Shout at the Eta

BY

MÄRTEN STENMARK
Quantum chromodynamics has interesting limits both in the low and the high-energy region. In the low energy region one has phenomenology of meson interactions which are still not clearly understood. In the high-energy region one wants to find a new theory which will envelope gravity and the standard model in the quantum framework, possibly via some kind of string theory.

In this thesis some aspects are touched upon including both these limits. On the one hand we look at meson scattering close to threshold and try to describe cross sections via phenomenological models such as the two-step model. We then go on and dwell upon noncommutative geometry, a framework which has been successful in describing certain aspects of the theory of strings.

The low-energy calculations gave some insight into the need for finding better understanding of the theories of mesons. The work on noncommutative geometry was on the other hand fruitful in gaining understanding of certain connections between different star products and their relations on a local level.

Keywords: Theoretical Physics, Meson production, Noncommutative geometry

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Baby
I’ve been breaking glass
In your room again
Listen.

*David Bowie*

Boy
Thanks for hesitating
You’ll never know the real story
Just a couple of dreams
You get up and sleep.

*David Bowie*
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I  The $pd \to ^3He\eta$ reaction near threshold  
   M. Stenmark  

II The $NN \to NN\eta$ reaction revisited  
   M. Stenmark  
   Submitted for publication, TSL/ISV-2004-0284.

III The production of theta baryons in proton-deuteron collisions near threshold  
   G. Fäldt and M. Stenmark  
   Submitted for publication, TSL/ISV-2004-0285.

IV Invariant $*$-products on $S^2$ and the canonical trace formula  
   K. Matsubara and M. Stenmark  
   Accepted for publication in Letters in mathematical physics, arXiv:hep-th/0402031.
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THEORETICAL PHYSICS TODAY is a broad subject, much too wide to be covered in one thesis. In this thesis, we will touch upon subjects which are seemingly far apart. The common denominator being quantization.

The theory describing nuclei is currently believed to be quantum chromodynamics, usually abbreviated QCD. For this theory, the low energy limit has problems, due to difficulties making consistent perturbative calculations. The large N expansion has proven useful when extracting some of the properties of QCD in this limit, but one is mainly left with phenomenological approaches.

Furthermore, QCD is a part of the standard model of elementary particles. The only interaction which has not been successfully included in the standard model is gravity, for which the exchange particle gives rise to too severe singularities in the Feynman amplitudes. This problem has been attacked using different string theories since the mid seventies. Many old subjects, such as higher dimensional gravity and noncommutative geometry, has had a new renaissance. In both these parts of theoretical physics, phenomenology plays an important role.

In this thesis we will begin within the framework of phenomenological low energy nuclear physics and study some different production scenarios, mostly with the help of different phenomenological couplings and parametrized wave functions. We will also study some aspects of “pen-taquarks”, a subject that will be our “red-thread” to approach, the topic at the other side of the energy spectrum, noncommutative geometry towards the end of the thesis.

All this will be expanded on as we go through the subjects one by one.
We begin with some classical mechanics since this is all too important in both quantum field theory and noncommutative geometry. Chapter 1, will be an introduction to the different aspects of theoretical physics contained in the thesis, hopefully in a manner that may be read by non specialists. In Chapter 2 we will go from classical mechanics to quantum field theory before turning to the phenomenology of low energy nuclear physics in Chapter 3. After this chapter we will step into the high energy framework and discuss noncommutative geometry in Chapter 4. Chapter 5 summarizes the included papers, and finally in Chapter 6 we give a summary in Swedish for non-specialists.
Chapter 1

Introduction

The theoretical understanding of physics on the boundary between nuclear and particle physics remains, to some extent, unknown within the atlas of theoretical frameworks of modern physics today. Although quantum chromodynamics has been tested significantly and has stood up to these tests remarkably well, the problem of extracting many of the results, in both the high and the low energy regions, remain more or less unsolved.

One of the challenges for QCD is the appearance of physical nucleons and mesons instead of quarks and gluons. In the low energy region, these should become the relevant quanta, so how do they emerge from the highly complex structures of nonperturbative QCD? Secondly, how do they produce the bound states needed to describe composite nuclei in a relativistic fashion? Such problems can be attacked with some success within the framework of large N expansions. With the help of this one may see how, in this limit, mesons appear as the relevant structure, building the scattering amplitudes in this region.

To be able to understand the fundamental theory better one builds phenomenological models which describe those parts which the fundamental theory are unable to explain. Here one uses interactions that are inspired from QCD, to build models which reproduce experiments. One example is the use of physical mesons, roughly as they emerge from the large N structure of QCD. Hopefully these models may be able to give physical insight into the interactions they model, which could trigger new understanding also in QCD. The regions for the models stay close to experiment and there are often many competing models for the same physical reaction. One of the key issues is to determine which model is
most appropriate, which is one of the investigations done in this thesis.

In the end of the thesis we will go to the other extreme of QCD, the region of very high energies. At these energy regions one hope to find a unification between the standard model, which QCD is a part of, and gravity. This unification between quantum and gravity has long been one of the big unsolved problems of theoretical physics. Most effort has been put into a variety of theories known as string theories.

In the field of string theory, many new tools have appeared to tackle problems in the different regions which need to be understood therein. One of these are star products, coming from the field of noncommutative geometry. These are needed in order to explain certain limits of string theories.

In the coming chapters we will dwell on these subjects, both low energy effective theories for mesons, and some star products of noncommutative geometry.

Due to the different aspects of the thesis, the language may be a bit unfamiliar in some places. This is due to the two different target audiences for the thesis. On one side those interested in the phenomenology of nuclei, on the other side those interested in the more high energy parts of the thesis. Technicalities, such as some of the mathematical notation, are not important for the larger picture. It is probably only important for the last chapter, so just read on.
Chapter 2

Quantum Mechanics & Quantum Field Theory

IN THE LATE nineteenth century classical mechanics was a subject which had been put on a firm ground both physically and mathematically. On the physics side one let the Lagrangian system be a function of coordinates and momenta, usually defined as the difference between kinetic and potential energy [1]. By varying the action functional, the time integral of the Lagrangian, one gets a second order system of differential equations, which when solved gives the evolution of the system in time. Mathematically one lets the coordinates of the problem define a differentiable manifold and the Lagrangian function will then be a function from the tangent bundle, of the manifold, to the real numbers [2]. The Lagrangian function is often the first step towards quantization which will be expanded on when dealing with quantum field theory later on.

It is always preferable if a physical problem can be written down theoretically in different frameworks. Classical mechanics is noteworthy in this aspect, since the Lagrangian formulation of classical mechanics can be expressed in the Hamiltonian framework as well. The bridge between the two is well defined, the Hamiltonian function is obtained via a Legendre transformation of the Lagrangian. The second order differential equations breaks down into coupled first order differential equations. When defining this in a mathematical context one ends up with a treatment of symplectic manifolds. The number of coordinates of the manifold will then be $2n$, taking the ordinary coordinates $q_i$ together with the momenta $p_i$ ($i=1...n$). A closed nondegenerate 2-form $\omega$, i.e. $d\omega = 0$ and $\forall \xi \neq 0 \exists \eta : \omega(\xi, \eta) \neq 0, (\xi, \eta \in TM_x)$, is then defined as $\omega = \sum_i dp_i \wedge dq_i$. The manifold together with this symplectic 2-form is called a symplectic manifold [2]. This symplectic framework is used
in the context of deformation quantization, a topic which will be dealt with at the end of the thesis.

