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Drift-Type Waves in Rotating Tokamak Plasma

BY

ELINA ASP

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Abstract
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The concept of energy production through the fusion of two light nuclei has been studied since the 1950's. One of the major problems that fusion scientists have encountered is the confinement of the hot ionised gas, i.e. the plasma, in which the fusion process takes place. The most common way to contain the plasma is by using at magnetic field configuration, in which the plasma takes a doughnut-like shape. Experimental devices of this kind are referred to as tokamaks. For the fusion process to proceed at an adequate rate, the temperature of the plasma must exceed 100,000,000C. Such a high temperature forces the plasma out of thermodynamical equilibrium which plasma tries to regain by exciting a number of turbulent processes. After successfully quenching the lager scale magnetohydrodynamic turbulence that may instantly disrupt the plasma, a smaller scale turbulence revealed itself. As this smaller scale turbulence behaved contrary to the common theory at the time, it was referred to as anomalous. This kind of turbulence does not directly threaten existents of the plasma, but it allows for a leakage of heat and particles which inhibits the fusion reactions. It is thus essential to understand the origin of anomalous turbulence, the transport it generates and most importantly, how to reduce it. Today it is believed that anomalous transport is due to drift-type waves driven by temperature and density inhomogeneities and the theoretical treatment of these waves is the topic of this thesis.

The first part of the thesis contains a rigorous analytical two-fluid treatment of drift waves driven solely by density inhomogeneities. Effects of the toroidal magnetic field configuration, the Landau resonance, a peaked diamagnetic frequency and a sheared rotation of the plasma have been taken into account. These effects either stabilise or destabilise the drift waves and to determine the net result on the drift waves requires careful analysis. To this end, dispersion relations have been obtained in various limits to determine when to expect the different effects to be dominant. The main result of this part is that with a large enough rotational shear, the drift waves will be quenched.

In the second part we focus on temperature effects and thus treat reactive drift waves, specifically ion temperature gradient and trapped electron modes. In fusion plasmas the α-particles, created as a by-product of the fusion process, transfer the better part of their energy to the electrons and hence the electron temperature is expected to exceed the ion temperature. In most experiments until today, the ion temperature is greater than the electron temperature and this have been proven to improve the plasma confinement. To predict the performance of future fusion plasmas, where the fusion process is ongoing, a comprehensive study of hot-electron plasmas and external heating effects have been carried out. Especially the stiffness (heat flux vs. inverse temperature length scale) of the plasma has been examined. This work was performed by simulations done with the JETTO code utilising the Weiland model. The outcome of these simulations shows that the plasma response to strong heating is very stiff and that the plasma energy confinement time seems to vary little in the hot-electron mode.

Keywords: fusion, plasma, tokamak, rotation, drift, ship, ITG, TE, Landau, Te/Ti, hot-electron, confinement, stiffness, Weiland

Elina Asp, Department of Astronomy and Space Physics, Box 515, Uppsala University, SE-75120 Uppsala, Sweden

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“The most exciting phrase to hear in science,
the one that heralds new discoveries, is not
‘Eureka!’ (I found it!) but ‘That's funny ...’”

Isaac Asimov

Contents

Introduction

"*Fascinating*."
Spock (Star Trek)

In the diverse research area of plasma physics, ranging from the fundamental research of space physics to the applied research of sputtering, the fusion research presented in this thesis lies somewhere in the middle. With the final goal of fusion, to develop a fully operational nuclear power plant, the research is in some sense applied but the understanding of the physics that governs the fusion plasma is often of a very fundamental character.

The development of a power plant of a totally new concept has many aspects. Not only do you need new technical solutions for operational purposes, but you also have to have a clear understanding of how to maintain and control the nuclear process, in this case the fusion of deuterium and tritium nuclei. The latter involves controlling and confining an extremely hot ionised gas, i.e. a fusion plasma.

Invariably when you heat a fusion plasma, the edge will be cooler than the interior and as the plasma resides in a vacuum chamber to minimize impurities, the plasma density is also higher in the interior than at the edge. From the world around us, we know that whenever we have temperature or density inhomogeneities nature tries to smooth them out by some sort of mixing. Just think of the convection occurring in a boiling saucepan, the mixing the cold fluid on top with the hot fluid at the bottom.

The steeper the variations of the inhomogeneities, the more violent processes occur to even them out. The force of an avalanche sliding down a mountain is immense. The avalanche can teach us something more about the character of these smoothening processes. Not all snow covered mountains sport avalanches, only those on which the layer of snow has a critical steepness or, from a more mathematical point of view, a critical gradient. From the above we can draw the conclusion that the mixing processes do not take place at random, but they are excited by large enough inhomogeneities. In a sense you could venture to say that nature detests gradients.

From where does nature's aversion for gradients come? The answer to that question lies in nature's tendency to always assume a state in which the total energy is a minimum. This is the fundamental law that governs everything from the erosion of mountains to the decay of radioactive atoms.

With this in mind, we can, without difficulty, understand that the confinement of a fusion plasma is not an easy task. To force two positively charged nuclei so close together that they can overcome the repelling forces pushing them apart and fuse, requires an immense pressure. The fusion processes occurring in stars have the entire mass of the star to pressurise the core to this level. Here on earth we have to make use of other means to confine and pressurise the particles. The most common method of fusion experiments of today is confinement by magnetic field configurations and the most successful so far is the tokamak. The largest fusion experiment in the world, JET situated outside of Oxford, UK, has such a configuration.

Using the knowledge of elementary electromagnetic field theory, that charged particles gyrates around magnetic field lines and thus become trapped, the plasma is moulded into the shape of a doughnut by a magnetic field. This kind of device, with the toroidal geometry of a doughnut, is referred to as a tokamak. The word tokamak originates from Russia and is an abbreviation for toroidal magnetic bottle. This 'bottling up' of the plasma comes with a drawback. As the plasma is constrained by the magnetic field it is also forced out of equilibrium and plasma inhomogeneities arise. Subsequently, the plasma responds by exciting all kinds of turbulent processes to regain its equilibrium state.

To sustain the fusion reaction, it is not enough to simply keep the plasma in place; it also has to be hot and dense enough. For reference, the core of the sun has a temperature of about 15,000,000C and with the entire mass of the sun to compress it, the core is too dense to make it possible to reproduce its density in a fusion reactor here on earth. To increase the pressure in an ideal gas you have two options, either you enhance the density or you raise the temperature. For feasible fusion reactor densities we have to raise the plasma temperature up to around 100,000,000C to overcome the repulsion of the fusing nuclei. With the previous account of nature's aversion to steep inhomogeneities, we now realise it is a major task to maintain a stable plasma with a core temperature of 100,000,000C falling off to a few hundred degrees at the edge one or two meters away.

Normally, the instabilities that arise in fusion plasmas are divided into two major fractions depending on the size of the turbulence excited. Magnetohydrodynamic (MHD) instabilities have a size comparable to the dimension of the plasma and these instabilities were encountered at the earliest phases of experimental fusion research in the 1950's. As time passed, a greater understanding of how to quench MHD activities hazardous to plasma confinement emerged. To the scientist astonishment, there seemed to be some other phenomenon occurring in the plasma which enhanced transport. According to MHD theory the confinement should increase with higher magnetic field, which it did up to a point and then it started to deteriorate again[1]. Since, at the time, no one could explain this sudden onset of turbulence it was called anomalous. Today we ascribe this anomalous turbulence to small scale drift-type waves. This is the second of the two major fractions of plasma turbulence and as it has a size much smaller than the plasma, it is also referred to as micro-turbulence.

Even if the scale of this turbulence does not threaten the immediate confinement of the plasma as MHD-type turbulence might do, it still enhances the transport of heat and particles to the edge of the plasma where it is lost to the vacuum chamber walls. This decay of the plasma is fast enough to inhibit the ignition of the plasma, i.e. the sustaining of continuous fusion reactions. To understand what governs micro-turbulence and how to suppress it, are thus of the utmost importance to the success of a future fusion reactor and this is the subject of this PhD thesis.

The micro-turbulence considered in this thesis is of so called drift type and is thus excited through the onset of drift-type waves. These waves which are driven by temperature and density inhomogeneities got their name from their propagation velocity, the electron diamagnetic drift velocity. To understand how waves can create turbulence, think of the surface of a pond. If you throw in a stone, a well defined circular wave starts to propagate from the point where the stone hit the water. If instead, you throw in a bunch of stones, the water surface becomes jumbled. Even though each stone still create a well defined wave, the merging of all the waves from all the stones renders the surface chaotic. In a similar fashion, a multitude of drift waves causes the turbulent motion of the particles in a plasma. Hence, if we can improve the understanding of the excitation and driving mechanisms of these waves we might be able to suppress them and the turbulence they create.

In the first three papers we examine drift waves solely driven by density inhomogeneities and in the forth and last paper we also include temperature differences in the plasma. Moreover, the method of study differs as well in the last paper, where numerical methods based on the Weiland model[2], are used to directly calculate the diffusion the waves create. The first papers investigate the waves analytically and through the derived dispersion relations (equations that describe wave properties) we obtain a feeling for under which circumstances turbulence may arise.

J. B. Taylor showed in 1977[3] that drift waves could be damped due the tilted magnetic field lines in a tokamak plasma (magnetic shear damping) but that there where also a counteracting effect by the coupling of modes (toroidal mode coupling). The first three papers are in fact continuations of this work in rotating tokamak plasma and we study how these two features are modified by additional effects. The Weiland model used in paper IV, contains these elementary effects of magnetic shear damping and toroidal mode coupling too, but they will not be as explicitly studied as in paper I-III.

