Monetary Policy in Closed and Open Economies
Two DSGE models are calibrated and simulated to investigate how the role of monetary policy differs between a closed and an open economy. The central bank conducts monetary policy according to a Taylor (1993) rule, reacting to inflation- and output deviations. Prices are sticky and there are habit components which slow down adjustment of consumption and exports. The models are subjected to shocks in the interest rate, inflation, technology and consumption. In most of the cases the shocks have a bigger and quicker affect on output and employment in the open economy. In connection with positive consumption- and interest rate shocks inflation is big and negative at first but gets positive already two quarters after the shock, due to effects in the exchange rate channel. In closed and open economies, a stronger reaction to output, than in the standard Taylor (1993) rule, decreases welfare losses dramatically.
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In recent years, the economies of the world have become more and more integrated both in the financial– and goods market. Many small Western countries with earlier restricted and controlled capital flows have become “small open economies” making them very dependent on what happens to the world economy.

Due to the fact that countries have opened up their borders on the capital markets, more countries have chosen to make their currency fully flexible. According to the Mundell-Fleming model this implies that fiscal policy is no longer efficient in trying to stabilize the economy, instead it is monetary policy that’s efficient. In order for the countries to conduct a credible monetary policy, central banks have been made more independent from their governments and they also have a clearer goal; to stabilize inflation.

The purpose of this thesis is to compare the role that monetary policy plays in an open and a closed economy respectively. Does monetary policy have a smaller/larger impact in the open economy? Are some variables relatively more affected in the open economy? Is the speed of adjustment faster or slower?

To answer questions such as these, dynamic stochastic general equilibrium (DSGE) models of a closed- and small open economy have been created. DSGE modelling was first proposed in Christiano et al (2001) where Real Business Cycle\(^1\) (RBC) theory, with its introduction of stochastic shocks, and New Keynesian economics\(^2\), with rigid prices, rational expectations and monopolistic competition, are the key ingredients.

In the models, the central bank conducts monetary policy according to a Taylor (1993) rule with weights on inflation- and output deviations. The individuals maximize utility with respect to consumption and leisure where consumption has a habit component as in

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\(^1\) Kydland and Prescott (1982) was the starting point to RBC theory

\(^2\) See; Ball, Mankiw and Romer (1988)

The models are simulated with the Matlab application Dynare\(^3\). Numerical values are chosen for the parameters (calibration) and the models are subjected to stochastic shocks in the interest rate, inflation, technology and consumption.

The results of the simulations show a number of differences that come with openness. A shock in the interest rate gives a bigger decline in output and employment while inflation drops immediately the first quarter but becomes positive thereafter. The inflation shock creates a larger decline in output and employment, as a consequence of increased interest rate, appreciation and a decline in the net export. The technology shock has a larger and quicker effect on production but consumption is smoother because households can lend abroad. The consumption shock has a much smaller impact on both production and employment since part of the demand shock is directed towards foreign goods.

The results also suggest that, in order to minimize welfare losses, the central bank should react more to output deviations than in the standard Taylor (1993) rule.

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\(^3\) See; Juillard (1996) for details about the software
2 CLOSED MODEL

2.1 Important notations
When the word “log linear” is used in the thesis, it implicitly means logarithmic deviations from steady state using a first order approximation. For instance, \( Y_t = \beta X_t \) in log linear form becomes \( y_t = x_t \), since the constant disappears when looking at deviations. Small letters are defined as \( x_t = \ln X_t - \ln X \) and the absence of the time subscript \( t \) defines the variable’s steady-state value. Another useful manipulation is that \( x_t = \log X_t - \log X \) can be rewritten as \( X_t = Xe^{x_t} \approx X(1 + x_t) \).

2.2 Households
The consumer gets utility according to a CES utility function with external habit formation. The habit property is based on the assumption that the consumer gets utility relative to a previous level of consumption. A previous high consumption encourages the consumer to keep up this level which delays the adaption to a change in income. These delayed changes appear as hump shaped curves in the dynamics. This have been shown to improve the realism of economic models where the permanent income hypothesis fails to model fluctuations in practice, where hump shaped curves tend to occur; Fuhrer (2000).

In our case a utility function using external habit as in Abel (1999) is used. The term “external” refers to the fact that “aggregated” consumption is the reference level. Internal habit uses the consumers “own” previous consumption as a reference level instead. The weight that is put on the habit is decided by \( \gamma \in [0,1] \). The utility function in period \( t+j \) is given by

\[
\frac{1}{1-\theta} \left( \frac{C_{t+j}^r}{C_{t+j-1}^r} \right)^{1-\theta}
\] (1)
where \( \bar{C} \) is the aggregated consumption level and \( \theta \) is the measure of absolute risk aversion, which makes \( \sigma = \frac{1}{\theta} \) the intertemporal elasticity of substitution, i.e. how willing the consumer is to smooth consumption over time. With the form of habit formation used in (1) we must have \( \theta \neq 1 \) because otherwise the habit will disappear from the equation and we’ll end up with a regular CES function. More precisely \( \theta > 1 \) is assumed from now on, which is usually the case for functions with habit formation; This will be discussed further in the calibration chapter.

The individual also care about the disutility from work, \( N_{t+j} \), according to (2).

\[
\frac{1}{1 + \varphi} N_{t+j}^{\theta \varphi}
\]

(2)

Where \( \varphi \) is the elasticity of labour supply.

The individual maximizes (3) subject to the budget constraint (4)

\[
U_{\text{Max}} = \sum_{j=0}^{\infty} \beta^j \left( \frac{1}{1 - \theta} \left( \frac{C_{t+j}}{C_{t+j-1}} \right)^{1-\theta} - \frac{1}{1 + \varphi} N_{t+j}^{\theta \varphi} \right)
\]

(3)

\[
R_{t+j} B_{t+j} + W_{t+j} N_{t+j} = B_{t+j+1} + C_{t+j} \quad \forall_j
\]

(4)

\( \beta = e^{-\rho} \) where \( \rho \) is the discount rate and \( B_{t+j+1} \) is the amount of assets carried over to the next period. \( R_{t+j} \) is the real interest rate realized in period \( t+j \) and defined as

\[
R_{t+j} = \frac{1 + \hat{i}_{t+j-1}}{1 + \pi_{t+j}}.
\]

The individual’s maximization problem is set up over an infinite time horizon, as a lagrangean function in (5)

\[
L = \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1 - \theta} \left( \frac{C_{t+j}}{C_{t+j-1}} \right)^{1-\theta} - \frac{N_{t+j}^{\theta \varphi}}{1 + \varphi} \right] + \sum_{j=0}^{\infty} \lambda_{t+j} \left[ R_{t+j} B_{t+j} + W_{t+j} N_{t+j} - B_{t+j+1} - C_{t+j} \right]
\]

(5)
The individual optimizes with respect to this quarter \((t)\) and the next quarter \((t+1)\), every time a decision is made, which gives the following first order conditions

\[
\frac{\partial L}{\partial C_t} : \frac{C_t^{\gamma}}{C_{t+1}^{\gamma(1-\theta)}} = \lambda_t
\]

(6)

\[
\frac{\partial L}{\partial C_t} : \beta \frac{C_{t+1}^{\gamma}}{C_t^{\gamma(1-\theta)}} = \lambda_{t+1}
\]

(7)

\[
\frac{\partial L}{\partial N_t} : N_t^{\gamma} = \lambda_{t} W_t
\]

