Numerical models of salt diapir formation by down-building: the role of sedimentation rate, viscosity contrast, initial amplitude and wavelength

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SUMMARY
Formation of salt diapirs has been described to be due to upbuilding (i.e. Rayleigh–Taylor like instability of salt diapirs piercing through a denser sedimentary overburden) or syndepositional down-building process (i.e. the top of the salt diapir remains at the surface all the time). Here we systematically analyse this second end-member mechanism by numerical modelling. Four parameters are varied: sedimentation rate $v_{sed}$, salt viscosity $\eta_{salt}$, amplitude $\delta$ of the initial perturbation of the sedimentation layer and the wavenumber $k$ of this perturbation. The shape of the resulting salt diapirs strongly depends on these parameters. Small diapirs with subvertical side walls are found for small values of $v_{sed}$ and $\eta_{salt}$ or large values of $\delta$, whereas taller diapirs with pronounced narrow stems build for larger values of $v_{sed}$ and $\eta_{salt}$ or small values of $\delta$. Two domains are identified in the four-parameter space, which separates successful down-building models from non-successful models. By applying a simple channel flow law, the domain boundary can be described by the non-dimensional law $v'_{sed} = C_1 \frac{\delta'_{0} \rho'_{sed} k^2}{\rho_{sed} h^2 + \rho_{sed} g}$, where $\rho'_{sed}$ is the sediment density scaled by the density contrast $\Delta \rho$ between sediment and salt, the wavelength is scaled by the salt layer thickness $h_{salt}$, and velocity is scaled by $\eta_{salt} / (h_{salt}^2 \Delta \rho g)$, where $\eta_{salt}$ is the salt viscosity and $g$ is the gravitational acceleration. From the numerical models, the constants $C_1$ and $C_2$ are determined as 0.0283 and 0.1171, respectively.

Key words: Numerical solutions; Sedimentary basin processes; Diapir and diapirism; Mechanics, theory, and modelling.

1 INTRODUCTION
Although formation of salt diapirs as a product of upbuilding has been studied extensively as an application of the classical Rayleigh–Taylor instability (e.g. Ramberg 1967; Woidt 1978; Schmeling 1987), it has long been clear that many salt diapirs formed already during sedimentation by down-building in a brittle overburden (e.g. Barton 1933; Parker & McDowell 1955; Seni & Jackson 1983; Vendeville & Jackson 1992a, b; Talbot 1995; Koyi 1998). A number of syndepositional diapirism models have been formulated during the past 50 years. One of the first was the model by Biot & Ode (1965), who related sedimentation and compaction (i.e. thickening of the overburden above a salt layer) to the time-dependence of the characteristic wavelength of the Rayleigh–Taylor instability. This behaviour has been confirmed by the numerical models of van Keken et al. (1993), who obtained shorter wavelengths for syndepositional diapirs compared to post-depositional diapirs given the same thickness of the source layer. However, as in these models, the diapiric growth started some finite time after the formation of the overburden, the shape of the diapirs was not significantly affected by sedimentation. Poliakov et al. (1993) included the effect of fast erosion (redistribution of material at the surface so that the surface remains flat) and found that after a first stage of diapiric growth similar to the classical Rayleigh–Taylor instability, the shape of the diapirs is strongly affected by erosion once they approach the surface.

To our knowledge, Chemia et al. (2007) were the first to skip the Rayleigh–Taylor like first phase of the previous numerical models and started their down-building models with a salt-anhydrite layer reaching the surface at the onset of their models. A stiff sedimentary overburden was successively added from above to simulate sedimentation, and, depending on the rate of sedimentation and the salt viscosity, a variety of salt diapir shapes were found (Chemia et al. 2007). Furthermore, they identified a regime of down-built salt diapirs at slow sedimentation or low salt viscosity, and separated it from a regime where the salt layer was buried at fast sedimentation or high salt viscosities.

In this study we will generalize these results for a pure salt layer (without an extra anhydrite layer as in Chemia et al. 2007) and systematically study the effect of the sedimentation rate $v_{sed}$, the...
viscosity ratio \( m \) between salt and overburden, the wavelength \( \lambda \) and the initial amplitude \( \delta \) of the perturbation on formation of down-built salt diaps and their geometry.