In paper I we study ship-type drift waves. These waves have the same wave characteristics as the ordinary bow waves of a boat traversing water and can be excited by measuring rods piercing the rotating plasma.

Paper II contains an investigation of how a sheared rotation affects drift waves. The underlying hypothesis is that turbulent eddies can be torn apart by a sufficiently strong sheared rotation of the plasma[4]. In solid body rotation the turbulent whirls would simply follow the plasma around the tokamak. If, on the other hand, the rotation is sheared then the different parts of the whirls move at different speeds with the end result that the eddy is stretched and for a large enough velocity shear, torn apart. This analysis also contains a peaking of the diamagnetic frequency. It was previously shown[5] that in a plasma without rotation, such a peaking localises drift waves and inhibits the waves from being damped by magnetic shear damping.

The beneficial influence of the sheared rotation on the Landau resonance is examined in paper III. The Landau resonance describes an interaction of waves and particles and its effect on waves is two-fold, it can either stabilise or destabilise them. Thus, it is interesting to investigate the possibility to turn the destabilising properties of the Landau resonance into stabilising properties by adding a sheared rotation as the results from paper I and II point towards drift waves being damped by a sheared plasma rotation.

In Paper IV we use the Weiland model to examine the influence of the temperature on drift-type turbulence. In particular two modes are of interest here, the ion temperature gradient mode (ITG) and the trapped electron mode (TE). Through simulations carried out with the JETTO code provided by the JET facility at the Culham Science Centre, UK, studies of the effects of the temperature ratio between electrons and ions on plasma confinement where performed. It has been known for quite some while, that a low temperature ratio is beneficial for plasma confinement[6]. The fusion research has now reached the point where studies of the impact of the fusion reactions take place. In these experiments it has been observed that the helium particles created in the fusing of deuterium and tritium nuclei, transfer most of their energy to the electrons. This implies that the electron temperature rises, which leads to an increase of the electron to ion temperature ratio. For this reason it is important to theoretically clarify how the temperature ratio influences the plasma confinement.

In the ensuing chapters I illustrate the various phenomena which are the foundation of the attached papers. Chapter 1 describes drift waves driven by density inhomogeneities and how they are affected by the Landau resonance, magnetic shear damping and toroidal mode coupling. Chapter 2 contains an account of the Weiland model and chapter 3 summarises the articles with a short introduction to the effects of a sheared rotation velocity.

1. Description of drift waves in static equilibrium

"We are at the very beginning of time for the human race. It is not unreasonable that we grapple with problems. But there are tens of thousands of years in the future. Our responsibility is to do what we can, learn what we can, improve the solutions, and pass them on."

Richard Feynman

For a plasma, or any gas for that matter, to be completely stable it has to be in thermodynamical equilibrium. This requires Maxwell distributed plasma particles (see Fig. 3) and a spatially homogeneous plasma density. If we restrict the mobility of the particles and confine the plasma we invariably break these conditions. The plasma then tries to regain equilibrium by exciting turbulent processes and the in this chapter we will look closer at drift waves driven solely by density inhomogeneities.

Drift waves are low frequency waves[*] and have an electric field component along the magnetic field, unlike the ideal MHD-modes. This parallel electric field allows the electrons to flow freely along the magnetic field lines and cancel any space charge in the plasma. In addition, low frequency modes often have quite a slow variation along the magnetic field lines and it is common to assume that $k_\perp \gg k_\parallel$, where $k_\perp$ and $k_\parallel$ are the wave vectors perpendicular and parallel to the magnetic field, respectively.

The reason for studying low frequency waves are that they are considered the most dangerous for plasma confinement. The periodicity of the wave due to the real eigenfrequency causes a reversible behaviour that reduces transport and for lower frequencies this effect becomes progressively more negligible. The imaginary part of the eigenfrequency, i.e. the linear growth rate generates the irreversible transport we are seeking to reduce to improve

* With low frequency waves we refer to waves with frequency $\omega \ll \Omega_{ci}$, were the ion cyclotron is defined by $\Omega_{ci} = eB/m_i$. For these waves the ion and electron cyclotron motions averages out and we only have to consider the gyro-centre motion of the particles.

plasma confinement properties.

Since, the pressure inhomogeneities which drive drift waves are inherent to confined plasmas, this type of instability is often referred to as the universal instability. Throughout this chapter we assume that the temperatures are constant and thus the drift waves will be driven exclusively by density inhomogeneities[†]. Hence, this chapter provides a basis for the first three publications.

1.1 The Universal Instability

From rudimentary electromagnetic field theory we know that charged particles are trapped around a magnetic field line due to the Lorentz force, but can move freely along the magnetic field line. Moreover, any gas which contains particles which move without constraint tends to equilibrate the temperature and so called thermalisation of the gas takes place. The long wavelength characteristic along the magnetic field lines of drift waves allows the electrons to flow freely parallel to the magnetic field. Hence thermalisation of the electrons occurs, which thus obey the Boltzmann relation,

$$n = n_0 \exp(e\phi_1 / T_e) \tag{1}$$

Here n and n_0 are the total and ambient electron density, e is the magnitude of the electron charge, ϕ_1 is the perturbed electrostatic potential and T_e is the electron temperature.

If the perturbation in the potential is small, we can Taylor expand Eq. (1) around zero and the perturbed electron density, n_1 becomes

$$\frac{n_1}{n_0} = \frac{e\phi_1}{T_e} \tag{2}$$

Hence n_1 and ϕ_1 are in phase and the density fluctuation in Fig.1. also depicts a potential variation.

[†] The pressure, p, of an ideal gas is given by $p = k_B nT$, with the Boltzmann constant k_B, number density n and temperature T.

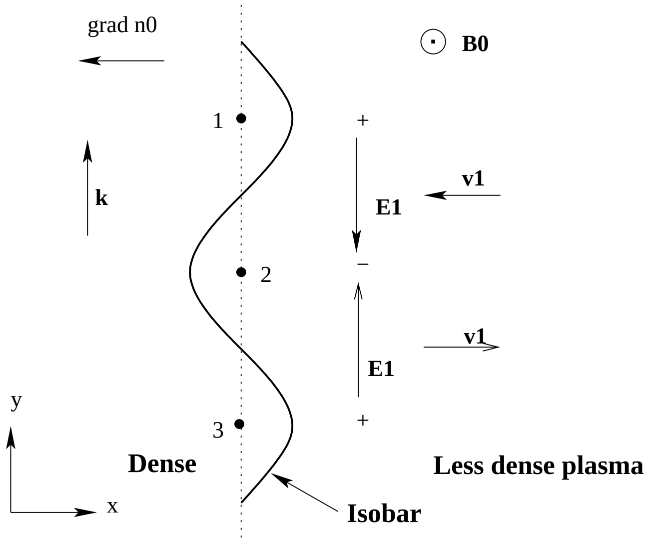

Figure 1: A density perturbation in a plasma depicted by a fluctuation of the isobar. In point 1(3) density is larger than in equilibrium and in 2 it is less. The perturbation propagates in the **k** direction.

The fluctuating potential in Fig. 1. gives rise to an electric field $\mathbf{E}_1$. Together with the background magnetic field $\mathbf{B}_0$, $\mathbf{E}_1$ creates a plasma flow with the $\mathbf{E}\times\mathbf{B}$ - velocity $\mathbf{v}_1$. We can see that this flow pushes the plasma in the x direction between point 1 and 2 and then pulls the plasma x direction between point 2 and 3, i.e. the perturbation starts to oscillate. So, if we stand in point 1, we can observe plasma of different density move back and forth. This results in the density perturbation propagating in the y direction. A drift wave, therefore, has a motion such that the plasma moves back and forth in the x direction although the wave travels in the y direction[7].

The instability of drift waves comes about when we realize that the velocity, $\mathbf{v}_1$ for the ions is not quite the $\mathbf{E}\times\mathbf{B}$ - velocity. Corrections must be made for drifts arising from temporal and/or spatial variations in the electric fields. As a result of these drifts the electrostatic potential ϕ_1 always lags behind the perturbed density n_1. Plasma already shifted outwards then has a velocity $\mathbf{v}_1$ also pointing outwards and vice versa. Hence the amplitude of the perturbation increases and the waves becomes unstable.

The electric field $\mathbf{E}_1$, in the above model (Fig. 1), would be short-circuited by the unobstructed electron flow along the magnetic field lines. In reality, there are mechanisms which impede the electron motion such as electron-ion collisions, magnetic mirror trapping of the electrons and Landau damping(see section 1.2). When the electrons can no longer provide quasineutrality, a phase shift between the perturbed density and electrostatic potential emerges and we can rewrite Eq. (2) into

$$\frac{n_1}{n_0} = \frac{e\phi}{T_e}(1 - i\delta) \tag{3}$$

The resulting resistivity, depicted by the phase shift δ, in conjunction with the long wave length along the magnetic field lines creates a potential drop and makes a finite value of $\mathbf{E}_1$ possible.

From this section we can conclude that drift waves are always present in plasmas and that these waves are intrinsically unstable. The question is; are there other mechanisms in plasmas which may stabilise drift waves? Before answering that question, we take a look at the analytical derivation of drift waves.

