Quantum Optics in Constrained Geometries

BY

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When light exhibits particle properties, and when matter exhibits wave properties quantum mechanics is needed to describe physical phenomena.

(i) A two-photon source produces nonmaximally entangled photon pairs when the source is small enough to diffract light. It is shown that diffraction degrades the entanglement. Quantum states produced in this way are used to probe the complementarity between path information and interference in Young’s double slit experiment.

(ii) When two photons have a nonmaximally entangled polarization it is shown that the Pancharatnam phase is dependent on the entanglement in a nontrivial way. This could be used for implementing simple quantum logical circuits.

(iii) Magnetic traps are capable of holding cold neutral atoms. It is shown that magnetic traps and guides can be generated by thin wires etched on a surface using standard nanofabrication technology. These atom chips can hold and manipulate atoms located a few microns above the surface with very high accuracy. The potentials are very versatile and allows for highly complex designs, one such design implemented here is a beam splitter for neutral atoms. Interferometry with these confined de Broglie is also considered. These atom chips could be used for implementing quantum logical circuits.

Key words: Quantum optics, atom optics, complementarity, magnetic traps, laser cooling, quantum information, quantal phases.
Till min syster
List of publications

This thesis is based on the following papers, which will be referred to in the text by their Roman numerals:

I Quantum entanglement and partial path information for a double slit
   B. Hessmo and A. Mair
   Manuscript

II Quantal Phase for nonmaximally entangled photons
   B. Hessmo and E. Sjöqvist
   Accepted for publication in Physical Review A

III Trapping Neutral atoms with a wire
   A. Haase, D. Cassettari, B. Hessmo, and J. Schmiedmayer
   Accepted for publication in Physical Review A.

IV Controlling cold atoms using nanofabricated surfaces: Atom chips
   R. Folman, P. Krüger, D. Cassettari, B. Hessmo, T. Maier, and J. Schmiedmayer

V A beam splitter for guided atoms on an atom chip
   D. Cassettari, B. Hessmo, R. Folman, T. Maier, and J. Schmiedmayer
   Accepted for publication in Physical Review Letters.

VI Micromanipulation of neutral atoms with nanofabricated surfaces
   D. Cassettari, A. Chenet, R. Folman, A. Haase, B. Hessmo, P. Krüger, T. Maier, S. Schneider, T. Calarco, and J. Schmiedmayer

VII Nanofabricated atom optics: Atom chips
   K. Brugger, T. Calarco, D. Cassettari, R. Folman, A. Haase, B. Hessmo, P. Krüger, T. Maier, and J. Schmiedmayer
   Accepted for publication in Journal of Modern Optics

VIII Geometry effects in van der Waals forces
   B. Hessmo and J. Babb
   Manuscript
Other papers not included in the thesis are:

i) ”Atoms and Wires: Towards Atom Chips”,
   Accepted for publication in Journal of Quantum Electronics

ii) Comment on ”Precision Neutron Interferometric Search for Evidence of Nuclear Quantum Entanglement in Liquid H$_2$O-D$_2$O Mixtures”
   C. A. Chatzidimitriou-Dreismann, T. Abdul Redah, R. M. F. Streffer, and B. Hessmo

iii) ”Indirect Measurements and the Mirror Theorem”
   E. Brändas and B. Hessmo
   Quantum Theory in Rigged Hilbert Spaces, Eds. A. Bohm et al., Springer Verlag (1997)
Preface

There are in my opinion two main aspects of quantum optics. Light optics, such as described by Maxwell’s equations, becomes quantum optics when the particle concept (photons) is needed to explain phenomena. Matter dynamics becomes quantum optics when the wave nature (wave functions) is needed to explain phenomena. This view is strictly based on the complementarity principle [1], assuming that classical light is described by waves and classical matter is considered to be particles.

The work presented here is in the domain of quantum optics. In the first part single photons and photon pairs are studied (papers I and II) and in the second part atomic de Broglie waves are studied, or rather guides for these matter waves (papers III to VII). Paper VIII deals with the van der Waals force, which is a direct consequence of how vacuum is described in quantum mechanics.

The experimental work on photons was done in close collaboration with A. Mair at University of Innsbruck during the early part of 1998. The double slit experiment was a precursor to his thesis work, where the slits were replaced by holograms to induce an orbital angular momentum in single photons [2]. Our intention was to understand certain aspects of diffraction and complementarity. This work is presented in paper I. Much of inspiration to this complementarity experiment comes from a recent realization of a Heisenberg microscope done by B. Dopfer [3].

The work on atoms was performed in the group of J. Schmiedmayer, also in Innsbruck. Magnetic guides and traps has been studied by JS since an original experiment in 1992, when hot sodium atoms where guided along a current carrying wire [4]. These experiments where continued together with J. Denschlag using laser cooled lithium atoms. They realized a $1/r^2$-potential for neutral atoms using a thin charged wire [5]. One year later, a free-standing current carrying wire was used to confine atoms in a Kepler potential. The same wire could be used to realize a side guide capable of guiding atoms along one side of the current carrying wire [6]. This work has formed the basis for the more recent atomic experiments performed in the Innsbruck group. Some of these results is a part of this thesis. The work on magnetic traps has been performed in collaboration with T. Calarco, D. Cassettari, R. Folman, A. Haase, P. Krüger, and J. Schmeidmayer.

In the context of surface mounted magnetic traps it should be noted that the work of the Innsbruck group is not unique. There was an original theoretical proposal by Weinstein and Libbrecht in 1995 [7]. Experimentally, there has been a number of beautiful experiments since, mainly from MPQ in Munich [8, 9, 10, 11], Harvard [12, 13, 14], and Boulder [15]. A personal favourite in this context is the atomic motor implemented by J. Reichel et al. consisting of a movable magnetic trap transporting atoms from one region to another [11].

The part of the thesis concerning van der Waals forces has been done in collaboration with J. Babb of Harvard-Smithsonian Center for Astrophysics. The main motivation for this collaboration was that an ongoing experiment by A. Chenet and E. Hartungen in

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1A Kepler potential has a repulsive $1/r^2$ part and an attractive $1/r$ part giving a total potential on the form $V(r) = -A/r + B/r^2$. The repulsive part comes from the angular momentum barrier and the attractive part from the attractive force. (In celestial mechanics this force is gravity.)
Innsbruck is heading for accurate measurements of the weak van der Waals force using cold atoms as probes. This experiment was also initiated by J. Denschlag’s work on charged wires [6]. Another motivation for this study is to understand what limits magnetic trapping close to surfaces, and if dispersive forces are the main problem in reaching truly small and tight traps [16].

The purpose of making small magnetic traps for neutral atoms is that this is a promising technique for performing coherent manipulation of neutral atoms. This may form extremely sensitive detectors, also these atom chips may become capable of performing quantum logic.

The topic of this thesis is centered on quantum optics, with some theoretical perspectives and some experimental applications. The research presented here is in no way complete and not even representative for this rapidly growing research field. The work on photons is quite limited to entangled states and does not consider statistical properties of quantum light, neither did any of the experiments consider the polarization of photons. The theoretical work is even more limited, it only considers some aspects of complementarity and non-maximally entangled photons. The work on atoms completely ignores many of the last years’ achievements in the field of Bose-Einstein condensation. Also for the atom guides and traps considered, many theoretical considerations are missing. I must apologize for all the topics that should have been treated, but are not present in this thesis. Some things are intentionally left out for space and time reasons, some things are unintentionally left out because of lack of knowledge.

I thank A. Mair for all the wonderful hours we spent in the photonics lab, and the hours spent in front of the computer together. I am also grateful to the people of Institut für Experimentalphysik, both visitors and permanent physicists for the many illuminating discussions on photonics, especially worth mentioning are D. Bouwmeester, M. Horne, C. Simon, H. Weinfurter, A. Zeilinger, and M. Zukowski. Also, I thank AZ so much for inviting me to Innsbruck at the first occasion. In Innsbruck I also had the luck to meet J. Schmiedmayer with whom I since have had nice (at least for me) collaboration with. It is difficult to express how stimulating this collaboration has been, nearly all ideas presented in papers III to VIII are either directly from JS or has their origin in some discussion with him. His importance for my scientific training cannot be underestimated. In many aspects he has been one of my thesis advisors (together with E. Brändas and O. Goscinski). In this collaboration I also had the pleasure to work with T. Calarco, D. Cassetari, A. Chenet, R. Folman, A. Haase, and P. Krüger either in the lab, or in front of the computer, or fist-fighting in the library over some physics problem. A very special thank to D. Cassetari for introducing me to the experimental work in such a nice way.

In Innsbruck there was a constant flux of visitors, one of them was special. I thank J. Babb for teaching me how to think about van der Waals forces, and for the great hospitality he has shown me during my visits to his group. I also thank him for a nice collaboration.

Thanks to G. Björk, H. Blom, M. Bourennane, A. Karlsson, and J. Söderholm from the quantum optics group in Stockholm for making me feel always welcome there. Uppsala is full of very special people as well, they make daily work ”at home” a pleasure. Many thanks to E. Sjöqvist with whom paper II was written during our many travels. I hope
this collaboration will continue now when we are geographically closer to each other. I am most grateful to M. Andersson for giving me a highly effective and very useful piece of software, and for being a great officemate. Other people who have taught me much during my time at the department are: J. Brännlund, H. Carlsen, E. Engdahl, M. Ericsson, S. Jonsell, and A. Runehov. I also thank the astronomer E. Olsson for many interesting discussions. I thank A. Dreismann and E. Karlsson for allowing me to participate in their research on decoherence and for teaching me countless of physics.

I thank my thesis advisors E. Brändas and O. Goscinński for giving me plenty of freedom during my time as their student and always encouraging me in times of need. Finally I thank two of my best friends P. Andersson and L. Thalmann for constantly asking the simple questions with the difficult answers.

B. Hessmo
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Chapter 1

Important concepts in quantum optics

In quantum mechanics the concepts of complementarity and entanglement are of central and fundamental importance. These two concepts have no classical counterparts, therefore all physics based on these two principles is of purely quantum nature\(^1\).

1.1 Complementarity

The limitation of language and logic has been studied by philosophers since Aristotle. When language is used to describe Nature this limitation is called complementarity, defined and probably best explained by Niels Bohr.

