

System identification and control of an irrigation channel with a tunnel

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Abstract

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The demand on water is high but the supply of water is low due to the dry and warm climate in Australia. This is especially difficult for farmers that do not get enough water delivered in the irrigation channels which has dramatic consequences. A high percentage of the water lost in the channels can be saved if the systems are managed better.

Using system identification techniques to develop a mathematical model which describes the dynamics in the irrigation system is a helpful tool for control system design. The model needs to be able to describe the downstream water level of a pool, containing a tunnel, in a satisfying way. The model was built using data from experiments to estimate unknown parameters in a chosen model structure. A feedforward PI controller with a low pass filter was designed using frequency response techniques and the results were simulated with varying parameters and disturbances.

The result of the modeling was a first order OE grey box model. The model performed well on validation data and was therefore used in the controller design. The controller using both feedback and feedforward could in a satisfying way reduce offtake disturbance, track water level set points and limit the excitation of waves in the channel, and therefore also reduce the water wastage.

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Populärvetenskaplig sammanfattning

Efterfrågan på vatten är stor medan tillgången på vatten är låg på grund av det torra och varma klimatet i Australien. Detta påverkar speciellt lantbrukare då otillräcklig bevattning av markerna kan ha förödande konsekvenser för dem. Vatten levereras ofta på beställning genom stora nätverk av bevattningskanaler, men i dessa försvinner en stor andel av vattnet delvis p.g.a. att man inte har tagit hänsyn till olinjäriteter i systemet, vilket t.ex. en tunnel utgör. Med modeller som beskriver flödet i kanalerna mer noggrant och med regulatorer som baseras på dessa modeller kan man minska volymen vatten som idag går förlorad i systemet.

System identifiering kan användas som ett verktyg för att ta fram en matematisk modell som beskriver dynamiken i bevattningskanalerna vilket är användbart vid regleringen av systemet. Modellen behöver kunna beskriva vattennivån nedströms i en bassäng som innehåller en tunnel på ett tillfredsställande sätt. Modellen utvecklades baserat på data från experiment och de okända modellparametrarna estimerades i den valda modellstrukturen. En PI-regulator med lågpasfilter designades med hjälp av frekvenssvarteknik och resultatet simulerades med varierande parametrar och störningar.

Resultatet av modelleringen är en första ordningens OE grey boxmodell. Denna modell kunde beskriva valideringsdata väl och används därför till designen av regulatorn. Regulatorn använder både återkoppling och framkoppling och kan minska störningarna från sidokanalerna, följa önskad vattennivå och minskar förstärkningen av vågor i kanalen, vilket bidrar till att mindre vatten slösas.

Preface

This master thesis is the result of a project carried out at the University of Melbourne in Australia.

I would like to thank everyone that have inspired and helped me throughout this thesis work. A special thanks to my supervisor Dr Erik Weyer at the Department of Electrical and Electronic Engineering, University of Melbourne, for giving me the opportunity to do the project at the department and for the invaluable discussions during the time of my work. I would also like to thank Valerie (Ping) Zhang at Rubicon Systems Australia Pty Ltd, who helped me collect data and supported me in my work.

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1 Introduction

1.1 Background

Due to a sharp rise in water demand in many parts of the world, water is becoming an increasingly scarce resource and water management has become a very important issue. Australia is a very dry continent that presumably cannot sustain the present exploitation levels of its natural water resources. Climate change, population and industrial growth pressures compound the problem.

The distribution of fresh water for irrigation in Australia is achieved via an extensive civil infrastructure, reservoirs, approximately 15,000 km of channel network and 17,000 channel pools¹. On a global scale, agricultural irrigation accounts for approximately 70% of all fresh water usage.² It is not always the low supply of water that causes the problem, but the disability to fully and efficiently utilize the available quantities³. Large amounts of water, typically 25% in the Australian irrigation systems, are wasted due to poor management and control⁴. The water losses are evenly split between the large scale distribution losses and the on-farm losses. The on-farm losses are to a large extent due to poor timing of irrigation, a consequence of manual water scheduling on the side channels leading to the farms, called offtakes. Most of the large scale distribution losses are due to natural tendency to oversupply water, meaning that water passes the last gate in the irrigation system and since there are few opportunities to recapture water that is not used, the water is considered wasted. The channels tend to be operated with relatively large volumes of water, oversupplied, in order to avoid the dramatic consequences on the rural sector when water is not delivered on time. With better models available, the channels do not have to be oversupplies and the losses of water could be reduced.

Australia's channel infrastructure is 100 years old and consists mainly of open water channels, as opposed to closed channels which are covered. Clearly, there are problems with open water channels, such as evaporation and seepage, with an estimated loss of 10-15% of supply. This could be avoided with closed channels but at present, these losses are insufficient to warrant consideration of replacing the current channel infrastructure with closed channels in a piped water network, which have other problems such as leaks.⁵

The water levels can be controlled by installing gates along the channel, which can be opened and closed. With an IT infrastructure installed at the gates, data, such as water levels and gate

¹ Rubicon Systems

² Mareels, Weyer, Ooi, Cantoni, Li, Nair (2005)

³ Ooi, Weyer (2007)

⁴ Cantoni, Weyer, Li, Ooi, Mareels, Ryan (2006)

⁵ Mareels, Weyer, Ooi, Cantoni, Li, Nair (2005)

positions, can be communicated from the channels. System identification can then be used to exploit the data and find models that accurately describe the flow in the channel. Using control techniques, a controller can be designed, which can control the water levels. A controller has the potential to improve distribution efficiency and achieve near on-demand water delivery.

1.2 Channel description

Water runs from reservoirs through a series of open water irrigation channels to the farms. The large-scale distribution of water is powered purely by gravity. Most farms also have a gravity fed system, which means that the amount of farmland that can be irrigated is directly limited by the available water supply (potential energy) at the channel outlet (on-farm supply point). A number of gates are positioned along the channel, which work as controllers and restrict the water flow.

The measured variables are the water levels, measured with reference to a standard level, mAHD (meter Australia Height Datum) and the gate positions, given in meters. The head over gate, denoted by h , is the amount of water above the gate and is calculated as the upstream water level minus the gate position p as shown in figure 1.

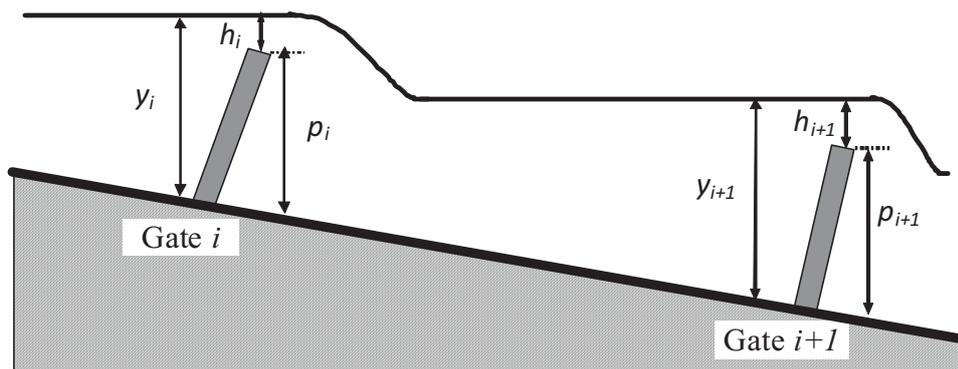


Figure 1: A section of an irrigation channel

A stretch of channel between two gates is referred to as a pool. The pools are named according to the upstream gate. y_i and y_{i+1} are the upstream water level of gates i and $i+1$ respectively, p_i and p_{i+1} are the position of gates i and $i+1$, and h_i and h_{i+1} are the head over gates.

Irrigation channels should be managed so that they can deliver water to the farms on demand and minimise water wastage. These both objectives are conflicting. Most Australian irrigation channels are demand driven. This means that farmers place orders of water in advance, rather than the water authority telling the farmers when they can take water⁶. Based on the orders of all farms along a section of a channel the water authority calculates the amount of water to be released into a channel from a dam or a reservoir. When the water is in the channel, the water

⁶ Weyer (2008)

levels are controlled and maintained around setpoints, which are determined depending on the water levels and the water demands. The water level is controlled by moving the gates and hence the water level is the output of the system. The input is the amount of water flowing into the channel and this is associated with the position of the gate. It is important to keep the water level as close to the setpoints as possible as the offtakes are gravity fed. This means that there is no way of pumping the water to feed the offtakes when the change in potential energy is not sufficient to feed the offtakes from gravity alone.

A pool can sometimes be several kilometres long and as the water takes some time to travel this distance, the system experiences a time delay. The time delay makes the control task harder because more water than necessary is released into the channel in order to make sure that the water is delivered to the farmers on demand. From a farmers perspective, the quality of the irrigation service is determined by the timing of the irrigation water and, since the on-farm irrigation is gravity fed, also by the water level at the on-farm inlet.

1.3 Aim

The aim of this project is to further improve the performance and thereby the efficiency of an irrigation system. More specific, this is done by controlling a part of the channel containing a tunnel. I intend to use system identification methods to find a model that predicts the water level in a satisfying manner and then with the help of controlling techniques design a controller that can regulate the flow.

1.4 Outline of the report

The thesis consists of 6 chapters, the contents of which is organised as follows. The first chapter gives an introduction to the subject of irrigation channels and the problem with water loss in irrigation channels that we are facing today. The second chapter provides the theory for system identification. First, some modelling techniques are introduced and then the system identification process, including experiment design and performance, model structure selection, parameter estimation and model validation, is described. Chapter three gives the basis of control theory considering the advantages of different controllers and how their parameters can be found. Some control related topics such as frequency response, stability and robustness of control systems are also discussed. In chapter four the modelling of the irrigation channel starts. In order to understand the complex dynamics in the system, the part of the irrigation channel considered in this report is introduced with its physical parameters. Thereafter, the modelling based on physical relations and system identification begins. A model structure is chosen, and a model and its predictor are found. Experiments are performed and analysed and the unknown parameters are estimated. Finally the models are validated. Chapter five involves the design of the controller starting with stating the objectives of the controller. Then the design process, using both feedback and feedforward controller, is described. Simulation results are provided to support the analysis and illustrate achievable performance. In the final chapter the main results of the thesis are outlined.

2 System identification theory

The use of system identification models for control has two obvious advantages. The models are simple discrete time difference equations and easy to use for control design, and since they are built from observed data, they often give an accurate description of the relevant dynamic.⁷

2.1 Modelling techniques

Constructing a model of a system is a good way of learning about the qualities of the system without having to perform experiments on it. A mathematical model is a description of the system where the relations between the model variables and the signals are expressed as mathematical relations. The laws of nature are examples of mathematical models, but also they are idealised and simplified systems. For real systems, the relations between the variables can be very complicated. Using a simplified model of a system, the qualities and behaviour of the system can be studied either analytically or with numerical experiments through simulations. The value of the results are however dependent on the quality of the model.⁸

2.1.1 Physical modelling

Using all the physical insights about the behaviour of a process to form a model containing both known and unknown parameters is called physical modelling. The principle of physical modelling is to use known relations to describe the systems. In many systems physical laws can be used to describe the system dynamics whereas in some systems hypotheses may also have to be used.

2.1.2 System identification methods

The other approach uses operational data, which is measurements of the behaviour of the system and the external influences, to try to determine a mathematical relation between them. This approach is called system identification and requires less physical a priori information of the system. The system identification method is often used as a complement to physical modelling.⁹ Two types of modelling techniques are common in the field of system identification; black box modelling and grey box modelling.

Black boxes are a family of (usually linear) models, whose parameters do not have physical significance but where the objective is to find a good linear system that fits the observed data. The advantage of black box modelling is that the demand on information about the system is small. Only the relation between the input and the output is described, without taking any notice

⁷ Weyer (2008)

⁸ Glad, Ljung (2004)

⁹ Glad, Ljung (2004)

of what happens inside the system. The description of the system can be a number of mathematical equations or a graph showing the relationships between input and output.

Between the two extremes on the design scale of a model structure there is a middle zone where considerable and important physical insight is used in the identification process, but not to the extent that a formal physically parameterized model is constructed. This is called Grey box modelling or semi-physical modelling and is used for partially known systems. This is the process of taking physical insight known about the behaviour of the system into account and use the insight to find adequate nonlinear transformations of the raw measurements so that the new variables stand a better chance to describe the true system when they are subjected to standard model structures. A grey box model should explicitly utilise the a priori knowledge such as the physical laws.¹⁰

2.2 Procedure of system identification

The system identification procedure usually goes through four different phases, as shown in figure 2, which can be considered as the method of using system identification. First of all the experiment needs to be planned so that as much information as possible will be collected and then the experiment can be performed. Using the information about the system the model structure is selected and the unknown parameters need to be estimated. When this has been done, the model needs to be validated. In this phase, one might find that the model is not good enough, if this is the case one or more of the prior phases need to be repeated in order to arrive at a satisfying model.