1.1.1 Analytical Description of Drift Waves in Slab Geometry

To describe drift waves analytically, we make use of a two-fluid model which depict the plasma as consisting of an ion and an electron fluid. Each fluid is governed by the equation of continuity,

$$\frac{\partial n_{e,i}}{\partial t} + \nabla \cdot \left(n_{e,i}\mathbf{u}_{e,i}\right) = 0, \tag{4}$$

the equation of motion,

$$n_{e,i}m_{e,i}\left[\frac{\partial \mathbf{u}_{e,i}}{\partial t} + \left(\mathbf{u}_{e,i} \cdot \nabla\right)\mathbf{u}_{e,i}\right] = n_{e,i}q_{e,i}\left(\mathbf{E} + \mathbf{u}_{e,i} \times \mathbf{B}\right) - \nabla p_{e,i} \tag{5}$$

and interacting with each other through the Poisson equation,

$$\nabla \cdot \mathbf{E} = -\frac{e}{\varepsilon_0}\left(n_e - n_i\right). \tag{6}$$

In Eqs (4)-(6), $\mathbf{u}_{e,i}$ are the velocities of the fluid elements and $q_{e,i}$ is the charge of the electrons and ions, respectively. For low frequency modes it is possible to replace Eq. (6) by the often used quasineutrality condition, $n_e = n_i$. The inertialess electrons then track the motion of the ions and cancel any space charge that may arise.

The above description can only be applied if the particles are localised. Perpendicular to the magnetic field we know this assumption holds, since the

Lorentz force restrains the motion in this direction. Parallel to the magnetic field we have to restrict our analysis to the following three cases:

i) The cold plasma approximation, $\omega \gg k_\parallel v_{th}$, where ω is the wave frequency with parallel wave vector $k_\parallel$ and v_{th} is the thermal velocity of the particles. For fast enough processes the plasma particles appear stationary and are thus localised.

ii) If collisions are frequent compared to the wave period, the plasma particles are not free to maintain communication between different parts of the perturbation which make them localised.

iii) $\omega / k_\parallel \approx 0$ and symmetric distribution. In this case non-local effects cancel.

For drift waves it is common to use case 1) for ions and case 3) for the, along the magnetic field, free flowing electrons. The latter is also a requirement for the Boltzmann distribution of Eq. (1) to be valid.

For cold ions it is widespread to neglect the ion pressure term. Then, in equilibrium, the ions are at rest and assuming constant electron temperature, the electron flow with the diamagnetic drift velocity,

$$\mathbf{v}_{*\mathrm{e}} = \frac{\kappa T_\mathrm{e}}{eB_0} \tag{7}$$

where $\kappa = -d \ln n_0 / dx$ is the inverse density inhomogeneity length scale.

Low frequency waves become electrostatic in a low-β[‡] plasma where we may neglect the magnetic field perturbations. The low frequency comes with another advantage as well, since it permits us to use perturbation analysis to solve our equations[§]. If, in addition, we assume that the wavelength of the perturbations are much smaller than κ^{-1}, we can represent them as plane waves of the form $\exp(-\mathrm{i}\omega t + \mathrm{i}\mathbf{k}\cdot\mathbf{r})$. The differential system of the fluid equations (4)-(5) together with the quasineutrality condition, then reduces to an algebraic system which can be readily solved. The resulting dispersion relation for the slab geometry of Fig. 1 is in this case,

$$\omega^2\left(1 + k_y^2\rho^2\right) - \omega\omega_{*\mathrm{e}} - k_\parallel^2 c_\mathrm{s}^2 = 0. \tag{8}$$

Here we have introduced the ion sound speed, $c_\mathrm{s} = (T_\mathrm{e}/m_\mathrm{i})^{1/2}$, the ion Larmor radius at the electron temperature, $\rho = c_\mathrm{s}/\Omega_\mathrm{ci}$, the perpendicular wave vector, k_y and the diamagnetic drift frequency $\omega_{*\mathrm{e}} = k_y v_{*\mathrm{e}}$. The term $k_y^2\rho^2$, originates from the ion polarisation drift and represents the influence of ion inertia. The dispersion relation (8) has a local character since it does

‡ $\beta = 2\mu_0 p / B_0^2$, is the ratio between the thermal and the magnetic pressure.

§ For low frequency waves the ion polarisation drift can be considered as a correction of the electric drift and thus we may make a perturbation analysis of the fluid equations.

not describe the wave structure in the x direction. If the plasma inhomogeneity is weak enough, k_x can be considered as constant. We can then replace k_y with $k_\perp$ in (8) to get the dispersion relation for weak inhomogeneity. In general Eq. (8) has two roots related to an accelerated and a retarded ion acoustic wave. The former is referred to as the electron drift wave and propagates in the direction of the electron drift. The latter, which propagates in the opposite direction, is the so called ion branch. In the limit of a uniform plasma density compared to the parallel wavelength ($\omega_{*e} \ll k_\parallel c_s$), the parallel ion motion gets increasingly important and the drift wave of Eq. (8) turns into a pure ion acoustic wave. For small $k_\parallel$ the frequency of the ion branch tends to zero, while the electron drift wave approaches the electron diamagnetic frequency corrected for ion inertia.

1.2 The Landau Resonance

In a plasma which is considered collisionless, dissipation of wave energy can still occur through wave-particle interactions. Since the particles in a plasma are charged they interact with the electric field generated by plasma waves, for instance drift waves. If this interaction leads to an acceleration of plasma particles the wave loses energy and dampens out. Paper III contains a thorough analysis of the influence of the Landau resonance on drift waves in rotating tokamak plasma.

Even though the Landau resonance can not be derived through fluid analysis, we may use the result from kinetic theory to modify the phase difference, δ, between density and electrostatic potential in Eq. (3). The reason why we need kinetic theory to describe the Landau resonance is twofold. First of all, fluid theory does not take into account the interaction of individual particles but averages out their properties in so call fluid elements. Secondly, the shape of the velocity distribution plays a crucial role when determining if the Landau resonance has a stabilising or destabilising effect on the wave, confer chapter 1.2.2.

1.2.1 A Qualitative Picture of Wave-Particle Interaction

Particle-wave interactions can only take place if the plasma particle and wave propagation speed is similar. Particles moving slower or faster than the wave will only experience a rapidly oscillating electric field, which effect on the particles effectively averages out. You can get a qualitative understanding of this by imagine to ride a boat on the sea. If you travel with the same speed as the waves you can be caught in a trough, but if you go faster or slower than the waves you will on average pass through as many wave tops as troughs. Although you may find the ride somewhat bumpy as you break the waves, not much net energy has been exchanged.

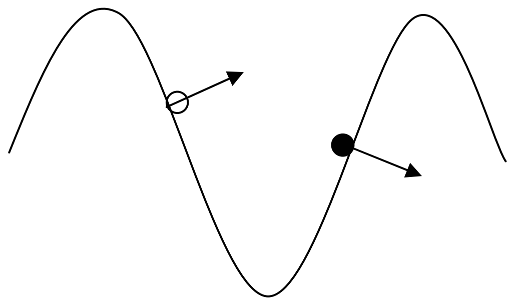

Figure 2: Wave-particle interaction. The transparent particle moves slower than the wave. The particle then gains energy as the wave pushes it along and the wave dampens out. The solid particle travels overtakes the wave and drives the wave in front of it. The particle thus transfers energy to wave which grows.

Return to the boat trapped in trough. If you slow down, the wave starts pushing the boat along, confer the transparent particle in fig. 2. Thus, the wave transfers energy to the vessel which accelerates. As the energy loss of the wave proceeds, it dampens out. Conversely, if the boat moves slightly fast than the wave(solid particle in fig. 2), the wave gains energy and its amplitude increases.

1.2.2 Physical Description of Landau Damping

The above description of the Landau resonance can only give an insight to the real process going on in a plasma. Instead of the single vessel above, there are a multitude of particles which simultaneously interacts with the wave. To clarify the net effect of the wave-particle interactions we thus need to look closer on the velocity distribution of the particles. Clearly, damping of the wave takes place if there are a majority of particles which moves slower than the wave, and thus gains energy from it. From section 1.1 we know that drift waves are intrinsically unstable and hence this damping mechanism can have a stabilising effect on these waves. On the other hand, if the wave on average gains energy from the particles, the wave further destabilises.

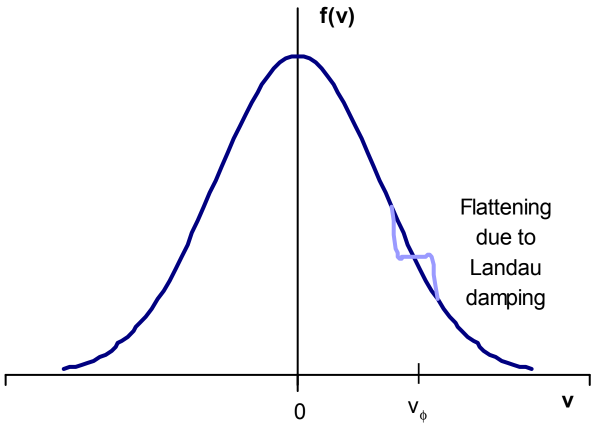

Figure 3: Maxwellian velocity distribution, f(v) with flattening (pale line) due to Landau damping.

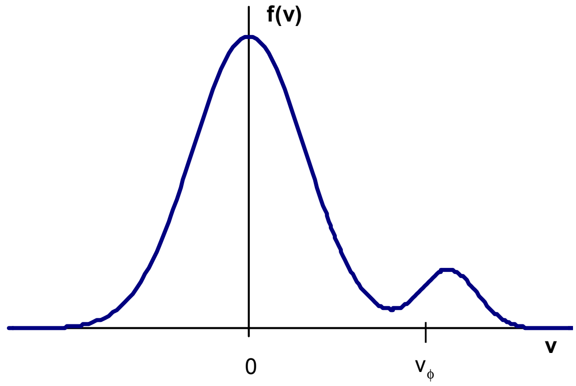

Figure 4: Bump-on-the-tail velocity distribution. At v_ϕ there are more particles which travels slower than the wave and thus a net transfer of energy to the wave takes place. This increase of wave energy destabilises the wave.