2.1 Quantum Mechanics

There are several ways to approach the topic of quantum physics. The route we have chosen to take here is the one using Feynman path integrals. Although never explicitly used in the actual calculations we believe it gives a smoother transition from the classical framework. As described above, the fundamental laws of classical physics can be understood in terms of the action functional. The key issue when making the transition between classical physics and quantum physics is the choice of path. Classically, the path travelled by a system is given by the solution of the equations obtained via variation of the action functional. In the quantum region our vision is blurred when approaching regions in the order of the Planck constant. The system is not bound to follow the classical path through space, instead all paths are possible but with different probability of them being actually taken by the system. Feynman solved this problem via summing over all paths while weighting the probability with the classical action \[3\]. Hence the classical path is the most probable but not necessarily the one actually taken.

In particle physics one is usually interested in how to find the transition function between two states, \(|q'_T\rangle\) and \(|q_T\rangle\). This function has a beautiful expression in terms of the Feynman path integral,

\[
\langle q'_T|q_T \rangle = \int Dq e^{iS}
\]

The measure means roughly integration over all paths, but certain care has to be taken when defining the measure \[4\].

2.2 Quantum Field Theory

When combining relativity with quantum mechanics we have the need to develop equations which are relativistically invariant. It is then necessary to put space and time on an equal footing. This leads us to consider Lagrangian densities which are written in terms of fields depending on space-time points instead of the usual coordinates and momenta. Hence we shift our attention to continuous media. When we have such a Lagrangian density we may apply the path integral formalism to obtain propagators between different state configurations. We propagate a set of initial fields into a new set of fields via an object related to the transition function given in the last section. This object is the generating
2.2 Quantum Field Theory

Once the correct generating functional has been written, the theory is quantized, thus the step from classical to quantum has been made. The oscillatory properties of this functional is remedied when going over to the Euclidean space formalism. With this functional one may determine the Green’s functions of the theory as,

\[ G^{(N)}(1,2,\ldots,N) = \frac{1}{i^N} \frac{\delta}{\delta J_1} \frac{\delta}{\delta J_2} \ldots \frac{\delta}{\delta J_N} W[J] \big|_{J=0} \]  

These are then, in momentum space, identified with the transition functions. Note that we, so far, have not done anything perturbatively. We still have the choice between a perturbative route or some other way of solving for the Green’s functions. Note also that the Green’s functions are determined in Euclidean space and cannot be identified with transition amplitudes unless we analytically continue back to Minkowski space. To perform this we have to know that there are no singularities in the way of the contour rotation. This problem is related to bound state problems, which will be mentioned later.

2.2.1 Perturbation Theory

The expression for the Green’s functions are difficult to solve analytically, here as in most other parts of physics, approximation is needed to extract results from the theory.

The higher order Green’s functions may be expressed in terms of the zeroth order one. One can thus rewrite the generating functional as \( W[J] = \exp(iZ[J]) \) and write the Green’s functions in terms of \( Z[J] \). This is particularly neat when dealing with perturbation theory since one can express \( W[J] \) as,

\[ W[J] = N \exp \left( - \int d^4x \left( \frac{\delta}{\delta J} \right) \right) \exp(-Z_0[J]) \]  

hence giving the explicit dependence on the potential of the Lagrangian density. Usually the interaction terms, which are present in this potential, depend on some parameter that is “small” enough to make the perturbative series convergent. One takes all terms generated by the above functional up to some order in this parameter. For each set of such interaction terms one can derive rules for how to construct these without having to derive them every time. These rules are called the
Feynman rules of the theory and the constituent Green’s functions are pictured by Feynman diagrams [5]. Each such diagram then symbolizes a part of the approximated total Green’s function.

2.3 Quantum Chromodynamics

The fundamental part of the standard model which describes nuclei is quantum chromodynamics. This is a gauge theory which has 8 gauge particles, called gluons. It is usually assumed, based on experimental results, that the number of flavors in the theory is 6, which gives 6 fermions called quarks. The Lagrangian is a Yang-Mills Lagrangian for these particles [6]. Perturbatively this gives rise to three fundamental diagrams and two propagators out of which one builds the Green’s functions. Quark-Quark-Gluon, 3-Gluons and 4-Gluons. It is a gauge theory and hence so as not to overcount one has to count each gauge family only once [7]. This is usually done by fixing a gauge, in which case new diagrams might arise that takes care of the overcounting.

In this theory a bound state of a quark an anti quark is identified with a meson. The bound state between three quarks is identified with a baryon. The problem is that this theory is all too complex to enable a calculation of phenomena in the region of nuclear physics. Although all of nuclear physics is a special case of this theory, we are unable to reproduce many of the fundamental parts of the low energy physics.

2.3.1 The Large N Expansion

These problems were attacked during the seventies via an approximation scheme called the large N expansion, from which attempts were made to extract results from the theory that could give a better understanding of QCD as a fundamental theory. Here N is the number of colors, i.e. the number of possible gauge charges that can be carried by the quarks. The coupling constant, g, of QCD is not really a free parameter since it is used to define the scale of masses within the framework of the renormalization group [8]. Solving the Callan-Symanzik equations one ends up introducing a length scale that is related to the coupling constant, hence the coupling constant is no longer free. This feature, that the theory appears to have no free parameters, makes approximations difficult. ’t Hooft suggested to look at the extension of QCD to N colors and SU(N) gauge group and treat this theory in the limit of N approaching infinity, arguing that the results are to some degree still be applicable for N=3 [9]. The original work done by ’t Hooft was carried out in two dimensions but it was later studied by Witten in four dimensions [10].

What makes it remarkable is, among other things, that the elastic
amplitudes for meson-meson scattering are given in terms of physical mesons and not by quarks and gluons. This is exactly what one does when building phenomenological models for meson interactions. Furthermore, interactions between mesons and baryons can be written down semi-classically giving integro-differential equations. The phenomenological potential models for meson interaction, such as the CD-Bonn potential that will be discussed later, can have their exchanges ordered by their importance in the large N limit.

2.3.2 Pentaquarks

In the early sixties Skyrme pointed out that the nucleon could be seen as a soliton wave emanating from the pion field [11]. The idea of quarks came soon thereafter and most work was put into this idea for a while. When the large N expansion surfaced in the seventies the Skyrme model was once again brought into the light, this time by Witten who showed that baryons behave as solitons. In the large N limit mesons can be described by an effective field theory, the baryons behave as solitons of this theory [10, 12].

The particles that have been observed in experiments are in two categories, two or three quark states. This is a bit of a surprise since QCD does not demand this structure of the physical particles. From the pure QCD point there could be five and six quark states (and so on...), why are they not observed?

This point has led to proposals of existence of exotic states throughout the history of QCD. In the late seventies a calculation within the MIT bag model proposed several exotics of different quantum numbers [13], but no specific response came. In the late nineties Diakonov et al. made a calculation within a Skyrme type of model. They predicted an exotic baryon having spin 1/2, isospin 0 and strangeness +1 with positive parity. The important issue with this was the ability to predict both the mass and put a upper bound on the width. The prediction of a mass of 1530 MeV and a width of less than 15 MeV [14] was certainly an interesting result. The small width would give it a clear signal and the relatively low mass should make it accessible to experiments, but nothing much happened for a few years.

