Department of Economics
Working Paper 2011:16

A Case for Interest Rate Inertia in Monetary Policy

Mikael Bask
A CASE FOR INTEREST RATE INERTIA IN MONETARY POLICY

MIKAEL BASK
A Case for Interest Rate Inertia in Monetary Policy

Mikael Bask

Version: October 17th, 2011

Abstract: We argue that it is not necessary for the central bank to react to the exchange rate to have a desirable outcome in the economy. Indeed, when the Taylor rule includes contemporaneous data on the variables in the rule, the central bank can disregard from the exchange rate as long as there is enough with interest rate inertia in monetary policy. The reason is that interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy. Hence, we give a rationale for the central bank to focus on the interest rate change rather than the interest rate level to have a desirable outcome in the economy, which we define as a determinate rational expectation equilibrium that is stable under least squares learning.

JEL codes: E52; F31.

Keywords: Determinacy; Foreign Exchange; Interest Rate Inertia; Least Squares Learning; Monetary Policy; Taylor Rule.

---

1 An earlier version of this paper has benefited from a presentation at CERGE-EI in Prague, Czech Republic. The usual disclaimer applies.

2 Department of Economics, Uppsala University, SE-751 20 Uppsala, Sweden. E-mail: mikael.bask@nek.uu.se
1 Introduction

It is argued in Taylor (2001) that it is not necessary to react to the exchange rate in monetary policy to have a desirable outcome in the economy. Taylor’s (2001) argument is that the indirect effects that the exchange rate has on monetary policy, via its effects on the inflation rate and output, are to prefer because they result in fewer and less erratic changes in the interest rate. Taylor (2001, p. 267) writes that “[r]esearch to date indicates that monetary-policy rules that react directly to the exchange rate, as well as to inflation and output, do not work much better in stabilizing inflation and real output and sometimes work worse than policy rules that do not react directly to the exchange rate”. The research that John Taylor had in mind include Ball (1999) and Svensson (2000) who investigate the role of the exchange rate in monetary policy in calibrated macroeconomic models, where they find little support for the view that the central bank should react to the exchange rate in its policy.

In more recent research, Alexandre et al. (2011) present a Markov-switching model in which the exchange rate is in one of two states: (i) in a regime in which the exchange rate randomly oscillates around its equilibrium; and (ii) in a bubble regime in which the deviations of the exchange rate from its equilibrium are persistent, possibly explosive. The authors augment different specifications of the central bank’s interest rate rule with the exchange rate and find limited welfare gains of reacting to the exchange rate in monetary policy. A model that is close in spirit to the one in Alexandre et al. (2011) is the Markov-switching model in Zampolli (2006). The author finds that when the central bank in the model economy knows the transition probabilities between the states, the exchange rate should be included in the interest rate rule because it improves macroeconomic performance.

Adolfson (2007) examines the performance of various interest rate rules in a macroeconomic model with incomplete exchange rate pass-through and finds limited welfare gains of including the exchange rate in the monetary policy rule. Batini et al. (2003) and Kollmann (2002) assume incomplete exchange rate pass-through in their macroeconomic models as well and also find limited welfare gains of reacting to the exchange rate in monetary policy. However, Wollmershäuser (2006) finds that a high degree of exchange rate uncertainty has the effect that the exchange rate should be included in the interest rate rule. Leitemo and Söderström (2005) examine the effect of exchange rate uncertainty on monetary policy design as well but find that the central
The main difference between the latter two papers is that the degree of exchange rate uncertainty is higher in Wollmershäuser (2006) than it is in Leitemo and Söderström (2005).

On the one hand, Divino (2009) derives optimal monetary policy rules, both under discretion and commitment in monetary policy, and finds that the central bank should not react to the exchange rate in its policy. On the other hand, Cavoli (2008) derives optimal monetary policy rules under different assumptions about the central bank’s loss function and finds that the exchange rate should be included in the interest rate rule in targeting regimes such as CPI and domestic inflation rate targeting regimes. Finally, Bask and Selander (2009) present a macroeconomic model with heterogeneity in currency trade and find that the central bank should react to the exchange rate in its policy. Specifically, currency trade is not only determined by fundamental analysis of the expected exchange rate but also by chartist analysis in the form of trend extrapolation of the exchange rate. The reason why the conclusions in the papers differ from each other is that the model economies, including the specifications of the central bank’s loss function, differ somewhat in the papers.

Of all the papers discussed above, only three of them come to the conclusion that monetary policy should be directly affected by the exchange rate. In Wollmershäuser’s (2006) case, a high degree of exchange rate uncertainty results into an exchange rate reaction in the monetary policy rule; in Cavoli’s (2008) case, some specifications of the central bank’s loss function result into an exchange rate reaction in the optimal monetary policy rule; and in Bask and Selander’s (2009) case, heterogeneity in currency trade results into an exchange rate reaction in the monetary policy rule. Undoubtedly, we have not discussed all the papers in the literature but it is anyhow alluring to agree with Taylor (2001, p. 267) when he writes that “monetary-policy rules that react directly to the exchange rate [...] sometimes work worse than policy rules that do not react directly to the exchange rate”, even though the arguments differ in the papers discussed herein.

We re-examine the question whether the central bank should react to the exchange rate in its policy by embedding three specifications of the monetary policy rule into the small open economy derived in Galí and Monacelli (2005). In the first specification, contemporaneous data on the output gap, the CPI inflation rate and the nominal exchange rate change are included in the rule, whereas, in the second specification, forward expectations of the same variables are included in the rule. In the third specification, both contemporaneous data and
forward expectations of the variables are included in the rule. The interest rate in the previous time period is also included in all three interest rate rules to allow for interest rate inertia in monetary policy. We are therefore able to answer the following question in the paper: Should the central bank focus on the interest rate level or the interest rate change to have a desirable outcome in the economy? Recall that monetary policy inertia sometimes is criticized in the financial press when interest rate adjustments are claimed to be too slow. The question is therefore relevant to answer.