Assume that the velocity distribution is Maxwellian (Fig 3.) and that the phase velocity of the wave is v_ϕ. Around v_ϕ in fig 3 we see that the slope is negative and that there are a surplus of particles which moves slower than the wave. Hence, we draw that conclusion that this configuration has a suppressing influence on the wave. If we modify the Maxwellian distribution somewhat, we might get a so called bump-on-the-tail distribution, see fig. 4. This adjustment of the velocity distribution can easily be achieved through pumping a beam of fast particles into the plasma. At v_ϕ, the majority of the particles now transfers energy to the wave and thus destabilises the wave. In the case of drift waves which are thought to be a major threat to plasma confinement, we accordingly prefer a pure Maxwellian velocity distribution, as in fig. 3. Such a distribution reduces the drift wave amplitudes and therefore suppresses the anomalous transport.

1.2.3 Analytical Description of Landau Damping

The Landau resonance bears the name of L.D. Landau who first discovered it in 1946. By a careful analysis of the contour integral arising in the Vlasov equation (Eq. (9)) he predicted that waves could be damped without energy dissipation from collisions[8]. Hence, the Landau resonance was a purely theoretical discovery and its existence was later proven in several experiments.

As the Landau resonance depicts an interaction between individual particles and the wave the fluid model used in section 1.1.1 to describe drift waves can not be applied to explain this phenomenon. Firstly, the fluid model only considers the average quantities of bundles of particles, so called fluid elements, and not the particles themselves. Secondly, since the fluid model only takes into account average quantities, the shape of the velocity distribution never enters into the equations. From the above illustration of the Landau resonance we can clearly see that the shape of the velocity distribution is of the greatest importance. Hence, to obtain an analytical description of the Landau resonance we have to apply kinetic theory, which considers the full phase-space distribution function, $\hat{f}(\mathbf{r},\mathbf{v},t)$, of the particles. To this purpose we use the Vlasov equation,

$$\frac{\partial \hat{f}}{\partial t} + \mathbf{v}\cdot\nabla\hat{f} + \frac{q}{m}\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial \hat{f}}{\partial v} = 0, \tag{9}$$

which takes into account the particles interaction with an electric and magnetic field, but do not treat particle collisions.

Doing a one-dimensional analysis neglecting the magnetic field, using plane waves, $\exp\left(-\mathrm{i}\,\omega t + \mathrm{i}\,kx\right)$ and perturbation analysis in conjunction with residue calculus of the resonance $v - v_\phi$, we find that

$$\omega = \omega_p\left(1 + \mathrm{i}\frac{\pi}{2}\frac{\omega_p^2}{k^2}\left[\frac{\partial f\left(v\right)}{\partial v}\right]_{v=v_\phi}\right). \tag{10}$$

Here $f(v)$ is the unperturbed Maxwellian of Fig. 3 and $\omega_p = \left(n_0 e^2 / \varepsilon_0 m_e\right)^{1/2}$ is the electron plasma frequency. Eq. (10) is valid for the simplest case of electron plasma waves, but more importantly it holds the main characteristic of the Landau resonance, namely the dependence on the derivative of $f(v)$ at the point of the particle-wave resonance. A negative slope of $f(v)$, as in Fig. 3 and we will have damping of a wave with an $\exp(-\mathrm{i}\omega)$ dependence in concurrence with the discussion in the previous section.

1.3 Magnetic Shear Damping

Thus far we have considered a system with a constant magnetic field. If we instead have a slightly twisted magnetic field, $\mathbf{B} = B_0\left[\hat{z} + (x/L_s)\hat{y}\right]$ (see Fig. 5) the drift modes may become damped[3]. The wave starts to propagate out from the magnetic surface (the dotted line in Fig. 1) on which it resides. The outwards propagating component can be regarded as loss of energy for the main wave, still localised on its magnetic surface, because it transfers energy from the mode to the Landau resonance[9]. The latter converts the outgoing wave energy into particle heating, as discussed in the previous section. A decrease of the main wave amplitude follows the energy loss and the wave is subsequently damped.

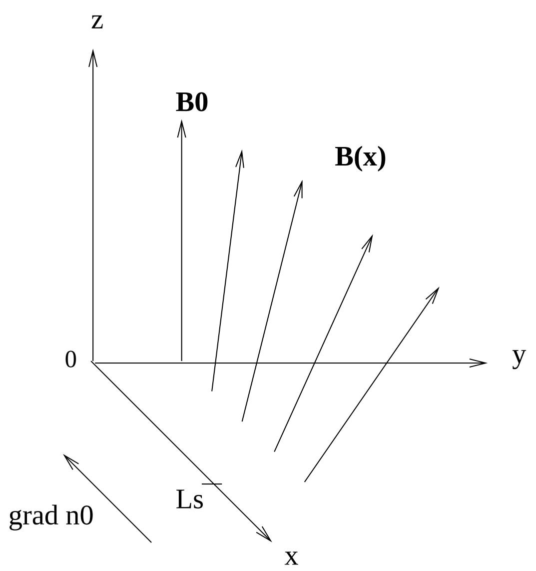

Figure 5: Sheared magnetic field in slab geometry. L_s is the characteristic scale length of the magnetic field variation.

To better understand this phenomenon, we take a closer look at the equation governing the system in Fig. 1 including the sheared magnetic field of Fig 5, i.e. a perturbed equilibrium plasma in a sheared magnetic field. In this model the amplitude, ϕ of the perturbed electrostatic potential $\phi_1 = \phi(x)\exp\left(-\mathrm{i}\omega + \mathrm{i}k_y y + \mathrm{i}k_\parallel z\right)$ is given by[2],

$$\rho^2 \frac{d^2\phi}{dx^2} + \frac{c_s^2}{v_{*e}^2}\frac{x^2}{L_s^2}\phi + \left(\frac{\omega_{*e}}{\omega} - 1 - k_y^2\rho^2\right)\phi = 0 \tag{11}$$

Here ρ is the ion Larmor radius at the electron temperature, $v_{*\mathrm{e}}$ is the electron diamagnetic drift velocity which defines the diamagnetic drift frequency as $\omega_{*\mathrm{e}} = k_y v_{*\mathrm{e}}$. It was assumed that the degree of twisting of the magnetic field lines equals the degree of twisting of the drift mode. Thus, $k_\parallel\left(x = 0\right) = 0$ and as the wave propagates out from the magnetic surface at x = 0, $k_\parallel = xk_y / L_s$ in the simplest case. This expression replaced $k_\parallel$ in the term proportional to x^2.

Eq. (4) has the form of the Weber equation[10],

$$\frac{d^2\psi}{dx^2} + \left(E - kx^2\right)\psi = 0, \tag{12}$$

which we know from quantum mechanics has either global (bound) or propagating solutions depending on whether the sign of k is positive or negative. Hence, in the case of global solutions we have a potential well in which the drift modes are trapped and a potential anti-well for propagating solutions with an outgoing energy flux for large x[11].

In Eq. (11),

$$k = -\frac{c_s^2}{v_{*\mathrm{e}}^2}\frac{1}{L_s^2\rho^2} < 0 \tag{13}$$

and we conclude that the magnetic shear, L_s gives rise to drift modes which are propagating out from the magnetic surface of their origin.

We notice that since $v_{*\mathrm{e}} \sim r_n^{-1}$, we have $k \sim -r_n / L_s$.A competition between the density inhomogeneity and magnetic shear thus determines the magnitude of the magnetic shear damping. The stronger the magnetic shear is compared to the density inhomogeneity (i.e. for smaller shear lengths, L_s the larger the component of the drift mode propagating out from the magnetic surface. This implies that the drift modes experience more efficient damping for stronger magnetic shear[12].

With the definition of the parallel wave vector as $k_\parallel = xk_y / L_s$, we observe that $k_\parallel$ increases as the wave propagates. At some point the wave has propagated far enough, for $k_\parallel$ to be sufficiently large, to make Landau damping effective (i.e. $v_\phi = \omega / k_\parallel \ll v_{T_e}$).

1.4 Toroidal Mode Coupling

The toroidal system differs from the previously studied slab geometry as the magnitude, the shear and curvature of the magnetic field are no longer uniform over a magnetic surface. This non-uniformity induces a coupling of drift modes[3] situated at different rational surfaces (at which $k_\parallel = 0$) which may inhibit the outward convection of energy by magnetic shear damping. As a result, drift modes can yet again cause turbulence detrimental to plasma confinement.

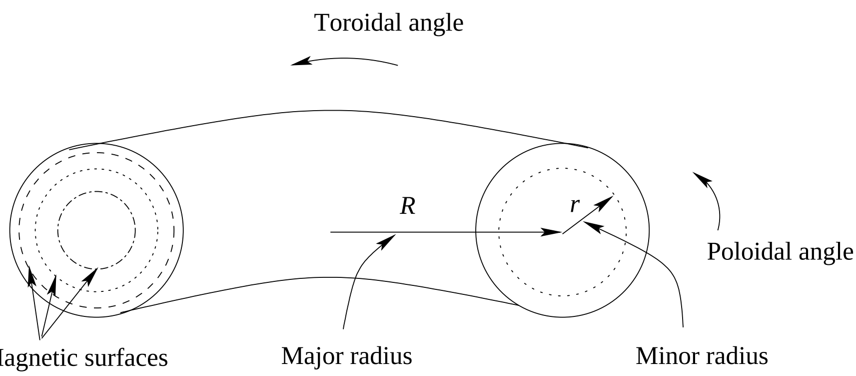

Figure 6: The toroidal geometry: Part of a full torus.

