Unemployment benefits or taxes: How should policy makers redistribute income over the business cycle?

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January 20, 2012

Abstract

This paper studies optimal unemployment benefit levels and optimal proportional income tax rates over the business cycle. Previous research suggests that policy makers should make unemployment insurance (UI) dependent on the business cycle because the UI system can be used to smooth consumption across different economic states. However, high benefits increase unemployment. An alternative way to redistribute income is to vary tax rates over the business cycle. In this paper, we develop an equilibrium search and matching model with risk-averse workers and two states, namely, a good and a bad state. The model yields potential ambiguity concerning the welfare effects of business cycle-dependent UI. The model is calibrated to United States (U.S.) labor market data. The numerical results suggest that higher benefits in the bad state are optimal, but the benefit differential is small. A more efficient way for policy makers to redistribute income over the business cycle is to decrease taxes in the bad state. Compared to an optimal uniform system, however, differentiation yields small welfare gains. Nevertheless, imposing two tax rates strictly dominates imposing two benefit levels. This finding is robust to a wide range of sensitivity checks.

JEL codes: E32, H24, J64, J65

Keywords: Job search, business cycles, unemployment insurance, time-varying benefits and taxes
1 Introduction

Unemployment insurance (UI) is an important labor market institution in most developed countries. When implementing UI, policy makers try to determine the optimal mix between the demands of risk-averse workers for insurance and the disincentives for job search (moral hazard). It might very well be that the optimal mix varies over the business cycle. Despite this, most of the literature on UI has so far ignored business cycles.

In bad times, more unemployed workers compete for jobs. As a result, the expected time for a person with a constant search effort to obtain a job offer is longer. Job offers are scarce, and the worker can do less to affect his/her own probability of finding a job. In addition, because workers are more discouraged (i.e., jobs are harder to find, have a shorter expected duration and are associated with lower wages), they could be more responsive to changes in UI benefits. The optimal benefit levels balance the value of insurance with the disincentives from it.

A good portion of the literature on business cycle-dependent UI concludes that UI should be more generous in bad times; for example, see Kiley (2003), Sanchez (2008), Andersen & Svarer (2010, 2011) and Landais, Michaillat & Saez (2010). On the contrary, Moyen & Stähler (2009) conclude that the U.S. should have counter-cyclical durations of UI while the EU should not, and Mitman & Rabinovich (2011) find pro-cyclical UI to be optimal overall. These papers focus on the optimal UI from the social planner’s point of view, but only Mitman & Rabinovich (2011) quantify the magnitude of the welfare gain. They find that the consumption equivalent gain is 0.28 percent compared to the current U.S. unemployment system. One question that remains to be answered is whether business cycle-dependent UI is associated with significant welfare gains compared to the optimal uniform system. In addition, except for Andersen & Svarer (2011), none of these papers consider taxes as a complementary way of redistributing income over the business cycle.

In this paper, we ask how policy makers should redistribute income over the business cycle. Redistribution of income redistributes worker welfare. If workers are risk averse, they dislike volatility in consumption. The policy maker has two (four) potential instruments: benefits and taxes (in the good and in the bad state). To this end, we develop a rich two-state general equilibrium search and matching framework with endogenous job search and Nash bargaining over wages and hours. Business cycles are modeled in
a stylized way; the economy moves between two economic states, a good and a bad state. Firms consider current and future economic conditions when opening up vacancies. Wages are set in a decentralized fashion through worker-firm bargaining. The outside options in the good and the bad state differ, resulting in both wage and working time dispersion across states. In the baseline case, benefits are financed by a proportional tax rate on all income, including benefits. This income tax is equivalent to a consumption tax when there are no savings or borrowing. The model is calibrated to match U.S. data. We argue that benefits generally affect search effort, job finding and unemployment more so in the bad state than in the good state. Therefore, the policy maker must balance the worker’s demand for insurance with the distorting effects of benefits on job search and unemployment in bad times. Taxes are neutral\footnote{By neutral, we mean that taxes do not affect the equilibrium system and can be solved residually after the level of unemployment is determined.} throughout the paper, which is an implication of the assumption that taxes affect all workers, regardless of income.

Similar to the previous literature, we find that, in general, the optimal system involves higher benefits in bad times than in good times. However, the policy maker can increase welfare by allowing for differentiated taxes. Higher taxes in good times redistribute income without affecting either job search or unemployment. The most general optimal UI system entails lower taxes and higher after-tax benefits in bad times. The welfare gain of workers compared to the current U.S. system is significant. However, compared to an optimal uniform system, differentiation entails small welfare gains.

The main novelty in this paper is the combined analysis of taxes and benefits as a means of improving welfare in a model with two business cycle states. We argue that varying tax rates instead of benefit levels over the business cycle increases worker welfare when workers are risk-averse. Contrary to most of the previous literature, we consider endogenous wages that depend on benefits. In addition, we quantify the optimal system and show that the gain from business cycle-dependent UI is negligible, but introducing two tax rates improves welfare more so than imposing two benefit levels.

The paper proceeds with a brief discussion of the related literature. The model is presented in section 3, and section 4 introduces the calibration that we use in section 5 to provide a welfare analysis of alternative UI systems. Section 6 concludes the paper.
2 Related literature

This paper is situated within the literature on the normative aspects of unemployment insurance (UI); see Fredriksson & Holmlund (2006) for a survey. It also relates to the growing literature on business cycle-dependent UI. The main focus of this literature is whether business cycle-dependent UI should be more or less generous in bad times. There are papers on benefit levels, including Andersen & Svarer (2010, 2011), and Landais, Michaillat & Saez (2010), and on the duration of benefits, such as Moyen & Stähler (2009). Most of the papers so far conclude that benefits should be more generous in bad times because the policy maker can use UI to smooth consumption over the business cycle.

Two early papers on state-dependent UI are Kiley (2003) and Sanchez (2008). Both set up partial search models with risk-averse workers in which the variation over the business cycle stems from job-finding probabilities. They rely on the assumption that benefits are more distortionary in good times and conclude that benefits should be more generous in bad times. Neither paper considers wage bargaining, endogenous job-finding probabilities or budget effects. As we show in this paper, it is not obvious that benefits are more distortionary in good times when workers choose their search effort optimally and the policy maker cannot observe their effort.

Moyen & Stähler (2009) study two-tier business cycle-dependent UI in a real business cycle framework. Productivity shocks follow an AR(1) process, and unions and firms bargain over wages. Benefits are financed by state-dependent lump sum taxes on wages. They find ambiguous welfare effects from differentiating over business cycles, and they calibrate the model to match the U.S. and European labor markets. They find that counter-cyclical UI is preferable in the U.S. but not in Europe. The intuition behind this difference is that counter-cyclical duration is optimal when the negative effects on the labor market are smaller than the positive effect of consumption smoothing. In that paper, the calibrations hinge on the assumption that the Hosios condition is not fulfilled; they assume that there are too many vacancies in the U.S., while there are too few vacancies in the EU.

This paper is most closely related to studies by Andersen & Svarer (2010, 2011). Andersen & Svarer (2010) set up a partial search and matching model with endogenous job search but rigid wages. Workers are risk-averse,

\footnote{The benefit levels take two values, which can be considered insured and non-insured levels.}
and bad times are characterized by higher job destruction. They focus solely on the job search margin and how it affects the design of optimal UI. In their paper, benefits are financed by a flat proportional wage tax across economic states together with a state-dependent lump sum income tax. They show that it is optimal for the policy maker to provide countercyclical benefits when workers search more in bad times than in good times (if the social planner balances the budget over time). Andersen & Svarer (2011) evaluate state-dependent benefits in a static search framework. They note that “the financing of the benefit scheme turns out to be very important, and in particular the ability to diversify risk not only between employed and unemployed but also across states of nature.” The tax rate is a proportional tax on wages. When the budget is balanced in each state, higher benefits in good times is the optimal policy due to the basic budget mechanism; that is, a high employment rate decreases the taxes needed to finance benefits. On the other hand, when risk diversification across states through the UI system is possible, the policy maker should set lower taxes and benefits in the state with the most distortions. In their model, this implies higher benefits and higher taxes in bad times. Although high (low) benefits and high (low) taxes in bad (good) times increase unemployment fluctuations across states, state-dependent benefits may lower average unemployment.

