Uppsala University

This is a published version of a paper published in *Geophysics*.

Citation for the published paper:

Pedersen, L., Beiki, M. (2011)

"Window constrained inversion of gravity gradient tensor data using dike and contact models"

Geophysics, 76(6): 159-172

URL: <http://dx.doi.org/10.1190/GEO2010-0368.1>

Access to the published version may require subscription.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-169382>

<http://uu.diva-portal.org>

Window constrained inversion of gravity gradient tensor data using dike and contact models

Majid Beiki¹ and Laust B. Pedersen²

ABSTRACT

We have developed a constrained inversion technique for interpretation of gravity gradient tensor data. For dike and contact models striking in the y -direction, the measured g_{xz} and g_{zz} components can be jointly inverted for estimating the model parameters horizontal position, depth to the top, thickness, dip angle, and density contrast. For a given measurement point, the strike direction of the gravity gradient tensor caused by a quasi 2D structure can be estimated from the eigenvector corresponding to the smallest eigenvalue. Then, the measured components can be transformed into the strike coordinate system. It is assumed that the maximum of g_{zz} is approximately located above the causative body. In the case of gridded data, all measurement points enclosed by a square window centered at the maximum of g_{zz} are used to estimate the source parameters. The number of data

points used for estimating source parameters is increased by increasing the size of the window. Solutions with the smallest data-fit error were selected as the most reliable solutions from any set of solutions. The gravity gradient tensor data are deconvolved using both dike and contact models within a set of square windows. Then, the model with the smallest data-fit error is chosen as the best model. We studied the effect of random noise and interfering sources using synthetic examples. The method is applied to a gravity gradient tensor data set from the Vredefort impact structure in South Africa. In this particular case, the dike model provides solutions with smaller data-fit errors than the contact model. This supports the idea that in the central dome area there is a predominance of vertical structures related to the formation of the transient crater and subsequent central uplift of the lower and middle crustal material.

INTRODUCTION

The 2D gravity gradient tensor (GGT) can be represented as the second derivatives of the earth's gravitational potential in two Cartesian directions. If the excess mass distribution is infinitely long in the y -direction and has a uniform cross section of arbitrary shape in the xz -plane, the gravitational potential U from excess density distribution ρ can be written as

$$U(\mathbf{r}) = 2\gamma \int_x \int_z \rho \ln(1/|\mathbf{r} - \mathbf{r}'|) dx dz, \quad (1)$$

where $\mathbf{r}$ and $\mathbf{r}'$ denote observation and integration points, respectively, and γ is the gravitational constant. The gravity gradient tensor Γ is then:

$$\Gamma = \begin{bmatrix} \frac{\partial^2 U}{\partial x^2} & \frac{\partial^2 U}{\partial x \partial z} \\ \frac{\partial^2 U}{\partial x \partial z} & \frac{\partial^2 U}{\partial z^2} \end{bmatrix} = \begin{bmatrix} g_{xx} & g_{xz} \\ g_{xz} & g_{zz} \end{bmatrix}, \quad (2)$$

where $\mathbf{g} = (g_x, g_z)$ is the gravity vector and g_{xx} , g_{xz} , and g_{zz} are first order derivatives of the gravity vector components with respect to x and z . Outside the source U satisfies Laplace's equation $\nabla^2 U(\mathbf{r}) = 0$; hence, the trace of the tensor is equal to zero, and then $g_{xx} = -g_{zz}$.

GGT data can be measured either using land, airborne, marine, or space platforms. In recent years, new techniques for processing and interpretation of GGT data have been developed (e.g., Pedersen and Rasmussen, 1990; Vasco and Taylor, 1991; Zhdanov et al., 2004; Mikhailov et al., 2007; FitzGerald et al., 2009; Beiki, 2010; Beiki and Pedersen, 2010). In 1990, Pedersen and Rasmussen studied

Manuscript received by the Editor 13 November 2010; revised manuscript received 30 June 2011; published online 30 December 2011.

¹Formerly Uppsala University, Department of Earth Sciences, Uppsala, Sweden; currently CSIRO Earth Science and Resource Engineering, North Ryde, NSW, Australia. E-mail: majid.beiki@csiro.au.

²Uppsala University, Department of Earth Sciences, Uppsala, Sweden. E-mail: laust.pedersen@geo.uu.se.

© 2011 Society of Exploration Geophysicists. All rights reserved.

gravity and magnetic gradient tensors and introduced scalar invariants to indicate dimensionality. They also showed that the eigenvector corresponding to the maximum eigenvalue points toward the center of a point mass source. Two decades later, [Beiki and Pedersen \(2010\)](#) use the eigenvector corresponding to the smallest eigenvalue to estimate the strike of quasi 2D structures. They also introduced a least-squares procedure to estimate the source point which has the smallest sum of squared distances to the lines defined by the first eigenvectors and position of measurement points.

[Beiki and Pedersen \(2010\)](#) apply their method to a GGT data set from the Vredefort impact structure, South Africa, for estimating depth to the center of mass as well as the strike direction of quasi 2D bodies related to the impact structure. In this paper, we show that the dike and contact parameters horizontal position, depth to the top, thickness/width, dip angle and density contrast can be estimated using joint inversion of the GGT components. The presented technique provides reliable information about physical and geometrical properties of dike and contact models in the presence of random noise and interfering sources. We can choose the most appropriate model type based on data-fit error criterion. The application of the method is demonstrated on the GGT data set from the Vredefort dome in South Africa.