Monetary policy inertia is a well-documented feature of central bank behavior in developed countries (see Bullard and Mitra, 2007). Rudebusch (1995), for example, finds that the Federal Reserve’s policy can be characterized by interest rate inertia. Moreover, the inclusion of the exchange rate in the interest rate rule is investigated in Lubik and Schorfheide (2007) and they find that the Bank of Canada and the Bank of England react to the exchange rate, whereas this is not the case for the Reserve Bank of Australia and the Reserve Bank of New Zealand.

What is a desirable outcome in the economy? First, we search for regions in the interest rate rule’s parameter space that generate a determinate and stable rational expectation equilibrium (REE). The reason is that the central bank would like to avoid coordination problems in the economy. For instance, without imposing additional restrictions into a rational expectation model, it may not be known in advance which of the REE that agents will coordinate on, if there will be any coordination at all. Second, a determinate REE should be stable under least squares learning. The reason is that rational expectation is a rather strong assumption because it assumes that agents have an outstanding capacity when it comes to deriving equilibrium outcomes of the variables in, say, a macroeconomic model (see Evans and Honkapohja, 2001, for an introduction to learning in macroeconomics).

There are two papers that are closely related to the present paper. The first is Llosa and Tuesta (2008) who examine the inclusion of the exchange rate in the interest rate rule in an open economy but neglects from interest rate inertia in monetary policy, and the second is Bullard and Mitra (2007) who allow for interest rate inertia in monetary policy but for a closed economy. Llosa and Tuesta (2008) use the Galí and Monacelli (2005) model in their analysis as also we do, and Bullard and Mitra (2007) use the closed economy version of the same model in the analysis.
It turns out that the present paper fills the gap between Bullard and Mitra (2007) and Llosa and Tuesta (2008) with three interesting findings. First, it is easier to achieve a desirable outcome in the economy when the central bank focuses on the change in the interest rate rather than the level. Second, the specification of the interest rate rule that includes contemporaneous data on the variables in the rule is more suitable to deliver a desirable outcome in the economy than the other two specifications of the interest rate rule. This finding is encouraging because it means that the central bank does not have to predict the outcome in the economy to be able to implement a successful policy.

Third, it is not necessary for the central bank to react to the exchange rate in its policy to have a determinate REE that is stable under least squares learning. The reason is that interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy. The intuition behind this finding is that a parity condition at the international asset market (i.e., uncovered interest rate parity, UIP) ties the current interest rate and the expected nominal exchange rate change together. Further, because this parity condition is assumed to hold in every time period, it also held in the previous time period, meaning that the parity condition also ties the interest rate in the previous time period and the current nominal exchange rate change together. The current nominal exchange rate change can therefore be replaced by interest rate inertia in the monetary policy rule without affecting the properties of the model economy.

We have the following link between Bullard and Mitra (2007), Llosa and Tuesta (2008) and the present paper: Llosa and Tuesta (2008) show that an interest rate rule that includes an exchange rate reaction can help alleviate the indeterminacy problem but at the cost of greater volatility in the economy (cf., Taylor, 2001). We show that the central bank can shift focus from an exchange rate reaction to interest rate inertia in its policy. Consequently, interest rate inertia in monetary policy should help alleviate the indeterminacy problem as well. This is also the main finding in Bullard and Mitra (2007) for a closed economy, meaning that we have broadened their finding to an open economy.

Hence, we give a rationale for the central bank to focus on the change in the interest rate rather than the level to have a desirable outcome in the economy. This is an important result from the point of view of practical policy-making: The central bank can, instead of focusing on a variable that it does not have direct control over (i.e., the exchange rate), concentrate on a variable that it does have direct control over (i.e., the interest rate in the
previous time period). Accordingly, the present paper adds to the literature that comes to the conclusion that it is not necessary to react to the exchange rate in monetary policy to have a desirable outcome in the economy.

The rest of this paper is organized as follows: The model economy is outlined in Section 2, whereas the properties of this economy are investigated in Section 3 under three specifications of the monetary policy rule. Section 4 concludes the paper with a discussion of its main findings.

2 A small open economy

The baseline model for a small open economy is outlined in Section 2.1. In Section 2.2, we present three specifications of the monetary policy rule that thereafter, in Section 3, are embedded into the baseline model.

2.1 Baseline model

A dynamic stochastic general equilibrium (DSGE) model for a small open economy with imperfect competition and nominal rigidities is derived in Galí and Monacelli (2005). Their model economy can be reduced to a dynamic IS-type equation and a new Keynesian Phillips curve:

\[ \begin{cases} \dot{x}_t = E_t[x_{t+1}] - \alpha(r_t - E_t[\pi_{H,t+1}] - \bar{r}_t), \\ \pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \gamma x_t, \end{cases} \]

where \( x \) is the output gap, \( r \) is the nominal interest rate that is controlled by the central bank, \( \pi_{H} \) is the domestic inflation rate and \( \bar{r} \) is the natural rate of interest. Moreover, \( E_t[\cdot] \) is the rational expectation of the variable in focus, where the dating of expectation is time period \( t \). All variables, except the interest rates, are in natural logarithms.