Other papers include Landais, Michaillat & Saez (2010) and Mitman & Rabinovich (2011). Landais, Michaillat & Saez (2010) analyze optimal UI when unemployment in good times is due to matching frictions and unemployment in bad times stems from job rationing. Benefits are financed by a proportional wage tax. They calibrate a DSGE model to U.S. data and find that the optimal replacement rate is higher in bad times. Taxes increase in bad times because more individuals are unemployed and thus obtain higher benefits. Mitman & Rabinovich (2011) set up an equilibrium search and matching model with risk-averse workers and model business cycles as aggregate shocks to labor productivity. They abstract from taxes by assuming that benefits are financed by a proportional lump sum tax on firm profits. Moreover, they characterize the optimal path of benefits; namely, benefits rise in the beginning of the unemployment spell to provide some short-term relief and then drastically fall to induce job search and shock recovery. Compared to the current U.S. system, they calculate the welfare gain from the optimal system to be 0.28 percent in consumption equivalent terms.

This paper differs in some important aspects from the previous research.
First, we consider the optimal UI jointly with optimal benefit financing. As Andersen & Svarer (2011) note, the optimal UI depends on the tax scheme. When taxes are allowed to vary over the business cycle, pre-tax benefits are sometimes higher in good times; after-tax benefits are, however, always higher in bad times. Because benefits affect search effort and bargaining outcomes while taxes do not, the tax rate is a more efficient instrument for policy makers who want to redistribute consumption over the business cycle. Second, we consider an equilibrium set up with worker-firm Nash bargaining over wages and hours. To facilitate this extension, we focus on steady states. In this model, workers search more in good times because the marginal revenue of job search is higher in good times. We show that levels of search effort, job finding and unemployment are more responsive to changes in benefits in bad times than to changes in benefits in good times. Finally, while most of the previous literature focuses on the qualitative question regarding whether benefit levels should be higher in bad times, we also focus on the quantitative issue regarding whether higher benefits have a welfare-enhancing effect and, if so, how much.

3 The Model

3.1 The labor market

The economy consists of identical individuals with infinite time horizons. Time is continuous. The model is set in a stochastic environment with a good and a bad state. Firms make decisions in the present, taking possible future upturns and downturns into account. This stochastic framework is similar (but not identical) to Anderson & Svarer (2010) and Cahuc & Zylberberg (2004). We focus solely on steady states to maintain analytical tractability of the model. This assumption implies that the economy shifts directly between the good and the bad state, which of course is not the case in reality. However, as long as the transition time is short enough, it should not qualitatively affect the model results. This simplification is necessary to allow for focusing on benefits, taxes and wages simultaneously.

Let \( i = G, B; \) denote the good state \( i = G \) and the bad state \( i = B. \) The bad state is characterized by higher job destruction than the good state. Let \( \phi_i \) denote the (Poisson) job destruction rate; thus, \( \phi_B > \phi_G. \) The good state is in general better than the bad state for both workers and firms\(^3\).

\(^3\)This statement is always true when benefits and taxes are constant across states. If
The transition rate from the good to the bad state is denoted by $\pi_G$, and from the bad to the good state, it is denoted by $\pi_B$. The average duration of the good (bad) state is $1/\pi_G$ ($1/\pi_B$).

Instead of shocks to job destruction, we could consider shocks to productivity. However, the use of varying job destruction matches labor flow dynamics, as well as wage responses, to the U.S. labor market. Because we consider Nash bargaining between workers and firms, the use of shocks to productivity leads to unrealistic wage dispersion between states and unrealistic labor flow dynamics.

All individuals participate in the labor force, and the total labor force is fixed and normalized to unity in both states. In each state workers can be either unemployed $u_i$ or employed $e_i$. The labor force identity is $u_i + e_i = 1$. All individuals are, at the same time, in either the good or the bad state.

All unemployed workers engage in search activity $s_i$. The job-finding rates are determined by state-specific CRS matching functions $M_i = m(v_i, S_i)$; $v_i$ is the number of vacancies in each state, and $S_i = s_i u_i$ is the number of effective job searchers. Labor market tightness for each state is defined as $\theta_i = v_i/S_i$. Unemployed workers with search effort $s_i$ receive job offers from firms at rate $s_i \alpha (\theta_i) = s_i m(v_i, S_i)/S_i = s_i m(\theta_i, 1)$, and hence, $\alpha'(\theta_i) > 0$. For individuals, it is easier to find jobs when there are many vacancies compared to the number of job seekers. Firms meet unemployed workers at the rate $q(\theta_i) = m(v_i, S_i)/v_i = m(1, 1/\theta_i)$, and thus, $q'(\theta_i) < 0$. For firms, it is more difficult to find workers when there are many vacancies in relation to job seekers. Note that $\alpha(\theta_i) = \theta_i q(\theta_i)$. $\alpha(\theta_i)$ will sometimes be abbreviated as $\alpha_i$, and $q(\theta_i)$ will sometimes be abbreviated as $q_i$. The equations for flow equilibrium in the labor market are given as:

$$u_i = \frac{\phi_i}{\phi_i + s_i \alpha(\theta_i)} \quad (1)$$

$$e_i = 1 - u_i \quad (2)$$

the policy maker redistributes welfare by increasing benefits or decreasing taxes in the bad state, this state might be preferred by workers.
3.2 Workers

All individuals are identical and have preferences for consumption \( c \) and leisure \( l \). Utility functions are isoelastic of the form:

\[
v(c, l) = \frac{(d^\delta)^{1-\rho} - 1}{1 - \rho}
\]

where \( \rho \) is the degree of relative risk aversion \( \rho \geq 0 \). For \( \rho = 0 \), workers have linear utility in consumption \( v(c, l) = cl^\delta \). When \( \rho \to 1 \), we obtain the logarithmic utility function \( v(c, l) = \ln c + \delta \ln l \). Individuals cannot access capital markets, and so consumption equals income at all times. If we allowed for savings and borrowing, that should decrease the need for business cycle-dependent benefits because workers could smooth consumption themselves. In this sense, the results in this paper can be seen as an upper bound on optimal UI.

Let \( b_i \) denote the flat rate benefits for an unemployed worker in state \( i \). Benefits are financed by a proportional tax rate \( t_i \) on all incomes, including benefits. Employed workers earn hourly wages \( w_i \) and work \( h_i \) hours. The individual’s time endowment is normalized to 1. Unemployed workers consume \( b_i (1 - t_i) \) and enjoy leisure \( 1 - s_i \), while employed workers consume \( w_i h_i (1 - t_i) \) and enjoy leisure \( 1 - h_i \). Wages and hours may differ between states. The instantaneous utilities in the various states are \( v(b_i, s_i) \) for the unemployed worker in state \( i \) (sometimes abbreviated as \( v_i^u \)) and \( v(w_i, h_i) \) for the employed worker in state \( i \) (sometimes abbreviated as \( v_i^e \)).

Consider the intertemporal objective functions for workers. Let \( U_i \) denote the expected discounted present value of utility for an unemployed worker in state \( i \), and let \( E_i \) denote the value for an employed worker in state \( i \). The value functions are:

\[
\begin{align*}
\text{r} U_G &= v(b_G, s_G) + s_G \alpha (\theta_G) (E_G - U_G) + \pi_G (U_B - U_G) \quad (3) \\
\text{r} E_G &= v(w_G, h_G) + \phi_G (U_G - E_G) + \pi_G (E_B - E_G) \quad (4) \\
\text{r} U_B &= v(b_B, s_B) + s_B \alpha (\theta_B) (E_B - U_B) + \pi_B (U_G - U_B) \quad (5) \\
\text{r} E_B &= v(w_B, h_B) + \phi_B (U_B - E_B) + \pi_B (E_G - E_B) \quad (6)
\end{align*}
\]

where \( r \) is the subjective rate of time preference. The flow value of unemployment includes instantaneous utility \( v(b_i, s_i) \), the probability of a job offer (thereby moving the worker into employment) and the probability that
the economy switches between states. Similarly, the flow value of employment involves instantaneous utility \( v(w_t, h_t) \), the probability of a job loss (thereby moving the worker into unemployment) and the probability that the economy switches between states.

