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Predicting future effects from nutrient abatement and climate change on phosphorus concentrations in Lake Bourget, France

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Abstract

Like many temperate European lakes, Lake Bourget (France) has suffered from eutrophication during the second half of the last century. Due to a remarkable policy restoration program, the lake has been recovering for the past 25 years after a massive decrease in total phosphorus (TP) loading. TP concentrations have decreased from about 100-120 to 20-25 $\mu\text{g/L}$. Additional efforts are, however, still required to obtain a perennially sustainable good ecological status and model parameterisation of fluxes can assist in predicting future outcomes, especially in the context of global warming. In this paper, a dynamic model (MeroLakeMab) was developed and tested with the purpose to reconstruct the loading history of Lake Bourget and to predict TP concentrations during scenarios of increased temperature, decreased water runoff and decreased P loading. Simulations suggested that the historical TP loading decrease may have been as extensive as 90%. Decreases in water discharge to Lake Bourget at magnitudes forecasted by the Intergovernmental Panel on Climate Change would not affect TP concentrations notably, but marked concentration changes could, however, occur if decreases in runoff would have a strong impact on the TP loading. Increasing temperature effects on yearly mean TP concentrations in the water column would be very small compared to effects from changes in the TP loading. Predictions such as these could be instrumental for future successful lake management.

List of Abbreviations. Additional explanatory material is provided in the article text.

A	area	MidA	mid-range accumulation sediments
Area	lake surface area	mix	mixing
C	concentration	MW	middle waters
CV	coefficient of variation	P	phosphorus
D_{crit}	critical depth	PF	particulate fraction
D_{m}	mean depth	Prec	precipitation
D_{max}	maximum depth	R	rate
DeepA	deep accumulation sediments	Q	water flux
diff	diffusion	sed	sedimentation
DR	dynamic ratio	SW	surface waters
DW	deep waters	TP	total phosphorus
ET	erosion and transport sediments	V	volume
F	phosphorus flux	Vd	volume development
IPCC	Intergovernmental Panel on Climate Change	Y	dimensionless moderator
Lat	latitude		

1. Introduction

Eutrophication of lakes has been a major environmental concern during recent decades. It has been attributed to increased anthropogenic phosphorus (P) loading to the lakes from domestic sewage, industries and runoff from fertilised agricultural areas, and is manifested as intensified algal (or cyanobacterial) blooms, decreased visibility in the water, oxygen depleted bottoms, and, in some cases, fish kills (Vollenweider, 1976; Sas, 1989; Chorus and Bartram, 1999; Jeppesen *et al.*, 2005). Recently, Schindler *et al.* (2008) presented results from a 37-year whole ecosystem experiment which confirmed a commonly held view that eutrophication of most lakes cannot be controlled by nitrogen inputs and that the focus in management must instead be on decreasing P inputs. It is now also well established that many eutrophicated lakes have recovered after significant decreases in P loading (Sas, 1989; Jeppesen *et al.*, 2005). One such example is Lake Bourget, the largest natural lake in France, located in the southern part of the Alps. Although a substantial re-oligotrophication has been observed in this lake after extensive measures to decrease the P loading (Vinçon-Leite *et al.*, 2002), eutrophication is still of great concern to lake managers and residents in the area. The water quality has been intensively surveyed; in the lake since 1981 and in the main tributaries since 2004 (Jacquet *et al.*, 2008). A recent sediment core study showed a particularly pronounced prevalence of large calcite crystals, diatoms, and total organic carbon deposited in the 1980s and 1990s, indicating high internal P loading during this period (Giguet-Covex *et al.*, in press). Complete mixing of the water column normally occurs in the winter but less frequently than once a year, which means that the lake can be viewed as meromictic (Jacquet *et al.*, 2005; Boehrer and Schultze, 2008).

Phosphorus abatement is often very costly, and one common way to estimate the future response of the water quality and ecosystem interactions of the lake before undertaking costly measures is to use predictive eutrophication models, although the uncertainty in many predictive models may discourage the use of them as a basis for action (Sas, 1989). Furthermore, the possible effects from climate change (i.e., global warming, changes in precipitation and changes in the length of drought periods) on various types of ecosystems have aroused widespread concern in the scientific community and elsewhere (IPCC, 2008). According to the A1B climate change scenario (200-300% increase in carbon dioxide emissions from 1990 to year 2100), runoff in the catchment area of Lake Bourget could decrease by about 10-20% until 2080-2099 (IPCC, 2007, 2008). Three climate scenarios (A2, A1B and B1; 0-600% increase in carbon dioxide emissions from 1990 to 2100) all indicate average temperature increases on the earth surface in south-eastern France by up to 4.5°C (IPCC, 2007).

Many previously published load-concentration models are unable to predict future P concentrations in meromictic lakes, for reasons which will be discussed in the following section. Therefore, MeroLakeMab, a dynamic model for deep alpine lakes, will be presented and tested against available data from Lake Bourget and the northern basin of Lake Lugano (Italy/Switzerland; Barbieri and Simona, 2001). Predictions will be compared to those made with LakeMab (a similar model suited for holomictic lakes; Håkanson and Bryhn, 2008) and the widely used Vollenweider model (Vollenweider, 1976).

All dynamic models contain many calibration constants, and calibration can be done more or less arbitrarily – different constant sets can give similar descriptions of the prevailing conditions, a phenomenon referred to as equifinality (Beven, 2006). The number of possible calibration constant sets decreases drastically if the same

model is tested for more than one system without re-calibration (Bryhn and Håkanson, 2007). Moreover, there are only five years (2004-2008) for which both TP concentration data and reliable TP loading data are available for Lake Bourget. By testing MeroLakeMab with a fixed set of algorithms and calibration constants against data from two lakes (Lake Bourget and Lake Lugano) instead of one, the model's predictions may be regarded as more reliable than if the calibration constant set would be valid for one lake only.

After output data from MeroLakeMab have been compared with field measurements (hereafter referred to as "empirical data") from monitoring programmes, the model will be used for reconstructing the loading history to Lake Bourget as well as for predicting total phosphorus (TP) concentrations in this lake during scenarios of increased temperature, decreased water runoff and decreased P loading. Such predictions should be essential for successful management of algal bloom intensity and water clarity in the lake.

2. Materials and methods

2.2. Study areas

MeroLakeMab was developed for conditions in the northern basin of Lake Lugano as described by Barbieri and Simona (2001) and for conditions in Lake Bourget 2004-2007 described in the present study. Additional data for Lake Bourget were collected during the calibration process in 2008. Basic data about the two lakes used for model development and testing are given in Table 1. TP loading, TP concentrations and water budget data for Lake Lugano were taken from Barbieri and Simona (2001). The total tributary TP loading and water inflow (Q) for Lake Bourget 2004-2008 (earlier loading data were either insufficient or based on estimations) are given in Table 2. The two main tributaries are the River Leysse and the River Sierroz (responsible for about 80% of the water inflow), while the three smaller tributaries are Tillet, Belle-Eau and Grand Canal (no data available) and also for a few days during the year the Savières channel (i.e. the only outflow from the lake contributing to <1% of water inflow; Jacquet *et al.*, 2008).

2.3. Models

There is a wide variety of eutrophication models available, the oldest ones being based on one or a few equations and yielding rather uncertain predictions (Sas, 1989). Although easy to use, these static models do not account for year-to-year variations in internal loading (nutrient transport from sediments to water through resuspension, bioturbation and diffusion) and may therefore comparatively poorly describe the recovery process of eutrophied lakes (Bryhn and Håkanson, 2007). One such well-known and widely used model that will be tested in this work is that of Vollenweider (1976), which calculates TP concentrations in lakes from two parameters; the TP concentration in the inflowing water and the water retention time.

Another model type is lake-specific and dynamic (i.e. time-dependent). Models in this category (of which there are several sub-types) are more complex than static models; they are based on differential equations to account for processes such as the internal loading, and they include lake-specific calibration constants. However, many of them (e.g., Vinçon-Leite *et al.*, 2002; Pers, 2005; Hu *et al.*, 2006) are driven by meteorological data, which can only be forecasted for a few days, while eutrophication management typically depends on which

environmental effects can be expected within the next year, decade, or decades. Furthermore, such models are calibrated and validated for site-specific conditions for which data must be available. The validity of their calibration constant matrix is thus rather limited and does not stretch towards TP loading conditions that differ greatly from those for which validation data are at hand. Therefore, lake-specific models may be good for descriptive purposes but because of their data requirements and their limited range of validity, they could substantially entail prediction of future changes and reconstruction loading histories with respect to phosphorus in lakes.

A third model type also produces dynamic predictions, but is, in addition, general, meaning that the model has been developed to contain a unique set of equations and calibration constants, and the output data have subsequently been tested against empirical data on TP concentrations from several lakes (Aldenberg *et al.*, 1995; Janse, 2005; Bryhn and Håkanson, 2007; Håkanson and Bryhn, 2008). Only easily accessible and measurable driving variables are used (e. g., lake area, mean and maximum depth and total P loading) and driving variable values should be site-specific for each lake. No meteorological data or other scarcely predictable data (on the annual or multiannual scale) are used as driving variables. There are at least two general TP models available; PCLake, whose output data have been tested against data from shallow European lakes (Janse, 2005), and LakeMab, which has been tested against TP data from deep and shallow lakes from northern Sweden to central Florida (Bryhn and Håkanson, 2007). However, neither of these models has been tested against data from meromictic lakes, which for long periods sustain three vertical water layers with limited exchange of water and P. PCLake simulates P masses and fluxes to and from a mixed water column (Janse, 2005) while LakeMab describes two communicating vertical water layers.

The model developed in this study is referred to as MeroLakeMab. It is based on LakeMab (Håkanson and Bryhn, 2008), but has been expanded to predict concentrations and fluxes in three different water layers instead of two layers. The time step (one month) and the length of predictions are equal in the two models. The depths at which the three water layers are typically separated, D_{crit1} and D_{crit2} (the critical depths, in m) are determined using two statistical models based on multi-lake surveys described by Håkanson *et al.* (2004) and Bryhn (in press). The models are defined as

$$D_{crit1} = 45.7 \cdot \sqrt{(\text{Area} \cdot 10^{-6})} / (21.4 + \sqrt{(\text{Area} \cdot 10^{-6})}) \quad (1)$$

$$D_{crit2} = 6.70 \cdot D_{max}^{1.23} / (21.4 + \sqrt{(\text{Area} \cdot 10^{-6})}) \quad (2)$$

where D_{max} is the maximum depth (in m) and Area is the lake surface area (in m²; Håkanson *et al.*, 2004; Bryhn, in press). It should be noted that the constants 45.7 and 21.4 in equation 1 and the constant 21.4 in equation 2 have the dimension [m] while the first constant in equation 2 (6.70) has the dimension [m^{0.77}]. D_{crit1} and D_{crit2} often show great areal and temporal variability in each lake (Håkanson *et al.*, 2004; Bryhn, in press), so equations 1 and 2 are particularly well-suited for nutrient modelling on monthly or yearly time scales using general model structures. The division of water layers and bottom areas according to vertical position in relation to the critical depths is illustrated in fig. 1. Shallow sediments above D_{crit1} are referred to as erosion- and

transport sediments and are highly exposed to wind and wave action so this is where sediment resuspension mainly occurs in a lake (Håkanson and Peters, 1995; Håkanson *et al.*, 2004). Below D_{crit1} , sediments are instead mainly exposed to processes such as particle accumulation, sediment burial and dissolved P diffusion. The higher the temperature, sedimentation or TP content, the more intensive the P diffusion (see the Appendix or Håkanson and Bryhn, 2008). ET, the dimensionless ratio of areal distribution of erosion- and transport sediments to the lake surface area, was calculated using an algorithm from Håkanson and Bryhn (2008):

$$ET = 1 - (D_{max} - D_{crit1}) / (D_{max} + D_{crit1} \cdot e^{(3 - Vd^{1.5})^{(0.5/Vd)}}) \quad (3)$$

where Vd is the volume development; a (dimensionless) mean depth (D_m) to D_{max} ratio which describes the shape of the hypsographic (cumulative area) curve (Håkanson and Peters, 1995). There are additional boundary conditions (see Appendix) that limit ET to remain between 0.15 and 0.95. Analogously with Equation 3, MidA, the algorithm for the dimensionless ratio of accumulation bottoms between D_{crit1} and D_{crit2} to the total lake area, was calculated as

$$MidA = 1 - (D_{max} - D_{crit2}) / (D_{max} + D_{crit2} \cdot e^{(3 - Vd^{1.5})^{(0.5/Vd)}}) - ET \quad (4)$$

Finally, DeepA, the dimensionless ratio of accumulation bottoms below D_{crit2} to total lake area is:

$$DeepA = 1 - ET - MidA \quad (5)$$

The volume of surface waters (SW; epilimnion), V_{SW} (in m^3), was calculated according to Håkanson and Bryhn (2008):

$$V_{SW} = Area \cdot (D_m - (MidA + DeepA) \cdot Vd \cdot (D_{max} - D_{crit1})/3) \quad (6)$$

Analogously, the volume of deep waters (DW; monimolimnion), V_{DW} (in m^3), is given by:

$$V_{DW} = Area \cdot A_{DW} \cdot Vd \cdot (D_{max} - D_{crit2})/3 \quad (7)$$

The middle water layer (MW; mixolimnion) volume, V_{MW} (in m^3), was then calculated as:

$$V_{MW} = Area \cdot D_m - V_{SW} - V_{DW} \quad (8)$$

Subscripts SW, MW, and DW used in equations 6-8 and elsewhere in this study refer to surface waters, middle waters and deep waters, respectively, as depicted in figure 1. The criteria for developing MeroLakeMab were the following:

- The structure should be as simple as possible and should to the greatest extent possible replicate LakeMab, whose detailed test results ($r^2=0.96$ for modelled TP concentrations against empirical data in

a cross-systems comparison) have been reported in Bryhn and Håkanson (2007) and Håkanson and Bryhn (2008).

