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New Techniques for Estimation of Source Parameters

*Applications to Airborne Gravity and Pseudo-Gravity
Gradient Tensors*

MAJID BEIKI

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Abstract

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Gravity gradient tensor (GGT) data contains the second derivatives of the Earth's gravitational potential in three orthogonal directions. GGT data can be measured either using land, airborne, marine or space platforms. In the last two decades, the applications of GGT data in hydrocarbon exploration, mineral exploration and structural geology have increased considerably.

This work focuses on developing new interpretation techniques for GGT data as well as pseudo-gravity gradient tensor (PGGT) derived from measured magnetic field. The applications of developed methods are demonstrated on a GGT data set from the Vredefort impact structure, South Africa and a magnetic data set from the Särna area, west central Sweden.

The eigenvectors of the symmetric GGT can be used to estimate the position of the causative body as well as its strike direction. For a given measurement point, the eigenvector corresponding to the maximum eigenvalue points approximately toward the center of mass of the source body. For quasi 2D structures, the strike direction of the source can be estimated from the direction of the eigenvectors corresponding to the smallest eigenvalues. The same properties of GGT are valid for the pseudo-gravity gradient tensor (PGGT) derived from magnetic field data assuming that the magnetization direction is known.

The analytic signal concept is applied to GGT data in three dimensions. Three analytic signal functions are introduced along x -, y - and z -directions which are called directional analytic signals. The directional analytic signals are homogenous and satisfy Euler's homogeneity equation. Euler deconvolution of directional analytic signals can be used to locate causative bodies. The structural index of the gravity field is automatically identified from solving three Euler equations derived from the GGT for a set of data points located within a square window with adjustable size.

For 2D causative bodies with geometry striking in the y -direction, the measured g_{xx} and g_{zz} components of GGT can be jointly inverted for estimating the parameters of infinite dike and geological contact models. Once the strike direction of 2D causative body is estimated, the measured components can be transformed into the strike coordinate system. The GGT data within a set of square windows for both infinite dike and geological contact models are deconvolved and the best model is chosen based on the smallest data fit error.

Keywords: Gravity gradient tensor, pseudo-gravity, magnetic field, eigenvector analysis, least squares algorithm, strike direction, source parameter estimation, analytic signal, Euler deconvolution, joint inversion, infinite dike model, geological contact model

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Dedicated to:
Hediyeh

List of Papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

- I. **Beiki, M.**, and Pedersen, L. B. (2010), Eigenvector analysis of gravity gradient tensor to locate geologic bodies. *Geophysics*, 75(6): I37-I49.
- II. **Beiki, M.**, Pedersen, L. B., and Nazi, H. (2011), Interpretation of aeromagnetic data using eigenvector analysis of pseudo-gravity gradient tensor. Submitted to *Geophysics*, accepted.
- III. **Beiki, M.** (2010), Analytic signals of gravity gradient tensor and their application to estimate source location. *Geophysics*, 75(6): I59-I74.
- IV. **Beiki, M.**, and Pedersen, L. B. (2011), Deconvolution of gravity gradient tensor data using infinite dike and geological contact models. Submitted to *Geophysics*, under review.

Reprints were made with permission from the respective publishers. Additional articles written during my PhD studies but not included in this thesis are:

- **Beiki, M.**, and Pedersen, L. B. (2011), Comment on “Depth estimation of simple causative sources from gravity gradient tensor invariants and vertical component” by B. Oruç. Submitted to *Pure and Applied Geophysics*, accepted.
- **Beiki, M.**, Bastani, M., and Pedersen, L. B. (2010), Leveling HEM and aeromagnetic data using differential polynomial fitting. *Geophysics*, 75(1): L13-L23.

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Abbreviations

2D	Two-dimensional
3D	Three-dimensional
AGG	Airborne Gravity Gradiometry
GGT	Gravity Gradient Tensor
LPI	Lunar and Planetary Institute
MGT	Magnetic Gradient Tensor
PDF	Planar Deformation Features
PGGT	Pseudo-Gravity Gradient Tensor
TIB	Trans-Scandinavian Igneous Belt

1. Introduction

1.1 Motivation and general objective

Studies of the Earth's gravity and magnetic fields are examples of modern applications of classical Newtonian physics. Gravity and magnetic fields are routinely measured in mineral exploration, hydrocarbon exploration, crustal studies, environmental engineering and exploration of geothermal resources. Gravity and magnetic surveys are carried out using ground, airborne, shipborne, satellite or borehole measurement systems. Among those, airborne is the most popular type of survey because of its cost-efficiency and rapid coverage of large areas.

Airborne magnetic is the oldest airborne geophysical technique which has been in use since 1943. The first application of airborne magnetic surveys was for detecting submarines in World War II. By advancements of instrumentations and developing navigation systems, airborne gravity measurements became feasible in the early 1990s.

Gradients of gravity and magnetic fields are much more sensitive to short wavelength anomalies than gravity and magnetic fields. In the last two decades, airborne and shipborne gravity gradiometers use developed for routine measurements and introduced to industry. At the time of writing this thesis, airborne magnetic gradiometry instruments are still under development. However, it is anticipated that in the near future, airborne magnetic gradiometers will be commercially available.

The gradient tensors of gravity and magnetic fields contain the first order derivatives of the gravity and magnetic vectors in three orthogonal directions. Mathematical properties of gravity and magnetic gradient tensors suggest new processing and interpretation techniques. In recent years, several new techniques for interpretation of GGT data have been introduced in the literature (e.g. Pedersen and Rasmussen, 1990; Vasco and Taylor, 1991; Edwards et al., 1997; Routh et al., 2001; Zhdanov et al., 2004; Droujinine et al., 2007; While et al., 2006; Mikhailov et al., 2007; Pajot et al., 2008; While et al., 2009; Beiki and Pedersen, 2010; Beiki, 2010). However, both processing and interpretation of GGT data are still challenging and requires further development.

The general objective of this thesis is to develop new techniques for processing and interpretation of airborne gravity gradient tensor data. In particular, this thesis includes four papers. In paper I, a new interpretation technique is introduced for GGT data based on Pedersen and Rasmussen

(1990). In paper II, it is shown that the same mathematical properties of GGT data can be used for interpretation of pseudo-gravity gradient tensor (PGGT) derived from magnetic field data assuming that the magnetization direction is known. Paper III describes that the analytic signal concept can be extended to 3D GGT. It is also shown that the amplitudes of the analytic signals satisfy Euler's homogeneity equation and they can be used to estimate source location. Paper IV contains a new deconvolution approach for GGT data using infinite dike and geological contact models. The applications of introduced methods are demonstrated on an airborne GGT data set from the Vredefort impact structure, South Africa and an aeromagnetic data set from the Särna area, west central Sweden.

1.2 Structure of the thesis

Before presenting the developed methods for interpretation of GGT and PGGT data, a brief description of potential field theory is provided in chapter 2 to explain theoretical basis of the thesis. Then, I shortly describe the commercially available airborne gravity and gradiometry instruments as well as different types of magnetometers adopted for airborne measurements in chapter 3. The source estimation techniques which are widely adopted for interpretation of gravity and magnetic data are described in chapter 4. In particular, those have capabilities to be employed for interpretation of GGT data are discussed in more details. Geological settings of the Vredefort impact structure, South Africa and the Särna area, west central Sweden are outlined in chapter 5. In order to have a better understanding of impact structures, additional notes on the structural geology of the impact structures are provided in Appendix A. The principles of the developed methods are summarized in chapter 6. Chapter 7 contains discussion and conclusions of the developed methods and their applications to the real data examples and finally a short summary of the thesis in Swedish is given in chapter 8.

2. Potential field theory

2.1 Potential field

A field can be defined as a function that might be a scalar or a vector which holds space and time variations of a physical quantity. Temperature distribution throughout space and magnetic field are examples of scalar and vector fields, respectively. A vector field $\mathbf{F}$ is said to be conservative if the work required to move a particle from one point to another is independent of the path connecting the two points. When $\mathbf{F}$ is conservative it is called a potential field if it is related to the scalar potential ϕ such that $\mathbf{F} = \nabla \phi$. This implies that potential fields are irrotational, $\nabla \times \mathbf{F} = 0$. The potential ϕ is a harmonic function and satisfies Laplace's equation $\nabla^2 \phi = 0$ outside the source of the potential field $\mathbf{F}$. The solutions to Laplace's equation are known as harmonic functions.

2.1.1 Green's identities

Like any differential equation, a complete solution of Laplace's equation is obtained only by application of boundary conditions. Some fundamental theorems and identities are derived from vector calculus and Laplace's equation to study the existence and uniqueness of solutions to these boundary value problems.

Green's first identity is derived from the divergence theorem applied to the vector field $\mathbf{F} = \Psi \nabla \phi$ where ϕ and Ψ are scalar functions defined on region R and continuously differentiable to second and first orders, respectively. The region R is bounded by the surface boundary S with outward normal $\hat{\mathbf{n}}$:

$$\int_R \Psi \nabla^2 \phi dv + \int_R \nabla \phi \cdot \nabla \Psi dv = \int_S \Psi \frac{\partial \phi}{\partial n} ds. \quad (2.1)$$

Green's second identity is obtained by interchanging ϕ and Ψ in equation 2.1 and subtracting the new equation from equation 2.1,

$$\int_R (\phi \nabla^2 \Psi - \Psi \nabla^2 \phi) dv = \int_S \left(\phi \frac{\partial \Psi}{\partial n} - \Psi \frac{\partial \phi}{\partial n} \right) ds. \quad (2.2)$$

Now, by choosing $\Psi = \frac{1}{r}$ in Green's second identity (equation 2.2), where r is the distance between integration (point Q) and observation (point P) in the region R, the potential $\emptyset$ at the observation point is

$$\emptyset(P) = -\frac{1}{4\pi} \int_R \frac{\nabla^2 \emptyset}{r} dv + \frac{1}{4\pi} \int_S \frac{1}{r} \frac{\partial \emptyset}{\partial n} ds - \frac{1}{4\pi} \int_S \emptyset \frac{\partial}{\partial n} \left(\frac{1}{r} \right) ds. \quad (2.3)$$

Equation 2.3 is known as Green's third identity. Kellog (1953) and Blakely (1995) present a detail proof of the Green's identities as well as theorems derived from them.

Several interesting theorems are derived from Green's identities to prove the uniqueness of solutions to Laplace's equation. Here, I briefly describe some important theorems which are useful to understand the potential field theory in general. One of the important theorems (Green's equivalent layer) derived from Green's identities remains to be discussed in the next section after describing the gravity field.

Theorem 1. If $\Psi = 1$ and $\emptyset$ is harmonic, then $\nabla^2 \emptyset = 0$ and $\nabla \Psi = 0$. Therefore, from equation 2.1 we have

$$\int_S \frac{\partial \emptyset}{\partial n} ds = 0 \quad (2.4)$$

while on the surface of the region

$$\frac{\partial \emptyset}{\partial n} = \mathbf{F} \cdot \hat{\mathbf{n}}. \quad (2.5)$$

By substituting 2.5 into equation 2.4 and applying the divergence theorem we get

$$\int_R \nabla \cdot \mathbf{F} dv = 0. \quad (2.6)$$

The condition $\nabla \cdot \mathbf{F} = 0$ within the region is sufficient to conclude that the region is free of any sources.

Theorem 2. If $\emptyset$ is harmonic and continuously differentiable in region R surrounded by surface S, and if $\emptyset$ vanishes at surface S, it vanishes at all points located within the region R.

Theorem 3 (Stokes' theorem). If $\emptyset$ is harmonic and continuously differentiable in closed region R, then $\emptyset$ can uniquely be determined in the region by its values on the boundary surface S.

Theorem 4. If $\emptyset$ and Ψ are both harmonic and $\Psi=1$, from Green's second identity (equation 2.2) we get the same result as theorem 1:

$$\int_S \mathbf{F} \cdot \hat{\mathbf{n}} ds = 0. \quad (2.7)$$

In words, the flux of a conservative field through a closed surface, S is equal to zero.

