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Interaction between Electromagnetic Waves and Localized Plasma Oscillations

BY

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Abstract

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This thesis treats interaction between electromagnetic waves and localized plasma oscillations. Two specific physical systems are considered, namely artificially excited magnetic field-aligned irregularities (striations) and naturally excited lower hybrid solitary structures (LHSS). Striations are mainly density depletions of a few percent that are observed when a powerful electromagnetic wave, a pump wave, is launched into the ionosphere. The striations are formed by upper hybrid (UH) oscillations that are localized in the depletion where they are generated by the linear conversion of the pump field on the density gradients. However, the localization is not complete as the UH oscillation can convert to a propagating electromagnetic Z mode wave. This process, termed Z mode leakage, causes damping of the localized UH oscillation. The Z mode leakage is investigated and the theory predicts non-Lorentzian skewed shapes of the resonances for the emitted Z mode radiation. Further, the interaction between individual striations facilitated by the Z mode leakage is investigated. The LHSS are observed by spacecraft in the ionosphere and magnetosphere as localized waves in the lower hybrid (LH) frequency range that coincides with density cavities. The localized waves are immersed in non-localized wave activity. The excitation of localized waves with frequencies below LH frequency is modelled by scattering of electromagnetic magnetosonic (MS) waves off a preexisting density cavity. It is shown analytically that an incident MS wave with frequency less than the minimum LH frequency inside the cavity is focused to localized waves with left-handed rotating wave front. In addition, the theory is shown to be consistent with observations by the Freja satellite. For frequencies between the minimum LH frequency inside the cavity and the ambient LH frequency, the MS wave is instead mode converted and excites pressure driven LH oscillations. This process is studied in a simplified geometry.

Keywords: Space physics, plasma physics, radio waves, ionospheric modification, density depletions, field-aligned irregularities, lower hybrid solitary structures, lower hybrid cavities, Z mode waves, magnetosonic waves, scattering, mode conversion

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[1] J. O. Hall and T. B. Leyser. Conversion of trapped upper hybrid oscillations and Z mode at a plasma density irregularity. *Phys. Plasmas*, 10:2509–2518, 2003.

[2] J. O. Hall, Ya. N. Istomin, and T. B. Leyser. Electromagnetic interaction of localized upper hybrid oscillations in a system of density depletions. Manuscript.

[3] J. O. Hall, A. I. Eriksson, and T. B. Leyser. Excitation of localized rotating waves in plasma density cavities by scattering of fast magnetosonic waves. *Phys. Rev. Lett.*, Accepted, 2004.

[4] J. O. Hall. Conversion of localized lower hybrid oscillations and fast magnetosonic waves at a plasma density cavity. Submitted to *Phys. Plasmas*, 2004.

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Introduction

Our understanding of the near-Earth space environment has dramatically increased since it became possibility to deploy scientific instruments directly into space. The first man-made satellite, Sputnik I, was launched in 1957. The year after, the satellite Explorer I, which was equipped with a Geiger counter, was launched. This was the start of a space race during which many fields of technology and basic science developed rapidly. Space Physics emerged as a new scientific discipline from the wealth of new and exciting observations. As a spin-off of this space race, today's society benefits from the use of satellites for many purposes, e.g., for communication, weather forecasts, and positioning. In addition, other scientific disciplines than space physics are using satellites. The Hubble space telescope takes breathtaking images of stars and galaxies, the COBE satellite is measuring the cosmic background radiation and gives us a picture of the early universe, just to mention two satellite missions which are not directly connected to space physics. Modern scientific satellites are equipped with an extensive payload of instruments. The instruments are measuring physical quantities to accurately describe the space environment. In a sense, a satellite can be compared with a weather station on Earth measuring the temperature, pressure, and wind speed. However, space is filled with plasma instead of neutral gas. A plasma is a mix of charged particles which can generate and respond to electromagnetic fields. It is therefore necessary to measure a larger set of parameters than in the weather station. Satellite observation has made it possible to determine the large scale structure of the near-Earth space environment. Sounding rockets have been, and still are, a useful complement to satellite measurements close to Earth, i.e., in the ionosphere. Besides the *in situ* measurements conducted by spacecraft, ground based radio methods are suitable for studies of the near-Earth plasma.

In addition to the large-scale structures and the boundary layers separating various regions in space, spacecraft measurements have shown that small-scale phenomena are common in space plasmas. An example of such small scaled phenomena is the localized bursts of lower hybrid waves, termed lower hybrid solitary structures (LHSS) or lower hybrid cavities (LHC), first detected in 1986 by the Marie sounding rocket [LKYW86]. The wave activity coincides with density depletions which are elongated along the geomagnetic field. The perpendicular (to the geomagnetic field) width is observed to be typically a few ion gyro radii and the parallel dimension is estimated to be several order of magnitudes larger than the perpendicular. Further, these structures are always observed to be immersed in nonlocalized wave activity.

The space environment provides many opportunities to make interesting observations of naturally occurring phenomena. From another point of view, the space plasma is also an excellent plasma laboratory in which we can perform controlled and designed experiments. The plasma is virtually unlimited in size in comparison to man-made plasma chambers used in laboratory studies. For example, by injecting a powerful electromagnetic wave, a pump wave, into the ionosphere we can study the plasma response to electromagnetic waves. This type of experiments, which is termed ionospheric heating experiments or ionospheric modification, can be conducted from a number of facilities around the world. The pumped plasma can be studied by using radio methods and spacecraft. Especially, by using coherent and incoherent radar measurements it is possible to study the plasma turbulence which is excited by the pump wave. A broad range of phenomena can be observed in the pumped plasma. One of the most important is the structuring into filamentary plasma irregularities (striations). The irregularities are stretched along the geomagnetic field, similar to the LHSS, and are generated by localized plasma turbulence which is driven by the pump wave.

This thesis treats interaction between electromagnetic waves and localized plasma oscillations. Two specific physical systems are considered, namely the artificially excited density irregularities and the naturally excited lower hybrid solitary structures. Both systems are characterized by field-aligned plasma density depletions in which localized wave activity is excited. These localized oscillations can interact with long wavelength electromagnetic radiation of various polarizations. For the striations, the localized oscillations have frequencies in the upper hybrid (UH) frequency range. The UH oscillations can interact with electromagnetic Z mode wave. One consequence of this interaction is that the localized UH oscillation can convert to propagating Z mode waves. The Z mode wave transports energy away from the cavity which causes damping of the localized UH oscillation. This phenomenon, termed Z mode leakage, is considered in Paper [1]. On the other hand, the Z mode radiation provides a channel of interaction between individual filaments and can be decisive for the global response of the pumped plasma. Paper [2] treats this Z mode interaction. For the LHSS, the localized oscillations have frequencies in the lower hybrid (LH) frequency range. The localized oscillations can interact with electromagnetic waves on the R whistler/magnetosonic (MS) dispersion surface. Paper [3] treats the excitation of localized oscillations with frequencies below the LH frequency. The theoretical results are shown to be consistent with spacecraft observations of LHSS. The treatment in Paper [4] describes the excitation of pressure driven LH oscillations. Here, the frequency of the incident MS wave matches the local LH frequency and mode converts to localized oscillations.

1.1 Outline

This thesis consists of two parts, namely the present summary and a collection of four papers. The summary is organized as follows: In chapter 2 some basic plasma physics is reviewed. Chapter 3 is devoted to wave phenomena relevant for ionospheric heating experiments. The contents of Paper [1] and [2] are summarized. Also, some aspects of important previous work are reviewed. Chapter 4 treats the interaction between magnetosonic waves and localized oscillations in the lower hybrid frequency range. The contents of Paper [3] and [4] are summarized. In chapter 5, some suggestions for future work are given. Chapter 6 contains a short summary in Swedish.

A Plasma Physics Primer

Plasma physics is the theoretical foundation of space physics and describes the collective behavior of quasi-neutral gases of charged and neutral particles, i.e., a gas consisting of an equal amount of positive and negative charges. The theory can be applied to describe various phenomena in space and laboratory plasma. Other interesting and important applications of plasma physics are found in the research in thermo nuclear fusion as a future source of energy. To quantitatively describe the dynamics of the plasma it is necessary to formulate the equations of motion. In the majority of applications of plasma theory in space physics it is sufficient to use classical mechanics to describe the particle motion and classical electrodynamics to describe the electromagnetic field. Relativistic effects can be ignored in most cases, but need to be considered in certain astrophysical systems. Instead of tracking each individual particle in the plasma it is often convenient to use statistical mechanics to describe the gas of electrons and ions. Statistical mechanics describes the plasma in terms of the distribution function f. The distribution function describes, in a probabilistic sense, how many particles there are in a certain volume element having a certain velocity at a certain time. Mathematically speaking, the distribution function depends on seven variables. If short range interactions, such as binary collisions, are neglected the time evolution of f is described by the Vlasov equation. As the particles in the plasma are electrically charged, any motions inside the plasma will result in currents and local space charge. The Vlasov equation must therefore be solved together with Maxwell's equations to describe the electromagnetic fields excited by the currents and space charge. This set of equations describes self-consistently the motion of the plasma and the electromagnetic fields. In addition to the high dimensionality of the Vlasov equation, the governing equations are nonlinear which makes the mathematical analysis difficult. Even with today's supercomputers it is only possible to solve relatively simple model problems. However, the complexity of the governing equations is only reflecting the great richness of physical phenomena in the dynamics of plasma.

Wave phenomena in plasma have been studied extensively both experimentally in laboratories and space plasma and by theoretical considerations. In addition to the transversal electromagnetic wave mode found in vacuum, a plasma can support many different modes of wave propagation. The theory for small amplitude waves in a homogeneous plasma is well developed. It is possible to solve the linearized equation of motions exactly for small fluctuations around a stationary Maxwell distribution in the velocity space. The solution describes the dispersive properties of the wave, i.e., the relation between wave frequency and wavelength, and the polarization of the electric and magnetic wave fields. Many interesting phenomena, such as collisionless damping of plasma waves, are described by the linear theory for homogeneous plasma. However, when plasma are encountered in nature they are always inhomogeneous to some extent. There is no general method in existence for analyzing waves in inhomogeneous plasma. Each physical system must be analyzed instead with specialized models and methods. When allowing for finite amplitude the various wave modes are coupled by nonlinear terms and energy can flow between the wave modes. This type of wave interaction is termed parametric process. As an example, a large amplitude transverse electromagnetic wave can decay into an electrostatic Langmuir wave and another electromagnetic wave. Also, a few forms of strong turbulence, e.g. Langmuir turbulence, are fairly well understood [Rob97].

The mathematical formulation of the Vlasov equation and Maxwell's equations are given in section 2.1. In the present thesis the Vlasov equation is not used directly, instead two types of approximations are used. The cold plasma approximation is used as a starting point for the analytical treatment presented in Paper [3] and is considered in section 2.2. The finite Larmor radius (FLR) approximation is important in the treatments in Papers [1,2,4]. This approximation is briefly discussed in section 2.3.