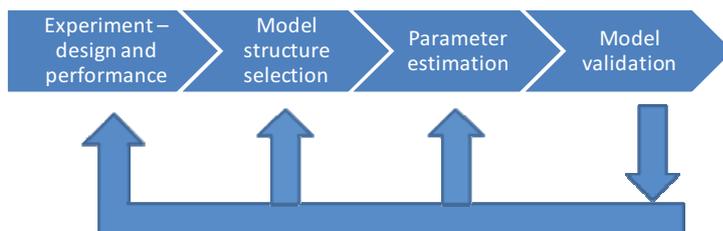


Figure 2: The system identification procedure

¹⁰ Lindskog, Ljung (1995)

2.2.1 Experiment design

The first step in the building of a model is to decide what quantities and variables that are important to describe the dynamics in the system. Simple experiments and observations of the true system are the main sources of information.

A common experiment in this phase of the model building is a step response analysis, also called a transient analysis. The input is changed from one constant value to another, as a step, and the other measurable variables are observed. A step response can give information about what variables that are affected by the input, time delays, time constants and the character of the step response, information which can be useful when studying the behaviour of the final model. The time delay can be found as the time difference from when the step is performed in the input until the effect of the step can be observed in the output. The time constant τ represents the time it takes for the system's step response to reach approximately 63% of its final asymptotic value. The time constant can change with flow conditions and when the experiment time is fairly long, one can also calculate the time constant with the so called 5τ -method. This method estimates the time constant as the time it takes for the system to reach steady state divided by five.

The experiment should be designed in such a way that it is as informative as possible since the more data that is collected during the experiment and the more variation in the data, the better model can be obtained.

2.2.2 Model structure selection

The second step in the system identification procedure is to obtain a useful model structure, which describes the dynamic relationships between the input signals and the output signals. Based on physical considerations a few equations can usually be obtained which are believed to pick up the essential features of the system in question. Most physical laws are continuous time differential equations and it is therefore natural to build a continuous time model. Instead of making experiments on the real system, they can be made on a model of the system. If the model will be used to design a controller, one has to take into account that if the controller is implemented on a computer, it works in discrete time which means that the continuous model needs to be sampled. The model will then only give information about the system at the sampling instances. It is therefore practical to use a discrete time differential equation model¹¹. A general linear, time discrete model can be written as

$$y(t) = \eta(t) + w(t) \tag{1}$$

$y(t)$ is the output, $w(t)$ represents the disturbance and $\eta(t)$ is the disturbance free output, which can be written as

¹¹ Glad, Ljung (2004)

$$\eta(t) = G(q, \theta)u(t) \quad (2)$$

where $u(t)$ is the input and $G(q, \theta)$ is a rational function of the shift operator q , also written as

$$G(q, \theta) = \frac{B(q)}{F(q)} = \frac{b_1 q^{-nk} + b_2 q^{-nk-1} + \dots + b_{nb} q^{-nk-nb+1}}{1 + f_1 q^{-1} + \dots + f_{nf} q^{-nf}} \quad (3)$$

which gives the relationship

$$\eta(t) + f_1 \eta(t-1) + \dots + f_{nf} \eta(t-nf) = b_1 u(t-nk) + \dots + b_{nb} (t - (nb + nk - 1)) \quad (4)$$

This is a differential equation with a time delay of nk samples. The disturbance $w(t)$ can be written in the same way as

$$w(t) = H(q, \theta)e(t) \quad (5)$$

where $e(t)$ is white noise and $H(q, \theta)$ is a rational function of the shift operator q

$$H(q, \theta) = \frac{C(q)}{D(q)} = \frac{1 + c_1 q^{-1} + \dots + c_{nc} q^{-nc}}{1 + d_1 q^{-1} + \dots + d_{nd} q^{-nd}} \quad (6)$$

By using the transfer functions defined above, the model given in equation (1) can be rewritten as

$$y(t) = G(q, \theta)u(t) + H(q, \theta)e(t) \quad (7)$$

The parameter vector θ contains the coefficients b_i, c_i, d_i and f_i from the transfer functions. nb and nf and nc and nd are the orders of the numerator and denominator respectively of $G(q, \theta)$ and $H(q, \theta)$.

A very common model structure is the Auto Regressive with External Input (ARX) model which is given by

$$A(q)y(t) = B(q)u(t) + e(t) \quad (8)$$

Figure 3 shows that the disturbance comes in early in the process in the ARX model structure.

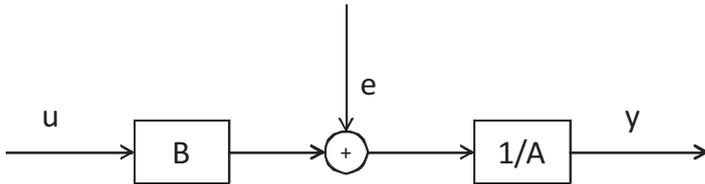


Figure 3: Block diagram of an ARX model structure

The ARX model is associated with the following predictor which uses old values of the output for the predicted output value $\hat{y}(t, \theta)$.

$$\hat{y}(t, \theta) = -a_1 y(t-1) - a_{na} y(t-na) + b_1 u(t-nk) + b_{nb} u(t-nk-nb+1) \quad (9)$$

Another common model structure is the Output error (OE) model

$$y(t) = G(q, \theta)u(t) + e(t) \quad (10)$$

In the OE model structure, as can be understood by its name, the noise comes in late in the process and only affects the output, which is shown in figure 4. No parameters are used for modelling the disturbance characteristics, hence $H(q, \theta) = 1$.¹²

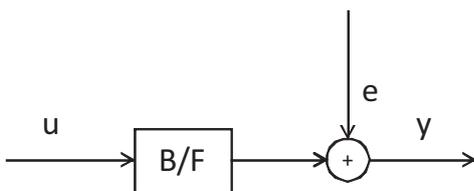


Figure 4: Block diagram of an OE model structure

The OE model is associated with the predictor

¹² Glad, Ljung (2004)

$$\hat{y}(t, \theta) = -f_1 \hat{y}(t-1) - f_{nf} \hat{y}(t-na) + b_1 u(t-nk) + b_{nb} u(t-nk-nb+1) \quad (11)$$

where \hat{y} represents the predicted output value. The OE predictor uses the previously predicted output at time t to predict the output at time $t+1$, as opposed to the ARX predictor which uses the measured output at t to predict the output at $t+1$.

From a system identification point of view the most important aspect of choosing the model structure is to make sure that it can be used to define predictors. How well and for how long a model can predict future values is crucial when choosing the model structure.¹³ The main differences between an ARX model structure and an OE model structure are that they model the noise differently and they represent different dynamics well. From linear theory it is known that an OE model gives a good description of the low frequency properties, whereas an ARX model gives a better description of the high frequency properties¹⁴.

Once the model structure has been chosen the model order has to be determined. A too low model order will not be able to describe the true dynamics of the system whereas a too high model order will adapt to the noise in the system. The choice of the model order can not always be made a priori, but have to be made along the way. The choices are also highly dependent on the future use of the model.

All models include simplifications of the real dynamics. A model is simplified because there is not enough information about the system, but even if there was, it would not be appropriate to try to model everything. There has to be approximations in the model for it to be easy enough to use. There will be a balance between complexity of the model and the accuracy of its predictions. This decision is made depending on what the model is used for. The model selection and the model order should be based on the principle, try simple things first, which means that only if simple models such as ARX and OE are not good enough, more complex models should be tested.

2.2.3 Parameter estimation

Some constants, called design parameters, can vary in the simulations and therefore be hard to evaluate. In order to predict future values of the output, estimations of the unknown parameter values in the model are needed. They can be estimated using the following model structure

$$\hat{y}(t) = \varphi^T(t) \theta \quad (12)$$

¹³ Eurén, Weyer (2006)

¹⁴ Glad, Ljung (2000)

where $\hat{y}(t)$ is the output of the model, $\varphi(t)$ is an n -dimensional column vector of the known variables, old inputs and outputs, and θ is an n -dimensional column vector which contains all the unknown parameters.

$$\varphi(t) = \begin{bmatrix} -y(t-1) \\ \vdots \\ -y(t-na) \\ u(t-1) \\ \vdots \\ u(t-nb) \end{bmatrix} \quad \theta = \begin{bmatrix} a_1 \\ \vdots \\ a_{na} \\ b_1 \\ \vdots \\ b_{nb} \end{bmatrix} \quad (13)$$

The aim is to find the unknown parameter vector θ . This can be done by “fitting” the model to the data so that the error,

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t, \theta) \quad (14)$$

becomes small. The model should hence give a good prediction of the measured data. This can be done with a common parameter estimation method called least squares method. Assuming that a data set from a system has been collected, this method finds estimates of the parameters by minimising the loss function $V(\theta)$, which is a function of the squared prediction errors.

$$V(\theta) = \sum_{t=1}^N (y(t) - \hat{y}(t))^2 = \sum_{t=1}^N (y(t) - \varphi^T(t)\theta)^2 \quad (15)$$

N is the number of measurements used in the estimation. $y(t)$ is the measured output and $\hat{y}(t)$ is the predicted output. It is common to write the loss function in normalised form as

$$V(\theta) = \frac{1}{N} \sum_{t=1}^N (y(t) - \hat{y}(t))^2 \quad (16)$$

which gives the minimizing θ . The θ vector can in some cases be found with an analytical solution. In more complicated cases an iterative search algorithm is needed in order to find an estimate. For every step in the search for θ , the prediction of the estimation set is calculated. The function of the squared prediction error, $V(\theta)$, is calculated and compared with the previous $V(\theta)$. The iterative search stops when a minimum has been found, but it is unknown whether this is a global or a local minimum. If the minimum is local it will not give the best model within the

particular model structure. This part of system identification can be very time consuming since many different starting values have to be tested to make sure that the minimum found is good enough.

The least square estimate is defined as

$$\theta = \hat{\theta} = R_N^{-1} f_N = \left[\frac{1}{N} \sum_{t=1}^N \varphi(t) \varphi^T(t) \right]^{-1} \left[\frac{1}{N} \sum_{t=1}^N \varphi(t) y(t) \right] \quad (17)$$

Assuming that the matrix R_N is invertible, this equation finds the θ that minimizes $V(\theta)$.

2.2.4 Model validation

Once the parameters have been estimated and the model has been constructed it needs to be validated. This is done to confirm that the estimated model meets the specifications required. The loss function does not give a true judgement of how well the model can describe the system, but gives a very good judgement of how well it can model the estimation set. The model may however have an overfit, meaning that it tries to describe the noise. The purpose of the validation is to find out if the model is valid for other empirical data sets than the estimation set. An accurate model will closely match the verification data even though this data set was not used to find the model's parameters. This practise is referred to as cross validation. The basic idea is that the available data is split into two sets, one estimation set and one validation set. The parameters are estimated using the estimation set and their performance is then controlled using the validation set. A good model should give similar performance from the estimation and validation set, which indicates that there is no overfit.

It is hard to say what a “good” model is. The demands for performance and quality on the model differ depending on the intended use of the model. Overall, a good model needs to be able to imitate a dynamic system closely and give good predictions of the observed data. A model needs to have robustness so that the simplified model gives good results even though the model does not match with the true system. The term stability robustness is used meaning that the stability in the closed system is not jeopardised although some modelling errors may be present.

The mismatch between the true plant and the model consists of two components, bias error and variance error. The variance error comes from the fact that noise cannot be reproduced and therefore two exact same performed experiments will not give the same output. The variance error can usually be reduced by performing longer experiments with more measurement data. Bias error arises when the wrong model structure is used. If there are unmodelled dynamics the model is simply not capable of describing the system. Thus, if the model does not live up to the

expectations the model construction procedure needs to be repeated. This is usually the case before a satisfactory model is obtained.¹⁵

¹⁵ Glad, Ljung (2004)

3 Control theory

3.1 Open loop vs. closed loop system

A system can be operated in either open loop or closed loop. In open loop, as shown in figure 5, there is no feedback mechanism from the output to the input. With other words, there is no correction for the error between the desired output and the observed output.

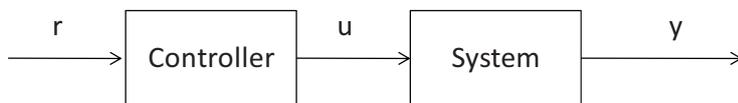


Figure 5: Block diagram of an open loop system

In closed loop, as shown in figure 6, the deviation between observed output y and the desired output r , called the error e is fed back into the system as the input to the controller. The controller can transform the input information to a value of the output of the controller u . If there are no model errors and no disturbances, there are no reasons to use feedback in a closed loop system.

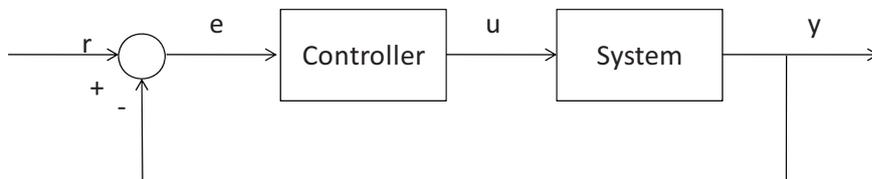


Figure 6: Block diagram of a closed loop system

The goal with a control system is for it to work in the real environment. The real environment may change with time, due to varying conditions, and the control system must be able to withstand these variations. Therefore, the particular property a control system must possess to operate properly in realistic situations is called robustness. Mathematically this means that the controller must perform satisfactorily not just for one plant, but for a family of plants.¹⁶

¹⁶ Stefani, Shahian, Savant, Hostetter (2002)

A feedback system must also possess reduced sensitivity and disturbance rejection. By sensitivity it is meant that using feedback the sensitivity of the closed loop system is reduced. Disturbance rejection refers to the fact that feedback can eliminate or reduce the effects of unwanted disturbances occurring within the feedback loop.¹⁷

3.2 PID controller

In order to design a controller the objective of the controller first has to be determined. A controller can be designed in many different ways to control the overall system behaviour. The most common controller is the PID controller, using proportional control (P), integral control (I) and derivative control (D). A PI controller is described by two parameters, the proportional gain (K_p) and the integral gain (K_I). An increase in the proportional gain will have the effect of reducing the rise time and will reduce, but never eliminate, the steady-state error. The integral gain will have the effect of eliminating the steady-state error, but it may make the transient response worse. Adding a derivative gain (K_D) will increase the stability of the system, reducing the overshoot, and improving the transient response. K_p , K_I and K_D are dependent of each other and changing one of these variables can change the effect of the other two.