## 2 Physical Approach

### 2.1 Governing equations

Assuming purely viscous flow, the evolution of a down-built salt diapir, differentially loaded by sediments with increasing volume bounded by a free surface can be described by the conservation equations of mass, momentum and composition:

\[
div(v) = 0, \tag{1}
\]

\[
0 = -\nabla P + \frac{\partial \tau_{ij}}{\partial x_j} - \rho g, \tag{2}
\]

\[
\frac{\partial C_k}{\partial t} + \nabla \cdot (\nu \cdot C_k) = 0, \tag{3}
\]

where \( \nu \) is the velocity of the fluid, \( P \) is the pressure, \( \tau_{ij} \) is the deviatoric stress, \( \rho \) is the density of the fluid and \( C_k \) is the material concentration of the \( k \)th composition. Here we assume incompressibility. The deviatoric stress \( \tau_{ij} \) is given by

\[
\tau_{ij} = \eta \left( \frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right), \tag{4}
\]

where \( \eta \) is a dynamic viscosity (constant in each material) and \( v_i \) the velocity in the \( i \)th direction.

The equations are non-dimensionalized using the following scaling laws:

\[
(x', z') = (x, z)/h_{salt}, \tag{5}
\]

\[
v' = v \cdot \frac{\eta_{salt}}{h_{salt} \Delta \rho g}, \tag{6}
\]

\[
t' = t \cdot \frac{\Delta \rho g h_{salt}}{\eta_{salt}}. \tag{7}
\]

The eqs. (1)–(4) are solved for a three component system, consisting of an initially flat salt layer with a viscosity ratio \( m \) times lower than that of the sediments, a denser sediment layer and a zero density and much softer ‘sticky air’ layer on top (see Schmeling et al. 2008). The Eulerian 2D Finite Difference code FDCON based on a stream function formulation is used in combination with a marker approach based on a predictor–corrector Runge-Kutta fourth-order scheme (Weinberg & Schmeling 1992). As the material interfaces move through the finite-difference (FD) grid, care has to be taken of how the viscosity is interpolated to the FD-grid. Following the discussion of Schmeling et al. (2008) geometric averaging has been chosen.

### 2.2 Model setup

The equations are solved in a 4 km \( \times \) 4 km box, with a resolution of 101 \( \times \) 101 equidistant grid points. No slip has been assumed at the bottom and top of the model (the later is not important as it is the top of the weak sticky air layer) and the sides are assumed free slip, which implies symmetry.

Model time step is chosen according to the following criterion:

\[
\Delta t \leq \min(\Delta t_{\text{Courant}}, 10 \cdot \tau_{\text{relax}}). \tag{8}
\]

where

\[
\Delta t_{\text{Courant}} = \frac{\min(\Delta x, \Delta z)}{|v|} \tag{9}
\]

is the Courant criterion with \( \Delta x \) and \( \Delta z \) as grid increments and \( v \) as velocity and the isostatic relaxation time

\[
\tau_{\text{relax}} = \frac{4\pi \eta_{\text{salt}}}{\rho_{\text{salt}} g^2 \lambda}. \tag{10}
\]

of a half-space with \( \lambda \) as wavelength (chosen equal to the box width). Tests showed that our layered system remains stable for time steps up to 10 times this relaxation time.

Initially no sediments are present. To initiate down-building, sedimentation is modelled by successively elevating the initially perturbed level of the sediment layer by a prescribed sedimentation rate and transforming all ‘sticky air’ particles below this level into sediments (Fig. 1). This initial perturbation is assumed to represent a small magnitude of laterally varying sedimentation or externally driven differential loading during the early stage of evolution. The perturbation amplitude of the sediment surface, \( \delta \), is assumed to decrease linearly from an initial value \( \delta_0 \) at the time the sediment layer first appears at the salt surface, to zero when the highest point of the sediment layer has reached a level \( h_{\text{decay}} = 2\delta_0 \) above the initial salt surface. This assumption assures that externally driven differential loading acts only during the early stage of evolution to possibly initiate down-building, but at later stages the dynamics is no longer influenced by any variation of surface topography of the sediment layer. However, the whole process is still driven by differential loading even at the later stages of model run because sediment subsidence varies laterally.

### 2.3 Definition of successful and failed down-building

Because of the small initial perturbation at the sides, differential sediment loading drives the salt laterally towards the centre causing a vertical ascent rate at the position of the future diapir. Down-building is defined to be successful, if the top of this diapir rises faster than the sedimentation level until the complete initial salt layer is exhausted. Failed down-building is defined when fast sedimentation buries the salt layer and instead of a diapir only a concordant pillow forms. Our definition of successful down-building is based on the condition that the salt source layer is squeezed out and displaced into the rising diapir. When the sediment basins sink to the bottom of the salt layer, the basal part of the new diapir cannot rise further and the source layer starts exhausting. Consequently, the diapir stops to rise even though sedimentation, that is differential loading, may continue to increase. Therefore, our diapirs inevitably become completely buried. Yet, we classify those ones as down-built diapirs, because during their formation they always have been at the surface. As our models represent the end member of a stiff sediment, in natural systems deformation of the sediments might occur due to water weakening, granular flow or lateral forces which shorten the diapir, and diapirs might continue to reach to the surface even after exhausting of the source layer.