2.1 Basic Equations

The evolution of the distribution function f_α for species α is described by the Vlasov equation. If all other forces but the electromagnetic are neglected the equation is [Che83]

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{x}} + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = 0 \tag{2.1}$$

where q_α (m_α) is the charge (mass) of the species α, $\mathbf{E}$ is the electric field, and $\mathbf{B}$ is the magnetic field. The distribution function f_α is a function of the spatial coordinates $\mathbf{x}$, the velocity $\mathbf{v}$, and time t, i.e., $f_\alpha = f_\alpha(\mathbf{x}, \mathbf{v}, t)$. The distribution function contains a statistical description of the microscopic state and all macroscopic quantities can be calculated as statistical moments of f_α. The number of particles per unit volume n_α, i.e., the number density, can be calculated by integrating f_α over the velocity space, we have that

$$n_\alpha(\mathbf{x}, t) = \int f_\alpha(\mathbf{x}, \mathbf{v}, t)\, dv^3 \,. \tag{2.2}$$

The macroscopic mean velocity is given by the integral

$$\mathbf{v}_\alpha(\mathbf{x},t) = \frac{1}{n_\alpha(\mathbf{x},t)} \int \mathbf{v}\, f_\alpha(\mathbf{x},\mathbf{v},t)\, dv^3 . \tag{2.3}$$

The electrical charge density ρ and current density $\mathbf{J}$ can be expressed in terms of the distribution function. The charge density ρ is

$$\rho(\mathbf{x},t) = \sum_\alpha q_\alpha \int f_\alpha(\mathbf{x},\mathbf{v},t)\, dv^3 = \sum_\alpha q_\alpha n_\alpha , \tag{2.4}$$

and the current density $\mathbf{J}$ is

$$\begin{aligned} \mathbf{J}(\mathbf{x},t) &= \sum_\alpha q_\alpha n_\alpha \int \mathbf{v}\, f_\alpha(\mathbf{x},\mathbf{v},t)\, dv^3 \\ &= \sum_\alpha q_\alpha n_\alpha \mathbf{v}_\alpha = \sum_\alpha \mathbf{J}_\alpha , \end{aligned} \tag{2.5}$$

where the sums are over all species in the plasma.

The fundamental equations in classical electrodynamics are the well known Maxwell equations which describe $\mathbf{E}$ and $\mathbf{B}$. The equations are [Jac75]

$$\nabla \cdot \mathbf{E} = \rho/\varepsilon_0 , \tag{2.6}$$

$$\nabla \cdot \mathbf{B} = 0 , \tag{2.7}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} , \tag{2.8}$$

$$\nabla \times \mathbf{B} = \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J} , \tag{2.9}$$

where $\mathbf{J}$ and ρ are given by Eqs. (2.4) and (2.5), respectively. The above set of equations describes the dynamics of the plasma and the electromagnetic field self-consistently.

2.2 Cold Plasma Approximation

It is often useful to consider various approximations of the Vlasov equation. A common and important one is the cold plasma approximation in which the thermal motion of the particles in the plasma is neglected. All particles in a volume dx^3 around $\mathbf{x}$ move with the same velocity $\mathbf{v}_\alpha$. By averaging Eq. (2.1) over the velocity space one obtains the continuity equation for each species, giving

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0 . \tag{2.10}$$

By calculating the first statistical moment of Eq. (2.1) one obtains a momentum balance equation, which is

$$\frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\mathbf{v}_\alpha = \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}) \,. \tag{2.11}$$

When thermal motion is included a term due to the pressure will appear in Eq. (2.11).

The conservation laws Eqs (2.10) and (2.11) are, despite their simple structure, difficult to solve because of the nonlinear terms. However, in many situations it is sufficient to consider small amplitude deviations from a stationary state which can be characterized by the magnetic field $\mathbf{B}_0$, electric field $\mathbf{E}_0$, density n_0, and velocity $\mathbf{v}_0$. The equations of motions can be linearized around the background quantities and all fluctuations can be assumed to have a harmonic time dependence, e.g., $\mathbf{E}(\mathbf{x},t) = \mathrm{Re}\left[\mathbf{E}(\mathbf{x})\exp(-i\,\omega t)\right]$ where ω is the wave (angular) frequency. For a magnetized plasma with $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, $\mathbf{E}_0 = 0$, $\mathbf{v}_0 = 0$, and $n_0 = n_0(\mathbf{x})$, Eq. (2.11) can be linearized and solved with respect to $\mathbf{v}_\alpha$ in terms of $\mathbf{E}$. The current of species α is in the linear approximation given by

$$\mathbf{J}_\alpha = i\omega\varepsilon_0 \frac{\omega_{p\alpha}^2}{\omega^2 - \Omega_\alpha^2}\left(\mathbf{E}_\perp - i\frac{\Omega_\alpha}{\omega}\hat{\mathbf{z}} \times \mathbf{E}_\perp\right) + i\omega\varepsilon_0 \frac{\omega_{p\alpha}^2}{\omega^2} E_z\hat{\mathbf{z}} \equiv \sigma_\alpha \mathbf{E}\,, \tag{2.12}$$

where $\mathbf{E}_\perp$ is the electric field component perpendicular to $\mathbf{B}_0$, $\Omega_\alpha = q_\alpha B_0/m_\alpha$ is the gyro frequency, and $\omega_{p\alpha}^2 = q_\alpha^2 n_0/(\varepsilon_0 m_\alpha)$ is the plasma frequency. For our purposes it is useful to define the susceptibility tensor χ_α in terms of the conductivity tensor σ_α as $\chi_\alpha = i/(\varepsilon_0\omega)\sigma_\alpha$. Further, the dielectric tensor ε is defined by

$$\varepsilon = I + \sum_\alpha \chi_\alpha\,, \tag{2.13}$$

where I is the identity matrix. These definitions allow us to write Eq. (2.9) as $\nabla \times \mathbf{B} = -i\omega/c^2\,\varepsilon\mathbf{E}$, where c is the vacuum light speed. The electric displacement, $\varepsilon\mathbf{E}$, can be written as

$$\varepsilon\mathbf{E} = S\mathbf{E}_\perp + iD\hat{\mathbf{z}} \times \mathbf{E}_\perp + PE_z\hat{\mathbf{z}}\,. \tag{2.14}$$

The elements are $S = (R+L)/2$, $D = (R-L)/2$, and

$$P = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}\,, \tag{2.15}$$

where

$$R = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}\frac{\omega}{\omega + \Omega_\alpha} \quad \text{and} \quad L = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}\frac{\omega}{\omega - \Omega_\alpha}\,. \tag{2.16}$$

In a rectangular coordinate system ε can be represented as

$$\varepsilon = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} . \tag{2.17}$$

Note that the cold plasma is temporal dispersive as ε depends on ω. Further, in an inhomogeneous plasma the elements of ε are functions of $\mathbf{x}$ because of the variations of $\omega_{p\alpha} = \omega_{p\alpha}(\mathbf{x}) \propto \sqrt{n_0(\mathbf{x})}$.

A wave equation governing $\mathbf{E}$ can be obtained by combining the Maxwell equations. We have that

$$\nabla \times (\nabla \times \mathbf{E}) - k_0^2 \varepsilon \mathbf{E} = 0 , \tag{2.18}$$

where $k_0 = \omega/c$. The wave magnetic field can be calculated from Eq. (2.8), i.e., $\mathbf{B} = -i/\omega \nabla \times \mathbf{E}$. A homogeneous plasma with constant n_0 supports plane waves of the form $\mathbf{E} = \exp(i\mathbf{k} \cdot \mathbf{x})\hat{\mathbf{e}}$ where $\mathbf{k}$ is the wave vector and $\hat{\mathbf{e}}$ describes the polarization of $\mathbf{E}$. For plane waves Eq. (2.18) is reduced to a set of algebraic equations. In order to have a non trivial solution of these equations $\mathbf{k}$ and ω must satisfy the dispersion relation

$$\det |N^2(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j - \delta_{ij}) + \varepsilon_{ij}(\omega)| = 0 , \tag{2.19}$$

where $N = k/k_0$ is the refractive index, $\hat{\mathbf{k}} = \mathbf{k}/k$, and δ_{ij} is the Kronecker delta. The dispersion relation can be solved with respect to ω or any components of $\mathbf{k}$, depending on the situation. The polarization vector for each solution can be calculated from the wave equation. The plane wave solutions describe all wave properties in a homogeneous plasma. However, when some of the background quantities are spatially inhomogeneous the wave equation (2.18) must be solved by some other method. In the above discussion we used a wave equation for $\mathbf{E}$, but it is also possible to derive a wave equation for $\mathbf{B}$. We have that

$$\nabla \times \left(\varepsilon^{-1} \nabla \times \mathbf{B}\right) - k_0^2 \mathbf{B} = 0 \tag{2.20}$$

and $\mathbf{E} = i\omega/k_0^2 \varepsilon^{-1}\mathbf{B}$. This formulation is applicable when ε is non singular, i.e., $\det|\varepsilon| = RLP \neq 0$.

2.3 Finite Larmor Radius Approximation

The cold plasma approximation describes many plasma wave phenomena with a good accuracy, even when the plasma is rather hot. However, the cold plasma approximation fails completely near certain resonance frequencies where the approximation predicts infinitely small wavelengths, or infinitely large k. However, a kinetic treatment based on the Vlasov equation shows that these resonances are artificial. The spatial dispersion described by the Vlasov equation becomes important for small length scales and allows us to describe the physics on the cold plasma resonances. Many important properties of the spatial dispersion can be described by the finite Larmor radius (FLR) approximation.

To see how these resonances arise in the cold plasma theory and how they are effected by FLR effects we consider wave propagation strictly perpendicular to the background magnetic field. The cold plasma theory admits two wave modes for a given ω. These modes are the O (ordinary) and X (extraordinary) mode. With $\mathbf{k} = k\hat{\mathbf{x}} \perp \mathbf{B}$ and $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ the O mode electric field is linearly polarized with $\mathbf{E} \parallel \mathbf{B}_0$ and the X mode electric field is elliptically polarized with $\mathbf{E} \perp \mathbf{B}_0$. For the X mode with $\mathbf{E} \perp \hat{\mathbf{z}}$ the wave equation (2.18) reduces to

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} - N^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = 0\,. \tag{2.21}$$

In order to have a non trivial solution with $\mathbf{E} \neq 0$ the determinant of the matrix on the left-hand side must equal zero. We obtain the dispersion relation

$$N^2 = \frac{\varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}\varepsilon_{yx}}{\varepsilon_{xx}}\,, \tag{2.22}$$

which in combination with Eq. (2.21) gives the polarization vector

$$\hat{\mathbf{e}} = \varepsilon_{xy}\hat{\mathbf{x}} - \varepsilon_{xx}\hat{\mathbf{y}}\,. \tag{2.23}$$

Note that $k = k_0 N$ tends to infinity, or the wavelength tends to zero, when $\varepsilon_{xx} = 0$. Further, $\mathbf{E}$ becomes parallel to $\mathbf{k}$ and describes a purely longitudinal wave. In addition, according to the Maxwell equation (2.8), the wave $\mathbf{B}$ vanishes and the wave is purely electrostatic. For a two component plasma with electrons and one ion species the resonances occur when ω is equal to the UH frequency,

$$\omega_{\mathrm{UH}}^2 = \omega_{\mathrm{pe}}^2 + \Omega_{\mathrm{e}}^2 \tag{2.24}$$

or equal to the LH frequency,

$$\omega_{\mathrm{LH}}^2 = \frac{\omega_{\mathrm{pi}}^2}{1 + \left(\omega_{\mathrm{pe}}/\Omega_{\mathrm{e}}\right)^2}\,, \tag{2.25}$$

where Ω_{e} is the electron gyro frequency and ω_{pe} (ω_{pi}) is the electron (ion) plasma frequency. To resolve the wave properties at ω_{UH} and ω_{LH} it is necessary to include corrections due to FLR. For a hot plasma in a magnetic field the susceptibility tensor χ depends both on ω and $\mathbf{k}$. The $\mathbf{k}$ dependence describes the spatial dispersion due to thermal motion in the plasma. In our case with $\mathbf{k} \perp \mathbf{B}_0$, $\chi_\alpha = \chi_\alpha(\omega, \lambda_\alpha)$ where $\lambda_\alpha = (\rho_{\mathrm{L}\alpha} k)^2/2$ and $\rho_{\mathrm{L}\alpha}$ is the Larmor radius. If we regard λ_α as a small parameter we can expand χ_α in a Taylor series. To the lowest non-vanishing order,

$$\chi_\alpha \approx \chi_\alpha^{(0)} + \lambda_\alpha \chi_\alpha^{(1)} . \tag{2.26}$$

$\chi_\alpha^{(0)}$ can be identified as the susceptibility tensor for a cold plasma. In the UH frequency range the ion dynamics can be neglected and the corrected dielectric tensor is

$$\varepsilon^{(1)} \approx \varepsilon + \rho_{\mathrm{Le}} \chi^{(1)} k^2/2 , \tag{2.27}$$

where $\varepsilon = I + \chi^{(0)}$ is the cold plasma dielectric tensor. In view of the dispersion relation Eq. (2.22), the most important correction is to ε_{xx} which is vanishing at ω_{UH} in the cold plasma approximation. With the corrected dielectric tensor $\varepsilon^{(1)}$ we obtain the dispersion relation

$$\Lambda_{xx} N^4 - \varepsilon_{xx} N^2 + \varepsilon_{xx} \varepsilon_{yy} - \varepsilon_{xy} \varepsilon_{yx} = 0 , \tag{2.28}$$

where $\Lambda_{xx} = -(\rho_{\mathrm{Le}} k_0)^2 \chi_{xx}^{(1)}/2$. Note that Eq. (2.28) is a quadratic equation in N^2 and describes two modes. The new mode, which is not described by Eq. (2.22), is a pressure driven oscillation which propagates for $\omega > \omega_{\mathrm{UH}}$. Both solutions N^2 are now finite, even for $\omega = \omega_{\mathrm{UH}}$ (see Figs. 2 and 3 in Paper [1]). The FLR expansion method is not limited to the considered case with $k_z = 0$. It is also possible to retain the full kinetic parallel dispersion [Bra98].