METHOD

Dike and contact models

Dike and contact models are widely used for the interpretation of magnetic and gravity field data (e.g., [Nabighian, 1972](#) and [1974](#); [Hsu et al., 1998](#); [Bastani and Pedersen, 2001](#)). GGT components for a dike can be derived from [Telford et al. \(1990\)](#) as

$$g_{zz} = 2\gamma\rho \left[\sin \alpha \cos \alpha \ln \frac{r_2}{r_1} + \sin^2 \alpha (\theta_1 - \theta_2) \right] \quad (3)$$

and

$$g_{xz} = 2\gamma\rho \left[\sin^2 \alpha \ln \frac{r_2}{r_1} - \sin \alpha \cos \alpha (\theta_1 - \theta_2) \right], \quad (4)$$

where α is the dip angle and parameters r_1 , r_2 , θ_1 , and θ_2 are described in Table 1 based on the parameters horizontal position x_0 , width $2b$, and depth to the top z . Figure 1b shows a dike with dip angle of 45° , depth to the top and width of 100 m, and density contrast of 500 kg/m^3 . The tensor components g_{xz} and g_{zz} are plotted in Figure 1a.

[Stanley and Green \(1976\)](#) show that a thick dike model can be expressed as the difference between two parallel contact models

Table 1. Parameters used in the nonlinear equations 3, 4, 5 and 6 for the dike and contact models shown in Figures 1 and 2.

	Dike	Contact
r_1	$[(x - x_0 + b)^2 + z^2]^{1/2}$	$[(x - x_0)^2 + z^2]^{1/2}$
r_2	$[(x - x_0 - b)^2 + z^2]^{1/2}$	$[(x - x_0 + t \cot \alpha)^2 + (z + t)^2]^{1/2}$
θ_1	$\tan^{-1} \frac{(x - x_0 + b)}{z}$	$\tan^{-1} \frac{(x - x_0)}{z}$
θ_2	$\tan^{-1} \frac{(x - x_0 - b)}{z}$	$\tan^{-1} \frac{(x - x_0 + t \cot \alpha)}{z + t}$

with the same dip angle. They also showed that GGT components for a contact model can be given as

$$g_{zz} = 2\gamma\rho \left[\sin \alpha \cos \alpha \ln \frac{r_2}{r_1} - \sin^2 \alpha (\theta_1 - \theta_2) \right] \quad (5)$$

and

$$g_{xz} = 2\gamma\rho \left[\sin^2 \alpha \ln \frac{r_2}{r_1} + \sin \alpha \cos \alpha (\theta_1 - \theta_2) \right], \quad (6)$$

where the parameters r_1 , r_2 , θ_1 , and θ_2 are described in Table 1 based on the parameters horizontal position x_0 , thickness t , and depth to the top z . Figure 2a shows the GGT components of a contact with depths to the top and thickness of 100 m and 250 m, respectively, dip angle 45° , and density contrast 500 kg/m^3 .

Nonlinear least-squares method

In practice, many anomalies caused by 2D structures can be approximated by either dike or contact models. In our approach we solve the nonlinear equations 3 and 4 for estimating dike parameters and equations 5 and 6 for contact in a line segment containing several data points measured approximately above the causative body. Assuming that the segment is sufficiently large, the parameter estimation problem is overdetermined. We used the MATLAB optimization toolbox (function “lsqnonlin”) for a nonlinear least-squares algorithm based on the Levenberg-Marquardt (LM) and the trust-region-reflective methods to solve for the best model subject to inequality constraints on the model parameters.

Figure 1. (a) Tensor components of g_{zz} and g_{xz} corresponding to a **(b)** dike model with depth to the top and thickness of 100 m and excess density of 500 kg/m^3 .

The LM algorithm is a combination of two minimization methods: the steepest descent and the Gauss-Newton. In the steepest descent method the sum of squared errors is reduced by updating the model parameters in the direction of the steepest descent of the least-squares objective function, whereas in the Gauss-Newton method it is assumed that the objective function is locally quadratic to estimate the updates. The LM method acts more like the steepest descent when the parameters are far from their optimal values. As the model approaches the optimal one the LM method becomes more like the Gauss-Newton method (Gavin, 2011). The Levenberg-Marquardt's relationship is given by

$$[\mathbf{J}^T \mathbf{J} + \lambda \text{diag}(\mathbf{J}^T \mathbf{J})] \Delta \mathbf{p} = \mathbf{J}^T [\mathbf{y} - \hat{\mathbf{y}}(\mathbf{p})], \quad (7)$$

where $\mathbf{J}$ is the Jacobian matrix, the superscript $\mathbf{T}$ denotes a transposed matrix, $\mathbf{y}$ is the measured data, $\hat{\mathbf{y}}(\mathbf{p})$ is the nonlinear function in the model parameters $\mathbf{p}$, $\Delta \mathbf{p}$ is the model perturbation, and the positive parameter λ is adjusted during the updates to ensure convergence. In the beginning, λ is set to a large value and then decreased by a multiplicative factor in each iteration. In the MATLAB function lsqnonlin, the Jacobian matrix $\mathbf{J}$ is calculated automatically using finite differences. Aster et al. (2005) and Madsen et al. (2004) give excellent overviews of the LM algorithm.