## 3 Model Results

We performed a total number of 171 model calculations varying the viscosity ratio, the sedimentation rate, the initial amplitude of the sediment perturbation and the wavelength (Table 1). To facilitate a presentation of the results, we give details of a reference model (Fig. 2a), starting with a 800-m-thick salt layer, a sedimentation rate
of 0.25 mm yr⁻¹, a cosine perturbation of 30-m amplitude and 4-km wavelength. Initially, sedimentation starts to cover the salt layer near x = 0 and 4 km and the edge of the sediment layer propagates laterally towards the centre of the box. The salt has a relatively low viscosity (5 × 10¹⁷ Pa s) and deforms by the sediments that sink into it and displace it towards the centre of the model where it rises to a level above the accumulating sedimentation level. As a result, sedimentation does not cover the top of the diapir, which continues to rise until the source layer is exhausted completely beneath the sedimentary basin. During this stage, further sedimentation results in increasing differential loading that accelerates the salt flow into the diapir. The lateral extent of the sediments placed on the top of the model is dictated by the amount of salt that is displaced into the diapir. A larger salt supply to the diapir thus results in an upward widening of the diapir that limits the lateral extent of the ‘deposited’ sediments. In contrast, a smaller salt supply that cannot outpace the sedimentation rate forms an upward-narrowing diapir, allowing the sediments to be deposited on a wider surface. This narrowing can be seen in the early stages of our reference model or in the case of the failed down-bulit diapir (Fig. 2d). According to our definition, the reference model is a successful case of down-building. As sedimentation continues, salt supply from the thinning source layer diminishes and the sediment surface starts to rise faster than the diapir. Hence, the diapir is covered with sediments after 5.99 Myr, which finally fills the box.

3.1 Model parameters

3.1.1 Variation of salt viscosity

Doubling the salt viscosity (10¹⁸ Pa s) with respect to the reference model and keeping the sediment viscosity constant slows down the deformation rate and the sediments do not sink as fast as in the reference model (Fig. 2b). Therefore, the diapir rises with a slower rate and a very narrow diapir forms providing only a narrow stem for subsequent salt flow of the source layer into the diapir. Even though the salt flows with a slower rate, the diapir reaches a higher final level than in the reference model. Because of its higher viscosity, the diapiric crest has a more pronounced dynamic topography (i.e. dynamic bulge; stages between 3.4 and 13.2 Myr in Fig. 2b), thus the final covering process of the mature diapir takes longer than in the reference model, and a small finger-like mini-diapir succeeds to reach an even higher level.

Decreasing the salt viscosity (10¹⁷ Pa s) allows fast flow of the salt by the sinking of the sediments (Fig. 2c) long before a broader area of the salt is covered with sediments as it was the case in the previous models. Interestingly, during the early stages of diapiric evolution (between 0.4 and 1.42 Myr), a slight (numerical) asymmetry strongly amplifies, and by a mutual feedback mechanism the sediment body on the left-hand side quickly grows inhibiting sediment accumulation on the right-hand side (Fig. 2b). Only at a later stage, when sediments on the left side has reached the bottom, the right-side sediments catches up and a wide, but short (1.92 km) columnar diapir forms in between and rises only a small distance exhausting the source layer. The flanks of this diapir are steeper (nearly vertical) than in the reference model and the model with higher viscosity.

With increasing salt viscosity the final height of the crest of the diapir is increasing strongly until the no-down-building regime is reached (Fig. 3).
3.1.2 **Variation of sedimentation velocity**

Keeping the salt viscosity as in the reference model ($5 \times 10^{17}$ Pa s) but increasing the sedimentation velocity to 0.4 mm yr$^{-1}$ covers the salt layer so fast, that it does not form an outlet (Fig. 2d). Instead, a sharp peak forms which finally is covered by the accumulating sediments. The differential loading building up at this stage is not strong enough to displace the salt with a rate faster than sedimentation. Because of the high viscosity of the overburden, the salt layer is not allowed to deform further during the remaining time of the model. However, it should be noted that this model will evolve into a classical Rayleigh–Taylor type diapir on a very long timescale because we considered linear viscous material behaviour. For geological timescales, this would be unrealistically too slow and is not directly relevant to the diapiric evolution we are focusing on in this paper.

3.1.3 **Variation of perturbation amplitude**

Different perturbation amplitudes lead to different diapir shapes and a shift of the boundary between the successful and failed down-built diapirs. Perturbation amplitudes smaller than that of the reference model (i.e. 30 m) lead to broad sediment coverage and a narrow salt
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outlet and ultimately formation of the narrow diapir stem. Large perturbation amplitudes produce broad and symmetric diapirs with subvertical side walls (Fig. 4, right).

