document in mathematics was established in 1878 (Prytz, 2015). This gives instead a long national tradition of steering documents to build upon, which may explain why textbook authors are not considered as important actors from a Swedish curriculum theory perspective. This section discusses how different actors in a curriculum reform may be discerned and what discernments are needed for this thesis.

According to Remillard (2005, p. 213),

[Curriculum] is used to refer to overarching frameworks that specify what should be taught or to guides or other resources that teachers use when designing instruction and deciding what will be enacted in the classroom.

This is ‘curriculum’ in a broad sense, involving texts directed both at the teacher and at the student. With this conceptualization of the curriculum, a curriculum reform may be discussed in terms of changes that teachers implement in the classroom, as opposed to changes in students’ achievements; but a discrimination is not so easily done between the intentions of policy makers and the intentions of teachers. Gehrke, Knapp and Sirotnik (1992) instead speaks about ‘curriculum’ in three parts:

- the planned curriculum – encompassing what is formulated by policymakers, in textbooks and by teachers,
- the enacted curriculum – encompassing what goes on between teachers and students in the classroom,
- the experienced curriculum – encompassing what is received by students, though admittedly this may be hard to account for.

This conceptualization of the curriculum enables a discussion of both what is planned or intended, and what goes on in classrooms. However, a discrimination between reform processes introduced by policymakers, textbook authors or teachers is still not so easily made. Therefore, it is not useful for discussing the textbooks and ILSAs in a curriculum reform.

The Valverde et al. (2002) model which was shortly described in the Introduction chapter may be used to discuss textbooks and ILSAs, because it differentiates both the potentially implemented curriculum from the intended, and the attained from the implemented. The model builds on a view of educational opportunity as a network of social, political and pedagogical conditions, which provide students with opportunities for learning. It is claimed to be compatible with

classical and contemporary ideas from the sociology of education regarding schooling as a process in which both teachers and students are primary actors conditioned by the social organization of school systems, schools and classrooms (Valverde et al., 2002, p.6).
In relation to this view, textbook authors and authors of test items are in this thesis considered as actors involved in constructing and shaping social, political, and pedagogical conditions for teachers and students. These conditions are visible in the textbook tasks and the test items they create. As stated in the Introduction chapter, these actors and their tasks and test items are important during different stages of a curriculum reform.

5.2 A model for understanding a curriculum reform

Drawing on the Valverde et al. (2002) curriculum model and Howson et al. (1981), this thesis develops a model for understanding a curriculum reform as concerning different actors, in different layers of the curriculum, during different stages of a curriculum reform (Figure 1, p. 48). The model is developed in this section.

In the intended curriculum (Valverde et al., 2002), intentions are restricted to those intentions held by policy makers and educational authorities. It thus comprises formal, national curriculum documents such as the syllabus and commentary material. The identification (point 1 in Figure 1) of a need for curriculum reform can come from different actors. Howson et al. (1981) mention pressures from society and politics due to e.g. economic and technological development, from mathematics due to developments in the scientific field, and from the educational system and developments in educational research. In the case of the 2011 school algebra reform, the pressure can be said to come both from ILSAs and from politicians.

The formulation (point 2 in Figure 1) of a plan for action concerns the scope of a reform. A problem which is identified locally – in a school or by a teacher – does not need a plan of action formulated regionally or nationally. Howson et al. (1981) also give the example of plans for actions which are proposed, in contrast to plans which are imposed. In the case of the present study, the plan of action was imposed through the new national syllabus in 2011, which in its turn was established by a government decision. This is what Howson et al. (1981, p. 11) call a power-coercive strategy.

While the model may seem flat, hierarchical and with no connections between different curricular levels, such is not the intention. The levels are not to be interpreted as parallel Euclidian surfaces. They may be entangled due to actors operating on more than one level. For instance, there are textbook authors who also work as teachers. For the thesis, this is not relevant though, since texts in textbooks and syllabi are produced at specific levels.

Other strategies may be emotive or other kinds of pressure, the use of ratio-empirical arguments, or re-educative strategies. For instance, other coercive strategies may use arguments building on ethos or more substantially, give increases in salary for teachers who adopt a new curriculum. A ratio-empirical strategy rather builds on arguments of logos, implying for instance a more effective or better outcome of knowledge through adopting the new curriculum. Finally, a re-educative strategy aims at changing attitudes and beliefs of those working within education so that they become actors themselves in driving the curriculum change (Howson et al., 1981).
The development (point 3 in Figure 1) of new materials is central in Howson et al.’s (1981, p. 14) account of curriculum development:

Such developmental work may well choose to build on existing research findings. In other cases research work in specific areas will have to be commissioned. During this stage, formative evaluation of the innovation will take place: the testing and assessing of the project’s draft materials as they are used in the ‘pilot’ experimental institutions.

It is clear from this quote how the authors understand the development of new materials as based on both research and experience. It is less clear who is to be responsible for this development. In the 2011 school algebra reform, national educational authorities did not develop materials of their own in order to concretize the syllabus. Given the position of textbooks in the Valverde et al. (2002) model – between the intended and the implemented curriculum – newly published textbooks may be considered as new materials in a curriculum development. The notion of the potentially implemented curriculum (Valverde et al., 2002), is used in this thesis for textbooks with the comment that we cannot assume that textbook authors mirror the syllabus in an unproblematic way. Given the deregulated textbook market and previous findings on textbook authors’ as actors in curricular reforms in Sweden (Johnsson Harrie, 2009; Prytz, 2017), there is no reason why textbook authors would simply mediate what is intended by policymakers, without holding any intentions of their own.

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15 The model is developed by Palm Kaplan in this thesis from Valverde et al. (2002) and Howson et al. (1981). Arrow icon made by Linh Pham, Miu Icons, from www.flaticon.com.
The dissemination and implementation (point 4 and 5 in Figure 1) of the reform ideas and new materials are the stages which address the work of realizing the reform in classrooms. In-service education and scaffolds to sustain the reform ideas are needed here (Howson et al., 1981). These parts of the 2011 school algebra reform are not studied in this thesis, but Boesen et al. (2014), described in the chapter Previous research, may be said to study the dissemination and implementation stages. According to Remillard (2005), the choice of the word ‘implemented’ in Valverde et al.’s (2002) curriculum model may carry an assumption of the view of the teacher as a passive transmitter and not as an actor. However, Valverde et al. (2002) point out that the implemented curriculum is negotiated by teachers and students through the strategies, practices, and activities which occur in the classroom.

Finally, in the Valverde et al. (2002) model the attained curriculum refers to student achievements. As stated in the Introduction, we know that results on ILSAs, among other things, are used in curricular reform. According to Howson et al. (1981), the evaluation (point 6 in Figure 1) of a curricular reform may be a more or less formal, summative judgment. These evaluations can “lead to the identification of new problems and so help initiate yet another round of innovation” (Howson et al., 1981, p. 15). Thus, results on ILSAs may be used for evaluating a reform and for drawing inferences about students’ knowledge in order to take further action (point 7 in Figure 1). The validity of this use is discussed in the next section.

5.3 Using results from ILSAs: aspects of validity

As stated in the section Research on large-scale assessments, critical studies of ILSAs often investigate test constructs, methodological questions and other aspects of how test scores may be interpreted in the same way for all test-takers, regardless of linguistic, cultural, or other differences. These studies thus appear to reflect a traditional view of test validity where the interpretation of test results in relation to the test questions is in focus; but they also reflect validity as a question of being relevant across, or in, different contexts. The concept of validity has thus evolved.

One concept of test validity which carries a scope that is different from many others, is Messick’s (1989). Besides comprising the validity of the test itself, his concept encompasses the use and the interpretations which can be made from test results, and what consequences this use and interpretation may have. The evidence for investigating validity lies thus not only in test results in relation to the test items, but in test results in relation to specific persons in a specific context. In general, test validity hinges upon how scores in a test are interpreted; upon the value implications of the scores; upon the relevance of using test scores; and upon the social consequences of using the scores (Messick, 1989). Value implications of the test scores concern how issues in
a test may affect the view of the subject being tested. Relevance of the test scores concerns the test results in relation to the specific context and specific persons. The social consequences of using test scores concern how a test may affect the behaviour of test-takers. In a sense, the latter is what Johansson and Hansen (2018) find when they see how the curricula of countries participating in TIMSS seem to harmonize over time with the TIMSS test.

In this thesis, aspects of TIMSS’ validity are used to understand in what ways inferences may be drawn about students’ knowledge in algebra and in what ways TIMSS results may be used in relation to the 2011 school algebra reform. In relation to the 2011 school algebra reform in Sweden, the following aspects of validity are considered:

- the interpretation of TIMSS results over time as a way to understand knowledge trends,
- the relevance of using TIMSS results over time for evaluating curricular reform and identifying needs for curricular change,
- the relevance of using TIMSS results to understand students’ achievements,
- value implications of TIMSS results.

To assess the validity in how a test is used in a specific way, it is important to rely on several sources of evidence. Messick (1989) distinguishes how different sources of evidence may be used for validation.

1. The internal structure of test responses can be examined through the relationships among responses compared with test items. This is the type of critical research on ILSAs which investigates the knowledge constructs of the tests. For example, Hopfenbeck et al. (2018) mention studies aiming at investigating whether the cognitive constructs in PISA measure intelligence or academic literacy.

2. The external structure of test responses can be examined through the relationships among responses compared to measures such as background variables. This is the type of critical research on ILSAs which investigates measurement invariance, sampling, translation, and other technical questions. For example, Hopfenbeck et al. (2018) mention studies aiming at investigating biases due to e.g. poor translation, or differences in language and culture.

3. The content of a test can be compared to content in a domain of reference. This is for instance the case with the comparisons by Sollerman and Pettersson (2016) and Lindström (2006). Their domains of reference are tasks from Swedish National tests for Year 3, 6 and 9 together with the syllabus, and one textbook series for Year 7 and 8, respectively. The general correspondence for these domains with ILSAs was raised in the Background chapter.
4. Differences in the structures and processes of tests can be investigated. Differences in structures may involve comparisons across groups and contexts as well as over time. Differences in processes may involve the practice of test-taking, e.g. the study by Krstić et al. (2018) who eye-track differences in how students with low and high achievements read PISA items.

5. The (intended or unintended) consequences of a test can be considered through a discernment of the effects of using or interpreting the test scores. According to Messick (1989, p. 20) it is important that “theoretical implications and the value implications of the test interpretation are commensurate”. In other words, this concerns an appraisal of the value implications and theories underlying the test construct.

In the present thesis, differences in the structure (point 4 above) of TIMSS over time are investigated through the analysis of the algebra characteristics in the four consecutive TIMSS tests. We know that previous research on IL-SAs seem to have prioritized comparisons cross-country, and that changes in discourse and language may carry unwanted consequences (cf. Morgan & Sfard, 2016a). This addresses validity as the interpretation of TIMSS results over time, and as the relevance of using TIMSS results over time in curricular reform.

A comparison of TIMSS’ algebra items to the domain of reference – the textbooks’ algebra tasks in Year 8 – is included in the Discussion and conclusions (point 3 above). Previous comparisons such as Sollerman (2019) have not compared the content of TIMSS with the potentially implemented curriculum as a domain of reference. Such an investigation may help in understanding the discrepancies between what TIMSS’ national research coordinators and teachers in Sweden report as a part of the curriculum (cf. Grønmo et al., 2014). Because there is less pertinence in drawing inferences from TIMSS results if the content of the test and its domain of reference are not fairly similar, this aspect of validity addresses the relevance of using TIMSS’ test scores as a way to understand Swedish students’ achievements in algebra. The comparison includes a diachronic aspect, since it involves textbooks as well as test items from both before and after 2011, when the school algebra reform was launched.

Moreover, this diachronic comparison of textbook tasks and test items, may give evidence concerning value implications of the TIMSS’ test scores (point 5 above). As reported by Johansson and Hansen (2018), mathematics curricula in many countries are harmonizing with topics in the TIMSS framework over time. If the textbooks’ algebra characteristics become more similar to TIMSS’ algebra characteristics over time, it may be a sign of a harmonization with TIMSS at the level of the potentially implemented curriculum.
5.4 Discourse and language

Discourse analysis is used in a wide range of studies and for several purposes. Common traits in these studies are the investigation of how language functions in social practices and the assumption that language is constructive of social actions (Morgan, 2016a; Wetherell, Taylor, & Yates, 2001; Winther Jørgenssen & Phillips, 2000).

For example, Morgan (2016b, p. 138) suggests that

If changes to school mathematics discourse increase human participation at the expense of a reduction in reification there is a risk that students will be less well prepared to higher level study of mathematics.

The reification mentioned in the quote is Sfard’s (2008) concept. It refers to a discursive move where references to actions or processes are replaced with references to objects. A reduction of reification thus means that mathematics is described as a process and not as an object, which in turn contributes to mathematics being depicted as less abstract and structural, less objectified – or less reified. Hence, the ontological view of mathematics, what it is, is a feature constructed in discourse. Morgan’s quote above implies that reification is important for students’ possibilities to learn (higher) mathematics. Furthermore, a certain topic can be constructed, for instance, with or without human participation. Then discourse expresses meaning about e.g., the view of the nature of this topic and how the reader, in this case the student, is invited to participate.

These aspects of language – how it functions in social practices and is constructive of social action – are used in the thesis to understand and analyse offers of meaning (Englund, 1998) in textbooks’ algebra tasks and TIMSS’ algebra items. The aspects will be developed below. For the sake of brevity, ‘tasks’ is used to denote both textbook tasks and TIMSS’ test items in this section and the next, unless otherwise stated.

Broadly, three central areas for discourse analysis can be distinguished by looking at how language functions in social practices: about the nature of social action, the production of social actors, and the organisation of social relations. The first area concerns mainly interaction between people, the second sense-making and construction of identity, while the third area focusses upon historical and institutional features of discourse (Wetherell et al., 2001; Winther Jørgenssen & Phillips, 2000). In this thesis, the constructors of tasks can be said to draw upon institutional and historical features of discourse when producing tasks. Additionally, the reader, or the imagined student, is supposed to answer tasks by performing some kind of action which is legitimized by the constructor of the task in question. By looking at the language in the tasks, it is possible to identify the nature of this social action.
The constructive aspect of language refers to that language does not reflect the world in a neutral way. As a consequence, language builds versions of the world, as well as of individuals (Luke, 1995; Morgan, 2016a; Wetherell, 2001). These constructions function to achieve certain things and not others, and thus tell us something about power relations and social formations as products of a wider context (Luke, 1995; Morgan, 1998; Ryve, 2011). In the thesis, these power relations and social formations concern how the imagined student is positioned by the text and what actions are legitimized, or required, by the text.

A discourse can be understood as “recurrent statements and wordings across texts” (Luke, 1995, p. 15). Discourses are constructed through patterns in the choices of linguistic resources across texts, and they construct and steer the linguistic possibilities for the production of new texts within the discourse. Since language constructs rather than reflects phenomena (Morgan, 2016a), these linguistic patterns make up systems of meanings and versions of the world. This, in its turn, excludes other systems of meanings and versions of the world (Englund, 1998).

By analysing linguistic resources, it is possible to identify what in the thesis is called school algebra discourses. The school algebra discourses consist of recurring patterns of semiotic resources across texts, in this case textbook tasks and test items, which are identified through an iterative process. They construct certain versions of algebra and of the student instead of other versions, and thus tell us something about what is valued and what is not. They offer different views of the nature of algebra, its relation to the world and other phenomena, and different types of social action for the student.

5.4.1 Discerning school algebra discourses in tasks

In this thesis, a social-semiotic perspective on language establishes a foundation for the analytic framework used to discern school algebra discourses. It has been developed by proponents of Systemic Functional Linguistics, SFL (cf. Halliday & Hasan, 1989; J. R. Martin, Matthiessen, & Painter, 1997). From this perspective texts are semantic units that express meaning, at the same time as texts are formed in and shaped by social interaction and context. The linguistic choices made in a text fulfil three different functions for expressing meaning: the ideational function, the interpersonal function and the textual function (Halliday & Hasan, 1989).

The ideational function realizes the field, which answers to questions about what kind of participants are present in the text, where the participant can be phenomena and/or objects as well as persons. Additionally, the ideational

---

16 Mathematics is considered to also use resources other than written language, e.g., graphs, tables and alpha-numerical symbols (O’Halloran, 2015). Therefore, other semiotic resources are also considered as taking part in constructing discursive patterns.
function is expressed through the processes that are taking place between the participants, e.g., actions, communication and relations (Halliday & Hasan, 1989; J. R. Martin et al., 1997).

In this thesis, the ideational function is analysed to grasp the context of and the relation between humans, the world, and algebra, as these are expressed in school algebra tasks. Thus the ideational function says something about how the nature of algebra is constructed and what doing algebra ‘is’, and to what extent human beings are doing it. For example, algebra can be constructed as taking place or being used in different contexts depending on the choices of participants, and as absolutist or more fallible depending on the choices of processes. Six types of processes distinguish experience of different types of events: material, mental, relational, behavioural, verbal and existential (J. R. Martin et al., 1997). Three of these have been discriminated in the present thesis. They were chosen because they turned out to be suitable for the texts which were to be analysed. Material processes construct doings and events, both concrete and abstract. Relational processes construct being, both attributive and identifying. Verbal processes construct ‘saying’ – communicative acts where not only humans may participate but also objects, phenomena and other semiotic resources, e.g. graphs and tables.

The interpersonal function realizes the tenor, which answers questions about how information, goods and services are negotiated, and by whom. This is achieved through linguistic choices of speech acts, i.e., if the text gives information as in statements, or demands information as in questions and requests. It is also achieved through choices in modifying expressions, which show if a statement is factual or is nuancing the grade of probability, usuality, obligation, and readiness. Finally, it is achieved through how the reader is addressed, e.g., as a beginner or a specialist, through processes indicating different actions, or with a formal or more personal tone through choices in pronouns (Halliday & Hasan, 1989; J. R. Martin et al., 1997).

In this thesis, the interpersonal function is analysed in order to grasp the social interplay between reader and author, and reader and the subject content. It thus says something about how the reader/student is invited to participate, and in what actions.

The textual function realizes mode, which answers questions about what role the writer and reader expect language to fulfil in the given situation. This is achieved through choices of how the information is organised, e.g., thematic

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17 From an absolutist view of mathematics, mathematical knowledge is a body of certain and unchallengeable truths, while a fallible view of mathematics implies that mathematical knowledge is corrigible and open to revision (Ernest, 1991).

18 Appraisal, i.e. expressions of engagement and attitude, also functions to realize the tenor. A pilot study indicated that this type of expressions was very rare in the material. An appraisal analysis would not substantially help in discerning and identifying school algebra discourses. Therefore, this option is not used in the thesis.
choices or cohesive structures (Halliday & Hasan, 1989; J. R. Martin et al., 1997).

In this thesis, the textual function is not analysed. The texts are very short. The information in the tasks is typically organised in a statement and a question, in a statement and a request, or simply a request. Some tasks, e.g. many test items, have multiple-choice answers to choose among for the reader. A pilot study of the textual function in the tasks showed no patterns in linguistic choices, which could help identifying and discerning school algebra discourses besides what was already accomplished through the ideational and interpersonal functions.

5.4.2 School algebra discourses vs mathematical discourse

School algebra discourses are not to be confused with Sfard’s (cf. Morgan & Sfard, 2016a; Sfard, 2008) view of mathematics as a discourse, though they have features in common. According to her communicational perspective, mathematical objects are discursive constructs, which are learned through participating in mathematical discourse. Therefore, the social aspect of Sfard’s concept of discourse concerns the interaction between people, focussing on what mathematics the students may learn when participating in mathematical discourse. The analysis of school algebra discourses in this thesis draws attention to the potentially implemented curriculum and the meanings which are offered in tasks. It has, however, not been developed for or tested in classroom situations.

Sfard’s mathematical discourse is distinguished by vocabulary and syntax, visual mediators, narratives, and routines. Narratives and routines are discursive features which distinguish different qualities about the view of mathematical knowledge and what actions the student is supposed to engage in (Morgan & Sfard, 2016a; Sfard, 2008). They can thus be understood to regard views on the nature of, e.g., algebra, and of the student, as the school algebra discourses do. However, Sfard’s visual mediators, vocabulary, and syntax are discursive features which say something about the extent to which a discourse is mathematical and thus, what demands are put on a student in order to participate in mathematical discourse (Morgan & Sfard, 2016a; Sfard, 2008).

School algebra discourses in this thesis have another scope. Drawing on Luke (1995) and Halliday and Hasan (1989), the analysis of school algebra discourses discerns patterns of different semiotic features regardless of whether these features are mathematical or not. This is done in order to understand in what ways algebra is constructed, and what relations are constructed to the imagined student. School algebra discourses as treated in this thesis thus tell other stories about algebra in school. They discern different views on what algebra ‘is’ and who does it and with what rather than what is mathematical discourse and what is not. In these respects, the concept of school algebra discourses differs from Sfard’s work.
5.5 Views of school algebra

There are several views of school algebra. Whether or not alphanumerical symbols are necessary or sufficient to capture the ‘essence’ of algebra is debated (Kieran, 2018; Kieran et al., 2016). This section discusses advantages and disadvantages with different views of school algebra. It further conceptualizes what in the thesis is called algebraic activities.

