A praxeological comparison of algebra content in vocational and academic preparatory programmes in Sweden

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We compare the algebra content in the Swedish upper secondary syllabi for higher educational preparatory (HEP) and vocational educational and training (VET) programmes. The study is theoretically embedded in the Anthropological Theory of the Didactic (ATD), where praxeology is used as an analytical tool. The results reveal that the algebra content in the VET programmes is more focused on praxis ('know-how') aspects compared to the HEP programmes, in which the balance between praxis and logos ('know-why') aspects is more even. We discuss the results in view of students' opportunities to develop algebraic knowledge within the framework of the given syllabi.

Keywords: Algebra, ATD, praxeology, syllabus.

In 2011, Swedish upper secondary school underwent a major reform, with the implementation of new curriculum documents and school organisational structures (Swedish National Agency for Education, 2011). An essential consequence of this was a stronger separation between vocational educational and training (VET) and higher educational preparatory (HEP) programmes (Lindberg & Grevholm, 2013), which has raised questions regarding social justice and purposes of education (Nylund et al., 2017). As part of the reform, three alignments in mathematics were introduced: for vocational, social science-oriented, and natural science-oriented programmes. Before 2011, all students, regardless of programme, took the same first course in mathematics. In this paper, we look more closely at the mathematical content in the syllabi for the three alignments in the 2011 curriculum, particularly the algebra content in the first two courses for each alignment.[1]

Algebra is often referred to as a gatekeeper, not only to more advanced studies in mathematics and science (Blanton et al., 2015) but also to participating in society and gaining full access to civil rights (Moses & Cobb, 2001). At the same time, algebra appears to be a problematic topic for students in several countries (Hemmi et al., 2021). One reason for this may be that algebra has traditionally not been introduced until secondary school, creating a gap between arithmetic and algebra (Linchevski & Herscovics, 1996). However, in recent decades, research has repeatedly shown that students benefit from being gradually introduced to algebra already from the earliest grades (Blanton et al., 2015). These findings have slowly made their way into the educational system (Hemmi et al., 2021).

In Sweden, algebra has been a difficult topic for students to manage for many years (Hemmi et al., 2021). In the international evaluation TIMSS (Trends in International Mathematics and Science Study), Swedish students' results in algebra have been below international average since the 1960s (Bråting, 2021). Research has revealed that generalised arithmetic is virtually absent in the last three Swedish curricula for compulsory school (Bråting, 2021), and Swedish students have trouble understanding the different roles of variables (Kilhamn, 2014) as well as using the relational property

[1] Since the analyses conducted in this paper, small revisions have been made to the syllabi. The analyses in this paper are based on the syllabi that were valid until 30 June 2021.

of the equal sign (Madej, 2021). However, this research, like most of the recent Swedish research on algebra learning, was conducted at compulsory school level. Research on algebra learning at upper secondary level in Sweden is lacking, particularly since the 2011 reform. One exception is Gustafsson's (2019) thesis on upper secondary students' difficulties with algebra, revealing that students still struggle to understand the different roles of variables and the invisible multiplication sign in expressions such as 4x (see also Hewitt, 2012).

This study is part of a larger project exploring socioeconomic aspects of the didactic transposition of algebra in Sweden. In the present study, the focus is on the transposition of algebra from scholarly produced knowledge to the educational system (Chevallard, 2006). Hence, the study is theoretically embedded in the Anthropological Theory of the Didactic (ATD), where praxeology is used as an analytical tool. The aim is to unpack differences in how algebraic content is formulated in the syllabi for VET and HEP programmes, and to discuss how these differences may affect students' opportunities to learn algebra. In forthcoming studies, the focus will shift towards aspects regarding social justice and purposes of education. We pose the following research question: *From a praxeological perspective, what characterises the algebra content in syllabi for VET and HEP programmes and what are the main differences?*

A praxeological perspective on school algebra

The Anthropological Theory of the Didactic (ATD) is particularly useful for studying teaching content, such as school algebra, from an institutional perspective. It acknowledges that humans and human activity are involved in the teaching and learning process, that syllabi do not appear ex nihilo, and that the process is an "exogeneous production" (Bosch & Gascón, 2006: p. 54). In other words, the process is "something generated outside school that is moved […] to school out of a social need of education and diffusion" (ibid.). Within the ATD, this process is expressed in terms of the *didactic transposition,* which describes how (mathematical) knowledge is transposed between different institutions (Figure 1). The outcomes of the teaching and learning process depend on the humans involved in it – starting with scholars developing and determining the content at one end, and ending with the students' learned knowledge on the other (Bosch & Gascón, 2006). From this perspective, it is thus possible as a researcher to study a content from an unbiased position. In this study, we are interested in investigating the transposition of algebra from 'Scholarly knowledge' to 'Knowledge to be taught' (Figure 1). This corresponds to the algebra content developed by professional mathematicians, selected by the educational system, and transferred into so-called school algebra (Bråting, 2021; Hemmi et al., 2021; Kilhamn, 2014).