**Complementarity:** In 1934 Niels Bohr [1] stated that:

*Any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena.*\(^2\)

Complementarity manifests itself in, for example, Heisenberg’s uncertainty principle and the wave-particle duality. Heisenberg’s uncertainty principle is of significant importance for quantum measurements. The wave-particle duality may be viewed as the only justification for a wave equation for matter. This justifies why the Schrödinger equation describes dynamics of matter. Historically, the complementarity principle was first discussed in the context of Young’s double slit experiment. In that experiment complementarity is applied to path information and interference. Namely, if it is in any way possible to tell which path (extract path information) particles travel through the slits, then there will be no interference between the two paths. Also the opposite is true, if interference is observed it will be impossible to tell which path the particles traveled.

Partial path information is available if it is possible to tell the path with some probability. Then there will be some interference, but the visibility [18] will be reduced depending

\(^1\)Cannot be explained using classical physics.

\(^2\)For example, in the Stern-Gerlach experiment [17] a particle’s spin is measured along a certain direction in space. For this a magnetic field gradient is needed in that direction. To measure spin in another direction, another gradient is needed. These two classical setups are not possible to realize simultaneously.
1.2 Entanglement

on how much information it is possible to extract. To clarify this, consider two narrow slits located a distance $d$ from each other with probability amplitudes $a_1$ and $a_2$ for photons to pass through slit 1 and 2, respectively. The pattern collected at a screen located distance $L$ far behind the two slits is given by

$$P \propto \left| a_1 e^{ikL_1} + a_2 e^{ikL_2} \right|^2 \propto 1 + 2a_1 a_2 \cos \left( \frac{kdL}{L} \right), \quad (1.1)$$

where $k = \frac{2\pi}{\lambda}$ is the wave number of the photon, $L_1$ is the distance the particle travels from the source to a certain point $x$ on the screen when passing through slit 1 and $L_2$ is the distance when passing through slit 2. Path information is defined as

$$D = |a_1|^2 - |a_2|^2 \in [0, 1]. \quad (1.2)$$

This is a measure of the distinguishability between the two paths first introduced by Jaeger et al. [19]. The visibility as defined by Michelson [18] is given by:

$$V = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} \in [0, 1]. \quad (1.3)$$

If the pattern is described by Eq. (1.1) the visibility becomes $V = 2|a_1 a_2|$. This gives the relation:

$$D^2 + V^2 = 1. \quad (1.4)$$

The visibility is maximal only when the two amplitudes satisfy $|a_1| = |a_2|$. In all other cases the visibility is less than unity. Equation 1.4 establishes a complementarity relation between visibility and path distinguishability.

1.2 Entanglement

Entanglement requires the (classical) superposition principle to occur, but is not a consequence of it. With entanglement defined in the following way

Entanglement: Entanglement is a relation of quantum nature between different degrees of freedom.

classical correlations are excluded. The concept of entanglement is best understood in terms of an example. Consider a quantum state vector describing the polarization of a two-photon state:

$$|\Phi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2). \quad (1.5)$$

where the subscripts 1 and 2 refer to the different photons and $H$ ($V$) refers to horizontal (vertical) polarization of the photons. This quantum state may be interpreted as describing:

3Technically: Classical correlations can be expressed with a separable density matrix. For entangled particles this is not possible.
"Photon 1 and photon 2 are found with the same polarization."

This statement can be justified by noting that a measurement of polarization on the photons in the horizontal-vertical basis (with eigenstates $|H\rangle_1$ and $|V\rangle_1$) will with probability unity give the same results. This is independent on the choice of polarization basis. By transforming to any other polarization basis the state will have the same generic form, for example a transformation into the basis of circular polarization gives

$$\frac{1}{\sqrt{2}} (|R\rangle_1 |R\rangle_2 + |L\rangle_1 |L\rangle_2). \quad (1.6)$$

Also in this case, the polarizations of the two photons are the same. Ignoring the fact that we have a two-photon state and consider only one of the two photons we would obtain a random result, described by the reduced density matrix $\rho_1 = \frac{1}{2} (|H\rangle \langle H| + |V\rangle \langle V|)$ yielding no information\(^4\).

In the above example there is perfect entanglement between the degrees of freedom for photon 1 and photon 2. It is also possible to have imperfect, or partial entanglement between the two systems. For instance such a state may have the form

$$|\Phi\rangle = \frac{1}{\sqrt{2}} |H\rangle_1 (\cos \alpha |H\rangle_2 + \sin \alpha |V\rangle_2) + \frac{1}{\sqrt{2}} |V\rangle_1 (\sin \alpha |H\rangle_2 + \cos \alpha |V\rangle_2). \quad (1.7)$$

Now only partial relations between the properties of photon 1 and photon 2 exist. For example, by detecting photon 1 in the state $|H\rangle_1$ then photon 2 is projected onto the state $\cos \alpha |H\rangle_2 + \sin \alpha |V\rangle_2$, providing a partial relation between the photons, which may be interpreted in the following way

"If photon 1 is found to be vertically polarized then photon 2 has probability $\cos^2(\alpha)$ to be found in the same polarization."

The partial entanglement is not independent of the basis in the same way as the maximally entangled state\(^5\). It also reduces to a completely classical relation (for $\alpha = \pi/4$) when Eq. (1.7) becomes a product state. Entanglement is in certain situations responsible for many counterintuitive effects:

**Bell’s Theorem:** It should be noted that the wave function described by Eq. (1.5) says nothing about the distance between photon 1 and photon 2, they should be found with the same polarization regardless of distance between them. Assuming local\(^6\) realism\(^7\) it is possible to derive Bell’s inequality [20] (not assuming quantum mechanics). Applying this inequality to quantum mechanics one finds a contradiction. Experiments [21, 22] however support quantum mechanics, not local realism. Even stronger arguments against local realism are found using several particles [23].

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\(^4\)Maximal entropy

\(^5\)Technically: A definite Schmidt form exists for these states, but not for maximally entangled states.

\(^6\)Locality: If two systems do not interact with each other, no real change can take place in the second system in consequence of anything that may be done to the first system.

\(^7\)Realism: If a physical quantity can be predicted with probability one, that quantity is considered real.
Schrödinger’s cat: It is possible to imagine entanglement between something small, say a radioactive nucleus, and something macroscopic, say a cat. Assume that a nuclear decay opens a container filled with poison that kills the cat. If the system $|\text{nucleus}\rangle |\text{cat}\rangle$ is isolated from the environment and left alone for a while, the nucleus will evolve into a superposition between $|\text{no decay}\rangle$ and $|\text{decay}\rangle$. These states are however entangled with the cat, yielding the state:

$$|\text{no decay}\rangle |\text{live cat}\rangle + |\text{decay}\rangle |\text{dead cat}\rangle \quad (1.8)$$

This puts the cat in a superposition between alive and dead. The cat is therefore neither dead nor alive. Some of the mystery of this paradox is resolved if one accepts a difference between macroscopic and classical. Experimentally, Schrödinger cats has not been found yet. However, a recent experiment by Arndt et al. [24] show interference between fullerene molecules, which in many ways behave like macroscopic objects.

### 1.3 Photonic example

Photons are suitable particles to use in experiments probing complementarity and entanglement. They are easy to detect and produce, also they are significantly easier to transport and manipulate than other particles (see chapter 3 for transport of atoms). Here, an experiment described in detail in paper I probing some features of complementarity and entanglement will be outlined.

Certain nonlinear optical materials have the property that they may "down-convert" one blue photon (the pump photon in Fig. 1.1) into a pair (or possibly several) of entangled red photons. Energy and momentum must be conserved in this process [25], so if one of the red photons (the idler photon) leaves the nonlinear material in a specific angle with respect to the propagation of the blue photon, it is possible to predict with certainty in which direction the other red photon (the signal photon) propagates. This is illustrated in Fig. 1.1. One application for these entangled photon pairs is to study the complementarity principle in double slit interference. A double slit is introduced in the path of the signal photons, with a detector recording the interference pattern behind the slits, while measurements are performed (or not performed) on the idler photon.

In the case of perfect entanglement between the two photons, a position measurement on the idler photon gives also the position of the signal photon. Note that it is actually not necessary to perform the measurement, it is sufficient to have the possibility to perform it. In the case of perfect entanglement it is possible to extract path information for the signal photon passing through the double slit. There should be no interference behind the double slit.

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8Using a very strict definition. A cat is something a child may play with. This excludes effects in liquid helium, superconductors, and crusts of neutron stars from the Schrödinger cat family.

9Such as $\beta$-Bariumborate (BBO).

10Assuming that the pump beam is sufficiently wide that diffraction effects can be neglected. For a narrow pump beam this source of the red photons will have properties similar to a pinhole diffracting light transmitted through it. This effect may be used to obtain non-perfectly entangled photons [26].
1.3 Photonic example

Figure 1.1: A typical setup for a parametric down-conversion experiment. A blue pump photon (from a laser) interacts with a nonlinear crystal and is down-converted into two red photons, one called idler and one called signal photon. These two photons are position/momentum entangled since energy and momentum must be conserved. If the transverse momentum (position) of the idler photon is $\mathbf{k}_\perp (-\mathbf{x}_\perp)$, then the transverse momentum (position) of the signal photon is $\mathbf{k}_\perp (\mathbf{x}_\perp)$.

If entanglement is partial, a position measurement on the idler can reveal partial path information for the signal photon, then there should be interference with reduced visibility. If momentum is measured on the idler photon, all knowledge of its position is destroyed since momentum is complementary to position. Also the possibilities of knowing the position of corresponding signal photon are lost\footnote{Only path information in the double slit is relevant for the interference. Therefore it is sufficiently to erase this information to obtain interference. It is not necessary to do this with a momentum measurement. For instance, to detect the idler photon on a screen behind a second double slit introduced in the idler’s path will also erase path information. See the work of Greenberger, Horne, and Zeilinger for more details on this setup\cite{27}.}, then there should be interference with full visibility behind the double slit.

Quantum states with partial entanglement may be obtained from a photon-pair source of the type described in Fig. 1.1. It will be argued that it is sufficient to adjust the size of the area producing the photon pairs\footnote{This area will correspond to the beam profile of the pump beam from the blue laser.} to achieve this. First recall the theory of diffraction, and consider one red photon passing through a rectangular slit with width $a$. The probability amplitude of finding the photon at position $x$ on a screen a distance $L$ from the aperture is given by

$$A(x) \propto \text{sinc} \left( \frac{\pi a x}{\lambda L} \right),$$

(1.9)

where $\text{sinc}(x) = \frac{\sin x}{x}$, and $\lambda$ is the wavelength of the photon. If two initially perfectly entangled photons leave the aperture from exactly the same point, they both diffract from the source reducing the correlation between them. Assume the idler photon is detected at a certain position $\hat{x}$ that in the case of perfect entanglement with certainty would tell us that the signal photon is at position $-\hat{x}$. For a small source the prediction less accurate since the position of the signal is smeared out by the diffraction amplitude $A(x + \hat{x})$. See Fig 1.2.