Notice that the closed economy version of the Galí and Monacelli (2005) model reduces to the typical prototype model that often is employed when examining monetary policy issues within the New Keynesian framework (see, e.g., Clarida et al., 1999, and Woodford, 2003). This means that the findings in this paper are not dependent on some peculiar formulation of a macroeconomic model that otherwise is rarely studied in the monetary policy literature. Besides, if we have the two papers that are closely related to the present paper in mind, Llosa and Tuesta (2008) use the Galí and Monacelli (2005) model in their analysis as well, whereas Bullard and Mitra (2007) use the closed economy version of the same model in the analysis.
Alas, (1) is not in a suitable form because there is no exchange rate in the equations. However, one can use the following equations that are derived in Galí and Monacelli (2005) to rewrite (1) into a suitable form:

\[
\begin{align*}
\pi_t &= \pi_{H,t} + \delta \Delta s_t, \\
s_t &= e_t + p_t^* - p_{H,t},
\end{align*}
\]

where \(\pi\) is the CPI inflation rate, \(s\) is the terms of trade, \(e\) is the nominal exchange rate that is the domestic price of the foreign currency, \(p^*\) is the index of foreign goods prices, \(p_{H}\) is the index of domestic goods prices and the asterisk refers to a foreign quantity. If we rewrite the equations in (1) with help of those in (2), we get the first two equations in the baseline model:

\[
\begin{align*}
x_t &= E_t[x_{t+1}] - \alpha \left( r_t - \frac{1}{1-\delta} \cdot \left( E_t[\pi_{t+1}] - \delta (E_t[\Delta e_{t+1}] + E_t[p_{t+1}^*]) \right) \right), \\
\pi_t &= \beta E_t[\pi_{t+1}] + \gamma (1-\delta) x_t + \delta (\Delta e_t - \beta E_t[\Delta e_{t+1}] + p_{t+1}^* - \beta E_t[p_{t+1}^*]).
\end{align*}
\]

where interpretations of the parameters in (3) are found in Galí and Monacelli (2005). The third equation in the baseline model, which also is derived in Galí and Monacelli (2005), is the UIP condition:

\[
r_t - r_t^* = E_t[\Delta e_{t+1}].
\]

To close the baseline model in (3)-(4), we augment it with a Taylor rule for the central bank.

2.2 Three specifications of the interest rate rule

The central bank uses a Taylor rule when setting the nominal interest rate, where three specifications of this rule are embedded into the baseline model in (3)-(4): (i) contemporaneous data on the variables in the rule:

\[
r_t = \zeta_x x_t + \zeta_\pi \pi_t + \zeta_e \Delta e_t,
\]

(ii) forward expectations of the variables in the rule:

\[
r_t = \zeta_x E_t[x_{t+1}] + \zeta_\pi E_t[\pi_{t+1}] + \zeta_e E_t[\Delta e_{t+1}],
\]

and (iii) both contemporaneous data and forward expectations of the variables in the rule:

---

3 First, write the second equation in (2) in relative form: \(\Delta s_t = \Delta e_t + \Delta p_t^* - \Delta p_{H,t} = \Delta e_t + p_t^* - \pi_{H,t}\). Second, substitute this equation into the first equation in (2) and solve thereafter for \(\pi_{H,t}: \pi_{H,t} = \frac{1}{1-\delta} \cdot \left( E_t[\pi_{t+1}] - \delta (E_t[\Delta e_{t+1}] + E_t[p_{t+1}^*]) \right) \) (3a). Third, forward (3a) one time period: \(E_t[\pi_{H,t+1}] = \frac{1}{1-\delta} \cdot \left( E_t[\pi_{t+1}] - \delta (E_t[\Delta e_{t+1}] + E_t[p_{t+1}^*]) \right) \) (3b). Finally, substitute (3a) into the first equation in (1), substitute (3a)-(3b) into the second equation in (1), and (3) is derived.
\( r_t = \zeta_r r_{t-1} + \zeta_x E_t[x_{t+1}] + \zeta_p E_t[\pi_{t+1}] + \zeta_e \Delta e_t, \)

where we have included the interest rate in the previous time period in all three specifications to allow for interest rate inertia in monetary policy. Two interesting special cases are when \( \zeta_r = 0 \) and \( \zeta_r = 1 \), respectively, in (5)-(7), because examining the properties of the model economy in these special cases allow us to answer a relevant question in practical policy-making: Should the central bank focus on the interest rate level or the interest rate change to have a desirable outcome in the economy? However, as a complement to this analysis, all degrees of monetary policy inertia are examined as well.

One could also imagine a specification of the Taylor rule that includes the output gap, the CPI inflation rate and the nominal exchange rate change in the previous time period. However, because Galí and Monacelli (2005) assume time-\( t \) dating of expectations in the derivations of their model economy, we neglect from such a specification in the analysis. To pose two rhetorical questions: (i) why should the central bank use time-\( t-1 \) dating of expectations when households and firms use time-\( t \) dating of expectations?; and (ii) why should the central bank react to old information if they use time-\( t \) dating of expectations? We will return to our interest in the Taylor rule in (7) and to a discussion of optimal interest rate rules in monetary policy in Sections 3.3 and 4, respectively.

Finally, the vigilant reader might object that Taylor (2001) refers to the real exchange rate in his discussion, whereas we have included the nominal exchange rate in all three specifications of the Taylor rule. However, it is an easy exercise to transform the Taylor rules in (5)-(7) to rules that are functions of the real exchange rate, \( q \), via the following identity:

\( \Delta q_t \equiv \Delta e_t + \pi_t^* - \pi_t. \)

3 Properties of the model economy

In Section 3.1, the Taylor rule in (5) is embedded into the baseline model that thereafter is analyzed. Specifically, the conditions for a determinate REE that is stable under least squares learning are examined. Thereafter, in Sections 3.2-3.3, the Taylor rules in (6)-(7) are embedded into the baseline model that thereafter is analyzed.
3.1 Contemporaneous data in the interest rate rule

After substituting the Taylor rule in (5) into the baseline model in (3)-(4), the model economy in matrix form is

\[ \Gamma \cdot y_t = \Theta \cdot E_t [y_{t+1}] + A \cdot y_{t-1} + \Xi, \]

where the state of the economy is

\[ y_t = [x_t, \pi_t, \Delta e_t, r_t]' , \]

and the coefficient matrices are

\[ \Gamma = \begin{bmatrix} 1 & 0 & 0 & \alpha' \\ -\gamma(1 - \delta) & 1 & -\delta & 0 \\ 0 & 0 & 0 & 1 \\ -\zeta_x & -\zeta_\pi & -\zeta_e & 1 \end{bmatrix}, \]

\[ \Theta = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & -\frac{\alpha \delta}{1-\delta} & 0 \\ 0 & \beta & -\beta \delta & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \]

and

\[ A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \zeta_r \end{bmatrix}. \]

The vector \( \Xi \) in (9) is a constant and is therefore not affecting our findings for a desirable outcome in the economy (see, e.g., Bullard and Mitra, 2002). The natural rate of interest, the foreign interest rate and the foreign CPI inflation rate are all parts of the constant but one could also assume that they are the exogenous driving forces in the model economy. Yet, they would not affect our findings for a desirable outcome in the economy, if we assume that these driving forces are stationary.