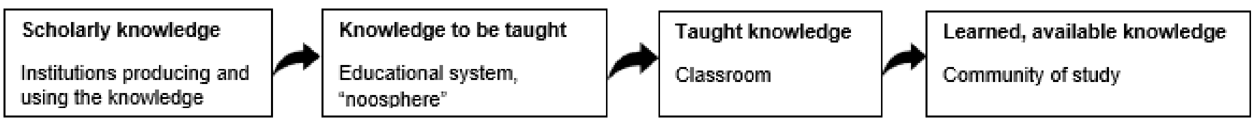

Figure 1: The didactic transposition from Bosch and Gascón (2006, p. 56)

To describe mathematical (indeed, any human) activity, ATD employs the notion of *praxeology* (Bosch & Gascón, 2006). Any praxeology is divided into two blocks: praxis ('know-how') and logos ('know-why'). The praxis block thus contains the practical part and consists of types of *tasks*, (mathematical) tasks or exercises to be done, and *techniques*, the method(s) connected to the type of task. The second block, logos, consisting of *technology* and *theory,* refers to human thinking and reasoning and is the 'explaining' part of the praxeology (Chevallard, 2006) (see Figure 2). The suffix -*logy* in technology indicates that this is a discourse on a given technique. This discourse is expected to justify the technique as a valid way of not only solving a particular type of task but also of clarifying the logic behind it. Theory can therefore be explained as an overarching theory or a set of underlying principles that justifies the technology (e.g. Chevallard, 2006).

PRAXIS	TASK
	TECHNIQUE
LOGOS	TECHNOLOGY
	THEORY

Figure 2: A model of the praxeology concept (Chevallard, 2006)

School algebra has been studied from an ATD perspective for many years. Summarising findings from such research, Bosch (2015) describes a situation in which a formal approach to algebra predominates, to the detriment of a functional view. In today's secondary schools, Bosch claims, algebra is largely identified with equation solving, and 'the language of algebra' is reduced to a formal structure whereby students are asked to manipulate algebraic expressions with little regard for what they represent. In contrast to this, she outlines a view of school algebra building on ATD principles, where it is instead interpreted as "a *process of algebraization* of already existing mathematical praxeologies" (ibid., p. 61, emph. in original). In other words, instead of being yet another piece of mathematical content, algebra appears as a general tool for modelling any school mathematical praxeology.

Method, material, and procedure

In this study, we have conducted a praxeological analysis of the syllabi for the first two mathematics courses in the national curriculum for upper secondary level in Sweden, Lgy11[2]. We view curriculum documents as indicative of the transposition from scholarly knowledge to knowledge to be taught (Figure 1); through this analysis we have been able to discern nuances in the written language and unpack implicit meanings behind the formulations in the syllabi, thus contributing knowledge of the transposition processes behind the algebra content in the upper secondary curriculum.

The first two mathematics courses in Lgy11 each have three alignments: a, b, and c, aimed at vocational (VET), social sciences-oriented (HEP), and natural sciences-oriented (HEP) programmes, respectively. In vocational programmes, Course 1a is compulsory and 2a is optional. In social sciences-oriented programmes, 1b and 2b are compulsory and 3b is optional (compulsory in economy). In natural sciences-oriented programmes, 1c, 2c, and 3c are generally compulsory, while 4 and 5 are optional (4 is compulsory in some programmes). The subject plan starts with an introduction to the subject and a formulation of aims. The syllabus for each course (1a, 1b, 1c etc.) consists of core content and knowledge requirements based on mathematical competencies. In this study, the sole focus of the analysis has been on the core content. This is organised in categories

[2] In the rest of the paper, we will use the abbreviation Lgy11 (Läroplan för gymnasieskolan [Syllabus for the upper secondary school], Swedish National Agency for Education, 2012).

written in bullet points, which constitute our unit of analysis. Before beginning the analysis, we extracted the algebra content from the core content, using Blanton et al.'s (2015) so-called big ideas of algebra consisting of I) variables, II) equivalence, expressions, equations, and inequalities, III) generalised arithmetic, IV) proportional reasoning, and V) functional thinking. The units of analysis containing algebra content were then classified in praxeological terms. Here it should be pointed out that Swedish school syllabi contain no explicitly formulated tasks, meaning that this part of the praxis block did not appear in the analysis.