The transfer function of a PID controller in continuous time looks like the following

$$K_p + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_p s + K_I}{s} \quad (18)$$

Referring to figure 6, the PID controller would work as follows. The error signal (e) will be sent to the PID controller which computes both the derivative and the integral of the error signal. The signal (u) just past the controller is now equal to the proportional gain (K_p) times the magnitude of the error plus the integral gain (K_I) times the integral of the error plus the derivative gain (K_D) times the derivative of the error. This signal (u) will be sent to the system, and the new output (y) will be obtained and sent back to the sensor again to find the new error signal. This process goes on and on.

When designing a controller, one usually starts with a proportional controller and then adds an integral, and derivative part if needed. There are other compensators that can be used in the controller as well, i.e. lead and lag compensation, or high and low pass filters. A low pass filter passes the low frequency components in the signals but attenuates (reduces the amplitude of) signals with frequencies higher than the cutoff frequency.

¹⁷ ibid

3.2.1 Frequency response

The frequency response is a representation of the system's response to sinusoidal inputs at varying frequencies. The frequency response is defined as the magnitude and phase differences between the input and the output sinusoids. The open loop frequency response of a system can be used to predict its behaviour in closed loop. One advantage of the frequency domain analysis compared to a step response analysis is that it considers a broader class of signals (sinusoids of any frequency). This makes it easier to characterise feedback properties, and in particular system behaviour in the crossover (bandwidth) region.¹⁸ Below some of the important frequency domain measures used to assess performance are defined and described.

Stability margins are measures of how close a stable closed loop system is to instability. Two commonly used quantities that measure the stability margin for control systems are gain margin and phase margin. These provide stability margins for gain and delay uncertainty. The gain margin is defined as the change in open loop gain required to make the system unstable. Systems with greater gain margins can withstand greater changes in system parameters before becoming unstable in closed loop. The phase margin is defined as the change in open loop phase required to make a closed loop system unstable. Again, the greater the phase margin, the greater changes the system can withstand.

The concept of bandwidth is very important when understanding the benefits and trade-offs involved when applying feedback control. The bandwidth is defined as the frequency at which the closed loop magnitude response is equal to -3 dB. It relates to the speed of the response and hence the system performance, in general. With a small bandwidth the time response will generally be slow, and the system will usually be more robust. Also here there is a trade off between the speed of the response and the robustness of the system. It is important to note that by decreasing the value of w_c (lowering the closed loop bandwidth, resulting in a slower response) the system can tolerate larger time delay errors.¹⁹

3.2.2 Anti windup compensation

All actuators have physical limitations. A gate cannot be more than fully open or fully closed and hence, the mechanical range of the actuators is limited. When this happens the feedback loop is effectively broken because the actuator will remain at its limit independently of the process output.²⁰ This can cause the behaviour of the system to deteriorate dramatically, or even become unstable²¹. If a controller with integral action is used, the error between the linear output and the actuator limit will continue to be integrated, the integrator output will grow, “wind up”, until the sign of the error changes and the integration turns around. The net effect is a large overshoot, as the output must grow to produce the necessary unwinding error, and poor transient response is

¹⁸ Skogestad, Postlethwaite (2005)

¹⁹ Chaudhry (1993)

²⁰ Åström, Häggglund (1988)

²¹ Skogestad, Postlethwaite (2005)

the result.²² However, there is a way to avoid integral windup, by using an extra feedback path shown in figure 7. This measures the actual actuator output and forms an error signal (e) as the difference between the output of the controller (u) and the actuator output (v). When the actuator saturates, the feedback signal (e) is fed to the integrator through a gain T_i and attempts to drive the error to zero. As soon as the actuator saturates, the feedback loop around the integrator becomes active and acts to keep the input to the integrator small.²³

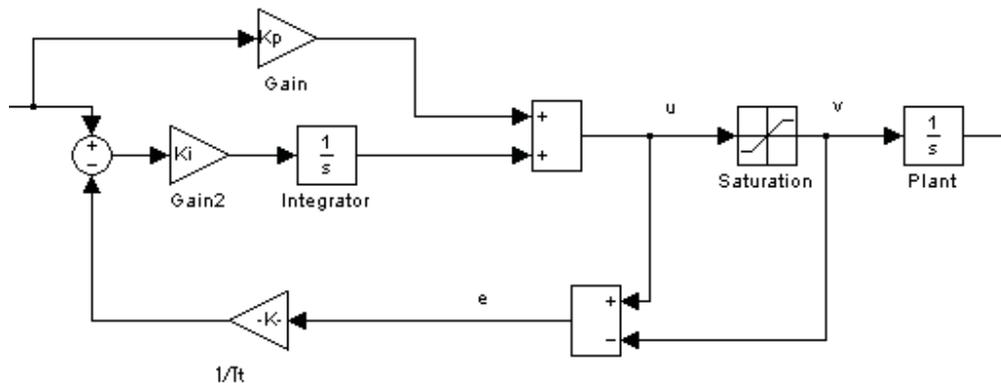


Figure 7: Block diagram of anti-windup compensation

The time constant T_i is called tracking constant and determines how quickly the controller output is reset. It should be chosen to be large enough so that the anti-windup circuit keeps the input to the integrator small under all error conditions. A common selection of T_i is the same value as the integral time.²⁴

3.3 Feedforward controller

Combining feedforward with feedback control can significantly improve performance over simple feedback control if there are measurements from the disturbance available. Feedforward control is often used along with feedback control because a feedback control system is required to track setpoint changes and to suppress unmeasured disturbances, which are always present in a real process. Even when there are modelling errors present, feedforward control can often reduce the effect of the measured disturbance on the output better than that achievable by feedback control alone. As seen in figure 8, the disturbance is fed into the feedforward, (K_{ff}) which can reduce the effect it has on the system (S). The decision to use feedforward control is however dependent on the degree of improvement achieved with feedforward in comparison with only feedback and if it justifies the added costs of implementation and maintenance.

²² Åström, Hägglund (1988)

²³ Franklin, Powell, Emanmi-Naeini (2006)

²⁴ Åström, Hägglund (1988)

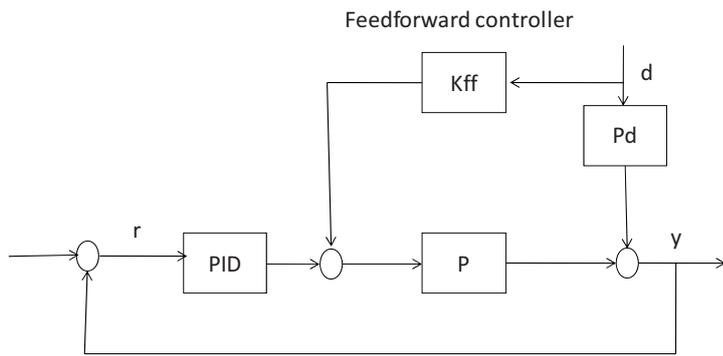


Figure 8: Block diagram of a combined feedback-feedforward control structure

4 Modelling of irrigation channels

This chapter describes the system identification procedure applied on a specific section of an irrigation channel. The results presented in this chapter are discussed straight away as this is easier to follow.

4.1 Introduction

The system identification is performed on a specific part of the irrigation channel containing a tunnel, two gates, 877A and 919, and an offtake to a farm, 880, illustrated in figure 9.

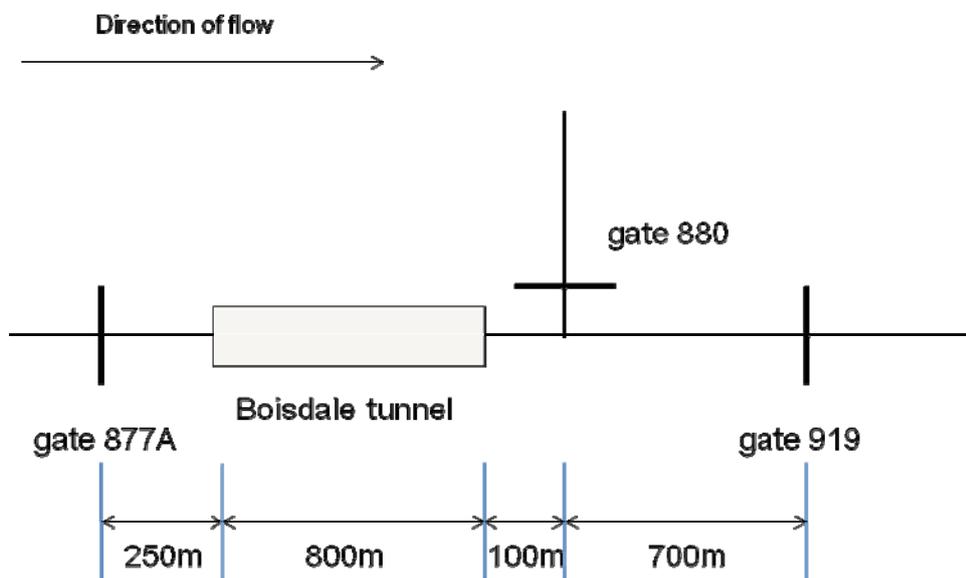


Figure 9: Section of channel including gates 877A, 880, 919 and tunnel

The water flows from gate 877A through the Boisdale tunnel and then out either over gate 919 or through offtake 880. From gate 877A to the inlet of the tunnel, there is a pool 250m long. The tunnel is 800m long and after the tunnel outlet another pool, 800m long before the water is let out through gate 919. 700 m before gate 919 and just 100 m after the tunnel outlet is another gate, 880 which leads off water to farms and is called an offtake, see figure 10.



Figure 10: Picture of the offtake at gate 880

Approximately 3950m upstream from gate 877A is gate 877 positioned. Before gate 877A was implemented, the pool before the tunnel was over 4000m.

The water levels are measured immediately upstream and downstream of each gate. All gates considered in this report are overshot and automatically controlled. The gates can operate in the free flow regime (figure 11) when the water level downstream of the gate is lower than the gate position, or submerged flow (figure 12), where the gate position is below the downstream water level and when this occurs, the flow properties changes.²⁵

²⁵ Eurén, Weyer (2006)

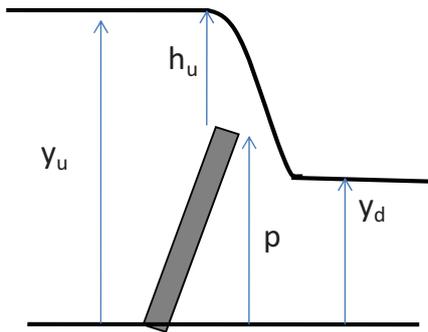


Figure 11: Free flow over gate

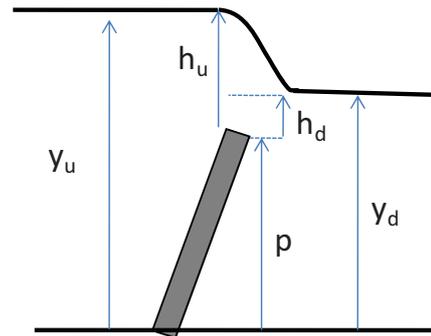


Figure 12: Submerged flow over gate

As can be seen in figure 13, there can be more gates operating at each site, in which case they operate in parallel, and the gates should always have the same position.



Figure 13: Picture of gates at 877A powered by solar panels

Table 1 presents information about the gates, how many there are at each site, the width of each gate and the summed width of all gates at each site.

Table 1: Information about the gates

Gate	Number of gates	Width per gate (m)	Width per regulator structure (m)
877A	3	1.372	4.116
880	1	1.56	1.56
919	2	1.37	2.74

A gate cannot be more than fully opened or fully closed, hence, the mechanical ranges of the gates are limited. In the case of gate 877A the mechanical range is 8.256 - 9.561 mAHD. A gate can easily get stuck in the extremes of the mechanical range and the controller, which is software, is therefore operated within a safety range in order to avoid the gates getting stuck. The controller range for the gate 877A is 8.27 - 9.3 mAHD.

As channels are located in rural areas, solar panels supply the electric power needed to move the gates and data communication takes places via a radio network. The required communication for control purposes is kept to a minimum by using decentralized control structure where a peer-to-peer session between the gates can pass the required information, such as the water demand, to the nearest upstream regulator. At each gate, all data are monitored on a regularly sampled basis.