So what is a pentaquark state and what makes it special? From the point of QCD a pentaquark state is just a state composed of five quarks. This puts it in a different representation than those of the ordinary mesons and baryons. The chiral soliton model can be extended to hyperons, in which case the lowest state is a spin 1/2 octet while the first excitations are within the spin 3/2 decuplet. The next rotational excitation is an anti-decuplet with spin 1/2 [15]. In the calculation of
Diakonov et al. the lightest of the pentaquarks in the anti-decuple has the structure of a bound state of a neutron and kaon with a mass of some 1530 MeV. In their model there is only one unknown parameter which is fixed by identifying the nucleon-like member of the anti-decuple with the observed spin 1/2 N(1710)-resonance. In this model all objects, octet, decuplet and anti-decuple, are excitations of the soliton. In the real world the objects are a bit more sophisticated than those of the large N limit so possible pentaquarks in reality would have corrections.

Nothing much happened for a few years but finally, in 2003, things started to heat up. During a few months several experiments reported evidence for an exotic state with mass of roughly 1540 MeV [16, 17, 18]. During the next year the physics community had a heated debate on whether or not there are pentaquarks. New theoretical proposals were made, in which different parities were used, so the parity of the state is also under debate. Several experiments reported not finding anything in the expected energy region. The question is still not settled at the moment, although there seems to be growing scepticism within the community [19].
Chapter 3

Phenomenology

The phenomenological description of nuclei and their interactions with mesons is a rich subject. We shall focus mainly on the phenomenological interactions that are dealt with in the papers of this thesis. For a short introduction to a renormalizable theory based on hadrons we refer to [20]. The discussion below will be related to the production of eta-mesons and pentaquarks, in nucleon-nucleon and nucleon-nucleus collisions.

The relevant object of study in particle collisions, in quantum field theory, is the reaction amplitude. This object describe the reaction through the summation of relevant diagrams of the model. In perturbative QCD one would just sum all diagrams generated by the Lagrangian, as explained in Chapter 2. In phenomenology, things are a bit more complex. No fundamental Lagrangian exists, one has to use phenomenological Lagrangians which describe a certain area of the physics one want to be able to describe.

The reaction amplitudes of the models are usually assumed to factorize. Assume that the total amplitude, call it $\mathcal{M}$, consists of three factors,

$$\mathcal{M} = \mathcal{M}_{FSI} \times \mathcal{M}_{A\rightarrow B} \times \mathcal{M}_{ISI} \quad (3.1)$$

The first and the last, final state interaction and initial state interaction, are elastic amplitudes of the form $A \rightarrow A$. The main amplitude $\mathcal{M}_{A\rightarrow B}$ is the inelastic one. Our models use interaction Lagrangians and phenomenological matrix elements to describe the main amplitude. The final state interactions are assumed to factorize from this amplitude so that we may describe this part separately.
For nucleon-nucleon collisions the cause for factorization is mainly large momentum transfer. The interaction takes place within a small ball around the interaction point which causes the contributions from small nucleon-nucleon separations to be emphasized [21]. This makes it possible to calculate the final state interaction factor for zero separation and just treat it as a multiplicative factor.

In the coming section we shall discuss how to build the main amplitude out of interaction Lagrangians between the nucleons and the mesons, in other words the phenomenological matrix elements. These type of calculations are almost the same as in ordinary quantum field theory, except that one replaces the vertex function by the physical amplitude. We then go through the models one by one to see what they are built of. In the last section we briefly describe the final state interaction part of the amplitude. First let us briefly mention the CD-Bonn potential.

3.1 The CD-Bonn Potential

When studying bound states in the low energy region of nuclear physics a problem appears. Bound states show up as poles in the scattering matrix. To produce such a pole one has to sum an infinite number of diagrams, since any finite sum of Feynman diagrams are unable to produce a pole [22].

To be able to sum an infinite number of diagrams one has to write down some kind of integral equation that sums the type of diagrams which we think will approximate the theory well enough. One can for example sum all ladder and crossed ladder diagrams. This leads to a Bethe-Salpeter type of equation [23], a relativistic two-body equation.

For nucleon-nucleon scattering one of the most commonly used Bethe-Salpeter type equations is the one used in the CD-Bonn potential. This potential actually consists of three potentials each describing proton-proton, neutron-neutron and proton-neutron scattering respectively. The three potentials are related though charge symmetry [24]. To be able to describe how the nucleons interact, interaction Lagrangians between nucleons and mesons are used. Exchange of $\pi^0$, $\pi^\pm$, $\omega$, $\sigma$ and $\rho$-mesons are put in with coupling constants in each interaction Lagrangian. These parameters are then fitted to the experimental data to obtain the physical values of the coupling constants. The model reproduces a bound state deuteron once the binding energy is inserted. From this potential we can use phenomenological couplings to model different interactions involving nucleons and mesons. Also wave functions, for the deuteron, can be obtained from this potential.
3.2 Interaction Vertices

The interaction vertices that we use can be divided into two categories. First those that are calculated from an interaction Lagrangian and secondly, those which are given by some specific phenomenological matrix element. Let’s start out with the interaction Lagrangians before we turn to the matrix elements.

The explicit form of these Lagrangians are,

\[ L_{\pi NN} = -ig_{\pi NN}\bar{\psi}\gamma_5\tau_i\psi \] (3.2)

\[ L_{\eta NN} = -ig_{\eta NN}\bar{\psi}\gamma_5\eta\psi \] (3.3)

\[ L_{\eta NN^*} = -ig_{\eta NN^*}\bar{\psi}\eta\psi \] (3.4)

\[ L_{\rho NN^*} = ig_{\rho NN^*}\bar{\psi}\gamma_5\left(\gamma_\mu - \frac{1}{M}\partial_\mu\right)\tau^i\rho_i^\mu\psi \] (3.5)

\[ L_{\rho NN} = -g_{\rho NN}\bar{\psi}\left(\gamma_\mu\rho_i^\mu - \frac{\kappa}{4m_N}\sigma_{\mu\nu}\rho_i^{\mu\nu}\right)\tau^i\psi \] (3.6)

where the derivatives acting on the rho are written \( \rho^{\mu\nu} = \partial^{\mu}\rho^{\nu} - \partial^{\nu}\rho^{\mu} \). The mass factor \( M = m_N^2/(m_\pi + m_N) \) [25]. \( \psi \) is the field of the nucleon and \( \psi^* \) is the field of the resonance, which in this case is the \( S_{11}(1535) \)-resonance. \( \pi, \eta \) and \( \rho^\mu \) are the fields for the pion, eta and rho-meson respectively. Here we use a \( \gamma_5 \)-interaction for both \( \pi NN \) and the \( \eta NN \) vertices.

The type of interaction terms in the Lagrangians are determined by parity, which is a symmetry of the Lagrangian. One has \( \mathcal{P}\mathcal{L}(x)\mathcal{P}^\dagger = \mathcal{L}(\mathbf{-r}, t) \). This implies \( \gamma_5 \)-interaction for \( \pi NN \) and the \( \eta NN \) vertices if no derivatives are present, while it gives 1 in the case of the \( S_{11}(1535) \)-resonance. \( \pi, \eta \) and \( \rho^\mu \) are the fields for the pion, eta and rho-meson respectively. Here we use a \( \gamma_5 \)-interaction for both \( \pi NN \) and the \( \eta NN \) vertices.