(8)

\[
\frac{\partial L}{\partial B_{t+1}} : \lambda_t = \lambda_{t+1} R_{t+1}
\]

(9)

Assuming symmetry among the individuals gives \(C_{t+j} = \bar{C}_{t+j}\)

Combining equation (6), (7) and (9) yields the Euler equation

\[
C_t^{\gamma(1-\theta)-\theta} = \beta \frac{C_{t+1}^{\gamma(1-\theta)}}{C_t^{\theta}} \frac{1 + i_t}{1 + \pi_{t+1}}
\]

(10)

By combining equation (6) and (8) we get the individuals optimal labour supply decision.

\[
N_t^{\gamma} C_{t-1}^{\gamma(1-\theta)} C_t^{\theta} = W_t
\]

(11)

Using the notations described in chapter 1 we can rewrite equation (11) as

\[
\varphi (\log N_t - \log N) - \gamma (\theta - 1) (\log C_{t-1} - \log C) + \theta (\log C_t - \log C) = \log W_t - \log W
\]

which then becomes
\[ \phi n_t - \gamma (\theta - 1)c_{t-1} + \theta c_t = w_t \]  

(12)

The equivalent expression for the Euler expression (10) is the dynamic IS equation (13)

\[ \left( \gamma (\theta - 1) + \theta \right) c_t = \gamma (\theta - 1)c_{t-1} + \theta c_{t+1} - i_t + \pi_{t+1} \]  

(13)

If \( \gamma = 0 \) there is no habit and we end up with the same solution as in Gali (2008)

2.3 Firms

A large part of the derivations in this section are based on Gali (2008).

The firm produces output \( Y_t \) whose level is decided by the technology variable \( A_t \) and the amount of labour \( N_t \) where capital is assumed to be fixed at all times and therefore omitted from the production function (14), which is assumed to be a Cobb-Douglas

\[ Y_t = A_t N_t^{1-\alpha} \]  

(14)

The accompanying marginal product of labour is

\[ MPN_t = (1 - \alpha) \frac{Y_t}{N_t} \]  

(15)

Rewriting (14) and (15) in logarithms gives

\[ n_t = \frac{y_t - a_t}{1 - \alpha} \]  

(16)

and

\[ mpn_t = y_t - n_t \]  

(17)
Then we define the real marginal cost to the firm as the difference between the real wage, $w_t$, and the marginal product of labour, $mpn_t$, which is also the real marginal cost to the economy since we have symmetry among firms.

$$mc_t = w_t - mpn_t \quad (18)$$

Substituting equation (12) and (17) into (18) gives.

$$mc_t = (\phi n_t + \gamma (1-\theta)c_{t-1} + \theta c_t) - (y_t - n_t) \quad (19)$$

Then substituting equation (16) to get rid of $n_t$.

$$mc_t = \frac{\phi + \alpha}{1-\alpha} y_t + \theta c_t - \gamma (\theta -1)c_{t-1} - \frac{\phi + 1}{1-\alpha} a_t \quad (20)$$

Where the term including $\gamma$ is negative for the reason that habit formation causes people to work more since they want to keep up a previous high level of consumption, which in turn reduces the marginal cost to the firm. The reason the term involving $a_t$ is negative is that higher technology makes production more efficient and thereby lowers costs.

Then use the New Keynesian Phillips curve derived in Gali (2008).

$$\pi_t = \beta \pi_{t+1} + \lambda mc_t \quad (21)$$

With $\lambda = \frac{(1-\omega)(1-\beta\omega)}{\omega} \frac{1-\alpha}{1-\alpha + \alpha\epsilon}$

$(1-\omega)$ comes from the formalism in Calvo (1983) and is the proportion of firms that are able to change their price to an optimal level each period to compensate for changes in marginal cost. $\omega$ is thus a measure of price stickiness. Firms are also doing this in a
forward looking way through $\pi_{t+1}$, which means that if they expect prices to be high in the future they will raise the prices already today.

Combining (20) and (21) we get a New Keynesian Phillips curve with habit formation.

$$\pi_t = \beta \pi_{t+1} + \kappa_y y_t - \kappa_a a_t - \lambda \gamma (\theta - 1)c_{t-1} + \lambda \theta c_t$$ (22)

Where $\kappa_y = \lambda \frac{\phi + \alpha}{1 - \alpha}$ and $\kappa_a = \lambda \frac{\phi + 1}{1 - \alpha}$

Now let’s comment on (22) going from left to right. First we have forward looking inflation, an important issue in New Keynesian Economics. Next the $y_t$ term simply stems from the fact that more production increases the cost. The $a_t$ term is negative since higher technology makes it possible to produce more efficiently and therefore lowers the cost. The consumption term involving habit formation is negative because the individual needs to work more in order to keep up a previous high consumption level, which benefits the firm in terms of lower costs. Current consumption is positive for the reason that the individual substitutes away work for consumption leading to decreased labour supply and thereby a higher equilibrium wage.

Furthermore, the economy is closed and there is no capital accumulation, implying that all production must be consumed each period meaning that $Y_{t+j} = C_{t+j}, \ \forall j$. The dynamics for $c$ and $y$ are therefore identical.

2.4 The Central Bank

During the 1990’s several countries such as Canada, New Zealand, UK and Sweden to mention a few, shifted to a new monetary policy regime. The central banks were made more independent from the government and got a precise goal, namely to keep inflation on a low and stable level. In practice however the central banks use a flexible inflation target, where the word flexible stands for the property that in addition to target inflation the central bank also aims to keep the output gap as small as possible. The inflation target is usually around 2-3 %, Sweden has 2 % plus/minus 1 %, just to mention an example; Svensson (2000)
To make the model realistic without complicating things, we assume that the central bank sets the interest rate according to a simple Taylor (1993) rule.

\[ i_t = \phi_x \pi_t + \phi_y y_t \tag{23} \]

Where \( \phi_x \) and \( \phi_y \) decides how the central bank reacts to inflation and the output gap respectively which makes \( \phi_x \) a measure of inflation aversion. Without any loss in generality the inflation target in (23) is assumed to be zero.

There is no intercept either since everything is expressed as logarithmic deviations from steady state.

*Summary of the closed model*

One of the key issues with a DSGE model is to analyze the dynamics of the variables when a number of shocks are added to the model. Shocks following AR(1) processes will be added to, the Phillips curve, the Taylor rule, the IS-equation and the production function. The production function is already containing a shock term through the technology parameter \( a \).

Substitution between labour supply and consumption

\[ \varphi n_t - \gamma (\theta - 1) c_{t-1} + \theta c_t = w_t \]

Dynamic IS equation

\[ (\gamma (\theta - 1) + \theta) c_t + z_t = \gamma (\theta - 1) c_{t-1} + \theta c_{t+1} - i_t + \pi_{t+1} \]

Production function

\[ (1 - \alpha) n_t = y_t - a_t \]

New Keynesian Phillips curve

\[ \pi_t = \beta \pi_{t+1} + \kappa \gamma y_t + \lambda \theta c_t - \lambda \gamma (\theta - 1) c_{t-1} - \kappa a_t + u_t \]
Taylor rule
\[ i_t = \phi_\pi \pi_t + \phi_y y_t + v_t \]

Technology shock
\[ a_t = \rho_a a_{t-1} + \nu_a^y \]

Taylor rule shock
\[ v_t = \rho_v v_{t-1} + \nu_v^i \]

Phillips curve shock
\[ u_t = \rho_u u_{t-1} + \nu_u^\pi \]

Demand shock
\[ z_t = \rho_z z_{t-1} + \nu_z^c \]
3 OPEN MODEL

This model can be referred to as a two country model consisting of one small open economy and the rest of the world. The size of the small open economy is assumed to be negligibly small compared to the rest of the world. The word “foreign” is used when talking about the rest of the world, while the word “domestic” or “home” is referred to the small open economy.