If the diffraction pattern is wide enough to cover both slits of a double slit inserted in the path of the signal photon, even an optimal position measurement on the idler will
1.3 Photonic example

Figure 1.2: Diffraction from the source will correlate the position of the idler photon with a diffraction pattern for the signal photon. Using the idler to extract path information for the signal photon is limited by the resolving power (as defined by Rayleigh [28]) of the aperture: $\theta = \frac{\lambda}{D}$, where $\theta$ is the angular resolution. A double slit may be beyond this resolution limit, yielding amplitudes on both slits.

not reveal with certainty the path of the signal photon. By adjusting the source size, the amount of path information stored in the idler may be controlled. Limiting the discussion to discrete positions (the two paths) a state described by Eq. (1.7) is obtained

$$| \Phi \rangle = \frac{1}{\sqrt{2}} [\text{signal should go } R]_{\text{idler}}$$

$$\times \left( \cos \alpha [\text{signal goes } R]_{\text{signal}} + \sin \alpha [\text{signal goes } L]_{\text{signal}} \right)$$

$$+ \frac{1}{\sqrt{2}} [\text{signal should go } L]_{\text{idler}}$$

$$\times \left( \sin \alpha [\text{signal goes } R]_{\text{signal}} + \cos \alpha [\text{signal goes } L]_{\text{signal}} \right)$$

(1.10)

where $L$ and $R$ refers to left and right path, respectively. The parameter $\alpha \in [0, \pi/4]$ is determined by the source size. An experimental implementation of the setup shown in Fig. 1.2 is described in paper I. In this experiment the pattern behind a double slit is measured in two different ways. First, the idler photon is ignored in the measurement (but can in principle be used to extract path information). Second, a momentum measurement is performed on the idler photon (destroying the possibility of using it for extracting path information). These measurements are performed while changing the apparent source size by varying the distance between source and double slit. The outcome of such an experiment is shown in Fig. 1.3. The one-photon interference depends strongly on the source size, whereas the interference obtained after momentum measurement on the idler photon is independent of source size. The first column of data in Fig. 1.3 shows the recorded pattern when no measurement is performed on the idler. The second column shows the measured pattern under the condition that momentum is measured on the idler photon.

The main interpretation of this experiment is that diffraction can be used to modify the degree of entanglement between the two photons. This changes the amount of available

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13Technically: A momentum measurement is performed by introducing a lens in the idler path with the detector in the focal plane of this lens.
1.3 Photonic example

Figure 1.3: The first column contain the pattern collected behind the double slit if the idler photon is ignored. The second column contains the pattern recorded in coincidence with a momentum measurement on the idler photon. For the first row of measurements the source is close to the double slit and in the last row the distance is large. The visibility for the one-photon pattern increases with distance since diffraction effects reduce the available path information. When the momentum measurement is made, the visibility is constant and high. The curves are obtained from theory assuming the visibilities shown in the upper right corner of each graph.

path information and determines the visibility of the interference pattern. It is seen in column one of Fig. 1.3 that the single photon pattern is consistent with the coherence of single-photon source of the same dimensions as the used two-photon source. This shows that diffraction destroys entanglement. Also the path information can be erased by a complementary measurement on the idler photon which otherwise could have been used to extract full or partial path information about the signal photon passing through the double slit. Another feature worth pointing out, is that the position entanglement between the two photons is degraded by propagation. This would make it harder to make a long distance experiment relying on conserved position/momentum entanglement.

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14The source used is Gaussian shaped (the beam profile of the pump beam) with $\sigma \sim 0.25 \text{ mm}$ at distances $d = 0.23 \text{ m}, 0.5 \text{ m}, 0.7 \text{ m}, \text{ and } 1 \text{ meter away from the double slit. The double slit has slit separation } 250 \mu\text{m} \text{ and slit width } 80 \mu\text{m. For these parameters one expects visibilities: 0.005, 0.5, 0.8, and 0.95 for the four graphs.}
Chapter 2

The Pancharatnam phase

In the previous chapter interference in a double slit was studied. Photons also have an internal degree of freedom (polarization) which can be used in interference experiments. The polarization of a photon is best represented on the Poincaré sphere, and manipulation of polarization is done by SU(2) transformations on this sphere. In interference experiments geometric properties of the sphere will influence the results. One such feature is the Pancharatnam phase difference.

2.1 One-photon interference

In 1956, S. Pancharatnam described the interference between two light beams in non-orthogonal states of polarization [29]. By measuring the polarization on a photon, one can partially tell from which of the two beams the photon came. This should lower the visibility of the interference pattern. Pancharatnam realized that there is an additional phase associated with the obtained interference pattern. In 1987 M. Berry [30] pointed out that this phase is closely related to the geometric phase.

To illustrate the Pancharatnam phase difference, consider a Mach-Zehnder interferometer such as the one illustrated in Fig. 2.1, where in one of the arms the incoming polarization state $|\mathbf{n}_0\rangle$ is transformed into another polarization state

$$|\mathbf{n}\rangle = e^{i\delta \cos \frac{\theta}{2}}|\mathbf{n}_0\rangle + e^{i\kappa \sin \frac{\theta}{2}}|\mathbf{n}_\perp\rangle,$$  \hspace{1cm} (2.1)

**Figure 2.1:** Mach-Zehnder interferometer: In one of the arms the initial polarization $|\mathbf{n}_0\rangle$ is transformed into $|\mathbf{n}\rangle$ by a SU(2) transformation. In the other arm a polarization independent U(1) phase shift is introduced. The output intensity is given by Eq (2.2).
with $\theta \in [0, \pi], \langle n_0 | \tilde{n}_0 \rangle = 0$, and $\delta$ and $\zeta$ real valued. In the other arm the polarization is unchanged, but an additional $\text{U}(1)$ phase shift $\chi$ is applied. The observed output intensity of the interferometer is

$$I = |e^{i\chi} |n_0 \rangle + |n \rangle|^2 \propto 1 + \mathcal{V} \cos (\chi - \Phi), \tag{2.2}$$

where $\mathcal{V}$ is the visibility of the interference pattern and $\Phi$ the Pancharatnam phase. These are defined by

$$\Phi = \text{arg} \langle n_0 | n \rangle = \delta,$$

$$\mathcal{V} = |\langle n_0 | n \rangle| = \cos \frac{\theta}{2}. \tag{2.3}$$

The visibility is reduced because it is possible to make a polarization measurement to extract path information about the photon. For orthogonal $|n_0 \rangle$ and $|n \rangle$ the visibility is zero since the polarization gives complete path information. The extra phase, $\Phi$, can be interpreted in terms of the nontrivial geometry of the projective Hilbert space (the Poincaré sphere) describing polarization.

### 2.2 Two-photon interference

For two-photon states the Hilbert space needed to fully describe polarization is richer than in the case of one-photon states since the two photons may be entangled. It is therefore expected that the corresponding expression for the two-photon Pancharatnam phase would have a nontrivial dependence also on the structure of this projective Hilbert space.

Consider a general two-photon state

$$|P\rangle = e^{-i\beta/2} \cos \frac{\alpha}{2} |n_0 \rangle_1 |m_0 \rangle_2 + e^{i\beta/2} \sin \frac{\alpha}{2} |\tilde{n}_0 \rangle_1 |\tilde{m}_0 \rangle_2, \tag{2.4}$$

where $|n_0 \rangle_1$ and $|\tilde{n}_0 \rangle_1$ is the initial polarization basis for photon 1 and $|m_0 \rangle_2$ and $|\tilde{m}_0 \rangle_2$ the initial basis for photon 2. By transforming this into a new two-photon state $|P'\rangle$ by means of local $\text{SU}(2)$ transformations on the two photons one obtains

$$|P'\rangle = e^{-i\beta/2} \cos \frac{\alpha}{2} |n \rangle_1 |m \rangle_2 + e^{i\beta/2} \sin \frac{\alpha}{2} |\tilde{n} \rangle_1 |\tilde{m} \rangle_2, \tag{2.5}$$

where $n$ and $m$ represent the new basis. Repeating the calculation described in Eq. (2.2) one finds:

$$I \propto |e^{i\chi} |P\rangle + |P'\rangle|^2 \propto 1 + \mathcal{V} \cos (\chi - \Phi), \tag{2.6}$$

From Eq. (2.4) and Eq. (2.5) we obtain the two-photon Pancharatnam phase difference $\Phi$ by writing $\langle P | P' \rangle = \mathcal{V} \exp (i\Phi)$. The phase $\Phi$ and interference amplitude $\mathcal{V}$ are

$$\Phi = \text{arg} \langle P | P' \rangle = \arctan \left( \frac{Y}{X} \right),$$

$$\mathcal{V} = |\langle P | P' \rangle| = \sqrt{X^2 + Y^2} \tag{2.7}$$
with

\[
X = \mathcal{V}_n \mathcal{V}_m \cos(\phi_n + \phi_m) + \sqrt{1 - \mathcal{V}_n^2} \sqrt{1 - \mathcal{V}_m^2} \sin \alpha \cos(\beta + \phi_n + \phi_m),
\]

\[
Y = \mathcal{V}_n \mathcal{V}_m \cos \alpha \sin(\phi_n + \phi_m)
\]  

(2.8)

the real and imaginary part, respectively, of \( \langle P | P' \rangle \). Here \( \phi_n = \text{arg}(n_0 | n) \), \( \phi_m = \text{arg}(m_0 | m) \), \( \mathcal{V}_n = |\langle n_0 | n \rangle| \), and \( \mathcal{V}_m = |\langle m_0 | m \rangle| \) are single-photon phases and amplitudes.