The relevant value equation differences are:

\[
E_G - U_G = \frac{(\pi_B + \phi_B + s_B \alpha_B) (v_G^e - v_B^e) + \pi_G (v_G^e - v_B^e)}{\pi_B (\phi_G + s_G \alpha_G) + (\pi_G + \phi_G + s_G \alpha_G) (\phi_B + s_B \alpha_B)} \tag{7}
\]

\[
E_B - U_B = \frac{\pi_B (v_G^e - v_B^e) + (\pi_G + \phi_G + s_G \alpha_G) (v_B^e - v_B^e)}{\pi_B (\phi_G + s_G \alpha_G) + (\pi_G + \phi_G + s_G \alpha_G) (\phi_B + s_B \alpha_B)} \tag{8}
\]

The present value difference between employment and unemployment in a given state is the discounted value of the utility differences between employment and unemployment in both states. The value difference \( E_i - U_i \) in state \( i \) is more affected by the immediate utility difference in the same state than the prospective difference in the other state.

The unemployed worker chooses search effort \( s_i \) to maximize \( rU_i \). The first-order conditions are:

\[
-\frac{\partial v(h_G, s_G)}{\partial s_G} = \alpha (\theta_G) (E_G - U_G) \tag{9}
\]

\[
-\frac{\partial v(h_B, s_B)}{\partial s_B} = \alpha (\theta_B) (E_B - U_B) \tag{10}
\]

These conditions state that the worker increases his/her search effort until the marginal cost equals the marginal gain of doing so.

### 3.3 Firms

Firms operate under constant returns to labor. Therefore, we use the job as the stand-in for the firm (Pissarides, 2000). Let \( y \) denote the constant level of labor productivity, which is equal across states, and let \( J_i \) be the present discounted value of a job in state \( i \). Recall that parameter \( \phi_i \) follows a Poisson process with two values, with \( \phi_B > \phi_G \). The value functions pertaining to occupied jobs are:

\[
rJ_G = (y - w_G) h_G + \phi_G (V_G - J_G) + \pi_G (J_B - J_G) \tag{11}
\]

\[
rJ_B = (y - w_B) h_B + \phi_B (V_B - J_B) + \pi_B (J_G - J_B) \tag{12}
\]

where \( r \) stands for the rate of interest, which is by assumption equal to the individual’s subjective rate of time preference. The value of a filled job includes the instantaneous profit \( (y - w_i) h_i \), the probability of job destruction
and the probability that the economy changes between states. The value of a filled job differs across states. The solutions to the value equations evaluated at \( r = 0 \) and \( V_i = 0 \) (free entry) are:

\[
J_G = \frac{(\pi_B + \phi_B) (y - w_G) h_G + \pi_G (y - w_B) h_B}{\pi_B \phi_G + \phi_G \phi_B + \pi_G \phi_B} \\
J_B = \frac{\pi_B (y - w_G) h_G + (\pi_G + \phi_G) (y - w_B) h_B}{\pi_B \phi_G + \phi_G \phi_B + \pi_G \phi_B}
\]  

(13) (14)

The value of an occupied job is given as the discounted present value of the profits in the current and future states. The flow value of keeping a vacancy is denoted by \( \kappa \), and the firm meets unemployed job seekers at the rate \( q(\theta_i) \). The value functions for vacancies \( V_i \) are:

\[
r V_G = -\kappa + q(\theta_G) (J_G - V_G) + \pi_G (V_B - V_G) \\
r V_B = -\kappa + q(\theta_B) (J_B - V_B) + \pi_B (V_G - V_B)
\]  

(15) (16)

We impose free entry, with \( V_i = 0 \), and obtain the following job creation equations when \( r = 0 \):

\[
\frac{(\pi_B + \phi_B) (y - w_G) h_G + \pi_G (y - w_B) h_B}{\pi_B \phi_G + \phi_G \phi_B + \pi_G \phi_B} = \frac{\kappa}{q(\theta_G)} \\
\frac{\pi_B (y - w_G) h_G + (\pi_G + \phi_G) (y - w_B) h_B}{\pi_B \phi_G + \phi_G \phi_B + \pi_G \phi_B} = \frac{\kappa}{q(\theta_B)}
\]  

(17) (18)

which indicate that firms open up vacancies until the expected profit equals the expected cost of a vacancy.

### 3.4 Worker-firm negotiations

Wages and hours are determined simultaneously by decentralized worker-firm Nash bargaining in each state. The worker and the firm negotiate over hourly wages and hours. As usual, the relevant threat point for the worker is the value of unemployment in the current state \( U_i \). Wages and hours are constantly renegotiated; therefore, new wages and hours are negotiated when the economy changes between states. Let \( \beta \in (0, 1) \) denote the worker’s bargaining power. The Nash products are:

\[
\Omega (w_G, h_G) \equiv (E_G - U_G)^\beta (J_G - V_G)^{1-\beta} \\
\Omega (w_B, h_B) \equiv (E_B - U_B)^\beta (J_B - V_B)^{1-\beta}
\]  

(19) (20)
The first-order conditions for wages and hours evaluated at $V_i = 0$ are as follows:

$$\beta J_G \frac{\partial v(w_G, h_G)}{\partial w_G} \frac{1}{h_G} = (1 - \beta) (E_G - U_G) \quad (21)$$

$$-\beta J_G \frac{\partial v(w_G, h_G)}{\partial h_G} \frac{1}{y - w_G} = (1 - \beta) (E_G - U_G) \quad (22)$$

$$\beta J_B \frac{\partial v(w_B, h_B)}{\partial w_B} \frac{1}{h_B} = (1 - \beta) (E_B - U_B) \quad (23)$$

$$-\beta J_B \frac{\partial v(w_B, h_B)}{\partial h_B} \frac{1}{y - w_B} = (1 - \beta) (E_B - U_B) \quad (24)$$

where eq. (21) and (23) determine the wage rate and eq. (22) and (24) determine hours.

Wages and hours are closely related. The contract curves for wages and hours can be obtained by combining (21) with (22) and (23) with (24). After some manipulations, we obtain for all values of $\rho$:

$$w_G = \frac{1 - h_G}{\delta h_G} \quad (25)$$

$$w_B = \frac{1 - h_B}{\delta h_B} \quad (26)$$

Wages can be expressed as a function of endogenous hours $h_i$ and exogenous variables $y$ and $\delta$. From the contract curves, it follows that wages are higher in the good state if hours are lower in that same state (and vice versa), that is:

$$w_G > w_B \Leftrightarrow h_G < h_B$$

### 3.5 Taxes

Recall that benefits are financed by a proportional tax rate $t_i$ on all incomes, including benefits. Let us consider the case when policy makers balance the budget across states. In this case, expected expenditures must equal expected revenues. Taxes are constant across states ($t = t_G = t_B$) in the baseline model. The expected revenue is:

$$T_x(t) = t \left[ \frac{\pi_B}{\pi_G + \pi_B} (u_G b_G + e_G w_G h_G) + \frac{\pi_G}{\pi_G + \pi_B} (u_B b_B + e_B w_B h_B) \right]$$

where $t$ is the proportional tax rate. Similarly, the expected expenditure is:

$$E(b_G, b_B) = \frac{\pi_B}{\pi_G + \pi_B} u_G b_G + \frac{\pi_G}{\pi_G + \pi_B} u_B b_B$$
The budget constraint $T_x(t) = E(b_G, b_B)$ yields the following tax rate:

$$t = \frac{\pi_G b_B u_B + \pi_B b_G u_G}{\pi_G b_B u_B + \pi_B b_G u_G + \pi_G e_B w_B h_B + \pi_B e_G w_G h_G} \quad (27)$$

Note that the tax rate is endogenous; the policy makers first set the benefit levels and then solve for the tax rate. The main advantage of this approach is that the tax rate is neutral\(^4\). Alternative tax schemes are introduced in section 5.