4.1.1 Tunnel

A channel transition is a local change in the channel geometry, which changes the flow from one state to another. Typical examples of channel transitions are contractions, expansions and bends. The inlet to the Boisdale tunnel, as can be observed in figure 14, is a channel contraction, which comprises a reduction in the channel width. Such a contraction may choke the flow if the channel width is reduced too much, since the energy may not be sufficient to pass the required amount of discharge per unit width.²⁶

²⁶ Chaudhry (1993)



Figure 14: Picture of the Boisdale tunnel inlet

A tunnel provides the channel with a possible non-linearity. Several different flow conditions may occur in a tunnel and these conditions depend on several parameters. A channel section at which there is a unique relationship between the depth and discharge is referred to as control. The control may be at the upstream end, called inlet control, where the flow mainly depends on the inlet conditions, e.g. area, shape and configuration at the inlet, or controlled at the downstream end, called outlet control. The tunnel may flow full or partially full throughout its length. The inlet and outlet may be submerged, partially submerged or unsubmerged. Hence, the computation of flow conditions through a tunnel can be somewhat complex.²⁷ From the pictures it can be seen that neither the inlet (figure 14) nor the outlet (figure 15) seem to be in submerged flow, meaning that the inlet/outlet is below the water level.

²⁷ Chaudhry (1993)



Figure 15: Picture of the Boisdale tunnel outlet

Previous work has shown that a section in an irrigation channel between two gates can be modelled with a first order non-linear equation. It has also been shown that when a pool contains a tunnel, the pool may have to be split up into two parts, before and after the tunnel and modelled separately. Therefore, the tunnel may change the dynamics of the channel and a higher order non-linear equation may need to be considered.²⁸

4.2 Modelling based on physical relations and system identification

Previous research has shown that models built from physical data such as the length, height, cross section area and side slope of the channel closely describes the relevant dynamics in the channel.²⁹ This information is used together with system identification methods to find a model that accurately describes the flow in the channel. As the system is partially known, a grey box modelling technique is appropriate to use.

4.2.1 St Venants Equations

A model that closely describes the dynamics of the irrigation channel can be obtained using two of the basic laws of physics, the conservation of mass and momentum. These two laws are used in the St Venant equations, which are nonlinear partial differential equations and represent a mass

²⁸ Ooi (2006)

²⁹ Weyer (2001)

and momentum balance along the length of each pool.³⁰ The first of the St Venant equations (11) is the continuity equation

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \quad (19)$$

and the second one is the momentum equation

$$\frac{\partial Q}{\partial t} + \left(\frac{gA}{B} - \frac{Q^2}{A^2} \right) \frac{\partial A}{\partial x} + \frac{2Q}{A} \frac{\partial Q}{\partial x} + gA(S_f - \bar{S}) = 0 \quad (20)$$

where A is the cross sectional area of the channel, B is the width of the water surface, $g = 9.81\text{m/s}^2$ is the gravity, \bar{S} is the bottom slope, S_f is the slope friction, t is the time, Q is the flow (discharge) and x is the distance along the channel.

Comparisons of the St. Venant equations against measured data have shown that they are capable of capturing the relevant dynamics of an irrigation channel for control purposes³¹. However, within the context of control design for setpoint tracking and load disturbance rejection, the value of a St Venant equation based model is limited because of its complexity, from both the perspective of system identification and closed loop analysis³². In the case when there is no operational data available or when the data fail to provide sufficient information about the system the St Venant equations can be used to obtain models of irrigation channels using a mix of both physical modelling and system identification techniques. This way, the first data set is obtained using the St Venant equations and then a simple mathematical model is estimated using system identification techniques based on the simulated data.³³

4.3 Model structure selection

Two model structures were introduced in the system identification chapter, the ARX model structure, and the OE model structure. In the case of modelling an irrigation channel reach, which has the interesting dynamics for control purposes, in the low to medium frequency area (changes due to dynamic properties are slow), the OE model is more appropriate since it gives a better

³⁰ Ooi, Weyer (2003)

³¹ Ooi, Weyer (2007)

³² Li (2006)

³³ Ooi (2003)

description of the relevant frequency properties.³⁴ Previous research has also shown that OE models can accurately describe the flow in irrigation channels³⁵.

The hypothesis is that due to the Boisdale tunnel a first order model will not be able to describe the flow accurately and therefore higher order models need to be tested. However, since it is advisable to try simple models first, the working process started with developing a first order model which was tested and thereafter also second and third order models were compared to the first order model to see if the achieved results were better.

4.4 Derivation of model structure for system identification

Previous work has resulted in several different model structures for an irrigation channel. The models are derived, from St Venant equation, by considering a simplified mass balance, which describe the relevant dynamics well.³⁶

$$\frac{\partial V}{\partial t} = Q_{in}(t) - Q_{out}(t) \quad (21)$$

In words, this equation says that the change in volume (V) in a pool is equal to the flow into the pool ($Q_{in}(t)$) minus the flow out ($Q_{out}(t)$). A common approximation³⁷ of the flow over an overshoot gate is

$$Q(t) = ch^{3/2}(t) \quad (22)$$

where Q (m^3/s) is the flow, h (m) is the head over the gate and c ($m^{2/3}/s$) is a proportionality constant which incorporates the geometric dimensions of the gate and the discharge coefficient and is in this case an unknown parameter.

A model of each pool can be identified using locally measured information. Assuming that the volume of water in a pool is proportional to the downstream water level, the following model for pool i is obtained

$$\dot{y}_{i+1}(t) = c_{in} h_i^{3/2}(t - \tau) - c_{out} h_{i+1}^{3/2}(t) - d_i(t) \quad (23)$$

³⁴ Ljung (1999)

³⁵ Weyer (2001)

³⁶ ibid

³⁷ Weyer (2001)

The equation means that the change in water level \dot{y} depends on the flow into the channel $c_{in}h_i^{3/2}$ minus the flow out $c_{out}h_{i+1}^{3/2}$ and disturbances d_i . τ represents the time delay associated with the time it takes for the water to travel the length of the pool and $d_i(t)$ models water offtake disturbances to farms and side channels and is the total water offtake from pool i including evaporation and seepage. In the case when the offtakes are not measurable or if they take very little water, they are included in the disturbance and are not used in the model. The offtakes should however be incorporated in the model if there are available measurements. Assuming that there is an offtake in the pool, which can be measured, equation (23) can be rewritten as

$$\dot{y}_{i+1}(t) = c_{in}h_i^{3/2}(t - \tau_1) - c_{offtake}h_{offtake}^{3/2}(t - \tau_2) - c_{out}h_{i+1}^{3/2}(t) \quad (24)$$

Another relationship that needs to be used in the model is that head over gate equals the upstream water level minus the gate position, $h_i = y_i - p_i$. Using this, the following first order model structure is obtained

$$\dot{y}_{i+1}(t) = c_{in}h_i^{3/2}(t - \tau_1) - c_{offtake}h_{offtake}^{3/2}(t - \tau_2) - c_{out}(y_{i+1}(t) - p_{i+1}(t))^{3/2} \quad (25)$$

When modelling water flow in a channel it is extremely important to base the model on appropriate flow conditions³⁸. As mentioned before, the gates can operate in free flow or submerged flow. In both data sets used, the gate position was below the downstream water level, which indicates submerged flow (figure 12). Since the flow conditions change in submerged flow, a correction factor needs to be added to the flow equation (14). The following factor³⁹

$$\left(1 - \left(\frac{h_d(t)}{h_u(t)}\right)^{3/2}\right)^{0.385} \quad (26)$$

is used to obtain

$$Q(t) = ch_u^{3/2}(t) \left(1 - \left(\frac{h_d(t)}{h_u(t)}\right)^{3/2}\right)^{0.385} \quad (27)$$

³⁸ Cunge, Holly, Verwey (1980)

³⁹ Eurén, Weyer (2006)

where Q is the flow, h_u is the upstream head over gate and h_d is the downstream head over gate. Note that gates with submerged flow require more information (the immediate downstream water level) than gates in free flow. Equation (18) is equal to the regular discharge equation (14) when $h_d = 0$, and to simplify the presentation h_d and h_u are redefined to

$$h_d(t) = \begin{cases} h_d(t) & \text{if } h_d(t) > 0, \\ 0 & \text{otherwise} \end{cases} \quad h_u(t) = \begin{cases} h_u(t) & \text{if } h_u(t) > 0, \\ 0 & \text{otherwise} \end{cases}$$

such that equation (18) can be used only. For simplicity h is sometimes used without subscript which means that the upstream head over gate is referred to.

Adding the correction factor for the submerged flow equation (25) becomes

$$\begin{aligned} \dot{y}_{i+1}(t) = & c_{in} h_i^{3/2}(t - \tau_1) \left(1 - \left(\frac{h_{i,d}(t - \tau_1)}{h_{i,u}(t - \tau_1)} \right)^{3/2} \right)^{0.385} - c_{offtake} h_{offtake}^{3/2}(t - \tau_2) \left(1 - \left(\frac{h_{offtake,d}(t - \tau_2)}{h_{offtake,u}(t - \tau_2)} \right)^{3/2} \right)^{0.385} \\ & - c_{out} (y_{i+1}(t) - p_{i+1}(t))^{3/2} \left(1 - \left(\frac{h_{i+1,d}(t)}{h_{i+1,u}(t)} \right)^{3/2} \right)^{0.385} \end{aligned} \quad (28)$$

4.5 Predictors

As stated above, the OE model gives a better description of the irrigation channels. Using an Euler approximation for the derivative

$$\dot{y}(t) = \frac{y(t) - y(t-1)}{T} \quad (29)$$

and a samplings interval T of 1 min the first order OE predictor can be derived from equation (28).

$$\begin{aligned} \hat{y}_{i+1}(t+1) = & \hat{y}_{i+1}(t) + c_{in} h_i^{3/2}(t - \tau_1) \left(1 - \left(\frac{h_{i,d}(t - \tau_1)}{h_{i,u}(t - \tau_1)} \right)^{3/2} \right)^{0.385} - c_{offtake} h_{offtake}^{3/2}(t - \tau_2) \\ & \left(1 - \left(\frac{h_{offtake,d}(t - \tau_2)}{h_{offtake,u}(t - \tau_2)} \right)^{3/2} \right)^{0.385} - c_{out} (\hat{y}_{i+1}(t) - p_{i+1}(t))^{3/2} \left(1 - \left(\frac{h_{i+1,d}(t)}{\hat{h}_{i+1,u}(t)} \right)^{3/2} \right)^{0.385} \end{aligned} \quad (30)$$

As can be seen in the OE predictor, the predicted water level $\hat{y}_{i+1}(t)$ at time t is used to predict the water level at time $t+1$ and since the upstream head over gate is calculated from the predicted water level $\hat{h}_{i+1,u}(t)$ is also predicted.

In order to obtain a second order model the simple mass balance presented in equation (21) is extended to

$$\ddot{y}(t) + a\dot{y}(t) = Q_{in}(t) - Q_{out}(t) \quad (31)$$

Using an Euler approximation for the second derivative is

$$\ddot{y}(t) = \frac{y(t+1) - 2y(t) + y(t-1)}{T} \quad (32)$$

The left hand side of equation (32) can be rewritten as

$$\ddot{y}(t) + a\dot{y}(t) = y(t+1) - 2y(t) + y(t-1) - ay(t) - ay(t-1) \quad (33)$$

and using the flow equations

$$Q_{in}(t) = c_{in} h_i^{3/2}(t - \tau_1) \quad (34)$$

and

$$Q_{out}(t) = c_{offtake} h_{offtake}^{3/2}(t - \tau_2) + c_{out} (y_{i+1}(t) - p_{i+1}(t))^{3/2} \quad (35)$$

the following model is derived

$$y(t+1) - 2y(t) + y(t-1) - ay(t) - ay(t-1) = c_{in} h_i^{3/2}(t - \tau_1 - 1) + b_1 c_{in} h_i^{3/2}(t - \tau_1) - c_{offtake} h_{offtake}^{3/2}(t - \tau_2 - 1) - b_2 c_{offtake} h_{offtake}^{3/2}(t - \tau_2) - c_{out} (y_{i+1}(t-1) - p_{i+1}(t-1))^{3/2} - c_{out} (y_{i+1}(t) - p_{i+1}(t))^{3/2} \quad (36)$$