In the classification, we made use of what we have called 'determining words'; that is, words that carry information on how the core content can be interpreted praxeologically. For instance, the core content in Course 1a regarding problem solving is formulated: "Strategies for mathematical problem solving including the use of digital media and tools" (Swedish National Agency for Education, 2012, p. 5). The word 'strategies' indicates an emphasis on different methods, which we interpret as *technique* (praxis). As another example, in the core content of Course 3b in the category 'Algebra', we find: "The concepts of polynomial and rational expressions, and generalisation of the laws of arithmetic for dealing with these concepts" (ibid., p. 27). The term 'concept' suggests that the teaching situation should offer a wider perspective on polynomials and rational expressions, i.e. to learn *why* they work. In learning a mathematical concept, definitions and underlying principles are conceivably offered, which we interpret as *theory* (logos). Contrastingly, the term 'generalisation' suggests drawing on established rules to justify new algebraic rules, and we have interpreted this as indicative of *technology* (logos).

Following the classification, the pieces of algebra content were sorted into tables with one column for each alignment. To help identify patterns in the content of the different courses and to recognise differences and similarities between the alignments, as a last step the praxeological 'determining words' were picked out and placed in a new summarising table. The analysis was done by the first author, but was discussed and reviewed by all authors throughout the process.

Results

We begin by presenting the results of the analysis of algebra content in all three alignments in the core content categories "Understanding of numbers, arithmetic, and algebra" and "Relationships and change". In order to be as transparent as possible, we provide the tables used in our analysis for Course 1 in both categories. Due to lack of space, the tables connected to Course 2 are not shown. In the following tables, green, purple, and yellow mark technique, technology and theory, respectively.

Table 1: The category 'Understanding of numbers, arithmetic, and algebra' in Course 1

Ma1a (VET)	Ma1b (HEP, social sciences)	Ma1c (HEP, natural sciences)
Handling algebraic expressions and formulae relevant in subjects typical of a programme, […]	Handling algebraic expressions and formulae relevant to subjects typical of programmes.	Generalisation of the rules of arithmetic to handle algebraic expressions.
	The concept of linear inequality.	The concept of linear inequality.
[…] as well as methods for solving linear equations.	Algebraic and graphical methods for solving linear equations and inequalities and exponential equations.	Algebraic and graphical methods for solving linear equations and inequalities and exponential equations.

As Table 1 shows, in the first category in Course 1 alignment *a* merely contains determining words indicating a focus on technique (praxis), whereas alignment *b* also contains wordings indicative of theory (logos). Words interpreted as indicative of technology (logos) were only found in alignment *c*. Furthermore, it is notable that the term 'generalised arithmetic' (technology) only occurs in alignment *c* (top right in Table 1) and that inequalities are only referred to in the HEP alignments.

Overall, Course 2 contains more algebraic topics in this category than Course 1 does. Some of the content focusing on praxis aspects is exactly the same in all three alignments, for instance certain methods for solving equations. However, the way linear equations are described differs between the alignments: While alignment *a* contains the phrase "use of linear equations in problem solving situations", in alignments *b* and *c* it is formulated as "the concept of linear equations". A similar distinction appears regarding logarithms: While alignment *c* introduces the *concept* of logarithms (theory), in alignment *b* the concept of logarithms is connected with *solving* exponential equations (technology). In alignment *a*, logarithms are not included at all.

Table 2: The category 'Relationships and change' in Course 1

Ma1a	Ma1b	Ma1c
The concepts of ratio and proportionality in reasoning, calculations, measurements, and constructions.		
Differences between linear and exponential processes.	The concept of a function, domain, and range of a definition, and also properties of linear functions and exponential functions.	The concept of a function, domain, and range of a definition, and also properties of linear functions and exponential functions.
	Representations of functions, e.g. in the form of words, shapes, functional expressions, tables, and graphs.	Representations of functions in the form of words, functional expressions, tables, and graphs.
	Differences between the concepts of equation, algebraic expression, and function.	Differences between the concepts of equation, algebraic expression, and function.

In the category "Relationships and change", wordings suggest a greater emphasis on logos compared to the previous one. This holds for both Courses 1 and 2, and for all three alignments. In Course 1, phrases indicative of technique (praxis) are only found in alignment *a* (in connection with ratio and proportionality; see top left in Table 2), while technology appears in all three alignments, either in the form of concepts in connection to specific techniques (in alignment *a*) or specific aspects of concepts, such as properties of or representations of functions (alignments *b* and *c*). Furthermore, in Course 1, a focus on theory can only be found in alignments *b* and *c*, in connection with the function concept. However, here it is important to emphasise that in alignment *a*, functions are not introduced until Course 2. The syllabus for Course 2a describes the function concept in almost the same way as in alignments *a* and *b*, except that "applications of functions" are added in Course 2a, indicating an emphasis on praxis. Finally, it is worth mentioning that in Course 2 the content regarding graph construction is phrased identically in all three alignments.