### 3.6 Equilibrium

The equilibrium model consists of endogenous labor market tightness, search effort, wages and hours. To solve the model, it is useful to focus on the job creation conditions (17)-(18) and the first-order conditions for search effort (9)-(10), along with the bargaining equations (21)-(24) and the value equation differences (7)-(8). The general equilibrium can be summarized as:

$$J_G = \frac{\kappa}{q(\theta_G)} \quad (28)$$

$$J_B = \frac{\kappa}{q(\theta_B)} \quad (29)$$

$$-\frac{\partial v(b_G, s_G)}{\partial s_G} = \alpha(\theta_G)(E_G - U_G) \quad (30)$$

$$-\frac{\partial v(b_B, s_B)}{\partial s_B} = \alpha(\theta_B)(E_B - U_B) \quad (31)$$

$$E_G - U_G = \beta J_G \frac{\partial v(w_G, h_G)}{\partial w_G} \frac{1}{h_G} \quad (32)$$

$$E_B - U_B = \beta J_B \frac{\partial v(w_B, h_B)}{\partial w_B} \frac{1}{h_B} \quad (33)$$

$$E_G - U_G = -\beta J_G \frac{\partial v(w_G, h_G)}{\partial h_G} \frac{1}{y - w_G} \quad (34)$$

$$E_B - U_B = -\beta J_B \frac{\partial v(w_B, h_B)}{\partial h_B} \frac{1}{y - w_B} \quad (35)$$

where $\hat{\beta} \equiv \beta/(1 - \beta)$ measures the worker’s relative bargaining power. Expressions for $E_i - U_i$ follow from (7)-(8). Eqs. (28)-(35) determine labor

\(^4\)Taxes do not enter into the job creation equations. Consider the first-order conditions for search effort (9)-(10) along with bargaining equations (21)-(24). In all of these equations, both the left- and right-hand sides are multiplied by $(1 - t)^{1 - \rho}$, which disappears from the equilibrium system.
market tightness, search effort, wages and hours in each state. Unem-
ployment in each state follows from (1) once tightness is determined. Vacancies
are obtained from $v_i = \theta_i S_i$. The numerical versions of the model always
deliver unique solutions.

Sometimes we are interested in average unemployment over the business
cycle, which is given as:

$$u = \frac{\pi_B}{\pi_G + \pi_B} u_G + \frac{\pi_G}{\pi_G + \pi_B} u_B$$

where $\frac{\pi_B}{\pi_G + \pi_B}$ is the probability of the good state and $\frac{\pi_G}{\pi_G + \pi_B}$ is the proba-
bility of the bad state.

As noted above, the first-order condition for search, wages and hours
(30)-(35) are independent of the tax rate. The budget constraint $T_x(t) = E(b_G, b_B)$ yields the tax rate as (27).

### 3.7 Wage differentials

Consider the wages in the two states. Because the contract curve has a
simple structure, we can obtain closed-form solutions for the wage equations
as a function of the exogenous variables and endogenous tightness. We
substitute the contract curves (25) and (26) in the job creation equations
(28) and (29) and solve for the two wages:

$$w_G = y \frac{\gamma G q_B + \kappa [\pi G q_B - (\pi G + \phi_G) q_B]}{\gamma q G q_B + \kappa \delta [(\pi G + \phi_G) q_B - \pi G q_G]}$$

$$w_B = y \frac{\gamma B q_B + \kappa [\pi B q_B - (\pi B + \phi_B) q_B]}{\gamma q B q_B + \kappa \delta [(\pi B + \phi_B) q_B - \pi B q_B]}$$

Wages depend on the arrival rates of workers $q(\theta_i)$ and the exogenous
variables. In general, wages differ even when workers have linear utility in
consumption (as long as benefits are positive). In the numerical exercises,
wages in the good state are higher than in the bad state as long as benefits
in the bad state are not significantly higher than benefits in the good state.
A necessary and sufficient condition for $w_G > w_B$ is that:

$$q(\theta_G)(\phi_B + \Pi) > q(\theta_B)(\phi_G + \Pi)$$

where $\Pi = \pi_B + \pi_G$. In general, it is easier for firms to fill vacancies in
the bad state because there are more unemployed workers around, that is
q(θ_B) > q(θ_G). However, the difference in job destruction rates φ_B > φ_G numerically outweighs the difference in the arrival rates of job seekers.

Because it is impossible to determine the sign of the wage relation (39) theoretically, we must resort to the calibrated version of the model to examine wage outcomes. The model is calibrated to U.S. labor market data. Section 4 presents the full calibration. Average job destruction is set to 40 percent; and ranges from 30 percent in the good state to 50 percent in the bad state. The baseline case involves uniform benefits, with b_G = b_B = 0.3. The average unemployment rate is 6.5 percent. Unemployment in the good state is 4.9 percent, while unemployment in the bad state is 8.1 percent. Output is 3.2 percent lower in the bad state. The baseline yields a wage differential of 1.1 percent between the good and the bad state.

Table 1 shows how wages and the wage differential depend on the level of risk aversion. Wages decline when workers become more risk-averse, because the marginal utility of a higher wage decreases when the utility function gets more concave. Wages in the good state are always higher than wages in the bad state, but the magnitude of the wage differential increases with risk aversion.

<table>
<thead>
<tr>
<th>Table 1. The impact of risk aversion.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>w_G</td>
</tr>
<tr>
<td>w_B</td>
</tr>
<tr>
<td>ln(w_G/w_B)</td>
</tr>
</tbody>
</table>

### 3.8 The effects of benefits

We are interested in the effects of flat benefits b and differentiated benefits b_i. It is useful to begin with a one-state model. The equilibrium system (28)-(35) simplifies to:

\[
\frac{(y - w) h}{\phi} = \frac{\kappa}{q(\theta)}
\]  

(40)

\[
-\frac{\partial v(B, s)}{\partial s} = \alpha(\theta)(E - U)
\]  

(41)

\[
\hat{\beta}\frac{(y - w) \partial v(w, h)}{\phi \partial h} = E - U
\]  

(42)

\[
-\beta h \frac{\partial v(w, h)}{\phi \partial h} = E - U
\]  

(43)
where \( E - U = \frac{v_u - v_w}{\delta + \alpha} \). We use the envelope property that \( E - U \) is constant to derivative changes in search effort when the latter is optimally determined. This property also implies that \( w, h \) and \( \theta \) are constant with respect to changes in optimal search effort.

If we substitute \( h(w) = y/(y + \delta w) \) from (43) into (42), the system is “almost recursive”, where (40) and (42) simultaneously determine \( w \) and \( \theta \). It is sufficient to differentiate (40) and (42) with respect to \( \theta, w \) and \( b \) such that \( \partial \theta/\partial b < 0 \) and \( \partial w/\partial b > 0 \). Because \( h'(w) < 0 \), it follows that \( \partial h/\partial b < 0 \). Higher unemployment benefits increase the worker’s threat point and reduce the utility difference between work and unemployment. An increase in the worker’s threat point increases worker’s relative bargaining strength; hence, workers receive higher wages and fewer hours. Higher wages and fewer hours reduce the value of a filled job for the firm. Firms open up fewer vacancies, and therefore, tightness decreases.