3.1 Some useful identities

A large part of the derivations in this section are based on Gali (2008).

The economy can now trade with the rest of the world and foreign currency is needed to do so. The nominal exchange rate is denoted by $E$ and is defined as the amount of domestic currency needed to buy one unit of foreign currency, implying that a rise in $E$ corresponds to a depreciation in the domestic currency and vice versa. Terms of trade is the foreign price level expressed in domestic currency divided by the price of domestic goods, namely

$$S_t = \frac{E_t \cdot \hat{P}_t}{P_{H,t}}$$

The * subscript is used to denote foreign variables, making $P_t^*$ the foreign consumer price index (CPI) and $P_{H,t}$ is the price of domestically produced goods. Terms of trade can be referred to as the relative price between foreign and domestic goods. In log linear form

$$s_t = e_t + P_t^* - P_{H,t}$$

(25)
Consumption, exports, imports and the CPI index

Since the domestic consumer can by foreign goods, consumption consists of both domestically produced goods and foreign goods according to a composite consumption index defined by (26)

$$C_i = \left(1 - \delta \right)^{\frac{1}{\eta}} C_{H,i}^{\frac{\eta - 1}{\eta}} + \delta^{\eta} C_{F,i}^{\frac{\eta - 1}{\eta}}$$  \hspace{1cm} (26)$$

Domestic (home) and foreign consumption uses the subscripts $H$ and $F$ respectively and $\delta$ is the share spent on foreign goods and thereby a measure of openness. $\eta$ is the elasticity of substitution between home and foreign goods.

The log linearised version of (26) is

$$c_i = (1 - \delta) c_{H,i} + \delta c_{F,i}$$  \hspace{1cm} (27)$$

Deriving an optimal expression for $C_{H,i}$ and $C_{F,i}$ respectively we get

$$C_{H,i} = (1 - \delta) \left( \frac{P_i}{P_{H,i}} \right)^\eta C_i$$  \hspace{1cm} (28)$$

$$C_{F,i} = \delta \left( \frac{P_i}{E_i P_i} \right)^\eta C_i$$  \hspace{1cm} (29)$$

(29) is the import function. Inserting equations (28) and (29) into (26) we get the domestic consumer price index (CPI).

$$P_i = \left[ (1 - \delta) P_{H,i}^{1-\eta} + \delta (E_i P_i)^{1-\eta} \right]^{\frac{1}{1-\eta}}$$  \hspace{1cm} (30)$$

---

* The derivations are shown in Appendix A
The log linearized versions of (28) and (29) are

\[ c_{H,t} = \eta \delta s_t + c_t \tag{31} \]

and

\[ im_t = c_t - \eta (1 - \delta) s_t \tag{32} \]

The export function depends on market shares together with a sticky component as in Gottfries (2002).

\[ \frac{EX_t}{Y_t} = \left[ \delta^* S_t^\eta \right]^{(1-\mu)} \left( \frac{EX_{t-1}}{Y_{t-1}} \right)^{(1-\mu)} \tag{33} \]

Where \( \mu \in [0,1] \) decides how the individual value between the lagged term and the rest of the right hand side as in (33).

In log linearised form (33) takes the form

\[ ex_t = y_t^* + \mu \eta^* s_t + (1 - \mu) (ex_{t-1} - y_{t-1}^*) \tag{34} \]

The net export

Net export is exports minus imports, (35).

\[ NX_t = EX_t - IM_t \tag{35} \]

Divide by steady state production, \( Y \), and use \( X_t \approx X (1 + x_t) \) gives

\[ nx_t = \frac{EX_t (1 + ex_t) - IM_t (1 + im_t)}{Y} \], where \( nx_t = \frac{NX_t}{Y} \). In steady state we have \( EX = IM \)

and \[ \frac{IM}{Y} = \delta \] which gives the log linear expression (36)

---

\(^5\) The full derivation is shown in Appendix C
\[ nx_t = \delta (ex_t - im_t) \quad (36) \]

**The trade clearing condition**

All domestic production is consumed each period which means that what’s left over after domestic consumption must be consumed from abroad. This leads to the domestic goods market clearing condition in (37).

\[ Y_t = C_{H,t} + EX_t \quad (37) \]

Using the expressions for domestic demand (28) and the export function (33) respectively in (38) gives

\[
Y_t = (1 - \delta) \left( \frac{P_t}{P_{H,t}} \right)^\eta C_t + \delta^* \mu \left( \frac{E_t P_t^*}{P_{H,t}} \right)^{\mu^*} \left( \frac{EX_{t-1}}{Y_{t-1}^*} \right)^{(1-\mu)} Y_t^* \quad (38)
\]

Using the following manipulation \( X_t = X e^{\delta t} \) allows us to rewrite (38) as

\[
e^{\gamma_i} = (1 - \delta) \left( \frac{P_t}{P_{H,t}} \right)^\eta e^{\eta (\mu - \mu_{gt})} C e^{\delta \gamma_i} + \delta^* \mu S^{\mu^*} e^{\mu^{*} \gamma_i} \left( \frac{EX}{Y^*} \right)^{(1-\mu)} e^{(1-\mu)(\varepsilon, \varepsilon_{t-1})} Y_t^* e^{\gamma_i}; \quad (39)
\]

In steady state, the economy can’t be in debt nor have a surplus which implies that \( Y = C^6 \) and exports must equal imports as in equation (40)

\[
\delta^* \mu S^{\mu^*} \left( \frac{EX}{Y^*} \right)^{(1-\mu)} Y^* = \delta \left( \frac{P}{EP^*} \right)^\eta C \quad (40)
\]

In steady state the cost for a domestic citizen of buying domestically, should be equal to the cost of buying abroad, which gives the equality in (41).

\[ P = EP^* = P_{H} \quad (41) \]

---

6 This equilibrium condition is also shown mathematically in the appendix.
\( P = P_t \) becomes clear when using the CPI equation (30) and setting \( P_t = EP^* \). In all, equation (40) pins down to

\[
\left( \frac{EX}{Y^*} \right)^{(1-\mu)} = \frac{\delta}{\delta^\mu} \]

(42)

Using (40) and (42) in (39) gives

\[
e^{y_t} = (1 - \delta) e^{\delta s_t} e^{\delta y_t^*} + \delta e^{\mu y_t^*} e^{(1-\mu) e_{t-1} - y_{t-1}^*} e^{y_t^*}
\]

Making a first order Taylor approximation around zero (since all the variables are zero in steady state) we get

\[
y_t = \left( (1 - \delta) \eta + \mu \eta^* \right) \delta s_t + (1 - \delta) c_t + \delta y_t^* + \delta (1 - \mu) \left( e_{t-1} - y_{t-1}^* \right)
\]

(43)

The higher the \( \delta \) the more is domestic production consumed by foreign consumers and less is consumed by domestic consumers. Terms of trade, \( s \), has two effects that’s working in opposite directions. The first effect depends on how much weight the domestic consumer puts on domestic goods in the consumption bundle, from equation (28), and is therefore decreasing with \( \delta \). The second effect comes from the fact that higher \( s \) means cheaper for foreigners to buy from the small open economy, this effect is increasing with the openness (\( \delta \)).