- TP concentrations should be predicted as accurately as possible in both lakes.
- TP concentrations in the top 10 centimetres in all accumulation area sediments should not exceed 2 mg/g dw (Håkanson and Bryhn, 2008) and not be lower than 0.4 mg/g dw, which is the lower end of the empirical data range from Lake Bourget.
- The particulate fraction (PF); i.e., the ratio of particulate to total phosphorus, should be near 60% and within the range of 25-86% (Bryhn *et al.*, 2007).
- All constants and equations should be identical for both lakes.
- The obligatory driving variables (see below) should be lake-specific.

Using identical constants and equations for both lakes could decrease the possibility of allowing an arbitrary calibration and model structure. Different combinations of equations and constants can produce similar output data (Beven, 2006), so an important aim in generalised lake modelling is to single out the combination of equations and constants which gives the most credible description of the system (Bryhn and Håkanson, 2007). The eight driving variables used for MeroLakeMab and LakeMab were Q, TP load, Alt (altitude, m. a. s. l.), Area, D_{max} , D_m , Lat (latitude, °N) and Prec (mean annual precipitation, mm) while only Q, TP load, Area and D_m were used to make predictions with the Vollenweider model. Alt and Lat are used in LakeMab and MeroLakeMab to estimate water temperatures, the seasonal pattern in tributary discharge, the duration of the growing season, and sedimentation rates. Predictions were compared to yearly mean TP values and to coefficients of variation (CV=standard deviation/mean value). Monthly or weekly predictions were not tested due to the scarcity of empirical data. Empirical CVs were calculated where data availability allowed this, but for TP data where standard deviations were not available, CV=35% was used, which is a typical CV value for TP in lakes on both monthly and annual scales (Håkanson and Peters, 1995). The 95% confidence level was used in all statistical calculations.

3. Results

3.1. Model structure

Fig. 2A shows D_{crit1} in relation to the temperature gradient in Lake Bourget, 1984-2006. Temperatures were rarely below 10°C in surface waters above D_{crit1} (11 m) and rarely above 20°C below D_{crit1} . Fig. 2B displays mean TP concentrations at various depths during the period 1999-2008 and clearly shows that there was no apparent TP gradient at D_{crit1} while a strong TP gradient could be found at D_{crit2} (111 m). The presence of a thermocline near D_{crit1} in Fig. 2A and a chemocline near D_{crit2} in Fig. 2B indicates that the division lines according to critical depths (Eqs. 1 and 2) would be acceptable for TP modelling in this lake. The DW temperature 1984-2006 (see Fig. 2A) showed low variability along the depth gradient, at 5.6°C (standard deviation 0.4 °C, mean value error 0.13%), so this value was used in the model as the bottom water temperature. At 115 m, the mean value was 5.7°C (standard deviation 0.3°C) while at 140 m, the mean temperature was 5.6 °C (standard deviation 0.4°C).

The algorithm for net diffusive P transport from bottom waters to middle waters (F_{DWMW}) was as follows:

$$F_{DWMW} = \text{if } (C_{DW} \cdot (1-PF_{DW}) < C_{MW} \cdot (1-PF_{MW})) \text{ then } (0) \\ \text{else } (M_{DW} \cdot R_{\text{diff}, DWMW} \cdot (C_{DW} \cdot (1-PF_{DW}) - (TP_{MW} \cdot (1-PF_{MW})))) \quad (9)$$

where C denotes TP concentrations (in $\mu\text{g/l}$), and M_{DW} is the TP mass in DW (in g) while MW and DW in subscript indicate middle and bottom waters, respectively. $R_{\text{diff}, DWMW}$ was calibrated to 0.008 to match empirical TP concentrations in both lakes. Thus, the diffusive P transport was assumed to be a function of the difference in concentrations of dissolved P between bottom waters and middle waters. This difference was never lower than 0, so no algorithm for diffusive P transport in the other direction was needed. The particulate fractions (see 2.3.) PF_{SW} , PF_{MW} and PF_{DW} were calibrated to 0.6, 0.5 and 0.3, respectively, in order to sustain concentrations in the different water layers of the two lakes which corresponded to empirical measurements. The deep water retention time (T_{DW} ; in months) was calculated using F_{DWMW} as a proxy for water flux from deep waters to middle waters:

$$T_{DW} = M_{DW} / (F_{DWMW} + 0.001) \quad (10)$$

The term 0.001 was added in order to avoid division by zero in Eq. 10. The middle water retention time (T_{MW} , in months) was calculated from V_{MW} , the water retention time in the whole lake (T , in years) and the water flux from SW to MW (Q_{SWMW} , in m^3/month):

$$T_{MW} = \text{if } (V_{MW}/Q_{SWMW} > T \cdot 12/4) \text{ then } (T \cdot 12/4) \text{ elseif } (V_{MW}/Q_{SWMW} < 0.5) \text{ then } (0.5) \text{ else } \\ (V_{MW}/Q_{SWMW}) \quad (11)$$

T_{MW} thus had a lower boundary condition of 0.5 months from Håkanson and Bryhn (2008) and T converted to months and divided by four was added as an upper boundary condition for T_{MW} .

The sedimentation algorithms in Håkanson and Bryhn (2008) have not been adequately tested for meromictic lakes with high dynamic ratios which may be assumed to provide particularly calm conditions for sedimentation. To correctly describe the difference in sedimentation and TP retention between the two investigated lakes, a dimensionless moderator ($Y_{DR, \text{sed}}$) that affected the sedimentation rate was, in the present study, multiplied with all of the sedimentation flux algorithms ($R_{\text{sed}, SW}$, $R_{\text{sed}, MW}$, $R_{\text{sed}, DW}$; see Appendix):

$$Y_{DR, \text{sed}} = 5.8 \cdot \sqrt{DR} \quad (12)$$

where DR is the dynamic ratio (dimensionless; see Appendix). The rate regarding wind- and wave driven mixing of the two upper water layers (R_{mix}) in Håkanson and Bryhn (2008) yielded much too high modelled TP_{SW} values and much too low TP_{MW} values in the two meromictic lakes. Instead of letting R_{mix} depend on temperatures, as in Håkanson and Bryhn (2008), R_{mix} was calibrated to $0.00008 \text{ months}^{-1}$ which allowed a realistic distinction between TP concentrations in SW and MW in both lakes.

Except for all details mentioned thus far, the rest of the model structure from Håkanson and Bryhn (2008) was kept and the full list of equations is provided in the Appendix. It is worth noting that equations concerning deep waters (DW) in Håkanson and Bryhn (2008) correspond to equations for middle waters (MW) in this work, and A-sediments in Håkanson and Bryhn (2008) correspond to middle water accumulation (MidA) sediments. Algorithms regarding burial, sedimentation and diffusion to or from DW and deep accumulation (DeepA) sediments in this work have been constructed from the corresponding well-tested algorithms regarding the MW layer and MidA sediments in Håkanson and Bryhn (2008).

3.2. Model results versus empirical data

Fig 3 compares modelled and empirical TP concentrations in Lake Bourget 2004-2008 (a period for which detailed concentration data and detailed loading data were both available). For TP in the whole water column, a lake-typical CV value of 0.35 was used to construct uncertainty bands, while annual CVs for SW, MW and DW were calculated from empirical data. Predictions regarding the whole water column (fig. 3A) from all three models occurred within the uncertainty bands for all modelled years.

Predictions for the different water layers were less accurate compared to predictions concerning the whole water column. Mean annual TP in SW (fig. 3B) was predicted within the uncertainty bands for the whole period in MW and DW (fig. 3C and 3D) but beyond the uncertainty bands during some months in 2004 and 2007 in SW (fig 3B) with MeroLakeMab. Although the model was calibrated for 2004-2007 data and not for 2008 data, the model error for 2008 was not particularly conspicuous among the years displayed in fig. 3. TP concentrations increased in 2008 compared to 2007 (fig. 3) despite the substantial decrease in TP input (table 2). The variability of TP in MW and DW was particularly great this year (figs 3C and 3D), which could be an effect of extraordinary meteorological or hydrological events. Predictions from LakeMab were poorer than those from MeroLakeMab and predictions from the former of these models were encompassed by uncertainty bands for about two years in DW (fig. 3D), five years in MW (fig. 3C) but for none of the years in SW (fig. 3B). LakeMab predicted higher TP concentrations in SW than in DW, which was not supported by empirical data.

Fig. 4 shows a time-series of yearly TP concentrations in surface waters of Lake Lugano's northern basin 1984-1997. Modelled concentrations from MeroLakeMab all appeared within the uncertainty bands, while some of the output data from LakeMab in the earliest part of the period exceeded data from Barbieri and Simona (2001) plus one standard deviation. Most of the output data from this model were, however, found within the uncertainty bands.

TP concentrations from the whole water column and deep waters in 1996 are displayed in fig 5. For both of these water masses, MeroLakeMab apparently produced concentrations that were lower than two given literature values, although within the uncertainty bands. LakeMab and the Vollenweider model yielded substantial underestimates (predictions from both models are represented by the grey line in fig. 5A) of TP concentrations in the whole water column and LakeMab also underestimated the TP concentration in deep waters. A comparison between figs 4B and 3 gives at hand that LakeMab predicted that concentrations would be much lower in deep

waters than in surface waters in this lake as well as in Lake Bourget. Higher SW concentrations than DW concentrations could, however, not be empirically supported for either of the two investigated meromictic lakes.

Thus, the Vollenweider model provided poor predictions in Lake Lugano but good predictions in Lake Bourget. LakeMab yielded good TP predictions in the SW of Lake Lugano and good productions of whole-lake TP in Lake Bourget but provided an erroneous picture of the vertical TP distribution in both lakes. Figures 3 and 5 demonstrated that although LakeMab may be useful for predicting trophic state changes in holomictic lakes (Bryhn and Håkanson, 2007), the model produced unacceptable predictions in both meromictic lakes of this study. Consequently, MeroLakeMab was chosen as a better alternative than the two others for use in the modelling of scenarios and loading history.

3.3. Sensitivity analysis for MeroLakeMab

Fig. 6 displays results from a Monte Carlo simulation analysis on the sensitivity of TP concentration in the whole water column of Lake Bourget at a stationary loading regime (at recent levels) of 30 tons TP per year to variations in a) TP concentration in the inflow (C_{in} ; in $\mu\text{g/l}$), b) eq. 12 ($Y_{DR, sed}$; dimensionless), and c) PF_{DW} (dimensionless). A variable (Rand, dimensionless) consisting of a normalised random series was created in the software Stella. The 100 positive values which were used from this series were normally distributed, had a mean value of 1 and a coefficient of variation (CV) of 0.4. All Rand values were consecutively multiplied with C_{in} , and the simulated lake TP concentration after 10 years (at stationary conditions) was recorded in each of the 100 runs. This procedure was subsequently repeated with the same Rand values multiplied by $Y_{DR, sed}$, and thereafter with Rand values multiplied by PF_{DW} . All other variables except for the investigated variable were kept constant. The resulting CV in lake TP concentration was 0.32 from variations in TP concentration in the inflow, 0.48 from variations in $Y_{DR, sed}$, and 0.31 from variations in PF_{DW} . This means that predictions were particularly sensitive to the uncertainty of the dimensionless moderator (eq. 12) applied on the sedimentation algorithms.

The sensitivity analysis with respect to TP concentrations in tributaries also conveys that if we accept higher model uncertainties than 32% with respect to TP concentrations, we must accept an uncertainty in TP loading higher than 40%. As motivated in the Materials and Methods section, the accepted model error in annual lake TP was 35%, so the modelled uncertainty in TP concentrations in tributaries can be approximated as $0.35/0.32 \cdot 0.4 = 44\%$. This figure was subsequently used in estimates of the TP loading.

3.4. Loading history and future scenarios

While winter TP concentrations in Lake Bourget have been obtained at several occasions since 1981, details in the loading history of Lake Bourget prior to 2004 are poorly known since the rivers Leysse and Sierroz were not regularly analysed and/or instrumented. TP concentrations have been measured once or twice a month since November 1999, and a comparison between this time series and winter concentrations gave at hand that yearly mean TP concentrations were generally 7.5% higher than winter concentrations. Annual mean TP concentrations are displayed in Fig. 7 together with modelled TP concentrations from MeroLakeMab.

The TP loading required to reconstruct the historical TP time series in fig. 7 is given in table 3. One can note that the loading in 1981-1984 may have been more than 8 times as high as in 1995-1999, although uncertainties are substantial, as demonstrated in the sensitivity analysis above and specified in the table. It is also worth noting that the loading in 1995-1999 (table 3) may have been lower than in 2004-2007 (table 2), although TP concentrations were slightly higher during 1995-1999 than in 2004-2007 (fig. 7). The modelling showed that high TP concentrations in waters and sediments in 1995-1999 were to some extent remainders from the preceding decades with intensive TP loading, which was supported by the sediment core analysis in Giguet-Covex *et al.* (in press).