According to Radford (2018), algebraic thinking concerns the use of indeterminate quantities analytically. This means operating with and handling unknowns, variables, parameters and so on, as if they were known though they are not. This analyticity is the key feature of Radford’s view of algebra. Furthermore, these indeterminate quantities and operations on them have been represented in different ways across history and culture so using alphanumerical symbols is not the only way to represent them. This view of algebra implicates possibilities for engaging in algebra without alphanumerical symbols, opening it up for younger children more so than what was thought earlier.

Alternatively, Kieran (2007) has developed the GTG-model, where algebra is viewed as an activity. Three types of activities are put forward: Generational, Transformational, and Global/meta-level activity. Generational activities are about forming expressions, equations, and other objects. Depending on whether a function-based or equation-based approach to algebra is used, different meanings may be enhanced in the activity. Transformational activities concern “changing the symbolic form of an expression or equation in order to maintain equivalence” (Kieran, 2007, p. 714) and can involve conceptual and theoretical aspects, not only those that are procedural. Global/meta-level activities use algebra as a tool and entail activities such as modelling and problem-solving, justifying and proving, generalizing e.g. number patterns, as well as predicting and making conjectures. The GTG-model implicates that these activities are intertwined in school algebra tasks, since a task might easily be about generating an expression, then transforming the expression and finally using it for problem-solving.

A symbolization perspective is proposed by Kaput (2008), where the core aspects of algebra are symbolizing generalizations, and reasoning and acting on generalizations. This is a double view of algebra – as something to acquire and as actions – implying that algebra is a question of both objects and humans’ doings with these objects. These aspects are embodied in three strands: as structures, generalized arithmetic, and quantitative reasoning; as functions and relationships; and as the use of modelling languages (Kaput, 2008). Hence, Kaput’s view of algebra partly resonates with the three main perspectives of algebra during the 20th century as mentioned in the Background chapter. It differs from them through introducing the two core aspects of algebra as both acquisition and action.
Building on Kaput (2008), Blanton et al. (Blanton, Levi, Crites, & Dougherty, 2011; Blanton et al., 2015) suggest a framework of ‘big ideas’ in algebraic thinking practices. These big ideas are

1. Equivalence, Expressions, Equations and Inequalities
2. Generalized Arithmetic
3. Functional Thinking
4. Variable and
5. Proportional Reasoning (Blanton et al., 2015, p. 43)

While the first idea concerns how the student can develop a relational understanding of the equal sign, the second takes a structural approach which focuses on properties of numbers and operations. The third, Functional Thinking, deals with generalizing relationships and different resources, which can be used for describing generalized relationships. The fourth idea, Variable, concerns the different aspects of variables, such as an unknown, a varying quantity, or a parameter. It is integrated in the other ideas in the sense that functional thinking entails the idea of variable as a varying quantity, generalized arithmetic entails the idea of variable as a generalized number (Blanton et al., 2015) and so on. The last big idea has to do with algebraic reasoning about generalized quantities, which relate to each other with an invariant ratio. Blanton et al. (2015) use these big ideas in an early algebra intervention to see whether or not it is possible for children to engage in and learn algebraic thinking practices.

Compared with the three main perspectives discussed above, variables and proportional reasoning are stressed in the Blanton et al. framework. However, these two ideas are not mentioned in more recent work by Blanton et al. (2018), which instead presents a framework of four algebraic thinking practices in three core content areas. The new version of the framework entails the thinking practices of generalizing, representing, justifying, and reasoning, as related to the areas of Equivalence, Expressions, Equations and Inequalities; of Generalized Arithmetic; and of Functional Thinking. This is a development which more clearly emphasizes both action and acquisition, more so than the ‘big ideas’ do. As mentioned above, the idea of Variable draws on a different type of those aspects, which are included when working with the other big ideas. It is not in itself a thinking practice, or core content area. Together the three core content areas pick up the three main perspectives of algebra described in the background. The activity-aspect is enhanced through the thinking practices in these areas. However, the development of the framework is made at the expense of Proportional Reasoning since this idea is left aside in Blanton et al. (2018).
5.5.1 The task as algebraic activities for the student

In this thesis a view of algebra is needed which enables the distinction of different algebra characteristics, and which accounts for what the student is supposed to do. Blanton et al.’s big ideas are more distinct from each other than are Kieran’s activities since they stress both the aspect of acquisition and the action, save for Variable. This difference facilitates a quantitative analysis. The idea of Variable is not distinct from the other ideas, since it has its aspects integrated within them. Nor can it be considered in terms of an activity. Moreover, Radford’s (2018) definition is more useful for discerning what algebra is against what it is not, than for discerning differences in algebra itself. The thesis also needs a view of algebra that fits the material. Procedural and manipulating activity is not viewed as algebra by Kaput (2008) and Blanton et al. (2018, 2015). It is not considered as a thinking practice and it concerns the same content area as Equivalence, Expressions, Equations and Inequalities. However, procedural and manipulating activities are included in textbook tasks, and so is proportional reasoning.

The thesis conceptualizes algebraic activities as being that algebra that the student is expected to engage with in order to solve a task. An algebraic activity is a combination of doing certain actions with certain objects. The conceptualization builds mainly on Blanton et al. (2018, 2015). In line with Blanton et al.’s big ideas, the activities are named Equivalence, Expressions, Equations and Inequalities; Generalized Arithmetic; and Functional Thinking.

To fit the material, two more algebraic activities are added. First, Blanton et al.’s (2015) idea of proportional reasoning is elaborated with Lundberg (2011) into Qualitative and Proportional Reasoning. Second, to acknowledge the transformational, or procedural and manipulating, activity put forward by Kieran (2007) in the GTG-model, the algebraic activity of Manipulation is included. Finally, other conceptualizations have been studied in order to adapt the algebraic activities to Year 8. For instance, a comparison of different algebra standards and frameworks by Eddy et al. (2015) gave examples of algebra content for secondary school, which were compatible with Blanton et al.’s (2015) big ideas. The different algebraic activities and the analysis of them are developed in the chapter on Analysis and methods.
6 Analysis and methods

Below, the selection of textbooks, tasks, TIMSS tests and TIMSS’ test items are described. It includes how the grouping of students’ achievements is done, in order to select groups of students with high and low achievements.

In the section called Systemic Functional Analysis, the SFL-analysis is presented. This is used to distinguish and identify school algebra discourses. Next the algebraic activities are presented, which are categories based on mainly Blanton et al. (2018, 2015).

Following this section, how student achievements in algebra are investigated with statistics, is accounted for. Descriptive statistics is used to understand the diachronic interplay of school algebra discourses and algebraic activities in textbooks and TIMSS tests, as well as differences in the structure of TIMSS over time. Inferential statistics is used to explore the relations between algebra characteristics in TIMSS and student achievements. The chapter ends with a description of how the diachronic interplay of the algebra characteristics in the textbook tasks and test items were compared, followed by methodological and ethical considerations.

The methods of analysis in the thesis are hence both qualitative and quantitative. Lilliedahl et al. (2016) claim that a mixed methods approach in studies of curricular reform produces a higher explanatory value. By studying algebra characteristics and adding a quantitative view on change, this thesis gains a fuller understanding of textbooks and TIMSS in the 2011 school algebra reform.

6.1 Materials

In the present thesis, textbook tasks, TIMSS’ test items and student achievements on TIMSS’ test items are analysed. The textbook tasks and the test items are selected from the period before and after the 2011 reform, and they concern algebra. These selections will be described in the first two sections below.

Student achievements on the test items are used in two ways in this thesis. First, student achievements on all test items are used for selecting groups of students with low and high achievements, in addition to all students. In the third section, is elaborated upon how and why the grouping of student achievements is done. Second, the student achievements in algebra for these groups
of students are explored statistically, in relation to different algebra characteristics in the structure of TIMSS (displayed by Table 11, p. 93). The statistical treatment of student achievements in algebra is explained in the section called Statistical analysis of student achievements.

6.1.1 The mathematics textbooks

The textbook series in mathematics were selected using the criterion that they should have been published both before and after the 2011 curriculum reform. This criterion discarded one of four possible publishers available on the Swedish market. The decision was also taken using an ethical concern: one of the textbook authors, working for the publisher which was left out, is a colleague and a good friend of the present author. Additionally, two of the selected series were written by authors who have been publishing textbooks in mathematics for a considerable time. Such continued publication indicates that their books have been popular, for otherwise, the publishing companies would not choose to continue publishing their works.

In addition to these considerations, three other textbook series from the chosen publishers were excluded because they were introduced for the first time on the publishing market after 2011. After this exclusion, three possible series remained. From these three series, the first published textbooks after the curriculum reform of 1994 were chosen along with the first published textbooks after the curriculum reform of 2011 (Table 2). As a comment to this choice, the MD textbook published in 2010 was designed to follow the 2011 syllabus (S. Carlsson, personal communication, September 9, 2015).

<table>
<thead>
<tr>
<th>Book Label</th>
<th>Publisher</th>
<th>Curricula</th>
<th>Number of tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matematikboken Y (2012)**</td>
<td>Liber</td>
<td>2011</td>
<td>455</td>
</tr>
<tr>
<td>Matte Direkt år 8 (2002)*</td>
<td>MD02</td>
<td>Bonnier/ Sannoma</td>
<td>1994</td>
</tr>
<tr>
<td>Matte Direkt år 8 (2010)*</td>
<td>MD10</td>
<td>Bonnier/ Sannoma</td>
<td>2011</td>
</tr>
<tr>
<td>Formula 8 Matematik (2007)*</td>
<td>F07</td>
<td>Gleerups</td>
<td>1994</td>
</tr>
<tr>
<td>Formula 8 Matematik (2013)</td>
<td>F13</td>
<td>Gleerups</td>
<td>2011</td>
</tr>
</tbody>
</table>

*Textbook entailing a separate chapter on relationships and functions.
**Textbook with two chapters on algebra.

The selection includes the same textbook series as in Johansson’s (2003) study, though from a different school year. This is also one of the series which
were analysed in the TIMSS’ textbook study (Valverde et al., 2002). Textbooks for Year 8 were singled out for analysis since this is the year Swedish students participate in TIMSS. Törnroos (2005) argued that long term data, i.e., textbooks from several school years, renders more consistency in the results than does data from one school year. The present thesis does not make a correlation analysis, as Törnroos did. It is assumed that the selection and analysis of six textbooks over time will give a somewhat similar consistency.

The tasks selected were all tasks in the algebra chapters, i.e., chapters called Algebra, Expressions and equations, Equations, Algebra and patterns, and so on. Chapters on functions were not chosen. Only three of the six selected textbooks include such chapters (Table 2), with the titles Relationships and Functions and graphs. Including these would have skewed the selection. The selection is developed in papers I and II and discussed in Methodological considerations, further below.

### 6.1.2 Selection of TIMSS’ test items for comparison with textbook tasks

Test items were chosen from four consecutive tests: TIMSS 2003, 2007, 2011 and 2015. The selection was made to match the time-period before and after the 2011 Swedish curriculum reform for the diachronic comparison with the textbook tasks.

The material included all test items marked by TIMSS as algebra, with a few exceptions. Two test items from 2015 were mistranslated and removed from the Swedish version of the test. Test items that lack valid responses in an assessment were discarded, since there were no student achievements related to them. Four trend items were not labelled as algebra in all of the assessments in which they appeared. These were left out for reasons of clarity. Furthermore, one item response was derived from another test item. This was not included since it lacked text to analyse. All exceptions are listed in Appendix A. Finally, the cyclic construction in TIMSS means that trend items reoccur in up to three tests. The student achievements on these items will be different for every test, so they were analysed as unique items in this study. In total, the material amounts to 232 unique items (Table 3).

<table>
<thead>
<tr>
<th>Assessment</th>
<th>Algebra items</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIMSS 2003</td>
<td>45</td>
</tr>
<tr>
<td>TIMSS 2007</td>
<td>62</td>
</tr>
<tr>
<td>TIMSS 2011</td>
<td>66</td>
</tr>
<tr>
<td>TIMSS 2015</td>
<td>59</td>
</tr>
</tbody>
</table>
6.1.3 Selecting groups of students for analysing student achievements

Based on the results of Bergvall (2016), it was decided to discriminate groups of students with different achievements for the statistical analysis. The results on all TIMSS items from the four tests, i.e. not only the algebra items, were adopted for doing this selection.

In the thesis, the group of students with low achievements is defined as the students with a total result below the lowest quartile, in the taken TIMSS test. The group of students with high achievements is defined as the students with a total result above the highest quartile, in the taken TIMSS test. This choice was made because the benchmarks form an absolute measure for cross-country comparison, while quartiles are relative to the population investigated. Compared with the benchmarks, there is a larger proportion of Swedish students with low achievements than with high achievements (cf. Mullis et al., 2012, 2008, 2016). Hence, using the quartiles enabled a selection that is not skewed, which it would be if TIMSS’ benchmarks had been used. The total result means that all test items were used in the calculation of the quartiles and not just the algebra items.

In TIMSS, plausible values are used to generalize the students’ achievements from the test items they actually solve in relation to the entire body of test items. It cannot be assumed that the theory deployed in TIMSS for this generalization will function similarly to the theoretical frameworks used in the present thesis. Therefore, in this thesis the raw scores were used.

Table 4. Number of test items in algebra selected for inferential analysis and average number of responses for Swedish students per selected test item (Paper III).

<table>
<thead>
<tr>
<th>TIMSS year</th>
<th>Number of algebra items in the test</th>
<th>Average number of responses per algebra item for different groups of students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>all students</td>
</tr>
<tr>
<td>2003</td>
<td>33</td>
<td>539</td>
</tr>
<tr>
<td>2007</td>
<td>52</td>
<td>732</td>
</tr>
<tr>
<td>2011</td>
<td>60</td>
<td>781</td>
</tr>
<tr>
<td>2015</td>
<td>52</td>
<td>574</td>
</tr>
</tbody>
</table>

The sum of one student’s raw scores on the test items answered, divided by the number of test items answered, was calculated to make students’ results comparable. This raw score quota enabled a comparison of students’ results even though they may not have answered the same number of test items. The comparison was done through a ranking of the raw score quotas for each TIMSS test, and then by calculating the quartiles for each test. The calculations were done in Excel. Since students in TIMSS have answered different numbers of
test items, the selection is described in terms of the mean number of student responses per algebra item in each group of students. This is displayed in Table 4.

6.2 Systemic Functional Analysis

The social-semiotic perspective on language establishes a foundation for the analytic framework used to discern school algebra discourses. As stated in the chapter Theoretical considerations, the linguistic choices in a text fulfil three functions for expressing meaning (cf. Halliday & Hasan, 1989; Morgan, 2006; Schleppegrell, 2004). The framework presented below in Table 5 consists of a range of analytic questions (cf. Alshwaikh, 2016) and is oriented towards the ideational and interpersonal function of language. It is explained and exemplified in the following sections.

Table 5. The analytic framework of SFL (Paper I).

<table>
<thead>
<tr>
<th>Field of discourse, realized by the ideational function</th>
<th>Questions guiding analysis</th>
<th>Indicators in text</th>
</tr>
</thead>
<tbody>
<tr>
<td>How is the nature of algebra construed?</td>
<td>Types of processes: relational, verbal, material. Verbs, e.g., is, explain, simplify.</td>
<td></td>
</tr>
<tr>
<td>What is happening and who are taking part?</td>
<td>Types of participants: personal pronouns or proper names; subject-specific or otherwise specialized vocabulary, mathematical symbols; images or tables with information needed to solve the task.</td>
<td></td>
</tr>
<tr>
<td>What processes are human agents engaged in, and to what extent do they do algebra?</td>
<td>Presence of human agents, passive verb form.Modifiers, e.g., can, will, may</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tenor of discourse, realized by the ideational function</th>
<th>Questions guiding analysis</th>
<th>Indicators in text</th>
</tr>
</thead>
<tbody>
<tr>
<td>What kind of action is the student expected to engage in?</td>
<td>Types of processes in imperatives: relational, verbal, material.</td>
<td></td>
</tr>
<tr>
<td>How is the student addressed?</td>
<td>Questions or requests, personal pronouns, e.g., you, we.</td>
<td></td>
</tr>
<tr>
<td>Are choices or decisions available to the student?</td>
<td>Modifiers, e.g., can, will, may</td>
<td></td>
</tr>
</tbody>
</table>

6.2.1 The ideational function

As can be seen in the framework presented in Table 5, there is a difference between what kinds of processes are used to present information. In Example
1, the relational process is identifies the participant one side with the participant 8 m longer than the other.

Example 1. A relational process.

In a rectangle one side is 8 m longer than the other. The circumference is 80 m. How large is the area of the rectangle?

I en rektangel är ena sidan 8 m längre än den andra. Omkretsen är 80 m. Hur stor area har rektangeln? (F07:211)

Nothing happens in this process, as opposed to a statement with material processes, e.g., can be calculated, or verbal processes, e.g., show. The view of mathematics which can be identified in the relational process in Example 1, is called atemporal by Morgan (2016b).

The use of a passive verb form in a process conceals human agency in mathematics and contributes to a view of mathematics as independent of humans (Morgan, 2006). This use is seen in Example 2, where the material process can be calculated contributes to concealing human agency through the absence of personal pronouns or named persons.

Example 2. Passive verb form and a material process.

The population of Tanzania in \( n \) years can be calculated with the expression \( 25\,000\,000 \cdot 1,028^n \). How large will the population of Tanzania be in 10 years? Round off to whole millions.

Tanzanias befolkning om \( n \) år kan beräknas med uttrycket \( 25\,000\,000 \cdot 1,028^n \). Hur stor kommer Tanzanias folkmängd att vara om 10 år? Avrunda till hela miljoner. (Y12:158)

It is not apparent which person it is who can calculate the population of Tanzania, so the nature of algebra is dehumanized. Beyond passive verb forms, the presence of given names and personal pronouns are signs of human activity, both generally for humans, and specifically for the student. As for verbal processes, they may both construct mathematics as a human enterprise through processes such as show, discuss, and express, and obscure mathematics as a human enterprise through combinations of choices in non-human participants, e.g. the table shows. Thus, choices in processes and participants realize different views of what algebra is.

Choices in participants also construct different relations between algebra and the world, or different contexts related to algebra. In Example 2, The population of Tanzania in \( n \) years is an example of a specialized vocabulary related to social science, or science. In Example 1 above, one side and 8 m longer than the other are examples of vocabulary specialized not only in mathematics, but also in geometry. These choices of participants draw on different social contexts and
therefore say something about what counts as algebra and what it can be used for: solving problems in social science and in geometry.

In the analysis of the ideational function, semiotic resources such as pictures, figures, and tables are taken into account if they function to enhance a participant in written text and in unclear cases. Mathematical symbols, e.g. variables and expressions of variables, are analysed as participants unless they function as processes in relation to the written text, e.g. an equal sign, or as neither participants nor processes.

6.2.2 The interpersonal function

In questions and requests in the imperative form, the student is addressed implicitly. The student may also be explicitly addressed by you or included in we. Choices in participants may add a more personal tone through personal pronouns when the student is addressed or a more formal tone if human beings are anonymized and the student is addressed implicitly.

The actions the student is supposed to engage in is realized by choices in processes, in the requests. In Example 3, simplify and calculate are material processes.

Example 3. Implicit requests to the student with material processes.

   a) Simplify the expression $6a - 2b - 2a + b$ as far as possible.
   b) Calculate the value of the simplified expression if $a = 1$ and $b = 4$.

   a) Förenkla uttrycket $6a - 2b - 2a + b$ så långt som möjligt.
   b) Beräkna värdet av det förenklade uttrycket om $a = 1$ och $b = 4$. (F13:203)

The address in the request is implicit, the student is supposed to engage in simplifying and calculating. Because no separate statements about algebra or anything else are given, the task in Example 3 consists of only a request.

Different processes give different possibilities for the student to make choices and take decisions. Material processes such as simplify and calculate, function as orders to carry out. However, verbal processes as, e.g., discuss, explain, or the process investigate instead invites the student to take control of his or her actions, since discussions, explanations and investigations are more open-ended.

According to Morgan (2006) there is a difference between requests which are subject-specific, e. g., calculate, and more everyday processes such as figure out. The former positions the student as a specialist, so the processes have different interpersonal functions. Thus, it is not enough to distinguish among

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19 As a reminder: definitions of different types of processes within SFL are given in the section Discerning school algebra discourses in tasks, in the chapter Theoretical considerations.
processes as being material, verbal, or relational when considering those processes the student is invited to engage in. The degree of subject-specific character must also be considered.

Finally, modifying expressions function to express degrees of probability, usuality, obligation, and readiness (Hellspong & Ledin, 1997). In Example 2 (p. 64), can indicates an assumption that it is possible to make calculations about societal phenomena with algebra, such as the population of Tanzania. Yet, in the question in Example 2, the student has no other choices but to follow the request and calculate, or to ignore the request. So modifiers can function to change both the view on what can be achieved with algebra, in general, and more specifically, the view on what algebra the student can achieve.