To be able to determine if the model economy has a determinate and stable REE, a first step is to rewrite it into first-order form and then to compare the number of predetermined variables with the number of eigenvalues of a certain matrix (that we soon will derive) that are outside the unit circle (see Blanchard and Kahn, 1980). Specifically, we use the following variable vector when rewriting the model economy in (9)-(13):
\( y_{d,t} = [y_t, r_t^L \equiv r_{t-1}]' \),

meaning that the coefficient matrices are

\[
\Gamma_d = \begin{bmatrix} 0_{(1x3)} & \Gamma & -A_4 \end{bmatrix},
\]

where the vector \( A_4 \) is the fourth column in the matrix \( A \) and

\[
\theta_d = \begin{bmatrix} \theta & 0_{(4x1)} \end{bmatrix},
\]

because the model economy in matrix form is now

\[
\Gamma_d \cdot y_{d,t} = \theta_d \cdot E_t[y_{d,t+1}].
\]

Alas, the matrix \( \Gamma_d \) in (15) (and \( \theta_d \) in (16)) is singular, meaning that the matrix \( \Gamma_d^{-1} \cdot \theta_d \) (and \( \theta_d^{-1} \cdot \Gamma_d \), which is essential when determining if the model economy has a determinate and stable REE, does not exist. One way to solve this problem is by substituting out the current and expected nominal exchange rate changes from the equations. That is, to use the UIP condition in (4) to substitute out \( E_t[\Delta e_{t+1}] \) and to use the same equation shifted one time period backwards in time to substitute out \( \Delta e_t \). In the latter case, the UIP condition is

\[
r_{t-1} - \hat{r}_{t-1} = E_{t-1}[\Delta e_t] = \Delta e_t + \varepsilon_t,
\]

where the dating of expectation is time period \( t - 1 \) and \( \varepsilon \) is the expectation error (that does not affect our findings for a desirable outcome in the economy). After doing these substitutions, the variable vector in (17) is

\[
y_{d,t} = [x_t, \pi_t, r_t, r_t^L \equiv r_{t-1}]',
\]

and the coefficient matrices in (17) are

\[
\Gamma_d = \begin{bmatrix} \frac{\alpha}{1-\delta} & 0 & 0 \\
-\gamma(1-\delta) & 1 & \beta \delta & -\delta \\
-\zeta_x & -\zeta_{\pi} & 1 & -(\zeta_{e} + \zeta_{\varepsilon}) \\
0 & 0 & 1 & 0 \end{bmatrix},
\]

and
\[ \theta_d = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \]

3.1.1 Main results

What we directly can see is that interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy because the parameters \( \zeta_r \) and \( \zeta_e \) in the Taylor rule in (5) appear at the same place in the matrix \( G_d \) in (20) and also with the same coefficient (i.e., \(-1\)). The intuition behind this finding is that the UIP condition in (18) ties the interest rate in the previous time period and the current nominal exchange rate change together. Thus, our finding is this:

**Proposition 1** When contemporaneous data are used in the Taylor rule, interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy.

It might be argued that Proposition 1 is not a surprising finding. Nevertheless, we are not aware that it has been emphasized in the literature before. Moreover, Proposition 1 is an important finding from the point of view of practical policy-making because it implies that the central bank can shift focus from an exchange rate reaction to interest rate inertia in its policy. Thus, Proposition 1 means that the central bank can concentrate on a variable that it does have direct control over in its policy (i.e., the interest rate in the previous time period), instead of focusing on a variable that it does not have direct control over (i.e., the exchange rate). Be aware that Proposition 1, and also Corollary 2 below, do not depend on the parameter values in the model economy, including the value of \( \zeta_r \). Thus, our findings in Proposition 1 and Corollary 2 are not restricted to the special cases \( \zeta_r = 0 \) and \( \zeta_r = 1 \); they are valid for all degrees of interest rate inertia in monetary policy.

If we continue with the derivations of the conditions for a determinate and stable REE, it is not always self-evident which variables in a model economy that are predetermined. However, by looking at the entries in the relevant matrix,

\[ G_d^{-1} \cdot \theta_d = \begin{bmatrix} - & - & 0 & - \\ - & - & 0 & - \\ 0 & 0 & 0 & 1 \\ - & - & 0 & - \end{bmatrix}, \]
we conclude that \( r \) in (19) is predetermined. Exactly one eigenvalue of the matrix \( \Gamma_d^{-1} \cdot \Theta_d \) must therefore be outside the unit circle to have a determinate and stable REE in the model economy. Thus, if more than one eigenvalue are outside the unit circle, we have an indeterminate REE in the model economy, and if no eigenvalue is outside the unit circle, there is no stable REE in the model economy.

