The vortex nature of the space-time curvature.

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1 Abstract

Fundamental physical phenomena, such as gravity, electromagnetism and light have been studied over the last centuries for their properties, mechanics and for their relationships to one-another. Bernoulli proposed in 1736 a vortex-sponge theory to describe light in mechanical terms which was later postulated by using Maxwell’s vortex equations. In this study we use a supersymmetric Hamiltonian recently developed for quantum systems to describe the vortex nature of space-time curvature arising at gravity sources. We use the solar system as model and describe the gravitational effect on space-time as a sheer result of vorticity in the gravity field, maintaining planets in their elliptic orbit and bending space-time in a similar fashion to Einstein’s gravity sink model. It is shown that from the Einstein equations for the post-Newtonian metric, the nonlinear diffusion equation and the Schrödinger equation can be derived. The results are analyzed

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numerically and the dynamics of the system studied.

2 Introduction

Vorticity is ubiquitous in nature and arises in all natural phenomena, from vortex eddies in water streams [1], to vortices in electromagnetic fields represented by superposition of wavefunctions [2, 3, 4, 5, 6], vortices in the atmosphere [7], vortices in superheated stellar plasma [8] and in many other phases of nature, including vortices in gravity waves [9, 10, 11]. Interestingly, Bernoulli proposed in 1736 that also light is behaving in a vortex-like manner, a theory which was further discussed by Kelly [12] where light was proposed to be a component of a continuous medium present in the universe. Descartes proposed even earlier, in his work "Principiorum Philosophiae" from 1644, that the universe was solely composed of vortices, also noted as Cartesian vortices. However, after Einstein’s general relativity theory, the view of the fundamental forces of the universe was greatly simplified by the theory of space-time curvature by gravity and its effect on cosmological objects.

In this study, we study the gravity sink (curvature) proposed originally by Einstein [13] as a pure consequence of gravitational space-time vorticity, leading to the circulation of planets and generation of the space-time curvature. We suggest hence a theory where gravity originates from a vortex in a space-time continuum, leading to the formation of the gravity curvature emerging from the center of the source of mass. Albert Einstein described this continuum in similar terms, naming it "the affine field" [14], a model which was founded on the idea of Arthur Eddington [15] who later critically discussed the affine field theory by Einstein [16]. The "affine field" theory was proposed to merge electromagnetism and gravity under one model, however, the theory was later abandoned and not pursued further. It is however of interest that the "affine field theory" bears similarities to Descartes model from 1644, and to the subsequently published vortex sponge model by Bernoulli. As defined by Eddington however [15], the necessity of pure mathematical models is deemed critical.
to describe a continuum in the universe, and the latter two fail therefore to
give rigorous mathematical modelling to show a reproducible description of the
universal continuum where space-time gravity vortices arise spontaneously in a
similar fashion to the atmosphere, fluids and superfluids.

Here, we devise a supersymmetric wave-equation which we developed recently
over several works [3, 4, 5, 6] when studying electron gas in quantized 2D sys-
tems. The Hamiltonian of the supersymmetric wave-equation is of particular
appeal to model vorticity in a space-time continuum, as it generates vortic-
ity spontaneously without need of auxiliary functions and has a particularly
appealing spectrum of eigenvalues, where the range of trivial eigenvalues in
its spectrum is positive infinite, and the evolution of the spectrum is slowly
crescent [17]. The algebraic properties of the operator of the supersymmetric
wave-equation [17] suggest therefore that it is valid for both microscopic and
quantized systems as well as for macroscopic continuous systems, yielding real
eigenvalues towards infinite levels of energy [17]. The equation, which reads:

$$\frac{1}{2m}(\hbar/i\nabla - (e/c)\vec{A}) \cdot (-\hbar/i\nabla - (e/c)\vec{A})\Psi = E\Psi$$

was developed using supersymmetry rules on its factors, generating the in-
verse sign on the the two inherent factors as given in [1]. The original form of
this equation developed by Fang and Stiles [18] differs from it as it does not
have supersymmetric form of the factors, and therefore generates different re-
results. We consider a dimensionless nonlinear form of [1] describing the vorticity
in a quantum system in the form

$$\frac{\partial \psi}{\partial t} = \frac{1}{2} \nabla^2 \psi - i\Omega(x \frac{\partial \psi}{\partial y} - y \frac{\partial \psi}{\partial x}) - \beta |\psi|^2 \psi + (\vec{A})^2 \psi$$