6.2.3 Discerning and identifying discourses

In the analysis, discourses emerge as patterns of the linguistic choices identified in the ideational and interpersonal function. For instance, a pattern which was identified early in the process was the absence of statements in some tasks, in combination with a frequent use of symbolic expressions as participants (Example 3, p. 65). A difference in linguistic features which emerged was the combination of geometric-specific participants and relational processes on one side (Example 1, p. 64), and science- or social science-related participants combined with material processes and often passive verb forms on the other side (Example 2, p. 64). The presence of proper names and pronouns emerged in two patterns: either in combinations of material processes with symbolic and everyday participants (Example 7, p. 83), or in combinations of mathematics-specific processes and participants (Example 6, p. 82). Finally, one pattern emerged with subject-specific participants enhanced in semiotic resources such as tables and graphs, together with verbal processes (Example 13, p. 92).

For most of the tasks, a unique discourse was identified. However, in some tasks the linguistic features drew upon more than one discourse. These tasks were labelled hybrids in order to indicate that they carry linguistic features from several discourses. Not all algebra tasks had linguistic features similar to other tasks, which implied that these had to be set aside. These tasks were labelled algebra outside the discourses. Only the textbooks displayed such tasks. Finally, in the textbooks, there were also tasks in the algebra chapters which simply were not algebraic: for instance, some tasks in special theme sections obviously involved statistics or arithmetic. In total, six school algebra discourses were identified in the analysis. They are named the symbolic discourse, geometrical discourse, arithmetical discourse, (un)realistic discourse, scientific discourse, and relational discourse. The descriptions of

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20 1398 out of 1557 textbook tasks and 197 out of 232 TIMSS-items
the first five discourses constitute results of paper I, and for the relational discourse, of paper III.

6.3 Algebraic activities

In this section, characteristics of the different algebraic activities are developed, those having been introduced in the chapter Theoretical considerations.

*Equivalence, Expressions, Equations, and Inequalities* (EEEI) is understood as the algebraic activity connecting to “a relational understanding of the equal sign, representing and reasoning with expressions and equations in their symbolic form and describing relationships between and among generalized quantities” (Blanton et al., 2015, p. 43). This entails, e.g., to evaluate equations as true or false; use variable expressions to model linear problem situations; and interpret an algebraic expression in the context of a given problem. It also entails solving equations and open number sentences, given that there is some kind of context represented in the task.

If there is no context represented in the task, the equation solving is understood as pertaining to the activity of *Manipulation* (M). *Manipulation* is an algebraic activity which instead draws on Kieran’s (2007) transformational activity in the GTG-model. This activity includes equation solving; using, simplifying, and calculating values of symbolic expressions without any modelling context; and equation-solving activities intended for use with lab material or with pictures of lab material, which quite literally requires the manipulation of objects. Hence, EEEI and M concern the same core content area, but while EEEI entail the thinking practices of representing, reasoning, generalizing and justifying, M entail solely transforming activity. They are not overlapping.

*Generalized Arithmetic* (GA) is the activity which explores properties which arithmetic and algebra have in common, by “generalizing arithmetic relationships […] and reasoning about the structure of arithmetic expressions” (Blanton et al., 2015, p. 43). This activity encompasses analysing information to conjecture an arithmetic relationship; to express the conjecture in words and/or variables; to identify values or domains of values for which a conjectured generalization is true; to justify an arithmetic generalization using either empirical or representation-based arguments; and to examine limitations of empirical arguments.

*Functional Thinking* (FT) is the activity which uses a manifold of semiotic resources, e.g., tables, graphs and symbols, to represent, reason and generalize “relationships between co-varying quantities” (Blanton et al., 2015, p. 43). It comprises actions which can almost be considered as steps in a process,
namely to generate data and to organise in a function table; identify and describe a recursive pattern; use a pattern to predict near data; identify and describe a co-varying relationship; identify and describe a function rule; use a function rule to predict far function values; and given a value of the dependent variable, to determine the value of the independent variable. Determining the domain and range of functions and constructing graphs (Blanton et al., 2015; Eddy et al., 2015) might also be included.

The idea of Proportional Reasoning is described by Blanton et al. (2015, p. 43) as “reasoning algebraically about two generalized quantities that are related in such a way that the ratio of one quantity to the other is invariant”. It entails, e.g., the rule of three. However, in an earlier version the idea is named ‘quantitative reasoning’ and described as to “extend relationships between and among quantities to describe and generalize relationships among these quantities” (Blanton et al., 2011, p. 39), and exemplified also with the relation between addition and subtraction. In Blanton et al. (2018) this idea is no longer mentioned. These changes renders this ‘big idea’ of algebraic thinking mildly confusing, and perhaps with a scope too far directed towards primary school. Hence, in this study Qualitative and Proportional Reasoning (QPR) is defined as an activity which involves performing qualitative prediction and comparisons without numerical values on variables (Lundberg, 2011), as well as the activity described by Blanton et al. (2015). A reason for keeping it within the framework established for this thesis is that proportionality is a topic both in the 1994 and 2011 syllabus (see Background chapter).

6.3.1 Categorizing algebraic activities

In the analysis, textbook tasks and TIMSS’ test items are analysed. As described in the chapter on Previous research, Törnroos (2005) finds that the task is a good unit for measuring when making comparisons with TIMSS’ test items. Therefore, in the present thesis written examples and theory sections are regarded as contexts to the tasks, and not analysed per se. They do not require the student to engage in algebraic activity by themselves, and there are no such sections in TIMSS tests. Still, written examples and theory sections may indicate what algebraic activity the student is supposed to engage with in a task, if such activity is not already made obvious within the task itself. The algebraic activity that is fair and reasonable to engage with, given the context, has been noted.

Mostly, only one activity was noted for each task. In the cases where several activities were found reasonable, the tasks were labelled more than one activity. 17 test items out of 232 and 33 textbook tasks out of 1557 entailed more than one activity. For some textbook tasks, no algebraic activity could be noted and the tasks were labelled not algebra. These were discerned with the aid of Radford’s (2018) definition when a task did not in any respect involve treating an indeterminate quantity as if it were known. How these tasks
relate to the *not algebraic*-tasks in the discourse analysis is developed in Methodological considerations, by the near end of this chapter. The diachronic interplay of the algebraic activities and the school algebra discourses in the textbooks and the TIMSS tests was investigated through cross-tables. How the comparisons in and between these tables were accomplished is described further below in the section Comparing algebra characteristics. The cross-tables are displayed in the Results and in Appendix C.

6.4 Statistical analysis of student achievements

As stated in previous research on large-scale assessments, test items with different characteristics may be related to differences in student achievements (Bergvall, 2016; Pedersen, 2015). In order to explore the extent to which significant differences can be identified in the students’ algebra achievements in TIMSS, statistical analysis is used. As a prerequisite, percent right values are calculated in order to make students’ achievements on test items with different maximum scores comparable. The percent right value for a test item is defined as the relative frequency of the students’ raw score, divided by the item’s maximum score (Bergvall, 2016). This means that a percent right value close to 0 indicates that few students solved the test item correctly, and a percent right value close to 1 indicates that many students solved the test item correctly. The percent right value is thus a measurement of student achievements for a particular test item.

Other prerequisites for statistical analysis are to decide whether the data consists of independent groups, what group size can be used without a loss of power, and if the dependent variable is normally distributed (Djurfeldt, Larsson, & Stjärnhagen, 2010). These are accounted for in the first section below. In the second section, the inferential statistical analysis is developed. All statistical calculations mentioned below are performed in SPSS Statistics 25. In the final section on statistical analysis, the selection of a test for inferential analysis is considered.

6.4.1 Descriptive statistics and prerequisites for inferential analysis

In the statistical analysis, percent right values on test items with different algebra characteristics are compared to each other group-wise. The test items which are *hybrids* of discourses and the test items with *more than one activity* are not independent of the school algebra discourses and algebraic activities, respectively. These groups of test items may pose a problem in the inferential analysis. The percent right values pertaining to test items in these groups are therefore not included, which resulted in 197 test items being possible for the
inferential analysis (Table 6). The impact of missing values on the inferential analysis is discussed in Methodological considerations. Descriptions of the identified school algebra discourses are given in the Results.

\begin{table}[h]
\centering
\caption{Distribution of the test items possible for inferential analysis (Paper III).}
\begin{tabular}{lcccc}
\hline
School algebra discourse & Algebraic activity & Items in total \\
 & $GA$ & $QPR$ & $FT$ & $EEEI$ & $M$ \\
\hline
Symbolic & 0 & 8 & 0 & 22 & 52 & 82 \\
Geometrical & 0 & 2 & 5 & 19 & 0 & 26 \\
Arithmetical & 0 & 0 & 13 & 6 & 0 & 19 \\
(Un)realistic & 1 & 0 & 32 & 13 & 0 & 46 \\
Scientific & 0 & 0 & 0 & 9 & 0 & 9 \\
Relational & 0 & 0 & 14 & 1 & 0 & 15 \\
\hline
Items in total & 1 & 10 & 64 & 70 & 52 & 197 \\
\end{tabular}
\end{table}

Please, note that the school algebra discourses in Table 6 constitutes results of papers I and III.

As for the size of the groups, the combinations of school algebra discourses and algebraic activities turned out to be distributed unevenly with some combinations including very few test items. Split up among the different test years, these groups became even smaller. Performing statistical analysis on too small samples may cause a loss of test power (Heiman, 1996). Thus, in this thesis the inferential analysis is done separately for school algebra discourses and algebraic activities. It is not split up for every possible combination of school algebra discourse and algebraic activity. For the same reason – i.e. not to lose test power – the inferential analysis is performed on all test items instead of being split up on test items from separate test years and only data groups of 15 or more test items are analysed. This restriction gives a selection of five school algebra discourses and three algebraic activities for the inferential analysis: the symbolic, geometrical, arithmetical, (un)realistic, and relational discourses, and the algebraic activities FT, EEEI and M.

Percent right values in this selection were studied separately for each TIMSS test, student group and algebra characteristics, to control to what extent their median and spread changed. Particular attention was given to the test items in the relational discourse, because of their containing a low number of test items before 2015. Considered over time, these percent right values did not deviate markedly from the trend measured by TIMSS (cf. Mullis et al., 2016, p. 32).

Depending upon what a data distribution looks like, different statistical tests can be used. Some tests demand normally distributed data and others do not (Heiman, 1996). In order to describe the data, tables and box plots of the
selected school algebra discourses and algebraic activities were made (see Appendix B). For some statistical tests, normality is a sufficient but not a necessary condition. If the data has an approximately normally distributed sample mean, a t-test is a valid method for drawing inferences about differences between data groups (Lumley, Diehr, Emerson, & Chen, 2002). For data groups with more than 30 observations, sample means are practically normally distributed, due to the Central Limit Theorem. For smaller groups of data, the t-test is valid if the distribution of the sample means is approximately normally distributed (Lumley et al., 2002).

The normality of sample means pertaining to this thesis are demonstrated below through a simulated example. In this example, a worst case scenario was created through a combination of the most extreme data distribution and the smallest sample size which were identified in the data.

Figure 2. Non-normal data distribution in the geometrical discourse for students with low achievements (Paper III).

The data distribution which showed the least resemblance to a normal distribution, of the data groups with fewer than 30 observations, was identified in the geometrical discourse for students with low achievements (Figure 2).
The smallest sample size which was identified in the data was 15 test items, in the relational discourse\(^{22}\). An example data distribution based on these data (Figure 3) was then simulated 1000 times. Sample means were calculated for each of these 1000 simulated samples. Their normality is demonstrated by the histogram and QQ-plots below (Figures 4 and 5), so the t-test can be used to draw inferences from the TIMSS data in this thesis.

\(^{22}\) This is a result of the selection, since groups with less than 15 items were excluded from the analysis. However, as stated earlier, keeping even smaller groups is not to be recommended due to losses in test power.
6.4.2 Inferential analysis

That the relation between student achievements and algebra characteristics is explored with inferential analysis means that no specific hypothesis is tested. The tests are performed to see if and where there are significant differences in student achievements, connected either to different algebraic activities (FT, EEEI and M), or to school algebra discourses (symbolic, geometrical, arithmetical, (un)realistic, and relational).

The t-test is a frequently used test for understanding whether or not two groups of data are similar. It can be used on independent groups of data where the dependent variable is numerical (Heiman, 1996) and if the sample means are normally distributed (Lumley et al., 2002). The t-test also requires a control of the homogeneity of variances in the groups tested. In SPSS Statistics, the t-test is automatically combined with Levene’s test to control for homogeneity of variances. When the variances are found to be unequal, SPSS automatically runs an unequal variance t-test, which presents the proper result for interpretation.

In this thesis, different algebra characteristics are compared pair-wise with the t-test. Between the algebraic activities, 3 t-tests are made and between the school algebra discourses, 10 t-tests are made.

It is well-known that multiple tests increase the risk for type I errors (Heiman, 1996), i.e., that the test does not reject the null hypothesis. In this thesis, that would mean finding a significant difference between two groups of algebra characteristics even though there is none, just because a repeated testing procedure makes finding differences more probable. To avoid this problem, the levels of significance are Bonferroni-adjusted in accordance to the number of tests performed. More specifically, the levels of significance are described by:

\[ \alpha_{\text{activities}} = 0.05/3 \approx 0.017 \quad \text{and} \quad \alpha_{\text{discourses}} = 0.05/10 = 0.005 \]
The results obtained by this correction are considered as robust. The significant differences in percent right values which are obtained in the inferential analysis are illustrated by the box plots in Figures 11–15 in Appendix B.

6.4.3 Considering the choice of test for inferential analysis

Pairwise t-tests are used in this thesis, though there are more than two groups which are tested against each other. The advantage of the t-test is that it is not particularly sensitive to different shapes of the data groups that are compared. As stated earlier there is a t-test for unequal variances, which is run automatically by SPSS when the assumption of homogeneity is not met. An alternative test might have been an analysis of variance, called ANOVA. This is a test designed for multiple comparisons, and not just pair-wise. However, the ANOVA assumes a homogeneity of variance and normally distributed data (Heiman, 1996) and if the data does not meet this requirement, other tests will have to be applied. Most other tests for multiple comparisons are non-parametric, and using a non-parametric test instead of a parametric – such as the ANOVA – causes a loss of power (Heiman, 1996). Hence, as long as the t-tests are adjusted for multiple testing, e.g., with the Bonferroni-correction, the t-test will do the same job as the ANOVA.

6.5 Comparing algebra characteristics

The second, fourth, and last research questions involve to what extent the algebra characteristics change over time, and in what ways the algebra characteristics differ in textbooks and TIMSS. To answer these questions, quantitative and qualitative comparisons were made in order to understand similarities and differences of the algebra characteristics, as well as changes over time in those characteristics.

The school algebra discourses and the algebraic activities were organised into cross-tables for the textbooks (Tables 8–10, pp. 87–88) and the TIMSS tests (Table 11, p. 31 and Appendix C), respectively. To visualize the difference between different textbook series, the standard deviation was calculated and added in the cross-table for the algebra characteristics in the textbook tasks (Table 10, p. 88). The comparisons were guided by the questions:

- What algebra characteristics are usual in the materials?
- What algebra characteristics are unusual?

These questions led to a discrimination of the most, and second most, common combination of school algebra discourse and algebraic activity. Moreover, deciding what constitutes a ‘change over time’ turned out to be problematic.
When looking at the changes over time in the textbook material, the comparison in the end regarded three aspects. First, the percentage points were compared. Then the absolute number of tasks were compared, because some changes in percentage points which looked similar entailed very different numbers of tasks. Third, the relative change for each school algebra discourse and algebraic activity in each textbook series was calculated as a quota. The relative change was compared with the quota between the numbers of algebra tasks in each textbook series before/after 2011. This was done in order to understand how the proportion of the algebra characteristics changed, compared with how the number of algebra tasks in total changed. During the process of looking into these three aspects of change, it was realized that they mapped considerably well to changes in the cross-table for the textbooks, within five percentage points or more. Therefore, the limit of five percentage points was taken as a measure of ‘change over time’ for the textbook material. Because the TIMSS tests do not entail as many items, this limit was deemed too low for the TIMSS material. Here, a limit of more than ten percentage points was applied instead.

The comparison over time of the textbook tasks in Table 8–10 forms a basis for the discussion of how textbooks change in response to the reform ideas on algebra in the 2011 school algebra reform.

The comparison over time of the structure of TIMSS in Table 11 includes the 197 test items, which is the selection that the inferential analysis is grounded in. The comparison forms a basis for the discussion on validity over time.

Had the last research question been quantitative, asking to what extent the algebra characteristics differ, a statistical test could have given an answer. Instead, a qualitative approach was applied in order to understand what the differences were. Accordingly, the comparison between the textbook tasks in Tables 8–10 and the test items in Appendix C was guided by the questions:

- What algebra characteristics do the materials have in common?
- What algebra characteristics stand out as differences?
- If there are changes over time, are these in common for the materials?

The table in Appendix C includes all 232 test items. The results of the comparison between the textbook tasks and the test items in Appendix C are given in the first section of the Discussion and conclusions (Table 12, p. 99). It forms a basis for discussing the relevance of using TIMSS results for drawing inferences about Swedish students’ knowledge in algebra.
6.6 Methodological considerations

In this section, quality in research is discussed. The intention in this thesis has been to account for quality throughout the text. However, a few issues need to be addressed separately. Below, validity and reliability are discussed. To address precision and rigor, some comments are also made concerning the understanding of concepts.

6.6.1 Validity in research

Research validity can be discussed as internal and external validity. The internal validity concerns whether the phenomena under scrutiny is matched by the methods and results of the study. The external validity concerns whether the generalizations made from the study are reasonable inferences (Kilpatrick, 1993).

In this thesis, textbook chapters on functions and relationships are not a part of the selected material. This may seem odd. The selection affects the proportion of FT-tasks, since it can be expected that the algebraic activity Functional Thinking, FT, is prevalent in chapters named Relationships or Functions and graphs. The problem with including these chapters is that only one of the textbook series features such a chapter both before and after 2011 (Table 2, p. 60). Another of the series includes a chapter on functions before 2011, but not after. Including these chapters would then affect the comparison over time. Including textbooks for Year 9 in the selection could have been an alternative, since all of the three series include functions and graphs in their Year 9 textbooks. Such a selection would allow for a broader comparison of tasks over time, and thus give a better insight into algebra in Swedish textbooks. Furthermore, it would give percentages of FT in Swedish textbooks, which are closer to the proportions of FT in TIMSS. Still, Year 9 is not the same year as the TIMSS tests are taken. Including Year 9 textbooks in the selection would therefore not give a good picture of what algebraic activities Swedish students have most likely encountered before they take the TIMSS test. Such a selection would affect the comparison between textbook tasks and test items, and the relevance of using TIMSS results for drawing inferences. The choice of including or not including textbook chapters on functions in the selection of material thus creates a tension between understanding Swedish textbooks per se, as against assessing the relevance of using TIMSS results to understand Swedish students’ achievements. Compared to the aim of the thesis, the latter is seen as a better choice.

Both hybrids of discourses and tasks or items entailing more than one algebraic activity were included in the analytical frameworks. This decision was taken because of the material: the textbook tasks and the test items did not always pertain to only one discourse, nor did they always entail only one algebraic activity. However, because of this inclusion, in the inferential analysis only 197
of 232 test items could be used. This selection had to be made in order to achieve independent variables, since interdependency is not an option when performing inferential analysis. 35 of the test items are left aside in the inferential analysis. An account for the algebra characteristics of these test items and their percent right values is given in the chapter Discussion and conclusions. They do not alter the results or the conclusions in the thesis.

Regarding the methods used, the process of the discourse analysis was iterative and took place over a long period when new tasks were regularly compared with previously analysed tasks in order to understand their differences and similarities when it comes to linguistic features. Parts of the textbook material were thoroughly analysed during a course in SFL during the spring of 2016. When the TIMSS’ test items were analysed, they were compared to textbook tasks which were already analysed, in order to clarify if further discourses could be identified. The analysis was frequently discussed with a researcher experienced in SFL. Moreover, the SFL framework was presented at several seminars and colleagues had empirical data to check for reliability.

The analysis of the algebraic activities was discussed at several times with researchers familiar with Blanton et al. (2018, 2015). It was also raised at seminars and at two mini-conferences with the Nordic Network for Algebra Learning, n2AL. At these occasions, colleagues discussed the framework, helped in elaborating the category *Manipulations*, and subsequently helped in checking data for reliability.

### 6.6.2 Some comments on rigor and precision

As a part of the working and learning processes, the understanding of some concepts have shifted along the way in this thesis. Writing a thesis is a long process and when it includes writing papers, it is not always an option to go back and make changes. This section is written in order to clarify such matters.

The first shift in this thesis concerns what algebra is. In paper I, what is *not algebraic* is described as tasks where symbolic expressions neither are given in the tasks, nor needed for solving them (see endnote 4, paper I). Hence, the concept builds on the use of alphanumerical symbols. According to Radford (2018) the use of these symbols is merely one way to represent algebraic thinking. This notion became obvious during the work on paper II, for which reason Radford’s definition of algebra as described in the Theory chapter, was used to discriminate what is *not algebra*. This means that 98 tasks of 1557 are viewed as *not algebraic* in the analysis of school algebra discourses in paper I, while 80 tasks of 1557 are viewed as *not algebra* in the analysis of algebraic activities in paper II. The difference – 18 more algebra tasks – mostly consists of number sentences. If analysed within the SFL framework, they would likely not be included in any of the school algebra discourses but rather left aside in the group *algebra outside the discourses*. The percentages in papers I and II are calculated in relation to all of the tasks in the chapters. This means that the
proportions of the school algebra discourses and algebraic activities are not affected by this change in the understanding of algebra.

