Dynamic cost-benefit analysis of large projects: The role of capital cost

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Abstract

This paper derives a dynamic cost-benefit rule for evaluating large projects. We show that, in addition to the conventional income and consumer surplus measures, the rule also entails an extra term involving capital cost changes.

JEL: D6, D9, H4.
Keywords: cost-benefit rule; large project; and capital cost.

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1 Introduction

Structural transformations occur frequently in real life, such as the replacement of an old road passing through a large number of cities by a large-capacity freeway moving the heavy traffic away from the urban areas. National environmental policy may change by discrete increases in emission taxes of pollutants and decreases in taxes on labor. One intention of such a green tax reform is to create large changes in equilibrium prices. Every time a project is large enough to considerably affect the prices in an economy, the dynamic theory of cost-benefit analysis for a marginal variation (see Starrett, 1988; and Li and Löfgren, 2008) has to be modified. The general idea behind the necessary modification is not new as it dates back to the French economist Dupuit. However, a satisfactory theory in a growth theoretic context has not been available until recently.

A rigorous theory for dynamic welfare comparisons has been developed by Weitzman (2001) who shows that the difference in intertemporal welfare between two economies or two points in time of the same economy can be exactly measured by the difference in real national income plus a consumer surplus term. In addition, he mentions that the theory may also be used to conduct social cost-benefit analysis by comparing the welfare levels generated by "twin economies" with identical preferences and technology but different initial capital stocks. This paper explores this issue further. We show that while the theory is valid for this special case, the cost-benefit rule for a more general, dynamic project also entails an extra term reflecting the change in capital costs during the project period.

2 Model setup and the generic cost-benefit rule

We consider a multisector growth model with all consumption and investment goods taken into account. Let \( C = (C_1, C_2, ..., C_m) \) be a \( m \)-dimensional vector of consumption flows at time \( t \), which is supposed to exhaust all possible goods and services relevant to social welfare in a first best setting. The utilitarian measure of intertemporal welfare at time \( t = 0 \) can be expressed as

\[
W = \int_0^{\infty} U(C(t)) \exp(-\theta t) dt
\]

(1)
where $U(C)$ is a given concave, non-decreasing, instantaneous utility function with continuous second order derivatives defined for $C \geq 0$, and $\theta$ is the utility rate of discount. Let $K = (K_1, K_2, \ldots, K_n)$ be a $n$–dimensional vector of capital goods, which is assumed to contain all types of capital goods in the economy including natural resources and human capital. Net investments are, by definition, the change in capital stocks, i.e. $I_i = \dot{K}_i$, $i = 0, 1, \ldots, n$, which, in vector form can be expressed as $I = \dot{K}$, given $K(0) = K_0 > 0$. At each point in time $t$, consumption $C(t)$ and investment $I(t)$ are allocated within the $(m + n)$–dimensional attainable-possibility set $S(K(t); \alpha)$, conditional on a collection of “parameters”, $\alpha$, (Drèze and Stern, 1987), where the set is assumed to be strictly convex. The parameter $\alpha$ may represent any premise that modifies the feasible set for consumption and investment allocations. The decision-maker is assumed to maximize the current-value Hamiltonian at each $t$, i.e. $H(t) = U(C(t)) + \Psi(t)I(t)$ with respect to $\{C(t), I(t)\}$ subject to the initial condition and the attainability set, where $\Psi(t)$ is the $n$–dimensional vector of the utility prices of capital satisfying the no-arbitrage condition $\dot{\Psi} = \theta \Psi - \nabla H_K$ along the optimal trajectory. Let $\{C(\alpha, t), I(\alpha, t), K(\alpha, t)\}$ be the conditional optimum trajectory, then the maximized intertemporal welfare can be expressed as

$$W(\alpha) \equiv \int_0^\infty U(C(\alpha, t)) \exp(-\theta t) dt$$  \hspace{1cm} (2)

Now, consider a project $\Delta \alpha$ with a change in $\alpha$ from $a_0$ to $\alpha_1$ over a project period $t \in [0, T]$, for $T \geq 0$, which, if implemented, would result in changes in the stream of consumption both within and beyond the project period. The general cost-benefit rule can be stated as "If the project $\Delta \alpha$ leads to a positive change in the intertemporal welfare in (2) i.e.

$$\Delta W = W(\alpha_1) - W(\alpha_0) = \int_0^\infty [U(C(\alpha_1, t)) - U(C(\alpha_0, t))] \exp(-\theta t) dt > 0$$  \hspace{1cm} (3)

the project is socially profitable; otherwise not".