Further, a determinate and stable REE is E-stable and therefore also least squares learnable because time-\( t \) dating of expectations is assumed in the Galí and Monacelli (2005) model. Two theoretical results lie behind this finding. First, McCallum (2007) shows that for a broad class of linear rational expectation models, which includes the model economy in (9)-(13), a determinate REE is E-stable when the dating of expectations is time period \( t \). Second, Marcet and Sargent (1989) show that an E-stable REE is a necessary and sufficient condition for a REE to be stable under least squares learning. The determinacy regions found in the figures below are therefore regions for a least squares learnable REE as well. Thus, our finding is this:

**Corollary 2** When contemporaneous data are used in the Taylor rule and a main objective for the central bank is to achieve a determinate REE that is stable under least squares learning, interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy.

Thus, we have a case for interest rate inertia in monetary policy because Corollary 2 implies that the central bank can shift focus from an exchange rate reaction to interest rate inertia in its policy and, at the same time, preserve a desirable outcome in the economy. But how strong must the interest rate inertia be in the first place to have a desirable outcome in the economy?

### 3.1.2 Numerical analysis

Derivations of analytical conditions for determinacy and least squares learnability of the REE are not reachable in practice because these expressions would be too large and also too cumbersome to interpret. We therefore illustrate our findings numerically\(^4\) with the use of the following two parameter sets for the model economy:

\[
\begin{align*}
\alpha &= \frac{1}{0.157}, \\
\beta &= 0.99, \\
\gamma &= 0.024, \\
\delta &= 0.15, \\
\zeta_r &= 0, \\
\zeta_x &= 0.5,
\end{align*}
\]

and

\[^4\text{MATLAB routines for this purpose are available on request from the author.}\]
\[
\begin{align*}
\begin{cases}
\alpha &= 1, \\
\beta &= 0.99, \\
\gamma &= 0.3,
\end{cases} \\
\begin{cases}
\delta &= 0.15, \\
\zeta_r &= 0, \\
\zeta_x &= 0.5.
\end{cases}
\end{align*}
\]

See Woodford (1999) for the parameter values of $\alpha$, $\beta$ and $\gamma$ in (23), and see Clarida et al. (2000) for the parameter values of $\alpha$, $\beta$ and $\gamma$ in (24), where both sets of parameter calibrations are based on U.S. data. Clearly, even though the time periods in the Clarida et al. (2000) and the Woodford (1999) calibrations are somewhat different, there are large differences in the magnitudes of $\alpha$ and $\gamma$. We will not discuss possible reasons for these differences herein. Instead, we will utilize the fact that the parameter calibrations are very different to check the robustness of our findings. Have also in mind that the model economy we examine is not an approximation of reality but instead a caricature of it (see Gibbard and Varian, 1978). This fact is often overlooked in the literature when caricature models are fitted to data as they were approximations of reality.

The economy’s openness index, $\delta = 0.15$, approximates the import/GDP ratio for the U.S. economy. Regarding the parameter values in the Taylor rule, we set the output gap reaction to $\zeta_x = 0.5$ because this value is close to what is found in U.S. data (see Clarida et al., 2000). When it comes to the degree of interest rate inertia in monetary policy, we examine two special cases: (i) the central bank focuses on the interest rate level by setting $\zeta_r = 0$; and (ii) the central bank focuses on the interest rate change by setting $\zeta_r = 1$. However, as a complement to this analysis, all degrees of monetary policy inertia are examined as well.

3.1.3 No interest rate inertia in monetary policy

In Figure 1, which is based on the parameter set in (23), there is no interest rate inertia in monetary policy. Two findings in the figure are worth to be noted. First, if the inflation rate reaction in the Taylor rule is large enough, the central bank can disregard from the exchange rate in its policy and nevertheless achieve a determinate REE that is stable under least squares learning. Second, if the central bank increases the interest rate when the domestic currency depreciates in value (i.e., $\zeta_e > 0$), it is not necessary to increase the interest rate one-to-one in response to a higher inflation rate to achieve a determinate REE that is stable under least squares learning. Thus, the Taylor principle does not have to be fulfilled in monetary policy to have a desirable outcome in the economy.
Figure 1: Contemporaneous data in the Taylor rule, no interest rate inertia and the Woodford (1999) calibration.

The parameter set in (24) lies behind the construction of Figure 2. Moreover, as in Figure 1, there is no interest rate inertia in monetary policy. If we compare the main findings in Figure 2 with those in Figure 1, there are no qualitative changes in the findings: (i) if the inflation rate reaction in the Taylor rule is large enough, the central bank can disregard from the exchange rate in its policy and nevertheless achieve a determinate REE that is stable under least squares learning; and (ii) it is not necessary for the central bank to satisfy the Taylor principle in its policy to have a desirable outcome in the economy.
3.1.4 Interest rate inertia in monetary policy

In Figure 3, which is based on the parameter set in (23), the central bank focuses on the interest rate change in its policy,

\[ \Delta r_t \equiv r_t - r_{t-1} = \zeta_x x_t + \zeta_\pi \pi_t + \zeta_\epsilon \Delta e_t, \]

meaning that there is interest rate inertia in monetary policy. It turns out that the findings in Figure 1 still hold but with one important exception: Due to monetary policy inertia, the lower bound of the region (in Figure 3) for a desirable outcome in the economy shifts downwards. In fact, when the degree of interest rate inertia is \( \zeta_r = 1 \), this shift is so large that the central bank does not have to bother about the inflation rate or the exchange rate in its policy to achieve a determinate REE that is stable under least squares learning.
**Figure 3:** Contemporaneous data in the Taylor rule, interest rate inertia and the Woodford (1999) calibration.

The parameter set in (24) lies behind the construction of Figure 4. Moreover, as in Figure 3, the central bank focuses on the interest rate change in its policy rather than the interest rate level. If we compare the main findings in Figures 3-4, we have the same qualitative findings. For example, the central bank can disregard from the inflation rate and the exchange rate in its policy and nevertheless achieve a desirable outcome in the economy.
Even though Bullard and Mitra (2007) allow for interest rate inertia in monetary policy, they do not consider a contemporaneous data specification of the Taylor rule. This means that our analysis complements their analysis in two directions: (i) monetary policy inertia in an open economy; and (ii) monetary policy inertia in a contemporaneous data specification of the Taylor rule.