Another shift concerns the addition of *Manipulation* to the framework of algebraic activities. Initially, this category was understood very much as a procedural skill, relying on the argumentation of Kaput et al. (2008) who uses the metaphor ‘playing the scales’ for this kind of algebra. Simplifying expressions and solving equations without any further context then could be interpreted quite literally as something which is done with the hand, not the mind. Nonetheless, Kieran (2007) emphasizes that there are conceptual aspects of carrying out these procedures which rather point in the direction of a structural understanding of the algebraic objects which are being manipulated. Using the same nomenclature as Kieran, *transformation* could have been a better choice for this algebraic activity. By the time this possible improvement was realized, paper II was already in review and changing the name of the analytical category seemed to be a less good idea. Additionally, the difference between transformation and manipulation may in fact be something which plays out in the classroom, rather than being aspects of the type of tasks here categorized as *Manipulation*. This possibility is raised in the end of the Discussion chapter.

Finally, a reader of paper I may, when reading the summary of paper I, observe that what was called *activity* in paper I is called *action* in the thesis. In order to avoid confusions with *algebraic activity*, the description of what the school algebra discourses invite the student to engage in was altered after the first paper was published. As a category, an algebraic activity may ask the student to do a number of things, but an *action* is a specific action required of the student by a specific task or test item.

### 6.7 Ethical considerations

The considerations described below are made with the guidelines from the Swedish Research Council (2017) in mind. Most of them concern research on human beings and are not so relevant for this thesis.

Within the social-semiotic framework, texts are made up of linguistic choices which both construct meaning, and are themselves constructed through social interaction and context (Halliday & Hasan, 1989; Luke, 1995). This means that the meaning a reader construes is not necessarily the same as the author’s intentions. In the present thesis, author’s intentions are not studied or valued, but the text is examined for the meanings which are constructed as an operation of looking at them through the theoretical frameworks applied. Hence, the analysis is in no respect a critique of the authors of the analysed tasks and test items.
Student achievements are an ethical concern, if personal data is obtained. IEA gives public access to students’ achievements in TIMSS through their website. These are undisclosed, so no personal data is available or used.

When doing research on texts, ethical concerns may be raised about whether the material is public or not and whether to present the material anonymously or not. Two types of texts are concerned in the present thesis: textbook tasks and test items.

In studies of textbooks, it is possible to a certain extent to present data anonymously. For instance, Törnroos (2005) calls the textbook series in his study K, P, and MM. In the present thesis, the textbooks would still be identifiable through example tasks, since textbooks are public texts. For the sake of reliability in the analysis, the examples cannot be excluded from this thesis. At an early stage in the research process, the publishing companies were contacted and asked if they would consider sending free copies of the textbooks in question for the purpose of research. Two of the publishing companies did so. Even in the case where a free copy was not obtained, the publishers were at least informed of the research.

Unlike textbooks, TIMSS’ test items are classified materials. The trend items are reused in up to three tests and cannot be published. Therefore, permissions to do research on TIMSS’ test items and to publish released test items was obtained by the Swedish National Agency of Education and IEA (see Appendix E). The test items have been stored locked up and out of sight for those who are not included in the permissions granted by the Swedish National Agency of Education. When discussed at seminars or published, only released test items have been used, unless all of the participants at the seminar were included in the permission from the Swedish National Agency of Education. In two instances, released test items were complemented with textbook tasks similar to classified test items in order to enable a deeper discussion of the analysis and a more thorough check of data for reliability.
7 Results. Summary of the papers

This chapter summarizes the papers in the thesis and their contributions to answering the research questions. The first paper develops how linguistic tools within a social-semiotic framework can be adopted for identifying *school algebra discourses* in textbook tasks. It also describes algebra characteristics in textbook tasks through the school algebra discourses which are identified. This answers the first research question.

In paper II, algebra characteristics in the same textbook tasks as in paper I are categorized through a second framework of *algebraic activities*. By incorporating the results from paper I, the diachronic interplay of school algebra discourses and algebraic activities is studied in order to understand changes in the textbooks during the period 1995–2015. These changes are then related to main ideas on algebra in the 2011 syllabus. The paper answers the first and second research questions.

The third paper repeats the analyses of school algebra discourses and algebraic activities from papers I and II, on TIMSS’ algebra items from 2003–2015. This is done in order to understand differences in the structure of TIMSS tests over time. The paper further explores significant differences in students’ achievements on the algebra items, as related to school algebra discourses and algebraic activities respectively. These differences are then compared to changes over time in the structure of TIMSS. The paper answers the third, fourth and fifth research questions.

The final research question is answered in the Discussion and conclusions by a synthesis of the results from papers I–III. This is done through a comparison of the diachronic interplay of school algebra discourses and algebraic activities in the two materials.

7.1 Discourses in school algebra: the textbooks’ different views on algebra and the positioning of students. Paper I

The purpose of this study is to understand school algebra such as it is offered in textbooks. From a social-semiotic perspective, the study of discourse can provide opportunities for discussing questions of subject-specific language and epistemology, as well as ideological aspects and equal access to education.
(Herbel-Eisenmann, 2007; Morgan, 2016b; Schleppegrell, 2004). Paper I employs the social-semiotic perspective for exploring how the nature of algebra, as well as the actions and positions which students are invited into, are constructed in text. Algebra tasks in textbooks for Year 8 are analysed with Systemic Functional Linguistics (Halliday & Hasan, 1989; J. R. Martin et al., 1997). This is done in order to identify patterns in the use of linguistic features, which in turn construct school algebra discourses. In total, 1557 tasks in six textbooks from three textbook series are studied.

Five school algebra discourses are discerned through an iterative analysis of the material: the symbolic discourse, the geometrical discourse, the arithmetical discourse, the (un)realistic discourse, and the scientific discourse.

The symbolic discourse is characterized by subject-specific words, e.g., equation, expression, value, solution, method; and by the presence of alphanumerical symbols. There is a widespread absence of separate statements, personal pronouns, and proper names, as demonstrated by Example 4. In most instances, the student is required to engage in material processes, e.g., to solve, calculate, simplify, check, and use. These features construct algebra as depersonalized material or manipulating doings.

Example 4. The symbolic discourse.

Solve the equations  

\[ a) \ 3x + 11 = 5x - 3 \quad b) \ 7x - 20 = 2x - 10 \]

Lös ekvationerna  

\[ a) \ 3x + 11 = 5x - 3 \quad b) \ 7x - 20 = 2x - 10 \] (Y96, p. 219)

Other characteristics in the symbolic discourse, though less prominent, are separate statements about algebra, questions, pronouns, modifiers, verbal processes directed at the student, e.g., explain, discuss, think, describe, or the action to decide ‘true or false’. These less prominent features instead invite the student to engage in algebra through communicative and mental actions, and more specifically, through argumentation.

In the geometrical discourse, relational processes build connections between geometrical objects, measurements and alphanumerical symbols. Sometimes these connections are stated through pictures of geometrical objects. As in the symbolic discourse, proper names or pronouns are absent. These features together construct algebra as atemporal. Sometimes algebra is instead constructed as depersonalized through the use of passive verbs and modifiers, e.g., can be calculated. The student is mostly required, or asked, to engage in material processes, e.g., simplify, calculate, work out, but also in write, express and draw. The student is also addressed in questions, as in Example 5.

Example 5. The geometrical discourse.

In a triangle, angle A is 45°. Angle B is twice as big as angle C. How big are the angles B and C?
Other and less prominent characteristics are identified when the student is required to discuss or decide ‘true or false’ in a task. Hence, the student is mostly invited to engage in algebra through material doings, but to some extent also in communicative and argumentative actions.

The characteristics of the arithmetical discourse are subject-specific words such as number, sum, and product, and subject-specific processes such as is added and is multiplied. As opposed to the symbolic discourse, arithmetical operations or calculations are rendered in written language and not in alphanumeric symbols. This is true for the equal sign as well, constructed as the relational process is or becomes. The arithmetical discourse sometimes features proper names and pronouns, which indicate that algebra is something that humans do. As Example 6 shows, passive verbs may also be used, which hides human agency.

Example 6. The arithmetical discourse.

A number is added to 5, thereafter the sum is multiplied by 2. The product becomes equally large as when one multiplies the number by 3. Set up an equation and work out the number.


The student is supposed to engage in material processes such as simplify, use, and calculate, as well as in processes such as express, set up, and write, or to discover and think. These features together construct algebra as a human enterprise, which most often entail solving number riddles but sometimes exploring e.g. properties of numbers. Other and less prominent characteristics are processes such as think, explain, show, and create; subject-specific processes, such as multiply; or tables. These processes invite the student to act in exploratory and communicating actions.

The (un)realistic discourse mostly features statements about third persons, nutrients, and everyday goods. These enter into relational and material processes, e.g. costing, being, baking, swimming, driving, selling, buying, or earning, in relation to alphanumeric symbols or to each other. These statements describe situations where algebra in reality is more or less likely to be used; and often less likely, as in Example 7. This is what the name (un)realistic refers to, since these tasks can be placed on a gliding scale between unrealistic and realistic.
Example 7. The (un)realistic discourse.

The tickets to a circus cost $a$ SEK per item. Write an expression of how much
a) 5 tickets cost
b) Emma gets in return from a 500-SEK banknote when she buys 2 tickets
c) Sara gets in return from 400 SEK when she buys 3 tickets

Biljetterna till en cirkus kostar $a$ kr per styck. Skriv ett uttryck för hur mycket
a) 5 biljetter kostar
b) Emma får tillbaka på en 500-kronorssedel när hon köper 2 biljetter
c) Sara får tillbaka på 400 kr när hon köper 3 biljetter (F13, p. 200)

The student is supposed to engage in material processes such as to simplify, solve, work out, use, set up, express, and write, and to engage in the verbal process explain. These features together construct the world as inherently algebraic and it is a world where only the student is doing algebra. These doings concern manipulating and communicating. Other less prominent features are verbal processes, constructing more communicative and mental actions, e.g., to discuss, ask, investigate, and invent. Most of the time, the features in the (un)realistic discourse construct humans as consumers or participants in everyday situations.

The scientific discourse is only identified in one of the textbook series. It distinguishes itself through rather long noun phrases of physical or societal phenomena. These are typically described with the material processes is / can be / could be calculated. The obscuring of human agency is once again achieved by using passive verbs, as seen in Example 8. The student is supposed to engage in only a few material processes: calculate, use, and round off. This constructs algebra as a tool for science that the student is supposed to use. However, it is unclear by whom the tools are constructed.

Example 8. The scientific discourse.

The speed of sound at different temperatures is calculated with the formula
\[ v = 332 \times \left( 1 + 0.0018 t \right) \]
where
\[ v \] = the velocity of sound in meters per second
\[ t \] = the temperature in degrees Celsius

Calculate the velocity of sound at the temperature …
\begin{align*}
a) & \quad 0^\circ C \quad b) \quad 10^\circ C \quad c) \quad 20^\circ C \\
\end{align*}

Ljudets hastighet vid olika temperaturer beräknas med formeln
\[ v = 332 \times \left( 1 + 0.0018 t \right) \] där
\[ v \] = ljudets hastighet i meter per sekund
\[ t \] = temperaturen i grader Celsius

Beräkna ljudets hastighet vid temperaturen …
\begin{align*}
a) & \quad 0^\circ C \quad b) \quad 10^\circ C \quad c) \quad 20^\circ C \quad (Y96, p. 191) \\
\end{align*}
The symbolic and geometrical discourses encompass linguistic features that mostly position the student as a person who knows how to do algebra, while the less prominent features rather address the student as an apprentice. The arithmetical and (un)realistic discourses have a more personal address, but they mainly position the 14–15 year old student as a child solving riddles and as an artificial consumer, respectively. In the arithmetical discourse, processes describe how to operate arithmetically. These processes are linguistically less challenging than descriptions using arithmetical operations with symbols or noun phrases (Morgan & Sfard, 2016a). In the (un)realistic discourse, many humans participate. If this feature is emphasized at the expense of mathematical objects, the mathematics will also be less challenging (Morgan, 2016b). Hence, both of these discourses are linguistically less challenging than the symbolic and geometrical discourses. The scientific discourse, finally, has a more complex language because of the long noun phrases, but it invites students to calculate and to do little else so the range of actions is limited. The actions that the student is supposed to engage in, within the different discourses, are summarized in Table 7 below.

Table 7. Actions that school algebra discourses invite to (Paper I).

<table>
<thead>
<tr>
<th>school algebra discourse</th>
<th>main actions which the student is invited to</th>
<th>less prominent actions which the student is invited to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbolic</td>
<td>calculating values of and manipulating symbols</td>
<td>discussing and explaining symbolic expressions, deciding ‘true or false’</td>
</tr>
<tr>
<td>Geometrical</td>
<td>expressing with symbols, manipulating symbols and calculating values in geometrical relations</td>
<td>discussing geometrical relations, deciding ‘true or false’</td>
</tr>
<tr>
<td>Arithmetical</td>
<td>solving riddles, expressing relationships in symbols and solving riddles</td>
<td>posing riddles, explaining or creating with numbers and symbolic expressions</td>
</tr>
<tr>
<td>(Un)realistic</td>
<td>expressing with symbols, manipulating symbols, and calculating, in more or less artificial everyday situations</td>
<td>discussing and exemplifying symbolic expressions in more or less artificial everyday situations</td>
</tr>
<tr>
<td>Scientific</td>
<td>calculating in scientific situations</td>
<td>–</td>
</tr>
</tbody>
</table>

The method applied in this study gives an example of how SFL can be used to analyse discourses in textbook tasks. The results show that several discourses are at play, constructing algebra and the student in different ways and with different demands. These discourses are presented higgledy-piggledy on the pages
in the textbooks, so the picture of mathematics is incoherent\textsuperscript{23}. This kind of discord may cause a problem for the ways in which students may understand how the subject matter is built up (Schleppegrell, 2004) and thus influence students’ possibilities to learn algebra and to prepare for higher studies.


**Paper II**

The aim of this study is to understand the textbooks’ relation to the syllabus in contemporary Swedish school mathematics reform. Several Swedish curricular reforms during the 20\textsuperscript{th} century were imposed through the introductions of new syllabi. Still, textbooks in mathematics were an important part of curriculum development. New ideas were introduced through textbooks rather than syllabi before 1960 and later on, when New Math was being implemented, a large part of the state-driven project was to develop new textbooks (Prytz, 2017, 2018). Since the 1960s, policy governance has also gone from being centralized to being decentralized (Lindensjö & Lundgren, 2014). Governance is now based on educational goals and evaluation of these goals (Sundberg & Wahlström, 2012). Moreover, textbooks were reviewed until 1991 but are now published on a market which is very weakly regulated (M. Johansson, 2003; Johnsson Harrie, 2009). How textbooks change in curricular reform today seems less researched, however.

In the commentary material to the 2011 syllabus, algebra was stressed as an area where students needed better knowledge (Skolverket, 2011a). Therefore, lower secondary school algebra can be an interesting case for studying a reform.

Through a framework of algebraic activities, 1557 textbook tasks in three textbook series from before and after the introduction of the 2011 syllabus are categorized. This is the same material as in paper I. Drawing on the results from paper I, the diachronic interplay of school algebra discourses and algebraic activities in algebra tasks is studied. Changes in this interplay are then compared to reform ideas put forward in the 2011 syllabus\textsuperscript{24}.

The point of using two frameworks is to capture both changes which are linked to the 2011 school algebra reform and changes which are independent of the reform. The framework of algebraic activities builds on mainly Blanton et al. (2015, 2018) and Kieran (2007). It includes Equivalence, Expressions, Expressions,

\textsuperscript{23} It is not in itself a problem that there are several school algebra discourses in the textbooks, but the way they are presented that seems problematic. An alternative could have been to present tasks pertaining to one discourse at a time.

\textsuperscript{24} The syllabi of 1994 and 2011 are compared in the Background chapter of the thesis, so these reform ideas are not repeated in this summary.
Equations, and Inequalities (EEEI), Manipulation (M), Generalized Arithmetic (GA), Functional Thinking (FT) and Qualitative and Proportional Reasoning (QPR)\textsuperscript{25}.

The algebraic activities EEEI (e.g. Examples 5–8, p. 79–81) and M (Examples 3, 4, p. 63, 81) together dominate the textbooks’ algebra chapters. As for the distribution of the algebraic activities (Table 8, 10, p. 87, 86), EEEI increases slightly in the textbook series. The changes in M are difficult to account for. In one series, the distribution increases in absolute numbers, while it decreases in proportion. Overall, there is a decrease of the proportion of M; but then again, in another of the series, this decrease is just a question of four tasks fewer. GA (Example 9) and QPR (Example 10) are rare both before and after 2011.

Example 9. Generalized arithmetic. Rare in textbooks before and after 2011.

Write a two-digit number where the digits are not the same. Let the digits change place and calculate the difference between the largest and the smallest number. Try several different numbers.

The answer is always a part of the same multiplication table. Which?

Skriv ett tvåsiffrigt tal där siffrorna är olika. Låt siffrorna byta plats och räkna ut skillnaden mellan det största och minsta talet. Pröva flera olika ta.

Svaret ingår alltid i samma multiplikationstabell. Vilken? (MD02, p. 103)

It could be argued that Example 9 is purely arithmetic. However, all tasks are analysed in relation to their context. In the case of Example 9, the task is followed up by further tasks where the student is asked to do similar investigations with three-digit numbers and then with symbols. Therefore, GA is a fair and reasonable activity to engage in, in this task.

Example 10. Qualitative and proportional thinking. Rare in textbooks before and after 2011.

How many times as big is
a) the circumference of Square B compared to Square A
b) the area of Square B compared to Square A

Hur många gånger så stor är
a) omkretsen av kvadraten B jämfört med kvadraten A
b) arean av kvadraten B jämfört med kvadraten A (F13, p. 234)

There is a small increase in the algebraic activities within FT (Example 11), in two of the three textbook series. These tasks mostly concern patterns. In

\textsuperscript{25} The categories in the framework are described in the thesis’ chapter Analysis and methods.

The pictures show how many persons can sit at 1, 2 and 3 tables. How many persons can sit of one, in the same way, puts together a) four tables b) seven tables c) ten tables

Bilderna visar hur många personer som får plats vid 1, 2 och 3 bord. Hur många personer får plats om man på samma sätt sätter ihop a) fyra bord b) sju bord c) tio bord (Y12, p. 119)

Table 8. Distribution of algebraic activities per textbook (Paper II).

<table>
<thead>
<tr>
<th>Algebraic activity</th>
<th>Y96 (n=338)</th>
<th>Y12 (n=455)</th>
<th>MD02 (n=160)</th>
<th>MD10 (n=154)</th>
<th>F07 (n=196)</th>
<th>F13 (n=254)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA % within book</td>
<td>1 %</td>
<td>2 %</td>
<td>4 %</td>
<td>1 %</td>
<td>2 %</td>
<td>2 %</td>
</tr>
<tr>
<td>QPR % within book</td>
<td>4 %</td>
<td>1 %</td>
<td>1 %</td>
<td>3 %</td>
<td>4 %</td>
<td>4 %</td>
</tr>
<tr>
<td>FT % within book</td>
<td>1 %</td>
<td>10 %</td>
<td>0</td>
<td>5 %</td>
<td>3 %</td>
<td>2 %</td>
</tr>
<tr>
<td>EEEI % within book</td>
<td>42 %</td>
<td>47 %</td>
<td>48 %</td>
<td>57 %</td>
<td>55 %</td>
<td>57 %</td>
</tr>
<tr>
<td>M % within book</td>
<td>49 %</td>
<td>33 %</td>
<td>38 %</td>
<td>37 %</td>
<td>41 %</td>
<td>39 %</td>
</tr>
<tr>
<td>Not algebra</td>
<td>8 %</td>
<td>8 %</td>
<td>9 %</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* Please, note that several algebraic activities can be identified in one task. The percentages may not add to 100.

As for changes in the school algebra discourses, there is a doubling in two of the three textbook series of the (un)realistic discourse, a decrease of the symbolic discourse in one of the series, and an increase of the geometrical discourse in another series (Table 9, p. 88). Hence, in relation to the introduction of the new syllabus, there are no general changes in the discourses, i.e., in all of the textbook series. In Table 9, the most prominent changes (five percentage points or more) in the school algebra discourses are circumscribed.
Table 9. Distribution of tasks in school algebra discourse per textbook (Paper II).