Theorem 5. If $\emptyset$ is harmonic, then by substituting $\nabla^2 \emptyset = 0$ in equation 2.3, we get

$$\emptyset(P) = \frac{1}{4\pi} \int_s \left[\frac{1}{r} \frac{\partial \emptyset}{\partial n} - \emptyset \frac{\partial}{\partial n} \left(\frac{1}{r} \right) \right] ds \quad (2.8)$$

which means that having the values of a harmonic function $\emptyset$ and its normal derivatives on a closed surface S , one can calculate $\emptyset$ at any point inside the region R enclosed by S . Equation 2.8 is simplified by eliminating the term containing $\frac{\partial \emptyset}{\partial n}$ which leads to derivation of a practical formula. Assuming that $\emptyset$ and Ψ are both harmonic, the left side of equation 2.2 (Green's second identity) vanishes. By adding the resulting equation to equation 2.8, we have

$$\emptyset(P) = -\frac{1}{4\pi} \int_s \left[\emptyset \frac{\partial}{\partial n} \left(\Psi + \frac{1}{r} \right) - \left(\Psi + \frac{1}{r} \right) \frac{\partial \emptyset}{\partial n} \right] ds. \quad (2.9)$$

With $\Psi = -\frac{1}{r}$ at surface S , we get

$$\emptyset(P) = -\frac{1}{4\pi} \int_s \emptyset \frac{\partial}{\partial n} \left(\Psi + \frac{1}{r} \right) ds. \quad (2.10)$$

Assuming a half-space region (Figure 2.1) where $\emptyset$ is harmonic throughout, if we choose point P' such that it is located below the surface $z=0$ and has a distance $r' = r$ to point Q , by letting $\Psi = -\frac{1}{r'}$

$$\emptyset(P) = -\frac{1}{4\pi} \int_s \emptyset \frac{\partial}{\partial n} \left(\frac{1}{r} - \frac{1}{r'} \right) ds. \quad (2.11)$$

Equation 2.11 can be written as

$$\emptyset(x, y, -z) = \frac{z}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\emptyset(\xi, \eta, 0)}{r^3} d\xi d\eta \quad (2.12)$$

where $r = \sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2}$. Equation 2.12 is called upward continuation operation and states that the potential $\emptyset$ can be calculated at any point above the surface on which the potential is measured (Blakely, 1995). Upward continuation is one of the most practical implications of Green's identities.

Figure 2.1. A half-space region above surface $z = 0$ where $\emptyset$ is harmonic.

2.2 Gravity field

Newton's law of universal gravitation states that any two point masses in the universe exert a gravitational force of attraction on each other which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them (Figure 2.2).

Figure 2.2. The illustration of the Newton's law of universal gravitation.

Assuming that m is a unit mass then the gravitational attraction caused by point mass m_0 at P is

$$\mathbf{g}(P) = -\gamma \frac{m_0}{r^2} \hat{\mathbf{r}} \quad (2.13)$$

where $\gamma = 6.67 \times 10^{-11} \text{ m}^3/\text{kg.s}^2$ (SI system) is gravitational constant and unit vector $\hat{\mathbf{r}} = \frac{1}{r} \left[(x - x_0) \hat{\mathbf{i}} + (y - y_0) \hat{\mathbf{j}} + (z - z_0) \hat{\mathbf{k}} \right]$ is directed from the mass m_0 toward the unit mass m . Gravitational attraction is a conservative field and it is related to the gravitational potential U through

$$\mathbf{g}(\mathbf{P}) = \nabla U \quad (2.14)$$

with

$$U(\mathbf{P}) = \gamma \frac{m}{r}. \quad (2.15)$$

Note: Mass and potential are related through Poisson's equation

$$\nabla^2 U = -4\pi\gamma\rho \quad (2.16)$$

and Laplace's equation $\nabla^2 U = 0$ is only a special case of Poisson's equation valid in source free regions.

Theorem 6. If S is a closed equipotential surface caused by a 3D density distribution ρ and R is the region surrounded by S (Figure 2.3), Green's second identity (equation 2.2) can be simplified to

$$-\int_R \frac{\nabla^2 \emptyset}{r} dv = \int_S \left[\emptyset_s \frac{\partial}{\partial n} \left(\frac{1}{r} \right) - \frac{1}{r} \frac{\partial \emptyset}{\partial n} \right] ds \quad (2.17)$$

where $\emptyset$ is the potential of the mass, r denotes the distance from point P located outside the region, $\Psi = \frac{1}{r}$ and $\emptyset_s$ is the potential at the equipotential surface. By substituting equations 2.4 and 2.16 into equation 2.17, Green's equivalent layer is given as

$$\gamma \int_R \frac{\rho}{r} dv = -\frac{1}{4\pi} \int_S \frac{1}{r} \frac{\partial \emptyset}{\partial n} ds. \quad (2.18)$$

In words, a gravitational potential caused by a three-dimensional source distribution with arbitrary shape is equivalent to the potential produced by a surface density distribution over any of its equipotential surfaces (Ramsey, 1940).

Figure 2.3. Equipotential surface S caused by a point source. S is a closed surface but to have a better illustration it is cut off from sides.

2.2.1 Three-dimensional gravity field

Considering a three-dimensional mass of arbitrary shape, the potential and gravity field at a point outside the mass can be calculated by superposition of small elements. Hence, the potential of the total mass m is

$$U(\mathbf{r}) = \gamma \int_R \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dv' \quad (2.19)$$

where $\mathbf{r}$ and $\mathbf{r}'$ denote observation and integration points, respectively. Consequently, the gravity vector is given as

$$\mathbf{g}(\mathbf{r}) = -\gamma \int_R \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2} \hat{\mathbf{r}} dv'. \quad (2.20)$$

The potential of a solid sphere at points located outside the source is equal to the potential caused by a point source located at the center of the sphere with an identical total mass m .

2.2.2 Two-dimensional gravity field

In the case of a two-dimensional mass distribution striking in the y -direction with a cross section of arbitrary shape in the xz -plane, the potential and gravity vector are, respectively:

$$U(\mathbf{r}) = 2\gamma \int_x \int_z \rho(\mathbf{r}') \ln \frac{1}{|\mathbf{r} - \mathbf{r}'|} dx' dz' \quad (2.21)$$

and

$$\mathbf{g}(\mathbf{r}) = -2\gamma \int_x \int_z \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \hat{\mathbf{r}} dx' dz' . \quad (2.22)$$

A theoretical example for a 2D source is an infinite horizontal cylinder. Similar to the point mass described earlier, the potential of a solid horizontal cylinder at points located outside the source is equal to an infinite number of point sources with a mass per unit length m lying next to each other located on the axis of the cylinder. Then, the potential and gravity vector associated with an infinite horizontal cylinder is given as

$$U(\mathbf{r}) = -2\gamma m \ln r \quad (2.23)$$

and

$$\mathbf{g}(\mathbf{r}) = \frac{2\gamma m}{r} \hat{\mathbf{r}} . \quad (2.24)$$

2.2.3 Regional gravity field and gravity corrections

The vertical component of the gravity vector, g_z , was first measured by Galileo Galilei about 1589. In honor of Galilei, the unit of gravity field is called Gal=0.01 m/s². However, because of small variations of gravity fields caused by local masses and the sensitivity of gravimeters used in the field measurements, mGal=10⁻³ Gal is adopted as unit of gravity field (Telford et al., 1990).

In order to interpret short wavelength gravity anomalies caused by local masses with positive or negative density contrasts, a key step should be carried out to separate regional field from the anomaly caused by the target source prior to analysis of measured gravity field. The separation procedure involves a series of corrections to the measured gravity field. Blakely (1995) describes that the measured gravity field contains different effects of latitude, moving platforms, elevation above sea level, topography of the surrounding terrain and earth tides which require free-air, Bouguer, terrain, tidal, Eötvös and isostatic corrections. Readers are referred to text books e.g. Telford et al. (1990) and Blakely (1995) for details about gravity data corrections.

2.2.4 Gravity gradient tensor

The history of gravity gradiometry goes back to 1886 when Lorand Eötvös, the Hungarian scientist, introduced his first torsion balance gradiometer to

the oil industry (Bell and Hansen, 1998). After Eötvös, the unit of gravity gradients is called Eötvös which is equal to 10^{-4} mGal/m. In the 1970's new systems were developed to measure gradients of gravity vector components mainly for military purposes (Bell et al., 1997). About three decades later, new gravity gradiometry systems were introduced to industry for hydrocarbon exploration, mineral exploration, structural geology, environmental engineering and geoid computations in geodesy.

As was described, the gravity vector is the gradient of the gravitational potential Φ in three Cartesian directions. Similarly, the next order of spatial derivatives yields a second-rank tensor:

$$\mathbf{\Gamma} = \nabla \mathbf{g} = \begin{bmatrix} \frac{\partial^2 U}{\partial x^2} & \frac{\partial^2 U}{\partial x \partial y} & \frac{\partial^2 U}{\partial x \partial z} \\ \frac{\partial^2 U}{\partial x \partial y} & \frac{\partial^2 U}{\partial y^2} & \frac{\partial^2 U}{\partial y \partial z} \\ \frac{\partial^2 U}{\partial x \partial z} & \frac{\partial^2 U}{\partial y \partial z} & \frac{\partial^2 U}{\partial z^2} \end{bmatrix} = \begin{bmatrix} g_{xx} & g_{xy} & g_{xz} \\ g_{xy} & g_{yy} & g_{yz} \\ g_{xz} & g_{yz} & g_{zz} \end{bmatrix}. \quad (2.25)$$

Outside the source region, U satisfies Laplace's equation, and hence the trace of the tensor is equal to zero. Since $\mathbf{\Gamma}$ is symmetric, it only contains five independent components and can be diagonalized as:

$$\mathbf{V}^T \mathbf{\Gamma} \mathbf{V} = \mathbf{\Lambda} \quad (2.26)$$

where $\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3]$ and $\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$ contain eigenvectors and eigen-

values, respectively. Physically, with the origin of the coordinate system at any observation point, equation 2.26 means that one can find a new coordinate system with axes along the eigenvectors in which the gradient tensor is on diagonal form. Pedersen and Rasmussen (1990) introduced the following three invariants:

$$I_0 = \text{Trace}(\mathbf{\Gamma}) = \lambda_1 + \lambda_2 + \lambda_3 = 0, \quad (2.27)$$

$$I_1 = \Gamma_{11}\Gamma_{22} + \Gamma_{22}\Gamma_{33} + \Gamma_{11}\Gamma_{33} - \Gamma_{12}^2 - \Gamma_{23}^2 - \Gamma_{13}^2 = \lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_1 \lambda_3 \quad (2.28)$$

and

$$I_2 = \det(\mathbf{\Gamma}) = \lambda_1 \lambda_2 \lambda_3. \quad (2.29)$$

They showed that the invariant ratio $I = -\frac{27I_2^2}{4I_1^3}$ lies between zero and unity

for any potential field. When the causative body as seen from the observation point looks more and more 3D like, then I increases and eventually approaches unity and

$$\begin{cases} \lambda_2 = \lambda_3 \\ \lambda_1 = -2\lambda_2 = -2\lambda_3 \end{cases} \quad (2.30)$$

With 2D geometry of sources striking in the y direction, the number of non-zero GGT components reduces to four with two independent elements

$$\mathbf{\Gamma} = \nabla \mathbf{g} = \begin{bmatrix} \frac{\partial^2 U}{\partial x^2} & \frac{\partial^2 U}{\partial x \partial z} \\ \frac{\partial^2 U}{\partial x \partial z} & \frac{\partial^2 U}{\partial z^2} \end{bmatrix} = \begin{bmatrix} g_{xx} & g_{xz} \\ g_{xz} & g_{zz} \end{bmatrix}. \quad (2.31)$$

Then for a strict 2D case, I is equal to zero for all measurement points and

$$\begin{cases} \lambda_1 = -\lambda_3 \\ \lambda_2 = 0 \end{cases} \quad (2.32)$$

For a quasi 2D body, the projection of the third eigenvector corresponding to the minimum eigenvalue onto the horizontal plane is directed along the strike of the body while the eigenvector corresponding to the intermediate eigenvalue is along the maximum gradient of the field.

The potential U , the gravity vector and the GGT components corresponding to the theoretical examples of a mass point source and a mass line source striking in y direction are listed in Table 2.1 when $\Delta x = x_0 - x$, $\Delta y = y_0 - y$ and $\Delta z = z_0 - z$.

In reality, many of 2D geological structures can be approximated by infinite dike and geological contact models which make them more practical than the infinite horizontal cylinder model for interpretation of magnetic and gravity field data (e.g., Nabighian, 1972 and 1974; Hsu et al., 1998; Bastani and Pedersen, 2001; Beiki and Pedersen, 2010). Telford et al. (1990) and Stanley and Green (1976) have shown expressions for the gravity gradients g_{xz} and g_{zz} caused by infinite dike and geological contact models, respectively. Application of these models in interpretation of GGT data is demonstrated in paper IV.

Table 2.1. Gravity and GGT components of mass point and mass line sources.

	<i>Point source</i>	<i>Line source</i>
U	$\gamma m/r$	$-\gamma m \ln(r)$
g_x	$\gamma m \Delta x/r^3$	$\gamma m \Delta x/r^2$
g_y	$\gamma m \Delta y/r^3$	—
g_z	$\gamma m \Delta z/r^3$	$\gamma m \Delta z/r^2$
g_{xx}	$\gamma m (3\Delta x^2 - r^2)/r^5$	$\gamma m (2\Delta x^2 - r^2)/r^4$
g_{xy}	$\gamma m (3\Delta x \Delta y)/r^5$	—
g_{xz}	$\gamma m (3\Delta x \Delta z)/r^5$	$\gamma m (2\Delta x \Delta z)/r^4$
g_{yy}	$\gamma m (3\Delta y^2 - r^2)/r^5$	—
g_{yz}	$\gamma m (3\Delta y \Delta z)/r^5$	—
g_{zz}	$\gamma m (3\Delta z^2 - r^2)/r^5$	$\gamma m (2\Delta z^2 - r^2)/r^4$

2.3 Magnetic field

The word magnetism comes from the Greek ancient city of magnesia which was known because of certain magnetic rocks in the vicinity of this city (Roy, 2008). Oersted (1819) was the first to observe the connection between the flow of electric current through a wire and the generated magnetic field. One year later, Biot and Savart described the relation between the magnetic induction $\mathbf{B}$ generated by an electric current. In the same year, Ampere proposed his force law describing that there is an effective force between two parallel wires carrying an electric current.