The three investigated scenario categories were a) decreases in TP loading, b) decreases in runoff and c) temperature increases. Fig. 8 shows modelling results from MeroLakeMab simulating conditions in Lake Bourget during three different scenarios, all of which began with 24 months of TP loading corresponding to 35 tons per year, yielding lake TP concentrations around 29 µg/l (in agreement with recently measured values). After 24 months, the annual loading was decreased to 30, 25 or 20 tons in each of the respective scenario. After 8-10 years, the lake TP concentration stabilised around 26, 22 and 20 µg/l, respectively. The magnitude of seasonal variations in fig. 8 were smaller than what the empirical data showed. Using the previously mentioned empirical conversion factor of 1.075 (yearly mean to winter mean), winter concentrations were calculated at approximately 24, 21 and 18 µg/l for the three different scenarios. It should be noted that these are estimates for average meteorological and hydrological conditions; during years with unforeseen changes in other external factors apart from the TP loading, such as particularly pronounced weather events, concentrations could differ considerably from these modelled values.

Results from the first set of climate change scenarios where present and decreased runoff was simulated are illustrated in fig. 9. The figure describes two typical annual cycles. Scenario 1 represented similar conditions to scenario 1 in fig. 8 after 7-8 years, with runoff rates at present levels and a TP loading of 30 tons/year, which yielded lake TP concentrations near 26 µg/l. Scenario 2 included a 10% runoff decrease with the same TP loading as scenario 1, and TP concentrations evidently differed very little between these scenarios (fig. 9). Scenario 3 described a 10% runoff decrease, but with the same TP concentration in tributaries as in scenario 1, i. e. a 10% decreased loading, which would result in TP concentrations near 24 µg/l. In scenario 4, runoff was decreased with 20% but the loading remained at 30 tons/year, and fig. 9 shows that lake TP concentrations from this scenario were very close to those from scenarios 1 and 2. Finally, scenario 5 was run to investigate effects from a 20% decrease in runoff, but with similar TP concentrations in the inflow as in scenario 1. This scenario also decreased TP loadings with 20% compared to scenario 1 and fig. 9 shows that lake TP concentrations decreased in this scenario to levels near 22 µg/l. This whole simulation exercise, illustrated in fig. 9, suggested that decreases in water input to Lake Bourget at magnitudes forecasted by the IPCC (2007) would not affect TP concentrations notably *per se*. Marked changes in TP concentrations could, however, occur if decreases in runoff would affect the TP loading to a significant extent.

The second set of climate change scenarios included changes in surface water temperature with up to 4.5°C and is illustrated in fig. 10. The TP loading was kept constant at 30 tons per year in all scenarios. One can note that

the effects on yearly mean TP concentrations in the water column would be very small compared to effects from changes in the TP loading (fig. 8). A 4.5°C temperature increase would increase the TP concentration with 4.6% (fig. 10). At the beginning of the simulations (data not shown), temperature increases spurred an increase in sedimentation, which was followed by an increase in sediment diffusion of a slightly higher magnitude, and these compensatory effects prevented any substantial impact on yearly mean TP concentrations in the water column. Simulations also suggested that the seasonal variation in concentrations would be less pronounced, as shown in fig. 10, although it is important to bear in mind in this context that MeroLakeMab has been tested against annual mean concentrations and not for shorter time perspectives.

4. Discussion and conclusions

This work presents the foundation for MeroLakeMab, a model which was constructed and tested against data from two deep meromictic-like lakes. With the use of easily measurable driving variables and calibration constants that are valid for two different lakes, it is possible to circumvent some of the problems associated with lake-specific dynamic models described in section 2. No weather data are necessary for driving the model, and the matrix of calibration constants in MeroLakeMab may be seen as more reliable than if it would only have been applied to one lake. However, extensive tests against a large number of other well-researched meromictic lakes would be required before MeroLakeMab can be described as having truly generic attributes.

The comparisons between modelled and empirical data revealed that LakeMab and the Vollenweider model were poorly suited to predict TP concentrations in the whole water column of Lake Lugano's northern basin, and also that LakeMab predicted several times higher TP concentrations in SW layers than in DW layers, which was contradicted by empirical data. Limitations in MeroLakeMab were instead that TP concentrations in SW and MW were sometimes predicted more than one standard deviation higher than empirical annual means in Lake Bourget, and that seasonal variations were not tested. However, these limitations may be considered as less discouraging than those of the other two models.

Apart from the introduction of two additional state variables (TP in deep waters and TP in deep A-sediments) for MeroLakeMab, there were some notable differences in sedimentation algorithms between this model and LakeMab, which need to be reflected on. These changes were necessary to sustain predicted TP concentrations in all of the three water layers close to empirical data from Lake Bourget and the northern basin of Lake Lugano. Fig. 6 showed that increases in the magnitude of the sedimentation algorithms may have a substantial impact on modelled TP concentrations in Lake Bourget, as increased sedimentation clears the waters from particulate P. Conversely, the low values for the particulate fraction in MW and DW (0.5 compared to 0.6 in SW), work in the opposite direction, i.e. low PF values decrease sedimentation and increase the TP concentration in the lake. We would, however, argue that it is reasonable to use lower PF values in the MW and DW layers than in SW for the following reasons: 1) SW is where the algal blooms occur, yielding extensive autochthonous particulate matter, 2) the sediment processes that primarily affect SW are erosion and resuspension of particulate matter 3) MW and DW are to a greater extent affected by diffusion of dissolved P from accumulation sediments. PF values as well as the dimensionless moderator for the sedimentation algorithms (Eq. 12) which is based on the dynamic ratio

could be investigated empirically in a cross-system study of a large number of meromictic lakes. Alternatively, the model assumptions motivated in this study, could be tested for a set of other well-researched meromictic lakes.

The sensitivity analysis (Fig. 6) also revealed that it is of great importance to have high accuracy in TP loading data. While this analysis certainly supports the conventional wisdom about the importance of TP loading accuracy for the success of lake eutrophication modelling (Håkanson, 1999), it also implies that reconstructions of loading histories must include substantial uncertainties. The estimates from Vinçon-Leite *et al.* (2002), a 50% loading decrease in the 1980s, are within the uncertainty bands of the results from this study, which instead indicated a 66% loading decrease between the early and late 1980s (Table 2). Likewise, the estimated eight-fold loading decrease between the early 1980s until the late 1990s should be interpreted using the uncertainties given in Table 3.

Figure 7 shows that it is indeed possible to decrease TP concentrations further, although this would require additional efforts to decrease the TP loading. The estimates in Figure 7 may provide useful information about what could be achieved after such efforts, as well as information about how long it could take until the full effect from abatement may be observed, i.e. as long as 8-10 years depending on the internal loading from sediments and the limited P exchange between water layers in meromictic lakes compared to holomictic lakes.

Blenckner *et al.* (2007) found a decreasing general effect on winter and spring TP concentrations in lakes from climate change but no effects on summer TP concentrations, although it was stressed in this meta-analysis that any effects may vary from lake to lake depending on catchment area characteristics and differences in sediment-water dynamics. The climate change scenarios in the present study (Figs 8 and 9) showed that the projected changes in runoff would only marginally affect TP concentrations in Lake Bourget. Temperature increases could have a slightly greater effect (a possible 4.6% increase in TP concentrations from a 4.5°C increase in surface water temperature), although the discussed model uncertainties of up to 35% suggest that our projections associated with temperature increases should be interpreted with care. Both sedimentation and diffusion are spurred by increasing temperatures (Håkanson and Bryhn, 2008) and decreasing TP concentrations as suggested by Blenckner *et al.* (2007) would probably be more likely in shallow lakes where ET bottom types dominate and diffusion is less important than in Lake Bourget. The greatest effects from climate change on TP concentrations in this lake would most probably occur if the TP loading would be significantly affected by changes in runoff or temperature.

Temperature increases may, however, have greater impact on biota than on TP concentrations. This could include improved competitive strength of temperature-dependent cyanobacteria as well as of daphnids and cyclopoid copepods (Blenckner *et al.*, 2007; Jöhnk *et al.*, 2008; Shatwell *et al.*, 2008). Effects on these organisms and others from climate change are nonetheless difficult to predict because of the complexity of foodweb interactions in lakes (De Stasio *et al.*, 1996) and it has been beyond the scope of this work to make such predictions for Lake Bourget.

To conclude, this study has presented a dynamic TP model which has been tested against empirical data from two meromictic-like lakes. This model was subsequently used to estimate that there has been an eight-fold decrease in TP loading to Lake Bourget between the early 1980s and the late 1990s. Climate change scenarios from this model did not support any anticipation of sharp increases in TP concentrations from temperature increases or changes in runoff. A temperature increase would intensify P diffusion from deep sediments but would also increase P sedimentation and these two fluxes may have compensatory effects on the TP concentration. While climate change may pose a rather limited threat to Lake Bourget, it remains obvious from our results that external changes in TP loading have had, and may have, a massive impact on the ecosystem and that Lake Bourget still needs to be protected against extensive TP inputs, e. g., against erosion of P-rich material in the catchment.

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Appendix: Model equations

The dynamic model MeroLakeMab for meromictic lakes is based on the generic dynamic model LakeMab for holomictic lakes (for a motivation of its variables and constants, see Håkanson and Bryhn, 2008) and the adjustments made have been described and motivated in the Materials and methods of the present article.

Differential equations are illustrated in fig. A1. Water temperatures were predicted from altitude and latitude according to Ottosson and Abrahamsson (1998). General nomenclature for constants and variables: C = TP concentrations, D= depths or thicknesses, DC= distribution coefficients, F = TP fluxes, M= TP masses, Q=water fluxes, R = rates, T = time, t = time step (months), V = volumes, v = velocities, Y = dimensionless moderators. All of the variables, constants and equations below are illustrated in the supplementary material (Figs. S2–S15).

Obligatory lake-specific driving variables

Area [lake surface area, m²]

A_{catchment} [catchment area, m²]

Alt [altitude, m. a. s. l.; used in sub-models for temperature and water discharge]

C_{in} [flux-weighted TP concentration in tributaries, µg/l]

D_{max} [maximum depth, m]

D_m [mean depth, m]

Lat [latitude, °N]

Prec [annual precipitation, mm]

Q [mean annual water discharge, m³/year]

Surface-water (SW) compartment

$$M_{SW}(t) = M_{SW}(t - dt) + (F_{in} + F_{MWSWx} + F_{ETSW} + F_{prec} - F_{out} - F_{SWMW} - F_{SWET} - F_{SWMWx}) \cdot dt$$

$$INIT M_{SW} = V_{SW} \cdot 0.001 \cdot 100$$

$$F_{in} = (Q/12) \cdot Y_Q \cdot C_{in} \cdot 0.001$$

$$F_{MWSWx} = \text{if } V_{SW}/V_{MW} < 1 \text{ then } M_{MW} \cdot R_{mix} \text{ else } M_{MW} \cdot R_{mix} \cdot V_{SW}/V_{MW}$$

$$F_{ETSW} = M_{ET} \cdot R_{res} \cdot (1 - V_d/3)$$

$$F_{prec} = C_{prec} \cdot Area \cdot Prec \cdot 0.001 \cdot 0.001/12$$

$$F_{out} = M_{SW} \cdot R_{out}$$

$$F_{SWMW} = (1 - ET) \cdot M_{SW} \cdot R_{sed, SW} \cdot PF_{SW} \cdot ((1 - DC_{res, SW}) + Y_{res} \cdot DC_{res, SW})$$

$$F_{SWET} = M_{SW} \cdot R_{sed, SW} \cdot PF_{SW} \cdot ET \cdot ((1 - DC_{res, SW}) + Y_{res} \cdot DC_{res, SW})$$

$$F_{SWMWx} = M_{SW} \cdot R_{mix}$$

Middle-water (MW) compartment

$$M_{MW}(t) = M_{MW}(t - dt) + (F_{SWMW} + F_{ETMW} + F_{SWMWx} + F_{MidAMW} + F_{DWMW} - F_{MWSWx} - F_{MWMidA} - F_{MWDW}) \cdot dt$$

$$INIT M_{MW} = V_{MW} \cdot 0.001 \cdot 140$$

$$F_{DWMW} = \text{if } (C_{DW} \cdot (1 - PF_{DW}) < C_{MW} \cdot (1 - PF_{MW})) \text{ then } 0 \text{ else } M_{DW} \cdot R_{diff, DWMW} \cdot (C_{DW} \cdot (1 - PF_{DW}) - (C_{MW} \cdot (1 - PF_{MW})))$$

$$F_{ETMW} = M_{ET} \cdot R_{res} \cdot (V_d/3)$$

$$F_{MidAMW} = M_{MidA} \cdot R_{diff, MidA}$$

$$F_{MWMidA} = Y_{TMW, sed} \cdot M_{MW} \cdot R_{sed, MW} \cdot PF_{MW} \cdot ((1 - DC_{res, MW}) + Y_{res} \cdot DC_{res, MW}) \cdot MidA / (MidA + DeepA)$$

$$F_{MWSWx} = \text{if } V_{SW}/V_{MW} < 1 \text{ then } M_{MW} \cdot R_{mix} \text{ else } M_{MW} \cdot R_{mix} \cdot V_{SW}/V_{MW}$$

$$F_{MWDW} = Y_{TMW, sed} \cdot M_{MW} \cdot R_{sed, MW} \cdot PF_{MW} \cdot ((1 - DC_{res, MW}) + Y_{res} \cdot DC_{res, MW}) \cdot DeepA / (MidA + DeepA)$$

$$F_{SWMW} = (1 - ET) \cdot (M_{SW} \cdot R_{sed, SW} \cdot PF_{SW} \cdot ((1 - DC_{res, SW}) + Y_{res} \cdot DC_{res, SW}))$$

$$F_{SWMWx} = M_{SW} \cdot R_{mix}$$

Deep-water (DW) compartment

$$M_{DW}(t) = M_{DW}(t - dt) + (F_{MWDW} + F_{DeepADW} - F_{DWMW} - F_{DWDDeepA}) \cdot dt$$

$$INIT M_{DW} = V_{DW} \cdot 0.001 \cdot 350$$

$$F_{MWDW} = Y_{TMW, sed} \cdot (M_{MW} \cdot R_{sed, MW} \cdot PF_{MW} \cdot (1 - DC_{res, MW}) + Y_{res} \cdot DC_{res, MW}) \cdot DeepA / (DeepA + MidA)$$

$$F_{DeepADW} = M_{DeepA} \cdot R_{diff, DeepA}$$

$$F_{DWMW} = \text{if } (C_{DW} \cdot (1 - PF_{DW}) < C_{MW} \cdot (1 - PF_{MW})) \text{ then } 0 \text{ else } M_{DW} \cdot R_{diff, DWMW} \cdot (C_{DW} \cdot (1 - PF_{DW}) - (C_{MW} \cdot (1 - PF_{MW})))$$

$$F_{DWDDeepA} = Y_{TDW, sed} \cdot (M_{DW} \cdot R_{sed, DW} \cdot PF_{DW} \cdot (1 - DC_{res, MW}) + Y_{res} \cdot DC_{res, MW}) \cdot DeepA / (DeepA + MidA)$$