Data and model parameters are related through a nonlinear system of equations $\mathbf{g}(m) = \mathbf{d}$ and the objective function to be minimized is

$$s = \sqrt{\frac{\sum_{i=1}^n r_i^2}{n - m}}, \quad (8)$$

where

$$r_i = g(\mathbf{m}^{\text{est}})_i - d_i; \quad i = 1, 2, \dots, n, \quad (9)$$

and n and m are the number of data points and model parameters, respectively, $\mathbf{m}^{\text{est}}$ is the estimated model parameters and the data vector $\mathbf{d}$ include both g_{xz} and g_{zz} .

Implementation of the method and its sensitivity to random noise

To study the effect of random noise on the estimated model parameters, we return to the synthetic model shown in Figure 1. Gaussian noise with zero mean and standard deviation of 1 E is added to the GGT components shown in Figure 1a. Assuming that the maximum of g_{zz} component is approximately located above the causative body we form a square window centered at the maximum of g_{zz} . Then the data points enclosed by this window are used in the inversion scheme for estimating the source parameters. In Figure 3, the data-fit errors and the estimated model parameters are plotted versus the windows centered at maximum of g_{zz} component (945 m). By increasing the window length, more data points are used in the algorithm and data-fit error decreases. Figure 3 shows that using large windows the estimated model parameters are less scattered and they are close to the true values. In this example, we have used initial model parameters horizontal position of 750 m, depth to the top and width of 200 m, dip angle of 90° , and density contrast of 1000 kg/m^3 in the constrained inversion technique. Our investigation of different generic examples shows that the estimated model parameters using the presented constrained inversion

technique are relatively robust to deviations of the initial model parameters from true values. However, in the case that the initial model parameters are several times greater or smaller than true values, the estimated model parameters will be far from true parameters. Our experience with different synthetic examples shows that the introduced method is robust to the initial model parameters two or three times greater or smaller than true values.

It is well known that for thin sheets the product of thickness and density contrast is linearly related to the measured gravity field. Thus, it is impossible to distinguish between a dense thin dike/contact and a thicker less dense dike/contact. As the thickness exceeds a certain fraction of the depth, the data contains sufficient information to resolve both thickness and density contrast. Our synthetic examples illustrate this point.

Interfering sources

In practice, gravity anomalies suffer considerable interference from neighboring sources. In the next example, we study the effect of interfering sources on parameter estimations. Figure 4b shows a model which involves a dike and a contact. Gaussian noise with zero mean and standard deviation 2 E is added to the GGT components. Figure 4a illustrates the GGT components with a sampling interval of 10 m. As was mentioned earlier, a window is formed around the maximum of g_{zz} (at 500 m) to reduce the effect of interfering sources. The source parameters cannot be estimated accurately when the window length is very small. As we showed earlier (Figure 3), in the presence of random noise a large window size is preferred to overcome the effect of local high-frequency

Figure 2. (a) Tensor components of g_{zz} and g_{xz} corresponding to a (b) contact model with finite depth extent, depth to the top 100 m, thickness 250 m, dip angle 45° , and excess density 500 kg/m^3 .

noises. However, use of a very large window is also risky because we may include data points which are more affected by neighboring bodies. We define an optimum window length based on the data-fit error.

In Figure 5a, the data-fit errors for the model shown in Figure 4 are plotted versus the length of the windows centered at the maximum of the g_{zz} component. As expected, by increasing the length of the window, the data-fit errors increase. Figure 5 shows that the estimated parameters using a window length of 140 m is the best solution. For small windows the estimated parameters are more scattered as they are more influenced by random noise than the interfering sources. To estimate the contact parameters the same procedure is repeated using a contact model for body two. The data-fit errors and estimated parameters are plotted in Figure 6 for different windows centered at the maximum of the g_{zz} component (1750 m). As can be seen in Figure 6, the estimated parameters using a window length of 180 m are close to the true values. The initial and estimated source parameters for the window lengths of 140 m and 180 m for infinite dike and contact models are listed in Table 2.

We have also studied the effect of two interfering bodies on estimated model parameters and data-fit error for each body as a function of the separation-to-depth ratio. Figure 7c shows two

vertical dikes with depth to the top and width of 100 m and density contrast of 500 kg/m³. In Figure 7b, the absolute difference of the estimated parameters depth to the top, width, dip angle, and density contrast normalized to their true values for body one are plotted versus different separation-to-depth ratios such that normalized estimated parameters are defined as $\frac{|m^{est} - m^{true}|}{m^{true}} \times 100$. A maximum window size of 200 m is used to estimate the model parameters. The deviation of the estimated horizontal position from the true position and data-fit errors are illustrated in Figure 7a. In this particular example, when the separation-to-depth ratio is equal to one, the data-fit error is equal to zero as the anomaly caused by bodies 1 and 2 is approximated by a single vertical dike with depth to the top 100 m, width 200 m, and density contrast 500 kg/m³. As expected, when the causative bodies are located close to each other, the data-fit error increases. By increasing the separation between the sources, the estimated model parameters approach true values and data-fit error decreases.