Interaction Between Z mode Waves and Localized Upper Hybrid Oscillations

One of the most important effects of injecting large amplitude electromagnetic waves from the ground into the ionosphere is the structuring of the plasma into filamentary plasma irregularities. The structures are stretched along the ambient geomagnetic field because of the much larger thermal conductivity along the magnetic field than across. The perpendicular (to the magnetic field) length scale of the small scale irregularities is smaller than the ion Larmor radius but much larger than the electron Larmor radius. The density depletions are a few percent deep. These striations are central to our understanding of a number of phenomena, including anomalous absorption of radio waves [SKJR82], stimulated electromagnetic emissions (SEE) [TKS82, Ley01], Langmuir turbulence evolution [ND90], and field-aligned scattering of radio waves [MKW74]. The structures are formed by UH oscillations that are trapped in the depletion where they are generated by linear conversion of the pump field on the density gradients [GZL95, IL97]. As a consequence, the striations are only observed when the electromagnetic pump wave is in O mode which is reflected above the layer where the wave frequency ω matches the UH frequency. The X mode is reflected at a lower altitude and does not reach the UH layer. Figure 3.1 shows a schematic picture of the experiment.

The dynamics of the plasma density and the plasma temperature when exposed to a powerful O mode pump wave can be described by [IL97]

$$\left(\frac{\partial}{\partial t} - D_{1,2}\nabla_{\perp}^{2}\right)\Psi_{1,2} = Q\,, \tag{3.1}$$

where Ψ_1 and Ψ_2 are linear combinations of the relative density profile η and the plasma temperature. The right-hand side describes the plasma heating and is proportional to the squared amplitude of the high frequency electric field $\mathbf{E}_\mathrm{h}$, i.e., $Q \propto |\mathbf{E}_\mathrm{h}|^2$. The O mode pump wave $\mathbf{E}_\mathrm{O}$ is scattered off the density irregularity and excites electrostatic UH oscillations with electric field $\mathbf{E}_1$. This scattering process is described by

$$\nabla\cdot[(\mathcal{H}-\eta)\mathbf{E}_1] = \nabla\eta\cdot\mathbf{E}_\mathrm{O}\,, \tag{3.2}$$

where $\mathcal{H}$ is a differential operator describing UH oscillations in a homogeneous plasma. The above system of equations describes self-consistently the evolution of the electrostatic field $\mathbf{E}_1$, the plasma density, and the temperature of the plasma. The large amplitude pump wave excites UH waves by scattering off a density perturbation η which initially is small. This scattering process is described by Eq. (3.2). The excited UH wave is damped by weak collisional damping. In the collisional process the energy in the UH wave is dissipated into heat. As a result of the heating, the electron temperature will increase in regions where $|\mathbf{E}_\mathrm{h}|^2 = |\mathbf{E}_\mathrm{O} + \mathbf{E}_1|^2$ is large. If the amplitude of the pump wave is sufficiently large, an initially shallow density cavity will start to grow. This is the so called thermal parametric instability (TPI). The TPI [GT76] is generally accepted in the community as the initial state of the striations.

The filamentary density irregularities with their trapped UH oscillations constitute a strongly inhomogeneous form of plasma turbulence. However, the short wave UH oscillations are not completely trapped, but are partially transmitted through the trapping depletion into the long wave electromagnetic Z mode, a phenomenon which has been termed Z mode leakage [DMPR82, Mjø83]. The Z mode wave transports energy away from the cavity and decreases the amplitude of the localized oscillation that sustain the cavity. The Z mode radiation can therefore directly affect the dynamics of the density cavity by decreasing the locally dissipated energy. The Z mode leakage from an isolated density cavity is considered in section 3.1.

The Z mode leakage from the individual filaments also facilitates an interaction mechanism [Mjø83] between different filaments. The energy lost from one filament is gained by its neighbors. This mechanism reduces the total outflux of energy from the collective of striations. Hence, the Z mode radiation acts as an agent for long range interaction between regions of localized UH turbulence. With this interaction the small scaled turbulence is coupled to the large scale behavior of the turbulent plasma. This Z mode interaction between density cavities is considered in section 3.2.

3.1 Z Mode Leakage From an Isolated Density Cavity

To investigate the Z mode leakage from one isolated density cavity we use the simplest possible model. The O mode pump wave is assumed to propagate parallel to $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ so that the circular polarized field $\mathbf{E}_\mathrm{O}$ is perpendicular to $\mathbf{B}_0$. The leakage process is considered in the stationary state and a preexisting stationary density cavity is assumed. The cavity is modeled by a one dimensional structure so that the density is varying in one direction perpendicular to $\mathbf{B}_0$, $n(x) = n_0^\infty[1+\eta(x)]$ where $|\eta| \ll 1$ is the density variation associated with the cavity and n_0^∞ is the density of the ambient plasma. The depletion is assumed to be small scale in the sense that the characteristic width $L_\perp$ perpendicular to $\mathbf{B}_0$ is much smaller than the wavelength of the freely propagating Z mode wave, i.e., $L_\perp \ll L_\mathrm{Z} \equiv 2\pi/k_\mathrm{Z}$. This condition is realized when ω is

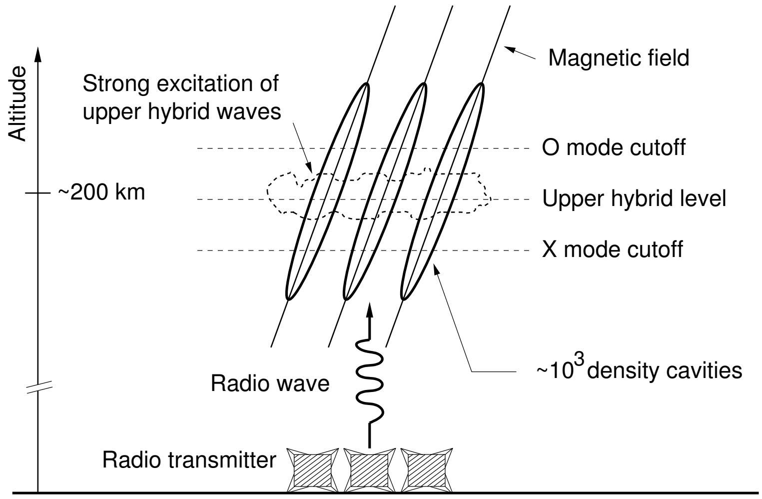

Figure 3.1: Schematic illustration of ionospheric heating experiments.

below and not too close to $\omega_{\text{UH}}^{\infty}$ (UH frequency of the ambient homogeneous plasma). For simplicity, the cavity is considered to be symmetric around $x = 0$, i.e., $\eta(-x) = \eta(x)$. Further, the wave properties of the O mode pump along $\mathbf{B}_0$ are neglected and the pump is scattered off the cavity to excite the electric field $\mathbf{E}$ which propagates in the direction of the density gradient, i.e., $\mathbf{E} = \mathbf{E}(x)$. The model must include FLR effects in order to describe UH waves since the cold plasma theory predicts a resonance at the UH frequency, as discussed in section 2.3. The dielectric properties can be approximately described by the dielectric tensor operator

$$\mathcal{E} = \varepsilon^{\infty} + \eta\chi^{(0)} - 1/2\rho_{\text{L}}^{2}\chi^{(1)}\frac{d^2}{dx^2}\,, \tag{3.3}$$

which was derived using the lowest order FLR approximation in Paper [1]. The first term $\varepsilon^{\infty} = I + \chi^{(0)}$ is the dielectric tensor for a cold magnetized plasma evaluated for the plasma parameters outside the cavity. The second term is a correction due to the density variation across the magnetic field. Thus, $\varepsilon^{\infty} + \eta\chi^{(0)}$ is the local cold plasma dielectric tensor. The FLR correction is described by the third term. By combining Maxwell's equations and the dielectric tensor operator, Eq. (3.3), we obtain a vector wave equation governing $\mathbf{E}$, giving

$$\left[k_0^{-2}\Lambda\frac{d^2}{dx^2} + \varepsilon^{\infty} + \eta\chi^{(0)}\right]\mathbf{E} = -\eta\chi^{(0)}\mathbf{E}_0\,, \tag{3.4}$$

where the tensor $\Lambda = \hat{\mathbf{y}}\hat{\mathbf{y}} + \hat{\mathbf{z}}\hat{\mathbf{z}} - (\rho_{\text{L}}k_0)^2\chi^{(1)}/2$ contains terms originating from the Maxwell equation $\nabla \times \mathbf{E} = i\omega\mathbf{B}$ and the FLR correction. The right-hand side of Eq. (3.4) describes the scattering of the O mode pump field off the density cavity. The E_z component is, due to the structure of the tensors Λ, ε^∞, and $\chi^{(0)}$, not coupled to E_x or E_y and describes O mode waves propagating perpendicular to $\mathbf{B}_0$. This component is not excited by the pump and will not be considered further. The two perpendicular components are coupled and describe two wave modes.

In the homogeneous plasma outside the cavity the wave modes are uncoupled and the general solution to Eq. (3.4) can be written as a superposition of plane waves. We have that

$$\mathbf{E}(x) = (A_\mathrm{X}^+ e^{ik_\mathrm{X}x} + A_\mathrm{X}^- e^{-ik_\mathrm{X}x})\hat{\mathbf{e}}_\mathrm{X} + (A_\mathrm{Z}^+ e^{ik_\mathrm{Z}x} + A_\mathrm{Z}^- e^{-ik_\mathrm{Z}x})\hat{\mathbf{e}}_\mathrm{Z}\,, \tag{3.5}$$

where $A_I^\pm$ are some constants. The wave numbers k_X and k_Z can be obtained from the dispersion relation

$$\Lambda_{xx}N^4 - \varepsilon_{xx}^\infty N^2 + \varepsilon_{xx}^\infty\varepsilon_{yy}^\infty - \varepsilon_{xy}^\infty\varepsilon_{yx}^\infty = 0\,, \tag{3.6}$$

where $N = k/k_0$ is the refractive index. The polarization vector $\hat{\mathbf{e}}_\mathrm{X}$ and $\hat{\mathbf{e}}_\mathrm{Z}$ can be calculated from the wave equation. For $\omega < \omega_\mathrm{UH}^\infty$, we have $N_\mathrm{X} \approx N_\mathrm{UH}$ where $N_\mathrm{UH}^2 = \varepsilon_{xx}^\infty/\Lambda_{xx} < 0$ is the refractive index for an UH wave in the electrostatic FLR approximation. The other solution is $N_\mathrm{Z}^2 \approx (\varepsilon_{xx}^\infty\varepsilon_{yy}^\infty - \varepsilon_{xy}^\infty\varepsilon_{yx}^\infty)/\varepsilon_{xx}^\infty$ which is the refractive index for a Z mode wave in the cold plasma approximation. For $\omega > \omega_\mathrm{UH}^\infty$, we have $N_\mathrm{Z} \approx N_\mathrm{UH}$. For our case with $\omega < \omega_\mathrm{UH}^\infty$, the X mode polarization vector $\hat{\mathbf{e}}_\mathrm{X}$ is essentially parallel to $\hat{\mathbf{x}}$, i.e., in the direction of propagation, and describes a longitudinal electrostatic wave. With an appropriate normalization, $\hat{\mathbf{e}}_\mathrm{X} \approx \hat{\mathbf{x}}/\sqrt{\Lambda_{xx}}$. The Z mode polarization vector for $\omega < \omega_\mathrm{UH}^\infty$ is $\hat{\mathbf{e}}_\mathrm{Z} \approx -\varepsilon_{xy}^\infty/\varepsilon_{xx}^\infty\hat{\mathbf{x}} + \hat{\mathbf{y}}$. In the inhomogeneous plasma inside the cavity the two modes are coupled and mode conversion can occur.