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The second category of interactions is a bit more complex. It describes phenomenological vertices in which several contributions are incorporated into a single point interaction.

First off is the short range contribution in the nucleon–nucleon interaction. This is a type of interaction which is due to approximations done in the CD-Bonn potential. The approximation made is to use only the positive spinor part of the nucleon wave function, which breaks Poincare invariance of the five-point functions [25]. To restore this invariance, and to include possible effects in physical reactions, one has to add the diagrams that have only negative spinor part in the exchange nucleon propagator. The two incoming nucleons are indexed by 1 and 2, and the outgoing by 1’ and 2’. The contributions added correspond to scalar, \( S \), or vector, \( V \), exchange, where the corresponding Fermi operators are
\[ \mathcal{O}_S = 1 \cdot 1 \quad \text{and} \quad \mathcal{O}_V = \gamma_1 \cdot \gamma_2. \] We use the expression,

\[
M(\alpha) = \bar{U} \gamma_\alpha (V \alpha) U \tag{3.7}
\]

where \( V \alpha \) is given by \( V \alpha = (v_\alpha^+ + v_\alpha^- (\vec{r}_1 \cdot \vec{r}_2)) \mathcal{O}_\alpha \). Here \( \bar{U} \) and \( U \) are four component spinors for the nucleons.

The expressions for the \( \pi N \rightarrow \eta N \) vertex must also be added. For these, as well as the \( \eta N \rightarrow \eta N \) vertex, we use the phenomenological \( M \) matrices,

\[
M(\pi N \rightarrow \eta N) = \bar{U}'(-i\lambda_\pi \tau_3 \pi^\dagger)U \tag{3.8}
\]

\[
M(\eta N \rightarrow \eta N) = \bar{U}'\left(-i\frac{1}{2}\lambda_\eta\right)U \tag{3.9}
\]

The \( M \) matrices for the \( \pi N \rightarrow \eta N \) as well as the \( \eta N \rightarrow \eta N \) vertices contain a \( \lambda \) factor which has to be defined. We use the expressions,

\[
\lambda_\pi = 4\pi \frac{m_N + m_\eta}{m_N} f(\pi N \rightarrow \eta N) \tag{3.10}
\]

\[
\lambda_\eta = 4\pi \frac{m_N + m_\eta}{m_N} f(\eta N \rightarrow \eta N) \tag{3.11}
\]

where \( f(\pi N \rightarrow \eta N) \) and \( f(\eta N \rightarrow \eta N) \) are extracted from the model of Batinic et al. [26]. They include contributions from the \( S_{11}(1535) \), \( P_{11}(1440) \) and \( D_{13}(1520) \) resonances. To see how \( f(\pi N \rightarrow \eta N) \) and \( f(\eta N \rightarrow \eta N) \) are related to the cross section for the respective reaction, look at for example [27].

When dealing with bound states such as the deuteron and helium nuclei we use relativistic expressions which are reduced to a form containing two-component spinors. The spinors given an index \( c \) are charge conjugate spinors while those without are ordinary ones. The relevant matrix elements in our cases are,

\[
M(pp \rightarrow \pi^+d) = \xi_p (A \hat{p}_\pi \cdot \epsilon_d^\dagger + iB \sigma \cdot (\epsilon_d^\dagger \times \hat{p}_p)) \eta_p \tag{3.12}
\]

\[
M(d \rightarrow pn) = -\zeta_n (2\pi)^{3/2} \sqrt{m_d} (\sigma \cdot \epsilon_d) \phi_d(q) \xi_{p_c} \tag{3.13}
\]

\[
M(\pi^+n \rightarrow \eta n) = \xi_p [f + i(\sigma \cdot (\hat{p}_\pi \times \hat{p}_n))g] \zeta_n \tag{3.14}
\]

\[
M(pd \rightarrow ^3He) = -\eta_h (2\pi)^{3/2} \sqrt{\frac{2m_d}{3}} (\sigma \cdot \epsilon_d) \psi^\dagger_h (k) \xi_p \tag{3.15}
\]

The parameters \( A \) and \( B \) are invariant functions of the total energy while \( f \) and \( g \) are from the same model as \( \lambda_\pi \). \( \phi_d(q) \) and \( \psi^\dagger_h (k) \) are the nonrelativistic momentum space wave functions of the deuteron and helium nuclei, which are in S-wave.
3.3 The Two-Step Model: The $pd \rightarrow ^3He \eta$ Reaction

We also study some interactions dealing with the pentaquark state, also known as the $\Theta$-baryon. For this model the additional matrix elements for the $K^+d \rightarrow p\Theta^+$ and $\pi^+n \rightarrow \Lambda K^+$ reactions are needed. The first of these is obtained through the one-nucleon exchange approximation. The other reaction is close to threshold and it is sufficient to use the S-wave production amplitude in this case. Again we use bound state wave functions for the deuteron and we give the additional relevant matrix elements,

$$M_1(K^+d \rightarrow p\Theta^+) = \frac{\xi_{\Theta}^\dagger}{\sqrt{2}} \frac{g_{Kd\rightarrow p\phi S}(q)}{\sigma \cdot p_K \sigma \cdot \epsilon_d} \eta_p,$$  \hspace{1cm} (3.16)

$$M_3(\pi^+n \rightarrow \Lambda K^+) = \eta_{\Lambda}^\dagger [g_{\pi n\rightarrow \Lambda K}] \eta_n.$$  \hspace{1cm} (3.17)

which are used in addition to the previous ones. Again $\varphi_S(q)$ is the wave function of the deuteron, nonrelativistic and in S-wave. Here, $q$, is the relative momentum for the deuteron.

3.3 The Two-Step Model: The $pd \rightarrow ^3He \eta$ Reaction

The reaction $pd \rightarrow ^3He \eta$ has through its experimental measurements produced quite a few surprises. In the mid eighties the first measurements revealed a large cross section at threshold and a strong oscillatory pattern as energy increased [28, 29]. Theoretical results soon became available, showing that this could be explained by a double scattering mechanism [30]. Some fifteen years later new experiments showed once more new features of the reaction, this time through measurements of differential cross sections [31].

This specific reaction is one of the reactions were the so called two-step model is applicable. Here an entire model is built out of one contributing diagram. The reason for the dominance of this contribution is the “kinematical miracle”, this diagram is kinematically favorable to others because the probability of a proton and a deuteron fusing together into a helium bound state is increased by kinematical matching within the diagram.

In paper I, we use this model to calculate the differential cross section. The main problem lies in calculating the $\mathcal{M}$-matrix for the diagram. Since the diagram is a two-loop diagram, see fig. 3.1, one has to integrate out the two loop-momenta.
We use an integral identity to reduce the two-momentum integrations to one integration in coordinate space. An identity is obtained for calculating the contribution for arbitrary loop momenta, but the final result is calculated for the lowest order contribution only. The results are in reasonable agreement with the experimental data. The angular dependence is given by the diagram but the energy dependence is reproduced only if an additional model for final state interaction is included in the calculation.