Geologic constraints and discrimination of solutions

The 2D GGT components are deconvolved using both dike and contact models by minimizing the data-fit within windows centered

Figure 3. (a) Data-fit error, estimated, (b) horizontal position, (c) dip angle, (d) width, (e) depth to the top, and (f) density contrast for different window lengths for the dike model shown in Figure 1b.

at the maxima of g_{zz} . By increasing the window length and repeating the procedure, the estimated parameters corresponding to the minimum data-fit error are chosen as the best solution. In practice, it is hard to select which model is most appropriate to describe the anomaly caused by an unknown source. Investigating different synthetic data sets shows that we normally are able to exclude inappropriate models based on a data-fit criterion or because of scattered or unrealistic solutions. To avoid unrealistic estimates, we define constraints with lower and upper bounds on the model parameters so that solutions are always in the trust region.

We now return to the dike and contact models shown in Figure 4b and try to interpret their responses with an incorrect model. We have defined bounds on the model parameters such that the density contrast is between 100 and 700 kg/m³, depth to the top and width/thickness are positive, and dip angle is between 0° and 180°. In addition to these constraints we use a criterion to reject spurious solutions located outside the window beyond a certain distance from the center of the window. Figure 8a and b shows the data-fit errors for the anomalies caused by the dike (body one) interpreted using the contact model and the contact (body two) interpreted using the dike model, respectively. Comparing these results with Figures 5a and 6a shows that the use of inappropriate models gives rise to large data-fit errors and unreliable estimates. For instance, the estimated density contrasts corresponding to the case shown in Figure 8a for most of the windows that are located at the upper bound (700 kg/m³) and the estimated source locations outside the profile. For real cases any other a priori information on the geology can be used either as constraints or criteria to reject spurious solutions.

Strike correction

In practice, profiles are not generally orthogonal to the strike direction of the 2D structure. This causes errors in the estimation of source parameters depending upon the deviation from normal of the angle between the strike and profile directions. The measured GGT components can be corrected for the effect of this deviation if the strike direction is known. Beiki and Pedersen (2010) show that the strike of quasi 2D bodies can be obtained from the eigenvector corresponding to the minimum eigenvalue. Defining the strike angle ψ as the angle between the strike direction and the x -axis counted positive from x to y , the GGT tensor $\mathbf{\Gamma}'$ in the new coordinate system (Figure 9) is

$$\mathbf{\Gamma}' = \mathbf{R}^T \mathbf{\Gamma} \mathbf{R}, \quad (10)$$

where

$$\mathbf{R} = \begin{bmatrix} -\sin \psi & -\cos \psi & 0 \\ \cos \psi & -\sin \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Subsequently, the coordinates of the measurement points in the new coordinate system can be expressed as

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -\sin \psi & \cos \psi \\ -\cos \psi & -\sin \psi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \quad (11)$$

Once the measured GGT data in a given window are corrected for the strike angle, the presented nonlinear least-squares method is used for estimating the source parameters.

In practice, GGT data are measured along parallel lines perpendicular to the dominant strike direction in the survey area. To improve the implementation of the presented algorithm, we include all data points in a square window centered at the maxima of g_{zz} (see Figure 9). The procedure is repeated until the window size exceeds a predefined limit. Then the solutions corresponding to the window with minimum data-fit errors are chosen as the final result.

REAL DATA EXAMPLE

We applied our method to a gravity gradient tensor data set from the Vredefort impact structure in South Africa. The Vredefort impact structure is known as the largest and oldest impact structure on earth, with a diameter of 250–300 km. The age of the Vredefort structure is estimated to be 2.02 Ga, meaning that it impacted during the paleoproterozoic era (Kamo et al., 1996). The center of the impact structure, the Vredefort dome, is located about 120 km south-west of Johannesburg within the Witwatersrand basin of the Kaapvaal craton. Reimold and Gibson (1996) give an excellent review of the geology of the Vredefort impact structure. According to Salminen et al. (2009) the basement of the Kaapvaal craton consists of Archean granotoid, gneiss, and migmatite, which are covered with neoproterozoic volcanic and sedimentary rocks. Figure 10 shows a simplified geology map of the Vredefort dome (after Lana et al., 2003).

The crystalline Archean basement in the central part of the Vredefort dome is a complex high-grade metamorphic terrane. In

Figure 4. (a) Tensor components g_{zz} and g_{xz} corresponding to a (b) model with a dike and a contact in presence of Gaussian noise of zero mean and standard deviation of 2 E with sampling interval of 10 m.

outcrops, most strata of the dome are overturned by up to 90° . Such rotation is generally explained as being associated with the collapse of the transient crater (Grieve and Theriault, 2000). Stepto (1990) describes the center of the Vredefort dome as comprising an uplifted granitic basement ringed by upturned sedimentary strata. He showed that detailed geologic mapping of the basement core has delineated three concentric zones of outer granite gneisses (OGG), Steynskraal Formation (SF) and Inlandsee Leucogranofels (ILG). Based on the chemical variations of the rocks across the core of the dome, the OGG and ILG represent upper and lower crust rocks, respectively (Hart et al., 1990).