### 3 A new result

Since the cost-benefit rule involves an integral over an infinite time horizon, the project would be rather difficult to evaluate in practice. By using the notion of
social profit (Dixit et al., 1980), we show how to transform the rule to a finite time horizon version and then derive a new dynamic cost-benefit rule as an extension of Weitzman’s twin economy parable. First, we consider a large project $\Delta \alpha$ as a sequence of marginal projects $d\alpha$ to motivate our use of differentials (Starrett, 1988).

Let $C_\alpha(\alpha, t) = \partial C(\alpha, t)/\partial \alpha$, $I_\alpha(\alpha, t) = \partial I(\alpha, t)/\partial \alpha$ and $K_\alpha(\alpha, t) = \partial K(\alpha, t)/\partial \alpha$ denote the changes caused by a marginal project $d\alpha$ within the project period $t \in [0, T]$. Then, its net social profit at time $t$ can be expressed as

$$B(\alpha, t) = \Pi(\alpha, t)C_\alpha(\alpha, t) + \Psi(\alpha, t)I_\alpha(\alpha, t) + \Omega(\alpha, t)K_\alpha(\alpha, t)$$

(4)

where $\Pi(\alpha, t) = \nabla U [C(\alpha, t)]$ is the utility price of consumption, $\Psi(\alpha, t)$ that of investment, and $\Omega(\alpha, t) = \dot{\Psi}(\alpha, t) - \theta \overline{\Psi}(\alpha, t)$ the cost-of-holding capital. Following Asheim (2000) and Arrow et al. (2003), we can now re-express the welfare change in (3) in the following manner:

**Lemma 1** The welfare effect due to changes in future consumption as in (3) is equivalent to the present discounted value of social profits within the project period i.e.

$$\Delta W = \int_0^T \int_{\alpha_0}^{\alpha_1} B(\alpha, t) \exp(-\theta t) d\alpha dt$$

(5)

**Proof.** Since $[\Psi(\alpha, t)I_\alpha(\alpha, t) + \Omega(\alpha, t)K_\alpha(\alpha, t)] \exp(-\theta t) = \frac{d\Psi(\alpha, t)K_\alpha(\alpha, t) \exp(-\theta t)}{dt}$,

the welfare change in (5) can according to (4) be written as

$$\Delta W = \int_0^T \int_{\alpha_0}^{\alpha_1} \nabla U [C(\alpha, t)] C_\alpha(\alpha, t) \exp(-\theta t) d\alpha dt$$

$$+ \int_{\alpha_0}^{\alpha_1} (\Psi(\alpha, T)K_\alpha(\alpha, T) \exp(-\theta T) - \Psi(\alpha, 0)K_\alpha(\alpha, 0)) d\alpha$$

(6)

For given $K(0)$, we have $K_\alpha(\alpha, 0) = 0$. By definition $\Psi(\alpha, T) = \frac{\partial \widehat{W}(K(\alpha, T))}{\partial K(\alpha, T)}$ for $\widehat{W}(K(\alpha, T)) = \int_T^\infty U(C(\alpha, t)) \exp(-\theta (t - T)) dt$ with $C(\alpha, t)$ as the optimal consumption path for $t \in [T, \infty)$, conditional on an "initial" capital $K(\alpha, T)$. Thus, the second line in (6) becomes

$$\left[ \widehat{W}(K(\alpha_1, T)) - \widehat{W}(K(\alpha_0, T)) \right] e^{-\theta T} = \int_T^\infty \left[ U(C(\alpha_1, t)) - U(C(\alpha_0, t)) \right] e^{-\theta t} dt$$

(7)
which together with the first line in (6) constitutes the welfare difference as in (3).

To arrive at a cost-benefit rule comparable to Starrett (1988, p236-237) and Weitzman (2001), we introduce the following lemma.