Rearranging equation (36) and multiplying the constants the second order predictor without the correction factor for submerged flow is found to be

$$y(t+1) = c_1 h_i^{3/2}(t - \tau_1) + c_2 h_i^{3/2}(t - \tau_1 - 1) - c_3 h_{offtake}^{3/2}(t - \tau_2) - c_4 h_{offtake}^{3/2}(t - \tau_2 - 1) - c_5 (y_{i+1}(t) - p_{i+1}(t))^{3/2} - c_6 (y_{i+1}(t-1) - p_{i+1}(t-1))^{3/2} + (2-a)y(t) + (a-1)y(t-1) \quad (37)$$

where c_1, c_2 are associated with the flow over gate 877A, c_3, c_4 with the flow over gate 880 and c_5, c_6 with the flow over gate 919. The correction factor for the submerged flow is finally added to arrive at the second order predictor

$$y(t+1) = c_1 h_i^{3/2}(t - \tau_1) \left(1 - \left(\frac{h_{i,d}(t - \tau_1)}{h_{i,u}(t - \tau_1)} \right)^{3/2} \right)^{0.385} + c_2 h_i^{3/2}(t - \tau_1 - 1) \left(1 - \left(\frac{h_{i,d}(t - \tau_1 - 1)}{h_{i,u}(t - \tau_1 - 1)} \right)^{3/2} \right)^{0.385} - c_3 h_{offtake}^{3/2}(t - \tau_2) \left(1 - \left(\frac{h_{offtake,d}(t - \tau_2)}{h_{offtake,u}(t - \tau_2)} \right)^{3/2} \right)^{0.385} - c_4 h_{offtake}^{3/2}(t - \tau_2 - 1) \left(1 - \left(\frac{h_{offtake,d}(t - \tau_2 - 1)}{h_{offtake,u}(t - \tau_2 - 1)} \right)^{3/2} \right)^{0.385} - c_5 (\hat{y}_{i+1}(t) - p_{i+1}(t))^{3/2} \left(1 - \left(\frac{h_{i+1,d}(t)}{\hat{h}_{i+1,u}(t)} \right)^{3/2} \right)^{0.385} - c_6 (\hat{y}_{i+1}(t-1) - p_{i+1}(t-1))^{3/2} \left(1 - \left(\frac{h_{i+1,d}(t-1)}{\hat{h}_{i+1,u}(t-1)} \right)^{3/2} \right)^{0.385} + (2-a)\hat{y}(t) + (a-1)\hat{y}(t-1) \quad (38)$$

The third order model was found in a similar way using

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_2 y(t) = Q_{in}(t) - Q_{out}(t) \quad (39)$$

The Euler approximation of the third derivative gave

$$\ddot{y}(t) = y(t+1) - 3y(t) + 3y(t-1) - y(t-2) \quad (40)$$

and the third order predictor is

$$\begin{aligned}
y(t+1) = & c_1 h_i^{3/2}(t-\tau_1) \left(1 - \left(\frac{h_{i,d}(t-\tau_1)}{h_{i,u}(t-\tau_1)} \right)^{3/2} \right)^{0.385} + c_2 h_i^{3/2}(t-\tau_1-1) \left(1 - \left(\frac{h_{i,d}(t-\tau_1-1)}{h_{i,u}(t-\tau_1-1)} \right)^{3/2} \right)^{0.385} \\
& + c_3 h_i^{3/2}(t-\tau_1-2) \left(1 - \left(\frac{h_{i,d}(t-\tau_1-2)}{h_{i,u}(t-\tau_1-2)} \right)^{3/2} \right)^{0.385} - c_4 h_{\text{offtake}}^{3/2}(t-\tau_2) \left(1 - \left(\frac{h_{\text{offtake},d}(t-\tau_2)}{h_{\text{offtake},u}(t-\tau_2)} \right)^{3/2} \right)^{0.385} \\
& - c_5 h_{\text{offtake}}^{3/2}(t-\tau_2-1) \left(1 - \left(\frac{h_{\text{offtake},d}(t-\tau_2-1)}{h_{\text{offtake},u}(t-\tau_2-1)} \right)^{3/2} \right)^{0.385} - c_6 h_{\text{offtake}}^{3/2}(t-\tau_2-2) \left(1 - \left(\frac{h_{\text{offtake},d}(t-\tau_2-2)}{h_{\text{offtake},u}(t-\tau_2-2)} \right)^{3/2} \right)^{0.385} \\
& - c_7 (\hat{y}_{i+1}(t) - p_{i+1}(t))^{3/2} \left(1 - \left(\frac{h_{i+1,d}(t)}{\hat{h}_{i+1,u}(t)} \right)^{3/2} \right)^{0.385} - c_8 (\hat{y}_{i+1}(t-1) - p_{i+1}(t-1))^{3/2} \left(1 - \left(\frac{h_{i+1,d}(t-1)}{\hat{h}_{i+1,u}(t-1)} \right)^{3/2} \right)^{0.385} \\
& - c_9 (\hat{y}_{i+1}(t-2) - p_{i+1}(t-2))^{3/2} \left(1 - \left(\frac{h_{i+1,d}(t-2)}{\hat{h}_{i+1,u}(t-2)} \right)^{3/2} \right)^{0.385} \\
& + \hat{y}(t) + (1-a_1)(\hat{y}(t) - 2\hat{y}(t-1) + \hat{y}(t-2)) + (1-a_2)(\hat{y}(t) - \hat{y}(t-1))
\end{aligned} \tag{41}$$

c_1, c_2, c_3 are associated with the flow over gate 877A, c_4, c_5, c_6 with the flow over gate 880 and c_7, c_8, c_9 with the flow over gate 919.

4.6 Experimental data

Previous work has shown that good models can be obtained using system identification techniques based on collected data from systems operating in open loop. To perform an open loop experiment in the channel will however often cause deviations from normal operating conditions and are therefore rarely allowed. This leaves the alternative of performing the experiments in closed loop, causing only minor disruptions to the normal operations in the channel.⁴⁰ It is important to be aware of the limitations of the system. In theory, the system does not have any limitations, but in reality the irrigation channel has limitations such as maximum water level in the channel.

Data from two experiments performed in open loop and one experiment in closed loop were available for studying. The open loop data had been collected with the non-parametric method step test. In a step test the reference input is changed from one constant value to another constant value at one time instance. The purpose of the step test was primarily to find estimates of the time delays, time constants and the static gain. The data sets available from open loop conditions come from experiments carried out prior to this thesis work and were collected at two different times,

⁴⁰ Ooi (2003)

the 28th to the 29th of September 2006 and the 2nd to the 3rd of October 2006. Both experiments gave around 30 hours of data but had different flow conditions. The flows in the channel during the experiment in September were in the range of 30 to 100 ML/day, where as the experiment in October had approximately 115 to 160 ML/day. The offtakes and flows to the secondary channel were small (between 5 and 30 ML/day) compared to the flow in the main channel. The closed loop data was collected in November during 45 hours from the 28th to the 29th of November 2007 with flows varying between 60 and 190 ML/day.

The steps were performed when the system was in steady state. The gate to the offtake, 880, was held as constant as possible, though moved a couple of centimetres during the experiments. When an overshot gate position is lowered, the flow over the gate increases. This makes the upstream water level decrease, and if the downstream gate positions are constant, the down stream water level increases. The water level upstream a gate will increase when a gate shuts, which will decrease the downstream water level. This relationship is more complicated with a further gate at the offtake moving, which also affects the water levels.

Since the experiments are performed during normal operations, that is, the channel is in use, the inputs were not allowed to be changed as much as would be wished for. Data was collected when 877A performed with around 70% of its maximum flow, and 880 with around 40% of its maximum flow. The flow over gate 919 was very close to its maximum flow. All three gates were not installed at the same time and therefore the maximum flows do not match. The maximum flow of 880 + 919 should be less than the maximum flow of 877A, but does not need to be equal.

4.6.1 Step test analysis

It is practice to inspect the data before using them. An experienced engineer should have a good idea of the relationships between the length of the pool and its time constant, time delay and wave period and should be able to determine the quality of the data.

The raw measurements of the variables of interest were collected with a non-uniform sampling interval. Matlab was used as a tool to examine the data and naturally all three data sets available were used. The raw data were first plotted and examined for outliers and then interpolated to get a uniform sampling of 1 minute. There were no outliers and virtually no difference in the plots of the raw data and the interpolated data, which was expected since the raw data was collected more frequently than every minute and from here on only the interpolated data will be used. Since there are three gates at 877A and two gates at 919 the positions of the gates at each site were controlled to make sure that they had the same values and really worked in parallel. It was found that the positions of the parallel gates only differ a few millimetres and a simplification was made so that the gate positions, at 877A and 919 respectively, could be considered to be exactly the same.

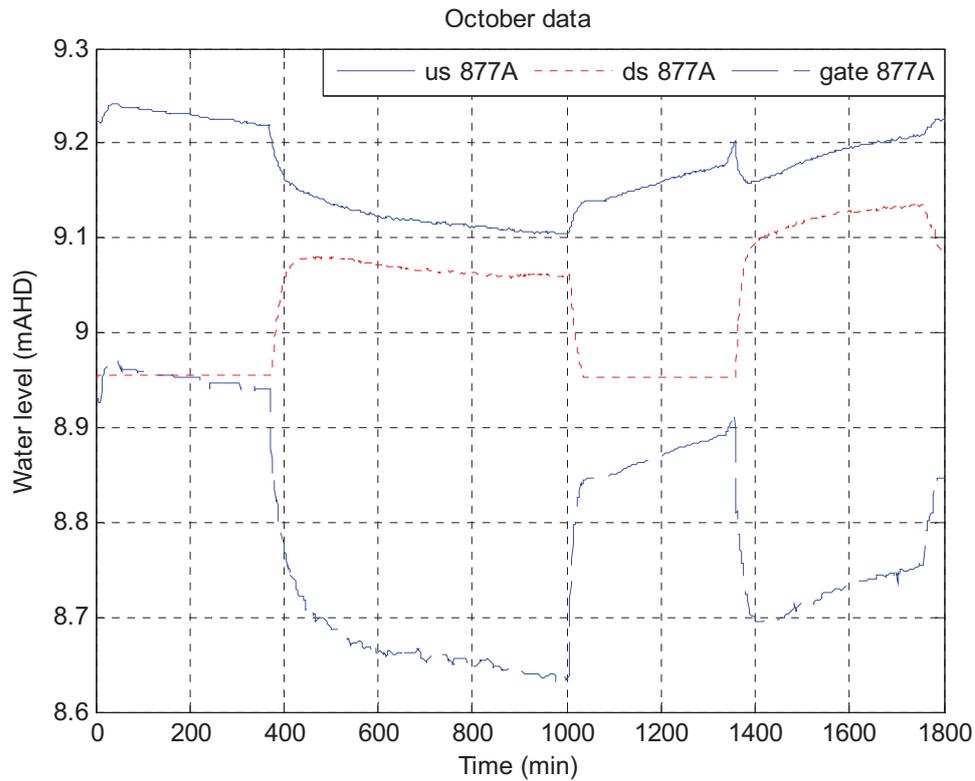


Figure 16: Plot of gate and upstream- and downstream water level from 877A in October

Figure 16 shows the step test performed at the experiment in October. The step was performed when the downstream water level was in steady state, and the upstream water level was almost in steady state, at $t=380$ min. In steady state the inflow and outflow of a pool is equal. During the test the gate at the downstream gates 919 was held at a fixed position and the gates at 877A opened up, the position was lowered, from 8.94 to 8.63 mAHD which caused a step in the water level upstream of gate 919 shown in figure 17. The gate at offtake 880 was moving slightly, but the change in water flow was so low, that it will not have any significant impact on the step results. When the gates at 877A opens the upstream water level lowers since more water flows over the gate and the downstream water level experiences a step, a significant rise in water level.

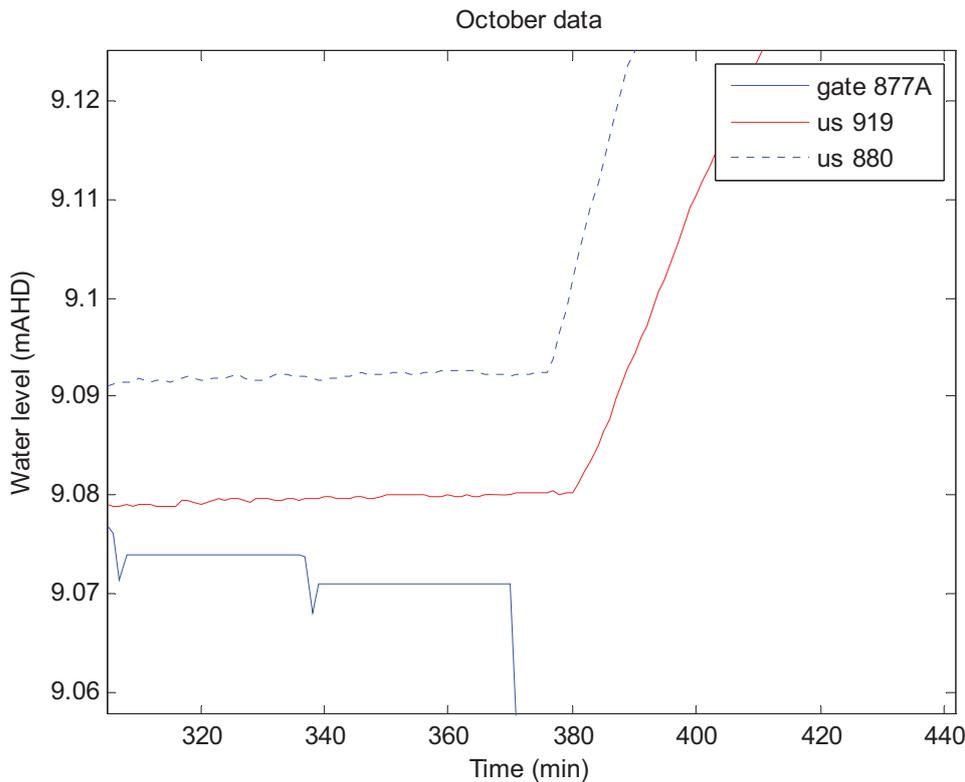


Figure 17: Magnified plot of gate 877A, upstream waterlevel of 880 and 919 at a step test performed in October

The step from the experiment in October is magnified to show how the time delays from gate 877A to gate 919, and gate 877A to gate 880 were found. The solid line is gate 877A which position is changed at $t=370$ min, creating a step. The dotted line is the upstream water level at gate 880, which starts the step at $t=376$ min. The dashed line represents the upstream water level at gate 919 and the water reaches this part of the channel at $t=380$ min, which indicates that the time delay from gate 877A to gate 919 is 10 min. The time delay from 877A to 880 was 6 minutes and from this information the time delay from 880 to 919 was calculated to be 4 min. No waves could be observed in the step test.