The toroidal geometry (Fig. 6) of plasma confinement was promoted by L.A. Artzimovich in the former USSR as a mean to suppress plasma loss through the ends of cylindrical devices. Inside the doughnut shaped plasma, magnetic field lines wind around so called magnetic surfaces. These can be of two kinds, rational or irrational depending on whether the field line closes on itself or continues to wind around the magnetic surface, never to follow in its own track.

In the system of coordinates commonly used for toroidal devices, the coordinate axes are not strictly independent of each other and as a consequence the parallel wave vector,

$$k_{\parallel} = \frac{\mathbf{k} \cdot \mathbf{B}}{B} = \frac{m - nq(r)}{Rq(r)} \tag{14}$$

is a function of the minor radius *r*. Here the safety factor $q(r) = m(r)/n$, with poloidal and toroidal mode numbers *m(r)* and *n*, respectively. The safety factor expresses how many turns the magnetic field line winds around poloidal direction in one turn along the toroidal direction and is thus closely connected to the magnetic field configuration. For instance, *m* and *n* integers defines a rational magnetic surface. The name safety factor originates from the realm of magnetohydrodynamic (MHD) instabilities for which a $q > 1$ indicates stable tokamak operation.

Through the *r* dependence of $k_{\parallel}$, wave modes have the ability to sense other wave modes, i.e. they become coupled. Looking into the equations governing the toroidal drift modes[13], one can see that this coupling has its origin in the ∇B-drift, i.e. in the variation of the magnetic field and thus in the mathematical description of the plasma confinement device.

1.4.1 Propagating or localised drift modes?

As one can comprehend from above, drift modes in a tokamak can be either propagating or localised, depending on whether the magnetic shear damping or the toroidal mode coupling dominates the mode structure. To get a better understanding of the influence of these effects on drift modes, we make a brief analytical derivation of the modes in toroidal geometry. The corresponding analysis for ship waves in rotating tokamak plasmas can be found in paper I.

In a torus, the fluctuations of the electrostatic potential, ϕ is governed by the differential equation[13],

$$\begin{aligned} &-\left[\frac{\rho^2}{n_0 r}\frac{\partial}{\partial r}\left(rn_0\frac{\partial\phi}{\partial r}\right) + \rho^2\left(\hat{e}_\perp\cdot\nabla\right)^2\phi\right] + \left(1 - \mathrm{i}\delta\right)\phi \\ &+\frac{\mathrm{i}h\rho}{\omega}\left[\hat{e}_\perp\cdot\nabla\phi - 2\varepsilon_n\left(\sin\theta\frac{\partial\phi}{\partial r} + \frac{\cos\theta}{r}\frac{\partial\phi}{\partial\theta}\right)\right] + \frac{c_s^2}{\omega^2}\left(\hat{n}\cdot\nabla\right)^2\phi = 0, \end{aligned} \tag{15}$$

where a harmonical time dependence, $\exp(-\mathrm{i}\omega t)$ of ϕ was assumed.
In Eq. (15) $h = c_s/r_n$, $\varepsilon_n = r_n/R$, $\hat{e}_\perp$ and $\hat{n}$ are the unit vectors perpendicular and parallel to the magnetic field respectively. Moreover,

$\hat{e}_\perp \cdot \nabla = (B_T / B)\partial / \partial\theta - (B_P / BR)\partial / \partial\varphi$ and $\hat{n} \cdot \nabla = (1/qR)(\partial / \partial\theta + q\partial / \partial\varphi)$, with B_P and B_T the poloidal and toroidal components of the magnetic field and θ and φ the poloidal and toroidal angles (see Fig. 6). The $\cos\theta$ and $\sin\theta$ terms of Eq. (15) arises from the magnetic field configuration.

Let us consider a solution of Eq. (15) on the form

$$\phi(r,\theta,\varphi) = \exp(\mathrm{i}m\theta - \mathrm{i}n\varphi)\sum_l \phi_l(r)\exp(\mathrm{i}l\theta) \tag{16}$$

which is centred around the magnetic surface with $r = r_0$ and $m(r_0) = m_0$. When we apply this solution to Eq. (15), the $\cos\theta(= \exp(\mathrm{i}\theta) + \exp(-\mathrm{i}\theta))$ and $\sin\theta(= \exp(\mathrm{i}\theta) - \exp(-\mathrm{i}\theta))$ terms couple the *l*:th mode to the adjacent modes with $l \pm 1$. Expanding Eq. (15) around the rational magnetic surface $r = r_0$, yields the differential-difference equation

$$\begin{aligned} &\left[\frac{\partial^2}{\partial x^2} - 1 - k_y^2\rho^2 + \mathrm{i}\delta\right]\phi_l \\ &+ \frac{hk_y\rho}{\omega}\left[\phi_l - \varepsilon_n(\phi_{l+1} + \phi_{l-1}) - \frac{\varepsilon_n}{k_y\rho}\left(\frac{\partial\phi_{l+1}}{\partial x} - \frac{\partial\phi_{l-1}}{\partial x}\right)\right] \\ &+ \left(\frac{c_s}{\omega qR}\right)^2 (l - k_y\rho s x)^2 \phi_l = 0. \end{aligned} \tag{17}$$

Here the dimensionless deviation from the magnetic surface $x = (r - r_0)/\rho$, $k_y = m_0 / r_0$ and the magnetic shear parameter $s = r_0 q'(r_0)/q(r_0)$.

Eq. (17) can analytically be solved in two different limits; the strong and weak coupling approximation in which the main mode at $r = r_0$ couples to numerous and few neighbouring modes, respectively. In both cases we require as boundary conditions of Eq. (17) that waves are outgoing and that the wave amplitudes decreases with the distance from the rational surface. Mathematically this corresponds to assuming that $\Sigma|\phi_l|$ is finite and that $\phi_l(x)$ for fixed l and $x \to \pm\infty$, represents outgoing wave energy propagation.

In the limit of sufficiently small ε_n, we have by the definition of m_0, that $|\phi_{\pm 1}(r)/\phi_0(r)| \sim \varepsilon_n \ll 1$. If, for small enough ε_n, we can neglect all higher harmonics but $\phi_{\pm 1}$ in Eq. (17), it reduces to a set of three differential equations with three unknown. These can then be readily solved. We call this limit the weak coupling approximation.

When the amplitudes of the main harmonic and the side bands are similar, many modes couple to each other and hence we use what is known as the strong coupling approximation. In this case, $m_0 \gg \Delta_l \gg 1$ and the modes are so close to each other, we may replace the discrete set of functions $\phi_l(x)$ by the continuous function $\phi(x,l)$, and rewrite the differences in Eq. (17) as

$$\phi_{l+1} + \phi_{l-1} = 2\phi(x,l) + \frac{\partial^2 \phi(x,l)}{\partial l^2} \tag{18}$$

and

$$\phi_{l+1} + \phi_{l-1} = 2\phi(x,l) + \frac{\partial^2 \phi(x,l)}{\partial l^2} \tag{19}$$

The above in conjunction with the combined coordinate $y = x - l / k_y \rho s$, reduces Eq. (17) to the Weber equation Eq. (12), in the form

$$\left[1 + \frac{\varepsilon_n h(2s-1)}{\omega k_y \rho s^2}\right] \frac{\partial^2 \phi}{\partial y^2} + \left(\frac{k_y \rho h \varepsilon_n s}{\omega q}\right)^2 y^2 \phi = \left[1 + k_y^2 \rho^2 - \mathrm{i}\delta - \frac{k_y \rho h(1 - 2\varepsilon_n)}{\varepsilon_n}\right] \phi \tag{20}$$

From Eq. (20) and the analysis of the Weber equation in section 1.3 we can conclude that whenever

$$\eta = 1 + \frac{\varepsilon_n h(2s-1)}{\omega k_y \rho s^2} < 0, \tag{21}$$

and hence for $s < 1/2$, toroidal mode coupling localises the drift modes. In the opposite case of $\eta > 0$ and $s > 1/2$, shear stabilisation due to the outgoing wave energy propagation persists.

We have thus shown that a curvilinear magnetic field can localise drift modes in the case of sufficiently weak magnetic shear. The drift modes can then amplify unhindered and instabilities emerge.

2. The Weiland Model

"Perfect as the wing of a bird may be, it will never enable the bird to fly if unsupported by the air. Facts are the air of science. Without them a man of science can never rise."

Ivan Pavlov

In the current chapter we treat drift-type modes which, in contrast to the previous chapter, are driven by temperature inhomogeneities in addition to density inhomogeneities. This class of modes are called reactive drift modes and do not necessarily require dissipative effects to be unstable. As we saw in chapter 1, to destabilise drift waves driven solely by density inhomogeneities demands some kind of mechanism which inhibits the electrons from flowing freely along the magnetic field lines and neutralise space charge. The mechanism that renders the reactive drift modes unstable is the competition between the compression and expansion, driven by the density and temperature inhomogeneities, which enters into the energy equation. The energy equation plays an important part when it comes to closing the system of fluid equations introduced in section 1.1.1. Since we are dealing with temperature perturbations as well as density perturbations, we need an additional equation, in this case the energy equation, to close the system. In the energy equation, different fluid models use different methods to truncate the expression for the heat flow and this is what distinguishes one fluid model from another. The Weiland model[2] we have used in paper IV, inserts the expression of the diamagnetic heat flow into the energy equation to close the set of fluid equations.

From the perspective of plasma confinement, reactive drift modes are commonly considered the second most dangerous after MHD modes and are likely to be a significant source of anomalous transport.