**Lemma 2** The present discounted value of social profits in (5) is equivalent to

\[
\Delta W = \int_0^T [\Delta Y(t) + CS(t) - \kappa(t)] \exp(-\theta t) dt \tag{8}
\]

where \(\Delta Y(t) = Y_1(t) - Y_0(t)\) with \(Y_i(t) = \Pi(\alpha_i, t)C(\alpha_i, t) + \Psi(\alpha_i, t)I(\alpha_i, t)\) as the comprehensive income in situation \(i = 0, 1\). \(CS(t) = \int_{\Pi(\alpha_0, t)}^{\Pi(\alpha_1, t)} D(\Pi)d\Pi\) is the consumer surplus with \(D(\Pi(\alpha, t)) = C(\alpha, t)\) as the (compensated) consumption demand system, and

\[
\kappa(t) = \theta \int_{\alpha_0}^{\alpha_1} \Psi(\alpha, t)K_\alpha(\alpha, t) d\alpha \tag{9}
\]

is the cost of capital reallocation at time \(t\).

**Proof.** Integrating (5) by parts over \(\alpha\) for the first two terms of \(B(\alpha, t)\) gives

\[
\Delta W = \int_0^T [Y_1(t) - Y_0(t) + CS(t)] \exp(-\theta t) dt
+ \int_0^T \exp(-\theta t) \int_{\alpha_0}^{\alpha_1} [\Omega(\alpha, t)K_\alpha(\alpha, t) - \Psi_\alpha(\alpha, t)I(\alpha, t)] d\alpha d\tau
\tag{10}
\]

Defining the inverse capital demand function by the vector \(\hat{D}(K(\alpha, t)) = \Psi(\alpha, t)\), and recalling that \(\Omega(\alpha, t) = \hat{\Psi}(\alpha, t) - \theta \Psi(\alpha, t)\) and \(\hat{K} = I\). Then, the inner integral in the second line becomes

\[
-\kappa(t) = \int_{\alpha_0}^{\alpha_1} \left[ (K_\alpha \hat{D}_K \hat{K} - \theta \Psi K_\alpha - \hat{K} \hat{D}_K K_\alpha) \right] d\alpha = -\theta \int_{\alpha_0}^{\alpha_1} \Psi K_\alpha d\alpha \tag{11}
\]

where \(\hat{D}_K\) is a symmetric matrix following Young’s theorem such that the two scalars \(K_\alpha \hat{D}_K \hat{K}\) and \(\hat{K} \hat{D}_K K_\alpha\) are equal to each other. Note that the arguments \(\alpha\) and \(t\) are suppressed here. Thus (8) is true.

These two lemmas imply the following main proposition:
Proposition 1 Consider a project $\Delta \alpha$ over a time period $[0, T]$, $T \geq 0$, which, if implemented, would result in changes in income $\Delta Y(t)$, consumer surplus $CS(t)$, and capital reallocation cost $\kappa(t)$ for each $t \in [0, T]$. Suppose that the welfare difference in (8) involving these variables is positive, then the project is socially profitable; otherwise not.

This dynamic cost-benefit rule extends Weitzman’s (2001) “twin-economy” parable in two aspects. First, it is applicable to projects of structural reforms over any discrete time period rather than an once-for-all reshuffle in the initial capital. While Weitzman considers the welfare difference between two economies with different initial capital structure, we are concerned with an economy with a given initial capital but "managed" by two different regimes. If an alternative regime would result in a higher intertemporal welfare than the base one, then it is worth the effort to shift from the base regime to the alternative one. Second, we obtain an extra term $\kappa(t)$ reflecting the cost of capital reallocation in addition to the income difference and consumer surplus terms. Weitzman’s comparison of twin economies can in our framework be interpreted as a project that can costlessly and instantaneously transform the initial capital composition. His reason is sound since the cost of the transformation is "sunk cost". We explicitly take into account the cost of capital reallocation. More exactly, by treating Weitzman’s parable as a permanent project over $t \in [0, \infty)$, with $\lim_{T \to \infty} \exp(-\theta T) = 0$, we can use (10) to derive the following version of our formula

$$
\Delta W = \int_0^{\infty} \int_{a0}^{a1} \Pi(C(\alpha, t)) C_\alpha(\alpha, t) \exp(-\theta t) d\alpha dt - \int_{a0}^{a1} \Psi(\alpha, 0) K_\alpha(\alpha, 0) d\alpha
$$

(12)
as $\Delta Y(0) + CS(0) \equiv \Delta H(0) = \theta \int_0^{\infty} \int_{a0}^{a1} \nabla U(C) C_\alpha e^{-\theta t} d\alpha dt$ is the well-known stationary equivalent change of future welfare. Multiplying both hand-sides of (12) by $\theta$, we obtain $\theta \Delta W = \Delta Y(0) + CS(0) - \kappa(0)$ where $\kappa(0)$ was assumed to be zero in Weitzman (2001). The relationship between our model and Starrett (1988) is less subtle, since he studies a discrete static project without any change in capital stocks.
For a project extended over $t \in [0, T]$, we have to calculate the net benefit $\Delta Y(t) + CS(t) - \kappa(t)$ for each $t$, and then calculate the present value of this stream over $t \in [0, T]$ to assess the overall welfare effect.