<table>
<thead>
<tr>
<th>School algebra discourse</th>
<th>Y96 n=338</th>
<th>Y12 n=455</th>
<th>MD02 n=160</th>
<th>MD10 n=154</th>
<th>F07 n=196</th>
<th>F13 n=254</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Symbolic discourse</strong></td>
<td>54%</td>
<td>36%</td>
<td>53%</td>
<td>56%</td>
<td>45%</td>
<td>44%</td>
</tr>
<tr>
<td><strong>Geometrical discourse</strong></td>
<td>12%</td>
<td>10%</td>
<td>18%</td>
<td>16%</td>
<td>20%</td>
<td>28%</td>
</tr>
<tr>
<td><strong>Arithmetical discourse</strong></td>
<td>9%</td>
<td>8%</td>
<td>9%</td>
<td>7%</td>
<td>16%</td>
<td>12%</td>
</tr>
<tr>
<td><strong>(Un)realistic discourse</strong></td>
<td>12%</td>
<td>25%</td>
<td>7%</td>
<td>15%</td>
<td>15%</td>
<td>13%</td>
</tr>
<tr>
<td><strong>Scientific discourse</strong></td>
<td>5%</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td><strong>Hybrids</strong></td>
<td>0%</td>
<td>0%</td>
<td>3%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td><strong>Algebra outside discourses</strong></td>
<td>0%</td>
<td>7%</td>
<td>0%</td>
<td>0%</td>
<td>1%</td>
<td>0%</td>
</tr>
<tr>
<td><strong>Not algebraic</strong></td>
<td>9%</td>
<td>10%</td>
<td>13%</td>
<td>1%</td>
<td>1%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 10. The interplay of the school algebra discourses and algebraic activities in the textbooks (Paper II). The standard deviations are given in parenthesis.

<table>
<thead>
<tr>
<th>School algebra discourse</th>
<th>Curriculum Algebraic activity in percent of Total*</th>
<th>Discourses in % of Total*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GA</td>
<td>QPR</td>
</tr>
<tr>
<td><strong>Symbolic</strong> 1994</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>2011</td>
<td>0</td>
<td>1.1</td>
</tr>
<tr>
<td><strong>Geometrical</strong> 1994</td>
<td>1</td>
<td>0.6</td>
</tr>
<tr>
<td>2011</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Arithmetical</strong> 1994</td>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>2011</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>(Un)realistic</strong> 1994</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>2011</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Scientific</strong> 1994</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2011</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Hybrids</strong></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Outside the discourses</strong></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Algebraic activity in % of Total</strong></td>
<td>1994</td>
<td>2 (1.6)</td>
</tr>
<tr>
<td>2011</td>
<td>2 (0.7)</td>
<td>2 (1.4)</td>
</tr>
</tbody>
</table>
* The row percentages do not add up to the total, since a task can entail more than one algebraic activity.
Table 10 above presents the diachronic interplay of the algebraic activities and the school algebra discourses. The most prominent changes (five percentage points or more) are circumscribed. Notably, approximately 30 to 40 percent of all algebra tasks in the textbooks consist of M in the symbolic discourse. The largest increase pertaining to more than one textbook series is the increase of the (un)realistic discourse.

The standard deviations change over time for some of the algebra characteristics. For instance, there is a small increase in the standard deviation for EEEI in the symbolic discourse (Example 12).

Example 12. EEEI in the symbolic discourse, increasing in one textbook series.

The equation \(5x + 2y = 29\) contains two different unknowns.
   a) Can you find any solution to the equation?
   b) How many solutions do you think there are?

Ekvationen \(5x + 2y = 29\) innehåller två olika obekanta.
   a) Kan du hitta någon lösning till ekvationen?
   b) Hur många lösningar tror du att det finns? (Y12, p.238)

These increases indicate ways in which the textbook series make different interpretations of the new syllabus\(^{26}\). There is also a decrease in the standard deviation for M in the symbolic discourse. This indicates how one series’ decrease renders the series more similar to each other with respect to M in the symbolic discourse.

Regarding the new ideas in the syllabus, there are no changes in common to all of the textbook series. The increasing standard deviations instead show how the textbooks make somewhat different interpretations of the new syllabus.

There are small changes in the textbook series which may be associated with the enhanced competence perspective in the 2011 syllabus. For instance, the decrease of M in one series, which comprise transformations of symbolic expressions and equations. The small increase of EEEI to some extent takes place in the algebra outside the discourses or the symbolic discourse (Example 12). These tasks comprise reasoning, communicating, and more conceptual thinking. Yet, it is a more ‘traditional curriculum’ which dominates, with lots of manipulating in M and equation solving in EEEI.

The increase of the combination EEEI and the (un)realistic discourse may be related to the new stress on the role of problem-solving in the 2011 syllabus\(^{27}\).

\(^{26}\) The different increases involve EEEI in the symbolic discourse in one series, EEEI in the geometrical discourse in another series, and EEEI in the (un)realistic discourse in two series.

\(^{27}\) In the 1994 syllabus, problem-solving was characterized as a competence but in the 2011 syllabus it was depicted both as a core content and a competence (Skolverket, 1994, 2011b).
The progression in the 2011 syllabus and the stress on FT does not appear to be enhanced in the textbooks. There may be more FT in lower secondary school textbooks, but in Year 8 algebra chapters, the proportion of FT is still small. Furthermore, even if FT increases in two textbook series, patterns as in Example 11 (p. 87) are mentioned in the syllabus for grade 1–6 and not for grade 7–9 (Skolverket, 2011b).

A potential for the competence perspective are provided by the activities GA and QPR since the 2011 syllabus mentions generalization and reasoning as desirable aims. However, the textbooks do not acknowledge this potential. The low proportion of GA is in line with results from Bråting et al. (2019) and Kongelf (2015).

The results are in line with the results of Boesen et al. (2014) who argues that a competence-oriented reform is harder to accomplish than the introduction of a new topic. In the 2011 curriculum reform, textbooks were conservative with respect to the algebra introduced in the new syllabus. Even the largest change – the increase in the (un)realistic discourse – may be called conservative since the tasks in the (un)realistic discourse have more in common with the long-established tradition of word problems than with problem-solving in real world situations. These tasks can also be seen as a transformation of the reform idea about problem-solving.28

Hence, power-coercive strategies in curricular reform may not function as planned when combined with a free textbook market and no textbook review or other support for textbook authors. Investigating how these conservative and transformative textbooks play out in the classroom is therefore an important step toward further understanding changes in the 2011 mathematics curriculum reform.

7.3 The test structure of TIMSS’ algebra varies between years and may influence the interpretability of Swedish students’ test scores over time. Paper III

The aim of this study is to understand the rationale of using student achievements on TIMSS in curriculum reform. Critical research on ILSAs has focussed on the validity of cross-country comparisons and comparisons of ILSAs’ frameworks against national contexts (Hopfenbeck et al., 2018; Lindblad et al., 2015) but comparisons over time seem to be absent. The part of the Swedish curriculum reform in 2011 which concerns school algebra provides an interesting case of ILSAs’ use in reform. Low results on TIMSS and PISA

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28 This argument is developed in the thesis’ chapter Discussion and conclusions.
were stated as one of the reasons for the reform, where especially better knowledge in algebra was requested (Skolverket, 2011a).

The material in this study encompasses the test items in algebra of four consecutive TIMSS tests from 2003, 2007, 2011 and 2015, as well as Swedish students’ achievements on these test items. The test items in algebra are analysed similarly to the textbook tasks in papers I and II, with algebra characteristics conceptualized as school algebra discourses and algebraic activities. To create independent groups for the analysis, test items which include hybrids of discourses or more than one algebraic activity were excluded from further analysis. The algebra characteristics of 197 out of 232 algebra items were used as a means to understand the structure of TIMSS’ algebra over time²⁹.

The student achievements are explored through t-tests with Bonferroni-corrected significance levels. The findings are then discussed in relation to the algebra characteristics in the structures of the TIMSS tests. The exploration can show whether or not there are significant differences in student achievements related to different algebra characteristics. The analysis is done separately for the group of students with low and high achievements, respectively, and for all students. If significant differences in results related to different algebra characteristics are identified, changes in the structure of TIMSS over time may affect the difficulty of the test.

The results show that TIMSS’ test items mainly draw on the same school algebra discourses and algebraic activities as the textbook tasks in papers I and II. However, a relational discourse is identified in the test items, which was not identified in the textbook tasks. This discourse is described in the first part of the results. In the second part of the results, changes in the test-structure are described. Finally, results of the exploratory inferential analysis are given.

The relational discourse features subject-specific words such as line, graph, point, coordinates and function, and semiotic resources such as tables, graphs, and alphanumerical symbols. These participants relate to each other in verbal processes, e.g., shows and relational processes, e.g., is. No names or pronouns are used. The student is invited to engage in answering questions about identifying relationships, such as in Example 13 below, and to some extent invited to engage in the material process to fill in.

²⁹ The effect of the missing values is raised in the thesis chapter Discussion and conclusions.

The table below shows a relation between x and y. Which of the following equations expresses this relation?

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

Vilken av följande ekvationer uttrycker detta samband?

A. \( y = x + 4 \)
B. \( y = x + 1 \)
C. \( y = 2x - 1 \)
D. \( y = 3x - 2 \)

The semiotic features in the relational discourse construct algebra as objects that are communicated and created, though by whom is obscured. The student is mainly invited to engage in actions such as identifying and understanding these objects through switching between semiotic resources. Less prominent actions in the relational discourse are to calculate and manipulate. This partly differs from what actions are afforded by the other school algebra discourses\(^{30}\), since they do not invite the student to understand and identify algebra as objects.

The algebra characteristics in the structure of TIMSS is given by Table 11 below. The proportions of algebraic activities do not change much over time. Although the proportion of items with M appears to show a decrease, there are in fact 11 M items in 2003, 14 in 2007, 15 in 2011 and 12 in 2015. However, the proportions of several school algebra discourses change over time. Changes of more than ten percentage points are circumscribed in Table 11. Most prominent are the increases in the relational and geometrical discourses and a decrease in the (un)realistic discourse. Notably, between one third and one fourth of the algebra items in the four tests combine M and the symbolic discourse.

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\(^{30}\) These actions are presented in Table 7 (p. 84), in the summary of Paper I.
The proportion of test items having the algebraic activity FT does not change, but there is a shift of focus within FT items from the (un)realistic to the relational discourse. This means that test items such as Example 13 become more frequent at the expense of test items such as Example 11 (p. 87). The proportion of test items with EEEI does not change either, but there are changes within EEEI between the discourses. This means that test items such as Example 5 (p. 81) in the geometrical discourse become more frequent, while test items such as Examples 6 and 7 (p. 82–83) in the arithmetical and (un)realistic discourses become less frequent. The structure of TIMSS does not stay the same. During the period 2003–2015 it has developed.

The inferential analysis found one significant difference in results for the group of students with low achievements, and several significant differences were found for the group of students with high achievements. No significant differences in results between different algebra characteristics were found for all students.
Students with low achievements on TIMSS performed significantly better on test items in the arithmetical discourse, than on test items in the (un)realistic discourse. Conversely, students with high achievements on TIMSS performed significantly better on test items in the (un)realistic discourse, than on test items in both the symbolic and the relational discourse. Students with high achievements also performed significantly better on test items in the arithmetical discourse compared with the relational discourse, and on EEEI and FT items compared with M items. Some of these differences are graphically visualized in Figure 6.31 For instance, the box plots show how the percent right values for the (un)realistic and arithmetical discourses are relatively high, compared with the percent right values in the relational discourse.

Figure 6. Percent right values of school algebra discourses for students with high achievements.

Because the proportions of M, FT and EEEI items are fairly the same over time, the differences in results between the algebraic activities do not affect the degree of difficulty of the assessment. Nor does the significant difference in results for students with low achievements affect the degree of difficulty of the assessment, since both the arithmetical and (un)realistic discourses decrease. However, the relational discourse increases over time while the (un)realistic and arithmetical discourses decrease. At the same time, students with high achievements have significant differences in percent right values between these discourses. This means that during the period 2003–2015 and for

31 The box plots of the remaining percent right values of the algebraic activities and school algebra discourses, which were tested, are given in Appendix B.
students with high achievements, the proportion of relatively difficult algebra items in TIMSS grows.

The results of the study thus implicate that TIMSS’ validity, with respect to the interpretation of TIMSS’ algebra results over time, is in some measures low. IEA claims it can provide countries with trends over time due to a strict design in TIMSS with trend items that recur in up to three tests (Mullis & Martin, 2013). Changes in proportions of algebra items with different discursive features, related to significant differences in algebra results for students with high achievements, contradict this.

Interestingly, students with low achievements have not only fewer significant differences in results than students with high achievements, but they also display another significant difference – test items in the (un)realistic discourse have significantly lower percent right values than do test items in the arithmetical discourse. These discourses employ the same algebraic activities, FT and EEEI. Bergvall (2016) finds that while students with high achievements perform low on TIMSS’ algebra items with complex symbolic expressions, students with low achievements perform low on TIMSS’ algebra items with a low proportion of symbols compared with the number of words. Moreover, Krstić et al. (2018) sees how students with low achievements in PISA lack reading strategies. Drawing on these findings, it is suggested in the paper that students with low achievements may be affected more by textual differences than by different algebraic activities. Therefore, linguistic matters should be paid more consideration in mathematics education. It is suggested that the methods for the discourse analysis used in papers I and III can be applied to other national languages, to make comparisons between developments in national curricula and TIMSS’ test items in algebra.

When results in ILSAs are used in educational or curricular reform, it is seldom ‘mathematics’ which changes. Instead, more geometry and problem-solving (Štraus, 2005) or more algebra and mathematical literacy (Nortvedt, 2018) are mentioned as areas where improvement is needed. In the 2011 school algebra reform in Sweden, better knowledge in algebra was requested. This study shows how the interpretation of TIMSS results in algebra over time in terms of a knowledge trend, may not be a relevant rationale for initiating and evaluating curricular reform.
8 Discussion and conclusions

In this thesis, the aim has been to understand in what ways mathematics textbooks change, and in what ways international large-scale assessments may be used, in a reform process of lower secondary school algebra. As already stated, textbooks in mathematics seem overlooked as a part of contemporary curriculum reform processes. Yet, they have played an important part in curriculum development during a large part of the 20th century (Prytz, 2017). Based on data from TIMSS’ test-curriculum matching analysis, Grønmo et al. (2014) claim there are reasons to believe that textbooks have a powerful influence on students’ achievements in algebra. Moreover, the interpretability of TIMSS results over time appears to be unquestioned. At the same time studies of another large-scale assessment suggest that the complexity of the mathematics through the years decreases, since there are changes where mathematics is constructed as processes and with human participation instead of as objects (Morgan & Sfard, 2016b).

What characterizes algebra tasks in Swedish textbooks and the ways in which these characteristics change during the period 1995–2015 are results of papers I and II. These results form a starting point for a discussion of the textbooks in relation to the syllabus, in the 2011 school algebra reform.

The characteristics of TIMSS’ algebra items, the ways in which these characteristics change over time, and their relation to results on TIMSS’ algebra items, are results of paper III. More precisely, are considered the interpretation of TIMSS results over time and the use of these results in curriculum reform, the relevance of using TIMSS results for understanding the knowledge in algebra of Swedish students, and value implications of using international large-scale assessments in curriculum reform.

This chapter starts with summarizing the main similarities and differences between algebra characteristics in the textbook tasks and TIMSS’ algebra items. This comparison answers the last research question – in what ways the algebra characteristics in the textbook tasks and the test items differ. The comparison includes a diachronic aspect since changes in the algebra characteristics over time are considered.

The second part of this chapter constitutes a discussion on the changes in textbooks of mathematics in the 2011 school algebra reform, while the third part constitutes a discussion on the ways in which TIMSS results may be used in relation to the 2011 school algebra reform.
In the fourth part of the chapter, conclusions are drawn concerning the teaching and learning of algebra. In the last parts of the chapter, missing values, the contribution of the thesis, and further research are discussed.

8.1 Comparing algebra characteristics in textbooks and TIMSS’ test items

Drawing inferences from results on TIMSS tests for curriculum reform builds on the prerequisite that these results are a relevant measure of the curriculum in question. There must be a fair representation in TIMSS of the algebra in the domain of reference. Reports from IEA and the Swedish National Agency of Education tell us that TIMSS does give a fair representation of the Swedish curriculum, as a domain of reference, and that the differences between the TIMSS’ test items and the Swedish curriculum do not impact the students’ average results (Lindström, 2006; Mullis et al., 2012, 2008, 2016, 2004; Sollerman & Pettersson, 2016). In these reports, the curriculum is understood as the intended curriculum, including syllabi and National tests.

Teachers however, report a lower percentage of algebra in the Swedish curriculum, than what the Swedish national research coordinators report in TIMSS’ test-curriculum matching analysis (Grønmo et al., 2014). This discrepancy may indicate a difference in the understanding of ‘curriculum’, which stresses the relevance of comparing TIMSS’ test items with textbooks as the potentially implemented curriculum, rather than the intended curriculum.

Therefore, a comparison over time is made of the potentially implemented curriculum in terms of textbook tasks and test items, and in what ways their algebra characteristics change over time. It is a prerequisite for drawing valid inferences from TIMSS results between 2003 and 2015. The comparison involves algebra characteristics in textbooks’ algebra tasks for Year 8 (Table 10, p. 88) and in TIMSS’ algebra items (see Appendix C).

The textbook selection consists of those series which were on the market both before and after 2011, which makes the results representative for algebra chapters in textbooks Year 8 during the period 1995–2015. Similarities and differences are summarized in Figure 7 below, where the arrow indicates the main results of the comparison.
Generally speaking, there are more changes in the school algebra discourses in both materials, than in the algebraic activities. There are also more changes over time in the proportions of the school algebra discourses in TIMSS than in the textbooks. The algebraic activities are more evenly distributed in the test items than in the textbooks’ algebra chapters.

The most common type of task in both textbooks and TIMSS’ algebra items is a combination of *Manipulation* and the symbolic discourse (Example 3, 4, p. 65, 81). This combination is even more common in the textbooks than in TIMSS, regardless of what series or year is studied. In TIMSS, this combination ranges from 24 % to 27 %; in the textbooks, it ranges from 32 % to 42 %. Algebra characteristics which are rare are also the same in the two materials, namely *Generalized Arithmetic*, *Qualitative and Proportional Reasoning*, and the scientific discourse. Differences between the materials’ second most common combinations of algebra characteristics are captured by Table 12 below. As seen in the table, somewhat different characteristics are enhanced both over time and across the different materials.

The proportions of *Functional Thinking* (Example 11, 12, p. 87, 89) differ between the materials. It represents the largest discrepancy in proportions of the algebraic activities between the materials. This activity is represented only moderately in the textbooks, while in the TIMSS tests *Functional Thinking; Equivalence, Expressions, Equations and Inequalities* (Examples 5–7, p. 81–83); and *Manipulation* (Example 3, 4, p. 65, 81) constitute roughly equal proportions. The proportion of *Functional Thinking* in all of the TIMSS tests constitutes roughly a third of the algebra items, while the proportion in the textbooks increases from 1 to 7 % of the algebra tasks during the time-period.
The relational discourse (Example 12, p. 89) is not prevalent in the textbook tasks, only in the TIMSS’ test items. This difference grows larger over time, and in TIMSS 2015, the relational discourse constitutes 19% of the algebra items. The changes in the (un)realistic discourse (Example 7, p. 83) also stand out since they go in opposite directions in the two materials: an increase from 12 to 20% in the textbooks and a decrease from 33 to 12% in the TIMSS tests.

Table 12. Comparison of the materials’ 2nd most common characteristics over time.

<table>
<thead>
<tr>
<th></th>
<th>Textbooks</th>
<th>TIMSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 2011</td>
<td>EEEI and the geometrical discourse scores on average 15% in the textbooks (Examples 1, 5, p. 64, 81). FT and the (un)realistic discourse scores on average 1% in the textbooks (Example 11, p. 87).</td>
<td>EEEI and the geometrical discourse scores on average 3.5% in TIMSS 2003 and 2007. FT and the (un)realistic discourse scores 24% in 2003 and 15% in 2007.</td>
</tr>
<tr>
<td>2011–2015</td>
<td>Different characteristics are enhanced in different textbooks: either EEEI and the symbolic discourse (20% in one book), EEEI and the geometrical discourse (26% in one book), or EEEI and the (un)realistic discourse (19% in one book) (Examples 12, 1, and 7, p. 89, 64 and 83, respectively).</td>
<td>Proportions in TIMSS 2011 and 2015 are on average similar, 10–13%, for combinations of EEEI and the symbolic discourse, EEEI and the geometrical discourse, FT and the (un)realistic discourse, and FT and the relational discourse (Example 12, p. 89).</td>
</tr>
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</table>

Moreover, the interpersonal function is slightly different in the textbooks and the test items, regardless of what discourse is considered. The less prominent actions which school algebra discourses invite to in textbooks (Table 7, p. 84) are not prevalent in TIMSS’ test items, save for in some test items which invite the student to decide ‘true or false’. The less prominent actions which are not identified in TIMSS are mostly verbal processes, e.g., explain and discuss. Considering the different purposes of textbooks and assessments, this difference was expected. The textbook may aim at developing students’ abilities for reasoning and arguing, but assessing a student’s reasoning ability may be too complex a task to fit into a large-scale assessment, which uses many multiple-choice answers.

Taken altogether, the comparison shows that there are differences and similarities between the textbooks’ algebra tasks and the TIMSS’ algebra items. How the differences may affect TIMSS’ validity in relation to the 2011 school algebra reform is discussed further below. Preceding that, changes in the textbooks in relation to this reform are discussed.
8.2 Transformative and conservative textbooks in the 2011 school algebra reform

The 2011 curriculum reform did not radically change the view of mathematics in the syllabus, as for example the New Math reform did in the 1960s and 70s: it aimed instead at enhancing problem-solving in everyday situations and different contexts. It also aimed at concretizing the competence perspective, enhancing the understanding of concepts, and promoting better knowledge in algebra (Skolverket, 2011a). Speaking with Howson et al. (1981), this can be understood as a formulation of a plan of action for the reform, at the level of the intended curriculum. The Valverde et al. (2002) model describes the potentially implemented curriculum as mediating. The word ‘mediating’ however implies that textbooks quite simply reflect ideas expressed in the syllabus. In fact, textbooks may as well modify or ignore reform ideas in a new syllabus, or introduce new ideas of their own. In the 2011 school algebra reform, the textbooks were partly transformative and conservative. These two ways of taking up reform ideas will be further discussed below.