Consider the right-hand side of (41). An increase in benefits decreases the difference between employment and unemployment \( E - U \) as well as the arrival rate of job offers \( \alpha(\theta) \); therefore, \( \partial s/\partial b < 0 \). Workers remain unemployed longer because they search less and receive job offers at a slower rate; thus, \( \partial u/\partial b > 0 \). To sum up the comparative statics we have obtained the following for the one-state model:

\[
\frac{\partial \theta}{\partial b} < 0, \quad \frac{\partial s}{\partial b} < 0, \quad \frac{\partial w}{\partial b} > 0, \quad \frac{\partial h}{\partial b} < 0, \quad \frac{\partial u}{\partial b} > 0
\]

Let us go back to the two-state model and consider differentiated benefits. Consider the partial effect of benefits on the value equation differences (7)-(8). If we only consider the effects of benefits and hold all other variables constant, then it is clear that \( \frac{\partial (E_i - U_i)}{\partial b} < 0 \) for \( i = G, B \) (the same holds for \( E_B - U_B \)). The difference between employment and unemployment decreases when benefits in the same state increase. This is also true when benefits in the other state increase because individuals are forward-looking. Therefore, we expect (at least in the partial model) that search effort decreases when benefits increase because the marginal gain of job search decreases. Additionally, consider the worker’s bargaining position. A decrease in the difference between unemployment and employment improves the worker’s threat point. Recall that the value difference is more heavily weighted toward the current state. Consider an increase in benefits in the good state. Workers in the good state have a stronger bargaining position, thus receiving higher wages and fewer hours in the good state. Higher wages and fewer
hours decrease the value of a job in both states. These responses lead to potential ambiguity on wages in the bad state because the value of a job is reduced for both workers and firms. In the numerical analysis, the firm part dominates. Moreover, wages in the bad state fall.

Analytical results are difficult to obtain for comparative statics in the general equilibrium with two states. We must therefore resort to the calibrated model. Table 2 shows numerical comparative statics for uniform and differentiated benefits. We only consider the average unemployment because there are no cases where unemployment in one state increases while unemployment in the other state decreases.

Table 2. Comparative statics (calibrated model).

<table>
<thead>
<tr>
<th></th>
<th>sG</th>
<th>sB</th>
<th>wG</th>
<th>wB</th>
<th>ln (wG/wB)</th>
<th>hG</th>
<th>hB</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>bG</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>bB</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Note that most of the results from the simplified model hold for the full model. A rise in benefits always reduces the search effort in both states. Wages depend positively on benefits in the same state and negatively on benefits in the other state. The opposite is true for hours. Higher flat-rate benefits reduce the wage differential. Benefits in the good state increase the wage differential (because wages were already higher in that state), while higher benefits in the bad state reduce the wage differential. Unemployment always increases with higher benefits.

We are also interested in the marginal effect of benefits in the good state compared to benefits in the bad state. If benefits in state $i$ increase by 1 percent, how much will the level of search effort and unemployment change? Table 3 presents the marginal responses of changes in benefits around the baseline. Note that these effects are not linear but are local effects based on the calibration presented in the next section. The purpose of Table 3 is to show how responses differ between the good and the bad state. Search effort, job-finding and unemployment are clearly more affected by benefits in the bad state than in the good state. Recall the first-order condition for search effort (9)-(10). The marginal effect of benefits on search effort

$^5$The marginal responses vary depending on the baseline, but the relative responses remain the same; i.e., responses are bigger for benefits in the bad state.
in the other state is small. In the good state, arrival rates are higher, and therefore, workers continue to search at a high rate even when they receive benefits; it is still much better to have a job. In the bad state, however, it is not only hard to get a job, but the duration of the match is shorter than in the good state. Note that this reduction in search effort is slightly counteracted by an increase in wages, but this is not enough to reverse the response. Because benefits in the bad state distort the search effort and job finding rates, average unemployment also responds more to benefits in the bad state than to benefits in the good state. Unemployment in the bad state is also more responsive to benefits in the same state than to benefits in the other state. As noted above, the policy maker has to increase taxes more to finance benefits in the bad state than in the good state.

Table 3. Estimated marginal responses with respect to differentiated benefits.

<table>
<thead>
<tr>
<th></th>
<th>1% increase in $b_G$</th>
<th>1% increase in $b_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Search good state</td>
<td>-0.08</td>
<td>-0.00</td>
</tr>
<tr>
<td>Search bad state</td>
<td>-0.00</td>
<td>-0.09</td>
</tr>
<tr>
<td>Job finding good state</td>
<td>-0.32</td>
<td>-0.02</td>
</tr>
<tr>
<td>Job finding bad state</td>
<td>-0.02</td>
<td>-0.33</td>
</tr>
<tr>
<td>Average unemployment</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>Taxes</td>
<td>0.50</td>
<td>0.80</td>
</tr>
</tbody>
</table>

4 Calibration

The model is calibrated to replicate some key features of the U.S. labor market. Following Shimer (2005), we assume a Cobb-Douglas matching function $M = aS^\eta v^{1-\eta}$ with $\eta = 0.72$. Like Shimer, we also set $\beta = \eta$. Productivity is normalized to unity, with $y = 1$. The time period is a quarter. The rate of interest, which is equal to the rate of time preferences, is set to zero. Constant relative risk aversion is set to $\rho = 1$, which is in the lower range suggested by Szpiro (1986). This level is also in the lower range of the values used in the previous literature on differentiated UI; for example Andersen & Svarer (2010) assume $\rho = 4$, and Moyen & Stähler (2009) assume $\rho = 1.5$. 

The average annual separation rate has historically been approximately 40 percent in the U.S. (Shimer, 2005). However, separation rates vary substantially over the cycle. The annual separation rate in the good state is set to 30 percent, and in the bad state, it is set to 50 percent. It is not clear from the data how to choose values for the transition $\pi_B$ and $\pi_G$. Luckily, neither the conclusions nor the calibrations are substantially affected by the choice of separation rates. We assume that both states last an average of three years, and we set $\pi_G = \pi_B = 1/12$.

Unemployment benefits in the baseline case are uniform and set to 30 percent of productivity, with $b/y = 0.3$. This level corresponds to a replacement rate slightly above 30 percent. Although replacement rates in the U.S. are higher than 30 percent, a good portion of the unemployed do not receive unemployment benefits at all. Replacement rates of approximately 30 percent should be a reasonable uniform characterization for the representative U.S. worker.

Three parameters remain to be calibrated: $\delta$, $a$ and $\kappa$. These parameters are calibrated to match unemployment levels, vacancy levels as well as unemployment and vacancy durations. According to Shimer (2005), vacancy rates are approximately 2 percent. Because the duration of the good state and the bad state is three years, we aim at matching the unemployment rates for the last six years (2005-2010). The average unemployment rate is 6.5 percent; the average unemployment in good times from 2005 to 2007 is 4.8 percent, and the average unemployment in bad times from 2008 to 2010 is 8.2 percent (Bureau of Labor Statistics). We let the vacancy duration and vacancy rates guide the choice of $\kappa$. The parameter for leisure $\delta$ is set to obtain reasonable responses in search effort to benefit changes. Last, the matching constant $a$ is chosen to obtain an average unemployment of 6.5 percent. Table 4 presents the exogenous parameter values in the model.
Table 4. Parameter values.

<table>
<thead>
<tr>
<th>Fixed parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching elasticity $\eta$</td>
<td>0.72</td>
</tr>
<tr>
<td>Bargaining power $\beta$</td>
<td>0.72</td>
</tr>
<tr>
<td>Separation rate $\phi_G$</td>
<td>0.075</td>
</tr>
<tr>
<td>Separation rate $\phi_B$</td>
<td>0.125</td>
</tr>
<tr>
<td>Constant relative risk aversion $\rho$</td>
<td>1</td>
</tr>
<tr>
<td>Transition probability $\pi_G$</td>
<td>$1/12$</td>
</tr>
<tr>
<td>Transition probability $\pi_B$</td>
<td>$1/12$</td>
</tr>
<tr>
<td>Unemployment benefits $b/y$</td>
<td>0.3</td>
</tr>
<tr>
<td>Vacancy cost $\kappa$</td>
<td>1.5</td>
</tr>
<tr>
<td>Matching coefficient $a$</td>
<td>2.31</td>
</tr>
<tr>
<td>Preferences for leisure $\delta$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 5. Calibrated outcomes.