If there’s no trade, \( \delta = 0 \) leading to the closed economy solution \( y_t = c_t \).

**Consumer inflation and the real exchange rate**

Inflation consists of price changes from both domestic- and foreign goods, whose weights depend on the CPI index which in turn depends in the Consumer index (26). Making a Taylor approximation of the CPI index gives

\[
p_t = (1 - \delta) p_{H,t} + \delta (e_t + p_t^*) = p_{H,t} + \delta (e_t + p_t^* - p_{H,t}) \]

where the terms of trade definition can be used to get
\[ p_t = p_{H,t} + \delta s_t \]  \hspace{1cm} (44)

Taking the first time difference of (44) we get the CPI inflation (consumer inflation)

\[ \pi_t = \pi_{H,t} + \delta \Delta s_t \]  \hspace{1cm} (45)

(45) is the inflation that the consumer faces when which is why it’s called consumer inflation, \( \pi_{H,t} \) is of interest to firms which will be discussed below.

From (46) it can be seen that inflation in the open economy is different than the closed economy due to price inequalities between foreign and domestic goods, which happens when \( s \neq 0 \), this effect is increasing with the openness (\( \delta \)).

When adding domestic and foreign price levels to the nominal exchange rate \( E \), we get the real exchange rate (46)

\[ Q_t = \frac{E_t P^*_t}{P_t} \]  \hspace{1cm} (46)

Log linearizing \( Q_t \) and rewrite as \( q_t = e_t + p^*_t - p_t = s_t - \left( p_t - p_{H,t} \right) = s_t - \delta s_t \) leads to equation (47)

\[ q_t = (1 - \delta) s_t \]  \hspace{1cm} (47)

3.2 Households

As in Post (2007) it’s assumed that the individual buys domestic bonds only, while the foreign consumer can buy both domestic and foreign bonds. This is an assumption that makes the calculations easier but it doesn’t cause any loss in generality. The first order conditions for the domestic individual is therefore identical to the ones derived in the closed economy. The difference however is that \( C_t \) now consists of both domestic and foreign goods which implies that, different from the closed economy, we generally have \( Y_{t+j} \neq C_{t+j} \).
This gives the same Dynamic IS equation as in the closed economy.

\[-(\gamma(\theta-1)+\theta)c_t = -\gamma(\theta-1)c_{t-1} - \theta c_{t+1} + i_t - \pi_{t+1}\]  \hspace{1cm} (13)

And the same labour supply function

\[\varphi n_t - \gamma(\theta-1)c_{t-1} + \theta c_t = w_t\]  \hspace{1cm} (12)

Although it’s important to keep in mind that \(c_t\) also depends on the consumer index (27) and \(\pi_t\) depends on openness and the foreign price level as in (45).

**Dynamic asset equation**

Despite the availability of both domestic- and foreign bonds it is assumed that this choice is decided by the foreigner consumer, a surplus and a deficit can still arise for the domestic individual; Post (2007). In case of a surplus the rest of the world is in debt to the small economy and vice versa. This is shown with a dynamic asset equation according to (48)

\[B_t = \frac{1+i_{t-2}}{1+\pi_{t-1}} B_{t-1} + P_{H,t-1}Y_{t-1} - P_{t-1}C_{t-1}\]  \hspace{1cm} (48)

Divide by steady state output times the domestic price level; \(YP_{H,t-1}\).

\[b_t = \frac{1+i_{t-2}}{1+\pi_{t-1}} b_{t-1} + \frac{Y_{t-1}}{Y} - \frac{P_{t-1}}{YP_{H,t-1}} C_{t-1}\]

Where \(b_t = \frac{B_t}{YP_{H,t}}\) and thereby deviates from the general notation, since \(b_t\) is not expressed in logarithms.
Then use the approximation \( \frac{1+i_{t-2}}{1+\pi_{t-1}} \approx 1+i_{t-2}-\pi_{t-1} \) together with \( X_t = X e^{x_t} \approx X \left(1+x_t\right) \)

to get \( b_t = (1+i_{t-2}-\pi_{t-1})b_{t-1} + \frac{Y(1+y_{t-1})}{P} - \frac{P C}{P^H} Y \left(1 + p_{t-1} - p_{H,t-1} + c_{t-1}\right) \) which together

with the steady state conditions \( P = P_H \) and \( C = Y \) gives us equation (49).

\[
b_t = (1+i_{t-2}-\pi_{t-1})b_{t-1} + y_{t-1} - c_{t-1} - \delta s_{t-1} \tag{49}
\]

**Uncovered interest parity condition (UIP)**

The uncovered interest parity (UIP) condition, comes from the fact that a border free financial market makes the yield between interest bearing accounts highly competitive since it’s possible to freely chose between a domestic- and a foreign bank account. The UIP condition is derived from the foreign individual’s first order condition as in Post (2007)\(^7\). The result is as follows.

\[
\left(1+i_t\right) = \left(1+i_t^\ast\right) \Phi_t \frac{E_{t+1}}{E_t} \tag{50}
\]

\( \Phi_t \) is a risk premium which is needed for the model to have a steady state.

An intuitive reason for such a premium is that if the foreign and domestic interest rates differ, despite real exchange rate parity, they most differ because saving domestically has a relatively higher/lower risk. According to the findings of Lane and Milesi-Ferretti (2001) the risk premium is correlated with domestic debt, implying that a domestic surplus indicates a lower risk, making the foreign investors accepting a lower yield compared to the foreign yield and vice versa. Besides, a pure UIP condition without any risk premium has shown to be rejected empirically, Adolphson et al (2007). The risk premium has the explicit expression (51)

\[
\Phi_t = e^{-\psi h} \tag{51}
\]

\(^7\) See Appendix A in Post (2007) for a derivation
Combining (50) and (51) and log linearize gives

\[ i_t = i_t^* - \psi b_t + \Delta e_{t+1} \]  

(52)

Then using the terms of trade definition (24) in order to get rid of the nominal exchange rate we get

\[ i_t = i_t^* - \psi b_t + \Delta s_{t+1} - \pi^*_{t+1} + \pi_{H,t+1} \]  

(53)

**3.3 Firms**

The assumptions for the firms are the same as in the closed economy with the addition that domestic firms sell everything in domestic prices while the workers care about inflation arising both from price changes in domestic as well as foreign goods.

The real marginal cost equation to the firm is given by (54).

\[ mc_i = w_t^\pi - p_{H,t} - mpn_t \]  

(54)

Where \( w_t^\pi \) is the nominal wage.

In order to include the labour’s decision we must rewrite (54) in a way that the consumer price level, \( p_t \), is included.

\[ mc_i = w_t^\pi - p_{H,t} - mpn_t \]  

\[ = (w_t^\pi - p_t) + (p_t - p_{H,t}) - mpn_t \]  

\[ mc_i = (w_t^\pi - p_t) + \delta s_t - mpn_t \]  

(55)

No we can use the \( mpn_t \) function (17) and the labour supply function (12) from the closed economy and combine these with the real marginal cost equation (55) to get

\[ mc_i = (\phi n_t - \gamma (\theta - 1)c_{t+1} + \theta c_t) + \delta s_t - (y_t - n_t) \]  

(56)
Use the production function to get rid of \( n_t \) and simplify the same way as in the closed economy, we end up with

\[
mc_t = \frac{\varphi + \alpha}{1 - \alpha} y_t - \frac{\varphi + 1}{1 - \alpha} a_t - \gamma (\theta - 1)c_{t-1} + \theta c_t + \delta s_t
\]

(57)

Further note, since inflation arises from stickiness in price settings among firms, it’s the inflation from their point of view that must be considered. The Phillips curve is therefore expressed in home inflation only

\[
\pi_{H,t} = \beta \pi_{H,t+1} + \lambda mc_t
\]

(58)

Where the parameter containing price stickiness, \( \lambda \), is defined the same way as in the closed economy.