A perturbation of 20 m, that is less than that of the reference model, causes a break in the symmetry of the shape of the diapir (Fig. 4, left). A marginal (numerical noise) excess in sedimentation in the left side amplifies the volumetric rate of sedimentation as the sediment surface level rises slowly with a constant rate. An imbalance of subsidence between left and right sides of the model is maintained until the left-side mini basin reaches the bottom. Only then, sedimentation on the right-hand side of the diapir takes over and finally an asymmetric diapir is generated.

3.1.4 Variation of the initial wavenumber

To check the role of wavenumber, the initial $k$ was varied between $3.75 \times 10^{-4}$ m$^{-1}$ and $3.125 \times 10^{-3}$ m$^{-1}$ [corresponding to wavelengths between 8 km (half box width) and 1 km]. Figs 5(a) and (b) show the regime boundaries in the $v_{sed}-k$–space and the $\eta_{salt}-k$–space, respectively, with the successful down-building regimes always below the curves. Clearly, within the wavenumber range of $6.25 \times 10^{-4}$ m$^{-1}$ to $1.875 \times 10^{-3}$ m$^{-1}$ down-building is most successful, which corresponds to a wavelength between three and nine times the thickness of the salt layer.

3.2 Down-building regimes in the parameter space

A series of models with four different model parameters $m = \eta_{salt}/\eta_{sed}$, $v_{sed}$, $\delta$ and $\lambda$ have been carried out to explore the four-parameter space. Fig. 6 shows a compilation of final stages for the subspace $\eta_{salt}-v_{sed}$ keeping $\delta = 30$ m and $\lambda = 4$ km constant. Small values of $\eta_{salt}$ and $v_{sed}$ result in small and broad diapirs, higher values lead to narrower and tall diapirs with slim waists (width of stem $< $ width of bulb). Failure of down-building occurs at high $\eta_{salt}$ or $v_{sed}$. The boundary between the two regimes is very sharp, failed cases do not form even salt pillows. Similarly, small $\delta$ values result in narrow stems and wide diapir bulbs, larger values lead to diapirs with subvertical side walls (cf. Fig. 4).

To generalize the result we use the non-dimensional parameters $m$, $v'_{sed}$, $\delta'$ and $k' = 2\pi h_{salt}/\lambda$, where $\delta$ and $v_{sed}$ are scaled using eqs (5) and (6), respectively. Near the transition from down-building to failure a dense set of models has been run to confine this transition range. Fig. 7(a) shows the regime boundaries in the $m-v'_{sed}$ space for different $\delta'$. Note that in contrast to Fig. 4, the boundaries are subhorizontal because $\eta_{salt}$ is used for scaling. Each curve, which stands for the non-dimensional critical sedimentation velocity, separates the down-building regime, for velocities smaller than the critical, from the regime where no down-building occurs, for velocities higher than the critical. The uncertainty is given by the vertical bar that indicates half the distance between the narrowest successful and narrowest failed model. Increasing $\delta'$ shifts the boundaries to higher sedimentation rates, with this shift depending linearly on the initial amplitude. This trend is quantified by taking the mean of the transition values of $v'_{sed}$ for the different $\delta'$ values and plotting them as a function of $\delta'$ (Fig. 7b). This indicates that for very small initial perturbations down-building will be inhibited even at slowest sedimentation rates.
Figure 5. Domain boundaries between down-building (below) and non-down-building (above) models for different initial wavenumbers. (a) $v_{\text{sed}} - k$ space with different salt viscosities in Pa s, (b) $\eta_{\text{salt}} - k$ space with different sedimentation rates in mm yr$^{-1}$.

3.3 A simple scaling law for the critical sedimentation velocity

A simple scaling law for the critical sedimentation rate separating successful and non-successful regimes can be derived by a simple channel flow model (Fig. 8). We define

$$I = \lambda c_1$$

as width of the early diapir or sediment free outlet ($c_1$ is a geometrical constant). Conservation of salt mass requires

$$v_{\text{diapir}} = \frac{h_{\text{salt}}}{T} \tilde{v},$$

where $v_{\text{diapir}}$ is the vertical salt velocity in the new diapir and $\tilde{v}$ is given by

$$\tilde{v} = -\frac{h_{\text{salt}}^2}{12\eta_{\text{salt}}} \frac{dP}{dx}.$$  \hspace{1cm} (13)

The average horizontal velocity of the channel flow, $\tilde{v}$, can be determined by using the continuity equation and integration from $x = 0$ to $L$, the point where the salt starts to rise. With this average velocity and eq. (13) it is possible to calculate the pressure that drives the flow of the salt diapir. Approximating this flow by a pure shear flow, diapiric rise velocity $v_{\text{diapir}}$ can be estimated (for a detailed derivation, see the appendix).