As expected we see that the structure of the two photon state Eq. (2.4) is present in the expression for the two-photon Pancharatnam phase. An experimental setup useful for studying this phase together with some interesting special cases are found in paper II. Here one special case is worth mentioning: If an SU(2) transformation is performed on only one of the two photons. Performing the transformation on say photon \( n \) Equation (2.7) gives the Pancharatnam phase

\[
\Phi = \arctan(\cos \alpha \tan \phi_n),
\]

(2.9)

where \( \alpha \) is the degree of entanglement and \( \phi_n \) is determined by the SU(2) transformation. The visibility of the interference pattern is given by

\[
\mathcal{V} = \mathcal{V}_n \sqrt{1 - \sin^2 \alpha \sin^2 \phi_n}
\]

(2.10)

where \( \mathcal{V}_n \) is the one-photon visibility. When \( \mathcal{V}_n = 1 \) and \( 0 < \alpha \ll 1 \) the visibility remain high independently of \( \phi_n \). When \( \phi_n \) assumes values below but close to \( \pi / 2 \) the Pancharatnam phase is \( \pi / 2 \). For \( \phi_n \) above but close to \( \pi / 2 \) the phase jumps to \( -\pi / 2 \). This may be used to implement a quantum gate for non-maximally entangled photons. This is described in more detail in paper II and in section 5.1.
Chapter 3
Atom traps and guides

Complementarity when put in the form of the wave particle duality expresses that particles, such as atoms, behave also as waves. We therefore expect that it is possible to perform wave-optics experiments with atoms. The dynamics of atoms is given by the Schrödinger equation, instead of the Maxwell’s equations describing photons. This has the advantage that atoms may be influenced by external potentials and may interact with each other much stronger than photons do. Also, atomic velocities can be changed over a quite large range.

The wavelength of an atom is given by its de Broglie wavelength: \[ \lambda_{dB} = \frac{\hbar}{p} \], where \( p \) is the momentum of the atom and \( \hbar \) Planck’s constant. To achieve long wavelengths it is desirable to work with as slow (cold) atoms as possible. For optics experiments, this poses a problem: A cold horizontal beam will fall because of gravity, and if directed along the field of gravity it will accelerate. One way to deal with this is to guide the atoms in tailored potentials analogously to fiber optic guiding of light. We may achieve this in some different ways.

**Magnetic potentials:** An atom with spin interacts with a magnetic field. The potential arises from the Zeeman shift of the internal levels given by: \( V_{\text{mag}}(x) = -\vec{\mu} \cdot \vec{B}(x) \), where \( \vec{\mu} \) is the magnetic moment of the atom and \( \vec{B}(x) \) the local magnetic field. In an adiabatic limit\(^2\) the potential becomes \( V(x) = \mu_B g_F M_F B(x) \) where \( \mu_B \) is the Bohr magneton, \( g_F \) the Landé factor\(^3\), and \( M_F \) is the magnetic quantum number of the hyperfine state. Very versatile state dependent potentials may be designed using this [31, 32].

---

\(^1\)Both wave properties and particle properties are needed to elucidate phenomena, these two concepts preclude each other.

\(^2\)A projection of the magnetic moment on the direction of the local magnetic field \( B(x) \) does not change during atomic motion in a non-uniform magnetic field.

\(^3\)The Landé factor for an atom with nuclear spin \( I \), orbital angular momentum \( L \), spin \( S \), and total angular momentum \( J \) is given by

\[
g_F = \left( 1 + \frac{J(\!J+1) - L(L+1) + S(S+1)}{2J(\!J+1)} \right) \frac{F(F+1) + J(\!J+1) - I(I+1)}{2F(F+1)}
\]

where \( F = J + I \).
Electric potentials: All atoms have an electric polarizability $\alpha$. This gives a potential given by the level shift due to the quadratic Stark effect: $V_\alpha(x) = -\frac{1}{2} \alpha E^2(x)$, where $E$ is the local electric field. This potential may also be used to trap atoms [32]. Experimentally this has been realized using hollow (blue detuned) laser beams repelling the atoms, or attracting (red detuned) laser beams.

Dispersion potentials: The Casimir-Polder interaction [33, 34] between a metal of a suitably chosen geometry and an atom may realize a trapping potential. This type of interaction relies on focusing of virtual photons thereby modifying the Casimir-Polder interaction. See [35, 36] for details. In general dispersion forces are of destructive nature making it more difficult to trap atoms near surfaces. Also, if trapping can be realized at all, it will be hard to modify the potential.

3.1 Magnetic traps and guides

Many experiments using magnetic trapping of atoms have been performed the last decade. The standard technique for producing a Bose Einstein condensate (BEC) is to load atoms into a magnetic trap from a magneto optic trap, evaporatively cool them until condensation occurs. For information on condensate physics, see [37]. The magnetic traps used in these experiments are normally formed by either electromagnetic coils, or permanent magnets if switching is not required.

Figure 3.1: a) A current carrying wire and an external bias field cancel out at some point. A two-dimensional magnetic quadrupole centered at this point is obtained. This implements a guide for atoms in a low field seeking hyperfine state. b) The wire may be mounted on a surface making the wire robust. An Ioffe-Pritchard guide is obtained when an additional bias field is added along the wire.

Here (and in papers IV to VII) surface mounted magnetic wire traps and guides suitable for miniaturization are studied. The main advantage with surface mounted traps is that robust and precise devices are feasible down to a mesoscopic scale using standard nanofabrication technologies.

A magnetic wire guide for atoms was first realized in 1992 in an experiment by Schmiedmayer who sent a beam of sodium atoms along a current carrying wire [4]. This
guide traps atoms in a high field seeking hyperfine state. The atoms are pulled closer to the wire by the magnetic interaction until the angular momentum barrier prevents them from approaching the wire further, this realizes a Kepler potential, $V(r) = -\frac{\Delta}{r} + \frac{\mu}{r^2}$ for the two dimensional motion around the wire. One obstacle for generalizing this type of potential is that local magnetic field maxima are impossible to obtain in free space (Ernshaw’s theorem). Atoms will be pulled towards any of the wires generating the field. Therefore it is significantly easier to control the potential for low field seeking atoms. These are attracted to regions of low magnetic field, such as the center of a quadrupole field where the field drops to exactly zero in the center, or a Ioffe-Pritchard field where the field is non-zero in the center.

One way to obtain a quadrupole field is to apply a homogeneous magnetic field perpendicular to a current carrying wire [38]. The field from the wire is circular around it and drops in intensity inversely proportional to the distance (Biot-Savart’s law). At some point the homogeneous field and the wire field cancel out giving a field minimum where atoms may be trapped. The two fields cancel out at position

$$r_0 = \left(\frac{\mu_0}{2\pi} \right) \frac{I_w}{B_b},$$

where $I_w$ is the current in the wire $B_b$ the homogeneous bias field. At this point the gradient of the field is\(^6\)

$$\frac{dB}{dr} \bigg|_{r_0} = \left(\frac{2\pi}{\mu_0} \right) \frac{B_b^2}{I_w}.$$  

If there is an additional field $B_{ip}$ along the wire a Ioffe-Pritchard configuration is obtained. The gradient is not suitable for describing such a guide. Around the minimum the potential is harmonic described by a frequency \(\omega\)

$$\frac{d^2 B}{dr^2} \bigg|_{r_0} = \left(\frac{2\pi}{\mu_0} \right)^2 \frac{B_b^4}{B_{ip} I_w^2},$$

yielding a transversal trap frequency \(\omega = \sqrt{\frac{\mu B_{ip} M_F}{m} \left(\frac{4\pi B}{x^2}\right)}\). This makes the theoretical analysis much easier since the harmonic oscillator is analytically solvable. From an experimental perspective it is also desirable to work in this regime since the (Ioffe-Pritchard) configuration suppresses unwanted transitions from trapped to untrapped hyperfine states.\(^7\) By changing $I_w$ and $B_b$ the position, gradients, and curvatures of the guide

---

\(^{4}\)A high field seeking atom is in an hyperfine state where the energy of the atom decreases with increasing magnetic field. The energy shift is given by $V = -\mu B_{ip} M_F B$ and the force on the atom is given by $F = -\nabla V = -m_F \nabla B$. A low field seeking atom will be pushed in the direction of weaker magnetic field. Low field seekers and high field seekers have magnetic quantum number of opposite sign.

\(^{5}\)A two-dimensional quadrupole field is given by $\vec{B} = (x, y, 0)$ and a two-dimensional Ioffe-Pritchard field is given by $\vec{B} = (-x, y, B_b)$.\(^\dagger\)

\(^{6}\)The gradient in the radial and the azimuthal directions are the same.

\(^{7}\)In an adiabatic limit, the spin of the trapped atom is aligned along the local magnetic field. When the atom moves nearby a point with zero magnetic field, the magnetic field may change direction very rapidly (from the atom’s reference system). This may provoke nonadiabatic Majorana transitions to other spin states.
may be controlled to high accuracy.

The wire can be mounted on a substrate forming a guide just above the surface. The support gives the wire mechanical stability and removes excess heat making the device suitable for nanofabrication. For these guides there is a favourable scaling property, lower wire current $I_w$ gives higher gradient according to Eq. (3.2). A lower current also brings the guide closer to the surface according to Eq (3.1). These scaling properties together with nanofabrication could bring guided atom optics into a one-dimensional regime if van der Waals forces were not present. The van der Waals force makes it hard to keep atoms in a magnetic guide closer than a few hundred nanometers from a nearby surface. Guiding atoms close to surfaces may become a very useful tool for studying atom-surface interactions. The magnetic guides can be transformed into magnetic traps quite easily.

![Figure 3.2: a) With a wire bent into a U-shaped structure a surface mounted quadrupole trap is obtained when an external bias field is applied. b) A Z-shaped wire and an external bias field yields a Ioffe-Pritchard field configuration.](image)

It is sufficient to terminate the guide by adding "endcaps" to it. These endcaps may be created by bending the wire into different shapes. In Fig. 3.2 two examples of traps suitable for surface mounting and nanofabrication are shown. Other design proposals may be found in an original paper by Weinstein and Libbrecht [7]. For long time magnetic trapping the Z-shaped wire is superior since it gives a Ioffe-Pritchard configuration. The U-shaped wire gives a quadrupole trap that is sensitive to Majorana transitions, it is however useful since this field is needed for a magneto optic trap.

In all the above traps and guides an external bias field was used for the confinement. Replacing the external field with fields from other surface mounted wires makes the chip self-containing with no need for external coils. The side guide mentioned above is made self-contained with one additional wire on each side of the guiding wire with currents in opposite direction with respect to the central wire. See Fig. 3.3 for surface guides. It is also possible to make surface traps using only surface mounted wires. The simplest example of such a trap is formed by an S-shaped wire which generates a Ioffe-Pritchard trap above the symmetry point when current is sent through the wire. Such a self contained trap, and another design yielding a quadrupole trap is shown in Fig. 3.4.

To use surface mounted wires for producing magnetic fields has a few clear advantages. They may be designed with far higher accuracy than traditional magnetic coils due to the precision in nanofabrication technology, they also require low currents to operate.

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8 A one dimensional system would have only one transverse state in the guiding potential.