ET-sediments

$$M_{ET}(t) = M_{ET}(t - dt) + (F_{SWET} - F_{ETMW} - F_{ETSW}) \cdot dt$$

$$INIT M_{ET} = 0.25 \cdot V_{ET} \cdot (1 - (W - 10)/100) \cdot BD \cdot 1.3 \cdot 1000$$

$$F_{ETMW} = M_{ET} \cdot R_{res} \cdot (V_d/3)$$

$$F_{ETSW} = M_{ET} \cdot R_{res} \cdot (1 - V_d/3)$$

$$F_{SWET} = M_{SW} \cdot R_{sed,SW} \cdot PF_{SW} \cdot ET \cdot ((1 - DC_{res,SW}) + Y_{res} \cdot DC_{res,SW})$$

A-sediments below MW (MidA)

$$M_{MidA}(t) = M_{MidA}(t - dt) + (F_{MWMidA} - F_{MidAMW} - F_{bur, MidA}) \cdot dt$$

$$INIT M_{MidA} = V_{MidA} \cdot (1 - W/100) \cdot BD \cdot 1000$$

$$F_{MWMidA} = Y_{TMW, sed} \cdot M_{MW} \cdot R_{sed,MW} \cdot (PF_{MW} \cdot (1 - DC_{res,MW}) + Y_{res} \cdot DC_{res,MW}) \cdot MidA / (MidA + DeepA)$$

$$F_{MidAMW} = M_{MidA} \cdot R_{diff, MidA}$$

$$F_{bur, MidA} = M_{MidA} \cdot (1.396/T_{MidA})$$

Deep A-sediments (DeepA)

$$M_{DeepA}(t) = M_{DeepA}(t - dt) + (F_{DWDDeepA} - F_{DeepADW} - F_{bur, DeepA}) \cdot dt$$

$$INIT M_{DeepA} = V_{DeepA} \cdot (1 - W/100) \cdot BD \cdot 1000$$

$$F_{DeepADW} = M_{DeepA} \cdot R_{diff, DeepA}$$

$$F_{bur, DeepA} = M_{DeepA} \cdot (1.396/T_{DeepA})$$

$$F_{DWDDeepA} = Y_{TDW, sed} \cdot M_{DW} \cdot R_{sed,DW} \cdot (PF_{DW} \cdot (1 - DC_{res,MW}) + Y_{res} \cdot DC_{res,MW}) \cdot A_{DW} / (MidA + A_{DW})$$

Additional model variables

$$Amp_{MidA} = C_{MidA} \cdot 25 \text{ [amplitude value for A-sediments above } D_{crit2}, \text{ dim. less]}$$

$$Amp_{DeepA} = C_{DeepA} \cdot 25 \text{ [amplitude value for deep A-sediments, dim. less]}$$

$$BD = 100 \cdot 2.6 / (100 + (W + IG \cdot (1 - W/100)) \cdot (2.6 - 1)) \text{ [bulk density, g/cm}^3\text{]}$$

$$BF_{DW} = \text{if Sedgram}_{DW} > 400 \text{ then } 1 \text{ else } (1 + D_{Ased})^{0.3} \text{ [bioturbation factor for deep A-sediments, dim. less]}$$

$$BF_{MW} = \text{if Sedgram}_{MW} > 400 \text{ then } 1 \text{ else } (1 + D_{Ased})^{0.3} \text{ [bioturbation factor for A-sediments above } D_{crit2}, \text{ dim. less]}$$

$$C_{DW} = 1000 \cdot M_{DW} / V_{DW} \text{ [TP concentration in DW, } \mu\text{g/l]}$$

$$C_{DeepA} = M_{DeepA} / ((V_{DeepA} \cdot 10^3 \cdot DeepA / (MidA + DeepA) \cdot BD \cdot (1 - W/100)) \text{ [TP-conc in deep A-sediments, mg/g dw]}$$

$$C_{lake} = 1000 \cdot (M_{SW} + M_{MW} + M_{DW}) / Vol \text{ [volume-weighted TP-conc in lake water, } \mu\text{g/l]}$$

$$C_{MidA} = M_{MidA} / ((V_{MidA} \cdot 10^3 \cdot MidA / (MidA + DeepA) \cdot BD \cdot (1 - W/100)) \text{ [TP-conc in deep A-sediments, mg/g dw]}$$

$$C_{MW} = 1000 \cdot M_{MW} / V_{MW} \text{ [TP-conc in MW, } \mu\text{g/l]}$$

$$C_{SW} = 1000 \cdot M_{SW} / V_{SW} \text{ [TP-conc in SW, } \mu\text{g/l]}$$

$$DC_{res, MW} = F_{ETMW} / (F_{ETMW} + F_{SWMW} + F_{MidAMW} + F_{DWMW} + F_{SWMWx}) \text{ [distribution coefficient for resuspended matter in MW, dim. less]}$$

$$DC_{res, SW} = F_{ETSW} / (F_{in} + F_{ETSW} + F_{prec} + F_{MWSWx}) \text{ [distribution coefficient for resuspended matter in SW, dim. less]}$$

$$D_{crit1} = 45.7 \cdot \sqrt{(Area \cdot 10^{-6})} / (21.4 + \sqrt{(Area \cdot 10^{-6})}) \text{ [critical depth 1 in m, see eq. 1]}$$

$$D_{crit2} = 6.70 \cdot D_{max}^{1.23} / (21.4 + \sqrt{(Area \cdot 10^{-6})}) \text{ [critical depth 2 in m, see eq. 2]}$$

$$D_{DeepA} = \text{if } (D_{max} - D_{crit2})/2 < 1 \text{ then } 1 \text{ else } (D_{max} - D_{crit2})/2 \text{ [depth of deep A-sediments, m]}$$

$$DeepA = 1 - ET - A_{MidA} \text{ [fraction of deep A-sediment areas, dim. less; eq. 5]}$$

$$D_{ET} = D_{crit1}/2 \text{ [depth of ET-sediments, m]}$$

$$D_{MidA} = \text{if } (D_{crit2} - D_{crit1})/2 < 1 \text{ then } 1 \text{ else } (D_{crit2} - D_{crit1})/2 \text{ [depth of A-sediments above } D_{crit2}, \text{ m]}$$

$$DR = \sqrt{(Area \cdot 10^{-6})} / D_m \text{ [dynamic ratio, dim. less]}$$

$$ET = \text{if } ET1 > 0.95 \text{ then } 0.95 \text{ elseif } ET1 < 0.15 \text{ then } 0.15 \text{ else } ET1 \text{ [fraction of ET-areas including boundary conditions, dim. less; see eq. 3]}$$

$$ET1 = 1 - (D_{max} - D_{crit1}) / (D_{max} + D_{crit1} \cdot e^{(3 - V_d \cdot 1.5)}^{(0.5/V_d)}) \text{ [fraction of ET-areas without boundary conditions, dim. less, see eq. 3]}$$

$$GS_{DeepA} = SMTH(\text{Sedgram}_{DeepA}, 60, \text{Sedgram}_{DeepA}) \text{ [gross sedimentation on deep A-areas, } \mu\text{g/cm}^2 \cdot \text{day]}$$

$$GS_{MidA} = SMTH(\text{Sedgram}_{MidA}, 60, \text{Sedgram}_{MidA}) \text{ [gross sedimentation on A-areas above } D_{crit2}, \mu\text{g/cm}^2 \cdot \text{day]}$$

$$IG = \text{if } W > 75 \text{ then } (1280 + (W - 75)^3)/207 \text{ else } W/11.9 \text{ [loss on ignition, \%]}$$

$$MidA = (1 - (D_{max} - D_{crit2}) / (D_{max} + D_{crit2} \cdot e^{(3 - V_d \cdot 1.5)}^{(0.5/V_d)})) - ET \text{ [fraction of A-sediments above } D_{crit2}, \text{ dim. less; eq. 4]}$$

$$MWT = [MW \text{ temperature, in } ^\circ\text{C; from temperature sub-model in Ottosson and Abrahamsson, 1998}]$$

$$Q_{SWMW} = F_{SWMWx} / (C_{SW} \cdot 0.001) \text{ [water flux from SW to MW, m}^3\text{/month]}$$

$$R_{diff, DeepA} = Y_{DR, diff} \cdot Y_{TDW, diff} \cdot R_{diff, default} \cdot Y_{sed, DW} \cdot (DWT/4) \cdot Y_{TPDeepA} \text{ [diffusion rate from deep A-sediments, 1/month]}$$

$$R_{diff, MidA} = Y_{DR, diff} \cdot Y_{TMW, diff} \cdot R_{diff, default} \cdot Y_{sed, MW} \cdot (MWT/4) \cdot Y_{TP, MidA} \text{ [diffusion rate from A-sediments above } D_{crit2}, \text{ 1/month]}$$