3.1.1 Electrostatic Treatment

Before considering the conversion process described by Eq. (3.4) it is instructive to consider the electrostatic approximation in the inhomogeneous plasma in some detail. By assuming that the excited field is purely electrostatic, i.e., $\mathbf{E} = -\nabla\phi = E_x(x)\hat{\mathbf{x}}$ where ϕ is the electrostatic potential, the vector wave equation (3.4) is reduced to a single equation, we have that

$$\mathcal{L}_\mathrm{UH}E_x = -Q_\mathrm{O}\eta E_\mathrm{O}\,, \tag{3.7}$$

where $\mathcal{L}_\mathrm{UH} = k_\mathrm{UH}^{-2}d^2/dx^2 + 1 + Q_\mathrm{U}\eta$, $Q_\mathrm{U} = \chi_{xx}^{(0)}/\varepsilon_{xx}^\infty$, and $Q_\mathrm{O} = \hat{\mathbf{x}}\chi^{(0)}\hat{\mathbf{e}}_\mathrm{O}/\varepsilon_{xx}^\infty$. For localized waves with $\omega < \omega_\mathrm{UH}^\infty$ the UH oscillation is evanescent outside the cavity and E_x must go to zero far away from the cavity. If $\omega > \omega_\mathrm{UH}^\mathrm{min}$ (the minimum UH frequency inside the cavity) the UH oscillations can propagate inside the cavity and are excited by the O mode pump. Equation (3.7) can be solved using the WKB approximation [DMPR82]. However, for our purposes it is more convenient to use a Green's function approach to "invert" the operator $\mathcal{L}_{\mathrm{UH}}$. The Green's function is determined by the equation $\mathcal{L}_{\mathrm{UH}} G_{\mathrm{UH}}(x,x_1) = \delta(x-x_1)$ and the boundary condition $G_{\mathrm{UH}} \to 0$ as $|x| \to \infty$, where $\delta(x)$ is the Dirac delta function. G_{UH} describe the excitation of UH waves by a point source located at $x = x_1$. An attractive representation of G_{UH} can be constructed as a superposition of all possible eigenmodes which can be excited. In a homogeneous plasma the eigenmodes are plane waves of the form $\psi_k = \exp(i\mathbf{k}\cdot\mathbf{x})$ and any wave field can be written as a superposition of such waves. In our case with an inhomogeneous plasma, the harmonic wave $\exp(i\mathbf{k}\cdot\mathbf{x})$ is no longer an eigenmode and we have to find a set of eigenmodes appropriate for the considered density profile. There are two types of eigenmodes, namely nonlocalized propagating waves with a continuum of possible frequencies and localized oscillations with a discrete set of eigen frequencies. The Green's function can be written as a superposition of these modes [Fri48]

$$G_{\mathrm{UH}}(x,x_1) = \sum_{n=0}^{M} \frac{\psi_n(x)\,\psi_n^*(x_1)}{1-\Delta_n/\Delta} + \frac{1}{i\pi}\int_{-\infty}^{\infty} \frac{\psi_k^+(x)\,\psi_k^-(x_1)}{1-k^2/\Delta}\,\frac{k\,dk}{W[\psi_k^+,\psi_k^-]}\,, \quad (3.8)$$

where the first term derives from the discrete spectrum of $\mathcal{L}_{\mathrm{UH}}$ and the second from the continuous spectrum. The parameter $\Delta \equiv -Q_{\mathrm{U}}^{-1} \propto \omega - \omega_{\mathrm{UH}}^{\infty}$ quantifies the deviation of ω from the ambient UH frequency. The normalized ψ_n which are associated with the discrete spectrum and the discrete eigenvalues Δ_n ($\min\eta < \Delta_0 < \Delta_1 < \cdots < \Delta_M < 0$) are determined by the equation $\mathcal{L}_{\mathrm{UH}}\,\psi_n = (1-\Delta_n/\Delta)\,\psi_n$ and the boundary conditions $\psi_n \to 0$ as $|x| \to \infty$. Due to the symmetry of $\eta(x)$ around $x = 0$, the eigenfunctions ψ_n are even for even n and odd for odd n. The eigenfunctions $\psi_k^{\pm}$ satisfy the equation $\mathcal{L}_{\mathrm{UH}}\psi_k^{\pm} = (1-k^2/\Delta)\psi_k^{\pm}$ and the boundary condition $\psi_k^{\pm} \propto \exp(\pm ikx)$ as $x \to \pm\infty$. $W[\psi_k^+,\psi_k^-]$ is the Wronskian.

For localized oscillations with $\omega < \omega_{\mathrm{UH}}^{\infty}$, or equivalently $\Delta < 0$, only the discrete part of the spectrum can be excited resonantly. The integrand in Eq. (3.8) has poles at $k = \pm i\sqrt{|\Delta|}$ and the integral term represents excitation of quasi modes. If the contribution from the continuous spectrum is neglected the solution to Eq. (3.7) can be written as

$$E_x(x)/E_{\mathrm{O}} \propto \sum_{n=0}^{M} \frac{\eta_n^*}{\Delta-\Delta_n}\psi_n(x)\,, \quad (3.9)$$

where $\eta_n = \int_{-\infty}^{\infty} \eta\psi_n\,dx_1$. Even though we have not specified ψ_n and Δ_n, the solution Eq. (3.9) gives a clear picture of the excitation of localized electrostatic UH oscillations. When ω is such that $\Delta \approx \Delta_n$ the amplitude of the UH oscillation will be large. When $\Delta = \Delta_n$ the pump wave is resonantly exciting the eigen mode ψ_n and the stationary solution Eq. (3.9) predicts an infinite amplitude at the resonance. Physically, the amplitude cannot be infinite and the oscillation must be damped by some linear or nonlinear mechanism. Further, the pump wave, which is assumed to be homogeneous, cannot excite the odd resonances. The UH amplitude is proportional to the overlap η_n which is vanishing when ψ_n is odd. The spectral representation of E_x has previously been applied to describe parametric decay of localized UH oscillations into lower hybrid waves [Mjø97].

3.1.2 Solution by the Scale Separation Method

In the electrostatic approximation discussed above it is possible to interpret the excitation of localized UH oscillations in terms of the cavities eigen modes. This interpretation is attractive, but in order to model the Z mode leakage we must include electromagnetic terms. Due to coupling between the components in Eq. (3.4) we are faced with a fourth order differential equation and the eigen function expansion method can not be applied directly. This is the topic of Paper [1] where we present an electromagnetic generalization of the electrostatic treatment above. Analytic expressions for the Z mode amplitude and the localized wave field are given in terms of the electrostatic eigenfunctions. The method developed in Paper [1] is similar to the source approximation first introduced to obtain analytic expressions for mode conversion of Langmuir and electromagnetic waves in unmagnetized plasma [HLFM89].

The components E_x and E_y are coupled in the wave equation (3.4) both inside and outside the cavity. The coupling is due to the magnetization of the plasma which implies that ε^∞ and Λ have off-diagonal elements. We know from the plane wave solution Eq. (3.5) that the two modes are independent of each other outside the cavity. Thus, by expressing $\mathbf{E}$ in terms of the polarization vectors the equations are uncoupled in the ambient homogeneous plasma. As $\hat{\mathbf{e}}_\mathrm{X}$ and $\hat{\mathbf{e}}_\mathrm{Z}$ are not parallel it is possible to describe any polarization as a superposition of these vectors. Without loss of generality we can write the solution to Eq. (3.4) as

$$\mathbf{E}(x) = E_\mathrm{X}(x)\hat{\mathbf{e}}_\mathrm{X} + E_\mathrm{Z}(x)\hat{\mathbf{e}}_\mathrm{Z}\,. \tag{3.10}$$

In this representation of $\mathbf{E}$, Eq. (3.4) is transformed to the component equations

$$\mathcal{L}_\mathrm{X} E_\mathrm{X} + Q_\mathrm{XZ}\eta E_\mathrm{Z} = -Q_\mathrm{XO}\eta E_\mathrm{O}\,, \tag{3.11}$$

$$\mathcal{L}_\mathrm{Z} E_\mathrm{Z} + Q_\mathrm{ZX}\eta E_\mathrm{X} = -Q_\mathrm{ZO}\eta E_\mathrm{O}\,, \tag{3.12}$$

which are uncoupled outside the cavity where $\eta = 0$. The operators are $\mathcal{L}_\mathrm{X} \equiv k_\mathrm{X}^{-2} d^2/dx^2 + 1 + Q_\mathrm{XX}\eta$ and $\mathcal{L}_\mathrm{Z} \equiv k_\mathrm{Z}^{-2} d^2/dx^2 + 1 + Q_\mathrm{ZZ}\eta$ where $k_\mathrm{X} \equiv k_0 N_\mathrm{X}$ and $k_\mathrm{Z} \equiv k_0 N_\mathrm{Z}$. The coupling constants are $Q_{IJ} = (\hat{\mathbf{e}}_I^\dagger \chi^{(0)}\hat{\mathbf{e}}_J)N_I^{-2}$ where $I,J = \mathrm{X},\mathrm{Z},\mathrm{O}$. It is important to note that the transformed equations (3.11) and (3.12) are equivalent to the original Eq. (3.4). The operator $\mathcal{L}_\mathrm{X}$ is approximately equal to $\mathcal{L}_{\mathrm{UH}}$ which describes localized UH oscillations. We can therefore identify Eq. (3.11) as a generalization of the electrostatic wave equation (3.7).

The above choice of basis vectors is rather natural and it decouples the equations outside the cavity. However, it is important to have a physical interpretation of $E_{\mathrm{X}}(x)$ and $E_{\mathrm{Z}}(x)$ for all x. Outside the cavity, E_{X} is evanescent since $k_{\mathrm{X}}^2 < 0$ and tends to zero far away from the density cavity. In this limit the wave is a pure Z mode wave since $\mathbf{E}(x) = E_{\mathrm{Z}}(x)\hat{\mathbf{e}}_{\mathrm{Z}} \propto \exp(\pm ik_{\mathrm{Z}}x)\hat{\mathbf{e}}_{\mathrm{Z}}$. Thus, in the region outside the cavity E_{Z} can be interpreted as the amplitude of the propagating Z mode wave. When approaching the cavity from the outside ω will be equal to or greater than the local ω_{UH} in some region inside the cavity. When passing through the transition region where $\omega = \omega_{\mathrm{UH}}$ the long wavelength Z mode wave is mode converted and $\mathbf{E}$ can be characterized as an UH oscillation. The UH oscillation is essentially an electrostatic wave. The local wave number for E_{X} is real inside the cavity, i.e., $k_{\mathrm{X}}^2[1+Q_{\mathrm{XX}}\eta(x)] > 0$, and E_{X} is a spatially oscillating quantity. The local wave number for E_{Z} remains real throughout the cavity, i.e., $k_{\mathrm{Z}}^2[1+Q_{\mathrm{ZZ}}\eta(x)] > 0$. Both E_{X} and E_{Z} are nonzero inside the cavity and both quantities are necessary to describe $\mathbf{E}$ inside the cavity. The polarization vector $\hat{\mathbf{e}}_{\mathrm{X}}$ is approximately parallel to the direction of propagation so that $E_{\mathrm{X}}(x)\hat{\mathbf{e}}_{\mathrm{X}}$ describes a localized electrostatic oscillation. However, $\mathbf{E}$ is not purely electrostatic and the contribution $E_{\mathrm{Z}}(x)\hat{\mathbf{e}}_{\mathrm{Z}}$ to $\mathbf{E}$ can be interpreted as an electromagnetic correction inside the cavity. In summary, we can interpret E_{X} as the electrostatic part of the localized UH oscillation. E_{Z} describes the Z mode wave field outside the cavity and an electromagnetic correction to the electrostatic UH field inside the cavity.