Combining (57) and (58) gives the *New Keynesian Philips curve with habit formation* for the open economy.

\[
\pi_{H,t} = \beta \pi_{H,t+1} + \kappa_y y_t - \kappa_a a_t - \lambda \gamma (\theta - 1)c_{t-1} + \lambda \theta c_t + \lambda \delta s_t
\]

(59)

Where \( \kappa_y = \lambda \frac{\varphi + \alpha}{1 - \alpha} \) and \( \kappa_a = \lambda \frac{\varphi + 1}{1 - \alpha} \)

The discussion about the five first the terms, on the right hand side, of (59) is the same as in the closed Phillips curve (22). The difference is the last term \( \lambda \delta s_t \), which is positive for the reason that higher foreign prices creates a domestic inflation pressure, which is increasing in \( \delta \). If \( \delta = 0 \) we end up with the closed Phillip curve (22).

### 3.4 The Central Bank

The central bank has a flexible inflation target according to the same rule as in the closed economy.

\[
i_t = \phi_\pi \pi_t + \phi_y y_t
\]

(23)
Summary of the open model

The same shocks that are added in the closed economy are also used here. The addition is that foreign output $y_t^*$ and foreign inflation $\pi_t^*$ are also treated as AR(1) processes.

CPI inflation
\[ \pi_t = \pi_{t-1} + \Delta s_t \]

Real exchange rate
\[ q_t = (1 - \delta) s_t \]

Substitution between labor supply and consumption
\[ \varphi n_t - \gamma (\theta - 1) c_{t-1} + \theta c_t = w_t \]

CES consumption
\[ c_t = (1 - \delta) c_{t-1} + \delta c_{t-1} \]

Domestic consumption
\[ c_{t-1} = \eta \delta s_t + c_t \]

Domestic output clearing condition
\[ y_t = \left( (1 - \delta) \eta + \mu \eta^* \right) \Delta s_t + (1 - \delta) c_t + \delta y_t^* + \delta (1 - \mu) (ex_{t-1} - y_t^*) \]

The export function
\[ ex_t = y_t^* + \mu \eta^* s_t + (1 - \mu) (ex_{t-1} - y_{t-1}^*) \]

Open dynamic IS equation
\[ \left( \gamma (\theta - 1) + \theta \right) c_t + z_t = \gamma (\theta - 1) c_{t-1} + \theta c_{t-1} - i_t + \pi_{t-1} \]

The dynamic asset equation
\[ b_t = (1 + i_{t-1} - \pi_{t-1}) b_{t-1} + y_{t-1} - c_{t-1} - \delta s_{t-1} \]
UIP condition
\[ i_t = i_t^* - \psi b_t + \Delta s_{t+1} - \pi_{t+1}^* + \pi_{H,t+1} \]

Production function
\[ (1 - \alpha) n_t = y_t - a_t \]

Open NKPC
\[ \pi_{H,t} = \beta \pi_{H,t+1} + \kappa_y y_t - \kappa_a a_t - \lambda \gamma (\theta - 1) e_{t-1} + \lambda \delta c_t + \lambda \delta s_t \]

Taylor rule
\[ i_t = \phi_n \pi_t + \phi_y y_t + \nu_t \]

Technology shock
\[ a_t = \rho_a a_{t-1} + \epsilon^a_t \]

Taylor rule shock
\[ \nu_t = \rho_v \nu_{t-1} + \epsilon^v_t \]

Demand shock
\[ z_t = \rho_z z_{t-1} + \epsilon^z_t \]

Phillip curve shock
\[ u_t = \rho_u u_{t-1} + \epsilon^u_t \]

Foreign output shock
\[ y^*_t = \rho_y y^*_{t-1} + \epsilon^y_{t,j} \]

Foreign inflation shock
\[ \pi^*_t = \rho_{\pi} \pi^*_{t-1} + \epsilon^*_{\pi,t} \]
4 CALIBRATION AND RESULTS

When finding values to the small open economy Sweden will be used as a reference economy when possible

4.1 Calibration

In order to make dynamic simulations we must find values for the different parameters that are used in the models.

There are basically two ways to find parameter values in DSGE models. One way is to use econometric methods where generalized Method of Moments (GMM), maximum likelihood and Bayesian methods are the most common ones; Dejong and Dave (2007).

The other way is to use calibration, where one looks through the literature and tries to find as reasonable values as possible to the model in question. Calibration is the chosen method in this thesis.

Households

The parameters to the household are $\phi$, $\theta$, $\beta$ and $\gamma$.

Meghir et al (1998) uses tax reforms as a way to estimate the labour supply of the UK. They get a value of about 1/5 implying $\phi = 5$.

The measures of $\gamma$ was in Fuhrer (2000) about 0.8 and so was Abel (1999). 0.8 is thus a reasonable value of $\gamma$. $\theta$ on the other hand has usually been proven to get high values when estimating utility functions with habit formation. Fuhrer and Abel got about 6 and 11 respectively. As a contrast, Hall (1988) and Atanasio and Weber (1993) got values around 3 when they used aggregated consumption data. Therefore we stick with Fuhrers
value of 6.11 for $\theta$ which gives an elasticity of substitution of 0.16 since $\sigma = \frac{1}{\theta}$, implying a low willingness to smooth consumption over time.

The time periods in this thesis are expressed as quarters, $\beta = e^{-0.01}$ will then be a reasonable value meaning that the individual requires an interest rate of 1 % each quarter to make up for the discount of waiting. This value is commonly used in the litterature; See Gali (2008) and Post (2007).

**Firms**

The firm parameters are $\alpha, \varepsilon$ and $\omega$.

Caselli and Feyrer (2007) used data recently compiled by the World Bank were they estimated the marginal product of labour in a number of countries. An estimation of the Swedish $\alpha$ was shown to be 0.23.

$\varepsilon = 6$ in accordance with Gali (2008).

In a survey answered by Swedish firms, Apel et al (2001) finds that companies change their prices once a year on average. When Korenok Radchenko and Swanson (2006) estimates the new Keynesian Phillips curve they find a price duration of almost 20 quarters for Sweden

Since

$$Duration = \frac{1}{1 - \omega}$$

these two findings imply a value of 0.9 and 0.95 respectively for $\omega$. Adolphson et al (2007) uses a value of 0.88 and 0.9 respectively in their DSGE models. In all $\omega = 0.9$ seems like a reasonable choice.
Open economy parameters

The open economy parameters are $\eta^*$, $\mu$, $\delta$ and $\psi$.

Gottfries (2002) estimates the export function and finds that $\eta^* = 3$ and $\mu = 0.1$. Since $\eta^*$ is the elasticity of substitution to the rest of the world, it should have a higher value than $\eta$. The reason is that it’s likely for the rest of the world to have more substitutes to choose from than the small open economy. For instance, in Sweden there is no home grown bananas, not even during summer time, which leaves the Swedes with the choice of importing bananas or not buying bananas at all. Collard and Dellas (2002) gets values of 1.3 and 2.3 for France and Germany respectively implying that $\eta = 2$ should work.