$R_{out} = 12 \cdot Q/V_{SW} \cdot Y_Q \cdot Y_{evap} \cdot Y_{prec}$ [outflow rate, 1/month]
 $R_{res} = 1/T_{ET}$ [resuspension rate; 1/month]
 $R_{sed, DW} = Y_{SPMDW} \cdot Y_{DR, sed} \cdot v/D_{DeepA}$
 $R_{sed, MW} = Y_{SPMMW} \cdot Y_{DR, sed} \cdot v/D_{MidA}$
 $R_{sed, SW} = Y_{SPMSW} \cdot Y_{DR, sed} \cdot v/D_{ET}$
 $Sedcm_{DeepA} = Sedgram_{DeepA} \cdot T_{dur} \cdot 10^{-6} \cdot (100/(100-W)) \cdot (1/BD)$ [mean annual deposition on deep A-sediments, cm/yr]
 $Sedcm_{MidA} = Sedgram_{MidA} \cdot T_{dur} \cdot 10^{-6} \cdot (100/(100-W)) \cdot (1/BD)$ [mean annual deposition on A-sediments above D_{crit2} , cm/yr]
 $Sedgram_{DeepA} = F_{DWDeepA} \cdot 10^5 / (30 \cdot 2 \cdot Area \cdot DeepA)$ [sedimentation on deep A-sediments, $\mu g / (cm^2 \cdot day)$]
 $Sedgram_{MidA} = F_{MWMidA} \cdot 10^5 / (30 \cdot 2 \cdot Area \cdot MidA)$ [sedimentation on A-sediments above D_{crit2} , $\mu g / (cm^2 \cdot day)$]
 $SPM = 10^{(1.56 \cdot \log(Clake) - 1.64)}$ [conc. of suspended particulate matter, SPM, mg/l]
 $SPM_{DW} = 10^{(1.56 \cdot \log(CDW) - 1.64)}$ [SPM-conc in DW, mg/l]
 $SPM_{MW} = 10^{(1.56 \cdot \log(CMW) - 1.64)}$ [SPM-conc in MW, mg/l]
 $SPM_{SW} = 10^{(1.56 \cdot \log(CSW) - 1.64)}$ [SPM-conc in SW, mg/l]
 $SWT = [SW \text{ temperature, in } ^\circ C; \text{ from temperature sub-model in Ottosson and Abrahamsson, 1998}]$
 $T = Vol/Q$ [theoretical water residence time, years]
 $T_{DeepA} = \text{if } 12 \cdot BF_{DW} \cdot D_{Ased} / Sedcm_{DW} > 3000 \text{ then } 3000 \text{ elseif } 12 \cdot BF_{DW} \cdot D_{Ased} / Sedcm_{DW} < 12 \text{ then } 12 \text{ else } 12 \cdot BF_{DW} \cdot D_{Ased} / Sedcm_{DW}$ [age of deep A-sediments, months]
 $T_{dur} = -0.058 \cdot Lat^2 + 0.549 \cdot Lat + 365$ [duration of growing season, days]
 $T_{DW} = M_{DW} / (F_{DWMW} + 0.001)$ [water residence time in DW, months; see eq. 10]
 $T_{ET} = \text{if } Y_{DR2} < 1 \text{ then } 1 \text{ else } Y_{DR2}$ [age of ET-sediments, months]
 $T_{MidA} = \text{if } 12 \cdot BF_{MW} \cdot D_{Ased} / Sedcm_{MW} > 3000 \text{ then } 3000 \text{ elseif } 12 \cdot BF_{MW} \cdot D_{Ased} / Sedcm_{MW} < 12 \text{ then } 12 \text{ else } 12 \cdot BF_{MW} \cdot D_{Ased} / Sedcm_{MW}$ [age of A-sediments above D_{crit2} , months]
 $T_{MW} = \text{if } V_{MW} / Q_{SWMW} > T \cdot 12 / 4 \text{ then } T \cdot 12 / 4 \text{ elseif } V_{MW} / Q_{SWMW} < 0.5 \text{ then } 0.5 \text{ else } V_{MW} / Q_{SWMW}$ [water residence time in MW, months; eq. 11]
 $T_{SW} = 12 \cdot V_{SW} / Q$ [surface water retention time, months]
 $v = v_{default} \cdot Y_{DR}$ [settling velocity of particles, m/month]
 $Vd = 3 \cdot D_m / D_{max}$ [form factor, dim. less]
 $V_{DeepA} = DeepA \cdot Area \cdot 0.01 \cdot D_{Ased} \cdot Vd / 3$ [volume of deep A-sediments, m^3]
 $V_{DW} = Area \cdot DeepA \cdot Vd \cdot (D_{max} - D_{crit2}) / 3$ [DW volume, m^3 ; eq. 7]
 $V_{ET} = ET \cdot Area \cdot 0.01 \cdot D_{Ased} \cdot 0.1 \cdot Vd / 3$ [volume of ET-sediments, m^3]
 $V_{MidA} = MidA \cdot Area \cdot 0.01 \cdot D_{Ased} \cdot Vd / 3$ [volume of A-sediments above D_{crit2} , m^3]
 $V_{MW} = (Area \cdot D_m - V_{SW} - V_{DW})$ [MW volume, m^3 ; eq. 8]
 $Vol = Area \cdot D_m$ [lake volume, m^3]
 $V_{SW} = Area \cdot (D_m - (MidA + DeepA) \cdot Vd \cdot (D_{max} - D_{crit1}) / 3)$ [SW volume, m^3 ; eq. 6]
 $W = \text{if } DR > 6 \text{ then } 65 \text{ elseif } DR > 0.5 \text{ then } 75 \text{ else if } DR > 0.045 \text{ then } 85 \text{ else } 95$ [water content in A-sediments, %]
 $Y_{DR} = \text{if } DR < 0.26 \text{ then } DR / 0.26 \text{ else } 0.26 / DR$ [dimensionless moderator with boundary conditions for settling velocity]
 $Y_{DR, diff} = \text{if } DR < 3.8 \text{ then } 1 \text{ else } 3.8 / DR$ [dim. less moderator for diffusion from A-sediments]
 $Y_{DR, sed} = 5.8 \cdot \sqrt{DR}$ [dim. less moderator for sedimentation; eq. 12]
 $Y_{DR2} = 12 \cdot DR / 0.26$ [dim. less moderator for T_{ET}]
 $Y_{evap} = \text{if } SWT < 9 \text{ then } 1 \text{ else } 1 - 0.4 \cdot (SWT / 9 - 1)$ [dim. less moderator for evaporation]
 $Y_{prec} = \text{if } Prec < 650 \text{ then } 1 + 1.8 \cdot (Prec / 650 - 1) \text{ else } 1 + 0.5 \cdot (Prec / 650 - 1)$ [dim. less moderator for the effect of precipitation on outflow from lake]
 $Y_Q = [seasonal \text{ moderator for } Q, \text{ from sub-model for water discharge in Håkanson and Bryhn, 2008}]$
 $Y_{res} = T_{ET} + 10$ [dim. moderator for resuspension]
 $Y_{sed, DeepA} = \text{if } GS_{DW} < 50 \text{ then } (2 - 1 \cdot (GS_{DW} / 50 - 1)) \text{ else } (2 + Amp_{DW} \cdot (GS_{DW} / 50 - 1))$ [dim. moderator for diffusion from deep A-sediments]
 $Y_{sed, MidA} = \text{if } GS_{MW} < 50 \text{ then } (2 - 1 \cdot (GS_{MW} / 50 - 1)) \text{ else } (2 + Amp_{MW} \cdot (GS_{MW} / 50 - 1))$ [dim. moderator for diffusion from A-sediments above D_{crit2}]
 $Y_{SPM, DW} = (1 + 0.75 \cdot (SPM_{DW} / 50 - 1))$ [dim. moderator for settling velocity in DW]
 $Y_{SPM, MW} = (1 + 0.75 \cdot (SPM_{MW} / 50 - 1))$ [dim. moderator for settling velocity in MW]
 $Y_{SPM, SW} = (1 + 0.75 \cdot (SPMSW / 50 - 1))$ [dim. moderator for settling velocity in MW]
 $Y_{TDW, sed} = \text{if } DR > 0.26 \text{ then } \sqrt{(T_{DW} \cdot 365 / 12)} \text{ else } \sqrt{(T_{DW} \cdot 365 / 12 \cdot DR / 0.26)}$ [dim. moderator for sedimentation in DW]
 $Y_{TDW, diff} = \text{if } Y_{TDW, diff1} < 1 \text{ then } 1 \text{ else } \sqrt{Y_{TDW, diff1}}$ [dim. moderator for diffusion from deep A-sediments]
 $Y_{TDW, diff1} = \text{if } T_{DW} \cdot 365 / 12 > 120 \cdot (0.26 / DR) \text{ then } 120 \cdot (0.26 / DR) \text{ else } T_{DW} \cdot 365 / 12$ [boundary conditions for $T_{DW, diff}$]

$Y_{TMW, sed} = \text{if } DR > 0.26 \text{ then } \sqrt{(T_{MW} \cdot 365/12)} \text{ else } \sqrt{(T_{MW} \cdot 365/12 \cdot DR/0.26)}$ [dim. moderator for sedimentation in DW]
 $Y_{TMW, diff} = \text{if } Y_{TMW, diff1} < 1 \text{ then } 1 \text{ else } \sqrt{Y_{TMW, diff1}}$ [dim. moderator for diffusion from A-sediments above D_{crit2}]
 $Y_{TMW, diff1} = \text{if } T_{MW} \cdot 365/12 > 120 \cdot (0.26/DR) \text{ then } 120 \cdot (0.26/DR) \text{ else } T_{MW} \cdot 365/12$ [boundary conditions for $T_{DW, diff}$]
 $Y_{TPDeepA} = \text{if } C_{DeepA} < 0.5 \text{ then } 0 \text{ else } C_{DeepA}(C_{DeepA}-0.5)/2$ [dim. moderator for diffusion from deep A-sediments]
 $Y_{TPMidA} = \text{if } C_{MidA} < 0.5 \text{ then } 0 \text{ else } C_{MidA}(C_{MidA}-0.5)/2$ [dim. moderator for diffusion from A-sediments above D_{crit2}]

Model constants

$C_{prec} = 5$ [typical TP-conc in precipitation, $\mu\text{g/l}$]
 $D_{Ased} = 10$ [typical depth of A-sediments in lakes, cm]
 $DWT = 5.6$ [typical DW temperature in Lake Bourget, $^{\circ}\text{C}$]
 $PF_{SW} = 0.6$ [particulate fraction in SW, dim. less]
 $PF_{MW} = 0.5$ [particulate fraction in MW, dim. less]
 $PF_{DW} = 0.3$ [particulate fraction in DW, dim. less]
 $R_{diff, default} = 2.5 \cdot 10^{-5}$ [default diffusion rate from A-sediments, 1/month]
 $R_{diff, DWMW} = 0.008$ [diffusion rate from DW to MW, 1/month]
 $R_{mix} = 0.00008$ [mixing rate, 1/month]
 $v_{default} = 6$ [default settling velocity of particles, m/month]

Tables

Table 1. Geographical and morphological information about the two lakes used for model development

Lake name	Bourget	Lugano (N. basin)
Latitude, longitude	45°44'N, 5°52'E	46°0'N, 9°0'E
Altitude (Alt)	231 masl	271 masl
Lake area (Area)	44.5 km ²	27.5 km ²
Mean depth (D _m)	80 m	171 m
Max depth (D _{max})	147 m	288 m
Drainage area	560 km ²	269.7 km ²
Water residence time	8.5 years	12.3 years

Table 2. Tributary inflow (Q) and TP load to Lake Bourget from 2004 to 2008. Minor tributaries contributed with a maximum of 6-7 tons TP per year while the remaining part was transported by Rivers Leysse and Sierroz.

Year	Q (10 ⁶ m ³)	TP load (tons)
2004	240	47-48
2005	220	27-28
2006	240	25
2007	380	33
2008	330	18

Table 3. Model estimates of annual TP loading to Lake Bourget using MeroLakeMab.

Time period	Loading (tons/year)
1981-1984	256±113
1985-1989	86±38
1990-1994	34±15
1995-1999	31±13
2000-2003	36±16

Fig. 1. The vertical distribution of bottom types, water layers and critical depths used in MeroLakeMab. Not to scale.

Fig. 2. Temperatures and total phosphorus concentrations at various depths in Lake Bourget. A. Temperatures 1984-2006. B. Mean total phosphorus concentrations during the period 1999-2008. Bars represent mean value errors. Dashed lines mark D_{crit1} and D_{crit2} .

Fig. 3. Modelled and empirical TP concentrations in Lake Bourget for the period 2004-2008. A: whole water column, B: surface waters, C: middle waters, D: deep waters. Dashed lines ($MV \pm SD$) denote mean annual values $\pm$ one standard deviation. Predictions from the MeroLakeMab, LakeMab and Vollenweider models are depicted with lines bearing the respective model name. The latter model is only suited for whole lake predictions and is therefore only included in (A).

Fig. 4. Modelled and empirical TP concentrations in surface waters of Lake Lugano (N. basin), 1984-1997. Dashed lines ($MV \pm SD$) denote mean annual values $\pm$ one standard deviation. Predictions from MeroLakeMab and LakeMab are depicted with lines bearing the respective model name.

Fig. 5. Modelled and empirical TP concentrations in the whole water column (A) and bottom waters (B) of Lake Lugano (N. basin) in 1996. Dashed lines ($MV \pm SD$) denote mean annual values $\pm$ one standard deviation. Predictions from MeroLakeMab and LakeMab are depicted with lines bearing the respective model name. Predictions from the Vollenweider model in (A) were very close to those from LakeMab and differences between output data from these two models cannot be visualised at the given scale.

Fig. 6. Sensitivity analysis of the effect on lake TP concentrations from variations in TP concentrations in the tributaries (TP_{in}), the five sedimentation algorithms (F_{sed}) and PF_{DW} in Lake Bourget.

Fig. 7. Modelled TP concentrations (thick line) in Lake Bourget (whole water column; values in $\mu\text{g/l}$) compared to empirical annual values (thin line) 1981-2008.

Fig. 8. Simulated decreases in TP loading in Lake Bourget. Scenarios: 1) A decrease in TP loading from 35 to 30 tons per year occurring in month 24 counted from the start of the simulation. Scenario 2: A decrease in TP loading from 35 to 25 tons per year occurring in month 24 counted from the start of the simulation. Scenario 3: A decrease in TP loading from 35 to 20 tons per year occurring in month 24 counted from the start of the simulation.

Fig. 9. Climate change scenarios describing long-term effects from changes in runoff on TP concentrations in Lake Bourget. Scenarios: 1) Runoff rates corresponding to present rates. 2) A 10% decrease in runoff rates but similar TP loading as in scenario 1. 3) A 10% decrease in runoff rates but similar TP concentrations in tributaries as in scenario 1. 4) Same as scenario 2 but with a 20% runoff decrease. 5) Same as scenario 3 but with a 20% runoff decrease.

Fig. 10. Four scenarios on long-term effects from surface temperature increases in Lake Bourget.

Supplement to the article “Predicting future effects from nutrient abatement and climate change on phosphorus concentrations in Lake Bourget, France” by Bryhn et al.

Introduction

This supplement describes the model MeroLakeMab and its 123 variables. MeroLakeMab is a dynamic (time-dependent) mass-balance model for total phosphorus (TP) in alpine meromictic lakes. This model type calculates time-dependent TP concentrations in lake water under stationary or changing temperature, water discharge and TP loading regimes. TP masses in different parts of the lake are represented by the state variables, which are defined by ordinary differential equations. Some general features of MeroLakeMab are described in table S1. Some of the equations have been motivated in the original article and the remaining parts have been motivated in Håkanson and Bryhn (2008).

The nomenclature of MeroLakeMab has been selected to make as much sense as possible to most readers; for instance, there are no Greek letters, depths are abbreviated D (and not Z, which is common in limnology) while fluxes are abbreviated F (and not Φ , which is used in some hydrodynamic lake models). The most common abbreviations in MeroLakeMab are given in table S2.

Table S3 lists all variables in alphabetical order, describes their meanings and units, and refers to the figure where the variable is put into context. Fig. S1 contains the legend of all subsequent figures. Driving variables are site-specific variables needed to run the model. State variables describe TP masses (always expressed in grams) in different parts of the lake. Flux variables describe TP fluxes (always expressed in grams per month) to and/or from the state variables. The “other variables and constants” constitute the majority of the model components; sub-models, factors, terms, dividends and divisors needed to calculate TP masses and fluxes.