In 2007, the Vredefort dome area was covered with a Falcon airborne gravity gradiometry survey conducted by Fugro Airborne Surveys. The Falcon Airborne Gravity Gradiometry (AGG) system is designed to measure the horizontal curvature components of the gravity gradients g_{xy} and $(g_{xx} - g_{yy})/2$. Then, the full gravity gradient tensor is derived from the measured components (Dransfield and Lee, 2004). The survey comprised two blocks covering the central and western parts of the Vredefort dome area. Here we apply

our method to the part of the data set that can be represented by quasi 2D geologic structures.

The study area was flown north-south with a line spacing of 1 km. The nominal height of the aircraft was 80 m above the ground. Multistep Falcon processing procedures were used to process the measured modulated differential curvature gradients. The data were demodulated and low-pass filtered with a Butterworth filter (of order six) with a cut-off frequency of 0.18 Hz. Also, terrain corrections were applied using a density of 2670 kg/m^3 . In the processing procedure, a low-pass filter with a cut-off wavelength of 1000 m was applied to the data set. Then the data were resampled with a new sampling interval of 250 m in x - and y - directions. Figure 11a shows the gridded g_{zz} data with a cell size of 250 m.

Beiki and Pedersen (2010) and Beiki et al. (2011) use the dimensionality indicator I to discriminate between quasi 2D and 3D geologic bodies. The dimensionality indicator I varies between zero (pure 2D) and unity (pure 3D). Figure 11b shows the regions with I greater than 0.5 in gray. The major 2D geologic structures of the study area are marked in Figure 11a as A, B, and C.

Figure 5. (a) Data-fit error, estimated, (b) horizontal position, (c) dip angle, (d) width, (e) depth to the top, and (f) density contrast for different window lengths for body one shown in Figure 4b.

For a given measurement point with I smaller than 0.5, the strike direction is estimated from the eigenvector corresponding to the minimum eigenvalue. Then a square window is formed around the maxima of g_{zz} to enclose the data points used in the analysis. Figure 11a shows that, in this particular case, the short wavelength anomalies caused by shallow sources have amplitudes smaller than 20 Eötvös. To avoid the effect of these anomalies, data points with

maxima of g_{zz} smaller than 20 Eötvös are rejected. The measurement points enclosed by the square window are transformed into the strike coordinate system, and the window constrained inversion method is applied to the rotated GGT for estimating the unknown parameters. A square window of 1000×1000 m is used as a starting window. Then the window length is increased by 250 m until it exceeds the window size of 2000×2000 m. The solution

Figure 6. (a) Data-fit error, estimated, (b) horizontal position, (c) dip angle, (d) thickness, (e) depth to the top, and (f) density contrast for different window lengths for body 2 shown in Figure 4b.

Table 2. Estimated parameters using the nonlinear least-squares method for the models shown in Figure 4.

	x_0	α (degree)	$2b$	z	t	$\rho(\text{kg/m}^3)$
True parameters of the model (body 1)	500	60	150	100	-	500
Initial model parameters (body 1)	450	45	100	150	-	400
Estimated parameters for body 1	495.5	59.5	146.2	81	-	445
True parameters of the model (body 2)	1500	75	-	150	400	300
Initial model parameters (body 2)	1400	60	-	100	350	400
Estimated parameters for body 2	1484	88	-	120	370	235

corresponding to the minimum data-fit error for both dike and contact models is chosen as the most reliable solution. Figure 11c illustrates the estimated strikes and dip angles of final solutions. We have used the following constraints (bounds) to differentiate spurious solutions from more reliable solutions:

- 1) The maximum depth to the top is 3000 m for both models.
- 2) The maximum width for dike model and thickness for contact model are 3000 and 5000 m, respectively.
- 3) The dip angle varies between 30° and 90° .
- 4) The density contrast varies between 100 and 400 kg/m^3 .

These constraints are chosen based on the available geologic information of the study area described in the literature (e.g. Stepto, 1990; Lana et al., 2003; Grieve and Theriault, 2000). The estimated parameters: depths to the top, dip angle, width, and density contrast

Figure 7. (a) The estimated parameters normalized by their true values, and (b) deviation of the estimated horizontal position from true position and data-fit errors plotted versus different separation-to-depth ratios for body one shown in (c).

are displayed in Figure 12. All final solutions shown in Figure 12 correspond to the dike model as inversion of the GGT data. Using the dike model provides smaller data-fit errors than the contact model. The estimated depths for dike-like bodies A, B, and C mainly vary between 500 and 1000 m. This agrees with results obtained from the eigenvector analysis method and Euler

Figure 8. (a) The estimated data-fit errors for body one using a contact model and (b) the estimated data-fit errors for body two using a dike model for different window lengths.