Here $\vec{A}$ is a dimensionless vector potential and $\beta, \Omega$ - parameters of the model
describing the quasiparticle interaction and the angular velocity, respectively.
Note that nonlinear Schrödinger equation (Gross-Pitaevskii model) follows from
[2] when replacing $t \rightarrow it$. Eq. (2) describes evolution of the wave function
from some initial state $\psi(x, y, z, 0) = \psi_0(x, y, z)$ and up to stationary state with
$E = 0$. 

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We show that the nonlinear diffusion equation \( \frac{\partial \phi}{\partial t} \) as well as the nonlinear Schrödinger equation can be derived from the Einstein equations for the post-Newtonian metric. Put \( \phi = \phi(t, x, y, z) \) as the gravitational potential, then the metric has the form (see the outline in [19]):

\[
    ds^2 = (1 + \frac{2\phi}{c^2}c^2 dt^2 - (1 - \frac{2\phi}{c^2})(dx^2 + dy^2 + dz^2),
\]

where \( c \) is the speed of light. We put \( c = 1 \) and consider the following generalization of the metric (3):

\[
    ds^2 = e^{h(t,x,y,z)}dt^2 - e^{-h(t,x,y,z)}(dx^2 + dy^2 + dz^2),
\]

where \( h(t, x, y, z) \) is some function that is a solution of the Einstein equation.

In metric (4), the Einstein tensor component \( G_{00} \) is reduced to the form

\[
    G_{00} = e^{2h}\nabla^2 h - \frac{1}{4}e^{2h}(\nabla h)^2 + \frac{3}{4}(h_t)^2
\]

Using the Einstein equation \( G_{ik} = 8\pi\gamma T_{ik} \), where \( T_{ik} \) is the energy-momentum tensor and \( \gamma \) is the gravitational constant, we find

\[
    8\pi\gamma T_{00} = e^{2h}\nabla^2 h - \frac{1}{4}e^{2h}(\nabla h)^2 + \frac{3}{4}(h_t)^2
\]

Without loss of generality, we set \( 8\pi\gamma T_{00} = \rho e^{2h} \), then equation (6) is reduced to

\[
    \nabla^2 h - \frac{1}{4}(\nabla h)^2 + \frac{3}{4}(\partial_t e^{-h})^2 = \rho
\]

In the static case, in the linear approximation, from equation (7) we have Newton’s theory of gravity

\[
    \nabla^2 h = \rho
\]

In another case, when \( h = h(t), h(0) = 0 \), equation (7) describes the collapse (-) or expansion (+)

\[
    \frac{3}{4}(\partial_t e^{-h})^2 = \rho \rightarrow h = -\ln(1 \pm \frac{\rho}{\sqrt{3}} \int_0^t \rho^{1/2}dt)
\]

Since equation (7) is not quasilinear, we reduce it to the form of a quasilinear equation using the hypothesis of the dominant influence of \( \rho \). Suppose that
If $|\rho| >> \max(\left|\nabla^2 h \right|, (\nabla h)^2)$, then it is possible to bring (7) to the form of a parabolic equation

$$\frac{\sqrt{3}}{2} \frac{\partial}{\partial t} (e^{-h}) = \sqrt{\rho - \nabla^2 h + \frac{1}{4} (\nabla h)^2} = \sqrt{\rho} (1 - \frac{1}{2\rho} \nabla^2 h + \frac{1}{8\rho}(\nabla h)^2 + ...) \tag{10}$$

There are two possible forms of equation (10), depending on the sign of $\rho$. If $\rho > 0$, then keeping only the first terms of the expansion on the right-hand side of (10), we find the diffusion equation

$$\frac{\sqrt{3}}{2} \frac{e^{-h} \partial h}{\partial t} = -1 + \frac{1}{2\rho} \nabla^2 h - \frac{1}{8\rho} (\nabla h)^2 \tag{11}$$