To calculate the time constant, the so called 5τ -method was used. This method is preferable to use when the experiments are fairly long. The time constant is calculated as the time it takes for the system to reach steady state divided by five. This is the preferable method to use when data is collected during a long time. From the step tests the time constant was found to be 124 min and 98 min from the September data and October data respectively.

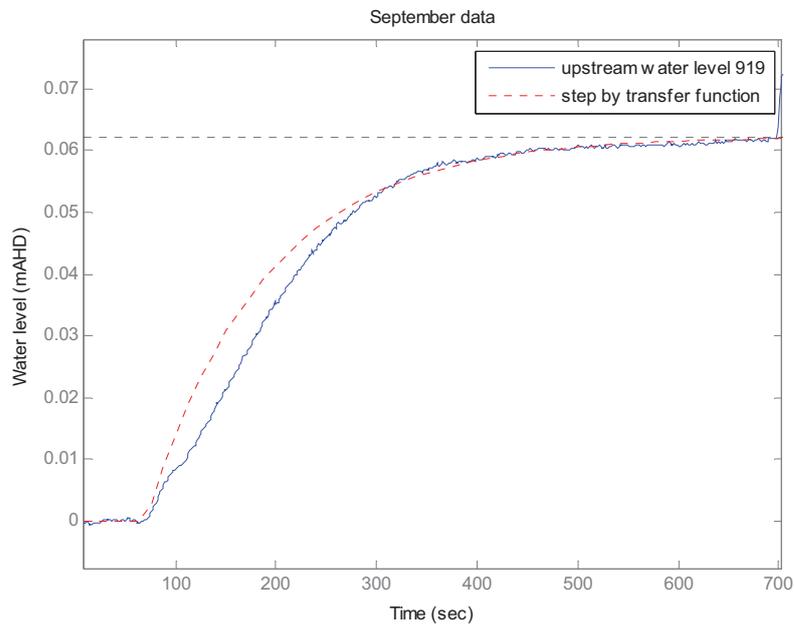


Figure 18: Plot of a step response showing upstream waterlevel of 919 together with a first order transfer function

Figure 18 shows a step response experiment using the data set collected in September. The measured water level (solid line) is plotted together with the simulated water level (dotted line) for a first order transfer function. The measured data resembles a first order system, but the tendency to an “S –shape” in the step suggest a second order system. The first order transfer function used here was achieved through testing different values until a step response resembling the step response form the experiment was achieved.

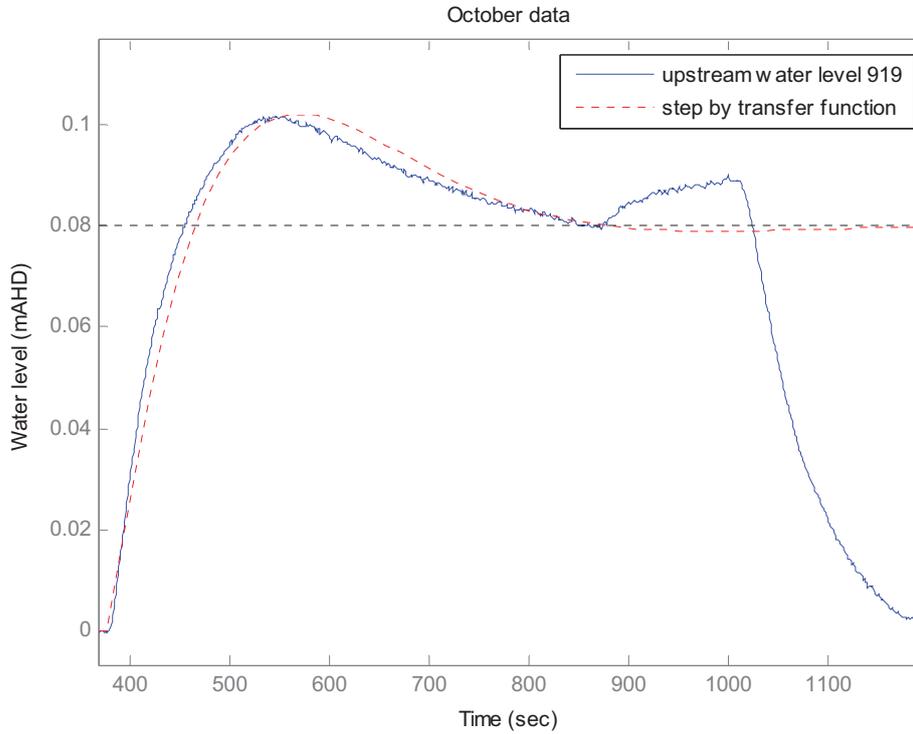


Figure 19: Plot of a step response showing upstream water level of 919 from the October data set together with a second order transfer function

Figure 19 shows a step response experiment using the data set from October (solid line). The measured water level is plotted together with the simulated water level (dotted line) for a second order transfer function. Examining the step response from October one can see that it resembles the step response of a second order system very closely. This indicates that a second order model is needed to describe the data. The transfer function giving this step was found through testing different values for the variables in a standard second order transfer function, equation (29).

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (42)$$

The damping ratio ζ was found to be 0.7 through comparing the step response from the measured data with plots of step responses of second order systems.

4.7 Parameter estimation

The data available from the open loop system (September and October data) and the data from the closed loop system (November data) were used to estimate the unknown parameters using the

parameter estimation method least squares. An iterative search routine in Matlab was developed for the parameter estimation to find the minimum prediction error of the non-linear model. The optimization algorithm Levenberg-Marquardt algorithm (LMA), which provides a numerical solution to the problem of minimizing a function, generally nonlinear, over a space of parameters of the function, was used. The LMA interpolates between the Gauss-Newton algorithm (GNA) and the method of gradient descent. It is more robust than the GNA which means that in many cases it finds a solution even if it starts very far off the final minimum. On the other hand, for well-behaved functions and reasonable starting parameters, the LMA tends to be a bit slower than the GNA.⁴¹

The minima that were found by the LMA algorithm differed a lot depending on what initial values were used, which indicated that local minima, and not global minima, were found. Due to this problem a further search routine was developed in Matlab which aim was to find reasonable starting values for the parameters in the θ vector to be used in the LMA algorithm. The search was performed with the data sets from the three different experiments resulting in three different θ vectors containing different optimal values on the unknown parameters.

The first order model, equation (19), has 3 unknown parameters, $c_{in}, c_{out}, c_{offlake}$ so in table 2, the vector has the parameters in the following order $\theta = [c_{877A}, c_{919}, c_{880}]^T$. The estimated parameters, the ones which gave the smallest value in the prediction error method with quadratic criterion, from the first order model are given below.

Table 2: Estimated parameters for the first order model using three different data sets

$$\theta_{Sep} = \begin{bmatrix} 0.0633 \\ -0.0397 \\ -0.0250 \end{bmatrix} \quad \theta_{Oct} = \begin{bmatrix} 0.0978 \\ -0.0655 \\ -0.0341 \end{bmatrix} \quad \theta_{Nov} = \begin{bmatrix} 0.0707 \\ -0.0540 \\ -0.0185 \end{bmatrix}$$

As expected the parameter estimate of c_{877A} is positive whereas c_{919} and c_{880} are negative, since c_{877A} is associated with inflow and c_{919} and c_{880} with outflow of the channel. As can be seen, the parameters in the θ vectors differ between the experiments. It is natural that the estimated parameters vary between the data sets and the reason for this can be that the data is not informative enough. More important is that the ratio between the parameters in the θ vector is fairly constant. Since the parameters are associated with the flow at each gate, it is only natural that the c values differ over the year, but the value of $c_{919} + c_{880}$ should approximately add up to the value of c_{877A} , and this is also true in all three cases.

⁴¹ www.wikipedia.org

The hypothesis that a higher order model is needed to capture the dynamics in the channel including the tunnel and the results from the step tests motivates that a second and third order models are examined to see if they can further improve the results. The same process as for the first order model was undertaken and the iterative methods in Matlab were extended to find parameter estimations for higher order nonlinear models. The second order model that was used, equation (27) has the following parameter vector $\theta = [c_1, c_2, c_3, c_4, c_5, c_6, a]^T$ which gave the following results for the second order model

Table 3: Estimated parameters for the second order model using three different data sets

$$\theta_{Sep} = \begin{bmatrix} 0.129 \\ 0.209 \\ -0.399 \\ 0.183 \\ -0.0612 \\ -0.0640 \\ 1.000 \end{bmatrix} \quad \theta_{Oct} = \begin{bmatrix} -0.669 \\ 0.779 \\ -0.675 \\ 0.600 \\ -0.438 \\ 0.403 \\ 0.994 \end{bmatrix} \quad \theta_{Nov} = \begin{bmatrix} 0.0642 \\ 0.0126 \\ -0.0605 \\ -0.0018 \\ -0.0359 \\ -0.0158 \\ 1.001 \end{bmatrix}$$

The results from the second order model shows parameters that differ more between the samples. c_1, c_2 are associated with the flow over gate 877A and should added together have a positive value indicating flow into the system. c_3, c_4 are associated with the flow over gate 880 and should added together have a negative value indicating flow out of the system and the same is true for c_5, c_6 which are associated with the flow over gate 919. All these are true for all three θ vectors. In all θ vectors a has received a value value very close to 1. This indicates that the term containing a in the equation (38) is not needed at all.

The third order predictor, equation (41), with the unknown parameter vector $\theta = [c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, a_1, a_2]^T$ gave the following results

Table 4: Estimated parameters for the third order model using three different data sets

$$\theta_{Sep} = \begin{bmatrix} 0.125 \\ 0.204 \\ 2.25e-14 \\ -0.397 \\ 0.186 \\ 0.00308 \\ -0.0627 \\ -0.0647 \\ -2.38e-4 \\ 1.000 \\ 0.999 \end{bmatrix} \quad \theta_{Oct} = \begin{bmatrix} 4.48e-7 \\ 6.09e-7 \\ 0.109 \\ -0.224 \\ -0.0274 \\ 0.178 \\ -0.168 \\ -0.0102 \\ 0.142 \\ 1.00 \\ 0.997 \end{bmatrix} \quad \theta_{Nov} = \begin{bmatrix} 6.87e-12 \\ 0.00400 \\ 0.0628 \\ -0.0840 \\ -0.0187 \\ 0.0514 \\ -0.105 \\ -0.00553 \\ 0.0939 \\ 1.000 \\ 0.999 \end{bmatrix}$$

c_1, c_2, c_3 are associated with the flow over gate 877A and should added together be positive, c_4, c_5, c_6 are associated with the flow over gate 880 and should added together be negative and c_7, c_8, c_9 are associated with the flow over gate 919 and should added together be negative. This is true also for the third order model. The values for a_1 and a_2 are all 1 or very close to 1, which also here indicates that a too high model order is being used.

4.8 Model validation

A validation method called cross validation was used because it is the best method to evaluate the different models average prediction error when confronted with new data sets⁴². The model that has the best capability of predicting a new data set should be considered the best model. Data set from September was collected under reasonably low flow and data set from October under reasonably high flow. The data set from November was collected under closed loop conditions using a controller. As mentioned when discussing the experiment design, using data sets collected under different flow conditions can give a better hint about what flow conditions the model is valid for, and hence serves as a quality measure of the model.

Since the three different parameter estimates have been used, three different models are obtained. With the cross validation method all data sets available can be used for model estimation and then cross validating these against each other, the model which best can predict the other two data sets can be found. Table 5 shows the squared averaged prediction errors from the first order model.

⁴² Glad, Ljung (2004)

Table 5: Squared average prediction error of the first order model using different data sets and different parameters

First order model	September parameters	October parameters	November parameters
September data		$3.1518 \cdot 10^{-5}$	$2.1687 \cdot 10^{-4}$
October data	$1.5915 \cdot 10^{-4}$		$2.1314 \cdot 10^{-4}$
November data	0.0010	$4.8982 \cdot 10^{-4}$	

The cross validation method gave similar results for all data sets, which is a good indication of the accuracy of the estimated parameters. Worth to be noted is that the parameters from November were able to predict the two other data sets equally well whereas the parameters from September and October did not predict the November data set well. Furthermore, the parameter vector θ from November is based on data collected during 45 hours whereas the parameters from September and October are based on data sets collected during approximately 30 hours. As mentioned before, the longer and the more varied the experiment, the more accurate will the estimated parameters be. Therefore, the choice was made to use the θ vector from November as an approximate of the unknown parameters in the control design process.

The simulated water level was calculated using a one step ahead predictor based on the estimates from the OE model. The OE model uses the input and the predicted water level from time t to predict the water level of time $t+1$. The predictor also used the correction factor for submerged water level.