3.4 NN-model: The $NN \rightarrow NN\eta$ Reaction

The $NN \rightarrow NN\eta$ reaction is quite difficult to describe theoretically. The amplitudes needed to give a theoretical description of the reaction involves many parameters whose values are not entirely well known. One example of this is the $\eta NN$ coupling. The value of the coupling constant is highly model dependent and varies roughly between between 2 and 6 depending on which model one chooses to look at [32, 33]. Hopefully this will be remedied in the future so that different mechanisms are not attributed importance just depending on which model we choose to describe the interaction. For the time being we will have to be content with what we got and do the best of the situation.

Our motivation for investigating this reaction came from the following peculiarity. In 2001 two papers appeared with somewhat different approaches to the calculation of the reaction. Fäldt and Wilkin found the exchange of rho mesons to be the dominating mechanism for the reaction [35]. They put in the exchange of rho, pion and omega mesons and calculated their cross sections. They then turned on the $\eta NN$-coupling to see how large this could be without giving a result that would disagree...
with experiments. They found that the exchange of eta was negligible within their model. Peña et al. on the other hand found eta exchange to be one of the more important contributions [25]. They also introduced the short range mechanism which was their dominant term, while the exchange of rho mesons was quite small. Both these models describe the cross sections quite well, and their choice of coupling constants are the same in those part of the models that coincide, so these two models is a good target for investigation of the peculiarity concerning the emphasis attributed to different mechanisms.

When we try to describe the nucleon scattering off nucleons we have to incorporate a lot of different interactions. Several exchange types may contribute and one has to decide which values to choose for the different couplings. We have in our calculations focused on contributions by \( \pi, \eta \) and \( \rho \) meson exchanges as well as the short range contributions of Peña et al. Let us go through the different contributions one by one.

First off are exchanges of pions and etas. The pion is the lowest quantum of the nuclear force and are hence expected to have a large contribution in the threshold region. Next is the eta, also expected to have some impact on results, although its coupling is much weaker. In the low energy region, that we are considering, the \( \pi N \rightarrow \eta N \) and \( \eta N \rightarrow \eta N \) reactions are well described by a 3-resonance model. The exchanges are assumed to be over the \( S_{11}(1535) \), \( P_{11}(1440) \) and \( D_{13}(1520) \) resonances, see fig. 3.2.

![Figure 3.2: The eta and pion exchange diagrams.](image)

The \( \pi N \rightarrow \eta N \) vertex as well as the \( \eta N \rightarrow \eta N \) vertex is included via the \( \lambda \)-factors given in the previous section. The \( \pi NN \) and \( \eta NN \) vertices are given by the Lagrangians for these interactions.
The short range contributions appear due to the approximations in the CD-Bonn potential. In this potential only the positive-energy spinors are included. But for the five point functions to be invariant, under actions of the Poincare group, the negative-energy parts of the spinors has to be included as well. This has been studied by Peña et al. and we have included these type of terms in the form given in fig. 3.3.

![Figure 3.3: The short range diagrams](image)

The significance of these contributions can be calculated from the framework of large N QCD[34]. The structure of the components of the CD-Bonn potential is consistent with their dependence on N in the large N limit. The short range terms depend on the scalar potential, $v^{\pm}_{\pm}$, and the vector potential, $v^{\pm}_{\mp}$. Both these scale as N in the large N limit.

The exchange of $\rho$ mesons is complex in comparison with the others. We keep only two diagrams for these exchanges, either with a $N \rightarrow \eta N^*$ or a $N^* \rightarrow \eta N$ vertex given in fig. 3.4.
For this exchange we use the resonance approach in the same way as was done by Peña et al. Fälldt and Wilkin on the other hand related photo-production amplitudes to describe this exchange.

In paper II, we calculate $d\sigma/d\Omega_\eta$. This is done via the calculation of the different diagrams given in Fig. 3.2-3.4. The final state nucleons are assumed to be in relative S-states, this since we are close to threshold where these states are dominant [36]. New experimental results seem to indicate that higher partial waves may be of importance even at relatively small excess energies [37], but since the original two models did not attempt to attack this issue neither do we. It would make a comparison harder to evaluate.

A final state interaction was needed to reproduce energy dependence and normalization. We find a fair agreement for the these in comparison with experiment, as did the two papers which motivated our calculation. The key result was that, in contradiction to Peña et al., we found the short range contributions to be small. Possibly due to the difference in final-state interaction.

### 3.5 The Two-Step Model: Pentaquark Production

The interest in the subject of pentaquark systems is not new, the theoretical approaches to different aspects of QCD has a long history. Pentaquark discussions surfaced in the late seventies, again now and then during the eighties, so the new aspect this time is that the new wave of pentaquark discussions was born out of experimental results. Indications for a new resonance was found at 1.54GeV.
In our model we use a two-step approach to the production of this baryon. We assume it to be of positive parity and of isospin 1/2. This makes it possible to describe the $\Theta$ baryon and its interaction with the kaon-nucleon system via the Lagrangian,

$$\mathcal{L} = -ig_{K\pi\Theta}\bar{\Theta}\gamma_5(K^+n - K^0p) + h.c. \quad (3.18)$$

This together with the decay rate,

$$\Gamma(\Theta \rightarrow KN) = \frac{Q^2\pi m_{\Theta}}{2\pi m_{\Theta}}(E_N(Q) - m_n)g_{K\pi\Theta}^2 \quad (3.19)$$

gives us the needed equations for using the appropriate matrix elements of the previous section to calculate the cross section for the reaction.

The dynamics of the model is in many ways similar to that of helium production. The main difference lies in the three-body final state. It is in this regard more like the breakup version of the helium reaction, i.e., where helium breaks up into a nucleon and a deuteron. The production is assumed to proceed via a series of intermediate steps: an initial $pn \rightarrow \pi d$ reaction, is followed by a $\pi n \rightarrow K^+\Lambda$ reaction, and finally a $K^+d \rightarrow p\Theta^+$ reaction. These sub reactions are determined by the phenomenological matrix elements given in Section 3.2. Again we have a two-loop diagram to begin with, and thus two momenta to integrate over. This is done in a similar manner as for helium production. In the end we have one integration left which is solved numerically.

In Paper III, we calculate the cross section for this reaction for both parities of the pentaquark state. Assuming a width of 2 MeV/c$^2$ for $\Theta^+$, this mechanism suggests a production cross section of 4 nb for a positive-parity thetas, and of 12 nb for negative-parity thetas, all at 1363 MeV, i.e., 30 MeV above threshold.

### 3.6 Final State Interactions

We use three different final state interaction models. They describe the interactions between $\eta$-helium, nucleon-nucleon and $\Lambda$-nucleon. Let's go through them one at a time.

#### 3.6.1 $\eta$—Helium FSI

The total cross section for the $pd \rightarrow ^3He\eta$ reaction varies rapidly with energy near its threshold. The model used by Fäldt and Wilkin [38] explains this with a final state interaction factor, a factor where the eta is supposedly rescattered repeatedly onto the $^3He$ nucleus.
It was assumed that the range of the $\eta$-nucleon interaction is small compared with the intermediate distances within the helium nucleus. Fälldt and Wilkin arrive at an expression,

$$ F = \int dl \left( \frac{G(l)}{1 - 2 \left( \frac{l}{\ell} \right)} \right) $$

(3.20)

which is valid for vanishing momentum for the eta. The parameters are those of [38]. $G(l)$ is a smearing function which gives a mean value over the inter-nucleon distances, $l$, and the function $1 - 2 \left( \frac{l}{\ell} \right)$ is what is left from the multi-scattering wave function after angular average. Here $f$ is the $\eta$-nucleon amplitude.