4 A money-metric analogy

To convert the welfare measure in (8) from utility to a money metric, we first define the nominal prices by $P(\alpha, t) = \Pi(\alpha, t)/\lambda(\alpha, t)$ and $Q(\alpha, t) = \Psi(\alpha, t)/\lambda(\alpha, t)$, where $\lambda(\alpha, t)$ is the marginal utility of income satisfying the no-arbitrage condition $\dot{\lambda}(\alpha, t) = \lambda(\alpha, t)(r(\alpha, t) - \theta)$ with $r(\alpha, t)$ as the nominal interest rate. Next, we define the real prices by $P(\alpha, t) = P(\alpha, t) = \Pi(\alpha, t)$ and $Q(\alpha, t) = Q(\alpha, t) = \Psi(\alpha, t)$ with $\pi(\alpha, t)$ as Weitzman’s ideal consumer price index

$$\pi(\alpha, t) = \frac{\tilde{P}(\alpha, t)C(\alpha_0, t)}{\hat{P}(\alpha_0, t)C(\alpha_0, t)}$$

(13)

where $\tilde{P}(\alpha, t)$ denotes the market clearing prices conditional on $\alpha$, for consuming the pre-project bundle $C(\alpha_0, t)$. By the assumption of an invariant utility function $U(C)$, we have $\nabla U(C(\alpha_0, t)) = \lambda(\alpha_0, t)P(\alpha_0, t) = \lambda(\alpha, t)\tilde{P}(\alpha, t) = \nabla U(C(\alpha, t))$, for all $C(\alpha, t) = C(\alpha_0, t)$, which implies that $\pi(\alpha, t)\lambda(\alpha, t) = \lambda(\alpha_0, t)$. In present value terms, this last equality becomes $\pi(\alpha, t)\lambda(\alpha, t)\exp(-\theta t) = \lambda(\alpha_0, t)\exp(-\theta t) = \lambda(\alpha_0, 0)\exp(-\int_0^t \bar{r}(s)ds)$ due to the no-arbitrage condition, where $\bar{r}(t) = r(\alpha_0, t) - \check{r}(\alpha_0, t)/\pi(\alpha_0, t)$ is the pre-project real interest rate at time $t$. Thus, the money-metric welfare change corresponding to (8) can be expressed as

$$\frac{\Delta W}{\lambda(\alpha_0, 0)} = \int_0^T \left[ \Delta \bar{Y}(t) + \bar{CS}(t) - \bar{\kappa}(t) \right] \exp\left( -\int_0^t \bar{r}(s)ds \right) dt$$

(14)

where $\Delta \bar{Y}(t) = \bar{Y}_2(t) - \bar{Y}_1(t)$ with $\bar{Y}_i(t) = \bar{P}(\alpha_i, t)C(\alpha_i, t) + \bar{Q}(\alpha_i, t)I(\alpha_i, t)$ as the comprehensive income in situation $i = 0, 1$, $\bar{CS}(t) = \int_{\tilde{P}(\alpha_0, 0)}^{\tilde{P}(\alpha_1, t)} D(\tilde{P})d\tilde{P}$ the consumer surplus, and $\bar{\kappa}(t) = \bar{r}(t)\int_{\alpha_0}^{\alpha_1} \tilde{Q}(\alpha, t)K_\alpha(\alpha, t)d\alpha$ the cost of capital reallocation, all in real terms$^1$.

$^1$A usual practice in cost-benefit analysis is to assume a constant marginal utility of income. However, Starrett (1988) shows that this is in general not consistent with utilitarian theory.
5 Conclusion

This paper has derived a dynamic cost-benefit rule for large projects from a multi-sector growth model, conditionally optimal for a given collection of parameters. As in Drèze and Stern (1987), we define a project as the change in the parameter in a given time period, involving changes in consumption, investment and capital stocks over time. By examining the change in the present value of net social profits, we find that the dynamic cost-benefit rule entails an extra term involving the cost of capital reallocation, in addition to the conventional income plus consumer surplus terms in Weitzman (2001). This cost component should be relevant for any large investment project that accumulates capital over time.

6 References


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