<table>
<thead>
<tr>
<th>Outcomes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average unemployment $\bar{u}$</td>
<td>0.065</td>
</tr>
<tr>
<td>Unemployment in the good state $u_G$</td>
<td>0.049</td>
</tr>
<tr>
<td>Unemployment in the bad state $u_B$</td>
<td>0.081</td>
</tr>
<tr>
<td>Vacancy rate in the good state $v_G$</td>
<td>0.012</td>
</tr>
<tr>
<td>Vacancy rates in the bad state $v_B$</td>
<td>0.018</td>
</tr>
<tr>
<td>Average duration unemployment</td>
<td>9 weeks</td>
</tr>
<tr>
<td>Average duration vacancies</td>
<td>2 weeks</td>
</tr>
<tr>
<td>Tax rate $t$</td>
<td>0.023</td>
</tr>
<tr>
<td>Wage differential $\ln (w_G/w_B)$</td>
<td>0.011</td>
</tr>
<tr>
<td>Output gap between the good and the bad state</td>
<td>-0.032</td>
</tr>
</tbody>
</table>

The calibrated outcomes are shown in Table 5. The model matches the U.S. labor market well on unemployment levels, unemployment duration and vacancies. Vacancies are slightly lower than what is observed in the data. At first glance, it might seem odd that the vacancy rate is higher in the bad state. It is possible because the economy jumps between two Beveridge curves. However, there are still more jobs in the good state (including both vacancies and filled jobs). The wage differential between the good and the bad state is approximately 1 percent. Output is 3.2 percent lower in the bad state. In the good state, there is more job creation; workers search more for jobs and earn higher wages. Workers also work slightly fewer hours, but this difference is trivial. The flat tax rate is 2.3 percent.
5 Welfare analysis

An important question is how unemployment benefits should depend on the state of the economy. Because workers are risk-averse, they prefer some form of UI. It is possible that workers prefer higher benefits in bad times when unemployment is relatively high, unemployment duration is longer, and job offers are harder to come by. In contrast, to finance benefits in bad times, the policy maker must increase taxes more than in good times because there are fewer employed workers around. Finally, benefits could affect worker behavior more (or less) in bad times. In bad times, workers can do less to affect their own job-finding probability, and therefore, labor demand is more important than labor supply. Andersen & Svarer (2010) show that benefits are more distortionary in good times when workers search more in bad times than in good times. In contrast, when workers search more in good times due to higher returns to search effort, benefits could be more distortionary in bad times. The payoff from search effort and the difference between employment and unemployment are both lower in bad times.

Another instrument to consider is taxes. The first argument for imposing two benefit levels is that policy makers want to redistribute consumption in bad times. A more efficient way of doing this could be to decrease tax rates for workers (including the unemployed) in bad times. Low tax rates in bad times give unemployed and employed workers more resources without increasing benefits. Because more individuals are employed in good times, taxes need to be raised less good times to finance benefits.

We use the model presented in the previous section to examine the welfare aspects of state-dependent benefits and taxes for the U.S. To facilitate the analysis of differentiated benefits and taxes we proceed under the assumption of constant relative risk aversion, with $\rho = 1$. The utility function of the workers then takes the following form:

\[ u_i^u \equiv v(b_i, s_i) = \ln (b_i) + \delta \ln (1 - s_i) + \ln (1 - t_i) \]
\[ u_i^c \equiv v(w_i, h_i) = \ln (w_i h_i) + \delta \ln (1 - h_i) + \ln (1 - t_i) \]

We focus on steady states and ignore discounting; i.e., we let $r \to 0$. We consider a utilitarian welfare function with the worker’s expected utility given as:

\[ \Lambda = \frac{\pi_B}{\pi_G + \pi_B} \left( [1 - u_G] v_G^c + u_G v_G^u \right) + \frac{\pi_G}{\pi_G + \pi_B} \left( [1 - u_B] v_B^c + u_B v_B^u \right) \]

(44)
The welfare effect $\Delta \Lambda$ of a specific UI regime is a compensating variation measure; namely, it is the equivalent of a consumption tax that equalizes welfare across policy regimes. Let $\Lambda^{BC}$ represent the welfare associated with the baseline case, and let $\Lambda^{A}$ represent the alternative policy. The welfare gain from policy $A$ compared to policy $BC$ is given by the tax rate $\tau$ that solves $\Lambda^{A}[1 - \tau] = \Lambda^{BC}$. For $\rho = 1$, it follows that $\Delta \Lambda = \tau \approx \Lambda^{A} - \Lambda^{BC}$.

Because welfare differs across states, it is also interesting to assess the welfare gains separately for each state. Let $\Lambda^{i}$ denote welfare in state $i$:

$$
\Lambda^{G} = u^{G}(b^{G}, s^{G}) + (1 - u^{G}) v(w^{G}, h^{G})
$$

$$
\Lambda^{B} = u^{B}(b^{B}, s^{B}) + (1 - u^{B}) v(w^{B}, h^{B})
$$

where $\Lambda = \frac{\pi_B}{\pi_G + \pi_B} \Delta \Lambda^{G} + \frac{\pi_G}{\pi_G + \pi_B} \Delta \Lambda^{B}$. As a third welfare measure, we use the difference $\Delta \Lambda^{B,G}$ as a measure of the welfare gain associated with the good state compared to the bad state.

The first policy (1) includes optimal uniform benefits $b$ and uniform tax rate $t$. The second policy (2) considers optimally differentiated benefit levels $b^{G}$ and $b^{B}$ and constant tax rate $t$. The third policy (3) studies the optimally differentiated tax rates $t^{G}$ and $t^{B}$ and constant benefit level $b$. The fourth policy (4) considers optimally differentiated benefits and taxes $b^{G}$, $b^{B}$, $t^{G}$ and $t^{B}$. Because we assume constant relative risk aversion, with $\rho = 1$, the tax is still neutral even when it is differentiated. The main advantage to this approach is that we compare apples with apples; the tax is solved residually throughout the paper. In this set up, taxes only affect worker welfare.

---

6The optimal system with budget balance in each state is not considered in this paper. For robustness and to relate this study to the previous literature, we can check this case. When taxes are equal across states, it trivially follows that $b^{G} > b^{B}$ simply because revenues are higher in the good state. By a similar argument, taxes in the good state are lower than in the bad state when benefits are equal across states. The optimally differentiated system entails $b^{G} > b^{B}$ and $t^{B} > t^{G}$ and yields similar welfare as the optimal uniform system. The result is similar as that in Andersen & Svarer (2011) when they consider budget balancing in each state.

7Consider the equilibrium system. The job creation equations are unaffected by taxes. Taxes are additive for log utility: $v(C, l) = \log((1 - t_i) C_i) + \delta \log(l) = \log(1 - t_i) + \log(C) + \delta \log(l)$. Therefore, taxes do not enter the left-hand side of the first-order conditions. Neither do taxes affect the right-hand side because $E_i - U_i$ is unaffected.
5.1 Business cycle-dependent UI levels

In section 3.7, we argue that the finding that unemployment benefits are less distortionary in bad times does not hold when workers have higher search effort in good times. Thus, the policy maker must meet worker demands for insurance by taking into account that search effort is more responsive to benefits in bad times. The social planner maximizes the social welfare function (44) with respect to the two benefit levels $b_B$ and $b_G$. It is impossible to solve this problem analytically because all variables, except the transition probabilities, depend endogenously on the benefit levels. However, it is still useful to consider the optimal benefit levels, holding all other variables constant. In this case, we can obtain explicit equations for the optimal benefit levels (see the Appendix for full derivation):

$$b_G = \frac{1 + \tau}{\pi_B + \pi_G} [\pi_G (1 - u_B) w_B h_B + \pi_B (1 - u_G) w_G h_G]$$  \hspace{1cm} (45)$$

$$b_B = \frac{1 + \tau}{\pi_B + \pi_G} [\pi_G (1 - u_B) w_B h_B + \pi_B (1 - u_G) w_G h_G]$$  \hspace{1cm} (46)$$

It is obvious that the optimal benefit level in the good state is equal to the optimal benefit level in the bad state, holding all other variables constant. However, benefits do affect all variables in the model. Nevertheless, this result suggests that the benefit differential between the good and the bad state might be small. It also suggests that full redistribution might not be optimal; on the contrary, these optimal levels do not imply full redistribution of welfare over the business cycle. To obtain the optimal benefit levels for the full model, we must resort to the numerical model, which is presented in section 5.3.