### 3.2 Forward expectations in the interest rate rule

Due to the fact that the analysis of the model economy herein is similar to the analysis in Section 3.1, it is not necessary to repeat every step in detail and we are therefore more concise in the presentation.

First, the model economy in matrix form is once more (9)-(13) with the exception that the elements in the last rows in the coefficient matrices instead reflect the Taylor rule in (6). After rewriting the model economy into first-order form, it is easily verified that we again have a problem with singular matrices when trying to derive the conditions for a desirable outcome in the economy. We therefore substitute out the current and expected nominal exchange rate changes from the equations by using the UIP conditions in (4) and (18), meaning that the model economy in matrix form is (17), the variable vector is (19) and the coefficient matrices are
\[ \Gamma_d = \begin{bmatrix} 1 & 0 & \frac{a}{1-\delta} & 0 \\ -\gamma(1-\delta) & 1 & \beta \delta & -\delta \\ 0 & 0 & 1 - \xi_c & -\xi_r \\ 0 & 0 & 1 & 0 \end{bmatrix}, \]

and

\[ \Theta_d = \begin{bmatrix} 1 & \frac{a}{1-\delta} & 0 & 0 \\ 0 & \beta & 0 & 0 \\ \xi_x & \xi_c & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \]

Second, the essential matrix for the determinacy and learnability results, \( \Gamma_d^{-1} \cdot \Theta_d \), has the same form as in (22). Consequently, \( r \) in (19) is predetermined in this case as well, which means that we have the same conditions as in Section 3.1 to have a determinate REE that is least squares learnable, an indeterminate REE and no stable REE in the model economy.

3.2.1 Main results

Interest rate inertia and a reaction to the expected nominal exchange rate change are not perfect substitutes in monetary policy. The reason is that the UIP conditions in (4) and (18) are not able to tie the interest rate in the previous time period and the expected nominal exchange rate change together. This is also apparent in the matrix \( \Gamma_d \) in (26) because the parameters \( \xi_r \) and \( \xi_e \) in the Taylor rule in (6) do not appear at the same place in the matrix. Thus, our two findings are these:

**Proposition 3**  When forward expectations are used in the Taylor rule, interest rate inertia and a reaction to the expected nominal exchange rate change are not perfect substitutes in monetary policy.

**Corollary 4**  When forward expectations are used in the Taylor rule and a main objective for the central bank is to achieve a determinate REE that is stable under least squares learning, interest rate inertia and a reaction to the expected nominal exchange rate change are not perfect substitutes in monetary policy.

Corollary 4 means that when the outcome in the economy is a determinate and least squares learnable REE, the central bank cannot necessarily preserve this desirable outcome simply by replacing the exchange rate term in the monetary policy rule with an interest rate inertia term.
3.2.2 No interest rate inertia in monetary policy

In Figure 5, which is based on the parameter set in (23), there is no interest rate inertia in monetary policy. One finding in the figure is worth to be noted: The central bank cannot disregard from the expected exchange rate in its policy and, at the same time, achieve a determinate REE that is stable under least squares learning. This finding also holds irrespective of the strength in the expected inflation rate reaction in the monetary policy rule. In fact, the central bank should decrease the interest rate when the currency is expected to depreciate in value (i.e., $\zeta_e < 0$) because, otherwise, there would be an indeterminate REE in the model economy.

\[ \text{Figure 5: Forward expectations in the Taylor rule, no interest rate inertia and the Woodford (1999) calibration.} \]

The parameter set in (24) lies behind the construction of Figure 6. Moreover, as in Figure 5, there is no interest rate inertia in monetary policy. If we have the aforementioned finding in Figure 5 in mind, it is no longer the case that the central bank must react to the expected exchange rate in its policy to achieve a determinate REE that is stable under least squares learning. Instead, if the strength in the expected inflation rate reaction in the monetary policy rule is not too weak or too strong, the central bank is able to secure a desirable outcome in the economy without bothering about the expected exchange rate. However, the lesson from Figures 5-6 is that this finding depends on the parameter calibration used in the numerical analysis.
3.2.3 Interest rate inertia in monetary policy

In Figure 7, which is based on the parameter set in (23), there is interest rate inertia in monetary policy. To be more precise, the degree of interest rate inertia is $\zeta_r = 1$. A shift in focus by the central bank from the interest rate level to the interest rate change has the effect that the region for a desirable outcome in the economy increases in size (cf., Figure 5 and Figure 7). In fact, the increase is so large that it is no longer necessary for the central bank to bother about the expected exchange rate to achieve a determinate REE that is stable under least squares learning. However, the expected inflation rate reaction in the monetary policy rule cannot be too strong to have a desirable outcome in the economy.
Figure 7: Forward expectations in the Taylor rule, interest rate inertia and the Woodford (1999) calibration.

In Figure 8, which is based on the parameter set in (24), the degree of interest rate inertia in monetary policy is $\zeta_r = 1$. A shift in focus by the central bank from the interest rate level to the interest rate change does not qualitatively affect the main finding in Figure 6. That is, it is still the case that the central bank is able to secure a desirable outcome in the economy without bothering about the expected exchange rate.
3.3 Both contemporaneous data and forward expectations in the interest rate rule

Can the central bank disregard from the exchange rate as long as there is enough with interest rate inertia also for other specifications of the interest rate rule than the specification in (5) and nevertheless achieve a determinate REE that is stable under least squares learning in its policy?

The specification of the Taylor rule that we examine herein is the rule in (7) and the reasons for our interest in this specification are twofold: (i) the current nominal exchange rate change is included in the rule; and (ii) interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy. Thus, the similarity between the Taylor rules in (5) and (7) is that both rules make it possible for the central bank to shift focus from an exchange rate reaction to interest rate inertia in its policy, whereas the difference between the rules is that the former rule includes contemporaneous data on the output gap and the CPI inflation rate, whereas the latter rule includes forward expectations of the same variables.