2.1 The Interchange Instability

The driving mechanism responsible for reactive drift type waves is the interchange instability[7]. This instability is closely related to the Rayleigh-Taylor instability responsible for the mixing of fluids when a light fluid supports a denser fluid. A classical example of this is the convection taking place in a saucepan. The heating of the fluid from below, causes the fluid to expand there and as a result it becomes less dense than the cooler fluid on top. When the density difference has reached a critical value, the lighter fluid rises and changes place with the cooler fluid, which heats up and the convectional motion of the fluid is excited. The driving force in this case is due to gravity.

In a plasma the magnetic field lines plays the part of the lighter fluid supporting the heavier plasma fluid. The gravitational force is replaced by the centrifugal force experienced by the plasma particles following the curved magnetic field lines of a toroidal plasma. Heating of the plasma is redundant. Since the magnetic field lines lie closer together at the inside of the toroidal plasma, compression which heats the plasma automatically occurs when the particles move along the field lines.

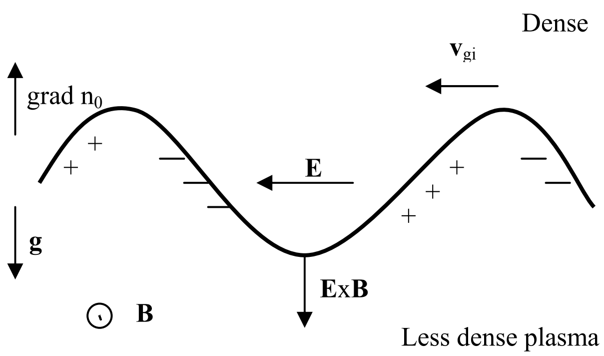

Figure 7: The interchange instability in slab geometry

Take a look at the slab case in Fig 7, where we have replaced the centrifugal force by the gravitational force, for simplicity. The ion-gravitational drift velocity,

$$v_{\mathrm{gi}} = -\frac{g}{\omega_{\mathrm{ci}}} \tag{22}$$

is a factor $m_{\mathrm{i}}/m_{\mathrm{e}}$ larger than the electron counterpart and hence we neglect the latter. Suppose that a ripple arises as depicted in Fig. 7, which for instance could be due to a random thermal fluctuation. As the ions drift, they cause a build up of charge on one side of the ripple and a depletion of charge on the other. This charge separation generates an electric field and the resulting $\mathbf{E}\times\mathbf{B}$-velocity displaces the plasma in the direction of the ripple, hence rendering it unstable. From this brief account we conclude that the instability arises because the there is a difference in the gravity (curvature) drifts of electrons and ions which in conjunction with a density perturbation cause a charge separation. Moreover, we can see that the instability only arise when the density gradient and the gravity force are antiparallel. In the realm of drift waves such an instability would not emerge, since the electrons are free to immediately cancel the charge separation. Note, that in a toroidal plasma, the full pressure gradient replaces the density gradient in Fig 7.

2.2 The Competition between Density and Temperature Inhomogeneities

In the next section where we derive the dispersion relations for reactive drift modes we will become aware of the importance of the ratio between the density and temperature inhomogeneity length scales for the stability of these modes. This fundamental property of plasmas can be easily understood by taking a closer look at the processes nonuniform densities and temperatures trigger.

The convective motion of the plasma, prompted by the density inhomogeneity, transfers plasma of higher density to areas of lower density. The following expansion of the plasma in the low density region cools it down, but has to compete with the simultaneous heating of the plasma through temperature convection. Accordingly the net temperature change, δT, depends on which of these two process is the strongest. As the strength depends on the slopes of the inhomogeneities, we may write

$$\delta T = -\boldsymbol{\xi}\cdot\nabla T + \alpha\boldsymbol{\xi}\cdot\nabla n.$$

Here α is a coefficient that determines the amount of cooling caused by the expansion. From this relation we immediately perceive that the ratio, $\eta = L_n / L_T$, between the density and temperature inhomogeneity length scales, $L_n = \nabla n / n$ and $L_T = \nabla T / T$, is highly significant to determine the outcome of the convection processes. Specifically, to drive an instability and get a net increase of δT, η has to exceed a certain value. Below this threshold value, the stability of the plasma is not threatened. Above it, turbulent processes are excited boosting the transport of heat and particles.

Temperature perturbations bring about density perturbations by varying the speed of the particles along their paths. When the velocity changes in its own direction, local bunchings and rarefactions of the particles arise. For example the curvature drifts and the motion along the magnetic field lines contains temperature dependencies that may compress the density. Hence, in the case of convective temperature perturbations the change in density,

$$\delta n = -\xi \cdot \nabla n + \beta \xi \cdot \nabla T,$$

were we again notice the contest between the inhomogeneity length scales.

In the dispersion relations of reactive drift waves derived in the next section, we will see that the density inhomogeneity produces a stabilising effect through the diamagnetic frequency ($\sim 1/L_n$) contribution to the real part of the frequency. A shorter length scale induces a more rapid periodic motion of the particles which reduces the harmful impact of the instability.

2.3 Reactive Drift Modes

The interchange instability of section 2.1 is essentially a magnetohydrodynamic instability but it can be recovered for drift-type waves in two cases. Firstly, if some of the electrons are trapped, e.g. due to the curvature of the magnetic field, they are not able to cancel space charge which implies that curvature drifts lead to charge separation[14]. Secondly, curvature of the magnetic field in combination with a temperature gradient can give an interchange type mode which does not correspond to charge separation but rather to the compressibility of the plasma[15]. We will start to consider the latter when an ion temperature gradient is present in a toroidal plasma[16].

2.3.1 Ion Temperature Gradient Modes

To derive a dispersion relation for the ion temperature (ITG) modes we reapply the two-fluid theory used in section 1.1.1 to find an analytical expression for drift waves. In the case of ITG modes the ion continuity equation (4) takes the form[2],

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \nabla \cdot \left(n_{\mathrm{i}} \mathbf{v}_E \right) + \nabla \cdot \left(n_{\mathrm{i}} \mathbf{v}_{*\mathrm{i}} \right) + \nabla \cdot \left[n_{\mathrm{i}} \left(\mathbf{v}_{\pi \mathrm{i}} + \mathbf{v}_{p\mathrm{i}} \right) \right] = 0, \tag{23}$$

where the $\mathbf{E} \times \mathbf{B}$-velocity,

$$\mathbf{v}_E = \frac{1}{B} \left(\mathbf{b} \times \nabla \phi \right), \tag{24}$$

the diamagnetic velocity including full pressure inhomogeneities,

$$\mathbf{v}_{*i} = \frac{1}{en_i B}\left(\mathbf{b}\times\nabla p_i\right),$$

the drift due to the off-diagonal terms of the stress tensor $\boldsymbol{\pi}$,

$$\mathbf{v}_{\pi i} = \frac{1}{n_i \omega_{ci} m_i}\left(\mathbf{b}\times\nabla\cdot\boldsymbol{\pi}\right),$$

the ion polarisation drift,

$$\mathbf{v}_{pi} = \frac{1}{B\omega_{ci}}\left[\frac{\partial}{\partial t} + \left(\mathbf{v}_i\cdot\nabla\right)\right]\mathbf{E}$$

and $\mathbf{b} = \mathbf{B}/B$. The density inhomogeneities in Eq. (23) are coupled to the temperature inhomogeneities through the energy equation,

$$\frac{3}{2}n_i\left(\frac{\partial}{\partial t} + \mathbf{v}_i\cdot\nabla\right)T_i + p_i\nabla\cdot\mathbf{v}_i = -\nabla\cdot\mathbf{q}_i. \tag{25}$$

In Eq. (25) the ion heat flux, $\mathbf{q}_i$ is given by the diamagnetic expression,

$$\mathbf{q}_{*i} = \frac{5}{2}\frac{p_i}{m_i\omega_{ci}}\left(\mathbf{b}\times\nabla T_i\right) \tag{26}$$

and with this truncation of the fluid hierarchy the Weiland model is closed. The divergence of the heat flux in Eq. (25) then takes the form,

$$\nabla\cdot\mathbf{q}_{*i} = \frac{5}{2}n\mathbf{v}_{*i}\cdot\nabla T_i + \frac{5}{2}n\mathbf{v}_{Di}\cdot\nabla T_i, \tag{27}$$

where $\mathbf{v}_{Di}$ is the total magnetic drift of the ions. Hence there is an additional contribution from $\mathbf{q}_{*i}$ to the curvature terms in Eq. (25). The diamagnetic part of $\mathbf{q}_{*i}$ (first term of Eq. (27)), actually cancels out the other diamagnetic terms after substitution of the continuity equation for $\nabla\cdot \mathrm{v}_{*i}$.

To derive the simple expressions for the ITG modes used in paper IV, we do not have to include the equation for parallel ion motion. This is also a rather good approximation of the most rapid growing modes for which it has been shown that $k^2\rho^2 \sim 0.1$. Using a kinetic description, smaller values of $k^2\rho^2$ lead to ion Landau damping with a following decrease of the growth rate. In the opposite limit of large $k^2\rho^2$, damping due to the perpendicular ion viscosity becomes increasingly important.