At high energies this expression simplifies to,

$$ |F|^2 = 1 - \frac{2\sigma_{\text{tot}}}{4\pi} \left\langle \frac{1}{\ell^2} \right\rangle $$

(3.21)

a result that reflects the losses into other production channels at higher momenta.

### 3.6.2 Nucleon-Nucleon FSI

For the $NN \rightarrow NN\eta$ reaction one has another model. Close to threshold one may calculate the influence of nucleon-nucleon rescattering, and include it as an enhancement factor as follows [35].

Due to equal sharing of energy between the two nucleon near threshold one gets a reduced mass of the exchanged particle, $x$, of the ladder diagrams included in the model. This reduced mass may be written as,

$$ m_x^* = m_x^2 - \frac{\omega^2}{4} $$

(3.22)

where $\omega$ is the total energy of the eta meson.

An interaction matrix is obtained by projecting out the threshold angular momentum,

$$ \mathcal{M}_x = \mathcal{M}_x C_x \int_0^\infty \psi_k^{(+)r}(r) Y_1(m_x^*, r) j_0(\frac{1}{2} p_0 r) \psi_{p}^{(+)}(r) r^2 dr $$

(3.23)

with $\mathcal{M}_x$ the plane wave matrix element calculated earlier and $C_x$ and $Y_1$ defined as follows,

$$ C_x = \frac{1}{p} m_x^2 (m_x^2 + p^2) $$

(3.24)

$$ Y_1(m_x^*, r) = \left( 1 + \frac{1}{m_x^* r} \right) \frac{e^{-m_x^* r}}{m_x^* r} $$

(3.25)
The two wave functions, \( \psi_k^{(-)}(r) \) and \( \psi_p^{(+)}(r) \), describe the final nucleon-nucleon S-wave and the incident nucleon-nucleon P-wave, respectively. One uses these expressions to get the final state enhancement factor \( E_x(k) \) as the ratio between \( M_x^{int} \) and \( M_x \).

\[ Z(Q) = \frac{\beta_i^2}{\alpha_i^2} \left( \frac{2}{1 + \sqrt{1 + Q/\epsilon_i}} \right)^2 \]  
\[ (3.26) \]

\( i \) is for total spin-zero or spin-one in the final lambda-nucleon state and \( Q \) is the relative kinetic energy in the lambda-nucleon final state.
Chapter 4

QCD Strings and Beyond

THE SEARCH FOR a theory which will envelope the standard model as well as general relativity has, since the mid seventies, focused mainly on the development of string theory. Although string theory was originally put forward as a model for the strong interaction, it has taken on a life of its own during the past thirty years. Below will be given a brief introduction to the history of string theory before turning to the noncommutative geometry, our final tropic. For further details on string theory, readable references are [39, 40].

It is quite amazing that so many developments within theoretical physics seem to converge to a common mathematical framework as it has done within string theory. First of all, noncommutative geometry was put forward as a way to introduce an effective ultraviolet cutoff in field theory [41]. This was largely ignored at the time since the renormalization of quantum field theories was beginning to produce numerical predictions. Later on, once quantum chromodynamics had been put forward, the large \(N\) expansion was seen to be identifiable with a theory for a dual string [9].

The motivation for studying strings is often put forward as a natural way to solve the problems of singularities in Feynman amplitudes. Since stringy amplitudes are extended in space, the usual divergences, should disappear since the amplitudes in \(x\)-space will not have the usual problems as two particles meet at a point. This solves the problem of smearing out the interaction needed to get rid of divergence problems for quantum gravity. In quantum field theory this is problematic due to Lorentz invariance, which gives smearing out in time as well [44].

During the eighties the study of strings had its second revolution.
One became able to construct consistent theories, introducing supersymmetry to reduce the number of needed dimensions. Several different string theories were found which seemed to describe quite different phenomena. The investigations focused mainly on perturbative approaches, which was about to end with the introduction of the dynamical objects called D-branes which open strings may end on. They are higher-dimensional objects, so that a D0 brane is a line, while a D5-brane is a six dimensional surface through spacetime.

Due to the finding of several different dualities, such as T-duality, it was found that all the old string theories was identifiable with an eleven dimensional theory for supergravity. The differences between the string theories are due to different ways to get rid of the eleventh dimension.

It was also found that string theory could be identified with a noncommutative field theory in certain limits, caused by the generalized string theory version of the magnetic field. One example is the action for a stack of N coincident D0-branes in the presence of a background magnetic field. This gives rise to a bound state between a D0 and a D2 brane which may be interpreted as a noncommutative two-sphere [42].

In the nineties a third revolution started, mainly due to the conjectured correspondence between conformal field theory and anti de Sitter spacetimes [43]. The correspondence became known as the AdS/CFT correspondence has stimulated a lot of research during the past ten years or so.

String theory as a mathematical construct can be used for different purposes which makes it important to distinguish between two points of view. As a mathematical construction the correspondence between conformal field theory on the boundary and strings in the bulk is just a, conjectured, correspondence which makes it possible to relate the conformal field theory to the string theory in a way which makes it possible to extract information from one using the other. The relation goes both ways which introduces a subtlety within string theory, namely whether we consider the bulk of the string theory as physical or not. Of course it does not make any difference from a mathematical point of view but it does from a physical one. One has to decide whether the gravity of the bulk is physical gravity or some other gravity-like object. An example of this is the proof in the beginning of the nineties that quantum chromodynamics in two space-time dimensions was in fact equivalent to a string theory [45]. Other QCD topics such as confinement has also been under fruitful investigation using the AdS/CFT correspondence [46]. Here one looks at QCD as the real world and the string theory is just a mathematical construct used to describe it. Hence one can say that the circle was completed when the AdS/CFT correspondence made it possible to write down QCD-like theories using strings, so in a way
string theory managed to elucidate what it once sought to explain.
At last, let us approach the last topic of the thesis, the subject known as noncommutative geometry.

4.1 Noncommutative Geometry

For a quantum phase space there is the Heisenberg commutation relation which smears out phase space and replaces it with a Planck cell. This can be taken as a motivation for noncommutative geometry, one recovers ordinary space in the $\hbar \to 0$ limit.

This is actually nothing new. There are many examples of theories which are deformations of others. As an example, special relativity is obtained from Newtonian physics via deformation of the invariance group. By letting the deformation parameter $c^{-1}$ be nonzero one gets the Poincare group, instead of the Galilei group which is obtained as $c^{-1} \to 0$. The idea is thus to get a quantized theory via deformation of a classical one.

When considering Hamiltonian mechanics there is a concept of Poisson bracket which is needed to construct constants of motion. Recall that a symplectic manifold is a pair, first the manifold with local coordinates $p_i$ and $q_i$, then the symplectic form such that $\omega = \sum_i dp_i \wedge dq_i$. The symplectic form induces an isomorphism $I$ between the cotangent bundle and the tangent bundle. One can thus identify vectors and one-forms. In these terms the Poisson bracket of functions can be written as \{F, G\} = $\omega(\text{Id}G, \text{Id}F)$.

We will in the following look at deformation quantization of symplectic manifolds. This will give a rigorous way to construct noncommutative products for the function algebra over the symplectic manifold. The new product will have some deformation parameter and we will recover the usual function algebra of the manifold when we let this deformation parameter go to zero.