First, the model economy in matrix form is once more (9)-(13) with the exception that the elements in the last rows in the coefficient matrices instead reflect the Taylor rule in (7). After rewriting the model economy into first-order form, it is easily verified that we again have a problem with singular matrices when trying to derive

Figure 8: Forward expectations in the Taylor rule, interest rate inertia and the Clarida et al. (2000) calibration.
the conditions for a desirable outcome in the economy. We therefore substitute out the current and expected nominal exchange rate changes from the equations by using the UIP conditions in (4) and (18), meaning that the model economy in matrix form is (17), the variable vector is (19) and the coefficient matrices are

\[
\Gamma_d = \begin{bmatrix}
1 & 0 & \frac{\alpha}{1-\delta} & 0 \\
-\gamma(1-\delta) & 1 & \beta\delta & -\delta \\
0 & 0 & 1 & -(\zeta_r + \zeta_e) \\
0 & 0 & 1 & 0
\end{bmatrix},
\]

and

\[
\theta_d = \begin{bmatrix}
1 & \frac{\alpha}{1-\delta} & 0 & 0 \\
0 & \beta & 0 & 0 \\
\zeta_x & \zeta_\pi & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

Second, the essential matrix for the determinacy and learnability results, \(\Gamma_d^{-1} \cdot \theta_d\), has the same form as in (22). Consequently, \(r\) in (19) is predetermined in this case as well, which means that we have the same conditions as in Sections 3.1-3.2 to have a determinate REE that is least squares learnable, an indeterminate REE and no stable REE in the model economy.

3.3.1 Main results

Due to the fact that the parameters \(\zeta_r\) and \(\zeta_e\) in the Taylor rule in (7) appear at the same place in the matrix \(\Gamma_d\) in (28) and also with the same coefficient (i.e., \(-1\)), interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy. This means that we can reformulate Proposition 1 and Corollary 2 somewhat:

**Proposition 5** Interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy.

**Corollary 6** When a main objective for the central bank is to achieve a determinate REE that is stable under least squares learning, interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy.
First, be aware that Proposition 5 and Corollary 6 do not depend on the parameter values in the model economy, including the value of $\zeta_r$. Thus, our findings are not restricted to the special cases $\zeta_r = 0$ and $\zeta_r = 1$; they are valid for all degrees of interest rate inertia in monetary policy. Proposition 5 means that the central bank can concentrate on a variable that it does have direct control over in its policy (i.e., the interest rate in the previous time period), instead of focusing on a variable that it does not have direct control over (i.e., the exchange rate). Corollary 6 means that the central bank can shift focus from an exchange rate reaction to interest rate inertia in its policy and, at the same time, preserve a desirable outcome in the economy. But, as we asked ourselves when monetary policy was conducted by the Taylor rule in (5), how strong must the interest rate inertia be in the first place to have a desirable outcome in the economy? Before we answer this question, let us start with the case of no interest rate inertia in monetary policy.

3.3.2 No interest rate inertia in monetary policy

In Figure 9, which is based on the parameter set in (23), there is no interest rate inertia in monetary policy. One finding in the figure is worth to be noted: The central bank cannot disregard from the exchange rate in its policy and, at the same time, achieve a determinate REE that is stable under least squares learning. Quite the opposite, the central bank should react strongly to the exchange rate to secure a desirable outcome in the economy; the central bank should either decrease or increase the interest rate considerably when the currency depreciates in value because, otherwise, there would be an indeterminate REE in the model economy.
Figure 9: Both contemporaneous data and forward expectations in the Taylor rule, no interest rate inertia and the Woodford (1999) calibration.

The parameter set in (24) lies behind the construction of Figure 10. Moreover, as in Figure 9, there is no interest rate inertia in monetary policy. Once more, the central bank cannot disregard from the exchange rate in its policy and, at the same time, achieve a determinate REE that is stable under least squares learning.
3.3.3 Interest rate inertia in monetary policy

In Figure 11, which is based on the parameter set in (23), there is interest rate inertia in monetary policy. To be more precise, the degree of interest rate inertia is $\zeta_r = 1$. As can been seen in the figure, it is again the case that the central bank cannot disregard from the exchange rate in its policy and, at the same time, achieve a desirable outcome in the economy. A difference now, however, is that the central bank does not have to increase the interest rate equally strongly when the currency depreciates in value (cf., Figure 9 and Figure 11).
Figure 11: Both contemporaneous data and forward expectations in the Taylor rule, interest rate inertia and the Woodford (1999) calibration.

In Figure 12, which is based on the parameter set in (24), the degree of interest rate inertia in monetary policy is $\zeta_r = 1$. A shift in focus by the central bank from the interest rate level to the interest rate change does not qualitatively affect the main finding in Figure 10. Thus, it is again the case that the central bank cannot disregard from the exchange rate in its policy and, at the same time, achieve a desirable outcome in the economy. However, as is also the case in Figure 11, the central bank does not have to increase the interest rate equally strongly when the currency depreciates in value. A modest increase in the interest rate is instead enough to secure a desirable outcome in the economy.
How strong must the interest rate inertia be in monetary policy so that the central bank can disregard from the exchange rate in its policy and, at the same time, achieve a desirable outcome in the economy? Based on the parameter set in (23), a degree of interest rate inertia that is just under $4$ is enough, which, of course, is strong interest rate inertia. However, based on the parameter set in (24), a degree of interest rate inertia that is just over $1$ is enough, which is only slightly stronger interest rate inertia than in Figure 12.