Equating $v_{\text{diapir}}$ with the critical sedimentation rate $v_{\text{sedcrit}}$ and using the scaling of eq. (6) we arrive at

$$v'_{\text{sedcrit}} = C_1 \frac{h_{\text{salt}}^2}{2\eta_{\text{salt}}} \frac{\rho'_{\text{sed}}}{k} \frac{\delta_0}{(kh_{\text{salt}})^2 (kh_{\text{salt}})^2 + C_2},$$

where all lengths are scaled by $h_{\text{salt}}$, the density is scaled by $\Delta \rho$ and $C_1$ and $C_2$ are constants to be determined by numerical models. In dimensional form, eq. (14) reads

$$v_{\text{sedcrit}} = C_1 \frac{h_{\text{salt}} \rho_{\text{sed}}}{2\eta_{\text{salt}}} \frac{\rho_0}{(kh_{\text{salt}})^2} \frac{(kh_{\text{salt}})^2}{(kh_{\text{salt}})^2 + C_2}.$$  \hspace{1cm} (14a)

Interestingly, the non-dimensional critical sedimentation rate is independent of the salt viscosity or the viscosity ratio $m$, whereas the dimensional critical velocity is inversely proportional to the salt viscosity. This is a consequence of the scaling law (6) and the assumption that the salt is significantly weaker than the sediment layer. This model (eqs 14 or 14a) is confirmed to first order by the numerical experiments (Figs 5, 6a and b). Evaluating the numerical results depicted in Figs 6 and 7 allows us to determine the constants $C_1$ and $C_2$ in eq. (14) as 0.0283 and 0.1171, respectively. Fig. 9 shows the analytical and numerical solutions for constant initial amplitude and the critical mean sedimentation velocities for different viscosities in the $v_{\text{sed}}-k'$ space. The mean velocities are calculated from the limit values for failed and successful down-building for different
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Figure 6. Final stages of the models in the viscosity – sedimentation rate space. Red symbols indicate cases where down-building failed, blue symbols (mostly behind the boxes) indicate successful down-building models. $\delta = 30$ m, $\lambda = 4$ km.

viscosities, which have been shown in Fig. 7(a). It should be noted that the analytical solution (14) breaks down at larger wave numbers because then the flow cannot be approximated by channel flow anymore. Thus, eq. (14) does not capture the decrease of the critical sedimentation velocity at higher $k'$ as observed in the numerical models (cf. Fig. 5a).

Eq. (14) allows to scale and apply our results to arbitrary scenarios. It reveals that the transition between down-building and burying of a salt layer is controlled quantitatively by the non-dimensional density of the sediment $\rho'_{sed}$, the non-dimensional magnitude of perturbations ($\delta'$) and the non-dimensional wavelength or wavenumber ($k'$), using the scaling laws given in (6). The non-dimensional values are essentially given by the salt viscosity as scaling viscosity, the initial thickness as a length scale and the density contrast as scaling density. It should be noted that the assumptions behind this simple relationship are sediments that from the early stage have a high viscosity and a higher density than the salt (i.e. are compacted).

4 DISCUSSION

4.1 Comparison with previous results

Compared to the models with an included stiff anhydrite layer (Chemia et al. 2007), there are small differences for the critical sedimentation velocities. The critical sedimentation rate of the models of Chemia et al. (2007) was calculated by taking the mean velocity between the zone of diapirism and the zone of no dipirism (see their Fig. 21). For a viscosity smaller than $5 \times 10^{18}$ Pa s, the critical velocity is 14–35 per cent smaller than in our models with an uncertainty of the same order. This may be explained by an effectively higher average viscosity of the salt-anhydrite layer, leading to smaller critical sedimentation rates (cf. Fig. 6). Results of our study and the one by Chemia et al. (2007) agree within the bound of uncertainty for models considering a salt viscosity higher than $5 \times 10^{18}$ Pa s.