By using perturbation theory we may linearise Eq. (25) using Eq. (27) for the heat flux and the perturbed temperature equation thus becomes[2],

$$\frac{\delta T_{\mathrm{i}}}{T_{\mathrm{i}}} = \frac{\omega}{\omega - \frac{5}{3}\omega_{D\mathrm{i}}}\left[\frac{2}{3}\frac{\delta n_{\mathrm{i}}}{n_{\mathrm{i}}} - \frac{\omega_{*\mathrm{e}}}{\omega}\left(\frac{2}{3} - \eta_{\mathrm{i}}\right)\frac{e\phi}{T_{\mathrm{e}}}\right]. \tag{28}$$

Inserting Eq. (28) into the linearised ion continuity equation (23) reveals

$$\frac{\delta n_{\mathrm{i}}}{n_{\mathrm{i}}} = \frac{\omega\left(\omega_{*\mathrm{e}} - \omega_{\mathrm{De}}\right) + \left(\eta_{\mathrm{i}} - \frac{7}{3} + \frac{5}{3}\varepsilon_n\right)\omega_{*\mathrm{e}}\omega_{\mathrm{Di}} - k^2\rho^2\left(\omega - \omega_{*\mathrm{i}T}\right)\left(\omega - 5\omega_{\mathrm{Di}}\right)}{\omega^2 - \frac{10}{3}\omega\omega_{\mathrm{Di}} + \frac{5}{3}\omega_{\mathrm{Di}}^2}\frac{e\phi}{T_{\mathrm{e}}}, \tag{29}$$

where $\omega_{*\mathrm{i}T} = \omega_{*\mathrm{i}}\left(1 + \eta_{\mathrm{i}}\right)$, $\varepsilon_n = 2L_n / R$, $\eta_{\mathrm{i}} = d\ln T_{\mathrm{i}} / d\ln n_{\mathrm{i}} = L_{n_{\mathrm{i}}} / L_{T_{\mathrm{i}}}$ and $L_{n_{\mathrm{i}}}$ and $L_{T_{\mathrm{i}}}$ are the inhomogeneity length scales for the ion density and temperature, respectively. Eq. (29) has the correct isothermal limit, i.e. for $\omega_D \ll \omega, \omega_*$,

$$\frac{\delta n_{\mathrm{i}}}{n_{\mathrm{i}}} \rightarrow -\frac{e\phi}{T_{\mathrm{i}}}. \tag{30}$$

This limit is entirely due to the curvature part of $\nabla \cdot \mathbf{q}_{*\mathrm{i}}$ and we thus realise that a careful choice of $\mathbf{q}_{*\mathrm{i}}$ and rigorous treatment of the temperature perturbation is needed to make the fluid theory consistent with kinetic theory. The curvature part of $\nabla \cdot \mathbf{q}_{*\mathrm{i}}$ provides a higher order contribution to the pressure force that can either stabilise or destabilise drift modes. In the case of ITG modes it is usually stabilising[17].

To find the dispersion relation of the ITG modes we assume that quasineutrality holds and that the electrons are Boltzmann distributed. Replacing $\delta n_{\mathrm{i}} / n_{\mathrm{i}}$ in Eq. (29) with $\delta n_{\mathrm{e}} / n_{\mathrm{e}}$ from Eq. (2) then yields the dispersion relation

$$\omega = \omega_r + \mathrm{i}\,\gamma. \tag{31}$$

The real part of the frequency,

$$\omega_r = \frac{1}{2}\omega_{*\mathrm{e}}\left[1 - \varepsilon_n\left(1 + \frac{10}{3\tau}\right) - k^2\rho^2\left(1 + \frac{1+\eta_{\mathrm{i}}}{\tau} - \varepsilon_n - \frac{5}{3\tau}\varepsilon_n\right)\right] \tag{32}$$

and the growth rate,

$$\gamma = \frac{\omega_{*\mathrm{e}}\sqrt{\varepsilon_n / \tau}}{1 + k^2\rho^2}\sqrt{\eta_{\mathrm{i}} - \eta_{th\mathrm{i}}} \tag{33}$$

with the ITG threshold,

$$\eta_{th\mathrm{i}} = \frac{2}{3} - \frac{\tau}{2} + \varepsilon_n\left(\frac{\tau}{4} + \frac{10}{9\tau}\right) + \frac{\tau}{4\varepsilon_n} - \frac{k^2\rho^2}{2\varepsilon_n}\left[\frac{5}{3} - \frac{\tau}{4} + \frac{\tau}{4\varepsilon_n} - \left(\frac{10}{3} + \frac{\tau}{4} - \frac{10}{9\tau}\right)\varepsilon_n + \left(\frac{5}{3} + \frac{\tau}{4} - \frac{10}{9\tau}\right)\varepsilon_n^2\right]. \tag{34}$$

The expression (34) for the ITG threshold has been expanded in $k^2\rho^2$ and terms like $k^4\rho^4/\varepsilon_n$ and with higher powers of ε_n, have thus not been calculated consistently. The reason for this is that the $\mathbf{q}_{*\mathrm{i}}$ used here only is in the lowest order of $k^2\rho^2$. The trend of Eq. (35) is never the less consistent with kinetic theory.

The presence of ε_n in $\eta_{th\mathrm{i}}$, introduces an upper stability limit for large ε_n, see third term of Eq. (34). Usually large values of ε_n can be found in the bulk of a tokamak plasma and especially in the so called high confinement regime in which flat density profiles are common.

As we in paper IV study the influences of the temperatures on the plasma stability and confinement, we observe that the ITG growth rate (33) and threshold (34) depend on both the ion and electron temperatures explicitly. In the next section we will conclude that this is not the case for the trapped electron mode.

Finally, we note that in the derivation of this model we have not made any expansion of ε_n and therefore it is at times referred to as fully toroidal. The influence of ε_n is indeed one of the most important toroidal effects on drift-type waves.

2.3.2 Trapped Electron Modes

Another mode that occurs for $k^2\rho^2 \sim 0.1$ is the trapped electron mode. When the electrons are no longer free to move along the magnetic field lines and cancel the space charge caused by the difference between the ion and electron curvature drifts, the interchange instability is recovered. The magnetic configuration of a tokamak, with stronger magnetic field on the inside (plasma facing the 'hole' of the doughnut) than the outside, traps the electrons in so called banana orbits.

The trapped electrons do not contribute to the current parallel to the magnetic field, since their bounce averaged velocity cancels out in this direction and we may subsequently neglect the parallel electron motion. Comparison of the kinetic integrals for trapped electrons and ions without parallel motion shows that they are symmetric[16]. We may thus use the same model equations for trapped electrons as for the ions in section 2.2.2.

The Boltzmann equation (2), valid exclusively for free electrons is in the derivation of trapped electron (TE) modes replaced by

$$\frac{\delta n_{\mathrm{e}}}{n_{\mathrm{e}}} = f_{\mathrm{t}} \frac{\delta n_{\mathrm{et}}}{n_{\mathrm{et}}} + \left(1 - f_{\mathrm{t}}\right) \frac{\delta n_{\mathrm{ef}}}{n_{\mathrm{ef}}}, \tag{36}$$

were δn_{ef} is the density of the free electrons and f_{t} is the fraction of trapped electrons given by δn_{et}. Since there is nothing to prevent the free electrons from becoming thermalised, we suppose that they are Boltzmann distributed in accordance with Eq. (2).

If we assume quasineutrality, i.e. that $\delta n_{\mathrm{i}} = \delta n_{\mathrm{et}} + \delta n_{\mathrm{ef}}$, we can equate the expression for the ion density perturbation (29) with the equivalent expression for the trapped electrons and the Boltzmann equation for the free flowing electrons. This yields the dispersion relation,

$$\begin{aligned} &\frac{\omega_{*\mathrm{e}}}{N_{\mathrm{i}}}\left[\omega\left(1-\varepsilon_n\right) + \left(\eta_{\mathrm{i}} - \frac{7}{3} + \frac{5}{3}\varepsilon_n\right)\omega_{D\mathrm{i}} - k^2\rho^2\left(\omega - \omega_{*\mathrm{i}T}\right)\left(\frac{\omega}{\omega_{*\mathrm{e}}} + \frac{5}{3\tau}\varepsilon_n\right)\right] \\ &= f_{\mathrm{t}}\frac{\omega_{*\mathrm{e}}}{N_{\mathrm{e}}}\left[\omega\left(1-\varepsilon_n\right) + \left(\eta_{\mathrm{e}} - \frac{7}{3} + \frac{5}{3}\varepsilon_n\right)\omega_{D\mathrm{e}}\right] + 1 - f_{\mathrm{t}} \end{aligned} \tag{37}$$

where

$$N_j = \omega^2 - \frac{10}{3}\omega\omega_{Dj} + \frac{5}{3}\omega_{Dj}^2, \; j = \mathrm{i}, \mathrm{e}. \tag{38}$$

We have neglected the finite Larmor radius effects for the trapped electrons in the dispersion relation (37), which is equivalent to supposing inertialess electrons. The symmetry between the ions and the trapped electrons is readily conceivable and this symmetry is complete in the limit $k^2\rho^2 = 0$ and $f_{\mathrm{t}} = 1$. As the free electrons have a stabilising influence on the interchange instability, this is also the most unstable mode.

As we can see from Eq. (37), this is a quartic equation in ω. Hence, it can have two complex conjugate roots which imply it can cause two instabilities simultaneously. The dispersion relation (31), previously derived for the ITG modes is recovered for $f_{\mathrm{t}} = 0$. When $f_{\mathrm{t}} \neq 0$ and $N_{\mathrm{e}} \gg N_{\mathrm{i}}$, this ITG branch is modified due to the reduced free electron response.

The resonant denominators N_{e} and N_{i} plays a similar role as in the dispersion for the two-stream instability. Close to the resonances the modes due to the ions and trapped electrons decouple. This happens for $\varepsilon_n \sim 1$ and we might then get an good approximation of the dispersion relation by neglecting the part with the larger N_j. Hence, for $N_{\mathrm{i}} < N_{\mathrm{e}}$ the mode propagates in the ion drift direction (ITG mode) and in the opposite limit where $N_e < N_i$, the mode propagates in the electron drift direction (TE mode). Strong coupling of the modes occurs when ε_n is small, which might change the direction of propagation of the above modes.