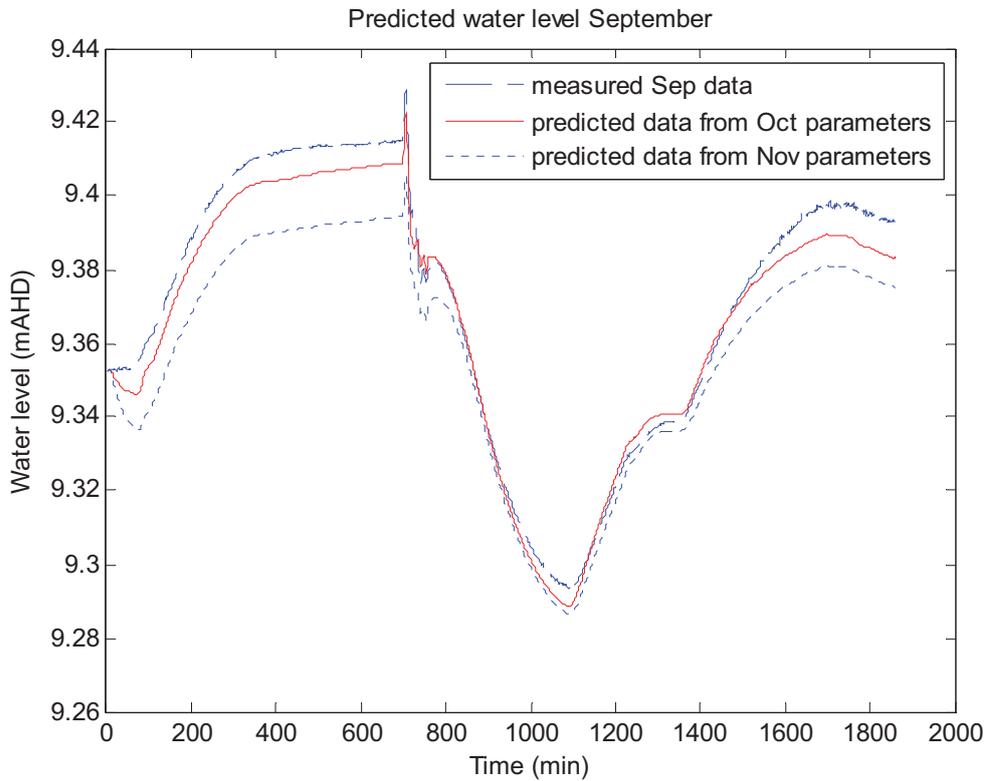


Figure 20: Plot of measured and simulated water levels using a first order predictor with correction factor for submerged flow and parameters from October and November data sets

Figure 20 shows how well the water level can be predicted using a first order nonlinear OE predictor with correction factor for submerged flow and the estimated parameters from October and November to predict the water level in September. The dashed line is the measured data from September plotted. The solid line is the predicted water level based on estimated parameters from October plotted and the dotted line is the predicted water level based on estimated parameters from November. The graph clearly shows that the parameters from October can better describe the September data than the parameters from November, predicting the water level only 1 cm away from the measured water level. The November parameters, although not as accurate as the October parameters in this case, can predict the water level with less than 2.5 cm error margin. Both parameter sets give very satisfying results.

Similar results are achieved using the September and October parameters on the November data set, and the September and November parameters on the October data set. Over all, the water level predictions are within 3 cm of the measured water level, and no prediction is further than 5 cm away at any time during the prediction period.

Table 6 gives the squared average prediction error of the second order model

Table 6: Squared average prediction error of the second order system using different data sets and different parameters

Second order model	September parameters	October parameters	November parameters
September data		$5.7806 \cdot 10^{-5}$	$2.1987 \cdot 10^{-4}$
October data	$1.6634 \cdot 10^{-4}$		$2.1059 \cdot 10^{-4}$
November data	$1.0525 \cdot 10^{-3}$	$6.305 \cdot 10^{-4}$	

Table 7 gives the squared average prediction error of the third order model

Table 7: Squared average prediction error of the third order model using different data sets and different parameters

Third order model	September parameters	October parameters	November parameters
September data		$3.7071 \cdot 10^{-5}$	$2.3116 \cdot 10^{-4}$
October data	$1.629 \cdot 10^{-4}$		$2.1055 \cdot 10^{-4}$
November data	$1.1481 \cdot 10^{-3}$	$5.3854 \cdot 10^{-4}$	

As the averaged square error indicates, the higher order models only improve the result slightly. In some instances the squared averaged prediction error even indicates that the prediction error grows with a higher order model. This is however not interpreted as that the higher model can describe the dynamics worse, but only that the parameters that were used were apparently not the ones that give the global minimum.

One of the basic principles in system identification is to keep the models as simple as possible and improved results need to be weighed against the greater complexity in the model. Therefore the improvements obtained with the second and third order models are not considered significant enough to imply a higher model order use.

The three data sets were also tested in a model without the correction factor for the submerged condition to make sure that this condition does improve the results. The average squared prediction errors showed clearly that the model with the correction factor for the submerged condition captured the dynamics in the pools better than the model without this condition.

The model of the system does not have to be an exact recreation of the true system. The aim of the model is for it to be accurate enough to give the user an idea of how the true system works with changing inputs and disturbances so that the controller can be designed and tested in a realistic way.

5 Controller design

This chapter describes how the controller was designed. A controller needs to be tested along the design process to make sure that the results are acceptable. It can be hard to make experiments on the true system and therefore the controller is often tested on a model of the system. Using frequency response techniques the work included testing many different parameters and simulating the results in Simulink. As in the previous chapter I find it more fruitful to discuss the results of the simulations straight away.

5.1 Controller objectives

The irrigation channel must be able to supply the farms along the channel with water. The main objectives of the controller can be summarised in three points.

1. **Offtake disturbance rejection**

The main disturbances are the scheduled and unscheduled offtakes (delivery of water) to farms and secondary channels. These load disturbances must be rejected.

2. **Water level setpoint tracking**

As the transport of water is powered by gravity, it is important to keep the water levels above certain supply levels in order to ensure the timely delivery of water to farms and secondary channels. Most offtakes are located at the downstream end of a pool and therefore it is important to keep the water level immediately upstream of the gates on setpoint which is the desired water level.

3. **Limited excitation of waves**

Gate movements can induce large waves in the pools which are undesired since it causes fluctuations in the flows at the offtake points and damage to the channel banks. Gate movements in the frequency range of the standing wave should therefore be avoided.

5.2 The system

The equation (23) for the first order model developed in chapter 4 can be rewritten as

$$\dot{y}_{919}(t) = c_{877A} h_{877A}^{3/2}(t-10) - c_{919} h_{919}^{3/2}(t) - d(t) \quad (43)$$

Substituting $u(t) = h^{3/2}$ we arrive at

$$\dot{y}_{919}(t) = c_{877A} u_{877A}(t-10) - c_{919} u_{919}(t) - d(t) \quad (44)$$

which Laplace transform is

$$sY_{919}(s) = c_{877A}u_{877A}e^{-10s} - c_{919}u_{919} - d \quad (45)$$

and ignoring the disturbance the system with input and output can be written as the transfer function

$$Y_{919}(s) = \frac{c_{877A}}{s}e^{-10s}U(s) \quad (46)$$

which means that an integrator with time delay model should be used for control design.

5.3 Feedback control

A decentralised controller in an open water channel has the same number of feedback controllers as the number of pools to be controlled and they should make sure that the water level is at setpoint. The most suitable decentralised controller configuration for demand driven irrigation channels is the distant downstream control. In this type of control the flow over a gate at the upstream end of each pool is calculated using information communicated from the corresponding downstream gate as can be seen in figure 21.

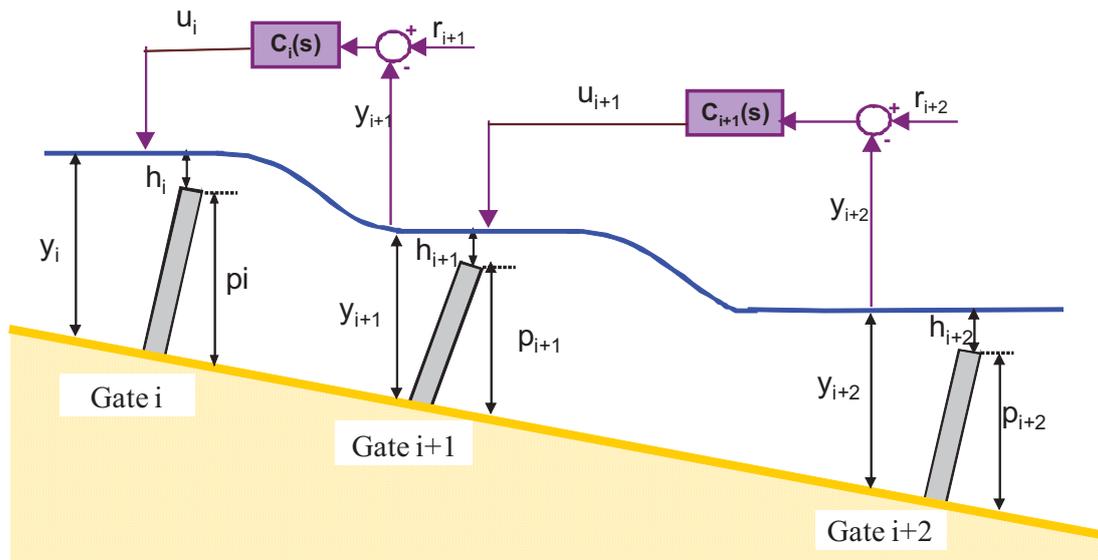


Figure 21: Diagram showing feedback control in the irrigation channel

Controller C_i is designed to control the water level of pool i , which is downstream of gate i , while C_{i+1} controls the water level in pool $i+1$. The control action at gate i is determined on the basis of the local water level setpoint r_i and the measured water level y_i , which is communicated from gate $i+1$. The controller C_i can transform the input information to a value u_i , from which the gate decides its position in order to get the right head over gate.

A decentralised control structure warrants consideration of how water level errors propagate through the system. Controlling the flow over the upstream gate results in the effect of any water level propagating to the upstream pools. That is, an offtake in a pool results in control action to increase the inflow to the pool for setpoint tracking. Since inflow to one pool means outflow from the upstream pool, a water level error occurs in the upstream pool, which triggers a new control action and so on along the string of pools. This is the reason for the importance of water level setpoint tracking.

With the control objectives in mind the control design started off with designing a feedback controller to see how well it could perform. A proportional controller will help the system stay on setpoint. As one of the main tasks of the controller is to reject load disturbances, integral action is

required. Adding a derivative control will increase the gain at a frequency where a low frequency is preferred, and therefore the derivative control is left out. The PI controller provides enough phase margin as it is without the derivative control.

Open water channels are not designed to cope with waves, so the wave dynamics should not be excited. Even though no waves could be observed from the step test, experience has shown that they are often present in the channel. Waves are also undesirable since they lead to fluctuations in the flow at offtake points onto farms and secondary channels. The waves are multiple of each other and the lowest frequency is usually the dominant one. In order to suppress the waves in the channel a lowpass filter is introduced so that the controller has a low gain at the wave frequency. This combination of proportional and integral action together with a low pass filter can fulfil the control objective; to keep water level at setpoint, reject load disturbances due to offtakes and suppress waves.

$$C(s) = \frac{K_p(1 + K_i s)}{K_i s(1 + T_f s)} \quad (47)$$

The feedback controller has proportional action + integral action + first order low pass filter.

5.3.1 Tuning of the PI controller

The controller was tuned using frequency response techniques⁴³. In the tuning, equation (30) was used as the process model with c_{877A} . The design process starts with ensuring stability and robustness for the local pool dynamics. The design specifications in this case were a phase margin of at least 45° and a gain margin of at least 6 dB. Since no waves were found in the channel the wave frequency had to be estimated as $2\pi(1/\text{wave period})$ where the wave period is approximately 3 times the time delay. Robust stability and performance is assured with a maximum controller gain at the wave frequency of -10 dB. The design criteria were met using Bode plots as representation of the system when performing frequency response methods. Subject to the design specifications the bandwidth needs to be as high as possible to give a fast response.

After the initial design specifications had been satisfied the tuning of the controller starts. This is a time consuming task where many different solutions are possible. They were all tested using simple simulations where a step is made and disturbance is added to see how the controller responds to setpoint changes and disturbances. Based on the simulation results the control parameters were fine-tuned.

It is important to get an understanding of how reliable the model is. This can be done by performing simulation experiments with the model. Figure 22 is a simulation test of an input step

⁴³ Franklin, Powell, Emanmi-Naeini (2006)

from 9.4 to 9.5 at $t=200$ min. and a disturbance of 20 ML/day at $t=1000$ min using the parameter values from the November experiment.