Regarding the openness, we’ll use the same value as in Gali (2008) i.e $\delta = 0.4$.

Beigno (2001) uses $\psi = 0.01$ and $\psi = 0.001$ whereas Post (2007) uses 0.01. We’ll use $\psi = 0.01$.

The Central Bank

$\phi_x$ and $\phi_y$ will use the standard values in Taylor (1993) which is 1.5 and 0.5 respectively. As will the foreign values $\phi_x^*$, $\phi_y^*$; Svensson (2000).

The shock parameters

$\rho^i$ and $\rho^c$ use the values from Gali (2008), which are 0.5 and 0.9 respectively.

The foreign output- and inflation shocks, as well as the domestic inflation- and demand shocks will use the values in Svensson (2000), namely $\rho^{i^*} = \rho^{c^*} = \rho^c = \rho^i = 0.8$.

The variances of the shocks will all have the same value following Svensson (2000), according to.

\[ \sigma_i^2 = \sigma_d^2 = \sigma_x^2 = \sigma_{\pi^*}^2 = \sigma_x^2 = \sigma_c^2 = 0.5 \]

In Table 1 follows a summary of the parameter values.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>5</td>
</tr>
<tr>
<td>( \theta )</td>
<td>6.11</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.16</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.8</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( e^{-0.01} )</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.23</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>6</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.9</td>
</tr>
<tr>
<td><strong>Open Economy parameters</strong></td>
<td></td>
</tr>
<tr>
<td>( \eta^* )</td>
<td>3</td>
</tr>
<tr>
<td>( \eta )</td>
<td>2</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.4</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Central Bank</strong></td>
<td></td>
</tr>
<tr>
<td>( \phi_{\pi} )</td>
<td>1.5</td>
</tr>
<tr>
<td>( \phi_{y} )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \phi_{\pi}^* )</td>
<td>1.5</td>
</tr>
<tr>
<td>( \phi_{y}^* )</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>AR(1)</strong></td>
<td></td>
</tr>
<tr>
<td>( \rho_{\pi} )</td>
<td>0.8</td>
</tr>
<tr>
<td>( \rho_{\pi}^* )</td>
<td>0.8</td>
</tr>
<tr>
<td>( \rho_{y}^* )</td>
<td>0.8</td>
</tr>
<tr>
<td>( \rho^C )</td>
<td>0.8</td>
</tr>
</tbody>
</table>
4.2 Results

This part will show the results in the order of the interest rate shock, the inflation shock, the technology shock and the consumption shock. The variables that are not graphically illustrated here can be found in Appendix E.

The nominal exchange rate $e_t$ is not explicitly included in the model but implicitly included in both the terms of trade $s_t$ and the real exchange rate $q_t$. $s_t$ and $q_t$ are therefore the referred variables when talking about appreciation and depreciation throughout the discussion.

Conducting monetary policy by changing the interest rate has a direct affect on consumption in both the open and the closed economy while the open economy is affected by an additional channel through the exchange rate.

*Interest rate shock*

A shock in the interest rate is what happens when the central bank conducts contractionary monetary policy and decreases the supply of money; See Figure 1.
FIGURE 1 Interest rate shock in the closed and open economy
To a given interest rate increase, output and employment declines more and recovers quicker since in addition to the decreased consumption, due to higher opportunity cost, the appreciation causes the net export to go down as well. The appreciation has a big and negative impact on inflation the first quarter but it also causes depreciation expectations, $s_t$ and $s_{t+1}$ respectively in equation (53), which increases inflation already the second quarter through expensive foreign goods; See equation (45). Furthermore, the exchange rate channel amplifies the habit affect the first few quarters in both consumption and the real wage.

\[
i_t - \pi_{H,t+1} = s_{t+1} - s_t + i_t^* - \psi b_t - \pi_{t+1}^*
\]

\[
\pi_t = \pi_{H,t} + \delta (s_t - s_{t-1})
\]

*Domestic inflation shock*

An inflation shock can for instance be caused by sudden expectations about price increases in the future, making firms increase prices already this period. The results are shown in Figure 2.
FIGURE 2 Inflation shock in the closed and open economy
Output and employment experience a larger decline since the interest rate, consumption, the real exchange rate and the net export all get negative values. The appreciation also dampens inflation the first quarter.

Consumption has a smaller decline with more habit the first few quarters and a smoother over all pattern stemming from the ability to borrow abroad through a negative $b_j$ and an appreciation in the currency; See the UIP equation (53). More habit in consumption gives the real wage a stronger habit as well through the labour supply equation (12).

So far output and inflation have been shown to be more reactive in the open economy. To investigate this finding further the values in the Taylor rule will be assigned, $\phi_j = 5$ and $\phi_{\pi} = 5$ respectively one at the time which will be evaluated with the intertemporal loss function (61).

$$\sum_{t=1}^{100} \beta^{t-1} \left( \pi_t^2 + 0.5 y_t^2 \right)$$

For simplicity $\beta$ is the discount rate and half as much weight is put on output deviations; Svensson (2000). In addition, the loss function has been summarized over 100 periods to make sure that the dynamics of the variables have reached zero. The results are shown in Table 2 where the last row shows the ratio between the open- and the closed economy losses. Higher ratio means relatively larger loss in the open economy and vice versa.

A high weight on $\phi_{\pi}$ gives a smoother inflation path while the interest rate and the real appreciation follow a much higher path in the inflation averse regime, resulting in a huge decline in output and a welfare loss of 394; see Figure 4 and Table 3.
The technology shock

A technology shock is an invention that makes production more efficient, allowing for a higher level of production with the same amount of labour; See Figure 4.
FIGURE 4 Technology shock in the closed and open economy
Output has a larger upturn which declines faster, where the surplus of goods is used to invest abroad to smooth consumption.

The welfare losses are in line with the previous findings. The open economy has a relatively larger loss when $\phi_x = 5$ and has a relatively smaller loss when $\phi_x = 5$ since the accompanying ratios are 1.62 and 1.00 respectively; see Table 4.

<table>
<thead>
<tr>
<th>$\phi_x = 1.5, \phi_y = 0.5$</th>
<th>$\phi_x = 1.5, \phi_y = 5$</th>
<th>$\phi_x = 5, \phi_y = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open</td>
<td>0.66</td>
<td>0.52</td>
</tr>
<tr>
<td>Closed</td>
<td>0.61</td>
<td>0.52</td>
</tr>
<tr>
<td>Open/Closed</td>
<td>1.08</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*All values except the ratios are multiplied by 100*

**Consumption shock**

This is a sudden willingness to increase consumption among domestic consumers. The results are shown in Figure 5.

Since part of the demand is directed towards foreign goods, domestic production and employment only goes up slightly while consumption goes up a lot. The big increase in consumption is boosted since the households can borrow abroad ($b_t \downarrow$).

The demand shock causes a higher equilibrium price on domestic goods through an increase in domestic inflation ($\uparrow \pi_H$) and through a real appreciation ($\downarrow s_t$). Since the latter affect is dominant, CPI inflation goes down.