4.2 Non-linear effects

It should be noted that despite constant viscosities and sedimentation velocity the chosen perturbation formulation results in a non-linear instability: lateral propagation of the edge of the early sediment layer is non-linear in time and space. For small perturbation amplitudes the lateral propagation velocity is fast, resulting in narrower outlet channels if the diapir is in the early stage. In turn

Figure 7. (a) Regime boundaries separating the successful down-building regime below from the failure regime above. The different curves are for different initial perturbations ($\delta$): blue: 20 m, black: 30 m, red: 40 m, green: 80 m. (b) Arithmetic mean of the transition values of $v'_{sed}$ for the different $\delta$ values as a function of $\delta$. The red data points were obtained with decay amplitude (see Section 2.2) increased and decreased by a factor of two and one half, respectively.
these narrow outlet channels result in narrow waists of the later diapirs or in complete failure of diapirism. These non-linearities may be the reason for the observed deviations from the simple relation (14), which are seen by the deviations up to a factor of 2 of the curves from the horizontal in Fig. 6a.

Another non-linear effect is associated with the volumetric sedimentation rates, which are not linear despite of the assumption of linear rise of sediment level. The base of the sediment basin subsides with a non-linear rate whereas the surface of the sediments rises with a linear rate. This leads to a non-linear increasing rate of the sediment volume, i.e. load. Small disturbances are communicated to neighbouring basins via viscous flow of the salt, which may lead to imbalances and negative feedback between neighbouring basins. Subsidence of one basin is accommodated by salt flow towards the evolving diapir. A small perturbation in one basin leads to salt flow towards the neighbouring basin impeding subsidence there. Such a perturbation will non-linearly grow and one basin will grow expedited on the expense of the neighbouring basin as observed in several cases (cf. Figs 2c and 4a).

4.3 Different decay amplitudes

In the previous models we used a decay amplitude of $h_{\text{decay}} = 2 \delta_0$ (see Section 2.2). The magnitude of this amplitude controls how long an increased differential loading lasts and therefore might affect the regime boundary between successful and failed downbuilding. We made some runs for two models with different viscosities for decay amplitude half and double of the reference amplitude to test the sensitivity of the regime boundary on the decay amplitudes. We found that there is a shift upward for higher amplitude by a factor of 1.62 and downwards by a factor of 0.84 (red stars in fig. 7b). Thus, although the regime boundary depends linearly on the amplitude of $\delta_0$, its dependence on the decay amplitude is significantly weaker than linear.

4.4 Surfacing or burying of well-developed diapirs

Our models predict that well developed or late stage salt diapirs may either reach close to the surface, sometimes even exceed it by their dynamic topography, or they may be buried by sediments. The latter case may easily be mixed up by incomplete diapiric rise. However, our models show that the reason instead may be exhausting the source layer followed by continuing sedimentation. Thereby the final thickness of the sedimentation layer is artificial as the model stopped after the sediments reached the top of the box. The higher the salt viscosity (with other parameters still putting it into the successful downbuilding regime), the taller and narrower the diapir will be (cf. Fig. 3). Thus, counter-intuitively, stiffer diapirs have a higher potential to rise close to the surface.
4.5 Neglecting compaction

In our models no compaction of the sediments has been assumed, whereas in nature unconsolidated sediments compact and gain strength upon burial. Thus, the high density needed for down-building is only reached after burial beneath a layer of certain thickness. We thus have to regard our models as non-compacting end-member models. However, we believe that models which would include compaction will not differ significantly. The main difference will be that in the early stage of sedimentation unconsolidated sediments of 10–20 per cent smaller density than in our models produce a slightly reduced load: This is equivalent to a slight reduction of perturbation amplitude in case of non-compacting denser sediments. At later stages we assume no differential sedimentation. Then, instead of a down-built diapir being always at the surface it will be covered by a lighter and softer layer of unconsolidated sediments. If the viscosity of these sediments would be of the order of $10^{16}$ Pa s, they might have a similar viscous friction effect on the diapir surface as the sticky air (which has a viscosity of $10^{16}$ Pa s in our models). Therefore, the unconsolidated sediments could have been modelled by a sticky air layer with a density 10–20 per cent smaller than the salt and a viscosity of the same order. In that case, due to the decreased density contrast at the interface between the surface of the salt diapir and the light, soft sediments its dynamic topography will increase, enhancing its rate of rise. Hence, the effect of compaction is probably captured by our models during the early phase, when a slightly reduced initial perturbation would be chosen, while at a later stage successful diapirs might rise to higher levels.

4.6 Variation of sedimentation rate

A main result of our study is a clear distinction between successful down-built diapirs with exhausted source layer and failed down-building cases not associated with any diapir. However, in nature, there are examples where well-developed salt diapirs are buried, although their feeding supply is not depleted (e.g. Nordkapp before the removal of 1- to 1.5-km-thick cover; ref. Richardsen 1992). According to our model predictions, such cases can be explained by a time-dependent increase in sedimentation rate. This case thus illustrates that the sedimentation rate had outpaced the rate of salt supply, that is the diapiric rise velocity.