Magnetic field is always a solenoidal field even within magnetic media

$$\nabla \cdot \mathbf{B} = 0. \quad (2.33)$$

This is the most significant difference between magnetostatic field and other fields like electrostatic field, gravity field, heat flow field etc, in which satisfy Laplace's equation only in source free regions. Unlike the gravity field, magnetostatic field is a rotational field and

$$\nabla \times \mathbf{B} = 4\pi C_m \mathbf{I}_t \quad (2.34)$$

where $C_m=10^{-7}$ Henry/m is a proportionality constant and I_t is current source. Assuming that there is no current source in the region, $\mathbf{B}$ is irrotational and it can be defined as the gradient of a scalar potential V

$$\mathbf{B} = \nabla V. \quad (2.35)$$

This approximation is valid only outside magnetic media where electric currents are negligible.

The potential of a magnetic dipole observed at point P is given as (Blakely, 1995)

$$V(P) = C_m \frac{\mathbf{M} \cdot \hat{\mathbf{r}}}{r^2} = -C_m \mathbf{M} \cdot \nabla_P \frac{1}{r} \quad (2.36)$$

where $\mathbf{M} = I \hat{\mathbf{n}} \Delta s$ is the magnetic dipole moment with unit of ampere.m² in SI system, I is the current in ampere, $\hat{\mathbf{n}}$ is a unit normal vector and Δs is the area of the loop in m². Poisson's equation for magnetic scalar potential can be written as (Roy, 2008)

$$\nabla^2 V = 4\pi \nabla \cdot \mathbf{M}. \quad (2.37)$$

The magnetic induction of a magnetic dipole is acquired by substituting equation 2.36 into equation 2.35:

$$\mathbf{B} = -C_m \frac{m}{r^3} [3(\hat{\mathbf{M}} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \hat{\mathbf{M}}]; \quad r \neq 0. \quad (2.38)$$

The magnetization of a volume V in ampere/m is defined as sum of individual dipole moments divided by the volume V (Blakely, 1995). In the SI system, the difference between magnetic induction normalized to the magnetic permeability of free space μ_0 and magnetization is called magnetic field intensity

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \quad (2.39)$$

where μ_0 is the magnetic permeability of free space in Henry/m.

Note: $\mathbf{H}$ and $\mathbf{B}$ in the cgs system are identical outside the source region whereas in the SI system they have the same direction but different magnitudes and dimensions. Their units in the SI system are ampere/m and weber/m²=Tesla, respectively. For more details about units and their conversions, readers are referred to Blakely (1995).

The induced magnetization of rocks in the direction of an inducing field $\mathbf{H}$ is

$$\mathbf{M} = \chi \mathbf{H} \quad (2.40)$$

where χ is the dimensionless magnetic susceptibility. The total magnetization of rocks, $\mathbf{M}$ is equal to the sum of induced and remanent magnetizations

and the ratio between their magnitudes is expressed by Koenigsberger ratio $Q = \frac{|M_r|}{|M_i|}$ where M_i and M_r are induced and remanent magnetizations, respectively.

2.3.1 Three-dimensional magnetic field

A uniformly magnetized volume of magnetic materials can be considered as a single dipole and subsequently its potential and magnetic induction at point P are (Blakely, 1995):

$$V(\mathbf{r}) = -C_m \int_R \mathbf{M}(\mathbf{r}') \cdot \nabla_r \frac{1}{|\mathbf{r} - \mathbf{r}'|} dv' \quad (2.41)$$

and

$$\mathbf{B}(\mathbf{r}) = \nabla_r V(\mathbf{r}) = -C_m \nabla_r \int_R \mathbf{M}(\mathbf{r}') \cdot \nabla_r \frac{1}{|\mathbf{r} - \mathbf{r}'|} dv' \quad (2.42)$$

where $\mathbf{r}$ and $\mathbf{r}'$ are observation and integration points, respectively. The magnetic potential corresponding to a uniformly magnetized sphere is equal to the potential of a dipole (equation 2.36) located at the center of the sphere with a magnetic moment equal to the magnetization multiplied by the volume of the sphere.

2.3.2 Two-dimensional magnetic field

In analogy with gravitational potential, the magnetic potential corresponding to a two-dimensional source striking in the y direction with arbitrary cross section in xz -plane is given as

$$V(\mathbf{r}) = 2C_m \int_x \int_z \frac{\mathbf{M}(\mathbf{r}') \cdot \hat{\mathbf{r}}}{|\mathbf{r} - \mathbf{r}'|} dx' dz' . \quad (2.43)$$

Consequently applying $\mathbf{B} = \nabla V$ provides:

$$\mathbf{B}(\mathbf{P}) = -2C_m \int_x \int_z \frac{\mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \left[2(\hat{\mathbf{M}} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \hat{\mathbf{M}} \right] dx' dz' . \quad (2.44)$$

The potential and magnetic field of an infinite horizontal cylinder with uniform magnetization can be written as

$$V(\mathbf{r}) = 2C_m \frac{\mathbf{m}' \cdot \hat{\mathbf{r}}}{|\mathbf{r} - \mathbf{r}'|} \quad (2.45)$$

and

$$\mathbf{B}(\mathbf{r}) = -\frac{2C_m m'}{|\mathbf{r} - \mathbf{r}'|^2} [2(\hat{\mathbf{m}}' \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \hat{\mathbf{m}}'] \quad (2.46)$$

where $\mathbf{m}'$ is dipole moment per unit length.

2.3.3 Geomagnetic field

William Gilbert (1540–1630) was the first to explain the existence of the earth magnetic field. He described that the field observed at the earth surface originates from the interior of the earth. Now using the satellites and space probe data it is understood that a significant part of the earth magnetic field has internal origin and only a small portion of the observed field (about 1%) is of extra terrestrial origin (Roy, 2008).

The magnetic field $\mathbf{B}$ can be described in terms of its deviation from the horizontal plane and the angle between its projection onto the horizontal plane (magnetic north) and the geographic north which are called inclination and declination of the field, respectively (Figure 2.4). The inclination and declination can be found from the horizontal and vertical components of the magnetic field such that

$$\begin{cases} I = \tan^{-1} \frac{B_z}{\sqrt{B_x^2 + B_y^2}} \\ D = \sin^{-1} \frac{B_y}{\sqrt{B_x^2 + B_y^2}} \end{cases} \quad (2.47)$$

The International Geomagnetic Reference Field (IGRF) is a mathematical model of the Earth's main magnetic field which is widely used in studies of the Earth's deep interior, the crust and ionosphere and magnetosphere. The IGRF is a result of an international collaboration between institutes and agencies responsible for collecting and publishing magnetic field data. The IGRF incorporates data from permanent observations at several base stations around the world as well as from land, airborne, marine and satellite surveys. The IGRF model is updated every five years by International Association of Geomagnetism and Aeronomy (IAGA) due to the time dependent variations of the geomagnetic field. Short period variations of the magnetic field are mainly caused by electric currents in the ionosphere. They are subdivided into different scales of solar diurnal variations with a period of 24 hours, lunar variations with a period of 25 hours and magnetic storms caused by sunspot activities. In addition there are longer period variations called geomagnetic secular variations which are due to the variation in fluid outer core of the earth (Telford et al., 1990).

Figure 2.4. Elements of the Earth's magnetic field. I and D are inclination and declination of the field and B_x and B_y are positive towards the north and east, respectively and B_z is positive downward.

Nowadays, total field magnetometers are widely used to measure the magnitude of the total magnetic field regardless of its direction. In order to study crustal anomalies, the total field anomaly is calculated by reducing the effect of regional field (usually IGRF) from the measured total field. The total field anomaly, ΔT is approximately equal to the component of the anomalous field $\Delta \mathbf{F}$ in direction of the regional field $\mathbf{F}$ (Blakely, 1995)

$$\Delta T \approx \hat{\mathbf{F}} \cdot \Delta \mathbf{F} . \quad (2.48)$$

3. Airborne gravity and magnetic instruments

In recent years due to the advancements of methodologies, improvement of instrumentations and development of navigation systems, airborne measurements are routinely carried out over vast areas of land and sea. Airborne surveys provide rapid and consequently low-cost coverage of large areas. Airborne measurements are usually carried out along parallel flight lines in which the target size and depth determines the line separation. The effects of high frequency anomalies caused by near surface features are considerably reduced by choosing a high flight altitude and coarse sampling interval to favour structures of certain size located at deep levels. In this chapter I briefly describe the commercially available systems used to measure gravity and magnetic fields as well as gravity gradients.

3.1 Airborne gravimetry

The largest market of airborne gravimetry is in oil exploration for identifying sedimentary basins. The latitude, free-air, Bouguer and terrain corrections are common processes applied to the airborne gravimetry and the ground gravity data. However, additional corrections are required in the airborne gravimetry to remove the effects from acceleration of the aircraft. Dransfield (1994) describes that there are two types of non-inertial effects to be corrected for; vertical acceleration of the aircraft and coupling between the aircraft velocity and the earth's rotation (Eötvös correction).

Wooldridge (2010) provides an overview on available commercial airborne gravity systems. At the time of writing this thesis, the commercially available airborne gravimeters are:

- 1- The LaCoste and Romberg–Air II system: It consists of a highly damped spring gravity sensor mounted on a two-axis stabilized platform. It has been commercially available since 1995.
- 2- The AIRGrav system: The system comprises a three-axis gyroscopically stabilized platform, with three orthogonal accelerometers. It is developed by Sander Geophysics and it has been surveying since 1997 (Sander et al., 2005).
- 3- The GT-1A and GT-2A systems: It was initially designed by Gravimetric Technologies in the Russian Federation. Later, this system was further developed by Canadian Micro Gravity (CMG). The

CMG GT-1A is an airborne single sensor gravimeter with a three-axis inertial platform. The GT-2A gravimeter is almost identical to the GT-1A but has a higher dynamic range which makes it a better gravimeter for drupe surveys. The GT-1A is commercially available since 2003. Studinger et al. (2008) has compared GT-1A and AIR-Grav systems for research applications from flight tests over the Canadian Rocky Mountains near Calgary.

- 4- The TAGS–Air III system: This system has been released by Micro-g LaCoste and Scintrex, recently. In fact, it's a modification of the original LaCoste and Romberg–Air II gravimeter and it consists of a two-axis gyroscopically stabilized platform.

3.2 Airborne gravity gradiometry

The major attraction of the airborne gravity gradiometry lies in the fact that it is insensitive to aircraft accelerations. Dransfield (1994) describes the ability of gravity gradiometry to provide better sensitivity and resolution than airborne gravimetry. In addition, airborne gravity gradiometry does not require the latitude, free-air or Bouguer corrections which are necessary for the airborne gravimetry. One of the most important corrections for the airborne gravity gradiometry is the terrain correction. The terrain corrections consist of building a model of topographic features and then removing the gravity gradient effects of the constructed model from observed data. This requires a priori knowledge about the shape of terrain as well as densities of surrounding rocks. The accuracy provided by the navigation system is crucial for terrain corrections. For more details about terrain corrections and elevation error, readers are referred to Drandfield and Zeng (2009).

At the present there are only three commercially available airborne gravity gradiometry systems and some others are either under test or under constructions. These gradiometers have been developed to commercial systems based on the first gravity gradiometers developed by Lockheed Martin between 1975 and 1990 (Difrancesco, 2007);

- 1- The Air-FTG[®] system: This is a full tensor gradiometry system which measures five independent GGT components, namely g_{xx} , g_{xy} , g_{xz} , g_{yy} , and g_{yz} . The g_{zz} component is calculated from Laplace's equation. The system is developed by Bell Geospace.
- 2- The Falcon[™] AGG system: This system was first developed jointly by BHP Billiton and Lockheed Martin. The Falcon AGG system is designed to measure the horizontal curvature components g_{xy} and $(g_{xx}-g_{yy})/2$ which are also called the horizontal directional tendency (HDT). Then, the complete GGT is calculated from the measured components. It is now owned by Fugro Airborne Surveys. Figure 3.1 shows the Falcon AGG gradiometer mounted on an aircraft together with magnetic sensor.

- 3- The BlueQube system: The ARKeX BlueQube is a full tensor gradiometer which measures five independent GGT components. This system is very similar to the Air-FTG[®] system. BlueQube is commercially available since 2004.

Figure 3.1. Falcon AGG system together with a cesium vapor magnetometer, Mongolia (with courtesy of Fugro Airborne Surveys).