The nine driving variables of the model are depicted in fig. S2. High accuracy of the data used to drive the model is decisive for model performance, and this particularly applies to accuracy in C_{in} . The driving variables in fig. S2 are relatively easily accessible from standard maps and monitoring programs. The time resolution for these variables (if applicable) is months.

MeroLakeMab has been developed for alpine meromictic lakes which sustain three distinct water layers for long periods. Consequently, MeroLakeMab contains three different state variables for each water layer, and three state variables for TP in sediments adjacent to each of the water layers. The six state variables are displayed and defined in fig. S3 alongside all flux variables to and/or from the state variables. The variable scheme in fig. S3 may be regarded as the “core” of the model; the differential equations are put into an interconnected context here.

Morphometric variables (figs S4 and S5)

Fig. S4 contains algorithms used for separating water and sediment volumes into the six state variable categories (SW, MW, DW, ET-areas, MidA-areas and DeepA-areas) used in fig. S3. The critical depths (D_{crit1} and D_{crit2}) are used for predicting the depths separating water layers and sediment categories (see equations 1 and 2 in the

article). ET, MidA and DeepA (see equations 3-5 in the article) are used for calculating the relative extent of bottom areas according to each of the three sediment categories. Two other morphometric variables described in fig. S4 are the dynamic ratio (DR) and the volume development (Vd, which is in some studies referred to as the form factor). The DR is a $\sqrt{\text{Area}}$ to D_m ratio while the Vd is a D_m to $D_{\max}$ ratio and both of these ratios are important determinants of many of the flux variables in the model. The variables expressing the average depth of the three bottom area categories, D_{ET} , D_{MidA} and D_{DeepA} , are depicted in fig. S5. These three variables are calculated from one or two of the variables $D_{\max}$, D_{crit1} , and D_{crit2} . Fig. S5 also describes the variables V_{SW} , V_{MW} , and V_{DW} , which denote the volumes of the three water layers. The equations for V_{SW} , V_{MW} , and V_{DW} correspond to equations 6-8 in the article. The total lake volume, V , is calculated as a product of the mean depth and the lake area (fig. S5).

Outflow, atmospheric deposition and inflow from the catchment (fig. S6)

Fig. S6 describes the determinants of TP loading from the catchment (F_{in}), TP outflow (F_{out}) and TP loading via precipitation (F_{prec}), three fluxes which directly affect the TP content in SW. F_{in} is controlled by two of the driving variables (C_{in} and Q) as well as a seasonal moderator (Y_Q) which captures the typical seasonal discharge pattern from the catchment based on two other driving variables, Lat and Alt. F_{out} is governed by Q , Y_Q , M_{SW} and V_{SW} , as well as a dimensionless evaporation moderator (Y_{evap}) which is in turn determined by the monthly typical surface water temperature (SWT) predicted from Alt and Lat. F_{prec} is calculated from the driving variables Prec and Area, as well as the model constant C_{prec} (typical TP concentration in precipitation, see Håkanson and Bryhn, 2008).

Mixing and resuspension (fig. S7)

Wind and wave action are main agents behind two major types of TP fluxes in the upper part of the lake; sediment resuspension, and mixing of water and substances between SW and MW (fig. S7). The resuspension rate (R_{res}) depends on the DR of the lake; i. e., R_{res} has a higher value for shallow lakes than for deep lakes because sediments in shallow lakes are to a greater extent exposed to wind and wave action. Resuspension fluxes also increase in magnitude when the TP content in shallow (ET) sediments increases. The resuspended matter is transported partly to SW and partly to MW. The volume development (Vd) expresses the typical sediment slope of the lake. A high Vd value means a steep sediment slope which implies that much of the resuspended matter is transported downwards to MW. Conversely, a low Vd value means gently sloping bottom areas from which much of the resuspended matter is transported upwards to SW. Mixing is the flux pair F_{SWMWx} and F_{MWSWx} which decrease the concentration gradient between SW and MW. Mixing depends on the stratification regime which in turn is influenced by wind and wave action. The two mixing algorithms are comparatively simple. F_{SWMWx} depends on M_{SW} and the calibrated model constant R_{mix} while F_{MWSWx} depends on M_{MW} and R_{mix} . If V_{SW} is smaller than V_{MW} , then F_{MWSW} is also multiplied with the $V_{SW} \cdot V_{MW}$ ratio.

Sedimentation (figs S8-S11)

Sedimentation is the downward gravitational transportation of particulate phosphorus (P) from upper water layers to lower water layers, and from waters to sediments. Sedimentation flux algorithms (F_{SWMW} , F_{SWET} , F_{MWDW} , F_{MWMidA} and $F_{DWDDeepA}$) have in common that they have a great magnitude when TP concentrations are

high, when a large part of the particulate matter has been compacted on ET-areas, when the turbulence is low and when the PF value is high. A high TP concentration also means high concentrations of suspended particulate matter (SPM). A high SPM concentration means intensive aggregation and flocculation of particulate matter. Large particle aggregations settle faster than small particles, and this is reflected in the moderators $Y_{SPM, SW}$, $Y_{SPM, MW}$, and $Y_{SPM, DW}$ which influence the sedimentation rates of the model. Sedimentation in SW, MW and DW is depicted in fig. S8, S9, S10, respectively. The turbulence is assumed to be particularly low when DR equals 0.26 (see Håkanson and Bryhn, 2008), so the product of Y_{DR} and $Y_{DR, sed}$ in fig. S8 and the sedimentation fluxes in figs S8-S10 attain high values when $DR = 0.26$, which implies that the lower the turbulence, the higher the sedimentation rate. The PF expresses the ratio of particulate P to TP. Particulate P is by definition the only P fraction which can settle due to gravity. Thus, the higher the PF, the larger the TP sedimentation fluxes.

The algorithms determining the distribution coefficients for resuspended matter ($DC_{res, SW}$ and $DC_{res, MW}$) are depicted in fig. S11. Resuspended particles from ET-areas have already been aggregated and compacted, and may also have been consumed and excreted by benthic organisms, which exert a “gluing effect” on particles in these sediments. Therefore, resuspended particles settle faster than the average particle and $DC_{res, SW}$ and $DC_{res, MW}$ are intended for quantifying the fraction of resuspended particles in the total particle pool. Both of the variables $DC_{res, SW}$ and $DC_{res, MW}$ are used for quantifying the sedimentation flux variables in figs S8-S10. $DC_{res, SW}$ is calculated as F_{ETSW} divided by the sum of all TP fluxes to the M_{SW} compartment. Analogously, $DC_{res, MW}$ equals F_{ETMW} divided by the sum of all TP fluxes to the M_{MW} compartment. Fig. S11 also describes how C_{lake} is calculated as the sum of M_{SW} , M_{MW} and M_{DW} , divided by V .

Diffusion (figs S12-S14)

Diffusion is the flux of dissolved P from DW to MW, and from accumulation sediments to DW and MW. Diffusion from DW to MW is driven by the dissolved P concentration gradient across D_{crit2} while diffusion from accumulation sediments is driven by sedimentation, temperature and P concentrations in active accumulation sediments. Fig. S12 illustrates the three diffusion flux algorithms in the model, F_{DWMW} , F_{MidAMW} , and $F_{DeepADW}$. The first one of these fluxes, F_{DWMW} , is calculated from M_{DW} , $R_{diff, DWMW}$ and the difference in dissolved P concentrations between DW and SW according to equation 9 in the paper. F_{MidAMW} is modelled as a product of M_{MidA} and the diffusion rate regarding MidA sediments, $R_{diff, MidA}$, while $F_{DeepADW}$ equals M_{DeepA} multiplied by $R_{diff, DeepA}$. Fig. S13 shows how $R_{diff, MidA}$ is calculated. Determinants are a default diffusion rate ($R_{diff, default}$), the predicted MW temperature (MWT; from Alt and Lat), and moderators for the dynamic ratio ($Y_{DR, diff}$), for the retention time in MW ($Y_{TMW, diff}$), for sedimentation on MidA areas ($Y_{sed, MidA}$) and for the TP concentration in MidA sediments ($Y_{TP, MidA}$). Fig S14 displays the calculation routine for $R_{diff, DeepA}$ which resembles the calculation routine in fig. S13. Determinants of $R_{diff, DeepA}$ are the deep water temperature (DWT), $R_{diff, default}$ and moderators for the retention time ($Y_{TDW, diff}$), for the dynamic ratio ($Y_{DR, diff}$), for sedimentation on DeepA areas ($Y_{sed, MidA}$) and for the TP concentration in DeepA sediments ($Y_{TP, DeepA}$).

Burial (fig. S15)

Burial is the transportation of particulate P from active (affected by biota) to passive (unaffected by biota) accumulation sediments; i. e., from the biosphere to the geosphere. This process takes place because previously precipitated particles are covered by more recently deposited particles, and particulate P is thus continuously compacted and “buried” and finally removed from the ecosystem. The burial algorithms ($F_{bur, MidA}$ and $F_{bur, DeepA}$) are illustrated in fig. S15. These algorithms are determined by sedimentation fluxes, the duration of the growing season (T_{dur} ; predicted from Lat), sediment water content (W) and sediment bulk density (BD). W and BD are predicted from the DR, as illustrated in fig. S13.

Table S1. Basic information about MeroLakeMab

Applicable to	Total phosphorus (TP) loading and concentrations in alpine meromictic lakes
State variable calculation by	Ordinary differential equations (ODE)
ODE solving by	Runge-Kutta iteration
Time scale	Monthly
Number of driving variables	9 (see fig. S2)
Total number of variables	123 (see table S3)

Table S2. Variable nomenclature and common abbreviations

A	area	MW	middle waters
C	concentration	P	phosphorus
D	depth	R	rate
DeepA	deep accumulation sediments	Q	water flux
diff	diffusion	Sed	sedimentation
DW	deep waters	SPM	suspended particulate matter
ET	erosion and transport sediments	SW	surface waters
F	phosphorus flux	TP	total phosphorus
M	phosphorus mass	V	volume
MidA	mid-range accumulation sediments	Y	dimensionless moderator
mix	mixing		

Table S3. Model variables and constants in MeroLakeMab.

Variable	Description	Unit	Defined in figure
$A_{\text{catchment}}$	catchment area	m^2	S2
Alt	altitude	m. a. s. l.	S2
$\text{Amp}_{\text{DeepA}}$	amplitude value for deep A-sediments	dimensionless	S14
Amp_{MidA}	amplitude value for A-sediments above D_{crit2}	dimensionless	S13
Area	lake surface area	m^2	S2
BD	bulk density	g/cm^3	S13
BF_{DW}	bioturbation factor for deep A-sediments	dimensionless	S15
BF_{MW}	bioturbation factor for A-sediments above D_{crit2}	dimensionless	S15
C_{DeepA}	TP concentration in deep A-sediments	$\text{mg}/\text{g dw}$	S14
C_{DW}	TP concentration in DW	$\mu\text{g}/\text{l}$	S10
C_{in}	flux-weighted TP concentration in tributaries	$\mu\text{g}/\text{l}$	S2
C_{lake}	volume-weighted TP concentration in lake water	$\mu\text{g}/\text{l}$	S11
C_{MidA}	TP concentration in deep A-sediments	$\text{mg}/\text{g dw}$	S13
C_{MW}	TP concentration in MW	$\mu\text{g}/\text{l}$	S9
C_{prec}	typical TP concentration in precipitation	$\mu\text{g}/\text{l}$	S6
C_{SW}	TP concentration in SW	$\mu\text{g}/\text{l}$	S7
D_{Ased}	typical depth of active A-sediments in lakes	cm	S7
$\text{DC}_{\text{res, MW}}$	distribution coefficient for resuspended matter in MW	dimensionless	S11
$\text{DC}_{\text{res, SW}}$	distribution coefficient for resuspended matter in SW	dimensionless	S11
D_{crit1}	critical depth 1	m	S4
D_{crit2}	critical depth 2	m	S4
D_{DeepA}	depth of deep A-sediments	m	S5
DeepA	fraction of deep A-sediment areas	dimensionless	S4
D_{ET}	depth of ET-sediments	m	S5
D_{m}	mean depth	m	S2
D_{max}	maximum depth	m	S2
D_{MidA}	depth of A-sediments above D_{crit2}	m	S5
DR	dynamic ratio	dimensionless	S4
DWT	typical DW temperature in Lake Bourget	$^{\circ}\text{C}$	S14
ET	fraction of ET-areas (including boundary conditions)	dimensionless	S4
ET1	fraction of ET-areas (without boundary conditions)	dimensionless	S4
$F_{\text{bur, DeepA}}$	TP burial in deep A-sediments	g/month	S15
$F_{\text{bur, MidA}}$	TP burial in A-sediments above D_{crit2}	g/month	S15
F_{DeepADW}	TP diffusion from deep A-sediments	g/month	S12
F_{DWDeepA}	TP sedimentation in DW	g/month	S10
F_{DWMW}	TP diffusion from DW to MW	g/month	S12
F_{ETMW}	TP resuspension to MW	g/month	S7
F_{ETSW}	TP resuspension to MW	g/month	S7
F_{in}	TP loading from catchment	g/month	S6
F_{MidAMW}	TP diffusion from A-sediments above D_{crit2}	g/month	S12
F_{MWDW}	TP sedimentation to DW	g/month	S9
F_{MWMidA}	TP sedimentation from MW to sediments	g/month	S9
F_{MWSWx}	TP mixing from MW to SW	g/month	S7
F_{out}	TP outflow	g/month	S6
F_{prec}	TP loading via precipitation	g/month	S6
F_{SWET}	TP sedimentation to ET sediments	g/month	S8
F_{SWMW}	TP sedimentation from SW to MW	g/month	S8
F_{SWMWx}	TP mixing from SW to MW	g/month	S7
GS_{DeepA}	gross sedimentation on deep A-sediments	$\mu\text{g}/(\text{cm}^2 \cdot \text{day})$	S14
GS_{MidA}	gross sedimentation on A-sediments above D_{crit2}	$\mu\text{g}/(\text{cm}^2 \cdot \text{day})$	S13
IG	loss on ignition	%	S13
Lat	latitude	$^{\circ}\text{N}$	S2
M_{DeepA}	TP in deep active A-sediments	g	S3
M_{DW}	TP in DW	g	S3
M_{ET}	TP in ET-sediments	g	S3
MidA	fraction of A-areas above D_{crit2} to lake area	dimensionless	S4
M_{MidA}	TP in active A-sediments above D_{crit2}	g	S3