5.2 Business cycle-dependent income taxes

An alternative way to redistribute incomes over the business cycle is to vary the tax rate. Consider benefit level $b$ and two tax rates, namely, $t_G$ in the good state and $t_B$ in the bad state. The policy maker’s budget constraint $T_x(t_G, t_B) = E(b)$ takes the following form:

$$b(t_G, t_B) = \frac{t_B \pi_G e_B w_B h_B + t_G \pi_B e_G w_G h_G}{\pi_B u_G (1 - t_G) + \pi_G u_B (1 - t_B)}$$

Because taxes do not affect the equilibrium system, we can obtain implicit equations for optimal taxes $t_G$ and $t_B$. We maximize the social welfare
function (44) with respect to $t_G$ and $t_B$ (for full derivation see the Appendix) and obtain tax rates from the following implicit functions:

$$t_G = 1 - \pi_B \frac{b(t_G, t_B)}{\pi_G u_B + \pi_B u_G} \left( \frac{\partial b}{\partial t_G} \right)^{-1}$$

(47)

$$t_B = 1 - \pi_G \frac{b(t_G, t_B)}{\pi_G u_B + \pi_B u_G} \left( \frac{\partial b}{\partial t_B} \right)^{-1}$$

(48)

It is clear from these equations that the tax rates depend on the transition probability between states and the first-order derivative of benefits with respect to taxes. As long as the employed worker’s income in the good state exceeds their income in the bad state, then $\partial b/\partial t_G > \partial b/\partial t_B$. In other words, taxes needs to be increased less in the good state than in the bad state to finance the same uniform benefit level.

This result suggests that it is more efficient to finance benefits from taxes in the good state; that is, the policy maker gets more “bang for the buck” in this state. Furthermore, the policy maker’s goal is to redistribute welfare across states, and because the good state involves higher income and worker welfare, this suggests higher taxes in the good state. However, if good times are more common than bad times (i.e., $\pi_B > \pi_G$), then it is no longer obvious that the policy makers want to redistribute income to workers in the bad state. It might be optimal to levy high taxes for a short period of time so that workers can enjoy low taxes in the good state for a long period of time. Indeed, the relationship might be reversed if $\pi_B$ is sufficiently higher than $\pi_G$. We can summarize these results in the following proposition:

**Proposition 1** Under the assumption that employed workers earn higher income in the good state than in the bad state, i.e., $w_G h_G > w_B h_B$, then the following is true regarding taxes in the good versus the bad state:

i) If $\pi_B \leq \pi_G$, then taxes in the good state will always be higher than taxes in the bad state, that is, $t_G > t_B$.

ii) If $\pi_B > \pi_G$, then the relationship between taxes in the good state and taxes in the bad state is ambiguous; if $\pi_B$ is sufficiently higher than $\pi_G$, the tax relation $t_G > t_B$ can be reversed.

**Proof.** See the Appendix. □
5.3 Numerical results

Table 6 shows the outcomes associated with the optimal uniform and optimally differentiated systems. Compared to the baseline system, there are substantial gains associated with all four systems of approximately 1 percent of consumption, but the large welfare gain stems from the overall increase in benefits. The welfare gain from differentiation is small. There are two important points here. The first is that it is not feasible to compare business cycle-dependent UI to the current system and draw the conclusion that business cycle-dependent benefits increase welfare. If we want to assess the gain in differentiated benefits, we must compare optimally differentiated benefits with optimal uniform benefits; otherwise, the gain from a business cycle-dependent scheme might be seriously overestimated. The second point is that there are no comparable results in the literature. Mitman & Rabinovich (2011) are the only researchers who quantify welfare gains, and they find that the optimal path of business cycle-dependent UI increases welfare by 0.28 percent compared to the current U.S. system.

The optimal system with uniform benefits and taxes involves flat benefits $b = 0.48$ (corresponding to a replacement rate approximately 49 percent of wages) and tax rate $t = 0.045$. Unemployment in the good state is 6.2 percent, and unemployment in the bad state is 10.1 percent. Workers prefer the good state; workers in the bad state would be willing to pay 2.8 percent of consumption to switch to the good state.

Business cycle-dependent benefits decrease benefits in the good state. So far, the results are similar to the literature, i.e., that $b_B > b_G$. When we estimate the optimal levels, however, benefits are roughly 1 percent higher in the bad state than in the good state. This result is in line with the findings in the previous section that optimal benefits would be equal across states if they did not affect worker behavior. Not surprisingly, the welfare gain from such a small differentiation is approximately zero. However, there is redistribution between states; workers gain as much in the bad state as they lose in the good state.

Taxes are neutral and do not affect the equilibrium system. Imposing business cycle-dependent tax rates is, in this sense, a costless way of redistributing welfare across states. Indeed, the optimal system entails significantly higher taxes in the good state $t_G = 0.057$ compared to $t_B = 0.033$ in the bad state. The welfare gain from differentiated taxes is much larger than the gain from differentiated benefits (almost 50 times as large) but
still small, amounting to 0.01 percent in consumption equivalence measures. Allowing for two tax rates almost closes the welfare gap between the good and the bad state; however, it is still 0.4 percent better to be in the good state.

The last exercise involves both tax and benefit differentiation. Taxes are still higher in the good state, but the benefit differential disappears. The gain compared to the optimal uniform system is 0.01 percent of consumption. However, when we split this gain, it is clear that the whole gain comes from tax differentiation; we find a 0.01 percent gain if we compare system (2) and (4) and no gain from adding differentiated benefits (4) to a system with already differentiated taxes (3). Imposing two tax rates always dominates a structure with two benefit levels.

<table>
<thead>
<tr>
<th>Table 6. Optimal unemployment insurance.</th>
</tr>
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<tbody>
<tr>
<td></td>
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<tr>
<td>-----------------------------------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>taxes (G)</td>
</tr>
<tr>
<td>taxes (B)</td>
</tr>
<tr>
<td>benefits (G)</td>
</tr>
<tr>
<td>benefits (B)</td>
</tr>
<tr>
<td>search effort (G)</td>
</tr>
<tr>
<td>search effort (B)</td>
</tr>
<tr>
<td>unemployment (G)</td>
</tr>
<tr>
<td>unemployment (B)</td>
</tr>
<tr>
<td>ΔΛ (%)</td>
</tr>
<tr>
<td>ΔΛBenefits (%)</td>
</tr>
<tr>
<td>ΔΛTaxes (%)</td>
</tr>
<tr>
<td>ΔΛB,G (%)</td>
</tr>
</tbody>
</table>

Notes: ΔΛ is the welfare gain in consumption tax measures compared to the uniform system; ΔΛBenefits is the gain associated with two benefit levels; ΔΛTaxes is the gain associated with two tax rates; and ΔΛB,G is the welfare surplus associated with the good state.

The results so far suggest that policy makers should use taxes if they want to redistribute consumption over the business cycle rather than imposing differentiated benefit levels; however, one should keep in mind that
the welfare gain from any level of differentiation is small. How robust is this finding to the key parameters in the model? There are three main parameters in the model that could affect the optimal UI system: the ratio between the two states $\phi_B/\phi_G$, the level of risk aversion $\rho$ and the coefficient capturing the value of leisure $\delta$. Across all sensitivity checks, two results stand out: (i) there is no significant welfare gain from differentiating benefits, and (ii) two tax rates always dominate two benefit levels.

The welfare gain from differentiation increases when we increase job destruction in the bad state (or decrease it in the good state). This result is intuitive; if we make the states more different, it is better to differentiate benefits and taxes. Increasing risk aversion yields similar results; risk-averse workers gain more from redistribution between states. As we would expect, the actual level of optimal benefits depends on risk aversion. Higher risk aversion results in higher benefit levels (i.e., more insurance). A higher value for leisure $\delta$ increases the moral hazard from UI in the economy. Workers who care more about leisure will adjust their search more in response to higher benefits. Therefore, the level of optimal benefits also depends on $\delta$ (i.e., low $\delta$ implies high benefits). However, the welfare gains have similar magnitudes, and tax differentiation results in even more gains because taxes do not affect incentives.