The seemingly large affect in the real exchange rate stems from the “producer currency pricing” (PCP) property meaning that the domestic firms set their prices in domestic currency which gives an immediate pass-through to the exchange rate. Engel (1999) have found that price setting in the currency of the customer, so called “local currency pricing” (LCP), is more empirically consistent. LCP generally dampens the changes in the exchange rate, suggesting that the big appreciation in Figure 5 is probably a bit smaller in reality.
FIGURE 5 Demand shock in the closed and open economy
Table 5 suggests the same pattern as previously which is lower welfare losses when $\phi_y = 5$ and higher losses when $\phi_x = 5$, only this time the open economy generates lower losses under the standard- and the inflation averse regime. Another deviating finding compared to previously is that $\phi_y = 5$ is relatively efficient in the closed economy.

TABLE 5 Intertemporal loss function from the demand shock

<table>
<thead>
<tr>
<th></th>
<th>$\phi_x = 1.5, \phi_y = 0.5$</th>
<th>$\phi_x = 1.5, \phi_y = 5$</th>
<th>$\phi_x = 5, \phi_y = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open</td>
<td>0.80</td>
<td>0.28</td>
<td>0.82</td>
</tr>
<tr>
<td>Close</td>
<td>4.46</td>
<td>0.17</td>
<td>3.64</td>
</tr>
<tr>
<td>Open/Closed</td>
<td>0.18</td>
<td>1.61</td>
<td>0.23</td>
</tr>
</tbody>
</table>

All values except the ratios are multiplied by 100.
This thesis investigates the role of monetary policy in a closed- and small open economy using DSGE modelling. The closed- and the open models are subjected to shocks in the interest rate, inflation, technology and consumption.

Subject to the interest rate- and the consumption shock inflation gets volatile which lasts for about two and three years respectively, caused by the exchange rate channel; Ball (1998) gets similar results. Subject to the other shocks, inflation has a lower response of about 30% the first quarter.

The consumption shock has a smaller affect on production and employment but a larger affect on consumption due to abroad loans and an appreciation of the currency.

In most of the shocks, output has a larger and quicker change stemming from the exchange rate channel which also amplifies the habit of consumption and the real wage rate in the inflation and interest rate shocks.

Furthermore, the parameters in the Taylor rule were changed to evaluate the welfare losses under an inflation averse- and an output gap averse regime respectively. A stronger reaction to output than in the standard Taylor (1993) rule was shown to give lower losses in both economies because output has such a big influence on the economy. This have been concluded in the literature before; See Ball (1999) and Svensson and Rudebush (1999).


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APPENDIX A Expenditure shares on home and foreign goods

Deriving $C_{H,t}$

The clearing condition in (A.1) holds.

$$P_i C_t = P_i \left[ \left( 1 - \delta \right) \frac{1}{\bar{p}} C_{H,t}^{\eta} + \delta^\eta C_{F,t}^{\eta} \right] = P_{H,t} C_{H,t} + E_{i,P,F} C_{F,t} \quad \text{(A.1)}$$

Using this in the Lagrangean

$$L = \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1 - \theta} \left( \frac{C_{t+j}^j}{C_{t+j+1}^j} \right)^{1-\theta} - \frac{N_{t+j}^{1+\varphi}}{1+\varphi} \right]$$

$$+ \sum_{j=0}^{\infty} \lambda_{t+j} \left[ (1+i_j) B_{t+j} + W_{t+j} N_{t+j} - B_{t+j+1} - P_t C_t \right] \quad \text{(A.2)}$$

And combine it with (A.1) we get

$$L = \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1 - \theta} \left( \frac{C_{t+j}^j}{C_{t+j+1}^j} \right)^{1-\theta} - \frac{N_{t+j}^{1+\varphi}}{1+\varphi} \right]$$

$$+ \sum_{j=0}^{\infty} \lambda_{t+j} \left[ (1+i_j) B_{t+j} + W_{t+j} N_{t+j} - B_{t+j+1} - P_{H,t} C_{H,t} - E_{i,P,F} C_{F,t} \right] \quad \text{(A.3)}$$

and

$$L = \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1 - \theta} \left( \frac{C_{t+j}^j}{C_{t+j+1}^j} \right)^{1-\theta} - \frac{N_{t+j}^{1+\varphi}}{1+\varphi} \right]$$

$$+ \sum_{j=0}^{\infty} \lambda_{t+j} \left[ (1+i_j) B_{t+j} + W_{t+j} N_{t+j} - B_{t+j+1} - P_t \left[ \left( 1 - \delta \right) \frac{1}{\bar{p}} C_{H,t}^{\eta} + \delta^\eta C_{F,t}^{\eta} \right] \right] \quad \text{(A.4)}$$
respectively. Then differentiate (A.3) and (A.4) w.r.t \( C_{ht} \) gives the following first order conditions.

\[
\frac{\partial L}{\partial C_i} \left( \frac{C_i^{-\theta_i}}{C_i^{-\theta_i}} \right) = \lambda_i P_H \tag{A.5}
\]

\[
\frac{\partial L}{\partial C_i} \left( \frac{C_i^{-\theta_i}}{C_i^{-\theta_i}} \right) = \lambda_i P_i \left[ (1-\delta)^{\eta_i} C_i^{\eta_i} + (1-\delta)^{\eta_H} C_{ht}^{\eta_H} \right] (1-\delta)^{\eta_H} C_{ht}^{\eta_H} = P_H \tag{A.6}
\]

Combine (A.5) and (A.6) and simplify pins down to (A.7)

\[
P_i \left[ (1-\delta)^{\eta_H} C_i^{\eta_H} + (1-\delta)^{\eta_H} C_{ht}^{\eta_H} \right] (1-\delta)^{\eta_H} C_i^{\eta_H} = P_H
\]

\[
P_i \left[ (1-\delta)^{\eta_H} C_i^{\eta_H} + (1-\delta)^{\eta_H} C_{ht}^{\eta_H} \right] (1-\delta)^{\eta_H} C_{ht}^{\eta_H} = P_H
\]

\[
P_i \left[ (1-\delta)^{\eta_H} C_i^{\eta_H} + (1-\delta)^{\eta_H} C_{ht}^{\eta_H} \right] (1-\delta)^{\eta_H} C_{ht}^{\eta_H} = P_H
\]

\[
P_i C_i^{\eta_H} (1-\delta)^{\eta_H} C_{ht}^{\eta_H} = P_H
\]

\[
\frac{1}{C_{ht}} = (1-\delta)^{\eta_H} \frac{P_i}{P_H} C_i^{\eta_H}
\]

\[
C_{ht} = (1-\delta) \left( \frac{P_i}{P_H} \right)^{\eta_H} C_i \tag{A.7}
\]

Using the similar methods we get an expression for \( C_{ht} \) in (A.8)
\[ PC_i = P_i \left[ \left(1 - \delta \right)^{\eta^{-1}} \frac{1}{\eta^{-1}} C_{\eta, \eta} \right]^{\eta^{-1}} = P_{H,i} C_{H,i} + E_i P_{F,i} C_{F,i} \]

\[ P_i \left[ \left(1 - \delta \right)^{\eta^{-1}} \frac{1}{\eta^{-1}} C_{H,i} + \delta^{\eta^{-1}} C_{F,i} \right]^{\eta^{-1}} = E_i P_{F,i} \]

\[ P_i \left[ \left(1 - \delta \right)^{\eta^{-1}} \frac{1}{\eta^{-1}} C_{H,i} + \delta^{\eta^{-1}} C_{F,i} \right]^{\eta^{-1}} = E_i P_{F,i} \]

\[ P_i \left[ \left(1 - \delta \right)^{\eta^{-1}} \frac{1}{\eta^{-1}} C_{H,i} + \delta^{\eta^{-1}} C_{F,i} \right]^{\eta^{-1}} = E_i P_{F,i} \]

\[ \frac{1}{\eta} \frac{1}{\eta^{-1}} \delta^{\eta^{-1}} C_{F,i} = E_i P_{F,i} \]

\[ C_{F,i} = \delta^{\eta} \frac{P_i}{E_i P_{F,i}} C_{\eta} \]

\[ C_{F,i} = \delta \left( \frac{P_i}{E_i P_{F,i}} \right)^{\eta} C_{i} \]  \( \text{(A.8)} \)
APPENDIX B Output equals consumption in steady state