M_{MW}	TP in MW	g	S3
M_{SW}	TP in SW	g	S3
MWT	MW temperature	°C	S13
PF_{DW}	particulate fraction in DW	dimensionless	S10
PF_{MW}	particulate fraction in MW	dimensionless	S9
PF_{SW}	particulate fraction in SW	dimensionless	S8
Prec	annual precipitation	dimensionless	S2
Q	mean annual water discharge	m ³ /year	S2
Q_{SWMW}	water flux from SW to MW	m ³ /month	S7
$R_{diff, DeepA}$	diffusion rate from deep A-sediments	1/month	S14
$R_{diff, default}$	default diffusion rate from A-sediments	1/month	S13
$R_{diff, DWMW}$	diffusion rate from DW to MW	1/month	S12
$R_{diff, MidA}$	diffusion rate from A-sediments above D_{crit2}	1/month	S13
R_{mix}	mixing rate	1/month	S7
R_{out}	outflow rate	1/month	S6
R_{res}	resuspension rate	1/month	S7
$R_{sed, DW}$	sedimentation rate in DW	1/month	S10
$R_{sed, MW}$	sedimentation rate in MW	1/month	S9
$R_{sed, SW}$	sedimentation rate in SW	1/month	S8
$Sedcm_{DeepA}$	mean annual deposition on deep A-sediments	cm/year	S15
$Sedcm_{MidA}$	mean annual deposition on A-sediments above D_{crit2}	cm/year	S15
$Sedgram_{DeepA}$	sedimentation on deep A-sediments	µg/(cm ² ·day)	S10
$Sedgram_{MidA}$	sedimentation on A-sediments above D_{crit2}	µg/(cm ² ·day)	S9
SPM_{DW}	concentration of suspended particulate matter (SPM) in DW	mg/l	S10
SPM_{MW}	SPM concentration in MW	mg/l	S9
SPM_{SW}	SPM concentration in SW	mg/l	S8
SWT	SW temperature	°C	S6
T	theoretical water residence time	years	S9
T_{DeepA}	age of deep A-sediments	months	S15
T_{dur}	duration of growing season	days	S15
T_{DW}	water residence time in DW	months	S12
T_{ET}	age of ET-sediments	months	S7
T_{MidA}	age of A-sediments above D_{crit2}	months	S15
T_{MW}	water residence time in MW	months	S9
v	settling velocity of particles	m/month	S8
Vd	form factor	dimensionless	S4
V_{DeepA}	volume of deep A-sediments	m ³	S14
$V_{default}$	default settling velocity of particles	m/month	S8
V_{DW}	DW volume	m ³	S5
V_{ET}	volume of ET-sediments	m ³	S7
V_{MidA}	volume of A-sediments above D_{crit2}	m ³	S13
V_{MW}	MW volume	m ³	S5
Vol	lake volume	m ³	S5
V_{SW}	SW volume	m ³	S5
W	water content in A-sediments	%	S13
Y_{DR}	moderator with boundary conditions for settling velocity	dimensionless	S8
$Y_{DR, diff}$	moderator for diffusion from A-sediments	dimensionless	S13
$Y_{DR, sed}$	moderator for sedimentation	dimensionless	S8
Y_{DR2}	moderator for TET	dimensionless	S7
Y_{evap}	moderator for evaporation	dimensionless	S6
Y_{prec}	moderator for the effect of precipitation on outflow from lake	dimensionless	S6
Y_Q	seasonal moderator for Q	dimensionless	S6
Y_{res}	moderator for resuspension	dimensionless	S8
$Y_{sed, DeepA}$	moderator for diffusion from deep A-sediments	dimensionless	S14
$Y_{sed, MidA}$	moderator for diffusion from A-sediments above D_{crit2}	dimensionless	S13
$Y_{SPM, DW}$	moderator for settling velocity in DW	dimensionless	S10
$Y_{SPM, MW}$	moderator for settling velocity in MW	dimensionless	S9
$Y_{SPM, SW}$	moderator for settling velocity in MW	dimensionless	S8
$Y_{TDW, diff}$	moderator for diffusion from deep A-sediments	dimensionless	S14
$Y_{TDW, diff1}$	boundary conditions for $Y_{TDW, diff}$	dimensionless	S14

$Y_{TDW, sed}$	moderator for sedimentation in DW	dimensionless	S10
$Y_{TMW, diff}$	moderator for diffusion from A-sediments above D_{crit2}	dimensionless	S13
$Y_{TMW, diff1}$	boundary conditions for $Y_{TMW, diff}$	dimensionless	S13
$Y_{TMW, sed}$	moderator for sedimentation in DW	dimensionless	S9
$Y_{TPDeepA}$	moderator for diffusion from deep A-sediments	dimensionless	S14
Y_{TPMidA}	moderator for diffusion from A-sediments above D_{crit2}	dimensionless	S13

Figures

Fig. S1. Legend to figs S2-S15.

Fig. S2. Driving variables for MeroLakeMab. Abbreviations are explained in tables S2 and S3.

$$\begin{aligned}
 \mathbf{M}_{\text{SW}}(t) &= \mathbf{M}_{\text{SW}}(t - dt) + (\mathbf{F}_{\text{in}} + \mathbf{F}_{\text{MWSWx}} + \mathbf{F}_{\text{ETSW}} + \mathbf{F}_{\text{prec}} - \mathbf{F}_{\text{out}} - \mathbf{F}_{\text{SWMW}} \\
 &\quad - \mathbf{F}_{\text{SWET}} - \mathbf{F}_{\text{SWMWx}}) \cdot dt \\
 \text{INIT } \mathbf{M}_{\text{SW}} &= \mathbf{V}_{\text{SW}} \cdot 0.001 \cdot 100 \\
 \mathbf{M}_{\text{MW}}(t) &= \mathbf{M}_{\text{MW}}(t - dt) + (\mathbf{F}_{\text{SWMW}} + \mathbf{F}_{\text{ETMW}} + \mathbf{F}_{\text{SWMWx}} + \mathbf{F}_{\text{MidAMW}} + \mathbf{F}_{\text{DWMW}} \\
 &\quad - \mathbf{F}_{\text{MWSWx}} - \mathbf{F}_{\text{MWMidA}} - \mathbf{F}_{\text{MWDW}}) \cdot dt \\
 \text{INIT } \mathbf{M}_{\text{MW}} &= \mathbf{V}_{\text{MW}} \cdot 0.001 \cdot 140 \\
 \mathbf{M}_{\text{DW}}(t) &= \mathbf{M}_{\text{DW}}(t - dt) + (\mathbf{F}_{\text{MWDW}} + \mathbf{F}_{\text{DeepADW}} - \mathbf{F}_{\text{DWMW}} - \mathbf{F}_{\text{DWDeepA}}) \cdot dt \\
 \text{INIT } \mathbf{M}_{\text{DW}} &= \mathbf{V}_{\text{DW}} \cdot 0.001 \cdot 350 \\
 \mathbf{M}_{\text{ET}}(t) &= \mathbf{M}_{\text{ET}}(t - dt) + (\mathbf{F}_{\text{SWET}} - \mathbf{F}_{\text{ETMW}} - \mathbf{F}_{\text{ETSW}}) \cdot dt \\
 \text{INIT } \mathbf{M}_{\text{ET}} &= 0.25 \cdot \mathbf{V}_{\text{ET}} \cdot (1 - (\mathbf{W} - 10)/100) \cdot \mathbf{BD} \cdot 1.3 \cdot 1000 \\
 \mathbf{M}_{\text{MidA}}(t) &= \mathbf{M}_{\text{MidA}}(t - dt) + (\mathbf{F}_{\text{MWMidA}} - \mathbf{F}_{\text{MidAMW}} - \mathbf{F}_{\text{bur, MidA}}) \cdot dt \\
 \text{INIT } \mathbf{M}_{\text{MidA}} &= \mathbf{V}_{\text{MidA}} \cdot (1 - \mathbf{W}/100) \cdot \mathbf{BD} \cdot 1000 \\
 \mathbf{M}_{\text{DeepA}}(t) &= \mathbf{M}_{\text{DeepA}}(t - dt) + (\mathbf{F}_{\text{DWDeepA}} - \mathbf{F}_{\text{DeepADW}} - \mathbf{F}_{\text{bur, DeepA}}) \cdot dt \\
 \text{INIT } \mathbf{M}_{\text{DeepA}} &= \mathbf{V}_{\text{DeepA}} \cdot (1 - \mathbf{W}/100) \cdot \mathbf{BD} \cdot 1000
 \end{aligned}$$

Fig. S3. State and flux variables in MeroLakeMab. Abbreviations are explained in tables S2 and S3.

$$\begin{aligned}
 \mathbf{D_{crit1}} &= 45.7 \cdot \sqrt[3]{(\text{Area} \cdot 10^{-6})} / (21.4 + \sqrt[3]{(\text{Area} \cdot 10^{-6})}) \\
 \mathbf{D_{crit2}} &= 6.70 \cdot D_{\max}^{1.23} / (21.4 + \sqrt[3]{(\text{Area} \cdot 10^{-6})}) \\
 \mathbf{DeepA} &= 1 - \text{ET} - A_{\text{MidA}} \\
 \mathbf{DR} &= \sqrt[3]{(\text{Area} \cdot 10^{-6})} / D_m \\
 \mathbf{ET} &= \text{if } \text{ET1} > 0.95 \text{ then } 0.95 \text{ elseif } \text{ET1} < 0.15 \text{ then } 0.15 \text{ else } \text{ET1} \\
 \mathbf{ET1} &= 1 - (D_{\max} - D_{\text{crit1}}) / (D_{\max} + D_{\text{crit1}} \cdot e^{(3 - Vd^{1.5})})^{(0.5/Vd)} \\
 \mathbf{MidA} &= (1 - (D_{\max} - D_{\text{crit2}}) / (D_{\max} + D_{\text{crit2}} \cdot e^{(3 - Vd^{1.5})}))^{(0.5/Vd)} - \text{ET} \\
 \mathbf{Vd} &= 3 \cdot D_m / D_{\max}
 \end{aligned}$$

Fig. S4. Morphometric variables. Critical depths, relative distribution of sediment areas according to bottom dynamics, the dynamic ratio and the volume development. Other morphometric variables are illustrated in fig. S5. Abbreviations are explained in tables S2 and S3.

$$\begin{aligned}
 \mathbf{D}_{\text{DeepA}} &= \text{if } (D_{\text{max}} - D_{\text{crit2}})/2 < 1 \text{ then } 1 \text{ else } (D_{\text{max}} - D_{\text{crit2}})/2 \\
 \mathbf{D}_{\text{ET}} &= D_{\text{crit1}}/2 \\
 \mathbf{D}_{\text{MidA}} &= \text{if } (D_{\text{crit2}} - D_{\text{crit1}})/2 < 1 \text{ then } 1 \text{ else } (D_{\text{crit2}} - D_{\text{crit1}})/2 \\
 \mathbf{V}_{\text{DW}} &= \text{Area} \cdot \text{DeepA} \cdot \text{Vd} \cdot (D_{\text{max}} - D_{\text{crit2}})/3 \\
 \mathbf{V}_{\text{MW}} &= (\text{Area} \cdot D_{\text{m}} - \mathbf{V}_{\text{SW}} - \mathbf{V}_{\text{DW}}) \\
 \mathbf{Vol} &= \text{Area} \cdot D_{\text{m}} \\
 \mathbf{V}_{\text{SW}} &= \text{Area} \cdot (D_{\text{m}} - (\text{MidA} + \text{DeepA}) \cdot \text{Vd} \cdot (D_{\text{max}} - D_{\text{crit1}})/3)
 \end{aligned}$$

Fig. S5. Morphometric variables (continued). Volumes of water layers and depths of sediment areas. Abbreviations are explained in tables S2 and S3.