9 It is possible to achieve high accuracy also by making very large coils of the type used in MRI. Very
3.1 Magnetic traps and guides

Figure 3.3: a) Three wires with alternating currents form a quadrupole guide above the central wire. The two wires on the sides substitute the external bias field. The thin lines show equipotential lines of the trapping potential and the arrows are magnetic field lines. b) Four wires with alternating currents also form a quadrupole guide. This quadrupole is rotated 45 degrees with respect to the one shown in a). Other designs using more wires are also feasible. See also [7, 12]

Figure 3.4: a) If current is sent through an S-shaped wire a Ioffe-Pritchard trap is obtained. The three parallel wires form a guide similar to the one in Fig. 3.3a). The guide is terminated by two endcaps formed by the two connections between the three wires. These endcaps give a field in the direction along the three wires, without dropping to zero. The field components in a plane containing the dashed lines are projected to the back and to the right of the substrate, respectively. In the right projection no field zero can be found. b) By bending the wire further the two endcaps can contribute in opposite directions with a zero field between the endcaps (indicated with the black dot in the projection). In this way a quadrupole is generated. Also here the field in the transverse direction is shown at the back. In both figures the approximate position of an atomic cloud is shown. For the quadrupole the trap is slightly displaced from the central wire.
3.1 Magnetic traps and guides

The above examples, illustrating how to make simple guides and traps, may easily be extended to much more elaborate designs implementing real optical components for atoms, like beam splitters\textsuperscript{10}, monochromators\textsuperscript{11}, and magnetic pumps\textsuperscript{12}. The main advantage is however due to the favorable scaling properties described by Eqs. (3.2) and (3.3), yielding field gradients and curvatures impossible to reach with traditional coils. Surface mounted structures may also be used to magneto-optically trap atoms close to surfaces. For further details see papers III to VII.

![Diagram of trap configurations](image)

**Figure 3.5:** a) The pyramid MOT is obtained when one single laser beam is retro-reflected by a four-sided pyramid in the centre of a magnetic quadrupole. The incoming laser beam, with $\sigma^+$-helicity, changes helicity to $\sigma^-$ after the first reflection and back to $\sigma^+$ after the second. The beam leaves the mirrors in the same direction as it entered, now counter-propagating the incoming $\sigma^+$-light. A quadrupole field (black arrows) with the same symmetry as the pyramid generates the MOT.

b) The surface MOT is generated by two trapping beams (I and II) impinging from opposite directions on a reflecting gold layer deposited on the substrate. Trap beam I (II) has helicity $\sigma^+$ ($\sigma^-$) before reflection off the chip and $\sigma^-$ ($\sigma^+$) after the reflection. This light configuration generates a MOT together with the magnetic field illustrated with the arrows. The magnetic field is obtained either by a set of external quadrupole coils, or by superimposing the field from a current carrying U-shaped wire underneath the chip with a homogeneous bias field. In both cases, the position of the atomic cloud is determined by the center of the quadrupole field. In both a) and b) only one direction is shown. Confinement in the third dimension is obtained by two additional mirrors in a) and by two laser beams along the surface in b).

**A surface MOT** The magnetic quadrupole has one important property. It can be used to generate a magneto optic trap (MOT) \cite{39} above the surface. This makes it easy to bring the atoms into the magnetic trap. In our experiments (described in paper IV-VII) large coils are precise because the errors made during their construction average out at large distances from the wires. This produces extremely homogeneous fields. In many experiments it is not very practical to use large coils.

\textsuperscript{10}A beam splitter may be formed by one guide that splits into two guides, or by two guides passing near each other allowing the atoms to tunnel between the two guides. This is described in paper V.

\textsuperscript{11}A periodic magnetic structure that match the de Broglie wavelength of the atoms acts in a similar way to an optical interference filter.

\textsuperscript{12}An atomic motor has been proposed by T. W. H"ansch. This consists of a set of moving potentials that actively transport atoms to some desired point above the surface. This device was recently demonstrated by J. Reichel \textit{et al.} \cite{11}. 
we used both external quadrupole coils, and a U-shaped wire placed underneath the chip together with an external bias field (Fig. 3.2), with four laser beams to generate a surface MOT. In addition a metal coated surface is needed. When light of $\sigma^+$ helicity is reflected from the metal surface (with a right angle between incoming and reflected beam) the helicity reverses to $\sigma^-$. This matches the quadrupole configuration. A similar scheme was invented by Lee et al. to form a MOT using only one laser beam together with a pyramid shaped mirror [40]. We needed four beams to compensate for the flat surface. In Fig. 3.5 the surface MOT and the pyramid MOT are shown.

3.2 Electric potentials

Electric potentials have been used for a long time to trap ions, see for instance the pioneering work by W. Paul [32], or more recent experiments in the context of quantum information [41]. Recently, also molecules with an electric dipole moment have been trapped using electric potentials [42]. Also, many experiments in high energy physics use electric potentials to accelerate particles. These topics will not be studied here. Instead, some applications in atom optics of the potential

$$V_\alpha = -\frac{1}{2}\alpha(\omega)E^2(\omega)$$

(3.4)

will be studied. The electric polarizability is given by $\alpha(\omega)$, and $E(\omega)$ is the local electric field oscillating with frequency $\omega$. The polarizability is present also for static electric fields. Using the classical Lorentz model

$$\alpha(\omega) = \frac{e^2}{m_e}\frac{1}{\omega_0^2 - \omega^2}$$

(3.5)

where $e$ is the electron charge, $m_e$ the electron mass, and $\omega_0$ the atomic resonance frequency one has a decent approximation of the polarizability. If higher accuracy is needed ab initio calculations give good results [43]. For zero frequency Eq. 3.5 yields a static polarizability $\alpha(0) = e^2m_e/\omega_0^2$. This determines the (approximate) potential an atoms experiences in presence of charges.

It is interesting to combine magnetic potentials of the type described in the previous section with potentials generated by static charges put on the surface. The magnetic potential depends on the internal hyperfine state of the atom whereas the electric potential is independent of hyperfine state. It is possible to combine electric and magnetic potentials in such a way that only certain hyperfine states remain trapped in a magnetic guide. A state selector may be designed using a magnetic side guide with a line charge put perpendicular to the current carrying wire. The potential from the line charge is given by

$$V_{\text{line}} = -\left(\frac{1}{2\pi\epsilon_0}\right)^2\frac{2\alpha(0)\sigma^2}{r^2}$$

(3.6)

where $\sigma$ is the line charge. By choosing the line charge in such a way that the attractive potential $V_{\text{line}}$ is weaker than the confining potential for an atom in the hyperfine state
3.2 Electric potentials

$|a\rangle = |F = 2, m_F = 2\rangle$ but also stronger than the confining potential for $|b\rangle = |F = 1, m_F = -1\rangle$, then only atoms in state $|a\rangle$ remain trapped. It is also possible to keep both states trapped by reducing the charge. (Fig. 3.2) More sophisticated designs usable in the context of quantum information are also feasible. For most of these applications only the static polarizability is needed.

**Optical traps** Using the dynamic features of the polarizability, we see from Eq. (3.5) that atoms will be attracted to high electric fields for field frequencies below the atomic resonance, and to low fields for frequencies above the resonance. A consequence of this is that a focused red-detuned laser beam may be used to trap atoms in the waist of the beam, also hollow blue-detuned beams can confine atoms in the dark region in the center of the beam. Combining these two features even more interesting applications are feasible [44].

In a Paul trap charged particles such as ions sit in an oscillating saddle potential $\Phi = \phi(t)(x^2 - y^2)$, depending on how the field vary with time stable guiding (quadrupole mass filter) may be obtained. The dynamics in the above potential with $\phi(t) = U + V \cos \Omega t$ is given by the Mathieu equations

\[
\begin{align*}
\ddot{x} + \frac{e}{m}(U + V \cos \Omega t)x &= 0 \\
\ddot{y} - \frac{e}{m}(U + V \cos \Omega t)y &= 0.
\end{align*}
\]

(3.7)

Stability is determined by $U$ and $V$. See [32] for further details. A similar situation may be realized for neutral particles using the potential described by Eq. (3.4). If blue-, and red-detuned lightfields alternate in forming a two-dimensional saddle potential the potentials become $V_{\text{blue}} = -\frac{1}{2}\alpha_b E_b^2(t)(1 + x^2 - y^2)$ and $V_{\text{red}} = -\frac{1}{2}\alpha_r E_r^2(t)(1 + x^2 - y^2)$,
3.3 Application of atom traps

After collecting atoms in a MOT atoms can be transferred to a surface trap. In the magnetic trap the atoms may be cooled further, possibly to reach quantum degeneracy. Then the atoms may be released towards other surface mounted devices in some different ways. The trap may be opened up in one end to form a guide, or the trap may be brought close

respectively\(^\text{13}\). Note that the polarizabilities have opposite signs. By choosing \(E_r^2(t) = \frac{\hbar}{\varepsilon_0 c} \sin^2 \Omega t = \frac{\hbar}{2}(1 - \cos 2\Omega t)\) and \(E_b^2(t) = \frac{\hbar}{\varepsilon_0 c} \cos^2 \Omega t = \frac{\hbar}{2}(1 + \cos 2\Omega t)\) the total potential becomes

\[
V_{\text{tot}} = V_{\text{blue}} + V_{\text{red}} = \left[ U_{\text{opt}} + V_{\text{opt}} \cos 2\Omega t \right](1 + x^2 - y^2),
\]

(3.8)

where \(U_{\text{opt}} = -\frac{\hbar k}{2}\) and \(V_{\text{opt}} = -\frac{\hbar k}{2}\). The equations of motion are the same as for a charged particle in a Paul trap. In this way an optical Paul trap is obtained. The trap illustrated in Fig. 3.7 is capable of holding Cs atoms with a temperature of 700 nK using 200 mW lasers detuned 100 nm from resonance. This is experimentally difficult to realize since the design has one major flaw, the trap size is not much larger than the de Broglie wavelength of the atoms at that temperature, so atoms may escape by tunneling. One advantage with the design is that the central part of the double slit may be used for magnetic trapping and can be used for bringing the atoms into the optical trap.

Figure 3.7: a) A schematic drawing of a Paul trap capable of holding ions. b) A schematic drawing of an optical dynamic trap capable of holding neutral atoms. Incoming coherent light diffracts in two openings, for example a double slit or two small apertures in the opening of an optical fiber. At the center of the plane holding the openings no E-field is present, the field increases in strength up to a maximum at distance \(y_0\) from the screen, after that point the field decreases again. Around \(y_0\) a quadrupole potential \((E^2 \propto 1 + x^2 - (y - y_0)^2)\) is obtained. For a slit distance of 1 \(\mu\)m and slit widths of 0.1 \(\mu\)m the quadrupole is located about 0.55 \(\mu\)m above the plane.