4 Discussion

It is natural to ask whether one should augment the Taylor rule with an exchange rate term to have a desirable outcome in the economy. Taylor’s (2001) answer to this question is no and his argument is that the indirect effects that the exchange rate has on monetary policy, via its effects on the inflation rate and output, are to prefer because they result in fewer and less erratic changes in the interest rate. We agree with Taylor (2001), even though our argument is different than his argument: If the Taylor rule includes contemporaneous data on the variables in the rule, the central bank can disregard from the exchange rate as long as there is enough inertial
terest rate inertia in monetary policy. The reason is that interest rate inertia and a reaction to the current nominal exchange rate change are perfect substitutes in monetary policy.

Bullard and Mitra (2007) show the merits of interest rate inertia in monetary policy for a closed economy, and Llosa and Tuesta (2008) examine the inclusion of the exchange rate in the interest rate rule in an open economy but neglects from interest rate inertia in monetary policy. Even though the inclusion of the exchange rate in the monetary policy rule can help alleviate coordination problems in the economy, it comes at the cost of greater volatility in the economy (cf., Taylor, 2001). As pointed out above, we show that the central bank can shift focus from the exchange rate to interest rate inertia in its policy. This is an important result from the point of view of practical policy-making: The central bank can, instead of focusing on a variable that it does not have direct control over (i.e., the exchange rate), concentrate on a variable that it does have direct control over (i.e., the interest rate in the previous time period).

The central bank uses a Taylor rule in policy-making. It would therefore be interesting to learn how the findings are affected if the central bank instead uses an optimal monetary policy rule. This kind of rule is derived in Bask (2009b) under discretion in policy-making, where the central bank minimizes the following objective function:

\[ \eta x_t^2 + \pi_t^2, \]

where \( \eta \) is the degree of flexibility in inflation rate targeting with \( \eta = 0 \) being strict targeting. A crucial difference between the model economy in the present paper and the one in Bask (2009b) is that trend extrapolation of the exchange rate is incorporated into the latter model economy. However, if there is no trend extrapolation in currency trade in Bask (2009b), the two model economies coincide with each other.

The optimal monetary policy rule in Bask (2009b) is on the following form:

\[ r_t = \text{const.} + \zeta_x E_t[x_{t+1}] + \zeta_\pi E_t[\pi_{t+1}] + \zeta_e \Delta e_t, \]

where \( \zeta_x, \zeta_\pi \) and \( \zeta_e \) are functions of the parameters in the model economy. Thus, the optimal monetary policy rule in (31) is not on the same form as the interest rate rules in (5)-(7). In particular, the rule in (31) does not include the interest rate in the previous time period, which means that there is no interest rate inertia in monetary policy when it is optimal. However, if \( \zeta_r = 0 \) in the rule in (7), the resulting rule coincides with the optimal monetary policy rule in (31), which means that the main finding in Section 3.3.2 applies for this rule as well: The
central bank cannot disregard from the exchange rate in its policy and, at the same time, achieve a determinate REE that is stable under least squares learning.

As mentioned in the introductory section, Bask and Selander (2009) conclude that the central bank should react to the exchange rate in its policy when currency trade not only is determined by fundamental analysis of the expected exchange rate but also by chartist analysis in the form of trend extrapolation of the exchange rate. Specifically, in their model economy, the market expectation \( m \) of the exchange rate is a weighted average of the fundamentalist expectation \( f \) and the chartist expectation \( c \) of the exchange rate:

\[
E_t^m[\Delta e_{t+1}] = \omega E_t^f[\Delta e_{t+1}] + (1 - \omega) E_t^c[\Delta e_{t+1}],
\]

where \( \omega \in [0,1] \) is the proportion of chartist analysis in the expectation formation. Moreover, Bask and Selander (2009) base their analysis on the Galí and Monacelli (2005) model as also we do in the present paper and they examine the properties of the model economy when the interest rate rule for the central bank is (5) but without interest rate inertia in monetary policy (i.e., \( \zeta_r = 0 \)). In particular, they examine the desirability of a determinate and least squares learnable REE in an inflation rate targeting regime and find, as we already have pointed out, that the central bank should react to the exchange rate in its policy.

Bask (2009a) examine the properties of the same model economy as in Bask and Selander (2009), but for the interest rate rules in (5)-(6) allowing for interest rate inertia in monetary policy, and find that a large proportion of chartist analysis in currency trade negatively affects the conditions for a determinate and least squares learnable REE. However, strong interest rate inertia in monetary policy positively affects the conditions for a determinate and least squares learnable REE. Be aware that the parameter sets utilized in the numerical analyses of the macroeconomic models in Bask (2009a, 2009b) and Bask and Selander (2009) are not entirely the same as the parameter sets utilized in the present paper.

Even though one of the main findings in the present paper is that interest rate inertia in monetary policy is beneficial for the economy’s outcome, we should end this discussion by mentioning the main findings in Bask (2011). The macroeconomic model for a closed economy that is presented in Bask (2011) contains stock traders who use a mix of fundamental and chartist analyses in stock trading in a similar fashion as in (32), and the author finds that the central bank should augment the interest rate rule with a term for stock price misalignments because a determinate REE that is stable under least squares learning is then easier to achieve. Another finding
is that interest rate inertia in monetary policy does not promote macroeconomic stability when chartist analysis plays a major role in stock trading. Even worse, if the central bank in its policy only indirectly responds to stock price misalignments, via its effect on the inflation rate, a combination of strong interest rate inertia in monetary policy and a significant role for chartist analysis in stock trading will lead to macroeconomic instability.

References


WORKING PAPERS*
Editor: Nils Gottfries


* A list of papers in this series from earlier years will be sent on request by the department.


2011:8 Niklas Bengtsson and Per Engström, Control and Efficiency in the Nonprofit Sector Evidence from a Randomized Policy Experiment. 24 pp.


2011:16 Mikael Bask, A Case for Interest Rate Inertia in Monetary Policy. 33 pp.

See also working papers published by the Office of Labour Market Policy Evaluation http://www.ifau.se/ ISSN 1653-6975