In Table 3 we have compiled all determining words found in the syllabi, and ordered them according to whether they are indicative of technique, technology, or theory. As the analysis revealed only small differences between the *b* and *c* alignments, in the table we have merged the two into the same column. In conclusion, Table 3 indicates that the praxeological organisation of the algebra content of the first two courses in the VET and HEP alignments differs in focus. The VET praxeology has a strong emphasis on praxis, while logos, in particular theory, is largely absent. The HEP praxeology, on the other hand, is more even in its emphases, with a fairly strong theoretical component.

Table 3: A compilation of the determining words that indicate technique, technology, or theory

	Ma1a	**Ma2a**	**Ma1bc**	**Ma2bc**
Technique	Methods	Methods	Methods	Methods
	Handling	Handling	Handling	Handling
	Strategies	Strategies		
	Calculations			
		Use of		
	Solution of	Solving	Solving	Solving
	Construction	Construction		Construction
		Applications		Applying
	Measurements			
Technology		Representations	Representations	
	Reasoning	Reasoning		Reasoning
		Properties	Properties	Properties
	Differences			
Theory	The concept	The concept	The concept	The concept
		Differences between concepts	Differences between concepts	Differences between concepts
			Motivation	Motivation
			Generalisation	
				Extension

Discussion

In response to our research question, through a praxeological analysis of the mathematics syllabi for the first two courses in the Swedish national curriculum for upper secondary school, we have concluded that the praxeological organisation of the algebra content in the VET alignment emphasises praxis (techniques), whereas the praxeological organisation in the HEP alignments is more evenly balanced between the praxis and logos blocks. However, although theory and technology (logos) are emphasised more in the HEP alignments, they are typically not connected to particular techniques. Indeed, connections between techniques and technologies are rarely explicit in the syllabi. The content regarding ratio and proportionality in Course 1a, generalisation of arithmetic laws in Course 1c, and complex numbers in Courses 2b and 2c are the only pieces of core content with formulations of technologies supporting techniques in the same core content. This lack of connection between technique and technology, as well as the limited role of theory in alignment *a*, makes it difficult to view the praxeologies outlined by the syllabi as complete, even allowing for the lack of explicit task formulations.

One of our main findings was that the praxeological organisation of school algebra in VET programmes syllabi emphasises several techniques, with little or no technology or theory justifying them. This resonates with the formal and technical approach that Bosch (2015) claims is characteristic of school algebra internationally: The 'language of algebra' is reduced to different techniques of equation solving and manipulation of algebraic expressions. It is not *per se* a problem that praxis is more emphasised in the VET programmes syllabi; given that the overall purpose of VET programmes is to act as preparational for the various future vocations, a more practical focus is needed. However, what is problematic is the inadequate connection between techniques and technologies in the core content. As technologies serve to justify the given techniques (see Figure 2), one apprehension regarding this – which is supported by Bosch's (2015) description of school algebra – is that the techniques are turned into mere 'recipes' for solving given tasks, without being grounded in knowledge of the underlying concepts. Such knowledge might, for instance, enable students to select the most efficient technique for solving a particular task. In the core content for HEP programmes, on the other hand, we saw more formulations that were only connected to the logos block (mostly theory), but with little or no connection to particular techniques.

As Gustafsson (2019), Madej (2021) and Hewitt (2012) have already stressed, students have difficulty understanding the meaning of different algebraic representations, such as the meaning of the equal sign and variables. From this study, we know that the algebraic knowledge to be taught at upper secondary level has a fairly insufficient organisation of praxis and logos, regardless of alignment. While it might be possible to build teaching situations merely around logos or praxis aspects of a concept, for a praxeology to hold and make sense, both aspects are needed, as "[praxis] entails logos, which in turn backs up praxis" (Chevallard, 2006, p. 3). Thus, one might ask whether the insufficient organisation observed in this study is connected to students' understanding of different algebraic representations, and whether the differences detected between alignments persist throughout the didactic transposition. Hence, as this small study focused only on formulations in the syllabi and not on students' knowledge, textbook content, or teaching situations, our next step is to examine how textbooks and teachers emphasise and organise algebra praxeologically. These forthcoming studies will also help us to dig more deeply into questions regarding social justice and purposes of education (Lindberg & Grevholm, 2013; Nylund et al., 2017).

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