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A strain gradient enhanced model for the phase-field approach to fracture

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Phase-field modelling has been shown to be a powerful tool for simulating fracture processes and predicting the crack path under complex loading conditions. Note that the total energy of fracture in the classical phase-field formulations includes the strain energy density from the linear elasticity theory resulting in singular stresses at the crack tip. Recently, we have demonstrated that integrating the strain gradient elasticity into the conventional phase-field fracture formulations may improve the final results by alleviating the effects of a singular stress field around the crack tip [1]. The current contribution focuses on a more general formulation of strain gradient elasticity.

1 Theory and Background

The strain gradient model in Ref. [1] was one of the simplest models possible which uses a single gradient parameter. However, there are more sophisticated models available in the literature which benefit from five gradient material parameters that can predict the behavior of the continuum in more details [2]. As a continuation of our previous work on a strain gradient enhanced phase-field fracture model [1,3], we present results for a fracture problem simulated using the fourth-order phase-field model. We assume small deformations and, therefore, we define the linear strain tensor as $E_{ij} := \frac{1}{2}\left(u_{j,i} + u_{i,j}\right)$, together with its gradient, $H_{ijk} := E_{jk,i}$, with $\boldsymbol{u}$ representing the displacement field. The stored elastic energy density is given as

$$\psi_{\text{el}} = \frac{1}{2} E_{ij} C_{ijkl} E_{kl} + \frac{1}{2} H_{ijk} D_{ijklmn} H_{lmn} = \frac{1}{2} E_{ij} S_{ij} + \frac{1}{2} H_{ijk} P_{ijk}, \tag{1}$$

where Einstein's summation convention is used, $\mathbf{C}$ and $\mathbf{D}$ are the fourth-order and sixth-order material tensors, and stresses and double-stresses are denoted by $\mathbf{S}$ and $\mathbf{P}$, respectively. For an elastic isotropic material, we can define the material tensors using seven material coefficients, c_1 to c_7: two Lamé constants and five gradient parameters (see also [3]). As for the total energy of the phase-field fracture part, we use the fourth-order formulation given by

$$\Psi_{\text{frac}} = \int_\Omega \left[s^2\,\psi_{\text{el}} + \mathcal{G}_{\text{c}} \left(\frac{(1-s)^2}{4\kappa} + \frac{1}{2}\kappa|\nabla s|^2 + \frac{1}{4}\kappa^3 (\Delta s)^2 \right) \right] \mathrm{d}V - \int_\Omega \mathbf{b} \cdot \mathbf{u}\, \mathrm{d}V, \tag{2}$$

where s is the phase-field parameter ($s = 1$: intact material), $\mathcal{G}_c$ represents the critical strain energy release rate, κ is the length-scale parameter for the crack, and $\mathbf{b}$ denotes the vector of body forces.

2 Numerical results

The structure is clamped at the bottom edge; and at the top edge, a displacement of $\bar{\boldsymbol{u}} = (0.1\,\text{mm}, 0.1\,\text{mm})$ is applied stepwise as a function of time. The model dimensions as well as the boundary conditions are provided in Fig. 1. The material properties and other related parameters are taken from Ref. [4]: $c_1 = 202$ MPa, $c_2 = 431$ MPa, $c_3 = 0.2$ N, $c_4 = 0.2$ N, $c_5 = 0.36$ N, $c_6 = 0.52$ N, $c_7 = 0.36$ N, $\mathcal{G}_c = 0.09$ N/mm. In addition, we set $\kappa = 0.02$ mm. The model is discretized using bi-quadratic NURBS elements providing the required C^1-continuity of the basis. The corresponding results are plotted in Fig. 2.

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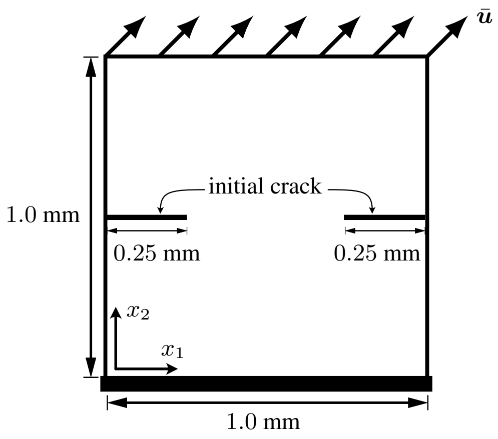

Fig. 1: Model dimensions, boundary conditions and the initial crack position.