4.1.1 Deformation Quantization

Let $(M, \omega)$ be a symplectic manifold and $C^\infty(M)$ the algebra of infinitely differentiable functions over $M$. Let $\hbar$ be a deformation parameter. A new product is defined, between functions expressed as formal series of the deformation parameter, called a star product [47]. For each pair of functions on the form $f(x, \hbar) = \sum_k^\infty \hbar^k f_k(x) \in C^\infty(M)[[\hbar]]$ the product
between the two is a new function,

\[ f \ast g = \sum_{k} h_{k}(x) \]  

\[ h_{0}(x) = f_{0}(x)g_{0}(x) \]  

\[ f \ast g - g \ast f = -i\hbar \{ f_{0}, g_{0} \} + \ldots \]  

the coefficients of the new series are polynomials in the old coefficients and their derivatives. The dots in the last equation means higher order terms. Note that the first order term of the deformed product is written in terms of the Poisson bracket.

There are of course many different deformations. One of the most important is the one usually associated with the Moyal product. As symplectic manifold take \( \mathbb{R}^{2n} \) together with the symplectic form \( \omega = \omega_{jk}dx^{j} \wedge dx^{k} \). Here, \( \omega_{jk} \) is a (skew-symmetric, nondegenerate, with constant entries) matrix. For elements of \( \mathcal{C}^{\infty}(\mathbb{R}^{2n})[[\hbar]] \) there is then a star product,

\[ f \ast_{m} g = \sum_{r=0}^{\infty} \left( \frac{i\hbar}{2} \right)^{r} \frac{1}{r!} \omega_{j_{1}k_{1}} \ldots \omega_{j_{r}k_{r}} \frac{\partial^{r} f}{\partial x_{j_{1}} \ldots \partial x_{j_{r}}} \frac{\partial^{r} g}{\partial x_{k_{1}} \ldots \partial x_{k_{r}}} \]  

which is usually called the Moyal product [48].

The fact that this particular product is one of the most famous ones stems from its origin. In quantum mechanics functions turns into operators which causes noncommutativity between these, since operator products not necessarily is commutative. One can define a map between functions and operators and use the fourier transform to get a correspondence between the operator product and the Moyal product. On ordinary phase space with coordinates \( p_{i} \) and \( q_{i} \) one would have,

\[ W : f \rightarrow \hat{f} = \int \hat{f}(k^{i}, l^{i}) \exp(-i\hbar(k\hat{p}^{i} + l\hat{q}^{i}))dk^{i}dl^{i} \]  

and one gets the star product for this construction as \( W^{-1}(\hat{f} \hat{g}) = f \ast_{m} g \). By suitably choosing coordinates one can show that this is the same as the ordinary Moyal product. Hence the Moyal product can be seen as a natural choice in many physics applications, although for many examples specific products may be more natural, just as a specific choice of local coordinates on a manifold may be more suitable for a specific problem in physics.

### 4.1.2 Fedosov, Nest-Tsygan Index Theorem

One of the most beautiful results on Deformation Quantization is the theorem by Fedosov, Nest and Tsygan [49]. This theorem connects an
algebraic index to the canonical trace of the manifold.

First of all, a star product is not unique, there can be many different possible quantizations for the same symplectic manifold. Two products are called equivalent if there exists a map, $\mathcal{F}$, such that $f \star_1 g = \mathcal{F}^{-1}(\mathcal{F}f \star_2 \mathcal{F}g)$ for all functions $f$ and $g$. If we disregard formal base changes, then the products are determined by a characteristic class $\theta(M) \in \omega/h + H^2(M)[[h]]$ which classifies star products up to global equivalence.

Other topological classes can be combined into the A-roof genus. This can in our case, i.e. a compact symplectic manifold, be expressed as $\hat{A} = \exp(c_1(M)/2)\text{Todd}(M)$, where $c_1(M)$ is the first Chern class and $\text{Todd}(M)$ is the Todd class of the manifold $M$. For a more elaborate description of these quantities, see for example [50].

For symplectic manifolds the Moyal product is of certain importance. With this product, as for all others, one can define a canonical trace $\text{Tr}_{\text{can}}$. The canonical trace density for any product can be obtained from the Moyal density. This since all products can be identified with the Moyal product, with an equivalence map $\mathcal{F}$, in some small neighborhood [51]. For the Moyal product one has the canonical trace density,

$$\mu_m = \left(\frac{1}{2\pi h}\right)^n \Omega$$

(4.6)

$$\Omega = \frac{\omega^n}{n!}$$

(4.7)

with which one has $\text{Tr}_{\text{can}}f = \int_M f \mu_m$. To obtain the canonical trace density of a product other than the Moyal, one uses $\int_U \mathcal{F}f \mu_c = \int_U f \mu_m$, where $U$ is the neighborhood, $\mu_c$ the canonical trace density of the product and $\mu_m$ the trace density of the Moyal product. This identification can be done globally since the trace densities agree on the overlaps. The procedure to identify two products in enough patches to cover the whole manifold is cumbersome and few canonical densities are known explicitly.

With the help of the above we may formulate the Fedosov-Nest-Tsygan index theorem,

$$\text{Tr}_{\text{can}}(1) = \int_M e^{\theta} \hat{A}(M)$$

(4.8)

For certain symplectic manifolds there is enough symmetries of the star product to be able to extract the algebraic content of this theorem from one point of the manifold. This is a consequence of the “almost” identical trace densities. When enough symmetry is at hand the canonical trace density will only differ from that of the Moyal product by an $\hbar$ dependent constant. It is then enough to study the trace of the delta function instead of the identity function.
In Paper IV, we show that two seemingly different star products on $S^2$ actually are the same. We then, by finding an isomorphism between this product and the Moyal product at the “north-pole” and use the Fedosov-Nest-Tsygan index theorem to calculate the characteristic class for the product.
Chapter 5

Summary of Papers

Paper I

The differential cross section for the reaction \( pd \rightarrow ^3He\eta \) in the energy region 930-1100 MeV is investigated in a two-step model. The model assumes a pi meson to be produced in a first \( NN \rightarrow \pi d \) collision and the eta meson in a second \( \pi N \rightarrow \eta N \) collision. Part of the angular dependence observed is ascribed to the \( \pi N \rightarrow \eta N \) amplitude, through its dependence on the \( P_{11} \) and \( D_{13} \) resonances, but the main structure comes from the \(^3He\) form factor. Fermi motion is taken into account, but does not seem to significantly change the results. The two-step model by itself is not completely successful in explaining the \( pd \rightarrow ^3He\eta \) differential cross section, but basic features are reproduced.

Paper II

The cross section for the \( pn \rightarrow pn\eta \) reaction is compared to that of the \( pp \rightarrow pp\eta \). Included are contributions from exchange of \( \eta^-\), \( \pi^-\) and \( \rho^-\) mesons as well as short range effects, from the negative energy pole of the nucleon, in \( \rho^-\) and \( \omega^-\) exchange contributions. Both angular and energy distributions are obtained. It is found that the short range contributions to this reaction is small in contrast to earlier investigations, while confirming the importance of eta exchange contributions.
Paper III

We estimate the cross section for production of theta baryons, $\Theta^+$, in the reaction $pd \rightarrow p\Lambda$ near threshold. The production is assumed to proceed via a series of intermediate steps: an initial $pN \rightarrow \pi d$ reaction, followed by a $\pi N \rightarrow K^+\Lambda$ reaction, and finally the $\Theta^+$ is formed in a $K^+d \rightarrow p\Theta^+$ reaction. Assuming a width of 2 MeV/c$^2$ for $\Theta^+$, this mechanism suggests a production cross section of 4 nb for a positive-parity thetas, and of 12 nb for negative-parity thetas, all at 1363 MeV, i.e., 30 MeV above threshold. The cross section is in our model directly proportional to the width of $\Theta^+$.