With the interpretations of E_{X} and E_{Z} given above we can regard Eq. (3.11) as the UH wave equation. It describes the interaction between the localized UH oscillation and the electromagnetic Z mode as well as the excitation by the O mode pump wave. Equation (3.12) describes excitation of Z mode waves by the localized source of UH oscillations and by scattering the pump wave directly into a Z mode wave. The interaction between the modes is not localized to any specific point. Instead, the wave interaction is effective in the whole cavity. This is an important conceptual difference between the present method and the previously used WKB method where the interaction is assumed to take place at some coupling points. A more detailed discussion of the WKB solution is given in section (3.1.3).

In analogy to the electrostatic treatment above, the operators $\mathcal{L}_{\mathrm{X}}$ and $\mathcal{L}_{\mathrm{Z}}$ can be inverted by using an appropriate Green's function. By inverting $\mathcal{L}_{\mathrm{Z}}$ we can calculate E_{Z} in terms of E_{X} and E_{O} from Eq. (3.12). Similarly, E_{X} can be calculated from Eq. (3.11). By combining the results one obtains a single integral equation governing E_{X}. We have that

$$E_{\mathrm{X}}(x) - \int_{-\infty}^{\infty} K_{\mathrm{X}}(x,x_1)E_{\mathrm{X}}(x_1)\,dx_1 = g(x)\,, \tag{3.13}$$

where the kernel $K_X(x,x_1)$ and the source $g(x)$ are given in terms of the Green's functions. The kernel is

$$K_X(x,x_1) \propto \int_{-\infty}^{\infty} G_X(x,x_2)\eta(x_2)e^{ik_Z|x_2-x_1|}\eta(x_1)\,dx_2\,, \tag{3.14}$$

where the exponential factor originates from the Z mode Green's function. By using the length scale separation $k_Z L_\perp \ll 1$, we can approximate

$$\eta(x_2)e^{ik_Z|x_2-x_1|}\eta(x_1) \approx \eta(x_2)\eta(x_1)\,, \tag{3.15}$$

which makes it possible to write the kernel as a product of one function of x and one function of x_1, i.e., $K_X(x,x_1) \approx K_1(x)K_2(x_1)$, which thus constitute a degenerate kernel approximation [Kre89]. With the degenerate kernel it is straightforward to solve the integral equation (3.13). When using the eigen function expansion of G_X the solution can be written as

$$E_X(x)/E_O \propto \frac{\sum_{n=0}^{M} \frac{\eta_n^*}{\Delta-\Delta_n}}{1+i\sum_{n=0}^{M}\frac{\nu_n/2}{\Delta-\Delta_n}}\,\psi_n(x)\,, \tag{3.16}$$

where $\nu_n \propto |\eta_n|^2$ and $\eta_n = \int_{-\infty}^{\infty}\eta\psi_n\,dx_1$. Equation (3.16) is the desired electromagnetic generalization of the solution Eq. (3.9), which was obtained in the electrostatic approximation. There is a clear correspondence between the electromagnetic and electrostatic results. The most important difference is that the electromagnetic solution is "normalized" such that the amplitude at the resonance is finite. The width of the resonance is determined by the real part of ν_n. The imaginary part of ν_n describes a shift of the resonance frequencies. The resonances are now determined by the condition $\Delta = \Delta_n + \mathrm{Im}(\nu_n)/2$. The frequency shift is due to interaction with the electromagnetic wave and can be compared to the frequency shift arising when coupling two harmonic oscillators. However, it can be shown that the shift is small in comparison to the resonance width and can therefore be neglected. Having solved for E_X it is possible to calculate the Z mode amplitude using Eq. (3.12), giving

$$E_Z(x)/E_O = -\frac{Q_{XO}}{Q_{XZ}}\left[\frac{a+i\sum_{n=0}^{M}\frac{\nu_n/2}{\Delta-\Delta_n}}{1+i\sum_{n=0}^{M}\frac{\nu_n/2}{\Delta-\Delta_n}}\right]\exp(ik_Z|x|)\,, \tag{3.17}$$

where $a \neq 1$ is a constant. The Z mode power spectrum $|E_Z|^2$ is not just a sum of Lorentzian resonances. Instead, the resonances are skewed and distorted as several terms in the sums can be important. However, if the spacing between the resonances $\Delta_{n+1} - \Delta_n$ is large in comparison to the resonance width ν_n only one term contributes and $|E_Z|^2$ is Lorentzian shaped around the resonances. Another interesting feature of the solution Eq. (3.17) is that it reduces to

$$|E_Z/E_O| = |Q_{XO}/Q_{XZ}| \tag{3.18}$$

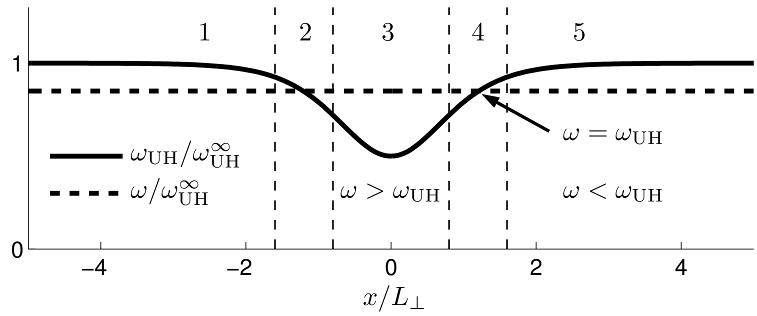

Figure 3.2: The local UH frequency as a function of x (solid line) and ω (dashed line).

at the resonances $\Delta = \Delta_n$. The ratio $Q_{\rm XO}/Q_{\rm XZ}$ does not depend on the specific shape of the cavity, it depends only on ω and the plasma parameters which characterize the ambient plasma. The resonantly excited localized UH oscillations facilitates a shape independent channel for the conversion from O mode to Z mode.

3.1.3 WKB Solution

The analytical treatment of the conversion between Z mode and UH wave discussed above is specially designed for localized UH oscillations. Previous treatments [DMPR82, Mjø83] are based on the WKB approximation. In order to compare the two methods, the WKB treatment is briefly described in this section.

The general idea is to formulate a set of simpler wave equations which are valid in limited regions. These approximate equations are solved in the regions where they are valid. The regions are constructed in such a way that each piecewise solution can be continued into neighboring regions. The solution is obtained by matching the asymptotical solutions. The method has a wide range of applications in plasma physics and other areas of physics. In the present mode conversion problem we can identify three different types of regions. Figure 3.2 shows the local UH frequency as a function of x (solid line) for an arbitrary density profile. It is assumed that ω (dashed line) is between the ambient UH frequency and the cavity's minimum UH frequency. The first type of region is characterized by $\omega < \omega_{\rm UH}$, regions 1 and 5 in Fig. 3.2, where the wave field can be regarded as a pure Z mode wave. As ω is not equal to the UH resonance frequency an approximate wave equation can be obtained from the cold plasma theory. We have that

$$\frac{d^2E}{dx^2} + k_Z^2(x)E = 0\,, \tag{3.19}$$

where $k_Z^2(x)$ is the local Z mode wavenumber. The second type of region is characterized by $\omega > \omega_{\mathrm{UH}}$ which corresponds to region 3 in Fig. 3.2. Guided by the dispersion relation for a homogeneous plasma the wave field is assumed to be a purely electrostatic UH oscillation. By using the FLR approximation we can derive

$$\frac{d^2E}{dx^2} + k_{\mathrm{UH}}^2(x)E = 0\,, \tag{3.20}$$

where $k_{\mathrm{UH}}^2(x)$ is the local UH wavenumber. The interaction with the O mode pump wave can be described by an additional term in the right-hand side of Eq. (3.20). The third type of region describes the transition between the two previous types and ω is allowed to be equal ω_{UH}. These transition regions correspond to region 2 and 4 in Fig. 3.2. It is not possible to describe the wave field as a single wave mode governed by a second order equation. We must use instead the full system of equations Eq. (3.4), or equivalently a single fourth order differential equation. However, the transition region can be made arbitrarily narrow around the point x_0 where $\omega = \omega_{\mathrm{UH}}$ and the coefficients in the governing fourth order equations can be Taylor expanded around the coupling point $x = x_0$. We obtain an approximate equation of the form

$$\frac{d^4E}{dx^4} + \kappa^2(x - x_0)\frac{d^2E}{dx^2} + \gamma E = 0\,, \tag{3.21}$$

where κ and γ are constants. The coefficient in front of the second derivative is vanishing at the coupling point where $\omega = \omega_{\mathrm{UH}}$. Equation (3.21) is sometimes referred to as the “Standard Equation” [Sti92]. The standard equation is a useful model in many mode conversion problems.

Equations (3.19) and (3.20) can be solved by using the WKB approximation. This is straightforward since $k_{\mathrm{UH}}(x)$ and $k_Z(x)$ are finite and nonzero in the regions where Eqs. (3.19) and (3.20) are defined. The WKB solutions can be continued into the transition region, but not across the coupling point. Equation (3.21) can be solved by contour integration [Sti92], giving

$$E(x) \propto \int_C du \exp\left[\frac{1}{\kappa^2}\left(-\frac{1}{3u^3} - \frac{\kappa^2 x}{u} + \gamma u\right)\right]. \tag{3.22}$$

The integration contour C in the complex u plane can be chosen in different ways to describe four linearly independent solutions to Eq. (3.21). The only restriction on C is that the integrand is vanishing at the endpoints or that C is closed. By choosing C appropriately the solution can be matched asymptotically to the WKB solutions outside and inside the cavity to obtain the solution.

Besides the technical difficulties of connecting the solutions across the $\omega = \omega_{\mathrm{UH}}$ layer, the method is easy to understand. The wave motion in the region inside and outside can be interpreted in terms of the WKB solutions. In this approximation the interaction between the modes is confined to the coupling points at the density slopes. In the mode conversion theory developed in Paper [1] the governing wave equations (3.11) and (3.12) and their solutions are defined everywhere. In addition, electromagnetic effects are retained inside the cavity, in contrast to the WKB method which considers the UH oscillations to be purely electrostatic. There are no distinct coupling points in the scale separation method. Instead the wave modes interact throughout the cavity. This interaction gives rise to the coupling between the electrostatic cavity modes and is responsible for the skewing of the resonances. The WKB method cannot describe this effect as the interaction is confined to the coupling points. As seen, the philosophy behind the two methods is completely different.

3.1.4 Numerical Solution

Even though the basic wave equation (3.4), which describes the mode conversion between localized UH oscillations and Z mode waves, has a simple mathematical structure it is difficult to solve analytically. Two different approximate solutions have been discussed above. In both the scale separation method and the WKB method it is necessary to introduce several simplifications and approximations. The scale separation method predicts that the resonances are skewed. As this feature of the conversion might be useful for experimental detection of localized UH turbulence by measuring the emitted Z mode it is valuable to compare the analytical result with a numerical solution. We do this in order to verify that the skewness effect is real and not an artifact of the approximations. A numerical solution can be obtained without introducing any further approximations or simplifications. Eq. (3.4) and the boundary conditions was solved in Paper [1] using a standard finite difference method [GO92]. The numerical procedure will be briefly described below.