On the other hand, if during successful down-building the sedimentation rate slows down or shuts off, the diapir will remain at the surface and due to its dynamic topography it will spread and develop into broad overhangs as it could be seen for salt glaciers like in the Gulf of Mexico, the Zagros and the Great Kavir.

4.7 Role of extension

Many studies have shown that salt diapirs are triggered by extension (Koyi 1991; Vendeville & Jackson 1992a, b; Koyi et al. 1993; Koyi 1998). Vendeville & Jackson (1992a, b) demonstrated that stiff overburden can bury a less dense rock salt which may never form a diapir unless the overburden is thinned and weakened by extension. In this study, the effect of lateral movement, extension or compression were not included in the modelling. However, speculating on the effect of extension on the diapirs, it is possible to argue that extension would have widened the space which the diapir occupied and as such assisted in exhausting the source layer; more salt would have been needed to fill the wider diapirs. Continued extension would have caused the diapirs to fall (Vendeville & Jackson 1992b; Koyi 1998). At the same time, by thinning and faulting of the overburden, extension would have assisted the buried narrow diapirs, which had sufficient supply to reactivate and rise further. On the other hand, compression would have pinched the feeding stem of the diapirs and thickened the overburden units (Koyi 1998).

4.8 Application to nature, case examples

Our models and the earlier anhydrite–salt models of Chemia et al. (2007) show upward widening due to horizontal spreading of the salt and the formation of a thin stem. In combination with the absence of upwarping of adjacent sediment layers, as one would expect it for active diapirism, these structures seem to be a good
identification for a down-built diapir. Several geometric features can be seen in some of the investigated diapirs in the Nordkapp region (Fig 10), which support the down-built nature of the diapiric evolution. Even though the exact boundaries of the diapir stem are not easy to outline in this example, the lower part of the diapir seems to have a constant width (Fig. 10). At the top, the diapir is widening and the salt spreads horizontally over the sediment layers, either due to a higher rising velocity (i.e. salt supply) or due to less sedimentation (Koyi et al. 1995). The sedimentary layers between the two diapirs are nearly horizontal, indicating that they do not seem to be pierced by a reactive salt diapir. Fig. 10(b) shows a diapir, which has not developed a wide overhang, that is the salt does not spread horizontally and does not cover the overburden layers. Besides, the intralower Triassic sediments of the western part of the diapir are thicker than their equivalents on the eastern part. A similar behaviour has been seen in our models, see for example Fig. 2(c) at 1.4 Myr, indicating an enhanced subsidence and sedimentation rate as salt of the source layer has been squeezed into the stem. An alternative explanation of this asymmetry could be the activation of a basement fault under the diapir. The sinking block (hanging wall block) of the fault created more space for sediments to accumulate and more salt has been withdrawn from this side.

5 CONCLUSION

The process of down-building of salt diapirs has been systematically modelled with emphasis on the physical conditions leading to successful down-building. These conditions as well as the geometry and the structure of the corresponding salt diapirs depend to first order on four parameters: salt viscosity, sedimentation rate, perturbation amplitude and wavelength. A scaling law relating the critical sedimentation rate to the other parameters has been found. The following conclusions can be drawn:

(1) With an increasing salt viscosity (keeping the sediment viscosity constant) tall and narrow diapirs are formed by down-building. However, above a critical viscosity down-building is not successful and the salt layer is buried.

(2) Increasing the sedimentation rate and keeping the salt viscosity constant has a similar effect: at high sedimentation rates down-building is not successful.

(3) A high perturbation amplitude forms broad and short diapirs, and shifts the critical sedimentation rate to higher values.

(4) As long as the salt is significantly weaker than the overburden layers down-building is most successful for wavelengths in the range three and nine times the thickness of the salt layer.

Within the chosen parameter space for our models we conclude that both our numerical experiments and the derived scaling law (eq. 14 or 14a) allow predicting whether a salt layer will evolve into a salt diapir by down-building during sedimentation or not. In reverse, the scaling law may be used to constrain past sedimentation rates.

ACKNOWLEDGMENTS

We are grateful to Zurab Chemia and Chris Talbot for fruitful and inspiring discussions on the down-building process. HK is funded by the Swedish Research Council (VR).