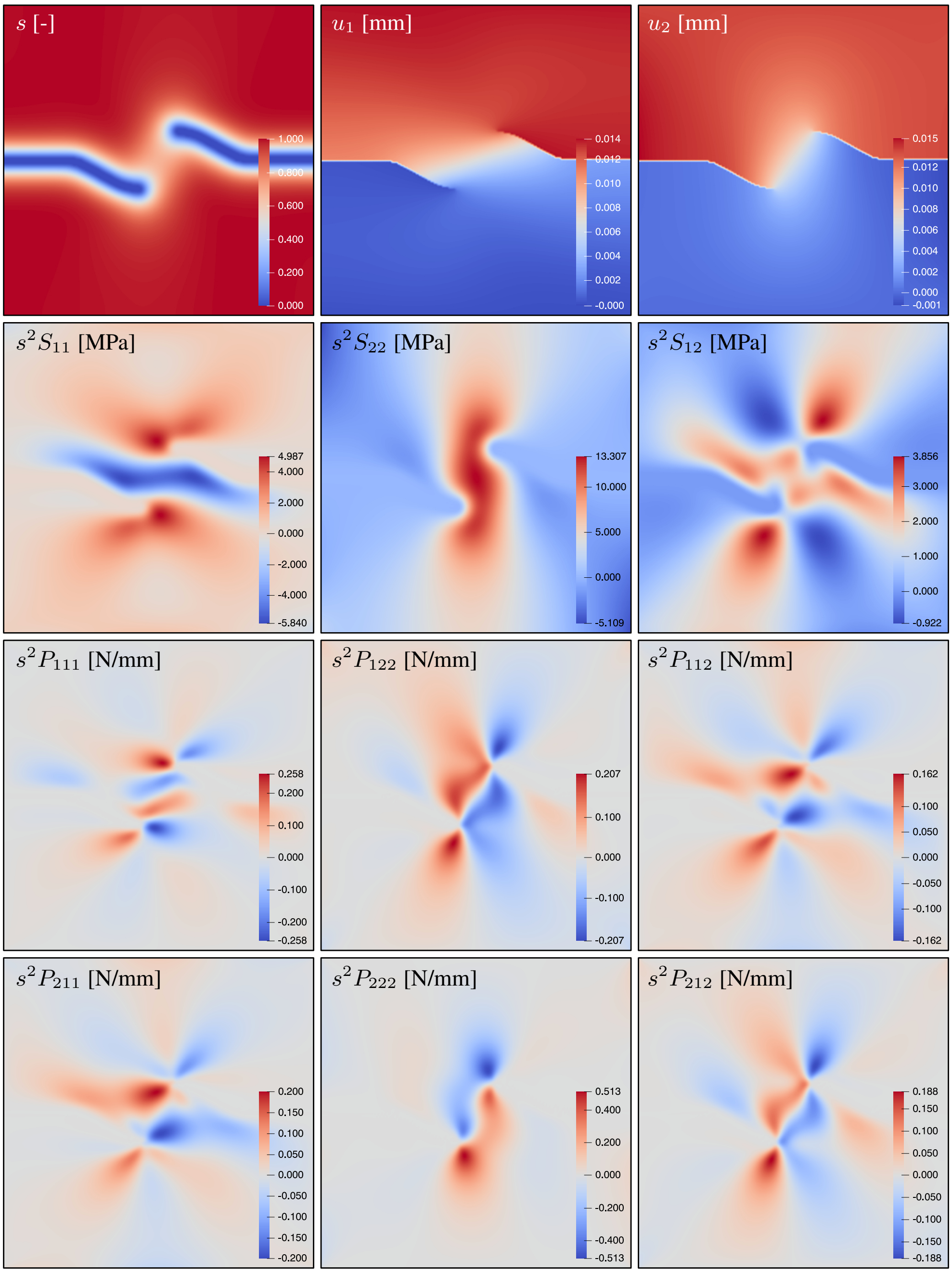

Fig. 2: Contour plots of the phase-field variable, displacement field, and effective (degraded) stress and double-stress components.