3.3 Aeromagnetic surveys

Aeromagnetic surveys are widely used as a primary mineral exploration tool for some specific types of ore bodies such as iron, base metals and diamonds and mineralization such as skarns, massive sulfides, and heavy mineral sands. Another important application of aeromagnetic surveys is in routine geological mapping of prospective areas with buried igneous bodies which generally have higher susceptibilities than their surrounding rocks and are frequently associated with mineralization. The aeromagnetic data can also be used in a variety of applications for example understanding tectonic settings, hydrocarbon exploration, modeling ground water and geothermal resources, mapping unexploded ordinances and environmental engineering.

Nabighian et al. (2005) gives an excellent overview on the developments of different types of magnetometers designed for ground, airborne, marine, space and borehole measurements. In general, types of magnetometers which have been used in airborne surveys are;

- 1- Fluxgate magnetometers: In fact, these instruments were the first instruments designed for airborne measurements. During World War II, they were utilized for detecting submarines. After the war, these magnetometers were introduced to the mining industry for geophysical applications. Typically, these magnetometers are designed to measure all three components of the earth magnetic field B_x , B_y

and B_z . The main disadvantage of this type of magnetometer for airborne applications is that since they measure the magnetic field components, they have to be oriented (Telford et al., 1990). Typical sensitivity of these magnetometers is about 1 nT.

- 2- Proton-precession magnetometers: These magnetometers were introduced to the industry in mid-1950s. Unlike fluxgate magnetometers, the proton-precession magnetometers do not require orientation. They measure the intensity of the earth magnetic field. The main disadvantage of the proton-precession magnetometer is that in order to have a reasonable signal strength, a large amount of sensor liquid as well as a large coil are needed (Nabighian et al., 2005). Furthermore, the sampling rate is limited if a reasonable sensitivity is required. It has a sensitivity of about 0.1 nT. However, new advancements in the instrumentation have increased the sensitivity to 0.05 nT (GEM magnetometers).
- 3- Alkali vapor magnetometers: This type of magnetometer was first designed for laboratory measurements. Then, at about the same time as the proton-precession it was introduced to industry (Nabighian et al., 2005). Nowadays, Alkali vapor magnetometers are popular for airborne measurements. These magnetometers have an alkali vapor pumped into their sensors. The alkali used in this instrument can be potassium, cesium 133, rubidium 85 or rubidium 87. Alkali vapor scalar magnetometers monitor the transitions in atoms to make calculations and reading of the total field magnetic intensity. The typical sensitivity of this magnetometer is about 0.01 nT.
- 4- SQUID magnetometers: The SQUID (superconducting quantum interference devices) is the most sensitive available magnetometer. This vector magnetometer requires cooling with liquid helium or liquid nitrogen to operate. By far, because of technical issues, SQUID magnetometers are not widely adopted for airborne measurements. However, it is anticipated that the use of SQUIDs may increase in the near future.

4. Source parameter estimation techniques

Early interpretation techniques were appeared in the literature in 1940s with the first gravity and magnetic surveys. They were mostly developed to estimate the depth to the source and thickness of sedimentary basins. Mapping basement structures was the preliminary application of gravity and magnetic data (Nabighian et al., 2005). These techniques were mainly based on curve matching, straight-slope, half-width amplitude and horizontal distance between various characteristic points.

In the 1970s, a new era was started in developing interpretation techniques by the appearance of digital systems and the collecting large amounts of gravity and magnetic data. The new automated inverse techniques were widely adopted for interpretation of gravity and magnetic profile data based on 2D models such as thin sheet, thick dike, geological contact and polygonal bodies.

In the 1990s, by advancement of computer systems, the 2D automated techniques were extended to 3D for application to gridded data. Since 1990, commercial software were introduced based on various methods developed for both 2D and 3D interpretation of gravity and magnetic data.

In this chapter, I review some of the inverse methods which are widely adopted by workers for estimating the geometry and physical source parameters from measured gravity and magnetic fields. Most of these methods were initially developed for interpretation of magnetic data due to the high availability of magnetic data in the public domain.

4.1 Werner deconvolution technique

Werner deconvolution has been widely used for four decades for rapid interpretation of gravity and magnetic data. Werner (1953) introduced a method for reducing the effect of neighboring sources on a magnetic anomaly. He assumed that the interference effect can be approximated by a polynomial of order n . Hartman et al. (1971) showed that the total magnetic field caused by an infinite thin dike striking along the y - direction can be written as:

$$F(x) = \frac{\mathbf{a} \cdot (\mathbf{r} - \mathbf{r}_0)}{|\mathbf{r} - \mathbf{r}_0|^2} \quad (4.1)$$

where $\mathbf{a} = A\hat{i} + B\hat{k}$, and $\mathbf{r}$ and $\mathbf{r}_0$ denote observation point and center of the top surface of the dike, respectively. A and B are constants that depend on the ambient magnetic field strength, direction of magnetization and the dike geometry and susceptibility, $\hat{i}$ and $\hat{k}$ are unit vectors in x and z directions. Combining the polynomial introduced by Werner and equation 4.1, the measured magnetic field can be approximated as:

$$F(x) = \frac{A(x - x_0) + B(z - z_0)}{(x - x_0)^2 + (z - z_0)^2} + C_0 + C_1x + \dots + C_nx^n \quad (4.2)$$

where $C_0, C_1, \dots, C_n$ are the coefficients of the polynomial.

In practice, a low order (first or second order) polynomial is used to approximate the interference effect. The unknown parameters are estimated using the least squares approach within a window with adjustable size enclosing several data points. The same analysis can be used for other model types such as thick dike, geological contact or fault.

Ku and Sharp (1983) introduced a deconvolution technique based on Werner (1953) using an infinite thin sheet model. They showed that the source parameters depth to the top, horizontal position, dip angle and susceptibility can be estimated using nonlinear least squares method in the presence of interfering sources. In 1993, Hansen and Simmonds generalized Werner deconvolution technique for multiple source bodies. Years later, Hansen (2005) extended the multiple source Werner deconvolution to 3D.

4.2 Naudy method

This technique is one of the earliest automatic methods developed for interpretation of magnetic data which is still in use today. In 1970, Koulomzine et al. introduced an analytic technique for interpretation of magnetic anomalies caused by an infinite dike. They showed that the magnetic field caused by a dike like body can be decomposed into a symmetric (tangential) and an antisymmetric (logarithmic) component.

One year later, their method was further developed for interpretation of magnetic anomalies caused by 2D structures. Naudy (1971) proposed an automated line based depth estimation technique in which anomaly type and location of the causative body are identified by cross-correlation of the symmetric component of the measured magnetic field with theoretical anomalies corresponding to infinite dike and a thin plate bodies. Depth to the source is then estimated from parameters relating width and depth of the source body and sampling interval.

4.3 Analytic signal techniques

The analytic signal $A(x)$ of a potential field $\phi(x)$ measured along the x -axis at a constant level caused by a 2D body striking along the y -axis can be written as a complex quantity

$$A(x) = \frac{\partial \phi(x)}{\partial x} + i \frac{\partial \phi(x)}{\partial z} \quad (4.3)$$

when $\frac{\partial \phi(x)}{\partial x}$ and $\frac{\partial \phi(x)}{\partial z}$ are a Hilbert transform pair. The amplitude of the 2D analytic signal is

$$|A(x)| = \sqrt{\left[\frac{\partial \phi(x)}{\partial x} \right]^2 + \left[\frac{\partial \phi(x)}{\partial z} \right]^2} \quad (4.4)$$

Nabighian (1972, 1974) showed the application of analytic signal techniques to 2D magnetic interpretation. He (Nabighian, 1984) generalized the 2D analytic signal to 3D and showed that the Hilbert transform of any potential field satisfies the Cauchy-Riemann relations. Roest et al., (1992) extended the definition of the analytic signal of the potential field $\phi(x)$ measured on a horizontal plane to 3D as

$$A(x, y) = \frac{\partial \phi(x, y)}{\partial x} + \frac{\partial \phi(x, y)}{\partial y} + i \frac{\partial \phi(x, y)}{\partial z} \quad (4.5)$$

and showed that the amplitude of $A(x, y)$ is given by

$$|A(x, y)| = \sqrt{\left[\frac{\partial \phi(x, y)}{\partial x} \right]^2 + \left[\frac{\partial \phi(x, y)}{\partial y} \right]^2 + \left[\frac{\partial \phi(x, y)}{\partial z} \right]^2} \quad (4.6)$$

Hsu et al., (1998) showed that vertical derivatives of the analytic signal amplitude can be used to estimate depth to the top surface of the dike like and step like structures. Bastani and Pedersen (2001) further developed the analytic signal techniques to estimate source parameters of dike like bodies. Li (2006) has provided an excellent overview on applications and limitations of the 3D analytic signal as an interpretive tool.

4.4 Euler deconvolution technique

In 1982, Thompson introduced a line based method for estimating magnetic source location based on Euler's homogeneity equation. Theoretically, the gravity and magnetic fields caused only by pure 2D and 3D sources (sphere, horizontal cylinder, thin sheet, and geological contact) satisfy Euler's ho-

mogeneity equation exactly. However, in practice, Euler deconvolution can be used to deconvolve the fields caused by sources of arbitrary shapes.

Reid et al. (1990) extended the 2D Euler deconvolution to 3D using square windows enclosing several grid data points;

$$(\mathbf{r} - \mathbf{r}_0) \cdot \nabla F = -N(F - B) \quad (4.7)$$

where $\mathbf{r}$ and $\mathbf{r}_0$ denote observation and source points, respectively, F is the measured gravity or magnetic field, N is the structural index and B is a constant level representing regional field within a sliding window with adjustable size. Equation 4.7 can be solved in a window enclosing several data points to estimate the source location, $\mathbf{r}_0$ and the regional field, B .

Barbosa et al. (1999) described a new criterion to select the best solutions from a set of previously computed solutions. They showed that only those solutions providing standard deviations for depth to the source smaller than a threshold value can be chosen as the most reliable solutions. Mushayandebvu et al. (2000) and Silva and Barbosa (2003) studied the theoretical basis of Euler deconvolution. Mushayandebvu et al. (2001) introduced an additional equation that expresses the transformation of homogenous functions under rotation. The combined implementation of the new equation together with the standard Euler deconvolution is called the extended Euler deconvolution. Nabighian and Hansen (2001) showed that the extended Euler deconvolution can be generalized using Hilbert transforms. They also showed their new technique is a generalization of the Werner deconvolution. Hansen and Suciú (2002) improved the standard Euler deconvolution for multiple sources to have better estimates in the presence of overlapping anomalies. Stavrev and Reid (2007) provided an excellent overview on degrees of homogeneity (structural index) of gravity and magnetic fields. They also studied Euler deconvolution of gravity anomalies caused by geological contacts and faults with negative structural indices (Stavrev and Reid, 2010).

Huang et al., (1995) showed that if a function is homogenous of degree N , then its analytic signal amplitude is homogenous of degree $N+1$. Salem and Ravat (2003) combined analytic signal and Euler deconvolution for estimating the location of magnetic source as well as its structural index. In 2004, Keating and Pilkington used Euler deconvolution of the analytic signal to remove effectively the regional value B . The advantage of the Euler deconvolution of analytic signal amplitude in contrast with the standard Euler deconvolution is that the structural index N can be directly calculated from the Euler's homogeneity equation instead of specifying N in the standard method. However, differentiation of the analytic signal amplitude may amplify high frequencies.

Zhang et al. (2000) employed the Euler's homogeneity equation for interpretation of GGT data and showed that the standard Euler deconvolution can be extended to

$$\begin{bmatrix} g_{xx} & g_{xy} & g_{xz} \\ g_{xy} & g_{yy} & g_{yz} \\ g_{xz} & g_{yz} & g_{zz} \end{bmatrix} \begin{bmatrix} x_0 - x \\ y_0 - y \\ z_0 - z \end{bmatrix} = -N \begin{bmatrix} g_x - B_x \\ g_y - B_y \\ g_z - B_z \end{bmatrix} \quad (4.8)$$

where B_x , B_y and B_z are the regional values of components of gravity vector g_x , g_y and g_z to be estimated. Their method uses the full GGT and the components of the gravity vector to provide additional constraints on the Euler solutions.

Mikhailov et al. (2007) combined scalar invariants of the tensor and Euler deconvolution to locate equivalent sources. They showed that their method, the so-called tensor deconvolution, is rather insensitive to random noise in the different tensor components and a solution is found at all individual observation points without use of sliding windows. They found that the structural index of Euler deconvolution for point and line sources was simply equal to the scalar invariant, I (section 2.2.4), plus one.