If $\rho < 0$, then setting $\rho = -m^2$, we reduce equation (10) to the form of the nonlinear Schrödinger equation

$$-i \frac{\sqrt{3}}{2} \frac{e^{-h} \partial h}{\partial t} = 1 + \frac{1}{2m^2} \nabla^2 h - \frac{1}{8m^2} (\nabla h)^2 \tag{12}$$

The nonlinear diffusion equation (11) is obviously related to equation (2), and equation (12) is related to the nonlinear Schrödinger equation. We pose the question if vorticity has arisen by virtue of equations (1-2), then how will this evolve by virtue of equations (11) or (12)? To simplify the problem, we put $\rho = \text{const}$ in eq. (11) and $m = \text{const}$ in eq. (12). But here other options are possible, which we will discuss below.

### 2.1 Numerical analysis

As the initial state for equations (11) and (12), we take the analytical solution obtained in [20]

$$\psi_m = C_m e^{-iE_mt + im\phi + \frac{1}{\sqrt{2}} \sqrt{m} L_{\kappa_1}^m (-i\sqrt{2}n^2)}, \tag{13}$$

Here $L_{\kappa}^m(x)$ is the generalized Laguerre polynomial, $\kappa = \frac{1}{2} (2i\sqrt{2m\Omega} - 4m + 2i\sqrt{2E} - i\sqrt{2k^2} - 4)$, and $C_m$ is constant. Here we set the constant from the condition $\max |\psi_m| = 1$. Consider a superposition of states with 3 different $m$ in the form

$$\psi = C_{m_1} \psi_{m_1} + C_{m_2} \psi_{m_2} + C_{m_3} \psi_{m_3} \tag{14}$$
Variants of the initial states for equations (11)-(12) are shown in Figure 1. For example we use the combination, including the first, seventh and thirteenth harmonics (bottom line in fig1). Figure 2 (top row left) shows the modulus of the function $\psi(0, x, y)$ for triplet with $m_1 = 1, m_2 = 7, m_3 = 13$. This triplet describes a system consisting of 6 vortices around a giant vortex. Figure 2, 3 shows the evolution of the initial state in a rectangular region with periodic boundary conditions and with initial data

$$h(0, x, y) = \pm |\psi(0, x, y)|$$  \hspace{1cm} (15)

**Figure 1.** The modulus of the wave function at different instants of time used as initial state for eqs. (11)-(12)
Figure 2. The evolution of the metric with an initial state in the form of a hexagonal flow with a negative sign in (15).

Figure 3. The evolution of the metric with an initial state in the form of a hexagonal flow with a positive sign in (15).
If we take the negative sign in equation (15), then the evolution of the metric proceeds while maintaining the hexagonal structure - see Figure 2. If we change the sign in the initial data (15) then the evolution of the metric is different - Fig. 3. This is obviously related to the property of the gravitational potential \( \phi \), which should be negative in Newton’s theory. Comparing metrics (3) and (4) we find that in linear theory \( h = \frac{2\phi}{c^2} \). Therefore, to be consistent with Newton’s theory, we must choose the minus sign in equation (15). In this case, the metric preserves some properties of the quantum micro-structure - Fig. 2.

This property is even more evident if, in the initial data, instead of magnitude, we use the square of the modulus of the wave function with a minus sign,

\[
h(0, x, y) = -|\psi(0, x, y)|^2
\]  

(16)

Figure 4 shows the evolution of the metric with initial data in the form (16). In this case, the hexagonal structure of the initial data is also preserved.

![Figure 4](image_url)

**Figure 4.** The evolution of the metric with an initial state in the form of a hexagonal flow using eq. (11) and (16).

We now turn to equation (12), which we could call the equation of quantum gravity. It could be assumed that for equation (12) it is necessary to use the
wave function as the initial data. However, equation (12) is very sensitive to boundary conditions, while function (13) \(- (14)\) does not completely satisfy the periodicity condition. This does not noticeably affect the solution of equation (11); therefore, the wave function was used as initial data in the form of (15) and (16). However, for equation (12) we use the following initial data:

\[
h(0, x, y) = \psi(0, x, y)e^{-x^2-y^2}
\]  

Figure 5 shows the evolution of the metric calculated using equation (12) with the initial data (17). We see here a completely different picture than in Fig. 2 and 4. Firstly, there is a loss of symmetry. Secondly, a peculiar structure is formed that is not similar to the original hexagonal.