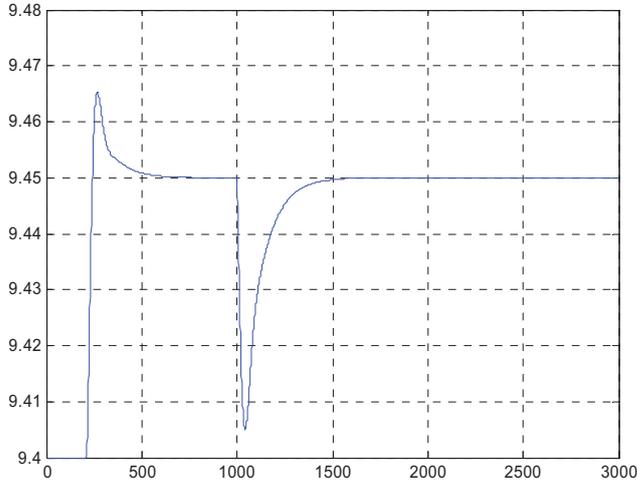


Figure 22: Simple simulation of feedback controller with step at $t=200$ and disturbance at $t=1000$

This simulation was compared to other simulations using different parameters, time delays, steps and disturbances. Most important is that the simulated water level should have fast responses and not have deviations from setpoint greater than 5 cm. This particular controller was selected based on these criteria and that it further satisfied the other design parameters required, based on gain margin, phase margin and the controller gain. The controller giving this step response had the parameter values $K_p=0.55$, $K_i=125$ and $T_f=5$, which gives the controller transfer function

$$C(s) = \frac{68.75s+0.55}{625s^2 + 125s} \quad (48)$$

The closed loop system gives satisfactory responses. The water levels recover smoothly from disturbances without large deviations from setpoint and without inducing excessive wave motions. Even greater variations, i.e. an input step from 9.4 to 9.5 and a disturbance of 50 ML/day was tested, and the results were considered good.

The controller should be tested under more realistic circumstances including saturating control signals, which should be included in the simulation experiments.⁴⁴ Therefore the simulation model was further developed to test the controller under the practical limitations in the system. In order to reduce the wear and tear of the gates, a 0.015 m dead-band on the gate position movements was used. That is, if the calculated new position of the gate is less than 0.015 m away

⁴⁴ Glad, Ljung (2000)

from the current position, the gate should not move. The simulation model was also extended to include the outflow of the water through both gate 880 and gate 919. Below are the simulation results using a feedback controller. The setpoint was changed at $t=500$ min from 9.4 to 9.45 mAHD. The system experiences a disturbance, an offtake at $t=1000$ min of 0.1 ML and the water level at gate 919 was stepped from 0.2 to 0.4 ML at $t=1500$ min.

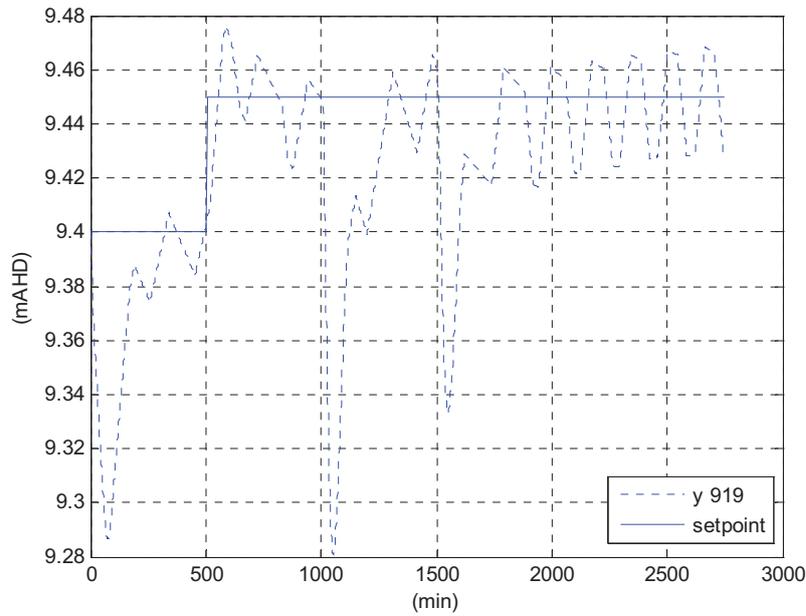


Figure 23: Simulation of feedback controller with reference signal and output signal with step at $t=500$ and step at $t=1000$ and $t=1500$

Figure 23 shows that due to the more realistic circumstances, including the disturbances in model, which makes the simulated water level oscillate more than the previous simulation result from the simplified model in figure 22. The oscillations are mainly due to the disturbances from the offtake which can be seen at $t=1000$ min.

5.4 Feedforward control

The result from the simulation with a feedback controller is not satisfying. The water level oscillates a lot and when the disturbance comes in the water level is 17 cm off setpoint. Since there are available measurements of the disturbance it should be incorporated into the model and used in a feedforward controller. The information about the disturbance in a downstream pool is then transmitted upstream faster and reduces the dynamic influence on the upstream pool. The model was further extended to incorporate also a feedforward loop and new simulations were performed.

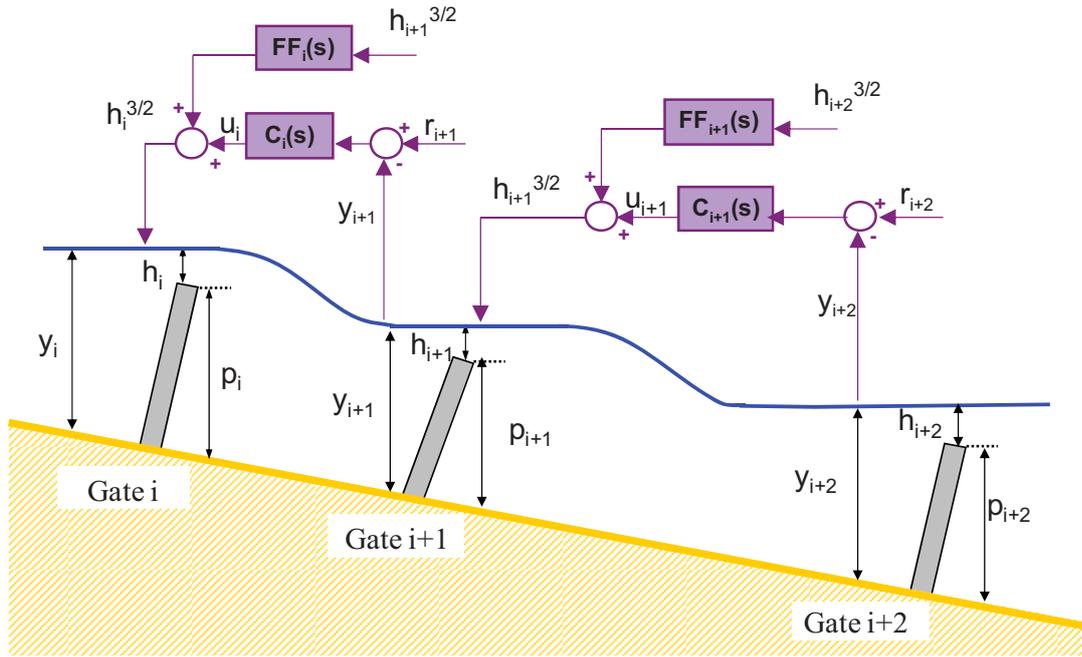


Figure 24: Block diagram showing feedforward control in the irrigation channel

The FF block in figure 24 includes the whole feedforward controller.

$$u_i(s) = C(s)(r_{i+1}(s) - y_{i+1}(s)) \quad (49)$$

$$h_i^{3/2}(s) = u_i(s) + K_{ff,i} \frac{c_{i+1}}{c_i} h_{i+1}^{3/2}(s) = u_i(s) + FF_i(s) \quad (50)$$

The gain K_{ff} is introduced to avoid large overshoots since the feedforward path cannot compensate for the time delay, and the gain is hence detuned from unity to 0.75 in order to avoid large overshoots in the water level responses. When the controller was designed with feedback and feedforward, both the disturbance from the offtake, and the flow out through gate 919, which in the system also can be considered as a disturbance, was used in the feedforward controller. The model was simulated using different water levels, disturbances and a variety of estimated parameters to make sure that the controller structure works in many different situations.

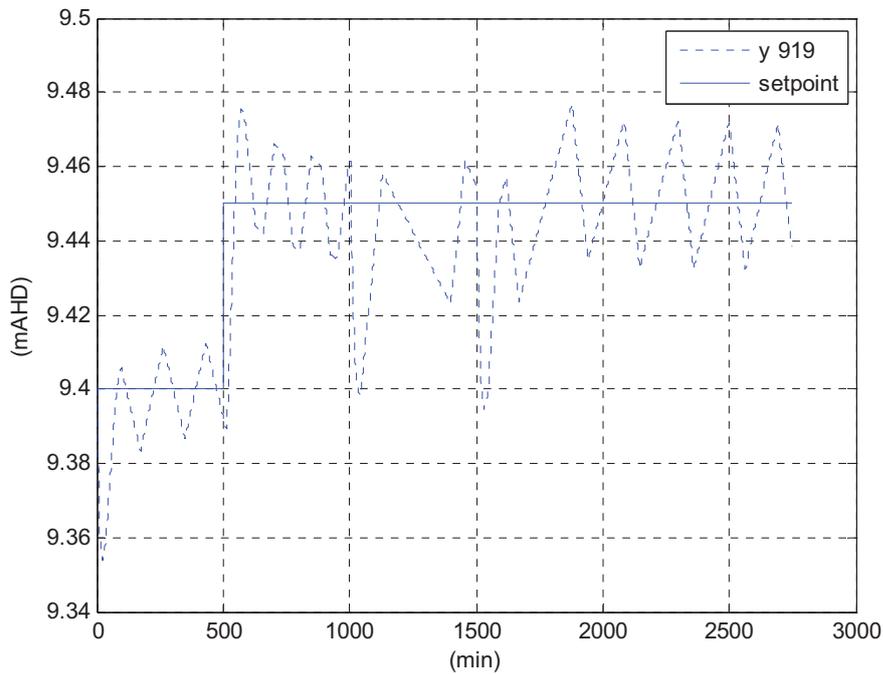


Figure 25: Simulation of feedback and feedforward controller with reference signal and output signal with step at $t=500$ and step at $t=1000$ and $t=1500$

The simulation result in figure 25 illustrates the improved controller performance with feedforward action. Both the disturbance from the offtake and the flow out, through gate 919 were included in the feedforward controller. Even with the disturbances the system now stays within 5 cm from setpoint. In this simulation the step and the disturbances were the same as in the feedback simulation to make it easier to see the improvements from figure 23.

6 Conclusion

Water has in the last few years become a hot topic in Australia, due to the increased demand, and the decreased availability. Everyone is affected by the water restrictions, but especially the farmers are experiencing an increasingly difficult situation when not enough water can be delivered to them. The huge amounts of water lost in the networks of irrigation channels can be reduced significantly with better control systems. Much research has been done to develop models and design controllers for irrigation channels. The results presented in this report are a further contribution to the knowledge of the systems.

The aim of this thesis was to find a model which describes the dynamics in the system and to design a controller which can regulate the flow in the irrigation channel. Starting with the modelling part, the main result in this report is that the pool from gate 877A to 919, including the Boisdale tunnel, can be accurately modelled with a first order non-linear equation. This result is somewhat surprising since the step tests from the open loop experiments indicated that at least a second order system was needed. The hypothesis that a higher order model was needed to model the flow from gate 877 to 919 was based on another infrastructure than the one that is currently in use in the channel and is therefore not valid anymore. The implementation of gate 877A close to the inlet of the Boisdale tunnel has clearly changed the former flow conditions.

The OE predictors of the first, second and third order all gave satisfactorily results but since the differences between them were small, there are more advantages to use a first order model. The first order OE predictor presented is able to capture the relevant dynamics in the irrigation channel and can predict the water level for 45 hours satisfactorily. The simulated water level was further improved when the correction factor for the submerged flow was added to the model. The model is also fairly robust since it can handle different parameter values and disturbances. The models are very simple and therefore ideally suited for control design.

Now focusing on the controller, by studying the simulation results presented it is clear that using only feedback to control the water level, the simulated water level oscillates around 3 cm off setpoint when there is no disturbance in the system. When the disturbance is added however, the system has obvious difficulties to keep the water level at setpoint, and it oscillates up to 17 cm off setpoint. When feedforward action is added to the controller, using both the disturbance caused by the offtake and the flow out over 919, the results are much improved in terms of response times and deviations from setpoints. The water level is now only oscillating around 2.5 cm off setpoint. Even at the instance when the disturbance is added, the maximum water level error is around 5 cm, which is considered very good since deviations of less than 10 cm are usually accepted. Note that these are simulations and the models only have access to the water level setpoint and the disturbances in the pools.

Due to the dead-band the water levels do not settle exactly on setpoints but this is a small problem in the matter. The greater oscillations are due to the disturbances in the system. As has been shown, the effect of the disturbance can be reduced if data from the offtakes are available.

This requires a gate to be implemented at the offtake and since gates are expensive, this is not always possible.

Rubicon Systems, which is responsible for the management of the gates considered in this report, are interested in using the results presented here to implement the controller in the channel and test its performance under “real” circumstances.

The results presented demonstrate that using system identification methods and control techniques on irrigation channels, the performance can improve significantly. This leads to large environmental benefits while still maintaining the same level of service to farmers. The economic benefits of better control of the irrigation channels can come from the lower operating costs, but most importantly, from less water wastage.

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