I described that in 3D case, the components of the third column of the GGT are Hilbert transform pairs of the components of the first and second columns (Beiki, 2010). Then I defined an analytic signal for every single row called directional analytic signals in x , y and z directions. The directional analytic signal in matrix form can be written as:

$$\begin{bmatrix} A_x(x, y, z) \\ A_y(x, y, z) \\ A_z(x, y, z) \end{bmatrix} = \begin{bmatrix} g_{xx} & g_{xy} & g_{xz} \\ g_{xy} & g_{yy} & g_{yz} \\ g_{xz} & g_{yz} & g_{zz} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ i \end{bmatrix} \quad (4.9)$$

Consequently, the amplitudes of the directional analytic signals are

$$|A_x(x, y, z)| = \sqrt{(g_{xx})^2 + (g_{xy})^2 + (g_{xz})^2} \quad (4.10)$$

$$|A_y(x, y, z)| = \sqrt{(g_{xy})^2 + (g_{yy})^2 + (g_{yz})^2} \quad (4.11)$$

and

$$|A_z(x, y, z)| = \sqrt{(g_{xz})^2 + (g_{yz})^2 + (g_{zz})^2}. \quad (4.12)$$

I also showed that the directional analytic signals are homogenous and satisfy Euler's homogeneity equation. Euler deconvolution of the analytic signal can be extended to 3D for the GGT as

$$\begin{bmatrix} \frac{\partial A_x}{\partial x} & \frac{\partial A_x}{\partial y} & \frac{\partial A_x}{\partial z} & A_x \\ \frac{\partial A_y}{\partial x} & \frac{\partial A_y}{\partial y} & \frac{\partial A_y}{\partial z} & A_y \\ \frac{\partial A_z}{\partial x} & \frac{\partial A_z}{\partial y} & \frac{\partial A_z}{\partial z} & A_z \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ N \end{bmatrix} = \begin{bmatrix} x \frac{\partial A_x}{\partial x} + y \frac{\partial A_x}{\partial y} + z \frac{\partial A_x}{\partial z} - A_x \\ x \frac{\partial A_y}{\partial x} + y \frac{\partial A_y}{\partial y} + z \frac{\partial A_y}{\partial z} - A_y \\ x \frac{\partial A_z}{\partial x} + y \frac{\partial A_z}{\partial y} + z \frac{\partial A_z}{\partial z} - A_z \end{bmatrix}. \quad (4.13)$$

4.5 Local wavenumber methods

In 2D, the local wavenumber k is defined as the derivative of the phase of the analytic signal with respect to distance (Bracewell, 1965);

$$k = \frac{\partial \theta}{\partial x} \quad (4.14)$$

where $\theta = \tan^{-1} \frac{\partial F / \partial z}{\partial F / \partial x}$. Thurston and Smith (1997) introduced a method

based on the local wavenumber of the magnetic field called Source Parameter Imaging (SPITM) for estimating source parameters depth, dip angle and susceptibility contrast using geological contact and thin sheet models. Salem et al. (2005) and Salem and Smith (2005) combined the local wavenumber in x and z directions with Euler deconvolution to estimate the source location and structural index, simultaneously.

Pilkington and Keating (2006) showed that the horizontal and vertical wavenumbers are equivalent to normalized vertical and horizontal derivatives of the analytic signal amplitude, respectively. They also showed that the same relations are hold for higher orders local wavenumbers and analytic signal amplitudes. In 2008, Salem et al. extended the local wavenumber method to 3D based on second derivative of magnetic field. Keating (2009) described that the local wavenumber of a potential field calculated at different levels above measurement height can be used to estimate depth to source and its degree of homogeneity. He showed that, in addition, ratio of the local wavenumber to its vertical derivative is independent of the degree of homogeneity.

4.6 Equivalent source techniques

Dampney (1969) described that gravity field measured on an irregular grid and at different elevations can be approximated by an equivalent source of discrete point masses on a plane located below the observation surface. He showed that once the equivalent source is obtained, the gravity field can be calculated at a regularly gridded horizontal plane. Emilia and Massey (1975)

showed that for a 2D causative body, magnetization inclination can be estimated by solving the nonlinear inverse problem of a horizontal equivalent source layer with unknown magnetization distribution.

Bhattacharyya and Chan (1977) and Bhattacharyya et al. (1979) studied the reduction of gravity and magnetic data measured on an arbitrary surface of high topographic relief with equivalent source representation at observation points. Hansen and Miyazaki (1984) introduced a new algorithm for continuing potential field data between arbitrary surfaces. In 1989, Leão and Silva (1989) presented a new approach to perform any linear transformation of gridded potential field data using the Green's equivalent layer method. In the last two decades, correction of topographic distortion in airborne gravity and magnetic data has been one of the most popular applications of the equivalent source technique (Pilkington and Urquhart, 1990; Xia and Sprowl, 1991; Xia et al., 1993; Meurers and Pail, 1998).

In 1991, Pedersen presented a new method for interpretation of aeromagnetic data based on equivalent source technique. He studied two types of equivalent sources in detail; horizontal thin sheet and uniaxially magnetized half space. He extended these simple models to a sandwich distribution model containing several layers with different magnetization distributions. He showed that the sandwich model can be refined by determining the thickness and rms magnetization of each layer from the decay of the power spectrum. Zhdanov et al. (2010) proposed a new technique to migrate (transform) gravity and GGT data into subsurface density distributions. They showed that migration of the gravity field requires only downward continuation operator whereas for its gradients an additional differential operator is involved in the transformation procedure. On the other hand, their migration technique requires equivalent source of observed field above their profile as mirror images of true sources.

4.7 Statistical methods

Another approach for estimating source parameters is statistical analysis of gravity and magnetic field anomalies which can be implemented in either space or wavenumber domains. In the magnetic method, an undulating magnetic basement can be represented by ensemble of prisms with finite depths or a slab. Bhattacharyya (1966) presented the magnetic anomaly caused by a prism in the wavenumber domain. Spector and Grant (1970) proposed a new technique for estimating depth to the top and bottom of a prism in wavenumber domain. In recent years, this approach is widely adopted for estimating Curie-isotherm depth of the crustal rocks (e.g. Hong et al., 1982; Okubo et al., 1994; Tanaka et al., 1999; Aydin et al., 2005; Ross et al., 2006; Aydin and Oksum, 2010). Abdeslem (1995) introduced a method for interpretation of gravity anomalies using prism model. He described

that the power spectrum of gravity anomaly is given by the product of functions that describe source parameters depth, thickness horizontal dimensions and density contrast of the causative body.

Maus (1999) and Maus et al. (1999) showed that model variograms describe the space domain statistics of gravity and magnetic data. They proposed variogram analysis of gravity and magnetic data as an alternative to power spectrum method. They used the space domain counterparts of a fractal power spectral model as model variograms.

5. Case studies

The application of the introduced methods in papers I, III, IV to real data is demonstrated on a GGT data set from the Vredefort impact structure in South Africa. The method introduced in paper I is extended to PGGT derived from measured magnetic field (paper II) and it is applied to an aeromagnetic data set from the Särna area, west central Sweden.

In this section, I briefly review the geological setting of the case studies shown in this thesis. Most emphasis is placed on the Vredefort impact structure because of its importance from a geological point of view and the number of papers included in this thesis related to this area. To have a better description of the structural geology of impact structures, some additional notes are given in section A.

5.1 The Vredefort impact structure, South Africa

The Vredefort structure is located within the Witwatersrand basin, South Africa. Boon and Albritton (1937) were the first to suggest an impact origin for this giant structure. Since 1960's many workers have studied the Vredefort structure and presented different hypotheses about its origin. After a few decades debate, the Vredefort structure is now accepted as an impact structure which has been eroded significantly. The 50-km diameter dome of Archean granotoid rocks represents the central uplift which is surrounded by Late Archean to Early Proterozoic meta-sediments (Grieve and Therriault, 2000).

5.1.1 The impact structure within the Witwatersrand basin

Based on concentric structural features reported by McCarthy et al. (1990), Grieve and Masaitis (1994) estimated the original size of the impact structure to about 300 km which is close to what Henkel and Reimold (1998) estimated based on geophysical modeling (Figure 5.1). Grieve and Therriault (2000) suggested that the diameter of the Vredefort impact structure can be 250–300 km. Henkel and Reimold (1998) estimated a central uplift of 13 km. They also described that the Vredefort dome may have experienced 7 to 10 km of erosion.

Pseudotachylite and shatter cones are widely accepted as traces of original impact structures. Shatter cones are evidence that the rock has experienced a shock with pressures of 2–30 GPa (French, 2005). They have a conical shape ranging in size from microscopic to several meters. Pseudotachylite is a fault rock which has a dark color and glassy appearance with very fine-grained material with radial and concentric clusters of crystals. Pseudotachylite is formed because of frictional effects. Usually pseudotachylite veins are much larger in impact structures than those associated with faults. In impact events, pseudotachylite is formed when the melting forms part of the shock metamorphic effect on the crater's floor. They can be seen only in impact structures that have been deeply eroded to expose the crater's floor.

Two sets of pseudotachylite occurrences are detected in the Vredefort impact structure. They occur along the contact of the inner and outer core rocks, 80 km from the center (Grieve and Therriault, 2000). The pseudotachylite and shatter cones occurrences in the Vredefort impact area are depicted in Figure 5.1.

In 1990, McCarthy et al. reported series of anticlines and synclines from the center to radius of 150 km (Figure 5.1) which can represent several rings around the core of the impact structure. There is still some discussion about the origin of these rings. Some studies (e.g., Spudis, 1993) have stated that there is no proof whether or not these rings correspond to a multi-ring crater. Some others (e.g., Brink et al., 1999) suggested that these rings are formed in excavation and modification stages of crater formation (section A). It seems that nowadays the latter hypothesis is more or less accepted to describe the multi-ring crater formation of the Vredefort impact structure.

The target rocks of the Vredefort impact structure have experienced at least two intense metamorphisms; one on a regional scale prior to the impact and a second with the impact event (Reimold and Gibson, 1996). Pseudotachylite and Planar Deformation Features (PDFs) are formed in the second stage of metamorphism (Grieve et al., 1990). Gibson and Reimold (1999) described that the metamorphism (caused by the impact event) centered on the core took place at a temperature of about 900°C.

5.1.2 The Vredefort dome or central part of the impact structure

In the central part of the impact structure (Figure 5.2), no melt sheet remains from the original transient cavity because of the post-impact erosion of the crater (McCarthy et al., 1990; Therriault et al., 1997). Instead, there are some dikes, the so-called Vredefort granophyre, exposed at the surface which can be related to impact melt rocks. In the core, these dikes are distributed radially to the center with up to 20 m width and 4–5 km length. In the collar rocks, they are concentric with widths greater than 50 m and lengths about 10 km (Grieve and Therriault, 2000). Walraven et al. (1990) estimated the

age of these granophyre dikes at 2.016 ± 0.024 Ga. Kamo et al. (1996) dated the impact event at 2.023 ± 0.004 Ga based on the estimated age of pseudotachylite in the core region.

Stepro (1990) subdivided the Archean basement into three concentric units; an inner zone of granulite facies mafic and felsic gneisses, known as Inlandsee Leucogranofels (ILG), an outer amphibolite facies granitoids composed of granite, granodiorites, tonalities and adamellites, known as Outer Granite Gneiss (OGG) and finally Steynskraal Formation (SF). Based on the chemical variations of the rocks along a radial traverse from the core, the OGG and ILG represent upper and lower crust, respectively.

The core of the Vredefort dome is surrounded by Archean to Proterozoic supracrustal Witwatersrand, Ventersdorp and Transvaal supergroup (Henkel and Reimold, 2002). These collar rocks which contain meta-sedimentary and meta-volcanic rocks are about 20 km wide. The impact related rocks are well exposed in the northern and western parts of the central uplift while the southern and southeastern parts are covered by Phanerozoic meta-sediments as well as dolerites of the Karoo supergroup (Henkel and Reimold, 2002; Figure 5.2).

5.1.3 AGG data from the Vredefort dome

In 2004 and 2007, Fugro Airborne Surveys covered the Vredefort dome area with airborne gravimetry and Falcon AGG surveys, respectively. The airborne gravity (GT-1A) survey was flown at a constant barometric altitude of 2100 m and a line spacing of 2 km in north-south direction (Ameglio, 2005; Dransfield, 2010). The AGG survey comprised two blocks covering the western and eastern parts of the Vredefort dome area. The eastern block is studied in papers I and III (the larger dashed rectangle in Figure 5.2) whereas paper IV focuses on the northeastern part of the Vredefort dome (the smaller dashed rectangle in Figure 5.2). This block was flown north-south with a line spacing of 1 km and sampling interval of about 7 m along the flight lines. The nominal height of the aircraft was 80 m above the ground.

The free-air ground gravity data are also made available by the South African Council for Geosciences with sample spacing varies from a few hundred meters to about 10 km. Dransfield (2010) showed that the DNSC08 (global gravity data released by the Danish National Space Center, 2008), ground, airborne gravity (GT-1A) can be used to improve the long wavelength anomalies of the gravity data derived from Falcon AGG data. The conformed vertical component of the gravity vector over the Vredefort dome area gridded with a cell size of 250 m is shown in Figure 5.3.

Figure 5.1. General geology of the Witwatersrand basin, South Africa, with distribution of large folds (after McCarthy et al., 1990), distribution of pseudotachylite, planar deformation features (PDFs) and shatter cones (after Theriault et al., 1997).

Figure 5.2. Geology map of the Vredefort dome (after Lana et al., 2003). The Vredefort discontinuity separating ILG and OGG (Hart et al., 1990) is shown with dashed line.

Figure 5.3. Conformed vertical component of the gravity vector, g_z .