REFERENCES


APPENDIX: DERIVATION OF EQ. (14)

Given a certain sedimentation load of varying thickness δ(x) distributed on top of a salt layer between x = 0 and x = L we wish to determine the rising velocity of the uncovered salt diapir between x = L and z (Fig. A1). This diapir velocity is equated with the critical sedimentation velocity to give eq (14). To derive this equation, we assume the salt layer of thickness h being loaded by a stiff sediment layer and being squeezed sideways towards the diapir. This sediment load generates a vertical velocity,
which is approximated by a linear function
\[ \nu_z(x, z = h) = \frac{v_1 - v_0}{L} x + v_0, \] (A1)
where \( v_0 \) and \( v_1 \) are the vertical velocities at \( z = h \) and at \( x = 0 \) and \( x = L \), respectively. Integrating the continuity equation from 0 to \( z = h \) and from \( x = 0 \) to \( x \) gives the vertically averaged horizontal velocity
\[ \overline{v}_z(x) = -\frac{1}{h} \left[ \frac{1}{2} \frac{v_1 - v_0}{L} x^2 + v_0 x \right]. \] (A2)
We assume horizontal channel flow with
\[ \overline{v}_z(x) = -\frac{1}{\eta} \frac{h^2}{12} \frac{dP}{dx}, \] (A3)
where \( \eta \) is the viscosity of the layer (salt) and \( P \) is the dynamic pressure. Equating (A2) and (A3) and integration with respect to \( x \) from 0 to \( x \) gives the pressure as a function of \( x \)
\[ P(x) = P_0 + \frac{12 \eta}{h^3} \left( \frac{v_1 - v_0}{6L} x^3 + \frac{v_0}{2} x^2 \right). \] (A4)
Because the normal strain rates are small compared to the horizontal shear strain rate, the normal deviatoric stresses in the layer can be neglected with respect to the pressure. Thus, the total vertical load is
\[ F_{load} = \int_0^L \delta z \rho_\text{sed} \rho' \text{d}x' = c_1 L \delta_0 \rho_\text{sed}. \] (A5)
where \( \delta_0 \) is the sediment thickness at \( x = 0 \) and \( c_1 \) is a geometric constant of the order 0.5. Integration of (A4) from 0 to \( x = L \) and equating the result with (A5) (i.e. applying force balance) allows us to solve for \( P_0 \). Inserting this expression back into (A4) and evaluating \( P(x) \) at \( x = L \) gives after some manipulations
\[ P(L) = c_1 \delta_0 \rho_\text{sed} + \frac{2 \eta L^2}{h^3} \left[ \frac{3}{4} v_1 + \frac{5}{4} v_0 \right]. \] (A6)
We now assume that the salt diapir is squeezed out where it is not covered with sediments, that is between \( x = L \) and \( L \). This deformation can be approximated by a pure shear flow, whose horizontal deviatoric stress \( \tau_{xx} \) is related to \( P(L) \) by
\[ P(L) = -c_2 \tau_{xx}, \] (A7)
where the geometric constant \( c_2 \) captures deviations due to the no slip bottom, which depends on the ratio \( (L - L)/h \). Because of simple numerical tests, we expect \( c_2 \) being of the order 1–10. Assuming homogeneous pure shear within the diapir region allows writing the rising velocity of the salt diapir
\[ v_{dia} = -\frac{h}{2\eta} \tau_{xx}. \] (A8)
The continuity equation can be evaluated for the salt diapir region giving
\[ \nu_z(x = L) h = v_{dia}(L - L). \] (A9)
Combining (A2) taken at \( x = L \) with (A6) to (A9) allows to eliminate \( v_0 \) yielding
\[ v_{dia} = \frac{c_1 \delta_0 \rho_\text{sed} h}{2c_2 \eta} \left[ \frac{1}{2} v_1 + 5c_3 v_{dia} \right], \] (A10)
where the geometric constant \( c_3 \) gives the width of the diapir relative to the width of the model
\[ c_3 = \frac{L - L}{L}. \] (A11)
As the vertical velocity \( v_1 \) at \( x = L \) at the point between the diapir and the sediment layer cannot be constrained by this simple analysis, we simply take
\[ v_1 = c_4 v_{dia}, \] (A12)
where \( c_4 = 1 \) represents the case in which the edge of the sediment layer rises with the diapir velocity, whereas \( c_4 = 0 \) represents the case where this point does not move vertically representing the boundary between sedimentary subsidence region to the left and the rising diapir region to the right. We expect \( c_4 \) to lie between 1 and slightly less than 0, probably close to 0. Inserting (A12) into (A10), defining the wavenumber
\[ k = \frac{2\pi}{\lambda} = \frac{\pi}{L}, \] (A13)
and using the scaling of (6) yields the final equation
\[ \nu'_{dia} = \frac{1}{2c_1 \delta_0 \rho_\text{sed} k^2 + C_2}, \] (A14)
where the two new constants relate to the former geometrical constants by
\[ C_1 = \frac{c_1}{c_2}, \quad C_2 = \frac{\pi^2 (1 - c_3)^2 c_4 + 5c_3 (1 - c_3)}{2c_2}. \] (A15)