In the finite difference method the spatial coordinate is discretized and the solution is calculated in N equally spaced points x_i. The differential operators in Eqs. (3.11) and (3.12) are approximated by finite differences. The boundary conditions $E_{\mathrm{X}} \to 0$ and $E_{\mathrm{Z}} \propto \exp(\pm i k_{\mathrm{Z}})$ as $|x| \to \infty$ give information about the asymptotic behavior at infinity. In a numerical calculation it is necessary to truncate the spatial interval and consider the solution in a finite interval between some points x_- and x_+. By choosing x_- (x_+) sufficiently far to the left (right) of the cavity the plasma density will be constant at these artificial boundaries. As the plasma is homogeneous near x_- and x_+, the solutions are independent propagating plane waves. The emitted Z mode satisfies the equation

$$\frac{dE_{\mathrm{Z}}}{dx} \mp ik_{\mathrm{Z}}E_{Z} = 0\,, \tag{3.23}$$

which can be used as a boundary condition at $x = x_{\pm}$. Similarly, a boundary condition for the evanescent quantity E_{X} is found to be

$$\frac{dE_{\mathrm{X}}}{dx} \pm |k_{\mathrm{X}}|E_{Z} = 0 \tag{3.24}$$

at $x = x_{\pm}$. The boundary conditions (3.23) and (3.24) can be discretized and are suitable for the numerical calculation. When discretizing the wave equations and the boundary conditions one obtains a set of $2N$ linear algebraic equations. The whole set of equations can be arranged in a matrix equation. We have that

$$\begin{pmatrix} A_{\mathrm{XX}} & A_{\mathrm{XZ}} \\ A_{\mathrm{ZX}} & A_{\mathrm{ZZ}} \end{pmatrix} \begin{pmatrix} a_{\mathrm{X}} \\ a_{\mathrm{Z}} \end{pmatrix} = \begin{pmatrix} b_{\mathrm{X}} \\ b_{\mathrm{Z}} \end{pmatrix}, \tag{3.25}$$

where $(a_{\mathrm{X}})_i = E_{\mathrm{X}}(x_i)$ and $(a_{\mathrm{Z}})_i = E_{\mathrm{Z}}(x_i)$ are column vectors, A_{IJ} are $N \times N$ block matrixes, and b_I are N components column vectors describing the excitation. The matrix equation can be solved numerically by using some iterative method. The solution gives an approximation of the wave field between x_- and x_+. To test the numerical solution it is useful to plot $|E_{\mathrm{Z}}|$ outside the cavity. As E_{Z} describes a propagating plane wave the amplitude $|E_{\mathrm{Z}}|$ should be constant. Any artificial reflections on the boundaries x_- and x_+ due to numerical inaccuracies or due to the truncation of the infinite interval will be visible as modulations of $|E_{\mathrm{Z}}|$.

3.1.5 A Quantitative Comparison Between the Methods

To quantitatively study the resonance features described by the scale separation method a specific density profile is considered. We consider $\eta(x) = -\tilde{\eta}\mathrm{sech}^2(x/L_\perp)$, where $\tilde{\eta} > 0$ quantifies the amplitude of the depletion and $L_\perp$ parameterizes the cavities perpendicular width. For this profile it is possible to calculate the eigenfunctions ψ_n and the eigenvalues Δ_n analytically (see Paper [1] for details). The Z mode wave field can be calculated by using Eq. (3.17). Figure 3.3 shows $|E_{\mathrm{Z}}|^2$ as a function of Δ (solid line) for a set of plasma parameters appropriate for the F region ionosphere. The even resonances at $\Delta = \Delta_0$, $\Delta = \Delta_2$, and $\Delta = \Delta_4$ are clearly seen. In addition, the resonances are slightly skewed as Eq. (3.17) indicates. To validate the accuracy of the scale separation method the basic wave equation is solved using the numerical method described in section 3.1.4. As seen from the figure, there is a good agreement between the scale separation method and the numerical solution (drawn with $\circ$-signs in Fig. 3.3). The numerical solution also exhibits the characteristic skew shaped resonances. The resonance amplitude is in good

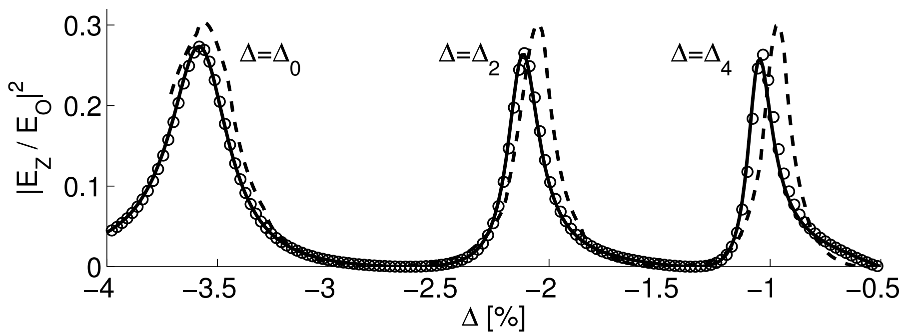

Figure 3.3: The Z mode wave field spectrum outside the cavity as a function of Δ from the length scale separation method (solid line) and from the WKB approximation (dashed line) and from the numerical solution ($\circ$) for $\tilde{\eta} = 4\%$, $L_\perp = 0.5$ m, $\omega_p/2\pi = 4.5$ MHz, $\Omega_e/2\pi = 1.35$ MHz, and $T_e = 1500$ K (electron temperature).

agreement with the simple formula $|E_Z/E_O| = |Q_{XO}/Q_{XZ}|$. The WKB solution (dashed line) reproduces the position of the resonances but does not describe the skewness.

3.2 Z mode Interaction Between Density Cavities

The Z mode leakage from an isolated cavity is strong and determines the amplitude of the localized UH oscillation. Another important implication of this leakage is that the individual filaments can interact through the Z mode radiation. The Z mode generated in one cavity can propagate to a neighbor cavity and mode convert to localized UH oscillations. The Z mode interaction between individual filaments is investigated in a one dimensional geometry in Paper [2]. The basic equations are the same as in Paper [1], i.e., Eqs. (3.11) and (3.12). This treatment is briefly discussed in this section.

To illustrate the physics it is convenient to first consider the simpler problem with the scattering of Z mode waves off a density cavity. This problem is almost the same as the O mode driven problem considered in Paper [1]. However, the incident Z mode has a long but finite wavelength so that it can excite odd modes as well as even modes. This is in contrast to the O mode case where only even modes are excited. It is shown in Paper [2] that the odd and even modes behave very different. In order to describe the excitation of odd modes and to obtain a finite amplitude the approximation Eq. (3.15) is not sufficiently accurate. Instead, it is necessary to expand the exponential factor to the second order in $k_Z L_\perp \ll 1$,

$$\eta(x_2) e^{ik_Z|x_2-x_1|}\eta(x_1) \approx \eta(x_2)\left[1 + ik_Z|x_2-x_1| - k_Z^2|x_2-x_1|^2/2\right]\eta(x_1)\,. \tag{3.26}$$

The zeroth order term does not give any contribution to the odd modes. The first order term gives rise to a shift of the system's resonance frequencies. The width of the resonance is described by the second order term. Physically, the frequency shift and resonance width is due to the interaction between the electromagnetic Z mode wave and the electrostatic UH oscillation. For the even modes the frequency shift is small in comparison to the width of the resonance and can therefore be neglected. For the odd modes the relation between shift and width is the opposite. The frequency shift is much larger than the width of the resonance. It is therefore necessary to include the shift in the calculations. The shift is small but the physics of the shifted resonance is different from the unshifted electrostatic resonances. The incident Z mode wave is totally reflected off the cavity at both even and odd shifted resonances. Due to the reflection, the total field for $x < 0$ will be a standing wave. However, the structure of the standing wave field is different for odd and even resonances. For even resonances the scattered field for $x < 0$ is out of phase with the incident wave and there will be a node on the location of the cavity. For the odd resonances the scattered field is in phase with the incident wave and there will be a maximum on the location of the cavity. The solution of the scattering problem shows that the Z mode field and/or its derivative can be discontinuous on a length scale larger than the width of the cavity.

Instead of describing E_Z in detail inside the cavity where it represents an electromagnetic correction to the localized UH oscillation, a set of conditions for the jump in E_Z and dE_Z/dx across the cavity can be derived. It can be shown that at an even resonance the mean value of E_Z near the cavity is $-Q_{XO}/Q_{XZ}E_O$, which is in agreement with results obtained in Paper [1]. At an odd resonance, the derivative of E_Z has opposite sign on the left and right side of the cavity. We have that

$$\frac{dE_Z}{dx}_{|x=x_j+\delta} + \frac{dE_Z}{dx}_{|x=x_j-\delta} = 0\,, \tag{3.27}$$

where $\delta > 0$ and x_j is the center of cavity j. These conditions can be used to calculate the Z mode wave field between the cavities and the amplitude of the localized UH oscillations. In addition to the one dimensional treatment, an extension to cylindrical geometry is indicated. However, the actual mode conversion mechanism is still considered as one dimensional.

Interaction Between Magnetosonic Waves and Localized Lower Hybrid Oscillations

An interesting example of naturally occurring small scale phenomena in space plasma is the localized bursts of waves in the lower hybrid (LH) frequency range. This phenomena was first observed by the Marie sounding rocket in the upper ionosphere [LKYW86]. The waves appeared as spikes in the measured spectrogram and the phenomena were therefore initially termed “spikelets”. It was later confirmed that the wave activity coincides with density depletions that are elongated along the geomagnetic field. The relative depth of the cavities is typically a few percent. The perpendicular width is observed to be typically a few ion gyro radii and the parallel dimension is estimated to be several order of magnitudes larger than the perpendicular. The localized waves are observed to have frequencies both above and below the LH frequency in the ambient plasma. However, the localized waves are not only observed where the frequency is larger than the local LH frequency. A large fraction of the wave power with $\omega < \omega_{LH}^{\infty}$ is located at frequencies well below the cavities minimum LH frequency. These elongated density structures with localized waves in the LH frequency range, now termed LH cavities (LHC) or LH solitary structures (LHSS) are frequently observed by spacecraft in the ionosphere [KVC+92, EHD+94, BSP+98] and magnetosphere [TEA03] (see also a recent review by Schuck et al. [SBK03]). Further, LHSS are always observed to be immersed in nonlocalized wave activity near ω_{LH}. The nonlocalized waves are collectively referred to as the “hiss” and are believed to be generated by precipitating electrons. The electric field associated with the hiss is strongest above ω_{LH} but wave activity below ω_{LH} is also observed. The wave activity above ω_{LH} is mainly attributed to LH resonance cone waves. The magnetic wave field has no sharp decrease in amplitude below ω_{LH} which is consistent with electromagnetic waves on the whistler/MS wave dispersion surface. It has been observed, by using interferometric *in situ* measurements, that the wave front of the localized waves is rotating around the ambient geomagnetic field [PKSB97, BSP+98, TEA03]. The wave front is rotating in a left-handed (right-handed) sense for frequencies ω below (above) the LH frequency ω_{LH}. The characteristic rotational properties are consistent with theoretical results from electrostatic cold plasma theory [Sey94, SSP+98].