Figure 5.4. Boundaries of rock units in simplified geological map (Figure 5.2) superimposed on measured g_{zz} component. The Vredefort discontinuity suggested by Hart et al., (1990) and its extension (introduced in this thesis) are shown with dashed red and white lines, respectively.

In the Falcon AGG system, the full GGT is derived from the measured horizontal curvature components g_{xy} and $(g_{xx}-g_{yy})/2$ (Dransfield and Lee, 2004). Terrain corrections are applied to the measured GGT components with a density of 2670 kg/m^3 . The multi-step Falcon AGG processing procedures were used to process the measured modulated differential curvature gradients. In the processing procedure a low-pass filter with cut-off wavelength of 1000 m was applied to the data set. To have a better correlation between gravity anomalies and geological units, the boundaries of the rock units in the geology map shown in Figure 5.2 are superimposed on the gridded g_{zz} component with a cell size of 250 m (Figure 5.4).

The trace of the alleged Vredefort discontinuity described by Hart et al. (1990) shown with a dashed line in Figure 5.2 coincides very well with the negative anomaly between the inner and outer ring structures. Hart et al., (1990) introduced a discontinuity southeast of Parys and south of Vredefort (Figure 5.2) which disappears beneath the Phanerozoic sediments based on a

broad negative magnetic anomaly that can give a good pattern for the core of the dome. This discontinuity separates the lower (ILG) and upper (OGG) crust. Based on the negative gravity anomaly separating the inner and outer ring structures (Figure 5.4), this discontinuity can be extended further to the south crossing the Greenstones in southeast of the Vredefort dome (Figure 5.4). In Figure 5.4, the Vredefort discontinuity suggested by Hart et al., (1990) and its extension (introduced in this thesis) are shown with dashed red and white lines, respectively.

5.2 The Särna area, west central Sweden

Särna is located 450 km north west of Stockholm. Most of the geological units of the study area are dated between Paleoproterozoic to Neoproterozoic. Bylund and Patchett (1977) describes that the age of the nepheline syenite of Särna exposed on the hills Siksjöberget located about 15 km west of Särna is estimated the Permian (0.292–0.250 Ga). This circular body which is the youngest geological unit in the area with about 4 km diameter is intrusive into the Dala porphyries, rhyolite and trackydacite rocks, dated at 1.87–1.66 Ga (Welin and Lundqvist, 1970). Other lithologies include sandstone and intrusive dolerite of Precambrian age. Figure 5.5 shows the simplified geology of the Särna area provided by the Geological Survey of Sweden (SGU) in 2010. This figure shows that dolerite dikes are dominant geological features. The Neoproterozoic dolerite dikes (1–0.54 Ga) are intrusive to most of the rock units except the young nepheline syenite. The dominant strike direction of the dolerite dikes is NW-SE. The most interesting geophysical feature of this area is the dolerite dike exposed as a ring structure in the center of the study area.

5.2.1 Aeromagnetic data from the Särna area

The Särna area was covered with aeromagnetic measurements during years 2004 and 2005 in three campaigns conducted by SGU. The nominal height of the aircraft was 60 m above the ground with a sampling interval of about 17 m along the flight lines. The flight lines were flown in E-W direction with spacing of 200 m, 400 m and 800 m for areas 1, 2 and 3, respectively (see Figure 6a in paper II). Figure 6a of paper II shows the gridded total magnetic anomaly with cell size of 200 m after applying the low pass filter of 400 m and re-sampling the filtered data with 200 m sample interval in both x and y directions.

Figure 5.5. Geology map of the Säma area (with permission from SGU).

6. Summary of papers

This thesis is based on four papers which are summarized in this chapter. The summary of papers involves a brief description of the main objectives, developed methods and their application to real data examples, conclusions and my individual contribution to each paper.

6.1 Paper I: Eigenvector analysis of GGT to locate geologic bodies

6.1.1 Summary

This paper introduces a new method for interpretation of GGT data which is developed based on Pedersen and Rasmussen (1990). They studied gradient tensors of gravity and magnetic fields and introduced scalar invariants to indicate their dimensionality. They also showed that the eigenvector $\mathbf{v}_1$ corresponding to the maximum eigenvalue of the GGT points towards the center of mass for a simple point source. In paper I, we show that the strike direction of a quasi 2D body can be estimated from the projection of the eigenvector corresponding to the smallest eigenvalue onto the horizontal plane.

For a compact body of arbitrary shape, the eigenvector $\mathbf{v}_1$ will be directed approximately towards the center of mass which can be approximated by an equivalent point source. Such an equivalent source location can then be estimated from a collection of observation points located within a given window whose gravity field is predominately generated from the same body. The eigenvector $\mathbf{v}_1$ which satisfies equation $(\mathbf{r} - \mathbf{r}') \times \mathbf{v}_1 = 0$ denotes a straight line passing through the observation point $P(x, y, z)$. The distance from the equivalent point source P_0 to this line is given by

$$d = \frac{|\mathbf{v}_1 \times (\mathbf{r}_0 - \mathbf{r})|}{|\mathbf{v}_1|} \quad (6.1)$$

where $|\mathbf{v}_1| = 1$. Then, the distance from the source point to the line passing through each observation point $P_i (x_i, y_i, z_i)$ parallel to the eigenvector $\mathbf{v}_1$ at P_i can be determined by minimizing the expression

$$Q = \sum_{i=1}^N d_i^2 \quad (6.2)$$

where N is the number of observation points in the window.

Assuming that maxima or minima of the g_{zz} component occurs approximately above the center of mass for positive or negative anomalies, respectively, a window centered on the located maximum or minimum is formed. With larger windows more eigenvectors are used to estimate the source location, and generally the accuracy of the estimated depth is improved significantly. However, in the case of real data, eventually eigenvectors may become more influenced by neighboring sources. The algorithm is improved by removing eigenvectors with distances to the estimated source location larger than a threshold value (20%) and the procedure is repeated.

In our algorithm, we increase the window size until the window size exceeds a predefined limit which is defined by user based on the bandwidth of the data. Solution corresponding to the window with minimum standard error is then chosen as the final source location. In order to study the effect of additive random noise and interfering sources, the method is tested on synthetic data examples and it appears that our method is robust to random noise in the different measurement channels. The application of the method is demonstrated on a GGT data set from the Vredefort impact structure, South Africa.

6.1.2 Conclusions

The application of the introduced method to a GGT data set from the Vredefort dome area provided very stable depth estimates along the ring structures. The estimated depths along the inner rings are predominantly in the range 1000 to 1500 m and outer rings are predominantly exceeding 1500 m. In the central uplift part they are somewhat more scattered with most estimates exceeding 1500 m.

Comparing the results of the method with the results of Euler deconvolution of GGT shows that Euler deconvolution by its very nature better outlines the edges of the causative bodies whereas our method is more focused on the center of mass. Using our method we can easily distinguish between positive and negative anomalies of g_{zz} and therefore produce two maps of depths and strikes. By this discrimination our maps become more easily interpretable. We conclude that the combination of Euler deconvolution and eigenvector analysis provides very useful complementary information which adds to the interpretative power of GGT data.

6.1.3 Contribution

I carried out the theoretical derivations of eigenvector analysis of GGT and developed the least squares algorithm by the help of second author. I did the programming and numerical tests illustrated in this paper. The discussion and conclusions drawn in this paper are result of a cooperative of both authors. I wrote the manuscript but it was largely improved by second author.

6.2 Paper II: Interpretation of aeromagnetic data using eigenvector analysis of PGGT

6.2.1 Summary

The pseudo gravity gradient tensor (PGGT) can be expressed in matrix form;

$$\Gamma = \frac{\gamma}{C_m} \frac{\rho}{M} \mathcal{F}^{-1} \left\{ \frac{1}{\theta_m \theta_f} \begin{bmatrix} -\frac{k_x^2}{|k|^2} & -\frac{k_x k_y}{|k|^2} & \frac{ik_x}{|k|} \\ -\frac{k_x k_y}{|k|^2} & -\frac{k_y^2}{|k|^2} & \frac{ik_y}{|k|} \\ \frac{ik_x}{|k|} & \frac{ik_y}{|k|} & 1 \end{bmatrix} \mathcal{F}[\Delta T] \right\} \quad (6.3)$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote Fourier transformation and inverse Fourier transformation, k_x and k_y are wavenumbers in x and y directions and $|k| = \sqrt{k_x^2 + k_y^2}$, θ_m and θ_f are phase factors of magnetization and induced field with vectors of $(\hat{m}_x, \hat{m}_y, \hat{m}_z)$ and $(\hat{f}_x, \hat{f}_y, \hat{f}_z)$, respectively (Blakely, 1995).

In paper II, we show that the same properties of the GGT are valid for the PGGT derived from magnetic field data. Assuming that the magnetization direction is known, we extended the method introduced in paper I to PGGT for interpretation of magnetic sources. Eigenvectors of the PGGT are used to estimate depth to the center of mass of geological bodies as well as the strike directions of 2D geological structures.

The dimensionality of the pseudo-gravity field is defined from the dimensionality indicator I , derived from the tensor components. In the case of a quasi 2D structure, the horizontal gradient of the field along the strike of the causative body is very small. Employing the eigenvectors located along the strike direction may increase the uncertainty of the least squares procedure. To improve the performance of the introduced algorithm for 2D structures, a rectangular window with a length a few times greater than its width and perpendicular to the strike of the 2D structure is used for estimating the

source location. The ratio between the length to the width of the rectangular window is defined based on the dimensionality indicator I .

To study the effect of remanent magnetization on depth estimates, the method is applied to synthetic examples. Our numerical tests show that depth estimates are more affected by the difference between the inclination of the induced field and magnetization vector ΔI , than the difference between their declinations ΔD . We have also studied the effect of remanent magnetization on strike estimation of 2D bodies by analysis of simple generic models. These generic models show that the strike estimates are affected by remanent magnetization direction with deviations from pure 2D conditions whereas in the case of a 2D structure with infinite depth and strike length (a purely 2D field) remanent magnetization has no effect on estimated strikes.

6.2.2 Conclusions

The method is applied to the magnetic field data from the Särna area, west central Sweden, to estimate depths to the center of mass of geological bodies as well as the strike direction of quasi 2D structure. Depth estimates along the linear features are very stable showing that most of the dolerite dikes are very shallow. This agrees with the geological information which show that these dikes outcrop on the surface.

As was discussed in paper I, comparing our method with Euler deconvolution shows that our method is rather insensitive to random noise while use of derivatives of the field in Euler deconvolution may amplify the random noise. Euler deconvolution gives better solutions for anomalies caused by sources suffering from interference effects whereas the eigenvector analysis method provides more reliable results for anomalies with small gradients of the field than Euler deconvolution. It is also shown that depth estimates derived from the proposed method are influenced by remanent magnetization when the solutions of Euler deconvolution are independent of magnetization direction

6.2.3 Contribution

I did the programming and numerical tests used in this paper. The geological description provided in this paper is result of cooperation between first and third authors. I wrote the manuscript but it was modified by second author.

6.3 Paper III: Analytic signals of GGT and their application to estimate source location

6.3.1 Summary

In the 3D case, the components of the third column of the GGT are Hilbert transform pairs of the components of first and second columns. Then we may define an analytic signal for every single row called directional analytic signals in x , y and z directions. The directional analytic signal in matrix form can be written as:

$$\begin{bmatrix} A_x(x, y, z) \\ A_y(x, y, z) \\ A_z(x, y, z) \end{bmatrix} = \begin{bmatrix} g_{xx} & g_{xy} & g_{xz} \\ g_{xy} & g_{yy} & g_{yz} \\ g_{xz} & g_{yz} & g_{zz} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ i \end{bmatrix}. \quad (6.4)$$

Consequently, the amplitudes of the directional analytic signals are

$$|A_x(x, y, z)| = \sqrt{(g_{xx})^2 + (g_{xy})^2 + (g_{xz})^2} \quad (6.5)$$

$$|A_y(x, y, z)| = \sqrt{(g_{xy})^2 + (g_{yy})^2 + (g_{yz})^2} \quad (6.6)$$

and

$$|A_z(x, y, z)| = \sqrt{(g_{xz})^2 + (g_{yz})^2 + (g_{zz})^2}. \quad (6.7)$$

It is shown that $|A_{x,z}|$ and $|A_{y,z}|$ (vertical derivatives of $|A_x|$ and $|A_y|$) enhance the edges of the causative body in x and y directions. Then, one can conclude that the amplitude of $|A_{x,z}|$ and $|A_{y,z}|$ can be a better function to detect the edges of the body:

$$|ED| = \sqrt{|A_{x,z}|^2 + |A_{y,z}|^2}. \quad (6.8)$$

Euler deconvolution of the analytic signal can be extended to 3D for the GGT as

$$(x_0 - x) \frac{\partial}{\partial x} A_\alpha + (y_0 - y) \frac{\partial}{\partial y} A_\alpha + (z_0 - z) \frac{\partial}{\partial z} A_\alpha = -(n+1) A_\alpha \quad (6.9)$$

where $\alpha=x, y$ or z . Equation 6.9 can be written in matrix form

$$\begin{bmatrix} \frac{\partial A_x}{\partial x} & \frac{\partial A_x}{\partial y} & \frac{\partial A_x}{\partial z} & A_x \\ \frac{\partial A_y}{\partial x} & \frac{\partial A_y}{\partial y} & \frac{\partial A_y}{\partial z} & A_y \\ \frac{\partial A_z}{\partial x} & \frac{\partial A_z}{\partial y} & \frac{\partial A_z}{\partial z} & A_z \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ N \end{bmatrix} = \begin{bmatrix} x \frac{\partial A_x}{\partial x} + y \frac{\partial A_x}{\partial y} + z \frac{\partial A_x}{\partial z} - A_x \\ x \frac{\partial A_y}{\partial x} + y \frac{\partial A_y}{\partial y} + z \frac{\partial A_y}{\partial z} - A_y \\ x \frac{\partial A_z}{\partial x} + y \frac{\partial A_z}{\partial y} + z \frac{\partial A_z}{\partial z} - A_z \end{bmatrix}. \quad (6.10)$$

By applying Euler's equation (equation 6.10) to the GGT data, the location and structural index of the causative body is estimated within a sliding window with adjustable size. Assuming that maxima of the edge detector function, $|ED|$ occurs approximately above the edges of causative bodies, a window centered at the located maximum is formed. The computational time and number of solutions decrease significantly by locating the center of the square window at the predefined maxima of the function $|ED|$.