8.2.1 The transformation of a reform idea

During the time 1995–2015, i.e., the period before and after the 2011 syllabus is introduced, the most prominent increase in the textbooks belongs to the (un)realistic discourse. It may be understood as an adaptation to the 2011 syllabus’ enhanced focus on problem-solving in everyday situations.

As described in the Background, one of the ideas in the 2011 syllabus may be phrased as mathematical literacy, or learning problem-solving in the real world with algebra. This is not the same thing as what the (un)realistic discourse offers, though. The (un)realistic discourse constructs algebra as a ‘gaze’ upon the world and positions the student as an artificial consumer. In Example 7 (p. 83), this is seen through the relation between tickets to a circus and a SEK per item. The cost for a ticket is never an unknown in an everyday situation, and as a buyer of tickets, one never need to form an algebraic expression for the change on a banknote. The underlying idea in these tasks may instead be phrased as looking at the ‘real world’ through algebra.

According to Jablonka (2015, p. 601f), many classroom activities and assessment tasks aiming at mathematical literacy “are in fact school mathematical recontextualizations of other practices and discourses, where recontextualization is understood as the process of subordinating one practice under the evaluative principles of another”. Put in other words: other practices and discourses do not build on the logic and knowledge-making of mathematics practices and discourses, but have their own logics and ways of knowledge-making. Hence, the reform idea understood as an emphasis on mathematical literacy, is recontextualized, or transformed into ‘school algebra’, in the sense that the textbook tasks
have a focus on algebraic representations connected to situations, whereas in the real world, algebra would not be used in the same situations (Figure 8).

The everyday connection in the (un)realistic discourse is then quite superficial. In this sense, textbook authors can be said to drive changes which were not initiated by educational authorities in the 2011 school algebra reform.

Figure 8. The (partly) transformative textbooks in the 2011 school algebra reform.

8.2.2 Resisting competences while changing content

There are signs of adaptation to the new syllabus in the textbooks’ algebra tasks, but the overall impression is that the changes are quite modest. The competence perspective is not really enhanced: in the period 1995–2015, the largest proportion of algebra tasks is Manipulation in the symbolic discourse. There are no changes in algebraic activities such as Generalized Arithmetic or Qualitative and Proportional Reasoning. These latter algebraic activities could easily draw on competences such as reasoning, justifying, and generalizing. In this respect, the textbooks are conservative in the school algebra reform.

In the 2011 syllabus, problem-solving is seen both as a core content and as a competence. Both are explicitly tied to situations relevant for the student, or everyday situations. Boesen et al. (2014) argue that teachers more easily respond to changes in content than to more complex reforms such as the competence perspective. The increase of tasks in the (un)realistic discourse provides a parallel to this claim. Other competences than problem-solving are not as visible since they may not be viewed as content, while the (un)realistic tasks may be understood as a sort of problem-solving connecting to everyday situations, which is a core content. In this sense, the textbooks resist including competences while they change content.

The increase in the (un)realistic discourse may also be interpreted as conservative, in the sense that ‘everyday situations’ is a theme which is not new in Sweden. Jakobsson-Åhl (2006) and Johansson (2003) find that problem-solving in everyday situations increased in algebra as well as generally, after
1980 and onwards. According to Prytz (2017), an everyday theme was already present as early as in the 1919 syllabus. The increase of the (un)realistic discourse in textbooks may in a sense illustrate the wording *Plus ça change, plus c’est la même chose*: the more things change, the more they stay the same.

The results discussed above indicate that the curriculum reform in 2011 could have benefited from clearer reform ideas. Still, achieving curricular change is not just about the clarity of the ideas. It is also about recognizing the reform as a process, where actors at all stages of the process must be involved. Recognizing this need is argued for below.

### 8.2.3 Implications for curricular reform

In Howson et al.’s (1981) model of curriculum development, the development of new materials either builds on research or involves a formative work with piloting new materials. These materials may then be used as guidelines for how to concretize the reform messages in a new syllabus.

Development of new curriculum material, initiated by national educational authorities, thus functions as an educational scaffold in a curriculum reform. The scaffold is directed at both textbook authors and teachers. A textbook review instead functions as educational control of new materials. In the 2011 reform, national educational authorities used neither such scaffolding development as mentioned above, nor control of curriculum materials in mathematics. State review of textbooks was abolished in 1991 (Johnsson Harrie, 2009) and the development of new materials for mathematics in the 2011 reform was managed by textbook authors and publishing companies. Thus, the development of new materials relied on mainly economic incitements.

In contrast, during the New Math reform, a long period for developing and trying out new materials in form of textbooks preceded the introduction of the 1969 syllabus (Prytz, 2018). Another feature of the New Math reform was that the textbook review made sure that the textbooks aligned with the reform ideas. Some may object to this description and say that the textbooks soon abandoned New Math. However, this abandonment may be explained by the central educational authorities’ lack of faith in the New Math reform, that the textbook review became voluntary in 1974, and a change towards less centralized governance (Prytz, 2018). As long as national educational authorities did scaffold and review the textbook production, the textbooks included changes in line with the 1969 syllabus’ new focus.

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32 Some may argue that the in-service education Boost for mathematics (2013-2016) for teachers functioned as a scaffold and development of new materials in the 2011 curriculum reform. However, the Boost was conducted two after the introduction of the 2011 syllabus, and did not explicitly aim at implementing the new syllabus.
In the textbooks, the changes are modest, conservative and transformative. As shown by the changes in standard deviations (Table 10, p. 88), the textbook series emphasize different algebra characteristics after 2011. The series thus display somewhat different interpretations of the reform ideas in the new syllabus. According to Howson et al. (1981), the stage in the curriculum development where new curriculum materials are established is crucial. This thesis exemplifies how leaving this stage to the publishing market results in very little change in the textbooks: at least there are no general changes throughout all textbook series. This lack of control may seem strange if the goal is to reform the curriculum and thereby achieve change.

Means of governance over, among other things, curriculum goals and selection of content, are discussed by curriculum theory, both as power-coercive rules and in terms of management processes where different groups are influential (Forsberg et al., 2017). However, textbook authors and publishing companies are not mentioned as such a group. This thesis shows that they do affect the selection of content, both by resisting change and by transforming a reform idea. According to Johansson (2003), power-coercive strategies have been used to develop curricula in Sweden. I would like to claim that when curricular development of new materials is left to the textbook market, the strategy does not appear to be coercive.

8.3 A lessened validity of TIMSS results in algebra

In this thesis, aspects of test validity are investigated in order to understand how ILSAs may be used in the 2011 school algebra reform. These aspects concern the interpretation of TIMSS results over time as a way to understand knowledge trends; the relevance of using TIMSS results over time for evaluating curricular reform and identifying needs for curricular change; the relevance of using TIMSS results to understand Swedish students’ achievements; and value implications using of TIMSS results in curriculum reform.

In the first section below, the first and second aspects are considered. In the second section, the discussion develops on the relevance of using TIMSS results to understand Swedish students’ achievements and in the third section, potential value implications of using results from ILSAs curriculum reform are highlighted.

8.3.1 Reduced interpretability and relevance over time

IEA claims that TIMSS gives evidence on the progression of student achievements over time, or about knowledge trends in countries’ educational systems (Pierre, 2017). These claims about students’ results are a presumption used by policy makers for evaluating curricular reform and identifying needs for curricular change.
The results in this thesis show that the interpretation of TIMSS’ algebra results over time as a knowledge trend is questionable (Figure 9). Empirical evidence does not support such an interpretation. The reader is asked to recall that the group of students with high achievements performed significantly lower on test items in the relational discourse, compared with test items in the arithmetical and (un)realistic discourses; and on test items in the symbolic discourse compared with test items in the (un)realistic discourse. Additionally, recall the increase of test items in the relational discourse and the decrease of test items in the (un)realistic and arithmetical discourses, which indicates how the structure of TIMSS changes over time. When viewed in relation to the significant differences in results on test items with different algebra characteristics, these changes in the structure of TIMSS show that the degree of difficulty of the tests increase over time for the group of students with high achievements.

As a contrast, the group of students with low achievements perform significantly lower on test items in the (un)realistic discourse, compared with test items in the arithmetical discourse. Since both of these types of test items decrease in TIMSS, it is not possible to say that the TIMSS tests’ algebra in 2003–2015 has become less, or more, difficult for students with low achievements. Similarly, significant differences in results for students with high achievements between items with Manipulation (low results) and items with Functional Thinking (high results), as well as between items with Manipulation (low results) and items with Equivalence, Expressions, Equations and Inequalities (high results), do not affect the degree of difficulty of the TIMSS tests. The proportions of items with these algebraic activities do not really change during the investigated time period.

Figure 9. The validity of TIMSS’ algebra results over time. Interpretation of results and the relevance of using them in a reform process.
It is notable that the changes in proportions of discourse in TIMSS increase the demands on the group of students with high achievements. The results of a research project on mathematical discourse in the British large-scale assessment GCSE, on the contrary point to a decrease in linguistic demands of the students (Morgan, 2016b; Morgan & Tang, 2016). The question of linguistic demands in relation to students’ achievements is not empirically investigated by the project, though. The claim that students perform differently on items with different linguistic features (cf. Morgan, 2016b) is supported by the results of the present thesis.

The results of this thesis show that linguistic differences such as those found in the different discourses may affect TIMSS’ ability to measure student achievements over time. Concerning Swedish students with high achievements, their algebra results on average might even increase. This is hard to establish, though, because there are not enough test items in the relational discourse from which to draw valid inferences over time as pertains to this possibility. The results also make the relevance of using TIMSS in curricular reform contestable. According to Messick (1989), test validity always concerns the responses to the tests. The question of validity should be addressed in specific cases and not in general. In the case of the 2011 school algebra reform, it would be more appropriate to use other instruments than TIMSS for evaluation.

8.3.2 Different results – in mathematics or in algebra – produce different relevance

So far, mathematics textbooks and TIMSS results in the 2011 school algebra reform have been discussed individually by focussing on their respective curricular levels in relation to the intended curriculum. Below, a discussion of the potentially implemented curriculum is developed, which builds on the comparison between textbook tasks and test items, described in the first part of the Discussion (Figure 7, p. 98). Discrepancies between the textbook tasks and test items may reduce the relevance of using TIMSS results to understand Swedish students’ achievements in algebra.

The comparison in the beginning of this chapter (Figure 7, p. 98; Table 12, p. 99) reveals that there are partly different conceptions of algebra in the TIMSS tests and in the textbooks. The relational discourse is not present in the textbooks’ algebra chapters, only in the ‘TIMSS’ algebra. The textbooks have a low proportion of tasks with Functional Thinking compared with the TIMSS tests, and while tasks in the (un)realistic discourse increases in the textbooks, they decrease in TIMSS. Using students’ results on TIMSS’ algebra for drawing conclusions about Swedish students’ algebra knowledge is then less relevant. Conclusions on Swedish students’ knowledge in algebra should not be based on TIMSS results on items with Functional Thinking or
in the relational discourse. However, conclusions might be drawn concerning those algebra characteristics where the textbooks and TIMSS tests have more similar proportions.

Sollerman’s (2019) findings appear to be in contrast to the findings of this thesis. As described in Previous research, he investigates the relevance of using ILSAs in Sweden through looking at their frameworks and test items, their implementation, and their results. These are compared to their equivalences in Swedish National tests, and Sollerman (2019) finds that ILSAs give a fair picture of the Swedish school mathematical context. This is similar to what the TIMSS’ test-curriculum matching analysis (Mullis et al., 2016) reveals. No big differences in results on average are found when results on all TIMSS’ test items are compared with results on the TIMSS’ test items which are, reportedly, similar to the Swedish curriculum. However, Sollerman’s (2019) claims are about the aggregate of TIMSS results in mathematics. He also limits his study to test items which are deemed as relevant by TIMSS’ test-curriculum matching analysis. As described in the Background, many algebra items in TIMSS 2015 are therefore considered as missing values in Sollerman’s (2019) study.

It is therefore suggested in the present thesis that the relevance of using algebra results does not necessarily follow from the relevance of using mathematics results. That TIMSS results in mathematics are relevant for measuring the Swedish context concerning mathematics does not warrant that conclusions about Swedish students’ knowledge in algebra can be drawn.

8.3.3 More influence from OECD than IEA?

When it comes to influences from ILSAs’ results, it is difficult to draw conclusions about causality and to verify that a certain condition is an effect of another. Still, Johansson and Hansen (2018) find that the mathematics curricula of countries participating in TIMSS develop over time towards becoming more similar to the TIMSS framework. As discussed in the Background chapter, Sollerman (2019) acknowledges the influence of ILSAs on the 2011 Swedish syllabus and argues that there is a risk that this influence – or adaptation to ILSAs – may occur in other parts of the educational system. It is likely then, that the increased importance given to ILSAs in educational reform, has affected the selection of content in the textbooks indirectly, through changes in the syllabus.

As shown in the comparison between the textbooks and the TIMSS tasks, the changes in the (un)realistic discourse constitute a large discrepancy. The changes go in *opposite* directions in the two materials – an increase in textbooks and a decrease in TIMSS. This may look contradictory to the statement that ILSAs have a large influence. However, in the Background it was discussed how the 2011 syllabus’ enhancement of problem-solving can be understood as an answer to the OECD focus on mathematical literacy. Problem-
solving as a competence in an everyday context, as well as in different subject areas (Skolverket, 2011b), fits well with OECD’s (2013) view of mathematical literacy as solving problems in the real world.

Perhaps then, PISA is more influential than TIMSS, in the Swedish context. That might explain the increase of textbook tasks in the (un)realistic discourse. If so, it is a value implication of the use of ILSAs’ results, and as such, an aspect of test validity according to Messick (1989). What is valued by OECD appears to be valued in Swedish textbooks as well. A possible consequence of the increase of the (un)realistic discourse in the textbooks is discussed further below.

8.3.4 Implications for curricular reform

In Howson et al.’s (1981) model of curriculum development, summative and comparative evaluation can be formal, and it can lead to identification of needs for further action. Such evaluation, in terms of ILSAs’ results, was partly a base for the Swedish curriculum reform in 2011. One of the needs identified was better knowledge in algebra (Skolverket, 2011a).

It is likely that national educational authorities will continue to use ILSAs’ results for evaluating the knowledge of students and for making plans for further action. It fits well with the current system of governing school by objectives and results (Sundberg & Wahlström, 2012) and Sweden has participated in both TIMSS 2015 and 2019 since the curriculum reform in 2011. It is quite unlikely though, that only the aggregate of TIMSS results in mathematics will be used in these manners, given that results in different cognitive domains and subject-specific areas, e.g. algebra, are available. Results in mathematics as a whole may also be a too crude measure to use, specifically if the aim is to identify needs for further action and to reform a curriculum.

The results of this thesis show that greater caution should be exercised when drawing conclusions from algebra-specific results and their changes over time. This thesis suggests that the validity of all subject-specific results in their specific cases be considered, instead of be generalized from the validity of using the aggregate of TIMSS results in mathematics. Investigating the validity for each of these cases may turn out to be a very time-consuming project, though. Perhaps the objectives and results in ILSAs are not the optimal choice for school governance, when it comes to the evaluation and identification of needs for further action in national curriculum reform.
8.4 Drawing conclusions from ILSAs’ results: implications for the teaching and learning of algebra

In this section, the comparison of the textbooks’ algebra tasks and the TIMSS’ algebra items is related to students’ results on the test, in order to understand their implications for the teaching and learning of algebra. Two implications are discussed below (Figure 10).

Figure 10. Implications of TIMSS results for the teaching and learning of algebra.

8.4.1 Excluding students with low achievements from learning algebra?

Tasks featuring problem-solving in the ‘real world’ may position students in ways which give unequal access to mathematics (Le Roux, 2008), and according to Dowling (1996, p. 410), “you cannot teach shopping within the context of school mathematics”. Ultimately, these kinds of tasks will then give students knowledge of neither mathematics nor of real life. In Swedish classrooms, students work a lot with textbook tasks, often silently on their own. The textbook affects what tasks the students work with and what tasks teachers choose for joint discussion in the classroom (M. Johansson, 2006).

The present thesis shows that the number of tasks in the (un)realistic discourse increase in the textbooks. As discussed above, the (un)realistic discourse can be understood as a transformation or recontextualization of problem-solving practices in the real world, into school algebra. Taking the TIMSS results into account, this is also a discourse where students with low achievements have relatively low performances, while students with high achievements have relatively high performances. These differences and the previous research mentioned above, implicate that using a high proportion of (un)realistic discourse in
the classroom may not scaffold students with low achievements in learning algebra. Instead, these students may be excluded since they will learn neither mathematics nor real life. This also makes questionable to what extent students who encounter tasks in the (un)realistic discourse will actually feel prepared for using mathematics outside of school.

As for teaching, there appears to be two options. Either do not use tasks in the (un)realistic discourse or ensure that all of the students in the classroom make the transition from the ‘real world’ to algebra. For instance, consider once again Example 7 (F13, p. 200):

The tickets to a circus costs \( a \) SEK per item. Write an expression of how much

a) 5 tickets cost
b) Emma gets in return from a 500-SEK banknote when she buys 2 tickets
c) Sara gets in return from 400 SEK when she buys 3 tickets

This problem could easily be rephrased by a teacher or a textbook author as

Suppose that tickets to a circus costs \( a \) SEK per item. Write an expression …

This may seem to be a small and insignificant change. Still, both the ideational and interpersonal function is different. The process *suppose* turns the statement *The tickets to a circus costs \( a \) SEK per item* into a request, directed at the student. Hence, the ideational meaning is not claimed to be about a real life situation, but about something imagined. In accordance with the SFL framework, the process *suppose* could be interpreted as verbal-mental. The student is then not just invited to write down an expression literally, but also to imagine that these types of expressions are possible to create: both by humans and – most importantly – by the student. The student might still not learn about real life situations or mathematical literacy, but at least an opportunity to learn algebra would be available.

8.4.2 Meaning-building not inherent in tasks with Manipulation

Both textbooks and TIMSS tests have a large number of tasks with *Manipulation* in the symbolic discourse. This is interesting both in relation to the formulations in the syllabus (Skolverket, 2011b) and to the early algebra research trend. Unlike the 1994 syllabus, the 2011 syllabus does not have explicit formulations about the transformation of algebraic expressions, so procedures and manipulations are downplayed. Blanton et al. (2018, 2015) do not consider manipulation as a big idea in algebra and Kaput et al. (2008) claim that manipulating algebraic symbols without generalizations, is not an algebraic activity. However, textbooks and TIMSS tests seem instead to draw on a view of algebra more in accordance with Kieran (2007, p. 714), who claims that in transformational activity, “meaning building [is included] for the use of properties and axioms in
the manipulative processes themselves”. Different conceptions of algebra may consequently be advocated at different curricular levels. So, how do we know which algebraic activities should be promoted in the classroom?

In Pedersen’s (2015) study, Norwegian students’ achievements in TIMSS Advanced are relatively low on items which require the manipulation of symbolic expressions and functions, and relatively high on items which require the representation of expressions and solving items in an applied context. Her conclusion is that more time in the classrooms needs to be allocated to manipulations of symbolic expressions.

One could have expected that a greater opportunity to engage in a specific kind of tasks would pay off in terms of better student achievements. However, as stated in the first part of the Discussion and conclusions, the combination of the symbolic discourse and Manipulation is even more common in Swedish textbooks than in TIMSS. Furthermore, Swedish students already allocate a lot of time – approximately 35% – to procedural activities (Boesen et al., 2014). Despite all this, Swedish students with high achievements still have significantly lower results on tasks with Manipulation than on tasks with both Functional Thinking and Equivalence, Expressions, Equations, and Inequalities, and on tasks in the symbolic discourse than in the (un)realistic discourse. This suggests that more tasks which stress manipulating symbolic expressions in the textbooks do not necessarily lead to better student achievements on similar items in the tests.

A tentative explanation for this could be that whether tasks with Manipulation lead to actual meaning building or not depends upon how tasks with Manipulation be approached in the classroom. Kieran’s claim above, that meaning-building is included in the manipulative processes themselves, should then rather be understood as opportunities for meaning-building included in tasks: it is not inherent in the task themselves. We know that teachers use textbooks in different ways, and that the ways in which they use textbooks in the classroom are related to both their mathematical knowledge, and to what scaffolds are afforded by the textbook (Hill & Charalambous, 2012; Neumann et al., 2015). This will of course influence how M tasks are approached in the classroom and to what extent properties and axioms are emphasized. Moreover, the tasks in the symbolic discourse which are not Manipulation, but draw on other algebraic activities, may provide more explicit opportunities for meaning-building concerning the use of properties and axioms. In Example 12 (p. 89), a task with Equivalence, Expressions, Equations, and Inequalities in the symbolic discourse, the question How many solutions do you think there are? to the equation $5x + 2y = 29$ does that by opening up for a property of equations with two unknowns; namely that there are more than one solution to this type of equation. Similar explicitness could be either a feature of tasks, or of joint classroom communication with the teacher. According to Naalsund (2012, p. 160), Norwegian students in lower secondary school, among other
things, show a weak understanding of the concept of equivalence and of “underlying operations in a given equation”. It is suggested here that emphasizing properties and axioms by making them more explicit, as in Example 12, could be a way to further scaffold students’ meaning-building.