\[
\text{Figure 5. The evolution of the metric with an initial state in the form of a hexagonal flow using eq. (12) with initial conditions in (17).}
\]

A remarkable property of equation (12) with initial data (17) is that the number of vortices increases from the initial $6 + 1 = 7$ to $15$ at $t = 1$ - see Fig. 5. This vortex growth becomes even more noticeable if we change the initial...
data to

$$h(0, x, y) = \psi(0, x, y) e^{-\frac{x^2 + y^2}{2}}$$  \hspace{1cm} (18)

Fig. 6 shows the evolution of the metric calculated using equation (11) with vital data (18). In this case, we see that the initial number of vortices (6 + 1) turns into 30 vortices arranged in pairs in a circle.

Figure 6. The evolution of the metric with an initial state in the form of a hexagonal flow using eq. (12) and (17).

We then put in the equations (11) and (12) $\rho = e^{2h}$ and $m = e^{2h}$ consequently, we then get the two equations:

$$\sqrt{3} \frac{\partial h}{\partial t} = -e^{2h} + \frac{1}{2} \nabla^2 h - \frac{1}{8}(\nabla h)^2$$  \hspace{1cm} (19)

$$-\frac{\sqrt{3}}{2} \frac{\partial h}{\partial t} = e^{2h} + \frac{1}{2} \nabla^2 h - \frac{1}{8}(\nabla h)^2$$  \hspace{1cm} (20)

Figure 7 shows the evolution of the metric by virtue of equation (20) with the initial data (17) for a hexagonal flow with periodic boundary conditions. We see that in this case too, the hexagonal structure is preserved.
**Figure 7.** The evolution of the metric with an initial state in the form of a hexagonal flow using eq. (19) and (16).

A similar calculation of the metric according to equation (20) with initial data (17) with periodic boundary conditions is shown in Fig. 8. In this case, the result is similar to that shown in Figure 5.
Figure 8. The evolution of the metric with an initial state in the form of a hexagonal flow using eq. (20) with initial conditions in (17) (magnitude of the wave function shown).

Thus, we have shown that both in the case of a homogeneous and in the case of a nonuniform density distribution, the evolution of the metric leads either to an increase in the initial quantum vorticity (equations (12) and (20)), or to its conservation (equations (11) and (19)). Finally, note that we have studied the evolution of various initial data indicated in Fig. 1. Some of these data are presented in Fig. 9-11 in 3D format. We see that the initial states with 6 (Fig. 9), 4 (Fig. 10) and 10 vortices (Fig. 11) around the central giant vortex evolve the same way while preserving the original structure. In this case, we used equation (11) with initial data in the form (16).

Figure 9. The evolution of the metric with an initial state in the form of a hexagonal flow using eq. (11) and (16).

Figure 10. The evolution of the metric with an initial state with $m_1 = 4, m_2 =$
8, \( m_3 = 12 \) using eq. (11) and (16).

The evolution of the metric with an initial state with \( m_1 = 3, m_2 = 8, m_3 = 13 \) using eq. (11) and (16).

2.2 Discussion

The evolution of the metric, shown in Figure 2 and 3 bears a series of important implications for the phenomenon of gravity. Taken from a spacetime curvature perspective, the positive sign in the metric attributes to gravity a geometry in spacetime which favors the formation of hexagonal shape, which we do not observe in gravitational orbits, but rather in electromagnetic systems, such as vortices in Bose-Einstein condensates [4, 5, 6, 21, 22, 23, 24] or for macroscopic fine medium (gases) governed by electromagnetic fields such as on the North Pole of Saturn [25]. We show therefore a preserved vortex model in eqn. (10) by assigning a positive sign to \( \rho \) which favors hexagonality in micro-gravity environments and electromagnetic fields of weak energy, which we describe as an equation for quantum gravity. The quantized state of gravity yields hence a hexagonal shape, composed of six angles that represent six discrete preferred positions for a quantum gravitational vortex model. This is indeed in agreement with that the hexagonal symmetry is the single symmetry that allows the highest number of vortices to arrange about a center and that overlap to form multi-hexagonal lattices, as also other have reported for electromagnetic systems and quantum gases [26, 27, 28]. The model of eqn. (10) with a negative sign explains...
also the inevitable emergence of quantum geometry, where all orientations of orbitals follow quantum rotation, as also proposed by Haldane \[29, 30\].