Figure 9. Plan view of a dike striking along the y' -direction but the measurement is supposed to be conducted along the x -direction.

deconvolution (Beiki and Pedersen, 2010). Figure 12b shows that the estimated widths of dike-like bodies mainly vary from 1000 m to greater than 2000 m. The estimated dip angles in Figure 12c are predominantly vertical to subvertical, except for shallower dip in the southern part of anomaly C. Figures 11c and 12c show that the estimated strike and dip angles are stable along the strike of the anomalies. An upper bound of 400 kg/m^3 is defined for the estimated density contrast of the dike model shown in Figure 12d.

DISCUSSION

Choice of the window size used in the LM algorithm

The dike and contact models can be said to represent two extreme, simplified models of real geologic structures, such as dike intrusions and faults in sedimentary basins. In general, such models can be used in a small neighborhood around prominent anomalies to estimate most important structural elements, such as depth and dip. For every single data point, GGT data contain information about the dimensionality of the underlying density distribution, whereby it becomes straightforward to identify the dimensionality of the gravity field for 2D analysis, such as the window constrained inversion approach presented here. Because the constrained inversion technique is only carried out in areas with quasi 2D behavior of the field, it makes sense to use a square window, such that all data points in the

window are given the same weight in the window constrained inversion process.

In our approach to finding the optimal source parameters, we minimize the data-fit error under a priori bounds on the parameters, using data in windows of variable sizes centered at the maxima g_{zz} . It is important to choose the minimum window size to be of the same linear size as the expected source depth, otherwise the minimum data-fit error may be attained for a very small number of data points such that the estimated source parameters become scattered. Similarly, the maximum linear size of the window should be chosen so that unnecessary calculations are avoided. With practical data, the window size corresponding to the best data-fit error will be determined from the degree to which interference effects from neighboring source bodies become dominant.

In practice, airborne gravity gradiometry surveys are collected along parallel lines crossing the dominant strike direction of 2D geologic structures. In our approach we use square windows centered at maxima of g_{zz} satisfying quasi 2D conditions. We thus include data points perpendicular as well as parallel to strike and therefore deviations from 2D conditions will be clearly reflected in the data-fit error, whereby such solutions can be deleted.

The strike direction of quasi 2D bodies could have been estimated as part of the nonlinear least-squares optimization. We have not used the nonlinear least-squares method for estimating the strike direction, because we noticed that estimated strike directions varied only little

Figure 10. Simplified geology map of the Vredefort dome area (after Lana et al., 2003). The study area is shown with a dashed rectangle.

Figure 11. (a) The g_{zz} component of the study area, (b) the calculated dimensionality indicator map (the regions with I greater than 0.5 are shown in gray), and (c) the estimated strike and dips superimposed on a contour map of g_{zz} .

Figure 12. The estimated (a) depth to the top, (b) width, (c) dip angle, and (d) density contrast acquired using the introduced method superimposed on gray scale map of g_{zz} . All solutions correspond to the infinite dike model.

over the window sizes (maximum 2000×2000 m). In addition, by focusing on the maximum of g_{zz} , we have selected the most favorable point for strike estimation: the one that is probably least affected by neighboring sources and is, thus, probably least biased.

Discrimination between model types

The discrimination between dike and contact models based upon the data-fit error criterion works well for the theoretical cases

Figure 13. Histograms of (a) data-fit errors for solutions of the dike model, (b) data-fit errors for solutions of the contact model, and (c) the ratio between the estimated data-fit errors of dike and contact models.

shown in Figures 5 and 6. For the GGT data from the Vredefort dome, the same criterion works surprisingly well. The distribution of the data-fit error for the two models and their ratio is shown in Figure 13 indicating mean data-fit errors of 2.87 ± 1.01 E (Figure 13a) and 10.70 ± 5.35 E (Figure 13b) for dike and contact models, respectively. For all solutions shown, the error ratio is less than unity with a peak around 0.25; thus, favoring dike models for the whole area.

Even though gravity gradient data are less perturbed by regional and interference effects than gravity data, we have investigated whether it is worthwhile to incorporate them as part of the constrained inversion scheme. We allowed g_{xz} and g_{zz} components to be contaminated by low order polynomials. We added a polynomial of first order to equations 3 and 4 for the dike model and equations 5 and 6 for the contact model. We found that, for small window sizes, as are often used in practice, these low order polynomials could accommodate large parts of the signal and, thus, make the discrimination between the two model types less distinct. They could even produce unreliable results in many cases, although in some cases for larger windows the results were good. For the real data example shown in this paper we have, therefore, not included polynomials in the models.

The estimated parameters: depth, width, dip, and density contrast, shown in Figure 12, are not typical for the dike intrusions identified on the surface of the dome. Their estimated widths, especially, are much larger than those shallow dikes. However, their strikes are consistently concentrically directed; those of anomaly A steeply dipping inward and those of anomaly B and C mostly steeply dipping outward. How these interpretations of structures close to the center of the dome structure can be related to deformations related to the rock flow during the cratering process remains to be understood.