6 Concluding remarks

We have proposed an equilibrium two-state search and matching model in which workers and firms face good and bad times. The model is calibrated to the U.S. economy to evaluate the optimal unemployment insurance system, including taxes. The optimal UI system involves lower taxes in bad times than in good times but almost no differentiation of benefits. There are small welfare gains for workers associated with the optimal system but substantial redistribution across states. We conclude that taxes are a more efficient way of redistributing income over the business cycle than benefits. The results also suggest that the welfare gains from business cycle-dependent UI are likely to be negligible.

There are, however, a few issues that are not explicitly addressed in this paper. The first is that it might be easier in real-world politics to change benefit levels than change income or consumption taxes over the business cycle. However, it might not be advisable to impose these changes through discretionary decisions; instead, they should follow some pre-determined
rules. Otherwise, it is likely that it is easier for policy makers to introduce higher levels of benefits or lower taxes in bad times than withdraw them in good times.

7 References


APPENDIX

Optimal benefit levels

We maximize the social welfare function with respect to the two benefit levels subject to the government budget constraint.

\[
\begin{align*}
\max_{tG,tB} \Lambda & = \frac{\pi_G}{\pi_B + \pi_G} \Lambda^G + \frac{\pi_B}{\pi_B + \pi_G} \Lambda^B \\
\text{s.t.} \quad t & = \frac{\pi_G b_G u_B + \pi_B b_G u_G}{\pi_G b_B u_B + \pi_B b_G u_G + \pi_G e_B w_B h_B + \pi_B e_G w_G h_G}
\end{align*}
\]

Define \( \tau = \frac{t}{1-t} \) to simplify the budget constraint to:

\[
\tau = \frac{\pi_G b_G u_B + \pi_B b_G u_G}{\pi_G e_B w_B h_B + \pi_B e_G w_G h_G}
\]

and the utility functions are:

\[
\begin{align*}
v(b_i, s_i) & = \ln (b_i) + \delta \ln (1 - s_i) - \ln (1 + \tau) \\
v(w_i, h_i) & = \ln (w_i h_i) + \delta \ln (1 - h_i) - \ln (1 + \tau)
\end{align*}
\]

We substitute the budget constraint into the welfare function and obtain the following FOCs:

\[
\begin{align*}
\frac{d\Lambda}{db_G} & = \frac{\pi_G}{\pi_B + \pi_G} \frac{\partial \Lambda^G}{\partial b_G} + \frac{\pi_B}{\pi_B + \pi_G} \frac{\partial \Lambda^G}{\partial b_G} = 0 \\
\frac{d\Lambda}{db_B} & = \frac{\pi_G}{\pi_B + \pi_G} \frac{\partial \Lambda^B}{\partial b_B} + \frac{\pi_B}{\pi_B + \pi_G} \frac{\partial \Lambda^G}{\partial b_B} = 0
\end{align*}
\]

Recall that:

\[
\begin{align*}
\Lambda^G & = u_G v (b_G, s_G) + (1 - u_G) v (w_G, h_G) \\
\Lambda^B & = u_B v (b_B, s_B) + (1 - u_B) v (w_B, h_B)
\end{align*}
\]

If we rewrite the FOC using these we obtain:

\[
\begin{align*}
-\pi_G \frac{1}{1 + \tau} \frac{\partial \tau}{\partial b_G} - \pi_B \frac{1}{1 + \tau} \frac{\partial \tau}{\partial b_B} + \pi_B u_G \frac{1}{b_G} & = 0 \\
-\pi_G \frac{1}{1 + \tau} \frac{\partial \tau}{\partial b_B} - \pi_B \frac{1}{1 + \tau} \frac{\partial \tau}{\partial b_B} + \pi_B u_G \frac{1}{b_B} & = 0
\end{align*}
\]
Solving for the benefit levels yields:

\[ b_G = \pi_B u_G \frac{1 + \tau}{\pi_B + \pi_G} \left( \frac{\partial \tau}{\partial b_G} \right) \]  \hspace{1cm} (49)  

\[ b_B = \pi_G u_B \frac{1 + \tau}{\pi_B + \pi_G} \left( \frac{\partial \tau}{\partial b_B} \right) \]  \hspace{1cm} (50)  

It is straightforward to differentiate \( \tau \) and substituting back into the benefit equations:

\[ b_G = \frac{1 + \tau}{\pi_B + \pi_G} (\pi_G e_B h_B w_B + \pi_B e_G h_G w_G) \]  \hspace{1cm} (51)  

\[ b_B = \frac{1 + \tau}{\pi_B + \pi_G} (\pi_G e_B h_B w_B + \pi_B e_G h_G w_G) \]  \hspace{1cm} (52)  

It is clear that \( b_G = b_B \) in optimum!

**Optimal tax rates**

We maximize the social welfare function with respect to the two tax rates subject to the government budget constraint.

\[
\max_{t_G, t_B} \Lambda = \frac{\pi_G}{\pi_B + \pi_G} \Lambda^B + \frac{\pi_B}{\pi_B + \pi_G} \Lambda^G \\
\text{s.t.} \quad b = \frac{t_B e_B w_B h_B + t_G e_G w_G h_G}{u_G (1 - t_G) + u_B (1 - t_B)}
\]

We substitute the budget constraint into the welfare function and obtain the following FOCs:

\[
\frac{d\Lambda}{dt_G} = \frac{\pi_G}{\pi_B + \pi_G} \frac{\partial \Lambda^B}{\partial t_G} + \frac{\pi_B}{\pi_B + \pi_G} \frac{\partial \Lambda^G}{\partial t_G} = 0 \\
\frac{d\Lambda}{dt_B} = \frac{\pi_G}{\pi_B + \pi_G} \frac{\partial \Lambda^B}{\partial t_B} + \frac{\pi_B}{\pi_B + \pi_G} \frac{\partial \Lambda^G}{\partial t_B} = 0
\]

If we rewrite these expressions we obtain:

\[
\pi_G u_B \frac{1}{b} \frac{\partial b}{\partial t_G} - \pi_B \frac{1}{1 - t_G} + \pi_B u_G \frac{1}{b} \frac{\partial b}{\partial t_G} = 0 \\
-\pi_G \frac{1}{1 - t_B} + \pi_G u_B \frac{1}{b} \frac{\partial b}{\partial t_B} + \pi_B u_G \frac{1}{b} \frac{\partial b}{\partial t_B} = 0
\]
Solving for the tax rates yields:

\[
t_G = 1 - \pi_B \frac{b}{\pi_G u_B + \pi_B u_G} 1/ \left( \frac{\partial b}{\partial t_G} \right) \tag{53}
\]

\[
t_B = 1 - \pi_G \frac{b}{\pi_G u_B + \pi_B u_G} 1/ \left( \frac{\partial b}{\partial t_B} \right) \tag{54}
\]

We differentiate the budget constraint to obtain \( \frac{\partial b}{\partial t_G} \) and \( \frac{\partial b}{\partial t_B} \):

\[
\frac{\partial b}{\partial t_G} = \frac{(1 - u_G) w_G h_G + u_G b}{u_G (1 - t_G) + u_B (1 - t_B)}
\]

\[
\frac{\partial b}{\partial t_B} = \frac{(1 - u_B) w_B h_B + u_B b}{u_G (1 - t_G) + u_B (1 - t_B)}
\]

It is clear that:

\[
\frac{\partial b}{\partial t_G} - \frac{\partial b}{\partial t_B} = \frac{w_G h_G - w_B h_B + (u_B - u_G) (w_G h_G - b)}{u_G (1 - t_G) + u_B (1 - t_B)} > 0
\]

as long as the income in good times is bigger than the income in bad times, that is \( w_G h_G > w_B h_B \).

Go back to the tax equations. As long as \( \pi_B \leq \pi_G \) then \( t_G > t_B \).
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