Output clearing condition

\[ Y_t = \left(1 - \delta\right) \left( \frac{P_t}{P_{H.t}} \right)^{\eta} C_t + \delta^* \left( \frac{E_t P_t^*}{P_{H.t}} \right)^{\eta} C_t^* \]  

(B.1)

Express (B.1) in steady state values, by taking away the time subscripts, and divide through by \(Y\).

\[ \frac{C}{Y} = \left[ \left(1 - \delta\right)^{\frac{1}{\eta}} \left( \frac{C_H}{Y} \right)^{\frac{\eta-1}{\eta}} + \delta^* \left( \frac{C_F}{Y} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \]  

(B.2)

In steady state \( \frac{C_H}{Y} = 1 - \delta \) and \( \frac{C_F}{Y} = \delta \) leading to (B.3).

\[ \frac{C}{Y} = \left[ \left(1 - \delta\right)^{\frac{1}{\eta}} \left(1 - \delta\right)^{\frac{\eta-1}{\eta}} + \delta^* \left( \delta \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \]

\[ \frac{C}{Y} = 1 \]

\[ C = Y \]  

(B.3)
APPENDIX C Different linearizations of the CPI formula

Divide the CPI index \( P_t = \left[ (1 - \delta) P_{H,t}^{\eta} + \delta \left( E_t P_t^* \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \) by \( P_{H,t} \) gives (C.1)

\[
\frac{P_t}{P_{H,t}} = \left[ (1 - \delta) + \delta S_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{C.1}
\]

Log linearizing around a symmetric steady state where \( P = EP^* \) gives

\[ p_t - p_{H,t} = \delta s_t \tag{C.2} \]

Dividing the CPI index by \( E_t P_t^* \) gives, \( \frac{P_t}{E_t P_t^*} = \left[ (1 - \delta) S_t^{1-\eta} + \delta \right]^{\frac{1}{1-\eta}} \) which in log linearized form becomes

\[ p_t - e_t - p_t^* = -\eta (1 - \delta) s_t \tag{C.3} \]

Log linearizing \( C_{H,t} = (1 - \delta) \left( \frac{P_t}{P_{H,t}} \right)^\eta C_t \) and \( C_{F,t} = \delta \left( \frac{P_t}{E_t P_t^*} \right)^\eta C_t \) and combine them with (C.2) and (C.3) gives

\[ c_{H,t} = \eta \delta s_t + c_t \tag{C.4} \]

and

\[ c_{F,t} = c_t - \eta (1 - \delta) s_t \tag{C.5} \]
var c c_h im s y y_star i i_star inflation inflation_star n w b a v nx inflation_h q ex u z;
varexo Taylor_schock output_schock philip_shock world_output_schock inflation_star_schock is_shock;
parameters theta phiy phi phiy_star phiy_star kappa_a kappa_y lambda
disc beta epsilon rhoi rhoy
alpha phi psi M mu theta_gamma psiy gamma etha delta rhoy_star
rho_inflation lambda_ex etha_star
rho_inflation_1 rho_is;

lambda_ex = 0.1;
etha_star = 3;
etha    = 2;
delta   = 0.4;
disc    = 0.01;
alpha   = 0.23;
gamma   = 0.8;
phi     = 5;
psi     = 0.01;
rho_inflation_1 = 0.8;
rhoi    = 0.5;
rhoy   = 0.9;
rhoy_star = 0.8;
rho_inflation = 0.8;
rho_is = 0.8;
beta    = exp(-disc);
phiy    = 0.5;
phiy_star    = 1.5;
phiy_star    = 0.5;
theta   = 6.11;
epsilon  = 6;
M       = epsilon/(epsilon-1);
mu      = log(M);
omega   = 0.9;
\[
\theta_{\gamma} = \gamma(\theta-1);
\]
\[
\lambda = \frac{(1-\omega)(1-\beta\omega)}{\theta}\frac{(1-\alpha)}{1-\alpha+\alpha\epsilon};
\]
\[
\kappa_y = \lambda\frac{\phi+\alpha}{1-\alpha};
\]
\[
\kappa_a = \lambda\frac{\phi+1}{1-\alpha};
\]
\[
\text{model(linear)};
\]
\[
ex = y_{\star} + \lambda_{\text{ex}}\theta_{\star}s + (1-\lambda_{\text{ex}})(ex(-1)-y_{\star}(-1));
\]
\[
q = (1-\delta)s;
\]
\[
inflation = inflation_h + \delta(s-s(-1));
\]
\[
c = (1-\delta)c_{\star} + \delta im;
\]
\[
c_{\star} = etha*delta*s+c;
\]
\[
y = ((1-\delta)etha + \lambda_{\text{ex}}etha_{\star})*\delta s + (1-
\delta)c_{\star} + \lambda_{\text{ex}}y_{\star} + \delta (1-\lambda_{\text{ex}})(ex(-1)-y_{\star}(-1));
\]
\[
i = phip*inflation + phi y + v;
\]
\[
i = i_{\star} - psi*b + s(+1) - s + inflation(+1) - inflation_{\star} (+1);
\]
\[
w = \phi n - \gamma c(-1) + \theta c;
\]
\[
-(\theta_{\gamma} + \theta)c + z = -\theta_{\gamma}c(-1) - \theta c(+1) + i - inflation(+1);
\]
\[
inflation_h = \beta inflation_h(+1) + \kappa_y y - \kappa_a a - \lambda_{\text{ex}}\theta_{\gamma} c(-1) + \lambda_{\text{ex}}\theta c + \lambda\delta s + u;
\]
\[
(1-\alpha)n = y-a;
\]
\[
inflation_{\star} = rho_{\text{inflation}} inflation_{\star}(-1) + inflation_{\star Schock};
\]
\[
i_{\star} = phip_{\star} inflation_{\star} + phi_{\star} y_{\star};
\]
\[
b = (1+i(-2)-inflation(-1))*b(-1) + y(-1) - c(-1) - \delta s(-1);
\]
\[
nx = \delta (ex-im);
\]
\[
v = rhoiv(-1) + Taylor_schock;
\]
\[
a = rhoa(-1) + output_schock;
\]
\[
z = rho_isz(-1) + is_shock;
\]
\[
y_{\star} = rhoy_{\star} y_{\star}(-1) + world_output_schock;
\]
\[
u = rho_{\text{inflation1}}u(-1) + philip_shock;
\]
end;

intval;
\[
y = 0;
\]
\[
inflation = 0;
\]
end;

steady;
check;

shocks;
var is_shock = 0.5;
var Taylor_shock = 0.5;
var output_shock = 0.5;
var philip_shock = 0.5;
var world_output_shock = 0.5;
var inflation_star_shock = 0.5;
end;

stoch_simul(periods=210, irf=25);
APPENDIX E Additional dynamics

Taylor shock

![Graphs showing economic variables over time](image-url)
Inflation shock

\[ \text{Inflation shock} \]
Technology shock
World inflation shock
World output shock
Consumption shock