The excitation of right-handed waves with $\omega > \omega_{LH}$ has been explained in terms of scattering LH resonance cone waves off the density cavities [SSP$^+$98]. The interaction between the nonlocalized LH resonance cone waves and the density cavity occurs through the Hall current due to the plasma inhomogeneity. This Hall current is decisive for the wave properties inside the cavity, as it accounts for the sense of rotation of the radially localized waves. This scattering process is not able to explain the excitation of left-handed waves for $\omega < \omega_{\mathrm{LH}}$ where the resonance cone waves are evanescent. In Paper [3] it is shown that the left-handed localized waves can be excited by MS waves which are incident on the cavity. The MS wave is focused by the Hall current mechanism to left-handed rotating oscillations. The treatment is valid for $\omega < \omega_{\mathrm{LH}}^{\mathrm{min}}$ (the minimum LH frequency in the cavity). The theoretical results are shown to be consistent with spacecraft observations of LHSS.

The theories presented in [SSP$^+$98] and Paper [3] describe the excitation of waves above the ambient LH frequency and below the cavity's minimum frequency, respectively. The transition regime between left-handed and right-handed with $\omega_{\mathrm{LH}}^{\mathrm{min}} < \omega < \omega_{\mathrm{LH}}^{\infty}$ is investigated in Paper [4] in a simplified geometry. In this regime the frequency of the incident MS wave matches the local ω_{LH} somewhere inside the cavity and the incident wave is mode converted to localized pressure driven LH oscillations.

This chapter is organized as follows: In section 4.1 the electrostatic description of rotating oscillations is discussed. Section 4.2 describes the electromagnetic treatment presented in Paper [3] and [4]. The excitation of rotating waves by focusing of MS waves is considered in section 4.2.1. In section 4.2.2 the transition regime $\omega_{\mathrm{LH}}^{\mathrm{min}} < \omega < \omega_{\mathrm{LH}}^{\infty}$ is considered where mode conversion occurs instead.

4.1 Electrostatic treatment

The first analytical treatment of rotating LH oscillations in a preexisting density cavity was based on the cold plasma approximation and the assumption that the electric field is purely electrostatic [Sey94]. Further, only wave propagation strictly perpendicular to the ambient magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ was considered. Using the same approximations in a homogeneous plasma gives the dispersion relation $\omega = \omega_{\mathrm{LH}}$ which describes non propagating LH oscillations. However, in a inhomogeneous plasma the dispersive properties are significantly altered by the Hall current. We assume a cylindrically symmetric plasma density variation, $n_0(r_\perp) = n_0^\infty[1+\eta(r_\perp)]$ where n_0^∞ is the density of the ambient plasma and $\eta = \eta(r_\perp)$ is the density variation associated with the preexisting density cavity. In the electrostatic approximation $\mathbf{E} = -\nabla_\perp\phi$, where ϕ is the electrostatic potential. The LH frequency domain is characterized by $\Omega_{\mathrm{i}} \ll \omega \ll \Omega_{\mathrm{e}}$ which allows us to evaluate the electron velocity in the drift approximation, giving the electron current

$$\mathbf{J}_{\mathrm{e}} \approx \frac{e^2 n_0(r_\perp)/m_{\mathrm{e}}}{\Omega_{\mathrm{e}}} \left(i \frac{\omega}{\Omega_{\mathrm{e}}} \nabla_\perp \phi + \hat{\mathbf{z}} \times \nabla_\perp \phi \right), \tag{4.1}$$

where the first term is the polarization current and the second term the Hall current due to the $\mathbf{E} \times \mathbf{B}$ drift. The ions can be regarded as unmagnetized as $\omega \gg \Omega_{\mathrm{i}}$, giving $\mathbf{J}_{\mathrm{i}} \approx -ie^2 n_0(r_\perp)/(m_{\mathrm{i}}\omega)\nabla_\perp \phi$. The fluctuations in the electron charge density can be calculated from the linearized continuity equation, giving

$$\rho_{\mathrm{e}} \approx \varepsilon_0 \frac{\omega_{\mathrm{pe}}^2(r_\perp)}{\omega \Omega_{\mathrm{e}}} \left[\frac{\omega}{\Omega_{\mathrm{e}}} \left(\nabla_\perp^2 \phi + \nabla_\perp \phi \cdot \nabla_\perp \eta \right) - i (\hat{\mathbf{z}} \times \nabla_\perp \phi) \cdot \nabla_\perp \eta \right], \tag{4.2}$$

where we have used $(\nabla_\perp n_0)/n_0 \approx \nabla_\perp \eta$. The polarization current describes a compressional motion of the electron fluid and give rises to the first two terms in Eq. (4.2). The second term is a correction due to the density gradient which is small in comparison to the first term and can therefore be neglected. In a homogeneous plasma the Hall current describes a non-compressional motion and the only contribution is the correction due to the density gradient. Similarly, the ion charge density has a negligible small correction due to the density gradient and $\rho_{\mathrm{i}} \approx -\varepsilon_0 \omega_{\mathrm{pi}}^2(r_\perp)/\omega^2 \nabla_\perp^2 \phi$. A wave equation for ϕ can be obtained by using the Poisson equation, $\nabla_\perp^2 \phi = -\rho/\varepsilon_0$. We have that

$$\left[1 + \frac{\omega_{\mathrm{pi}}^2(r_\perp)}{\omega^2} - \frac{\omega_{\mathrm{pe}}^2(r_\perp)}{\Omega_{\mathrm{e}}^2} \right] \nabla_\perp^2 \phi + i \frac{\omega_{\mathrm{pe}}^2(r_\perp)}{\omega \Omega_{\mathrm{e}}} [\hat{\mathbf{z}} \times \nabla_\perp \phi] \cdot \nabla_\perp \eta = 0 \,. \tag{4.3}$$

In a homogeneous plasma the second term, which is due to the Hall current in the inhomogeneous plasma, is vanishing and the wave equation is reduced to the well-known dispersion relation for LH oscillations, i.e., $\omega = \omega_{\mathrm{LH}}$.

The first bracket term in Eq. (4.3) is equal to $\varepsilon_{xx} = S$ and depends on $r_\perp$ through the local plasma frequencies. As a first approximation, the variation of S can be neglected in comparison to the Hall current term. Assuming a parabolic density profile it is possible to obtain a spectrum of localized waves, both below and above $\omega_{\mathrm{LH}}^\infty$. In addition, the obtained solutions are rotating in the same sense as in the observed LHSS. Thus, the Hall current interaction accounts for the sense of rotation and admits localized waves with ω ranging from well below to well above $\omega_{\mathrm{LH}}^\infty$.

The analysis outlined above describes localized waves with a rotating wave front. It is natural to ask if it is possible to excite these waves by a linear mechanism. The ambient hiss of nonlocalized propagating waves provides a source of energy. An extension of the above theory is presented in Ref. [SSP$^+$98] where the restriction of only perpendicular propagation is relaxed. With this generalization, the theory describes LH resonance cone waves which are propagating in the ambient plasma when $\omega > \omega_{\mathrm{LH}}$. The theory describes the excitation of right-handed waves by the scattering of LH resonance cone waves off the preexisting density cavity. In a homogeneous plasma the extension to $k_z \neq 0$ allows for propagating waves on the LH resonance cone, which is described by the dispersion relation $Sk_\perp^2 + Pk_z^2 = 0$. Taking the Hall current term into account the following generalization of Eq. (4.3) is obtained

$$\nabla_\perp^2 \phi + k_\perp^2 \phi + i\frac{D}{S}\left[\hat{\mathbf{z}} \times \nabla_\perp \phi\right] \cdot \nabla_\perp \eta = 0\,, \tag{4.4}$$

where $k_\perp^2 = -Pk_z^2/S$. This theory describes the excitation of right-handed waves above ω_{LH} by scattering LH resonance cone waves off the preexisting density cavity. The treatment in [SSP+98] is purely electrostatic and cannot describe the excitation of left-handed waves since the resonance cone waves are evanescent for $\omega < \omega_{\mathrm{LH}}$.

4.2 Electromagnetic treatment

In the following two subsections the work in Paper [3] and [4] is summarized. In Paper [3] we present the first analytic electromagnetic theory for the linear excitation of radially localized left-handed waves. It is shown that an incident MS wave is focused inside the cavity to excite left-handed localized oscillations. The interaction between the nonlocalized MS wave and the density cavity occurs through the Hall current mechanism discussed above. The developed theory is valid for frequencies below the cavity's minimum LH frequency. The results are shown to be consistent with observations of LHSS by the Freja satellite [EHD+94]. In Paper [4] the frequency range between the ambient LH frequency and the cavity's minimum LH frequency is considered. In this narrow frequency range an incident MS wave can mode convert to pressure driven LH oscillations and LH resonance cone waves. The mode conversion process between MS waves and pressure driven oscillations is investigated in a simplified geometry. It is shown that the MS wave excite localized LH waves for a set of resonance frequencies.

4.2.1 Excitation of Localized Rotating Left-handed Oscillations

To model the excitation of left-handed rotating oscillations it is necessary to include electromagnetic effects. For frequencies below ω_{LH}, the cold plasma theory describes an electromagnetic mode which is termed whistler wave or fast MS wave. In the following discussion the wave mode is referred to as the MS mode. In a homogeneous plasma the MS waves are described by the approximate dispersion relation

$$N_\perp^2 = \frac{(R-N_z^2)(L-N_z^2)}{S-N_z^2}\,. \tag{4.5}$$

The present treatment is restricted to $\omega < \omega_{\mathrm{LH}}^{\min}$ for which $R > 0$, $L < 0$, and $S < 0$ so that $S - N_z^2 \neq 0$ and $N_\perp$ is finite. Contrary to the previously considered LH resonance cone waves the MS waves can propagate outside the cavity when k_z is sufficiently small, i.e., $N_z^2 < R$.

It is convenient to start with the local dielectric tensor $\varepsilon = \varepsilon(\mathbf{x})$ as all dielectric properties are local in a cold plasma. The elements of ε are functions of $\mathbf{x}$ due to the density variation associated with the cavity. By using the Maxwell equations and the cold plasma ε one can derive a wave equation for $\mathbf{B}$, given by [Eps56],

$$\nabla \times \left(\varepsilon^{-1} \nabla \times \mathbf{B}\right) - k_0^2 \mathbf{B} = 0\,, \tag{4.6}$$

where ε^{-1} is the tensor inverse of ε. Equation (4.6) describes both the propagating MS mode and an evanescent mode. The evanescent wave mode can approximately be eliminated to obtain one single wave equation for B_z. The resulting wave equation depends on the parallel wave number but there is no qualitative difference between the oblique propagation and strictly perpendicular propagation. Therefore, the case $k_z = 0$ is studied in detail. The wave equation for B_z is

$$\nabla_\perp^2 B_z + k_\perp^2 B_z + i\frac{D}{S}\left[\hat{\mathbf{z}} \times \nabla_\perp B_z\right] \cdot \nabla_\perp \eta = 0 \tag{4.7}$$

where $k_\perp = k_0 N_\perp$, S, and D are calculated for the plasma parameters outside the cavity. The third term in Eq. (4.7) is due to the Hall current in the inhomogeneous plasma. The electric field can be calculated in terms of B_z by using Maxwell's equations, giving $E_z = 0$ and

$$\mathbf{E}_\perp = -\frac{i\omega}{k_0^2 RL}\left(iD\nabla_\perp B_z + S\hat{\mathbf{z}} \times \nabla_\perp B_z\right)\,. \tag{4.8}$$