$$\begin{aligned}
 C_{\text{prec}} &= 5 \\
 F_{\text{in}} &= (Q/12) \cdot Y_Q \cdot C_{\text{in}} \cdot 0.001 \\
 F_{\text{prec}} &= C_{\text{prec}} \cdot \text{Area} \cdot \text{Prec} \cdot 0.001 \cdot 0.001/12 \\
 F_{\text{out}} &= M_{\text{SW}} \cdot R_{\text{out}} \\
 R_{\text{out}} &= 12 \cdot Q/V_{\text{SW}} \cdot Y_Q \cdot Y_{\text{evap}} \cdot Y_{\text{prec}} \\
 \text{SWT} &= [\text{predicted from Alt and Lat using a temperature sub-model from Ottosson and Abrahamsson, 1998}] \\
 Y_{\text{evap}} &= \text{if SWT} < 9 \text{ then } 1 \text{ else } 1 - 0.4 \cdot (\text{SWT}/9 - 1) \\
 Y_{\text{prec}} &= \text{if Prec} < 650 \text{ then } 1 + 1.8 \cdot (\text{Prec}/650 - 1) \text{ else } 1 + 0.5 \cdot (\text{Prec}/650 - 1) \\
 Y_Q &= [\text{predicted from Alt and Lat using a seasonal water discharge sub-model from Håkanson and Bryhn, 2008}]
 \end{aligned}$$

Fig. S6. Total phosphorus loading from the catchment (F_{in}), loading via precipitation (F_{prec}), and total phosphorus outflow (F_{out}). Abbreviations are explained in tables S2 and S3.

$$\begin{aligned}
 C_{SW} &= 1000 \cdot M_{SW} / V_{SW} \\
 D_{Ased} &= 10 \\
 F_{ETMW} &= M_{ET} \cdot R_{res} \cdot (V_d / 3) \\
 F_{ETSW} &= M_{ET} \cdot R_{res} \cdot (1 - V_d / 3) \\
 F_{MWSWx} &= \text{if } V_{SW} / V_{MW} < 1 \text{ then } M_{MW} \cdot R_{mix} \text{ else } M_{MW} \cdot R_{mix} \cdot V_{SW} / V_{MW} \\
 F_{SWMWx} &= M_{SW} \cdot R_{mix} \\
 Q_{SWMW} &= F_{SWMWx} / (C_{SW} \cdot 0.001) \\
 R_{mix} &= 0.00008 \\
 R_{res} &= 1 / T_{ET} \\
 T_{ET} &= \text{if } Y_{DR2} < 1 \text{ then } 1 \text{ else } Y_{DR2} \\
 V_{ET} &= ET \cdot Area \cdot 0.01 \cdot D_{Ased} \cdot 0.1 \cdot V_d / 3 \\
 Y_{DR2} &= 12 \cdot DR / 0.26
 \end{aligned}$$

Fig. S7. Mixing and resuspension fluxes. Abbreviations are explained in tables S2 and S3.

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$$\begin{aligned}
 \mathbf{C}_{\text{MW}} &= 1000 \cdot \mathbf{M}_{\text{MW}} / \mathbf{V}_{\text{MW}} \\
 \mathbf{F}_{\text{MWDW}} &= \mathbf{Y}_{\text{TMW, sed}} \cdot \mathbf{M}_{\text{MW}} \cdot \mathbf{R}_{\text{sed, MW}} \cdot \mathbf{P}_{\text{FMW}} \cdot ((1 - \mathbf{DC}_{\text{res, MW}}) \\
 &\quad + \mathbf{Y}_{\text{res}} \cdot \mathbf{DC}_{\text{res, MW}}) \cdot \mathbf{DeepA} / (\mathbf{MidA} + \mathbf{DeepA}) \\
 \mathbf{F}_{\text{MWMidA}} &= \mathbf{Y}_{\text{TMW, sed}} \cdot \mathbf{M}_{\text{MW}} \cdot \mathbf{R}_{\text{sed, MW}} \cdot \mathbf{P}_{\text{FMW}} \cdot ((1 - \mathbf{DC}_{\text{res, MW}}) \\
 &\quad + \mathbf{Y}_{\text{res}} \cdot \mathbf{DC}_{\text{res, MW}}) \cdot \mathbf{MidA} / (\mathbf{MidA} + \mathbf{DeepA}) \\
 \mathbf{P}_{\text{FMW}} &= 0.5 \\
 \mathbf{R}_{\text{sed, MW}} &= \mathbf{Y}_{\text{SPMMW}} \cdot \mathbf{Y}_{\text{DR, sed}} \cdot \mathbf{v} / \mathbf{D}_{\text{MidA}} \\
 \mathbf{Sedgram}_{\text{MidA}} &= \mathbf{F}_{\text{MWMidA}} \cdot 10^5 / (30 \cdot 2 \cdot \mathbf{Area} \cdot \mathbf{MidA}) \\
 \mathbf{SPM}_{\text{MW}} &= 10^{(1.56 \cdot \log(\mathbf{C}_{\text{MW}}) - 1.64)} \\
 \mathbf{T} &= \mathbf{Vol} / \mathbf{Q} \\
 \mathbf{T}_{\text{MW}} &= \text{if } \mathbf{V}_{\text{MW}} / \mathbf{Q}_{\text{SWMW}} > \mathbf{T} \cdot 12 / 4 \text{ then } \mathbf{T} \cdot 12 / 4 \\
 &\quad \text{elseif } \mathbf{V}_{\text{MW}} / \mathbf{Q}_{\text{SWMW}} < 0.5 \text{ then } 0.5 \text{ else } \mathbf{V}_{\text{MW}} / \mathbf{Q}_{\text{SWMW}} \\
 \mathbf{Y}_{\text{SPM, MW}} &= (1 + 0.75 \cdot (\mathbf{SPM}_{\text{MW}} / 50 - 1)) \\
 \mathbf{Y}_{\text{TMW, sed}} &= \text{if } \mathbf{DR} > 0.26 \text{ then } \sqrt{(\mathbf{T}_{\text{MW}} \cdot 365 / 12)} \\
 &\quad \text{else } \sqrt{(\mathbf{T}_{\text{MW}} \cdot 365 / 12 \cdot \mathbf{DR} / 0.26)}
 \end{aligned}$$

Fig. S9. Sedimentation in middle waters (MW). Abbreviations are explained in tables S2 and S3.

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$$C_{lake} = 1000 \cdot (M_{SW} + M_{MW} + M_{DW}) / Vol$$

$$DC_{res, MW} = F_{ETMW} / (F_{ETMW} + F_{SWMW} + F_{MidAMW} + F_{DWMW} + F_{SWMWx})$$

$$DC_{res, SW} = F_{ETSW} / (F_{in} + F_{ETSW} + F_{prec} + F_{MWSWx})$$

Fig. S11. Distribution coefficients for resuspended matter ($DC_{res, SW}$ and $DC_{res, MW}$). Abbreviations are explained in tables S2 and S3.

$$F_{MidAMW} = M_{MidA} \cdot R_{diff, MidA}$$

$$F_{DeepADW} = M_{DeepA} \cdot R_{diff, DeepA}$$

$$F_{DWMW} = \text{if } (C_{DW} \cdot (1-PF_{DW}) < C_{MW} \cdot (1-PF_{MW})) \text{ then } 0 \\ \text{else } M_{DW} \cdot R_{diff, DWMW} \cdot (C_{DW} \cdot (1-PF_{DW}) - (C_{MW} \cdot (1-PF_{MW})))$$

$$R_{diff, DWMW} = 0.008$$

$$T_{DW} = M_{DW} / (F_{DWMW} + 0.001)$$

Fig. S12. Diffusion fluxes from sediments and deep waters. Abbreviations are explained in tables S2 and S3.

$$\begin{aligned}
 \text{Amp}_{\text{MidA}} &= \text{C}_{\text{MidA}} \cdot 25 \\
 \text{BD} &= 100 \cdot 2.6 / (100 + (W + \text{IG} \cdot (1 - W/100)) \cdot (2.6 - 1)) \\
 \text{C}_{\text{MidA}} &= \text{M}_{\text{MidA}} / ((\text{V}_{\text{MidA}} \cdot 10^3 \cdot \text{MidA} / (\text{MidA} + \text{DeepA}) \cdot \text{BD} \cdot (1 - W/100)) \\
 \text{GS}_{\text{MidA}} &= \text{SMTH}(\text{Sedgram}_{\text{MidA}}, 60, \text{Sedgram}_{\text{MidA}}) \\
 \text{IG} &= \text{if } W > 75 \text{ then } (1280 + (W - 75)^3) / 207 \text{ else } W / 11.9 \\
 \text{MWT} &= [\text{from Alt and Lat using a temperature sub-model from Ottosson and Abrahamsson, 1998}] \\
 \text{R}_{\text{diff, MidA}} &= \text{Y}_{\text{DR, diff}} \cdot \text{Y}_{\text{TMW, diff}} \cdot \text{R}_{\text{diff, default}} \cdot \text{Y}_{\text{sed, MW}} \cdot (\text{MWT} / 4) \cdot \text{Y}_{\text{TP, MidA}} \\
 \text{R}_{\text{diff, default}} &= 2.5 \cdot 10^{-5} \\
 \text{V}_{\text{MidA}} &= \text{MidA} \cdot \text{Area} \cdot 0.01 \cdot \text{D}_{\text{Ased}} \cdot \text{Vd} / 3 \\
 \text{W} &= \text{if } \text{DR} > 6 \text{ then } 65 \text{ elseif } \text{DR} > 0.5 \text{ then } 75 \\
 &\quad \text{elseif } \text{DR} > 0.045 \text{ then } 85 \text{ else } 95 \\
 \text{Y}_{\text{DR, diff}} &= \text{if } \text{DR} < 3.8 \text{ then } 1 \text{ else } 3.8 / \text{DR} \\
 \text{Y}_{\text{sed, MidA}} &= \text{if } \text{GS}_{\text{MidA}} < 50 \text{ then } (2 - 1 \cdot (\text{GS}_{\text{MidA}} / 50 - 1)) \\
 &\quad \text{else } (2 + \text{Amp}_{\text{MidA}} \cdot (\text{GS}_{\text{MidA}} / 50 - 1)) \\
 \text{Y}_{\text{TMW, diff}} &= \text{if } \text{Y}_{\text{TMW, diff1}} < 1 \text{ then } 1 \text{ else } \sqrt{\text{Y}_{\text{TMW, diff1}}} \\
 \text{Y}_{\text{TMW, diff1}} &= \text{if } \text{T}_{\text{MW}} \cdot 365 / 12 > 120 \cdot (0.26 / \text{DR}) \text{ then } 120 \cdot (0.26 / \text{DR}) \\
 &\quad \text{else } \text{T}_{\text{MW}} \cdot 365 / 12 \\
 \text{Y}_{\text{TP MidA}} &= \text{if } \text{C}_{\text{MidA}} < 0.5 \text{ then } 0 \text{ else } \text{C}_{\text{MidA}} (\text{C}_{\text{MidA}} - 0.5) / 2
 \end{aligned}$$

Fig. S13. The diffusion rate in MidA sediments ($\text{R}_{\text{diff, MidA}}$). Abbreviations are explained in tables S2 and S3.

$$\begin{aligned}
 \mathbf{BF}_{\text{DW}} &= \text{if Sedgram}_{\text{DeepA}} > 400 \text{ then } 1 \text{ else } (1 + D_{\text{Ased}})^{0.3} \\
 \mathbf{BF}_{\text{MW}} &= \text{if Sedgram}_{\text{MidA}} > 400 \text{ then } 1 \text{ else } (1 + D_{\text{Ased}})^{0.3} \\
 \mathbf{F}_{\text{bur, DeepA}} &= M_{\text{DeepA}} \cdot (1.396 / T_{\text{DeepA}}) \\
 \mathbf{F}_{\text{bur, MidA}} &= M_{\text{MidA}} \cdot (1.396 / T_{\text{MidA}}) \\
 \mathbf{Sedcm}_{\text{DeepA}} &= \text{Sedgram}_{\text{DeepA}} \cdot T_{\text{dur}} \cdot 10^{-6} \cdot (100 / (100 - W)) \cdot (1 / BD) \\
 \mathbf{Sedcm}_{\text{MidA}} &= \text{Sedgram}_{\text{MidA}} \cdot T_{\text{dur}} \cdot 10^{-6} \cdot (100 / (100 - W)) \cdot (1 / BD) \\
 \mathbf{T}_{\text{DeepA}} &= \text{if } 12 \cdot \mathbf{BF}_{\text{DW}} \cdot D_{\text{Ased}} / \text{Sedcm}_{\text{DeepA}} > 3000 \text{ then } 3000 \\
 &\quad \text{elseif } 12 \cdot \mathbf{BF}_{\text{DW}} \cdot D_{\text{Ased}} / \text{Sedcm}_{\text{DeepA}} < 12 \text{ then } 12 \\
 &\quad \text{else } 12 \cdot \mathbf{BF}_{\text{DW}} \cdot D_{\text{Ased}} / \text{Sedcm}_{\text{DeepA}} \\
 \mathbf{T}_{\text{dur}} &= -0.058 \cdot \text{Lat}^2 + 0.549 \cdot \text{Lat} + 365 \\
 \mathbf{T}_{\text{MidA}} &= \text{if } 12 \cdot \mathbf{BF}_{\text{MW}} \cdot D_{\text{Ased}} / \text{Sedcm}_{\text{MidA}} > 3000 \text{ then } 3000 \\
 &\quad \text{elseif } 12 \cdot \mathbf{BF}_{\text{MW}} \cdot D_{\text{Ased}} / \text{Sedcm}_{\text{MidA}} < 12 \text{ then } 12 \\
 &\quad \text{else } 12 \cdot \mathbf{BF}_{\text{MW}} \cdot D_{\text{Ased}} / \text{Sedcm}_{\text{MidA}}
 \end{aligned}$$

Fig. S15. Burial fluxes from accumulation sediments. Abbreviations are explained in tables S2 and S3.