In the case of the strong inequality $N_e \ll N_i$, the trapped electron branch of Eq. (37) is described by the dispersion relation

$$\omega^2 + \omega\omega_{*e}\left[\frac{f_t}{1-f_t}(1-\varepsilon_n) - \frac{10}{3}\varepsilon_n\right] = \frac{5}{3}\omega_{De}^2 - \frac{f_t}{1-f_t}\left(\eta_e - \frac{7}{3} + \frac{5}{3}\varepsilon_n\right)\omega_{*e}\omega_{De}. \tag{39}$$

Solving for the real and imaginary parts of the TE mode frequency yields,

$$\omega_r = -\frac{\omega_{*e}}{2}\left[\frac{f_t}{1\text{-}f_t}(1-\varepsilon_n) - \frac{10}{3}\varepsilon_n\right] \tag{40}$$

and growthrate,

$$\gamma = \sqrt{\omega_{*e}\omega_{De}(\eta_e - \eta_{the})} \tag{41}$$

with threshold

$$\frac{R}{L_{T_e}} \geq \frac{20(1-f_t)}{9f_t} + \frac{4}{3\varepsilon_n} + \frac{1}{2}\frac{f_t}{1-f_t}\left(1-\frac{1}{\varepsilon_n}\right)^2. \tag{42}$$

The real part of the frequency, Eq. (40) reveals that this mode propagates in the direction of the ion drift for small ε_n and in the direction of the electron drift for large ε_n. Furthermore, ω_r in both the ITG case, Eq. (32) and the TE case, Eq. (40) is directly proportional to the electron diamagnetic frequency $\omega_{*e} \sim 1/\varepsilon_n$. So, for small ε_n, the modes' real frequencies raise and the more rapid periodical motions exert a stabilising influence on the drift modes.

As we can clearly see from Eqs (40)-(42), the pure TE mode depends only on the electron temperature in contrast to the ITG mode, see Eqs. (32)-(34).The heavy ions have difficulty to keep up with the rapid movement of the much lighter electrons and therefore acts more like a stationary background with no influence on the trapped electron mode. The electrons on the other hand, may effortlessly track the ions and consequently interact with the ion temperature gradient mode. Moreover, the TE threshold (right hand side of Eq. (42)) is completely independent of any temperature. These features of the TE mode are important in the work performed in paper IV.

3. Summary of the Papers

"An expert is a man who has made all the mistakes which can be made, in a narrow field."
Niels Bohr

Since Lehnert's discovery of the anomalous transport properties of plasmas[1], many attempts to explain it have been presented. Nowadays, drift-type waves of the kind described in the two previous chapters are thought to be the main contributors to this anomalous transport. We have seen that these waves are affected by various confinement induced mechanisms, like e.g. magnetic shear damping, but also by intrinsic plasma properties like the Landau resonance. With the discovery of the high confinement mode (H-mode) in ASDEX**, a new transport reducing mechanism entered the scene: the sheared rotation. As H-mode plasmas have at least a factor two better confinement than what was normally achieved in tokamaks, theoreticians and experimentalists alike started to investigate the cause of this higher confinement and how it could be prompted. Observations made it clear that in the high confinement regime there was a radial electric field which was not present in low confinement plasmas. Moreover, this radial electrical field had strong radial dependence and together with the magnetic field confining the plasma it could give rise to a sheared $\mathbf{E}\times\mathbf{B}$-velocity (see Eq. (24)) with the ability to deform and tear apart turbulent cells, see review by Burell[4] and references therein. The effect of this rotational shear on the drift waves discussed in chapter 1 is the topic of paper I-III. In these papers the influence of a steady state electrostatic potential $\Phi_0(r)$ on drift waves in low-β tokamak plasmas is rigorously investigated. Only the larger poloidal rotation due to the toroidal magnetic field, is taken into account here as the toroidal magnetic field is much lager than the poloidal. In paper IV the rotational effect is neglected as we try to determine the influence of the temperatures alone on plasma confinement and the stiffness of the Weiland model.

** Tokamak located at IPP Garching outside Munich, Germany.

Paper I

Ship waves are the kind of waves that arise when a boat passes through water. Here though, we explore the opposite case of waves excited by a stationary object immersed in the rotating H-mode plasma, for instance a measuring rod penetrating the edge of the plasma. In the laboratory frame, such waves have zero frequency. The existence of ship waves in rotating tokamak plasma were first pointed out by Pavlenko, Yankov and Bondesson[18].

The analysis in paper I is based on a collisionless two-fluid model in a toroidal plasma with concentric circular magnetic surfaces. Due to the toroidal mode coupling we get a system of differential-difference equations which in the cases of weak and strong mode coupling are reduced to the Weber equation. For simplicity we restrict our consideration to the class of solutions which are functions of a single variable, combining the dependences on radius and poloidal angle. This limits the consideration of the boundary value problem, because the realistic boundary connected to a static obstacle is essentially not one-dimensional. Still, the one-dimensional variable allows us to study the general dispersion properties of drift waves disregarding the boundary.

From the Weber equation we get either global or propagating ship waves depending on the rotating plasma parameters. In the strong coupling approximation global ship eigenmodes exist for both positive and negative rotation velocities in contrast to propagating modes that are only present for negative velocities. The propagating modes experience magnetic shear damping as they leave the magnetic surfaces where they emerged. For the weak coupling case propagating ship wave eigenmodes do not exist, but global modes do for both positive and negative rotation velocities.

Paper II and III

The main setting is the same in paper II and III as in paper I, but here we in addition to the sheared plasma rotation due to a radial electric field, take into account a peaking of the diamagnetic frequency. Moreover, the analysis is done for drift waves of all frequencies, ω in the interval $k_{\parallel} v_{T_i} \ll \omega \ll k_{\parallel} v_{T_e}$. The peaking of the diamagnetic frequency arises from its inverse dependence on the particle inhomogeneity length scale and on the magnetic field. Hence, the diamagnetic frequency has a radial variation which it is common to neglect for simplicity. Horton et al[5] showed that in a toroidal plasma without rotation the peaking had a localising effects on two-dimensional drift modes and thus counteracts the magnetic shear damping.

The two-dimensionality of the problem calls for another strategy to find the solution compared to paper I. Instead of reducing two-fluid model to a system of differential-difference equations and further on to the one-dimensional Weber equation, we apply a prescribed solution[5] which resembles the solution to the Weber equation. By doing this we may retain the two-dimensionality of the problem and determine the mode structure in the radial and poloidal directions.

In paper II we add a sheared rotation to the model derived by Horton to study the change in the drift mode structure. Two limits are investigated, the edge and the bulk plasma. The plasma rotation is known to be larger in the edge plasma and thus we expect the effect of it to be more profound there. Another difference is that the density inhomogeneity length scale is shorter at the edge than in the bulk of the plasma. It is found that the rotational shear, to some degree, always suppresses the localisation due to the diamagnetic peaking and promotes magnetic shear damping.

The effect of the sheared rotation on instabilities causing a phase shift between the electrostatic potential and density, see Eq. (3) is investigated in paper III. Especially the Landau resonance has been studied. This is essentially a kinetic effect requiring kinetic theory to be derived. In our fluid description we have thus used the known result from kinetic theory to study the influence of the velocity shear on the Landau resonance. For both positive and negative rotation velocities it is concluded that the sheared velocity has a suppressing effect on the destabilising properties of inverse Landau damping. Moreover, in a case which without the beneficial influence of the velocity shear would be unstable, Landau damping of the drift modes is evoked.

Paper IV

With the prospect of substantial electron heating by the α-particles in fusion plasmas, an interest for the confinement properties of plasmas with the electron temperature greater than the ion temperature has emerged. In current experiments the ion temperature usually exceeds the electron temperature in the bulk of the plasma. The latter has been shown to be preferable for plasma confinement[6] in agreement with theory which predicts improved confinement for smaller T_e/T_i. For a series of hot-electron shots we have carried out simulations with JETTO[††] utilising the Weiland model. The outcome of these simulations indicates no significant dependence on the temperature ratio T_e/T_i. The short interval of T_e/T_i and that

[††] JETTO is a transport code developed at JET, Culham Science Centre, UK.

the total power was kept more or less constant in these shots may be the reasons for the lack of the T_e/T_i dependence predicted by theory.

In the second part of paper IV we attempt to evaluate if the Weiland model is stiff or not. Previous study by Dimits et al[19] suggests that the Weiland model is not stiff. A stiff plasma tries to maintain a certain temperature profile by adjusting the amount of heat flux, q, out of the plasma and this is closely related to the temperature inhomogeneity length scale, L_{T_e}. Subsequently, a plot of q versus $1/L_{T_e}$ showing a sharp increase of the heat flux for a nearly fixed value of $1/L_{T_e}$ indicates evidence of strong stiffness. Such a behaviour may as well be due to the changes in several plasma parameters in addition to the temperature inhomogeneity length scale. To investigate this behaviour, the heat flux in one channel was varied by increasing heating while keeping the temperature of the other species constant. With this procedure it is recovered that the ion channel is moderately stiff. However the electron response is still found to be stiff. Whether this is a consequence of the simultaneous variation in the temperature and its gradient remains to be clarified.

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"I am not young enough to know everything."
Oscar Wilde

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–How do you know so much?
–I asked them.
McCoy and Spock (Star Trek)

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