\(^{13}\)The coordinates are with respect to the saddle point.
to an already existing guide allowing the atoms to leak into the guide. The guide may then lead the atoms to other atom optical components.

**Beam splitters:** Magnetic guides of the type described in the previous section allows versatile trapping potentials to be designed. To realize mesoscopic atom optics certain optical components are needed. For instance, it is of interest to perform interferometry using atoms. This requires robust beam splitters. One way to implement such a beam splitter is to have one magnetic guide splitting into two guides. Another possibility is to have two guides approaching each other with a small barrier between the guides, allowing an atom in one of the guides to tunnel into the other guide with a 50% probability. The first realization has the advantage that it may operate for any transverse mode in the guide, whereas the tunneling barrier only can be adjusted well for one transverse mode.

![Diagram of beam splitters](image)

**Figure 3.8:** a) A one wire beam splitter may be constructed by dividing the incoming current between two outgoing wires. This potential is not very well-behaved close to the junction.

b) By applying a magnetic field perpendicular to a surface holding two wires with current flowing in opposite directions a guide is obtained. This guide can be split into two identical guides.

c) One single side guide may be formed between two wires. This guide splits smoothly into two side guides when the distance between the wires increase.

The easiest realization of a branching beam splitter is to send current through a Y-shaped wire and apply a bias field to the structure. Three side guides merging at a point close to the wire junction are formed. See Fig. 3.8a. This potential in not very smooth around the junction due to complicated field geometries. To realize a smoother potential it is possible to use a guide formed by two wires with current flowing in opposite directions and a bias field perpendicular to the surface holding the wires. The guide splits into two identical guides\(^{14}\) yielding a very smooth potential. This is illustrated in Fig. 3.8b. This type of beam splitter has the disadvantage that it is difficult to build an interferometer from it. Closing the two outgoing guides into one guide using a similar Y-structure makes it hard to feed current through the (circular) wire inside the interferometer. If a side guide is required it is possible to smooth out the potential by using two wires for the incoming guide. One single guide is obtained between the two wire with bias field \(B_{\text{bias}} = \frac{I_{\text{wire}}}{4\pi d^2}\), where \(d\) is the distance between the wires, and \(I_{\text{wire}}\) the current in the wires. When the two wires separate further one guide above each wire is obtained as shown in Fig. 3.8c.

\(^{14}\)The guides are identical only if there is no Ioffe-Pritchard field along the guide. Such a field would make the guides slightly different.
From an experimental perspective, the absolutely easiest one to realize is the one based on the one-wire configuration illustrated in Fig. 3.8a and in more detail in paper V. The one with the bias field from the top is more complicated to load, the one using the single minima above two wires is highly sensitive to variations in the bias field.

Figure 3.9: a) Shows a photograph of an atom chip before it is inserted to the vacuum chamber. b) The chip itself is mounted on a ceramics block containing a thick silver wire used to form the surface MOT. c) The chip is made out of GaAs coated with a 2.5 $\mu$m gold layer. The gold is removed along 10 $\mu$m etchings defining the wires. In the middle part of the chip two large U-shaped wires form a strong magnetic trap used to load a smaller magnetic trap defined by the thin wires between the two U. In the upper half a straight guide is shown. In the lower part a one-wire beam splitter is etched. d) This picture shows a CCD-image of atoms in the beam splitter. For the image a short laser pulse from the laser used to form the MOT is used to obtain fluorescence.

To send atoms through a surface mounted beam splitter the following steps are performed. A surface MOT is formed using four laser beams, two counter-propagating beams along the surface of the chip, and two beams impinging on the surface at 45 degree angle in such away that they are perpendicular to each other and to the beams along the surface. Two large external coils provide the quadrupole field necessary for the MOT. This MOT has large collecting capacity. In the second step the field from the external coils is switched off and replaced with the field from a U-shaped wire put underneath the chip superimposed with a homogeneous bias field. This forms a similar, but smaller, quadrupole field which is well aligned with the chip (Fig 3.5). Next the light is switched off and the atoms are magnetically confined in the U-field. The atoms are transfered to the chip wires by ramping up the current in the wires on the surface while ramping down the current in the large U underneath the chip. In this step few atoms are lost and the procedure can be repeated to transfer the atoms to even smaller wires carrying even less current. The atoms are in this way transfered either to a trap or to the guide leading to the beam splitter. After some period of time a CCD-image is taken. A typical image is shown in Fig. 3.9d. In paper IV plain traps are also studied using similar techniques.

15Atoms are however lost, but not because of the transfer method. During transfer to smaller traps the atoms are strongly compressed and lost because of inelastic collisions.
Interferometers: Two beam splitters can be combined to form an interferometer. This is easiest done in the following way. Atoms can be loaded into the single input guide of a Y-shaped beam splitter of the type described in Fig. 3.8a) or c). By combining the outgoing arms of the first Y-splitter with a second inverted Y-splitter, the two paths recombine to one guide again. See Fig. 3.3 for an illustration.

Sending single atoms from a point source, located on one side of the interferometer, and measuring the atomic distribution along the outgoing guide at the other side can be described by two interfering expanding Gaussian wave packets, one transported along each path. If the transverse confinement is strong, the guide might be treated as monomode, with no exchange between the different transverse levels. If there is a length difference \( \Delta L \) between the two paths, either because of an actual physical length difference, or because of an additional potential on one of the arms the outgoing probability distribution is given by

\[
P(x, t) = |\Phi(0, \sigma, x, t) + \Phi(\Delta L, \sigma, x, t)|^2
\]

where \( \Phi(L, \sigma, x, t) = \left( \frac{2}{\pi} \right)^{1/4} \sqrt{\frac{M \sigma}{2M \sigma^* + \hbar t}} \exp \left[ -\frac{1}{4} \frac{2M(x-L)^2}{2M \sigma^* + \hbar t} \right] \) and \( \sigma \) is the initial width of the wave packet. The coordinate \( x \) refers to the longitudinal position in the outgoing guide as illustrated in Fig. 3.3. Evaluating Eq. (3.9) one finds after some algebra

\[
P(x, t) = N \left( \exp \left[ -\frac{2M^2 \sigma^2 (x - \Delta L)^2}{4M^2 \sigma^* + \hbar^2 t^2} \right] + \exp \left[ -\frac{2M^2 \sigma^2 x^2}{4M^2 \sigma^* + \hbar^2 t^2} \right] + 2 \exp \left[ -\frac{M^2 \sigma^2 [(x - \Delta L)^2 + x^2]}{4M^2 \sigma^* + \hbar^2 t^2} \right] \cos \left[ \frac{\Delta L M (x - 2\Delta L) \hbar t}{2(4M^2 \sigma^* + \hbar^2 t^2)} \right] \right),
\]

where \( N \) is a normalization factor and \( M \) the mass. The two first terms are the constant background from the two wave packets, the third term is the interference contribution to the pattern. For large \( t \) the pattern is similar to that of a double slit. However, in most realistic situations this equation is very approximate. One necessary improvement is to consider a finite source. To obtain an analytical expression we assume that the single atom

\[16\] This will be the case if the atoms are condensed into the ground state, and sent through the interferometer with less kinetic energy than the transverse level spacing \( E_{\text{kin}, \text{exc}} \ll \hbar \omega_{\text{transverse}} \). If the energy is slightly higher but still in an adiabatic regime an atom initially in the ground state will couple with the first excited state. This situation may be described using Berry-Mead potentials for the two-level system formed by the two transverse states.
3.3 Application of atom traps

comes from a source with Gaussian intensity distribution $I_{\text{source}} = \frac{1}{\sqrt{4\pi\alpha^2}} \exp \left[ -\frac{x^2}{2\alpha^2} \right]$, where $\alpha$ is the width of the source (Fig. 3.3). Convoluting the intensity distribution with the pattern from a point source yields

$$P_{\alpha}(x, t) = \mathcal{N} \left( \exp \left[ -\frac{2M^2\sigma^2(x - \Delta L)^2}{4M^2\sigma^2(\sigma^2 + \alpha^2) + \hbar^2t^2} \right] ight.$$  

$$+ \exp \left[ -\frac{2M^2\sigma^2x^2}{4M^2\sigma^2(\sigma^2 + \alpha^2) + \hbar^2t^2} \right]$$  

$$+ 2\exp \left[ -\frac{M\Delta L^2\alpha^2 + M^2\sigma^2[(x - \Delta L)^2 + x^2]}{4M^2\sigma^2(\sigma^2 + \alpha^2) + \hbar^2t^2} \right]$$  

$$\times \cos \left[ \frac{\Delta LM(x - 2\Delta L)\hbar t}{2(4M^2\sigma^2(\sigma^2 + \alpha^2) + \hbar^2t^2)} \right] \right).$$  

(3.11)

Here we note that the visibility of the interference pattern is lowered due to the extra term in the exponent of the interference term. The visibility at some point along the outgoing guide is recovered with time evolution. This can be seen in the following way. An atom with short wavelength ($\lambda_{\text{dB}} \sim \sigma \ll \alpha$) originating from a random point $P$ in the source is likely to be out of phase with another point $P'$ in the source (located half a wavelength away). For long wavelengths the probability for starting at $P'$ is exponentially small since this point is most likely located far out in the exponential tail (of the function describing the source). At one given point in the outgoing guide one first finds the fast atoms with short wavelengths, then the slower arrives corresponding to longer wavelengths. This situation is also found in optics when a finite thermal light source illuminates a double slit, but since the speed of light in vacuum is independent on wavelength the coherence is of a spatial nature instead. See [25] for detailed discussions on coherence properties of light.

Experimentally, the above regime requires a cold BEC put in a very tight guide with a trap frequency well above 1 MHz. This remains an experimental challenge. In the case when the kinetic energy is above the level spacing the above interferometer works differently. Consider an interferometer with a length difference between the two paths that is equal to half the de Broglie wavelength of an incoming atom. If the kinetic energy is below the level spacing of the guide there will be destructive interference in the outgoing guide. The atom will not be transmitted. However, if the kinetic energy is larger than the level spacing there is energy available to make a transverse excitation. The atom is transmitted and excited to the first excited transverse state. Figure 3.11 shows the result of a numerical simulation illustrating the above transition using a Pseudo-spectral Split operator method [45] for propagating the wave packet.