Paper IV

We calculate the canonical trace and use the Fedosov-Nest-Tsygan index theorem to obtain the characteristic class for a star product on $S^2$. We show how, for this simple example, it is possible to extract the relevant information, needed to use the Fedosov-Nest-Tsygan index theorem, from a local calculation.
I har nu gått igenom de olika delarna av avhandlingen och det är dags att runda av. Till detta ändamål passar en populärvetenskaplig sammanfattning bra.

Figure 6.1: En helt vanlig hand, min.

Vi börjar vår resa genom den här sammanfattningen med en helt
vanlig hand, som i det här fallet räkar vara min egen, se Fig. 6.1.

Vi är alla uppbyggda av partiklar. Så tror vi i alla fall just nu. Om hundra år kanske vi har en helt annan bild av hur världen fungerar, så vi kan lika väl att vara lite skeptiska. Men vi kan med säkerhet säga att det är en ganska bra approximation, att vi är gjorda av partiklar alltså.

När vi tittar närmare på handen och förstorar upp den börjar nya strukturer framträdna. Till en början ser vi kanske mest rynkor och ett och annat härstrå, men efter att förstorat upp tillräckligt stort, ser vi molekyler. Det är av dessa molekyler som t.ex. DNA-bandet som vi ser i Fig. 6.2 är uppbyggt.

![Figure 6.2: En del av en dna-kedja.](image)

Dessa molekyler är i sin tur uppbyggda av ännu mindre beståndsdelar. Varje molekyl består bland annat av en samling ihopklumpade kärnor, som i sin tur består av ännu mindre beståndsdelar som vi kallar partiklar. De lättaste kärnorna och en del partiklar, nämligen nukleonerna, är en del av de saker som behandlats i den här avhandlingen.

I stora acceleratorer runt om i världen sitter experimentalister och utför experiment genom att krocka partiklar med andra partiklar, samt partiklar med kärnor. Ett exempel på hur detta kan se ut finns i Fig. 6.3.


Vid krockarna splittras partiklarna upp och bildar temporära partiklar som sedan återförenas för att bilda sluttillstånden. En bild av hur dessa splittras och återförenas kallas för Feynman diagram, på vilka många exempel finns i Kapitel 3. Dessa Feynman diagram är bilder av hur vi approximerar den totala övergången mellan två tillstånd. Man kan jämföra Feynman diagrammen med vågnätet på en karta. Egentligen

Figure 6.3: Partiklar och kärnor som krockar.
finns det massor med möjliga vägar att åka mellan två olika städer. Men de flesta åker ju längs vägar som de tycker är smidigast. Därför ritar vi bara ut de stora vägarna som partiklarna tar och hoppas att dessa räcker för att beskriva hur partiklarna färdas. De diagram vi tar med i en teoretisk modell ger en bild av vad vi tror är viktig för att återge den verkliga händelsen.

Om vi tar partikeln och förstorar upp den så finns det många experimentella resultat som tyder på att de är uppbyggda av mindre partiklar som vi kallar kvarkar. Om vi går vidare och förstorar upp kvarken för att se om den har någon mindre struktur lämnar vi den av fysiken som är experimentellt belagd. Vi kan nu endast spekulera om vad vi tror att de är uppbyggda av, och hoppas att experiment i framtiden kan visa om vi trodde rätt eller fel.

Den teori som har verkat lovande de senaste tioåren är strängteorin. I Fig. 6.4 ser vi hur vi tror att det kan se ut. Vi förstorar upp kvarken och vi ser att den egentligen är en sträng. Om strängen har ändpunkter så tror man att de måste sluta på vissa typer av ytor som man kallar för D-bran. Dessa är speciella delrum av den totala rumstiden.

I vissa situationer kan D-branen bilda bundna tillstånd. Ett exempel på detta är hur ett bundet tillstånd mellan ett 2-bran och ett 0-bran kan bildas i närvaro av ett starkt elektrisk fält. Detta bundna tillstånd,
som kan ses som en sfär, är lite säreget, koordinaterna kommutterar inte! Om vi t. ex. har två funktioner som beror på var du befinner dig på jorden. $T(x)$ för hur mycket klockan är när du befinner dig på punkten $x$, samt $L(x)$ för hur långt det är hem. I vanliga fall gäller $TL = LT$, funktionerna kommutterar, men på icke-kommutativa ytor gäller detta inte, $T \ast L \neq L \ast T$. Den nya produkten som har denna egenskap, $\ast$, kallar vi för stjärnprodukten. I sista kapitlet diskuterar vi några av de matematiska verktyg som kan tänkas vara till hjälp vid beskrivning av sådana produkter, på en sfär. I Fig. 6.5 illustreras denna sfär.

![Diagram](image)

*Figure 6.5: Den icke-kommutativa sfären.*

Jag hoppas att denna korta sammanfattning varit någorlunda begriplig samt att läsare haft en angenäm lästund.
I can’t understand why people are frightened of new ideas. I’m frightened of the old ones.

*John Cage*

A finished work is exactly that, requires resurrection.

*John Cage*
Acknowledgements

W RITING ACKNOWLEDGEMENTS IS a painful business. If you feel you are forgotten, read it one more time. Thanks to my supervisor Göran Fälldt for taking me on as a Phd-student and for being really helpful during the writing of the thesis. Thanks to Anton Yu. Alekseev, for inviting me to Geneva which was one of the most stimulating periods of my time as a graduate student, and for teaching me a lot in general. Thanks to Keizo Matsubara for letting me work with him on one of the papers in this thesis, I had a great time eating all those fondues, we should do it again some time! Thanks to Inger Ericsson, for valuable help during the last four years. Thanks to Jan Conrad, Keizo Matsubara, Göran Fälldt, Bengt Karlsson, Bo Höistad and Pierre Bäcklund for reading the manuscript. Thanks to Johan Rathsman for discussions on QCD. Thanks to Jan Conrad, who was the only one who could cope with my coffee habit, for valuable help and endless discussions...

Thanks to my parents, Agneta and Olle, for being there whenever I need them, wouldn’t have made it without you! Thanks also to my brother Marcus for talking me into going to Uppsala in the first place. Thanks to my sister Maria and to my grandparents Astrid and Einar for being there, maybe now I will find time to actually explain what I’ve been doing. Thanks to Ruff and Iris who are no longer with us, you are still in my head and I hope the memory of you will stay forever bright.

Rakastan sinua Jaana, minun kullanmuru! I hope you will always stay with me, although I’m a nutcase.

My sincere thanks goes out to all friends, you know who you are, I wouldn’t have stayed (in?) sane without you!
I slept with faith and found a corpse in my arms on awakening;
I drank and danced all night with doubt and found her a virgin in the morning.

Aleister Crowley
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48 References


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