Choice of initial model parameters

Our investigation, using different synthetic examples, shows that nonlinear least-squares optimization, using inequality constraints, is relatively insensitive to initial models. For the real data from the Vredefort dome we have tested two different initial models for the whole data set listed in Table 3. The initial model one is defined based on the available geologic information, as well as the results acquired using other interpretation techniques. We defined an initial dike model with its depth to the top of 1000 m (based on the estimated depths using Euler deconvolution), width of 1500 m (based on the wavelength of the anomalies A, B, and C), dip angle of 90° (based on the symmetric shape of the anomalies A, B, and C) and density contrast of 300 kg/m^3 (based on the petrophysical samples acquired for OGG and ILG reported in Henkel and Reimold, 1998). Initial model two was chosen to be different, but within the prede-

Table 3. Initial model parameters used for the real data example.

	α (degree)	$2b$	z	ρ (kg/m ³)
Constraint	30–90	0–3000	0–3000	100–400
Initial model 1	90	1500	1000	300
Initial model 2	30	200	500	100

Figure 14. Logarithms of the absolute differences of the estimated model parameters (a) depth to the top, (b) width, (c) dip angle, and (d) density contrast acquired using the initial models one and two listed in Table 3.

finer bounds as seen in Table 3. In Figure 14a and d, the absolute difference between the estimated model parameters, depth to the top, width, dip angle, and density contrast using initial models one and two are shown, corresponding to the same maxima of g_{zz} . The scatter between the estimated model parameters using different initial models is generally very small compared with their absolute values, although in a few cases the initial model can have substantial influence on the source solutions.

Comparison with conventional methods

The interpretation of gravity and magnetic field data provides important information about geologic bodies that are not exposed. Many automated methods have been developed to estimate the depth, width, susceptibility/density contrast, and horizontal position of causative bodies. The most widely used of the interpretation techniques are Werner deconvolution, Euler deconvolution, and local wavenumber methods. In these methods, when estimating the source parameters, it is normally assumed that the source has a simple geometry to linearize the problem. This assumption limits the number of effective model parameters to be estimated. Using Euler deconvolution only the source location is estimated, whereas for Werner deconvolution the source parameters horizontal position, depth to the top, and dip angle can be estimated for a thin dike (Ku and Sharp, 1983). For gravity case, Werner deconvolution can be applied to a g_{zz} component, which is equivalent to a magnetic

field reduced to the pole (RTP). In the local wavenumber method, the depth, thickness, and susceptibility/density contrast of a vertical contact model are estimated. Our method combines dimensionality and eigenvector analyses of the GGT with nonlinear least-squares optimization under inequality constraints. Thus, we first estimate strike directions of quasi 2D bodies, and then we use dike and contact models to estimate parameters such as horizontal position, depth to the top, width/thickness, dip angle, and density contrast. The window constrained inversion technique enables users to discriminate between the most appropriate model types based on the data-fit error criterion. In addition, the joint inversion of g_{xz} and g_{zz} components of GGT provides more constraints on the estimated source parameters.

CONCLUSIONS

We have described a method for inversion of two-dimensional gravity gradient tensor data for estimating source parameters of dike and contact models. We use a nonlinear least-squares technique to estimate the horizontal position, depth to the top, dip angle, width, thickness, and density contrast of the quasi 2D geologic structures. The applications of this window-constrained inversion method on synthetic examples show that it is robust to Gaussian random noise in the case that the convolution window used in the algorithm is sufficiently large. It is also shown that the inversion technique using inequality constraints is relatively insensitive to initial models.

For the constrained inversion technique, we use data points located within a square window centered at maxima of g_{zz} . To improve the reliability of the estimated source parameters, some geologic constraints are used in the inversion scheme. The measured g_{xz} and g_{yz} components are jointly inverted for estimating the source parameters for dike and contact models. The ratio between the estimated data-fit errors for dike and contact models are used to discriminate the spurious model types.

The different parts of the Vredefort impact area, such as the dense central part, dense ring structures, and less dense metasediments are delineated well by deconvolving the GGT data. Using geologic constraints and discrimination criteria, all major 2D geologic structures in the dome area can be modeled as thick dikes. Depth estimates along the major geologic features are stable predominantly around 1000 m. Inferred strikes of the thick dikes are concentric to the center of the dome and are subvertical and outward dipping for the two inner ring structures and inward dipping for the outer ring. The results from the Vredefort dome show that most solutions have thicknesses of the same order as the depths. We also note that estimated density contrasts in many cases reach the upper bound of 400 kg/m^3 possibly indicating that the density contrast strongly couples to depth and width, whereas dip angles seem to be little constrained by the imposed bounds of $\pm 30^\circ$.

ACKNOWLEDGMENTS

The authors thank Fugro Airborne Surveys for permission to use their data from the Vredefort dome, associate editor Xiong Li, as well as reviewers Marc A. Vallée, Martin Mushayandebu, Desmond FitzGerald, Alan Reid, and an anonymous reviewer for their constructive comments. The first author (MB) would like to thank Hediye Nazi for her support.