The relative estimated error, the standard error normalized by the corresponding estimated depth, is used to provide a better explanation of the error distribution for solutions with respect to the estimated depth. The following criteria are used in the presented algorithm to discriminate between solutions;

- 1- The estimated source locations with standard deviation-to-estimated depth ratio greater than a threshold which is defined by user are rejected.
- 2- Only those solutions with horizontal coordinates located within the window are accepted.
- 3- Solutions with estimated depths greater than a predefined threshold (by user) are rejected.
- 4- Solutions with unreliable estimated structural indices are rejected.

The application of the method is demonstrated on gravity gradient tensor data in the Vredefort impact structure, South Africa.

6.3.2 Conclusions

The introduced directional analytic signals can be used to detect the edges of causative bodies. It is shown that the edge detector function $|ED|$ enhances the edges of causative bodies better than the horizontal gradient amplitude, HGA when the causative body is interfered by neighboring sources laterally and vertically.

The amplitudes of directional analytic signals are homogenous and satisfy the Euler's homogeneity equation. The advantage of this method is that the constant base level is removed from the Euler's equation. Then the structural index and source location can be estimated from Euler deconvolution of the directional analytic signals directly. This improves the conventional

Euler deconvolution using three gradients of the vertical component of the gravity vector. It is also shown that the Euler deconvolution of the directional analytic signals estimates the source location better than the conventional Euler deconvolution of GGT when the causative is interfered by neighboring sources. The disadvantage of the presented method is that they are more sensitive to noise than the conventional ones. Using upward continuation of the gravity gradient tensor data, the noise effect can be reduced.

6.3.3 Contribution

In this work, I developed the idea of combining analytic signals of GGT and Euler's homogeneity equation to estimate source location. I did programming, numerical tests and manuscript writing alone. However, the comments made by Professor Laust B. Pedersen and other reviewers improved this work considerably.

6.4 Paper IV: Deconvolution of GGT data using infinite dike and geological contact models

6.4.1 Summary

For infinite dike and geological contact models striking in the y direction, the measured g_{xz} and g_{zz} components can jointly be inverted for estimating the five model parameters horizontal position, depth to the top, thickness, dip angle and density contrast for infinite dike and geological contact models.

In the introduced technique, we solve the nonlinear equations of infinite dike and geological contact models for estimating corresponding model parameters in a line segment containing several data points measured approximately above the causative body. Assuming that the segment is sufficiently large, the parameter estimation problem is overdetermined. The Levenberg-Marquardt (LM) algorithm is used to minimize the nonlinear objective functions describing data fit over parameter space together with a quadratic programming approach allowing for upper and lower bounds on parameters.

Data and model parameters are related through a nonlinear system of equations $\mathbf{g}(\mathbf{m})=\mathbf{d}$ and the objective function to be minimized is

$$s = \sqrt{\frac{\sum_{i=1}^n r_i^2}{n-m}} \quad (6.11)$$

where $r_i = g(\mathbf{m}^{est})_i - d_i$; $i = 1, 2, \dots, n$, n and m are the number of data points and model parameters, respectively, $\mathbf{m}^{est}$ is the estimated model parameters and the data vector $\mathbf{d}$ include both $\mathbf{g}_{xz}$ and $\mathbf{g}_{zz}$.

To reduce the effect of interfering sources, a window is formed around the maximum of g_{zz} . Data points enclosed by this window are used in the inversion scheme for estimating the parameters and the position of the dike. In the case that the window length is very small, the dike parameters cannot be estimated accurately. In the presence of random noise, a large window size is preferred to overcome the effect of local high frequencies. However, use of a very large window is also risky as we may include data points which are more affected by neighboring bodies. We define an optimum window length based on the data fit error.

Investigating different synthetic data sets show that we normally are able to exclude inappropriate models based on a data fit criterion or because of scattered or unphysical solutions. To avoid unphysical estimates, we define constraints with lower and upper bounds on the model parameters so that solutions are always in the trust region. For real cases, any a priori information of the geology can be used either as constraints or criteria to reject spurious solutions.

As was shown in paper I, for a given measurement point, the strike direction of the GGT caused by a quasi 2D structure is estimated from the eigenvector corresponding to the smallest eigenvalue. Then, the measured components can be transformed into the strike coordinate system. In the case of gridded data and assuming that the maximum of g_{zz} is approximately located above the causative body, all measurement points enclosed by a square window centered at the maximum of g_{zz} are used to estimate the source parameters.

We applied our method to a GGT data set from the Vredefort impact structure in South Africa. The physical and geometrical parameters of the major geological features of dike like bodies and/or geological contacts in the study area are estimated using the deconvolution technique introduced in paper IV.

6.4.2 Conclusions

In practice, airborne gravity gradiometry surveys are collected along parallel lines crossing the dominant strike direction of 2D geological structures. For the deconvolution, we use data points located within a square window centered at maxima of g_{zz} . To improve the reliability of the estimated source parameters, some geological constraints are used in the inversion scheme. The measured $\mathbf{g}_{xz}$ and $\mathbf{g}_{zz}$ components are jointly inverted for estimating the source parameters for infinite dike and geological contact models.

The ratio between the estimated data fit errors for infinite dike and geological contact models are used to discriminate the spurious model types.

This discrimination between dike and geological contact models based upon the data fit error criterion works very well for both theoretical and real cases. In the real case shown in this paper, the distribution of the data fit error for the two models and their ratio indicate mean data fit errors of 2.87 ± 1.01 E and 10.70 ± 5.35 E for dike and contact models, respectively.

The different parts of the Vredefort impact area such as the dense central part, dense ring structures and less dense metasediments are delineated very well by deconvolving the GGT data. Using geological constraints and discrimination criteria, all major 2D geological structures in the dome area can be modeled as thick dikes. Depth estimates along the major geological features are stable predominantly around 1000 m. Inferred strikes of the thick dikes are concentric to the center of the dome and they are subvertical and outward dipping for the two inner ring structures and inward dipping for the outer ring.

6.4.3 Contribution

I carried out the theoretical derivations of the nonlinear least squares algorithm for inversion of two dimensional GGT components under supervision of second author. I did the programming and numerical tests illustrated in this paper. Both authors contributed equally to the interpretation of the real data example. I wrote the manuscript but it was improved by second author considerably.

7. Discussion and conclusions

This thesis focuses on the development of new processing and interpretation techniques for GGT data as well as PGGT derived from measured magnetic field data. The proposed methods in this thesis are applied to a recent GGT data set from the Vredefort impact structure, South Africa and an aeromagnetic data set from the Särna area, west central Sweden. The implementations of the introduced methods are improved using sliding windows, discrimination of solutions, incorporating geological constraints and combining with results of complementary methods. In this chapter I describe the most important aspects of the methods and results achieved.

7.1 Discussion

7.1.1 Application of sliding windows in the source parameter estimation techniques

Many processing and interpretation techniques (e.g. Euler deconvolution and Werner deconvolution) of gravity and magnetic data employ overlapping sliding windows to reduce the effect of random noise and interfering sources. Uncertainty of solutions acquired for every single window is increased when a window does not include any significant variations of the gravity or magnetic field or when it includes variations caused by different sources.

In the case that causative sources are close together, the smallest window possible is preferred. However, broad anomalies caused by deep sources are poorly estimated using a small window. In the introduced methods, we have shown that the optimum window size can be chosen based on the uncertainty of the solutions acquired for different window sizes. In paper II it is also shown that, in the case of 2D field, the results can be further improved by employing a rectangular window with its length perpendicular to the estimated strike direction of the causative 2D body.

To reduce the number of solutions and computational time, windows are centered at maxima of g_{zz} (papers I, II and IV) or maxima of the edge detector function introduced in paper III depending upon the method used for estimating the source location.

7.1.2 Discrimination of solutions and geological constraints

In paper I, II and III, It is described that the uncertainty of the estimated source location can be calculated from the least squares solution of the overdetermined set of equations. The total uncertainty of the estimated source location contains the estimated errors of x , y and z coordinates of solution. The total uncertainty normalized by the corresponding estimated depth is used as a criterion to discriminate more reliable solutions from spurious ones. Furthermore, solutions with horizontal coordinates located outside the window, estimated depths greater than a given threshold or solutions with negative estimated depths are rejected. In paper III, the estimated structural index is used as a discrimination criterion in addition to the criteria above.

In paper IV, we have used a quadratic programming allowing for upper and lower bounds on source parameters. To avoid unphysical estimates (e.g. negative depth, thickness, width and unreliable density contrast), some constraints are defined so that solutions are always in the trust region. In addition, the solutions with a horizontal distance to the center of the window greater than a given threshold are rejected. For real cases, any other a priori information of the geology can be used either as constraints or rejection criteria to refine solutions.

We have also shown that the discrimination between infinite dike and geological contact models based upon the data fit error acquired for each model type works surprisingly well. We also studied incorporation of low order polynomials representing interference effects in the presented deconvolution scheme. We found that the low order polynomials could accommodate significant parts of the signal and thus make the discrimination between the model types less distinct.

7.1.3 Improvement of results using complementary methods

Interpretation of GGT or magnetic data using the introduced methods can be significantly improved by incorporating complementary methods. In paper I, it is shown that tensor Euler deconvolution of GGT can be used to delineate the edges of causative bodies whereas eigenvector analysis of GGT data provides useful estimates of depth to the center of mass as well as strike direction of quasi 2D bodies.

In paper II, eigenvector analysis of PGGT derived from magnetic field data is compared with Euler deconvolution of magnetic field. Combining the results of these methods has considerably improved interpretation of the aeromagnetic data from the Särna area, west central Sweden. The Euler deconvolution employs first order derivatives of the magnetic field whereas the eigenvector analysis uses pseudo-gravity gradients. It is also described that Euler deconvolution provides more reliable solutions for anomalies with

high gradients of the field while the eigenvector analysis of PGGT data gives better estimates for long wavelength anomalies.

Euler deconvolution of directional analytic signals of GGT data (paper III) is compared with tensor Euler deconvolution. The advantages and disadvantages of each method are studied with synthetic and real data examples. Results of the deconvolution technique introduced in paper IV can be combined with analytic methods developed for estimating dike and contact model parameters (e.g. Hsu et al., 1998; Bastani and Pedersen, 2001). These methods were initially developed for the magnetic case but can easily be adopted for interpretation of GGT data.

7.2 General conclusions

The main objective of this thesis was to develop new processing and interpretation tools for GGT data. The eigenvectors of GGT provide useful information about depth to the center of mass for 3D causative bodies as well as the strike direction of quasi 2D structures. The same properties are valid for PGGT derived from magnetic field data assuming that magnetization direction is known. It is also shown that in the presence of interference effects estimation of source location can be improved using Euler deconvolution of directional analytic signals in x , y and z directions.

In 2D cases, the estimated strike direction using eigenvector analysis of GGT or PGGT can be used to correct the measured gravity or magnetic field for the effect of deviation of the angle between measured profile and strike of 2D sources from the normal. The source parameters horizontal position, depth to the top, dip angle, width, thickness and density contrast of infinite dike and geological contact models can be estimated using deconvolution of GGT components. To improve the reliability of the estimated source parameters, geological constraints can be used in the inversion scheme. The developed methods for interpretation of GGT data may also well be suited for magnetic gradient tensor.

Applications of the introduced methods (papers I, III and IV) to the GGT data set from the Vredefort dome shows that the different parts of the structure such as the dense central part, dense ring structures and less dense metasediments are delineated very well. Depth estimates along the major positive anomalies are stable around 1000 m. The estimated strikes of these 2D features are concentric to the center of the dome and they are subvertical and outward dipping for the two inner ring structures and inward dipping for the outer ring.