### Scalability

Using the configuration described in and around Fig. 3.4 one can generate the quadrupole field needed for forming a surface MOT just by using one single wire without need of any external coils. Such a MOT can probably not hold many atoms due to its small size (compared to a MOT generated with external coils). But it requires very little space on the surface and the structure may be repeated many times. In Fig. 3.12 a
3.4 Prospects

Figure 3.11: Numerical simulation of an interferometer: An initial wave packet propagates towards an interferometer defined by one incoming guide that is split into two similar guides which recombine into one outgoing guide. The wave packet is initially in the transverse ground state with velocity 5 cm/s towards the interferometer. The transverse potential has trap frequency $2\pi \times 40$ kHz. The two interferometer arms have a length difference corresponding to a phase shift $\pi$.

a) An incoming wave in the transverse ground state is split by the first beam splitter in the interferometer.
b) If the kinetic energy is larger than the transverse level spacing, the phase shift forces transitions to higher transverse modes. Here the two interferometer arms are combined into the first excited transverse mode.
c) In the outgoing guide the atoms are in the first excited state.

schematic design of such a chip is shown. Care must be taken choosing materials for a self contained MOT since many semiconductors become conducting when exposed to intense light. This would short circuit the wires used to generate magnetic fields rendering the chip useless.

3.4 Prospects

Using atom chips for atom optics has some clear advantages. The well-established techniques in nanofabrication technology makes it possible to build devices with remarkable precision at all relevant length scales. Magnetic trapping is feasible down to a few hundred nanometers from a surface, where the van der Waals interaction or other heating mechanisms make the traps unstable [16]. Today’s technology reaches this limit. Therefore it is possible to use the atom chips as probes to study surfaces and their interactions. For the construction of magnetic traps only very simple wires are used, but other devices

Figure 3.12: Realization of many surface MOTs on a surface. Using the self-contained quadrupole described in Fig. 3.4 many surface MOTs can be formed on the same surface opening up for parallel experiments. a) If current flows from the upper wire through the loop leaving the chip at output a) a quadrupole is obtained in the center. This may be used to form a MOT. b) If the current leaves the chip through the dashed line a Ioffe-Pritchard trap is created. Switching to this configuration from the quadrupole is easy.
can be microfabricated. It is of considerable interest to integrate light optics on the chip to build well-aligned high efficiency detectors. These could be made out of microfabricated surface mounted cavities. Light sources themselves can also be built on micrometer scale, opening up the possibility of having a truly self-contained atom chip capable of laser cooling atoms, magnetically trap them, cool the atoms down to quantum degeneracy, perform a complex coherent operation on the atoms, detecting the outcome of that operation with high accuracy. This is however beyond present day technology, also atomic behavior on this scale remains to be understood.
Chapter 4

van der Waals forces

The van der Waals and Casimir-Polder [33, 34] forces between macroscopic objects are determined by the electromagnetic eigenmodes of the system. The attraction is temperature dependent and its strength is determined by how the free energy of the system changes as function of the distance between the objects

\[ F(R,T) = k_B T \sum_n \ln \left[ \sinh \left( \frac{\hbar \omega_n(R)}{2k_B T} \right) \right], \quad (4.1) \]

where \( k_B \) is the Boltzmann constant and \( \omega_n(R) \) are the eigenfrequencies of the system as a function of the distance. At zero temperature the van der Waals potential is given as the sum of all zero-point energies. The potential depend on the parameter \( R \) according to

\[ V(R) = \frac{\hbar}{2} \sum_n \omega_n(R). \quad (4.2) \]

The parameter \( R \) can for example be the distance between two atoms or two macroscopic dielectric bodies. Equation (4.2) then describes how the two objects attract each other. Generally, the parameter \( R \) describes any geometrical feature of the system. The geometry determines the eigenfrequencies as a function of the parameter. This is the topic of paper VIII.

4.1 The dispersive origin of van der Waals and Casimir-Polder forces

Usually it is very hard to evaluate the sums in (4.1) and (4.2). A method developed by van Kampen [46] and Ninham [47] based on Cauchy’s integration theorem makes this evaluation much simpler. One can show [46] that a sum of an analytic function \( g(\omega) \) evaluated at certain points \( \omega_n \) can be written on the following integral form

\[ \sum_n g(\omega_n) = \frac{1}{2\pi i} \int_C g(\omega) \frac{d}{d\omega} \log D(\omega) d\omega, \quad (4.3) \]
where $D(\omega)$ is zero at $\omega_n$. The integration contour $C$ must exclude the poles of $g(\omega)$ and $D(\omega)$. The function $D(\omega)$ is a secular function determining the eigenfrequencies of Maxwell’s equations in the geometry of the studied system. This function is called the dispersion function. In the next section $D(\omega)$ will be evaluated for a cylindrical geometry by matching Bessel functions on the interfaces between different dielectric media in such a way that they satisfy Maxwell’s equations. Choosing $g(\omega) = \ln \left( \sinh(\beta \hbar \omega / 2) \right)$ will yield an evaluation of Eq. (4.1). If $g(\omega) = \omega$ the temperature independent potential given by Eq. (4.2) is evaluated. For the evaluation it is convenient to integrate by parts

\begin{align*}
V(R) &= \frac{\hbar}{4\pi i} \int_C \omega \frac{d}{d\omega} \log D(\omega) d\omega \\
&= -\frac{\hbar}{4\pi i} \int_C \log D(\omega) d\omega \\
&= -\frac{\hbar}{4\pi} \int_{-\infty}^{\infty} \log D(i\xi) d\xi \\
F(R; T) &= \frac{k_B T}{2\pi i} \int_C \ln \left[ \sinh \left( \frac{\beta \hbar \omega}{2} \right) \right] \frac{d}{d\omega} \log D(\omega) d\omega \\
&= -\frac{\hbar}{4\pi i} \int_C \coth \left( \frac{\beta \hbar \omega}{2} \right) \log D(\omega) d\omega \\
&= -\frac{\hbar}{4\pi} \int_{-\infty}^{\infty} \coth \left( \frac{\beta \hbar \xi}{2} \right) \log D(i\xi) d\xi.
\end{align*}

Since the eigenfrequencies are real and positive, the contour $C$ is chosen to be along the imaginary axis in the last step. Once the dispersion function $D(\omega)$ is known the van der Waals potential can be evaluated. It should be noted that quite often there is a second integration in Eqns. (4.4) and (4.5). This integration is over free $k$-vectors for surface modes in the relevant geometry (such as a surface plasmon propagating along a cylinder).

### 4.2 Geometric effects

The function $D(\omega)$ is obtained from the boundary conditions imposed on Maxwell’s equations, therefore it is expected that the geometry of the interacting bodies strongly influence the strength of the force since the eigenfrequencies will change. This geometry dependency is unique for the van der Waals force. It is in general wrong to obtain the total potential by integrating the atom-atom potential over the relevant geometries.\(^1\)

To illustrate this, consider an example first studied by Mitchell et al. [48]. Two cylinders with radii $R_1$ and $R_2$ with dielectric constants $\varepsilon_1$ and $\varepsilon_2$, respectively are separated a distance $d$ from each other. Assume that the interaction is non-retarded (i. e. $c \to \infty$). The electric potential then satisfies the Laplace equation $\nabla^2 \Phi = 0$, which in cylindrical coordinates becomes

$$
\nabla_r^2 \phi + k^2 \phi = 0,
$$

\(^1\)Contrary to the total potential for a set of charges which can be obtained by adding each charge-charge potential.
where $\phi$ is defined by the substitution $\Phi = \phi(r, \theta) e^{ikz-\omega t}$ and $k = 2\pi n / L$ where $L$ is the length of the cylinders. The solutions to this equation are given in terms of Bessel functions. These are

$$\phi = \phi_1 + \phi_3, \quad r_1 < R_1, \quad r_2 > R_2$$

$$\phi = \phi_2 + \phi_3, \quad r_1 > R_1, \quad r_2 > R_2$$

$$\phi = \phi_2 + \phi_4, \quad r_1 > R_1, \quad r_2 < R_2$$

where

$$\phi_1 = \sum_{m=-\infty}^{\infty} A_m \frac{I_m(kr_1)}{I_m(kR_1)} e^{im\theta_1},$$

$$\phi_2 = \sum_{m=-\infty}^{\infty} A_m \frac{K_m(kr_1)}{K_m(kR_1)} e^{im\theta_1},$$

$$\phi_3 = \sum_{m=-\infty}^{\infty} B_m \frac{K_m(kr_2)}{K_m(kR_2)} e^{im\theta_1},$$

$$\phi_4 = \sum_{m=-\infty}^{\infty} B_m \frac{I_m(kr_2)}{I_m(kR_2)} e^{im\theta_2}.$$

where $I_m$ and $K_m$ are modified Bessel functions. The coefficients $A_m$ and $B_m$ are to be determined. For this it is useful to expand the Bessel functions centered on cylinder 2 in terms of Bessel functions centered on cylinder 1 using the addition theorem for cylindrical Bessel functions

$$K_m(kr_2) = \sum_{n=-\infty}^{\infty} K_{n+m}(kd) I_n(kr_1) e^{in\theta_1}$$

The boundary condition that has to be satisfied is: The normal component of the displacement field $D = \varepsilon E = \varepsilon \frac{\partial \phi}{\partial n}$ must be continuous at the cylinder surface. From Eqns. (4.7) and (4.8) the following condition are obtained for cylinder 1

$$\varepsilon_1 \frac{\partial}{\partial r_1} (\phi_1 + \phi_3) = \varepsilon_3 \frac{\partial}{\partial r_1} (\phi_2 + \phi_3)$$

(4.13)
which becomes
\[ \varepsilon_1 \frac{\partial \phi_1}{\partial r_1} - \varepsilon_2 \frac{\partial \phi_2}{\partial r_1} = (\varepsilon_3 - \varepsilon_1) \frac{\partial \phi_3}{\partial r_1}, \]  
(4.14)

where \( \varepsilon_i \) are the dielectric constants for the different media. Using the addition theorem Eq. (4.12) and the solutions in Eq. (4.11) one arrives at the condition
\[ A_n \left\{ \varepsilon_1' I_n'(k R_1) - \varepsilon_3 K_n'(k R_1) \right\} = (\varepsilon_3 - \varepsilon_1) I_n'(k R_1) \sum_{m=-\infty}^{\infty} B_m \frac{K_{n+m}(kd)}{K_m(k R_2)}. \]  
(4.15)