8.5 A brief discussion on missing values

As mentioned in Methodological contributions, 35 algebra items from TIMSS out of 232 are not included in the inferential analysis. Their characteristics, percent right values, and effect on the results are described in this section.

There were 17 TIMSS test items which were hybrids of discourses and 18 which included more than one algebraic activity\textsuperscript{33}. As opposed to the test items selected for the inferential analysis, more than half of the hybrids entailed features from the scientific discourse or the (un)realistic discourse. Test items with more than one algebraic activity relatively often entailed QPR, GA and FT compared to the selected test items, though EEEI dominated. Compared with the textbook tasks, the scientific discourse, QPR, GA and FT were more stressed. Concerning the relevance of using TIMSS results to draw conclusions about students’ knowledge in algebra, the missing values then enhance the differences between the materials, so the conclusion holds.

Overall, the percent right values were relatively high for both the hybrids and the test items with more than one algebraic activity (Figures 16–18, Appendix D). Actually, the hybrids displayed higher percent right values than did any of the individual school algebra discourses. This was the case for the groups of students with low and high achievements, as well as for all students. The number of hybrids as well as the number of tasks with more than one algebraic activity decrease over time in the TIMSS tests, so the differences in percent right values strengthen the conclusions of the thesis. The possibility to interpret TIMSS’ algebra results over time as a knowledge trend is questionable, and it is therefore less relevant to use TIMSS results in algebra over time for evaluating or initiating curriculum reform.

8.6 Contributions

The contributions of the thesis are manifold. Because of the discourse analysis supported by the SFL framework, plausible consequences of addressing students through a certain discourse – in this case the (un)realistic – could be highlighted when compared with the student achievements. The framework allowed the capturing of different contexts for algebra, which led to the conclusion that the mathematical literacy put forward in the syllabus is transformed into the

\textsuperscript{33} In addition, one of the hybrids also included more than one algebraic activity.
(un)realistic discourse in the textbooks. Mathematical discourse does not account for participants that are not mathematically subject-specific (Morgan & Tang, 2016). Therefore, it cannot capture in what contexts algebra is supposed to take place, or be used. In this respect, the school algebra discourses are better tools for identifying stories about algebra. The results of the thesis would not have become visible if mathematical discourse (cf. Sfard, 2008) had been studied instead. This is a methodological contribution.

In this thesis, textbook tasks are compared to test items. Therefore, the relevance of using TIMSS results is studied on another curricular level than for instance in previous Swedish studies (Sollerman, 2019; Sollerman & Pettersson, 2016) and IEA’s own reports. The thesis may illustrate the discrepancy identified by Grønmo et al. (2014) between what is reported as belonging to the Swedish curriculum by national research coordinators and by teachers.

Linguistic or discourse studies, e.g., the EDSM-project (Morgan & Sfard, 2016b), often do not consider students’ results. Just as Bergvall’s (2016) study, this study gives empirical evidence to the importance of considering language and discourse in mathematics tasks. Without looking into the results, it would not have been viable to discuss the validity of TIMSS. Since few studies on textbooks analyse discourse (Ryve, 2011), the thesis also contributes to filling this gap in textbook research. By studying lower secondary school textbooks, the thesis further gives results on new material since, according to Fan et al. (2013), most textbook studies are accomplished at the primary level.

As stated in Previous research, critical research on ILSAs has focussed on the traditional concept validity and reliability, and the validity of drawing inferences in cross-country comparisons (Hopfenbeck et al., 2018), but not comparisons over time. The present thesis contributes to filling this gap, through combining two analytical frameworks with student achievements and data over time, in order to explore changes in TIMSS’ difficulty.

Compared to Sollerman (2019), a contribution of this thesis is to examine the interpretation and use of TIMSS results concerning algebra instead of the aggregate of TIMSS results in mathematics. The thesis also confirms one of Sollerman’s results. The less prominent actions identified in the textbooks, which are not prevalent in the test items (Table 7, p. 84), can be said to concern the ability to communicate. This is something Sollerman (2019) finds is not covered by ILSAs, though it is included in the Swedish mathematics syllabus.

The results have practical relevance for actors operating at the level of the intended curriculum as well as of the implemented curriculum. The thesis shows how power-coercive strategies do not lead to the intended changes, in the free textbook market. New means of governance may be considered by actors at the level of the intended curriculum in addition to, or perhaps instead of, objectives and results. A better dialogue and scaffolding concerning the development of new materials is essential. Educational authorities could for instance go back to developing and trying out new textbooks before the launching of a new syllabus, as they did in the 1960s (cf. Prytz, 2018), or they
could include guidelines for textbooks as an appendix to the syllabus, as in China (cf. Jablonka, 2019). Regardless what way is chosen, an acknowledgement is needed of the importance of textbook authors and publishers in the curriculum reform process.

Self-knowledge and an awareness of the implications of the results in the thesis may help teachers to choose and use textbook tasks in a meaningful way in their classroom practice. As described in previous research, Hill and Charalambous (2012) hypothesize that teachers with high mathematical content knowledge might compensate for unsupportive teaching materials. Supportive teaching material which is followed closely, on the other hand, can lead to instruction of high quality even for teachers with low mathematical content knowledge (Hill & Charalambous, 2012). If so, then a teacher with high mathematical content knowledge might also be able to compensate less supportive tasks by identifying and making explicit properties and axioms that tasks with Manipulation may afford. A teacher with low mathematical content knowledge on the other hand might choose to use tasks which more explicitly point out properties or axioms, for improved support.

8.7 Further research
Every research project includes limitations and demarcations and alas, not all questions that pop up during a project be answered within it. In this section, some of these questions are described as suggestions for further research.

What do we know about textbook authors and their possibilities to affect the syllabus? The curriculum model in this thesis accounts for actors at different levels, but not for the actors’ possible movements between curricular levels. Additionally, the focus is not on actors but on texts. A more sociological analysis could clarify how different actors move between levels. For example, we know that some textbook authors have previously worked as teachers. If textbook authors also move to positions within, e.g., the National Agency of Education, they may affect the selection of content at the level of the intended curriculum.

What happens to the teachers’ activities and the students’ view of algebra, when functions are set apart from algebra as being a different subject area? The reader may recall that the 2011 syllabus discriminates between Algebra and Relationships and change as different subject areas, and that only one textbook series has a chapter on Relationships and changes in Year 8 after the 2011 reform. Moreover, tasks with Functional Thinking in the textbooks’ algebra chapters concern patterns and number sequences. However, both in the 1994 and 2011 syllabus, patterns are described as content for primary school and not for lower secondary. According to Pepin et al. (2013), both teachers’ activities in the classroom and students’ view of the subject are affected by textbooks. While their study pertains to geometry, their findings point to the importance of further
studies on textbooks related to also how these may affect the understanding of algebra. A cross-country comparative study between Sweden and countries where functions are not separated from algebra in the syllabus could shed light on in what ways teachers’ classroom activities or students’ meaning making differ, especially concerning the concept of variable.

**What could constitute a genre-pedagogical perspective on algebra teaching and learning and what would it afford?** One of the points of departure in the work with this thesis was to look at algebra tasks as texts. Often in mathematics education, reading demands are raised as a potential problem for students. However, texts are not only **read** in a classroom. Solutions to tasks are **written** by students, so they also should be provided with opportunities of learning how to write. Inspired by research on subject-specific language (Morgan, 1998; Schleppegrell, 2004), writing answers to textbook tasks may be understood as learning to write in specific genres, or types of texts. As Table 7 (p. 84) shows, the student is invited to somewhat different actions by the different school algebra discourses. Perhaps even different algebraic thinking practices (Blanton et al., 2018) are constructed through the discourses? Some of these actions, e.g. **think** and **explain**, draw on the thinking practice of reasoning, while **express** clearly has to do with representing, deciding ‘true or false’ with justifying, and **solve** and **simplify** with the transformation of expressions. From a social-semiotic perspective, these actions may be understood as different linguistic features: mental, verbal and perhaps material processes, respectively. An empirical study of genres in school algebra, or in mathematics, could pinpoint distinct linguistic features which are needed to construct mathematics in different ways. Further studies could then test the affordances of working with these genres in the classroom and in what ways it might scaffold students’ algebra learning.

### 8.8 Summary

At the beginning of this chapter, a comparison of the diachronic interplay of algebra characteristics in the two materials was made, to answer the last research question, namely in what ways algebra characteristics in the textbook tasks and the TIMSS’ test items differ. The most common algebraic characteristic in both textbook tasks and TIMSS’ test items is a combination of **Manipulation** and the symbolic discourse. Yet, partially different algebra characteristics are stressed in the materials: the relational discourse is prevalent only in the test items, the proportion of **Functional Thinking** differs between the materials, and the (un)realistic discourse increases in textbooks over time while it decreases in TIMSS tests. For both textbooks and TIMSS tests, there are larger changes over time in the proportions of the school algebra discourses, than in those of the algebraic activities.
In this chapter, changes in the textbooks related to reform ideas on algebra in the 2011 syllabus were discussed. It was argued that the (un)realistic discourse may be seen as a transformation of the reform idea on mathematical literacy into ‘school algebra’. This is achieved through requests of the students to engage in different actions with algebraic representations of situations in which algebra would not be used in the real world. Furthermore, it was argued that the textbooks are conservative, since many changes in them are quite modest or only occur in one textbook series. The textbooks still appear to resist the idea of a competence perspective on mathematics. Though there is an increase of tasks in the (un)realistic discourse in two of the textbook series, this change may also be interpreted as conservative: these test items seem to have more in common with an old tradition of word problems in everyday situations, than they have with the reform idea on mathematical literacy. It was concluded that in the 2011 school algebra reform, textbooks were important factors that affected the selection of content in unwanted ways. Aspects of TIMSS’ validity with respect to the 2011 school algebra reform were discussed in this chapter. The proportion of test items in the arithmetical and (un)realistic discourses decreases while it increases in the relational discourse. Hence, there are changes in the structure of TIMSS over time. As shown in the Results, students with high achievements on TIMSS have significant differences in results on test items with differing algebra characteristics. They have significantly lower results on test items in the relational and symbolic discourses than on test items in the (un)realistic discourse; and they have significantly lower results on test items in the relational discourse than on test items in the arithmetical discourse. These results support the claim that it is questionable to interpret TIMSS results in algebra over time as a knowledge trend. It is therefore less relevant to use these results over time for evaluating curricular reform or identifying needs for further action.

Moreover, the above mentioned comparison of textbook tasks with test items suggests that the relevance of using algebra-specific TIMSS results, may not necessarily follow from the relevance of using results in mathematics as a whole. This may be an unwarranted inference. Algebra-specific TIMSS results should therefore be interpreted with more caution.

In this chapter, implications were put forward concerning the teaching and learning of algebra. The (un)realistic discourse is an algebra characteristic for which students with low achievements have relatively low results in TIMSS. Its transformation of mathematical literacy into school algebra – which is more of an artificial gaze upon the ‘real’ world – resonates with findings from Dowling (1989) and Le Roux (2008). They claim that similar tasks may offer students to learn neither mathematics nor real life. The increase of the (un)realistic discourse in textbooks may then function to exclude students with low achievements. Suggestions for how to respond to this challenge were made.
Another striking finding was that both textbooks algebra tasks and TIMSS’ algebra items entail a large proportion of *Manipulation* – an even larger proportion in textbooks than in TIMSS. This finding contrasts with that the 2011 syllabus rather cut down the descriptions of this type of algebra. Additionally, Swedish students with high achievements in TIMSS still scored significantly low on items with *Manipulation*, compared to items with *Functional Thinking* and items with *Equivalence, Expressions, Equations, and Inequalities*. This may imply that more textbook tasks that stress *Manipulation* do not lead automatically to better student achievements on similar items. According to Kieran (2007), meaning-building is inherent in tasks that stress the transformation and manipulation of symbolic expressions. It is suggested in this thesis, that opportunities for meaning-building activities, may instead be highlighted for students by a more explicit address from teachers, or through textbook tasks in the symbolic discourse which are not combined with *Manipulation* but rather with other algebraic activities.
9 Sammanfattning på svenska

Introduktion


En läroplansreform kan alltså sägas omfatta mer än att bara lansera nya styrdokument. Genom att förstå läroplansreform som en process kan även andra aspekter än nya styrdokument belysas, såsom förändring i läroböcker samt utvärdering och identifiering av behov för vidareutveckling med hjälp av internationella storskaliga mätningar. I den här avhandlingen studeras därför algebrauppgifter från läroböcker och från den storskaliga mätningen TIMSS för årskurs 8, tillsammans med resultat på algebrauppgifterna i TIMSS. De förstås här som aspekter i den algebraspecifika delen av läroplansreformen 2011.

Syfte och forskningsfrågor

Forskningsfrågorna är:

1. Vad karaktäriserar algebrauppgifter i svenska läromedel för årskurs 8?
2. På vilka sätt och i vilken utsträckning ändras algebrakarakteristika i svenska läroböckers algebrauppgifter över tid?
3. Vad karaktäriserar algebrauppgifter i TIMSS-prov?
4. På vilka sätt och i vilken utsträckning ändras algebrakarakteristika i TIMSS-proven över tid?
5. I vilken utsträckning finns det signifikanta skillnader i svenska elevers resultat på algebrauppgifter i TIMSS-proven, i relation till olika algebrakarakteristika?
6. På vilka sätt skiljer sig algebrakarakteristika åt mellan algebrauppgifter i svenska läromedel åk 8 och algebrauppgifter i TIMSS-prov?

Avhandlingen omfattar av tre delstudier. I de första studierna beskrivs karaktäristika i algebrauppgifter i läromedel för årskurs 8, på två olika sätt. Dessa karakteristika länkas i delstudie två till reformidéer om algebra som fördes fram i kursplanen 2011. I den tredje studien beskrivs karakteristika i algebrauppgifter från fyra på varandra följande TIMSS-prov, för att sedan utforskas
statistiskt i relation till svenska elevers resultat på dessa provuppgifter. En syntes av delstudiernas resultat görs i avhandlingens kappa.

Forskningsfrågorna sätts även i det vidare sammanhanget av en läroplansreform. En teoretisk modell för detta utvecklas med hjälp av Howson et al. (1981) och Valverde et al. (2002), för att betrakta läroböcker och TIMSS-prov som delar av den process som en läroplansreform innebär.

**Teoretiska ramar**


**Analys och metod**


De analytiska ramverken används i avhandlingen för att förstå algebra-karakteristika i materialet i termer av skolalgebraiska diskurer och algebraiska aktiviteter. De skolalgebraiska diskurserna framträder som språkliga mönster. Lingvistiska val identifieras i algebrauppgifterna med det analytiska ramverket i systemisk funktionell lingvistik, SFL. Dessa val består av språkliga skillnader i processer, deltagare, typer av ämnesspecifika ord, modus, modalitet, ton och grad av ämnesspecifikt tilltal (se tabell 5, s. 63). SFL-analysen är alltså en iterativ process där språkliga skillnader urskiljs för att förstå diskursiva konstruktioner av vad algebra är, vem som gör algebra och vad eleven bjuds in att göra.

De algebraiska aktiviteterna är kategorier som beskriver vad elever förväntas delta i för aktivitet, för att lösa en algebrauppgift. De bygger på tidigare forskning om vad algebra är och omfattar ekvivalent, uttryck, ekvationer och olikheter (EEEI), manipulation (M), generaliserad aritmetik (GA), funktionellt tänkande (FT) samt kvantitativa och proportionella resonemang (QPR).

- **Ekvivalent, uttryck, ekvationer och olikheter** (exempel 2, s. 64; exempel 12, s. 89) omfattar aktivitet som har att göra med ”en relationell förståelse av likhetstecknet, att representera och resonera med algebraiska uttryck och ekvationer samt beskriva relationer mellan generaliserade kvantiteter” (Blanton et al., 2015, s. 43, egen övers.). Detta kan till exempel innebära att avgöra sant från falskt, använda variabler för att modellera linjära problem, tolka algebraiska uttryck eller lösa öppna utsagor och ekvationer som befinner sig i en problemkontext.
- **Manipulation** (exempel 3, s. 65, exempel 4, s. 81) omfattar aktivitet såsom ekvationslösning, förenkling av uttryck och beräkning av värden då det inte finns någon problemkontext. Det är en aktivitet med sin teoretiska bas i Kierans (2007) transformerande aktivitet, som kan förstås mer eller mindre proceduralt då den till skillnad från EEEI inte omfattar att generalisera, resonera, rättfärdiga eller representera. Innehållsligt är det alltså samma område som för EEEI, men vad eleven förväntas göra är olika.
- **Generaliserad aritmetik** (exempel 9, s. 86) är en aktivitet som utforskar egenskaper som aritmetik och algebra har gemensamt genom att generalisera aritmetiska samband och resonera om strukturer (Blanton et al.,
Aktiviteten omfattar att analysera information för att göra antaganden om aritmetiska samband och uttrycka dem med ord och variabler, att kunna uttrycka vilka begränsningar som sambanden har och att rättfärdiga en generalisering med empiriska eller representationsbaserade argument.

- **Funktionellt tänkande** (exempel 11, s. 87) är en aktivitet som utnyttjar många semiotiska resurser, såsom tabeller, grafer och symboler för att resonera kring, representera och generalisera samband (Blanton et al., 2015). Aktiviteten omfattar att undersöka och beskriva mönster, organisera data i tabeller, beskriva regler för det identifierade mönstret samt använda sådana beskrivna regler för att beräkna värden.

- **Kvantitativa och proportionella resonemang** (exempel 10, s. 86) omfattar att resonera algebraiskt kring två generaliserade kvantiteter, vars kvot inte varierar (Blanton et al., 2015, p. 43) samt att göra kvalitativa förutsägelser och jämförelser utan numeriska värden på variabler (Lundberg, 2011).

Dessa algebrakaraktäristika i böckerna och proven jämförs med avseende på likheter och olikheter. De jämförs också med avseende på förändringar i deras proportioner över tid. Jämförelsen sker med hjälp av korstabeller och deskriptiv statistik såsom relativ frekvens och standardavvikelse.

Vidare utforskas svenska elevers algebraresultat i TIMSS-prov med statistisk inferensanalyser av t-tester, i relation till algebrakaraktäristika i provuppgifterna. Resultatet av analysen diskuteras i förhållande till strukturen i TIMSS-provens algebra över tid. Här förstår provens struktur som de olika algebrakaraktäristika som ingår i ett TIMSS-prov, samt de olika andelar av algebrauppgifterna som dessa utgör. Ett urval på 197 av de 232 provuppgifterna görs för inferensanalysen, då en sådan analys förutsätter att de olika grupperna av algebrakaraktäristika är oberoende av varandra. Uppgifter som utnyttjar fler än en skolalgebraisk diskurs och uppgifter där mer än en algebraisk aktivitet erbjuds ingår därmed inte. Ytterligare en förutsättning för att kunna göra ett t-test är att medelvärdena av eleveresultaten i de grupper som testas är normalfördelade, varför detta också undersöks med hjälp av en datasimulering (figur 2–5, s. 71–73). Grupperna bör heller inte bestå av för få provuppgifter, varför typer av algebrakaraktäristika med färre än 15 uppgifter inte ingår i inferensanalysen (tabl 6, s. 70). T-testet kan visa huruvida det finns signifikanta skillnader i elevresultat mellan grupper av uppgifter med olika algebrakaraktäristika. Multipla tester görs för att utforska sådana skillnader. På grund av dessa multipla tester har signifikansnivån i t-testerna Bonferroni-justerats, så att inte signifikanta skillnader identifieras på felaktig grund genom ett så kallat Typ I-fel.
Resultat
Resultaten visar att fem skolalgebraiska diskurser kan identifieras i båda materialen: den symboliska, geometriska, aritmetiska, (o)realistiska och vetenskapliga diskursen. I TIMSS-proven kan dock även en sjätte relationell diskurs identifieras.