For macroscale gravity however, we observe directly an agreement with Newtons theory on gravity, which is only satisfied by having a minus sign in for $\rho$ in eqn. (10), where the continuum at macroscale yields a spherical shape with a growing elliptical pattern from the center of gravity in the space-dimensions (x,y). The model in (11) and (12) shows also a potential description on how gravity arises and how it evolves during a period of time. The evolution of the gravity potential in spacetime (fig 3) shows that the vortex nature of gravity assembles a hexagonal structure in its very infant states, and then develops an intense gravity-void around the gravity center (Fig 3, t=0.4) which then rapidly disappears as the gravity potential assumes an elliptical shape with a circular form the closer one reaches the center of the gravity. Attributing this model to our solar-system for instance, shows an excellent agreement with the orbit of the planets. Furthermore, the gravity-vortex model suggests also that the gravitational center increases in extension in the space plane (x,y), with a larger area of intensity arising with time (compare t=1.6, 2.0), which can explain the evolution of red giants and supergiants which develop into black holes. Figure 3, at time t=0.4 shows also a similarity to the behaviour of black holes at even horizon, when a superfine boundary of trapped light is contained outside the center of the black hole, hence neither escaping its gravity pull nor being compressed in the black hole. Figure 3 shows such a distinctive ring, which is in the periphery of the gravity center. The principal differences between macroscale gravity and quantum gravity, as described by the model showed above, is that macroscale gravity appears to be degenerate (elliptic) while quantum gravity appears to be orthogonal. To our knowledge, only one author has reported degeneracy of universal forces, where Kunz reported recently a model of dark energy with degenerate properties \[31\], which yields better agreement with the behaviour of supernovas and the angular velocities of rotating galaxies.

When we consider Figures 5-11, we focus on the evolution of the metric which explains preferred states of the initial symmetry for gravitational fields, by their
quantized parameters $m$. It is expected that the higher levels of $m$ will yield higher order symmetries which eventually will reach the circle symmetry, as its symmetry states jump from 4 (Fig 10) to 6 (Fig 5, 6 and 9), and further to 10 angles (Fig 11) in the symmetrical organization of the vortex shape. To our knowledge, only gravity waves display vortex phenomena at the macroscopic scale [32, 33, 34], and we can therefore extrapolate that gravity vortices in the atmosphere are the closest empirical example of the vortex signature of gravity at the universal scale. This study relates the supersymmetric wave-equation [3, 4, 5, 6] to the nonlinear diffusion equation in (2) and the Einstein equations. Equations (11) and (12) have been published recently by the corresponding author in [35], however the transition from equation (1) and (2) to (11) and (12) is here described for the first time. Studies to measure and model the vorticity of gravity fields should be emphasized in future astrophysics research, as it can explain several nonlinear astrophysical phenomena, such as the higher rotational velocities of galaxies compared to their total mass.

2.3 Conclusions

We have in this paper, presented a theory which suggests that gravity fields are governed by vorticity which generate the elliptic orbit of planets, and bear similarities to gravitational waves in the atmosphere, and yield quantum geometry by quantum gravity. We have also shown that the proposed model separate the quantum scale from gravity at the macroscopic scale by their geometry; the former is orthogonal, while the latter is degenerate and generates elliptic structures. The cause for this principal difference between quantum gravity and macroscopic gravity lies in the evolution of the respective wave-functions, which suggests that both gravity fields have the same original start geometry (hexagonal) at time zero, however due to the low-energy of particles governed by quantum gravity, they remain in their discrete levels and retain a quantum geometry. Inversely, macro-scale bodies, having a higher energy of macroscopic elements exert a dynamical component on their gravity pull (i.e. planets) that is
able to oscillate beyond the orthogonal framework of gravity (complete circular),
and without losing orbit, tend to oscillate in elliptic shapes.

3 Acknowledgements and correspondence

The authors declare that they have no competing financial interests.
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References


[10] Kaoru Sato and Motoyoshi Yoshiki. Gravity wave generation around the polar vortex in the stratosphere revealed by 3-hourly radiosonde observations.