To quantitatively study the coupling of localized and propagating MS waves we consider a cylindrically symmetric density cavity, with $\eta = \eta_0(1 - r_\perp^2/a^2)$ for $r_\perp < a$ and $\eta = 0$ otherwise, where $\eta_0 < 0$ characterizes the depletion amplitude and a is the perpendicular half-width of the cavity. The total wave field is a superposition of the incident wave and an outward propagating scattered wave. Equation (4.7) can be solved by using the method of separation of variables. With unit amplitude of the incident wave the solution is

$$B_z(r_\perp, \varphi, t) = \sum_{m=-\infty}^{\infty} i^m e^{i(m\varphi - \omega t)} B_m(r_\perp)\,, \tag{4.9}$$

where φ is the azimuthal angle. Each term in Eq. (4.9) can be interpreted as a wave with a wave front that rotates around the center of the cavity. The wave front rotates in a right-handed sense for $m > 0$ and in a left-handed sense for $m < 0$. The radial partial waves are continuous but defined piecewise inside and outside the cavity. They are

$$B_m(r_\perp) = \begin{cases} \alpha_m J_m(\kappa_m r_\perp), & r_\perp < a \\ \beta_m H_m^{(1)}(k_\perp r_\perp) + J_m(k_\perp r_\perp), & r_\perp > a\,, \end{cases} \tag{4.10}$$

where $\kappa_m^2 \approx -2D\eta_0 m/(Sa^2)$ describes the wave properties inside the cavity and J_m ($H_m^{(1)}$) is the Bessel (Hankel) function of the first kind and order m. It can be shown that $|\kappa_m| \gg k_\perp$ for $m \neq 0$ so that the length scale of the wave is much shorter inside than outside the cavity. The right-handed waves ($m > 0$) are evanescent inside the cavity since $\kappa_m^2 < 0$. These partial waves are defocused by the cavity so that the right-handed component will always be small inside the cavity. For left-handed waves κ_m is real and $\kappa_m \gg k_\perp$ so that the phase velocity is much smaller inside the cavity than outside. The amplitude inside the cavity is

$$\alpha_m = \frac{k_\perp J_m'(k_\perp a) H_m^{(1)}(k_\perp a) - k_\perp J_m(k_\perp a) H_m^{(1)\prime}(k_\perp a)}{\kappa_m J_m'(\kappa_m a) H_m^{(1)}(k_\perp a) - k_\perp J_m(\kappa_m a) H_m^{(1)\prime}(k_\perp a)}\,. \tag{4.11}$$

It is important to note that $\alpha_m \neq \alpha_{-m}$. The amplitude α_m is large when the denominator in Eq. (4.11) is small. The denominator does not have any roots for real ω and the amplitude will always be finite. However, the denominator has imaginary roots $\tilde{\omega}_{mn}$. The amplitude will be maximum at the resonance frequency $\omega = \mathrm{Re}(\tilde{\omega}_{mn})$ and the width of the resonance is described by $\mathrm{Im}(\tilde{\omega}_{mn})$. Due to the defocusing of the right-handed waves only the left-handed waves will be resonantly excited and B_z will have a left-handed rotating wave front. The sense of rotation is consistent with observations of LHSS. In addition, the theoretical predictions of $|\mathbf{E}|$ are demonstrated to be consistent with LHSS observations by the Freja satellite from a previously published event [EHD+94]. The theoretically calculated and the measured averaged $|\mathbf{E}|$ are compared and found to be in good agreement (see Fig. 2 in Paper [3]).

4.2.2 Excitation of Pressure Driven Lower Hybrid Oscillations

The focusing of MS waves considered in Paper [3] is valid for frequencies below the minimum LH frequency inside the cavity. The case $\omega > \omega_{\mathrm{LH}}^{\infty}$ has been considered previously by other authors. This leaves us with a transition regime between the left-handed and right-handed waves with $\omega_{\mathrm{LH}}^{\mathrm{min}} < \omega < \omega_{\mathrm{LH}}^{\infty}$. In this regime ω will be equal to the local LH frequency at some points on the density slopes of the cavity and the local variation of the plasma density must be considered.

The treatment in Paper [4] is a first attempt to study mode conversion inside LHSS in the transition regime $\omega_{\mathrm{LH}}^{\mathrm{min}} < \omega < \omega_{\mathrm{LH}}^{\infty}$. It is shown that an incident MS wave can effectively excite localized LH oscillations. The theory is fully electromagnetic and includes finite Larmor radius effects. To have an analytically tractable model of the conversion processes, only density variations in one direction perpendicular to the ambient magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ are considered. Further, the waves are considered to propagate parallel to the density gradient and hence perpendicular to $\mathbf{B}_0$. The basic equations are derived from a FLR approximation [CK83]. These equations have the same form as Eq (3.4). By using the same analytical method as in Paper [1] and [2] it is shown that the MS wave excites localized pressure driven LH waves for a set of resonance frequencies.

Outlook

In future work it is essential to extend the theory in Paper [1] to a cylindrical geometry. This would allow the localized wave to have an azimuthal component of the wave vector in addition to the radial component. Unfortunately, it is not clear how to generalize the scale separation method to cylindrical geometry. A crucial point in the scale separation method is that the wave field is expressed in terms of the polarization vectors. This is important since the equations are uncoupled outside the cavity in this representation. The polarization vectors for plane waves are constant in the homogeneous plasma outside the cavity. In a cylindrical geometry, with cylindrical waves, the polarization vectors are not constants but are a function of the distance from the center of the cavity. They will only agree with the plane wave polarization in the far field. As a consequence, the governing wave equations will be coupled not only inside the cavity but in the whole near field region. However, a deeper understanding of the coupling described by the coefficients Q_{XZ} and Q_{ZX} might give a hint about how to proceed.

Another important generalization is to allow for a nonzero wave vector component along the ambient magnetic field and a large scale density variation to model the plasma stratification in the ionosphere. With these generalizations it is possible to describe the absorption and scattering of the O mode pump wave. The present treatment of Z mode interaction should also be generalized. The treatment is confined to the resonance frequencies for a single density cavity at which total reflection occurs. However, the system as a whole has other resonance frequencies than those of each individual cavity. This means that the localized UH oscillations can be excited to even larger amplitudes at some other frequencies. It is also important to generalize the treatment to describe the mode conversion in a cylindrical geometry. This will give a more realistic model of the scattering processes.

Similar to the case of UH–Z mode interaction, the MS wave facilitates interaction between neighboring LHSS. The theory in Paper [3] should therefore be extended to describe the MS wave field in the presence of several cavities. This extension will describe multiple scattering of a MS wave incident on the system of cavities. The amplitude of the total MS wave field should, due to the multiple scattering, be larger in the region between the cavities than far away from the system. This global focusing of the MS wave field should possibly be observable by spacecraft measurements. The present treatment of excitation of pressure driven LH waves by scattering MS waves off a cavity is limited to a one dimensional density structure and wave vectors strictly perpendicular to ambient magnetic field. In future work it is essential to extend the treatment to a cylindrically symmetric density cavity. This is essential since the Hall current interaction becomes important and influences the local dispersive properties inside the cavity. Further, the formulation in Paper [4] should be extended to allow a nonzero wave vector component along the ambient magnetic field. This would allow the incident MS wave to mode convert to inertia driven resonance cone waves as well as to the considered pressure driven waves. With these two extensions of the theory it is possible to study the transition from the left-handed waves with $\omega < \omega_{\mathrm{LH}}^{\mathrm{min}}$ to the right-handed waves with $\omega > \omega_{\mathrm{LH}}^{\infty}$.

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Uppsala, April 2004

Summary in Swedish

Växelverkan mellan elektromagnetiska vågor och lokaliserade plasmaoscillationer

I denna avhandling studeras växelverkan mellan elektromagnetiska vågor och lokaliserade plasmavågor. Arbetet är i huvudsak teoretiskt men mätdata från satellitobservationer har också används i viss utsträckning. Två olika fysikaliska system har betraktats. Artikel 1 och 2 beskriver växelverkan mellan lokaliserade övrehybridvågor och Z-vågor. Speciellt har Z-våg emissioner från övrehybridvågor lokaliserade i plasma kaviteter studerats. Dessa emissioner är starka och bestämmer övrehybridvågornas amplitud. Dessutom, Z-vågorna kan propagera mellan kaviteterna och därmed koppla samman enskilda kaviteter. Denna växelverkan är viktig för vår förståelse och tolkning av radiovågsexperiment i jonosfären. Artikel 3 och 4 beskriver vågfenomen som är relevanta för solitära lägrehybridstrukturer som observerats i jonosfären och i magnetosfären. Exitation av lokaliserade vågor genom spridning av magnetosoniska vågor har studerats. Nedan följer en kort sammanfattning av de fyra artiklar som är inkluderade i denna avhandling.

Artikel 1: Konversion av infångade övrehybridoscillationer och Z-vågor vid en plasmakavitet

I denna artikel presenteras analytiska uttryck som beskriver konversion mellan kvasi-infångade övrehybridoscillationer och Z-vågor. Övrehybridoscillationerna är lokaliserade i en plasmakavitet där de exiteras av en extern våg med ordinär polarisation. Övrehybridoscillationerna är inte fullständigt infångade i kaviteten, istället emitteras Z-vågor från kaviteten. Detta problem är viktigt för en vidare utveckling av teorier som beskriver generering av plasmakaviteter under radiovågexperiment i jonosfären. Ett system av ekvationer som beskriver konversionen har härletts. Dessa ekvationer har lösts i en endimensionell geometri genom att nyttja att systemet har två olika längdskalor. Den elektromagnetiska Z-vågen har en mycket längre våglängd än kavitetens utsträckning vinkelrätt det geomagnetiskafältet. Resultatet är giltigt för godtyckligt låga resonanser av det kvasi-infångade elektriska fältet. Teorin förutsäger att resonanserna för Z-vågornas amplitud är skeva och inte Lorentzianska.

Artikel 2: Elektromagnetisk växelverkan mellan lokaliserade övrehybridoscillationer i ett system av plasmakaviteter

I denna artikel studeras elektromagnetisk växelverkan mellan plasmakaviteter som exiteras under radiovågexperiment i jonosfären. Dessa kaviteter skapas av lokaliserad övrehybridturbulens som drivs av en extern våg med ordinär polarisation. Radarmätningar och observationer från sondraketer visar att ett stort antal kaviteter genereras i den del av plasma som den externa vågen belyser. Det är välkänt att den lokaliserade turbulensen genererar Z-vågor som kan propagera fritt utanför kaviteterna, se artikel 1. Vågorna som emitteras från en kavitet faller in mot de närliggande kaviteterna. Denna växelverkan kopplar därmed samman turbulensen i de olika kaviteterna.

Artikel 3: Exitation av roterande lokaliserade vågor i plasmakaviteter genom spridning av magnetosoniska vågor

En analytisk beskrivning av elektromagnetiska vågor i ett icke-homogent plasma har använts för att undersöka exitation av lokaliserade vågor med roterande vågfront. Exitation är undersökt för frekvenser under lägrehybridfrekvensen. På grund av asymmetri i växelverkan mellan vågen och plasmat fokuseras den infallande magnetosoniska vågen till en lokaliserad våg vars vågfront roterar runt en magnetfältslinje genom kavitetens centrum. Resonansfrekvenser och resonansernas vidd beräknas. Teorin är viktig för förståelsen av solitära lägrehybridstrukturer som är vanligt förekommande i rymdplasmor. Genom att jämföra de teoretiska resultaten med observationer av lägrehybridstrukturer från Freja satelliten har vi visat att dessa är konsistenta med varandra.

Artikel 4: Konversion av lokaliserade lägrehybridoscillationer och magnetosoniska vågor vid en plasmakavitet

I denna artikel presenteras analytiska uttryck som beskriver konversion mellan lokaliserade tryckdrivna lägrehybridoscillationer och magnetosoniska vågor. De ekvationer som beskriver konversionen har lösts i en endimensionell geometri. Detta genom att använda samma skallängdseparationsmetod som tidigare använts för att beskriva konversion mellan övrehybridoscillationer och Z-vågor (se artikel 1). Analysen visar att en infallande magnetosonisk våg kan exitera tryckdrivna lägrehybridoscillationer i ett frekvensband mellan den omgivande lägrehybridfrekvensen och kavitetens lägsta lägrehybridfrekvens. Exitationen är speciellt stark då den infallande vågens frekvens sammanfaller med några resonanta frekvenser. Den teori som presenteras är viktig för förståelsen och tolkningen av observationer av solitära lägrehybridstrukturer i rymdplasmor.

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