- I den symboliska diskursen (exempel 4, s. 81) konstrueras algebra som opersonlig. Det är bara eleven som ska delta i att göra algebra och detta ska ske genom att eleven hanterar algebraiska uttryck i handlingar som att lösa, förenkla, beräkna, kontrollera och räkna ut. I viss mån förekommer även uppgifter med explicit elevtilltal i den symboliska diskursen. I dessa bjuds eleven in att delta i handlingar som tänka, förklara, diskutera. Även att avgöra ’samt från falskt’ erbjuds i viss mån.
- I den geometriska diskursen (exempel 5, s. 81) konstrueras algebra som tidlös och opersonlig. Passivformer döljer ibland annan mänsklig agens. Eleven ska förenkla, beräkna och uttrycka relationer mellan geometriska objekt och algebraiska uttryck, samt låta [något vara på ett visst sätt]. I viss mån förekommer även uppgifter där eleven bjuds in att diskutera. Även att avgöra sant från falskt erbjuds i viss mån.
- I den aritmetiska diskursen (exempel 6, s. 82) konstrueras algebra som en mänsklig aktivitet. Både namngivna tredjepersoner och eleven deltar i att mestadels lösa talgåtor, genom att förenkla, använda, beräkna, sätta upp, multiplicera, dubbla och addera. Explicit elevtilltal förekommer. I viss mån förekommer även uppgifter där eleven bjuds in att tänka, diskutera, visa och förklara.
- I den (o)realistiska diskursen (exempel 7, s. 83) konstrueras algebra som en artificiell blick på verkligheten. Många tredjepersoner deltar i mestadels vardagliga handlingar och situationer som att handla, köpa, betala, simma, baka och liknande. Eleven bjuds in att titta på dessa handlingar och uttrycka, förenkla, lösa, kontrollera, räkna ut och tolka dem med hjälp av algebra – samtidigt som det är mer eller (oftast) mindre realistiskt att algebra i verkligheten skulle användas på sådana sätt i dessa situationer. Explicit elevtilltal förekommer. I viss mån förekommer även uppgifter där eleven bjuds in att förklara och låta [något vara på ett visst sätt].
- I den vetenskapliga diskursen (exempel 8, s. 83) konstrueras algebra som opersonliga beräkningar kopplade till mestadels fysik och ibland samhällsvetenskap. Passivformer döljer ibland annan mänsklig agens. Eleven ska beräkna, använda, och avrunda, vilket är ett begränsat utbud av handlingar jämfört med inom de övriga diskurserna.
- I den relationella diskursen (exempel 13, s. 92) konstrueras algebra som opersonliga samband mellan matematiska objekt, såsom algebraiska uttryck, tabeller och grafer. Eleven bjuds in till både att fylla i tabeller och att svara på frågor, vilka ber eleven att identifiera samband mellan matematiska objekt för att svara korrekt. Mer eller mindre implicit är det alltså
handlingar som t ex att beräkna, tolka och förstå som eleven bjuds in att delta i. Den relationella diskursen är den enda skolalgebraiska diskursen som erbjuder algebra på detta sätt i form av relationella samband mellan olika typer av matematiska objekt, vilka eleven ska hantera.

Resultaten visar också på vilka sätt och i vilken utsträckning som algebraaktiviteter i läroböckerna före och efter reformen 2011 ändras. Den största förändringen består i en ökning av andelen uppgifter i den (o)realistiska diskursen (tabell 8–10, s. 87–88). Den ökningen återfinns i två av bokserierna och kan länkas till den nya kursplanens ökade betoning på problemlösning i vardagssituationer och algebra i situationer relevanta för eleven. För några algebraaktiviteter ändrar sig standardavvikelsen över tid, vilket kan förstås som att de olika läromedelsformer lyfter olika reformidéer från den nya kursplanen. Till exempel minskar uppgifter med kombinationen av den algebraiska aktiviteten manipulation och symboliska diskursen i en av bokserierna. Denna minskning kan länkas till att transformationer av uttryck inte längre nämns explicit i kursplanen 2011. Det finns däremot inga förändringar som är gemensamma för alla bokserier och förändringarna är i de flesta fall inte så stora. En kombination av uppgifter med manipulation och den symboliska diskursen dominerar läroböckerna både före och efter 2011 medan uppgifter med de algebraiska aktiviteterna generaliserad aritmetik eller kvantitativa och proportionella resonemang är ovanliga. Detta trevligt att kursplanen framhäver ett kompetensperspektiv på matematik där de två sistnämnda algebraiska aktiviteterna med fördel skulle kunna utnyttjas för att erbjuda eleverna möjligheter att resonera och generalisera i stället för aktiviteter som manipulation, som har en mer transformerande karaktär med fokus på omvandling av uttryck och ekvationslösning. Slutligen avspeglas inte den nya kursplanens betoning av variabler och progression i någon större utsträckning i läromedlens algebrauppgifter. Detta visar sig genom att andelen av den algebraiska aktiviteten funktionellt tänkande är låg även om den ökar från 1 % före 2011 till 7 % efter 2011 och genom att uppgifterna med funktionellt tänkande omfattar samband i mönster som snarare lyfts i kursplanen för årskurs 1–6, än för årskurs 7–9.

I TIMSS-provens algebrauppgifter kan, som nämnts, en relationell diskurs identifieras. När ändringarna i TIMSS-provens struktur under tidsperioden 2003–2015 studeras, märks särskilt att andelen provuppgifter i de (o)realistiska och aritmetiska diskurserna minskar, medan andelen provuppgifter i de geometriska och relationella diskurserna ökar. Däremot forekommer egentligen inga ändringar av andelen provuppgifter inom de olika algebraiska aktiviteterna. T-testerna visar att det finns signifikanta skillnader i resultat för elever med höga prestationer i TIMSS mellan å ena sidan provuppgifter i den relationella diskursen och å andra sidan provuppgifter i de aritmetiska och (o)realistiska diskurserna. För elever med höga prestationer har uppgifter i de två sistnämnda diskurserna en högre lösningsfrekvens, än uppgifter i den relationella diskursen. På liknande sätt finns för samma elevgrupp signifikanta
skillnader i resultat på uppgifter i den symboliska diskursen (relativt svåra) och på uppgifter i den (o)realistiska diskursen (relativt lätt). För elever med låga prestationer i TIMSS finns signifikanta skillnader i resultat på uppgifter i den (o)realistiska diskursen (relativt svåra) och på uppgifter i den aritmetiska diskursen (relativt lätt). Här är alltså en skillnad mellan elevgrupperna: elever med låga prestationer i TIMSS har relativt låga resultat på en uppgiftstyp som elever med höga prestationer har relativt höga resultat på. När resultaten från t-testen ställs emot ändringarna i TIMSS-provens struktur under tidsperioden 2003–2015, blir det tydligt att för elever med höga resultat i TIMSS, blir pro-

vken relativt sett svåra. Andelen uppgifter med en relativt hög lösningsfre-

kvens minskar samtidigt som andelen uppgifter med en relativt låg lösnings-

frekvens ökar (detta illustreras av tabell 11, s. 93, i förhållande till figur 6, s. 94 samt figur 11–15, Appendix B).

I jämförelsen av algebrauppgifterna i läroböckerna och TIMSS-proven framkommer att undantaget den relationella diskursen, så omfattar materialen samma karaktäristika. Däremot kan de förekomma i olika utsträckning. I TIMSS är de tre algebraiska aktiviteterna funktionellt tänkande, manipulation samt ekvivalens, uttryck, ekvationer och olikheter lika vanliga (se Appendix C). I läroböckerna är funktionellt tänkande betydligt ovanligare än manipulation och ekvivalens, uttryck, ekvationer och olikheter (tabell 10, s. 88). Att den (o)realistiska diskursen minskar i TIMSS är en ändring som går i motsatt rikt-
ning gentemot i läroböckerna. Jämförelse av algebrauppgifterna i läroböck-

erna och i TIMSS-proven visar slutligen att i båda materialen är en kombinat-

ion av den symboliska diskursen och manipulation absolut vanligast. Den är till och med något mer vanlig i böckerna och upptar ca 30–40 % av läroböck-

ernas algebrauppgifter, jämfört med ca 25 % i TIMSS-proven.

Diskussion och slutsatser
Om vi ser till 2011 års läroplansreform och algebraområdet, så är läroböckerna konserverande i reformprocessen, i bemärkelsen att förändringarna i algebra-

uppgifterna under perioden 1995–2015 är blygsamma. Kompetensperspekti-

vet som framhävs i kursplanen har inte resulterat i någon markant ökning av uppgifter som erbjuder algebraiska aktiviteter vilka kan tänkas utveckla kom-

petens att generalisera och resonera, såsom generaliserad aritmetik och kvantitativa och proportionella resonemang. Algebrauppgifter av kombinat-

ionen manipulation och den symboliska diskursen dominerar fortfarande. Eftersom uppgifterna i den (o)realistiska diskursen liknar sådan vardagsanknu-

ten matematik, som beskrivs redan i läroplanen 1919, kan även den största ändringen i böckerna – ökningen av andelen uppgifter i den (o)realistiska disk-

kursen – kallas konservativ.

Läroböckerna är också transformerande i 2011 års läroplansreform inom algebraområdet, i bemärkelsen att den reformidé som berör vad som kan kal-

las matematisk literacy, i böckerna har transformerats till tämligen ytliga var-

dagsanknutna uppgifter i den (o)realistiska diskursen. Detta syns i Exempel 7
(s. 83) på relationen mellan biljetter till en cirkus och a kr per styck, då kostnaden för en biljett inte är okänd i en vardags situation och den som köper en cirkusbiljett inte heller behöver formulera något algebraiskt uttryck för biljettkostnaden. På så sätt kan läroboksförfattarna sägas ha genomfört förändringar som inte hade formulerats i kursplanen 2011.

Enligt Howson et al. (1981) så är utvecklingen av nya undervisningsmaterial en ondänglig del i en reformprocess. Den här avhandlingen ger exempel på hur förändringarna i läroböckerna kan bli relativt små och obemärkta när utvecklingen av nya material helt överlämnas åt förlag och läroboksförfattare – åtminstone saknas förändringar av algebra-aktivistiska som går i samma riktning inom alla de undersökta läroboksserierna. På den punkten var reformidéerna i kursplanerna 2011 inte tillräckligt tydliga för att de skulle utvecklas åt samma håll i nästa steg av reformprocessen. Denna avsaknad av styrning kan tyckas märklig, om syftet med en läroplansreform är att faktiskt åstadkomma förändring.


material studeras, eftersom Sollerman (2019) jämför med de storskaliga mät-
ningarnas uppgifter emot uppgifter i de svenska nationella proven, det vill säga
på nivån för den avsedda läroplanen och inte som denna avhandling på nivån
för den potentiellt implementerade läroplanen. Vidare tittar den här avhand-
lingen specifikt på algebra, medan Sollerman (2019) studerar matematikämnet
i sin helhet. Avhandlingen föreslår därför att relevansen i att använda algebra-
resultat från TIMSS inte är avhängig relevansen i att använda matematikresultat
från TIMSS.

En aspekt av validitet inom prov och bedömning avser provs påverkan på
i länder som deltar i TIMSS över tid har utvecklats, så att de blir mer lika
TIMSS ramverk. Resultaten i den här avhandlingen visar hur utvecklingen i
svenska läroböcker går tvärt emot den i TIMSS-proven, då andelen uppgifter i
den (o)realistiska diskursen ökar i läroböckerna men minskar i TIMSS-pro-
ven. Samtidigt kan uppgifterna i den (o)realistiska diskursen förstås som svar
på OECDs fokus på matematik literacy. Avhandlingen föreslår därför att
PISA kanske har större inflytande över valet av matematikinnehåll i ett
svenskt sammanhang, än vad TIMSS har.

Det är troligt att resultat på internationella storskaliga mätningar kommer
att fortsätta användas för att utvärdera reformer och för att identifiera behov
för vidareutveckling av läroplaner. Sedan läroplansreformen 2011 har Sverige
deltagit i TIMSS 2015 och TIMSS 2019. Det är mindre troligt att enbart
TIMSS-resultat i matematik som en helhet kommer att användas för att dra
slutsatser om svenska elevers kunskaper: möjligheten ges ju av IEA att stu-
dera TIMSS-resultaten inom olika ämnesspecifika områden – som t ex algebra
– och inom olika kognitiva domäner. Den här avhandlingen visar att slutsatser
från algebraspecifika resultat borde dras med större försiktighet. Vad gäller de
svenska resultaten föreslås att TIMSS-proven justeras för ändringar i prov-
struktur innan deras resultat tolkas och används för att dra slutsatser om kun-
skapstrender i algebra. Fortsatta studier skulle kunna undersöka validiteten
hos TIMSS-resultat inom andra ämnesspecifika områden, men risken finns att
det blir ett tidsödande arbete.

Två didaktiska implikationer diskuteras i avhandlingen. För det första, så
riskerar elever med låga prestationer i TIMSS att exkluderas i algebraunder-
visningen. Avhandlingen visar att medan elever med höga resultat i TIMSS
har relativt höga resultat på uppgifter i den (o)realistiska diskursen, så har ele-
ver med låga resultat i TIMSS relativt låga resultat på sådana uppgifter. Vi vet
från tidigare forskning, såsom Dowling (1996) och Le Roux (2008), att uppgif-
ter med problemlösning och vardagsanknytning, som alltså liknar uppgifter
i den (o)realistiska diskursen, kan positionera elever så att de nekas tillgång
till matematiken. Uppgifterna transformerar reformidén om matematik lite-
racy, så att varken matematik eller verklighet erbjuds. Den här typen av upp-
gifter visar sig öka i två av tre läroboksserier i samband med skolalgebrafor-
men 2011. Att också använda en hög andel uppgifter i den (o)realistiska diskursen i undervisningen kan då leda till att elever med låga resultat inte erbjuds tillräcklig stödning för att lära sig algebra, eller för att lära sig om vardag och verklighet.

För det andra, så är det inte säkert att en ökad betoning av uppgifter som erbjuder manipulation av algebraiska uttryck leder till bättre elevresultat i algebra. Svenska elever ägnar en stor andel – ca 35 % – av sin lektionstid till enbart procedurala aktiviteter (Boesen et al., 2014) och det finns redan en större andel uppgifter i svenska läroböcker med kombinationen av manipulation och den symboliska diskursen, än vad det finns i TIMSS-proven. Trots detta har svenska elever med höga prestationer i TIMSS signifikant lägre resultat på uppgifter med manipulation, jämfört med uppgifter med funktionellt tänkande eller med ekvivalens, uttryck, ekvationer och olikheter. Om ordvitsen ursäktas: ekvationen går inte ihop. En förklaring kan vara att den mening som Kieran (2007) menar är inneboende i transformation och manipulation av algebraiska uttryck, snarare är en potential som kan behöva explicitgöras – antingen av lärare i klassrummet, eller genom uppgifter av en annan karaktär än manipulation, där denna mening endast är implicit. Här föreslås att sådana uppgifter i den symboliska diskursen som inte är kombinerade med den algebraiska aktiviteten manipulation, utan med andra algebraiska aktiviteter (se t ex uppgift 12, s. 75), kan utnyttjas för att göra denna potential explicit.


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34 Kieran (2007) skriver att i aktiviteten att omvandla uttryck och lösa ekvationer så är meningsskapande för användning av egenskaper och axiom är inneboende i de manipulativa processer man använder. Exempel på detta skulle kunna vara en strukturell förståelse av likhetstecknet, den distributiva lagen eller att en ekvation med två obekanta kan ha mer än en lösning.
10 References


11 Appendix A. Missing values in the TIMSS material

Test items entirely removed from the entire dataset:
- M032637 A, M032637 B (noted as Data and Chance in 2007)
- M032352 (noted as Number in 2003)
- M042229Z (derived item, no task to analyse)
- M042197 (noted as Number in 2007 and 2011)
- M052090 (mistranslated)
- M062237 (mistranslated)

Test items removed from one test:
- M042067 (system missing answer in 2007)
- M042103 (system missing answer in 2007)
- M032538 (system missing answer in 2007)
- M062342 (system missing answer in 2015)

Missing values in the inferential analysis are given by Table 13.

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<th>Item number</th>
<th>Prevalence</th>
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<td>M042248</td>
<td>In 3 tests</td>
</tr>
<tr>
<td>M012025</td>
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<td>M012040</td>
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<td>M032744</td>
<td>In 1 test</td>
</tr>
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<td>M022253</td>
<td>In 1 test</td>
<td>M042301A</td>
<td>In 1 test</td>
</tr>
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<td>M032047</td>
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</tr>
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Table 14. Distribution of percent right values for the school algebra discourses in TIMSS.

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Table 15. Distribution of percent right values for the algebraic activities in TIMSS.

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Figure 11. Percent right values in algebraic activities for all students.

Figure 12. Percent right values in school algebra discourses for all students.
Figure 13. Percent right values in algebraic activities for students with low achievements.

Figure 14. Percent right values in school algebra discourses for students with low achievements.
Figure 15. Percent right values in algebraic activities for students with high achievements.

Please, note that percent right values of school algebra discourses for students with high achievements, are already given by Figure 6 (p. 94) in the Results chapter.
### Appendix C. Table over the diachronic interplay of school algebra discourses and algebraic activities in the TIMSS tests

<table>
<thead>
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</table>

* The row percentages do not add up to the total since a task can entail more than one algebraic activity. Changes over 10 percentage points are circumscribed.
Appendix D. Box plots of percent right values, for missing values in the inferential analysis

Figure 16. Percent right valued for students with high achievements in the test items not selected for inferential analysis.
Figure 17. Percent right values for all students in the test items not selected for inferential analysis.

Figure 18. Percent right values for student with low achievements in test items not selected for inferential analysis.
Beslut om utlämnande av allmänna handlingar med förbehåll

Kristina Palm Kaplan, doktorand vid Institutionen för pedagogik, didaktik och utbildningsstudier, Uppsala Universitet har begärt att Skolverket lämnar ut:

- Sekretessbelagda provuppgifter i matematik från Trends in International Mathematics and Science Study (TIMSS) 2003 och 2015, årskurs 8.

I samarbete med den internationella forskningsorganisationen The International Association for the Evaluation of Educational Achievement (IEA) och övriga deltagande länder genomför Skolverket TIMSS i Sverige.

Provuppgifter i de internationella studierna är sekretesskyddade enligt 17 kap. 4 § offentlighets- och sekretesslagen (2009:400) så länge de inte är frisläppta från den internationella organisationen IEA. Sekretessen innebär att uppgifter i provet inte får röjas om det kan antas att syftet med provet motverkas om uppgiften röjs.

Enligt 10 kap. 14 § offentlighets- och sekretesslagen får uppgifter lämnas ut om risken för skada kan undanröjas genom ett förbehåll som inskränker den enskildes rätt att lämna ut uppgiften vidare eller utnyttja den.

Skolverket lämnar ut provuppgifterna till Kristina Palm Kaplan, handledarna Johan Prytz, Kajsa Bråting och Caroline Liberg, samt Kirsti Hemmi, Lars Madej, Yvonne Liljekvist och Johanna Pejlare vilka deltar i forskningsprojektet omnämnt nedan. Utlämnandet görs med följande förbehåll:

1. Provuppgifterna får endast användas inom forskningsprojektet ”Mot en forskningsbaserad undervisning i algebra – diakrona och synkrona analyser av styrdokument, läromedel och lärarens interaktion med dem”, och som bedrivs av berörd forskare.
2. Uppgifter om proven och deras innehåll får inte lämnas vidare till annan.
3. Vid eventuella skrivningar om proven får provens uppgifter inte röjas om inte IEA har gett tillstånd för detta (tillstånd kan ges om uppgifter är fri-släppta).

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Besöksadress: Flemingsgatan 14
Telefon: 08-527 332 00 vx Fax: 08-24 44 20
skolverket@skolverket.se www.skolverket.se

15 Appendix E. Permission to use test items
Skolverket

4. Provuppgifterna ska förvaras på ett sådant sätt att dess innehåll inte blir känt eller kan spridas.
5. Provuppgifterna ska förstöras när arbetet med forskningsprojektet är avslutat.

Genom att lämna ut proven med förbehåll uppkommer en tystnadsplikt för den person/de personer som beslutet riktar sig till. Om inte villkoren i förbehållet följs kan ansvar för brott mot tystnadsplikt enligt 20 kap. 3 § brottsbalken (1962:700) komma i fråga.

Skolverkets beslut kan överklagas till Kammarrätten i Stockholm, se Bilaga 2

På Skolverkets vägnar

Jonas Nordström  Maria Axelsson
Rättschef

I ärendets slutliga handläggning har Eva Durhán deltagit.

Bilaga 1 – Provuppgifter i matematik från TIMSS 2003 och TIMSS 2015, årskurs 8.
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To be completed by anyone seeking permission to reuse, reproduce, or translate IEA material.

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Please indicate the source of the IEA material:
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Title: TIMSS 2003 Assessment ____________________________
ISBN: ______________ Date of publication: 2005
Description of requested IEA material: the released items M012017 and M032044, English version

Author/editor: International Association for the Evaluation of Educational Achievement (IEA)/Pierre Foy, John F. Olson (eds.) Publisher: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.
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Information about where the requested content will appear/where you will use the requested content:

Author: Kristina Palm Kaplan _____________________
Title (preliminary): Reform on a shaky ground? A comparison of algebra tasks from TIMSS and Swedish textbooks _______________________
Language: English and Swedish _______________________
Publisher or sponsor: Acta Universitatis Upsaliensis _______________________
Intended audience: researchers, teachers, policy makers _______________________

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Additional comments: ___________________ ___________________________ ___________________________

3. Requestor information

First name: Kristina ___________________________ 
Last name: Palm Kaplan ___________ _____________ 
Name of institution or organization: Department of Education, Uppsala University _____ 
Address: Box 2136 _________ 
City and zip code: 750 02 _______ 
Country: Sweden _______ 
Phone: +46 18 471 16 78 Email: Kristina.palm.kaplan@edu.uu.se __
Signature: ___________________________ Date of request: 2019 02 11

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1 Cf. ECJ 16 July 2009, Case C-5/08 (Infopaq I).
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