This equation can be put on matrix form \( \mathbf{A} = \mathbf{M} \cdot \mathbf{B} \) with
\[ M_{nm} = \frac{(\varepsilon_3 - \varepsilon_1) I_n'(k R_1) I_n(k R_1) K_n(k R_1) K_{n+m}(kd)}{\varepsilon_1 I_n'(k R_1) K_n(k R_1) - \varepsilon_3 I_n(k R_1) K_n(k R_1)}. \]  
(4.16)

For cylinder 2 the matrix equation \( \mathbf{B} = \mathbf{N} \cdot \mathbf{A} \) is obtained, where \( \mathbf{N} = \mathbf{M} \) with the difference \( 1 \rightarrow 2 \)
\[ N_{nm} = \frac{(\varepsilon_3 - \varepsilon_2) I_n'(k R_2) I_n(k R_2) K_n(k R_2) K_{n+m}(kd)}{\varepsilon_2 I_n'(k R_2) K_n(k R_2) - \varepsilon_3 I_n(k R_2) K_n(k R_2)}. \]  
(4.17)

Combining the two matrix equations one finds the condition \( (1 - \mathbf{M} \cdot \mathbf{N}) \mathbf{A} = 0 \). The allowed frequencies are determined by the secular equation
\[ D(k, \omega_n) = \det (1 - \Omega) = 0, \text{ with } \Omega = \mathbf{M} \cdot \mathbf{N} \]  
(4.18)

The zero-point energy of the system is obtained after integration over \( k \) using Eq. (4.4) and Cauchy’s theorem Eq. (4.3) yielding
\[ V(d) = \frac{\hbar}{2} \left( \frac{L}{2\pi} \right) \int_{-\infty}^{\infty} dk \left( \sum_n \omega_n(k, d) \right) = \frac{\hbar L}{2\pi^2} \int_{0}^{\infty} d\xi \int_{0}^{\infty} dk \ln |D(k, i\xi)|. \]  
(4.19)

This expression determines the short range van der Waals force between two cylinders at zero temperature, including geometry effects. For a different geometry the Bessel functions and the boundary conditions have to be replaced.

The potential obtained in Eq. (4.19) should be compared to pairwise integration of the atom-atom potential over the two cylinders. The integration will not include non-additive effects present in Eq. (4.19). This is given by
\[ V_{\text{int}}(d) = -\frac{3\hbar \omega_0 \alpha_1 \alpha_2}{4} \int \frac{1}{\left[ (x_1 - x_2 + d)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \right]^{3/2}} d^3r_1 d^3r_2, \]  
(4.20)

where the integration is carried out over the two cylinders. The material properties are contained in the polarizabilities \( \alpha_i \) and in the main transition frequency \( \omega_0 \).
4.3 A proposed experiment

It is of interest to measure how much the van der Waals interaction is influenced by geometry effects since this is one unique feature of dispersion forces. Another motive for such a measurement is that the traps described in the previous chapter are all limited by the van der Waals force. By understanding the geometry effects properly it may become possible to eliminate the destructive nature of the force, perhaps in such a way as described in the papers by Ford and Spruch [35, 36].

One possible way for measuring the force would be by measuring the effective radius of a thin wire inserted in a MOT by monitoring the decay of the MOT See Denschlag et al. for details [5]. Atoms orbiting the wire would see an angular momentum barrier in addition to the van der Waals force. This barrier (repulsive and $\propto r^{-2}$) can be adjusted by putting charge on the wire yielding an additional $r^{-2}$-potential, see Eq. 3.6. The total potential becomes

$$V_{\text{eff}}(R) = \frac{A(Q)}{R^2} + V_{\text{vdW}}(R),$$

where $A(Q)$ can be controlled by the amount of charge put on the wire. The van der Waals potential is given by $V_{\text{vdW}}(R)$. Equation (4.21) determines the effective radius of the wire.

![Figure 4.2: Geometry dependent effective radius of a wire](image)

Atoms may be reflected from the wire by the angular momentum barrier (together with the electric potential from the charge), but if the atom comes too close to the wire they will fall onto the wire because of the dominating van der Waals force. The point of no return, indicated with dashed lines in Fig. 4.2, defines the effective radius of the wire. The decay rate of the MOT is determined by the absorbing diameter, $2R_{\text{eff}}$, of the wire.
Chapter 5

Quantum computing

To realize a quantum Turing machine [49] it would be required to implement robust quantum bits (qubits\(^1\)). These quantum bits constitutes the tape of a quantum Turing machine. The tape is read and written to according to some program. This manipulation must be coherent, otherwise superpositions would be destroyed. A quantum program would therefore be a sequence of unitary operators acting on the quantum states of qubits. One example of such a unitary operator is a control-NOT gate where the state of one qubit is left unchanged if a second qubit is found in state \(\psi\) and inversed if the second is found in state \(\bar{\psi}\). This can be represented in the following truth table

\[
\begin{array}{c|c|c}
| & |0\rangle & |1\rangle \\
\hline
|0\rangle & |0\rangle & |0\rangle \\
|1\rangle & |1\rangle & |0\rangle \\
\end{array}
\]

(5.1)

The qubit \(|q\rangle\) controls the evolution of qubit \(|q\rangle\) coherently. For example, if qubit 1 is in a superposition between \(|0\rangle\) and \(|1\rangle\) one has \(U_{\text{not}}(a|0\rangle_1 + b|1\rangle_1)|0\rangle_2 = a|0\rangle_1|0\rangle_2 + b|1\rangle_1|1\rangle_2\)

\[U_{\text{not}}|0\rangle_1|0\rangle_2 = |0\rangle_1|0\rangle_2\]
\[U_{\text{not}}|0\rangle_1|1\rangle_2 = |0\rangle_1|1\rangle_2\]
\[U_{\text{not}}|1\rangle_1|0\rangle_2 = |1\rangle_1|1\rangle_2\]
\[U_{\text{not}}|1\rangle_1|1\rangle_2 = |1\rangle_1|0\rangle_2\].

In paper II a proposal for such a gate is presented. This gate makes sense also classically. There are also gates without any classical correspondance like \(\sqrt{U_{\text{not}}}\). See Barenco for an overview of quantum computation[50]. For the manipulation of qubits it is also important to control the (non-classical) phase factors present in the superposition. Such a quantum gate is discussed at the end of paper VII. See also other work of T. Calarco et al. [51].

Atoms are promising candidates for implementing quantum gates since the atoms interact strongly with each other compared to photons. The atom chip described in chapter 3 and papers III-VII is a realistic candidate for implementing quantum logic using neutral atoms as qubits. For photons it is difficult to implement a quantum gate since photons interact very weakly with each other. Existing nonlinear media useful in this context are not very efficient either. In paper II it is shown that a weak photonic nonlinearity may be amplified using the two-photon Pancharatnam phase.

\(^1\)A qubit could be implemented in any two-level system. Such a quantum system can represent arbitrary superpositions of the logical states \(|0\rangle\) and \(|1\rangle\) yielding qubits on the form: \(|\psi\rangle = a|0\rangle + b|1\rangle\), where \(a\) and \(b\) are complex numbers satisfying \(|a|^2 + |b|^2 = 1\). Such a bit would represent the answer \textit{yes} or \textit{no} to one single question.
5.1 Photonic quantum gate

In section 2.2 the Pancharatnam phase difference for nonmaximally entangled photons was studied. Experimentally, the phase may be studied sending the two photons through a Franson-interferometer. Nonmaximally entangled photons can be generated by a light source invented by White et al. [52]. An experimental setup implementing this is illustrated in Fig. 5.1.

In the special case when only one of the two photons is manipulated, the Pancharatnam phase is given by Eq. (2.9)

$$\Phi = \arctan(\cos \alpha \tan \phi_n),$$

where $\alpha$ determines the degree of entanglement and $\phi_n$ is obtained from the SU(2) transformation. Note that this phase changes discontinuously for $\phi_n = \pi/2$ if $0 < \alpha \leq \pi/2$. In addition, the visibility of the interference pattern given by Eq. (2.10) can remain high for small $\alpha$. These two features may be used to amplify a conditional phase shift obtained from the interaction with a control photon in a nonlinear medium. Most photonic nonlinearities are very weak, but by using the singularity of $\tan \phi_n$ at $\phi_n = \pi/2$ one can obtain a conditional two-particle phase shift very close to $\pi$.

Assume that the nonlinear medium (See Fig. 5.2) induces a polarization shift yielding $\Delta \phi_n = 2\varepsilon$ to the signal photon if a control photon is also present in the nonlinearity, and $\Delta \phi_n = 0$ without control photon. By choosing the SU(2) transformation to be cyclic the visibility will be high, and with $\phi_n = \pi/2 - \varepsilon$, the weak nonlinearity will be enhanced significantly. The nonlinearity only has to be able to change $\phi_n$ from just below $\pi/2$ to just above $\pi/2$, then the Pancharatnam phase changes from $-\pi/2$ to $\pi/2$. Also, if the cyclic transformation is along geodesics certain types of errors can be eliminated [53]. See Fig. 5.2 for a possible realization. The large phase shift obtained from Eq. (2.9)
also exists for maximally entangled photons (See paper II), the visibility is however very low for a small nonlinearity. In the case of nonmaximally entangled photons both large phase shift and high visibility may be obtained. It is however clear that the experimental

![Diagram](image)

**Figure 5.2:** a) Amplification of a small nonlinearity: One photon (the signal photon) from the source described in Fig. 5.1 travels through its Franson loop, here realized as an imbalanced Michelson interferometer in order to cancel dynamical contributions. A small nonlinear polarization shifting medium NL (yielding $\Delta \phi_n = 2\varepsilon$) and a cyclic transformation generated by two $\lambda/4$-plates (yielding $\phi_n = \pi/2 - \varepsilon$) are placed in one of the arms. A control photon, also interacting with the nonlinearity, induces a two particle phase shift close to $\pi$ using the discontinuity of the phase in Eq. (2.9). The visibility may remain high for nonmaximally entangled photons. This implements an efficient quantum gate for photons.

b) On the Poincaré sphere: A cyclic transformation yielding large phase shift and high visibility. If no control photon is present in the nonlinearity the transformation will be along (i) yielding $\phi_n = \pi/2 - \varepsilon$ and with the control photon present along (ii) yielding $\phi_n = \pi/2 + \varepsilon$. Note that the transformation along (ii) is only approximately along a geodesic.

implementation of a quantum gate for photons using the two-photon Pancharatnam phase is nontrivial. The nonlinearities are very weak yielding a tiny $\varepsilon$. It is probably beyond present technology to align the $\lambda$-plates accurately enough to have a robust quantum gate. Atoms are much more promising for a robust implementation of quantum logic.
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