The method described in paper II is applied to magnetic field data from the Särna area to estimate depths to the center of mass of geological bodies as well as the strike direction of quasi 2D structure. Depth estimates along the linear features are very stable. The results of our method show that most

of the dolerite dikes are very shallow. This agrees with the geological information which show that these dikes outcrop on the surface.

7.3 Future developments

The eigenvector analysis introduced in papers I and II can be employed for interpretation of measured GGT or magnetic gradient tensor (MGT) data measured along profiles. GGT data can be directly used for estimating source locations dimensionality analysis. MGT data need to be transformed to PGGT data before source locations can be estimated. Such transformations can be carried out along profiles provided sufficiently long sections of 2D data.

To reduce the effect of the regional field on strike estimates, a high pass filter can be applied to data prior to calculation of eigenvectors. An alternative approach to the method proposed in paper III is Euler deconvolution of the Hilbert transform of GGT data. The constant base levels of gravity gradients are then removed from Euler's equation and the structural index and source location can be estimated directly from Euler deconvolution of Hilbert transformed GGT data.

The deconvolution technique presented in paper IV can be further developed using any simplified model such as sphere, vertical semi-infinite cylinder (pipe) or rectangular prism.

8. Summary in Swedish

I ett kartesiskt koordinatsystem innehåller gravitationsfältets gradienttensor (GGT) andraderivatorna av potentialen för jordens gravitationsfält i de tre ortogonala riktningarna. GGT-data kan mätas med antingen mark-, luft-, marin- eller rymdplattformar. Under de senaste två årtiondena har tillämpningar av GGT-data i prospektering efter kolväten och mineral och i strukturgeologiska undersökningar ökat avsevärt.

Detta arbete fokuserar på att utveckla nya tolkningstekniker för GGT-data samt för pseudo-gravitationsfältets gradienttensordata (PGGT-data) beräknade från uppmätta magnetfält. Tillämpningar av de utvecklade metoderna visas på GGT-data från impaktstrukturen Vredefort, Sydafrika, och på magnetfältdata från Särnaområdet i nordvästra Dalarna, Sverige.

8.1 Eigenvektoranalys av GGT- och PGGT- data (artiklarna I och II)

Eigenvektorer av den symmetriska GGT kan användas för att uppskatta såväl läget av en störkropp som dess strykningsriktning. För en given mätpunkt pekar den egenvektor, som motsvarar största egenvärdet, ungefär mot störkroppens tyngdpunkt. För en samling mätpunkter, som ligger inom ett kvadratisk fönster, har en robust algoritm, baserad på minstakvadratmetoden, utvecklats för att uppskatta störkroppens centrum som punkten med minsta kvadratsumma av avstånden till de linjer som går igenom mätpunkterna och är riktade som egenvektorerna. För en kvasi-2D störkropp kan strykningsriktningen beräknas från den egenvektor som motsvarar det minsta egenvärdet. Samma egenskaper som gäller för GGT gäller för pseudo-gravitationsfältets gradienttensor (PGGT) från magnetfältdata förutsatt att magnetiseringsriktningen är känd och är konstant.

8.2 Analytiska signaler av GGT för uppskattning av djup till störkroppar (artikel III)

Beräkning av analytisk signal tillämpas på GGT-data i tre dimensioner. Tre analytiska signaler införs i x-, y- och z-led och kallade riktade analytiska signaler. De riktade analytiska signalerna är homogena och uppfyller Eulers

homogenitetsekvation. Eulerdekonvolution av riktade analytiska signaler kan användas för att lokalisera orsakande störkroppar. Störningseffekter från angränsande källor minskas genom derivering av de riktade analytiska signalerna i x-, y- och z-riktningarna. Dessutom uppskattas det strukturella indexet för gravitationsfältet automatiskt genom att lösa de tre Eulerekvationer, som härrör från GGT för en uppsättning datapunkter inom ett kvadratisk fönster med justerbar storlek.

8.3 Dekonvolution av GGT-data med hjälp av modeller för oändliga geologiska gångar och kontakter (artikel IV)

För 2D störkroppar med strykning i y-riktningen kan g_{xz} - och g_{zz} -komponenterna av GGT inverteras tillsammans för att uppskatta parametrar i modeller för oändlig geologisk gång och kontakt. När strykningsriktningen av den orsakande störkroppen är beräknad, kan de uppmätta komponenterna av GGT transformeras till det strykningsorienterade koordinatsystemet. Mätpunkter inom ett kvadratisk fönster används för att uppskatta störkroppens parametrar. Efter dekonvolivering av GGT-data inom en uppsättning kvadratiske fönster med både gång- och kontaktmodeller, väljs slutligen den bästa modellen utifrån bästa dataanpassning.

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A. Additional notes on impact structures

A.1 Impact craters

The term impact crater is used for any approximately circular depression on the surface of a planet or any other solid body in the Solar System caused by the hyper-velocity impact of a bolide with the surface. In contrast to volcanic craters which are formed by explosion or internal collapse, impact craters typically have rims higher and floors lower than the surrounding terrain. On some planets like the Earth which has experienced active geological processes, impact craters are less visible as they are eroded, buried or transformed by tectonics over time. In such cases the original crater is destroyed, and the term impact structure is more commonly used. More than 170 impact structures with diameters ranging from a few tens of meters up to 300 km are identified on Earth. Their ages range from very recent impact craters in 1947, the Sikhote-Alin in Russia, to more than two billion years (in the paleoproterozoic era), the Vredefort impact structure in South Africa.

A.2 Impact cratering

In the case of hyper-velocity impacts, both impactor and target rocks are melted and vaporized partially. Impacts at speeds higher than velocity of sound produce shock wave in solid materials. Then impactor and the impacted surface materials are compressed considerably to form high density materials. Because of arbitrary size of impactors, the kinetic energy released into the Earth and subsequently geological processes may be extremely large. This is very different to normal geological process like volcanism as a huge amount of energy is released in a few seconds. Melosh (1989) divided the impact process into three distinct stages; contact and compression, excavation and modification.

A.2.1 Contact and compression

When a hyper-velocity projectile contacts the surface, target materials are pushed out of the projectile's path. Density of the materials between the projectile and surface is increased considerably because of the strong compression and subsequently shock waves are created around the contact area. At the same time, the resistance to penetration decelerates the projectile

(Melosh, 1989). When the projectile penetrates into the ground, shock pressure increases until it exceeds the yield strength of both the projectile and surface materials. In this stage they may melt or vaporize as the shock wave raises the temperature of the materials (Figure A.1). In the case of small impacts, this temperature is sufficient to melt the impactor. The contact and compression stage ends when the high pressure of the projectile decays.

Figure A.1. a) Contact and b) compression stage.

A.2.2 Excavation

After the contact and compression stage, the excavation begins by moving the target materials away from the contact point. By growing the cavity, a bowl-shaped crater is produced in which its center is pushed downwards, a large volume of the surface materials are ejected and its elevated crater rim is formed. In the case of vertical impacts, normally the produced transient cavity is hemispherical. Hyper-velocity impacts usually create crater's diameter much larger than sub-sonic impacts (Melosh, 1989).

During this stage, shock wave propagates into the target rocks and it diminishes by expanding into more materials. A small portion of the melt rocks are ejected while a significant part remains inside the transient cavity (Figure A.2). The melt rock zone follows the vaporized zone and further down when shock energy is decreased rocks are fractured and brecciated (Figure A.1b). The ejecta cover the terrain around the crater within a few crater radii.

A.2.3 Modification

In the modification stage, the transient crater remains as a simple bowl-shaped crater or develops into a complex crater exhibiting a central uplift or with several interior and exterior rings. This depends upon the size of the craters as well as physical properties of target materials. In this stage, the steep interior walls slide down and fill the transient cavity with collapsed breccia, ejecta and melt rocks.

For small transient craters, modifications are not significant. The original shape of the transient crater is preserved and the excavated crater remains as a bowl-shaped crater. In the case of large transient craters, the modifications are significant because of elastic rebound and collapse of the elevated rim under gravity. Then central uplifts and ring systems are formed a group of craters called complex craters (Figure A.2).

Figure A.2. Excavation and modification stages. (Image courtesy B. M. French and D. A. Kring, LPI).

A.3 Morphology of impact craters

The impact structures produced by hyper-velocity impacts are categorized into three types; simple crater, complex crater and multi-ring basins (Melosh, 1989). The difference between simple and complex craters is illustrated in Figure A.3. Complex craters have a central peak while simple craters are more bowl-shaped. According to this classification, most of the confirmed impact structures on the Earth are considered to be either simple or complex type. Hawke (2004) describe that the distinction between simple and complex craters is very straightforward but there is some confusion about definition of multi-ring structures. He clearly states that the only confirmed examples for true multi-ring basins are found on other planetary bodies in the Solar System. This is in contradiction with Grieve and Theriault (2000) which states that there are some confirmed multi-ring basins on the Earth e.g., the Vredefort, Sudbury and Chicxulub. They also mention that the only example with morphological ring features is the Chicxulub basin in Mexico.

A.3.1 Simple craters

Simple crater have the classic form of bowl-shaped depression (Hawke, 2004). Figures A.3a and b show an image of a typical simple crater in Arizona, USA and a schematic diagram of a simple crater, respectively. The raised rim of the crater usually overturns due to the excavation of the transient cavity. A layer of brecciated rocks resulting from slumping of the transient crater walls covers the floor of the crater (Westbroek and Stewart, 1996).

A.3.2 Complex craters

Generally, there is a transition diameter between simple and complex craters. An abrupt change in morphology of lunar craters takes place at diameters 10-20 km (Melosh, 1989) while on Earth this diameter is 2-4 km. This depends upon surface gravity and target rock type (Pilkington and Grieve, 1992). In the case of large craters, because of steepness of the walls, they collapse under gravity to form a more complex shape and their floors are filled with brecciated and melt rocks. Main characteristics of complex craters are shallow crater floors, central uplifts and terraces within the inner wall of the crater (Hawke, 2004). Figures A.3c and A.3d show an image of complex crater Tycho on the Moon and a schematic cross section of a complex crater, respectively.

Westbroek and Stewart (1996) describe that both simple and complex craters have a depth to diameter ratios of $1/4$ to $1/3$. They also state that because of the slumping, the transient crater diameter extends by almost 20% to form the final crater diameter. Grieve and Pilkington (1996) shows

that the terrestrial central peak is related to transient crater through the empirical relation

$$SU=0.086 D^{1.03} \tag{A-1}$$

where SU is the amount of structural uplift now exposed at the surface and D is the crater diameter.

Figure A.3. a) A simple crater in Arizona, USA, b) a simplified cross section of a simple crater, c) a complex crater, Tycho on the Moon and d) a simplified cross section of a complex crater. (Image courtesy NASA).

A.3.3 Multi-ring craters

Spudis (1993) defined multi-ring basins as large impact structures that initially had multi-ring morphology. Based on this definition, Grieve and Theriault (2000) categorized the large impact structures Vredefort (South Africa), Sudbury (Canada) and Chicxulub (Mexico) as multi-ring impact structures.

The Earth planet is the most geologically active planet in the Solar System. This is why the earliest morphology of most of the impact craters on Earth is not preserved. They are mainly eroded, covered with younger sediments or strongly tectonized. This makes the morphological study of the impact structures on the Earth very difficult. As an example of multi-ring basins, I show the 4000 km diameter Valhalla basin on Callisto, Jupiter’s moon (Figure A.4). Tens of rings are identified around the crater which is probably represented by the approximately 800 km diameter light gray area in the middle of the image (Hawke, 2004).

Figure A.4. Valhalla basin on Jupiter's moon, Callisto (Image courtesy NASA).

A.4 Gravity signature of impact craters

In the cases that morphology of impact craters are covered by post-impact sediments, geophysics can provide valuable information about physical and geometrical properties of impact structures. Many terrestrial impact craters are initially identified based on their geophysical signature. In 1992, Pilkington and Grieve provided an excellent overview on geophysical signature of terrestrial impact structures worldwide. To date, many new examples for geophysical studies including gravity, magnetic, electromagnetic and seismic methods are reported in the literature.

Gravity is an important geophysical tool for recognition of impact structures because of density changes of target rocks in the compression stage. For simple craters, gravity anomalies are generally circular with negative values which are caused by low-density brecciated and fractured rocks. The density contrast of the brecciated and fractured rocks depends on the diameter of crater as well as pre-impact density distribution of target rocks. For large complex craters e.g., the Vredefort dome in South Africa, the central part of impact structures show a relatively high gravity anomaly which is caused by central uplift.

Pilkington and Grieve (1992) described that the existence of a central uplift in complex craters is not a sufficient condition for producing a high gravity anomaly. They showed that for complex craters with diameter smaller than 30 km, the gravity anomaly caused by central uplift can be highly dependent to the pre-impact density distribution of target rocks. They also studied the relation between observed density values and their corresponding depths for a borehole at Ries impact structure, Germany. Pilkington and Grieve (1992) showed due to the decrease in the level of shock-induced stress and thus fracturing with depth, density increases with depth.

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