REFERENCES

- Aster, R. C., B. Borchers, and C. H. Thurber, 2005, *Parameter estimation and inverse problems*: Elsevier Academic Press.
- Bastani, M., and L. B. Pedersen, 2001, Automatic interpretation of magnetic dike parameters using the analytical signal technique: *Geophysics*, **66**, 551–561, doi: [10.1190/1.1444946](https://doi.org/10.1190/1.1444946).
- Beiki, M., 2010, Analytic signals of gravity gradient tensor and their application to estimate source location: *Geophysics*, **75**, no. 6, 159–174, doi: [10.1190/1.3493639](https://doi.org/10.1190/1.3493639).
- Beiki, M., and L. B. Pedersen, 2010, Eigenvector analysis of the gravity gradient tensor to locate geological bodies: *Geophysics*, **75**, no. 6, 137–149, doi: [10.1190/1.3484098](https://doi.org/10.1190/1.3484098).
- Beiki, M., L. B. Pedersen, and H. Nazi, 2011, Interpretation of aeromagnetic data using eigenvector analysis of pseudogravity gradient tensor: *Geophysics*, **76**, no. 3, L1–L10, doi: [10.1190/1.3555343](https://doi.org/10.1190/1.3555343).
- Dransfield, M. H., and J. B. Lee, 2004, The Falcon airborne gravity gradiometer survey systems, in R. Lane, ed., *Airborne Gravity 2004 Workshop*: Geoscience Australia, Extended Abstracts, 15–19.
- FitzGerald, D., D. Argast, R. Paterson, and H. Holstein, 2009, Full tensor magnetic gradiometry processing and interpretation developments: 11th South African Geophysical Association (SAGA).
- Gavin, H., 2011, *The Levenberg-Marquardt method for nonlinear least-squares curve-fitting problems*: Duke University.
- Grieve, R.A.F., and A. Theriault, 2000, Vredefort, Sudbury, Chicxulub: Three of a kind?: *Annual Review of Earth and Planetary Sciences*, **28**, 305–338, doi: [10.1146/annurev.earth.28.1.305](https://doi.org/10.1146/annurev.earth.28.1.305).
- Hart, R. J., M. A. G. Andreoli, M. Tredoux, and M. J. De Wit, 1990, Geochemistry across an exposed section of Archean crust at Vredefort: With implications for mid-crustal discontinuities: *Chemical Geology*, **82**, 21–50, doi: [10.1016/0009-2541\(90\)90072-F](https://doi.org/10.1016/0009-2541(90)90072-F).
- Henkel, H., and W. U. Reimold, 1998, Integrated geophysical modeling of a giant, complex impact structure: Anatomy of the Vredefort structure, South Africa: *Tectonophysics*, **287**, 1–20, doi: [10.1016/S0040-1951\(98\)80058-9](https://doi.org/10.1016/S0040-1951(98)80058-9).
- Hsu, S. K., D. Coppens, and C. T. Shyu, 1998, Depth to magnetic source using the generalized analytic signal: *Geophysics*, **63**, 1947–1957, doi: [10.1190/1.1444488](https://doi.org/10.1190/1.1444488).
- Kamo, S. L., W. U. Reimold, T. E. Krogh, and W. P. Colliston, 1996, A 2.023 Ga age for the Vredefort impact event and a first report of shock metamorphosed zircons in pseudotachylitic breccias and granophyre: *Earth and Planetary Science Letters*, **144**, 369–387, doi: [10.1016/S0012-821X\(96\)00180-X](https://doi.org/10.1016/S0012-821X(96)00180-X).
- Ku, C. C., and J. A. Sharp, 1983, Werner deconvolution for automated magnetic interpretation and its refinement using marquardt inverse modeling: *Geophysics*, **48**, 754–774, doi: [10.1190/1.1441505](https://doi.org/10.1190/1.1441505).
- Lana, C., R. L. Gibson, A. F. M. Kisters, and W. U. Reimold, 2003, Archean crustal structure of the Kaapvaal craton, South Africa — Evidence from the Vredefort dome: *Earth and Planetary Science Letters*, **206**, 133–144, doi: [10.1016/S0012-821X\(02\)01086-5](https://doi.org/10.1016/S0012-821X(02)01086-5).
- Madsen, K., H. B. Nielsen, and O. Tingleff, 2004, *Methods for non-linear least-squares problems*: Technical University of Denmark.
- Mikhailov, V., G. Pajot, M. Diamant, and A. Price, 2007, Tensor deconvolution: A method to locate equivalent sources from full tensor gravity data: *Geophysics*, no. 5, **72**, I61–I69, doi: [10.1190/1.2749317](https://doi.org/10.1190/1.2749317).
- Nabighian, M. N., 1972, The analytic signal of two-dimensional magnetic bodies with polygonal cross-section — Its properties and use of automated anomaly interpretation: *Geophysics*, **37**, 507–517, doi: [10.1190/1.1440276](https://doi.org/10.1190/1.1440276).
- Nabighian, M. N., 1974, Additional comments on the analytic signal of two-dimensional magnetic bodies with polygonal cross-section: *Geophysics*, **39**, 85–92, doi: [10.1190/1.1440416](https://doi.org/10.1190/1.1440416).
- Pedersen, L. B., and T. M. Rasmussen, 1990, The gradient tensor of potential anomalies: Some implications on data collection and data processing of maps: *Geophysics*, **55**, 1558–1566, doi: [10.1190/1.1442807](https://doi.org/10.1190/1.1442807).
- Reimold, W. U., and R. L. Gibson, 1996, Geology and evolution of the Vredefort impact structure, South Africa: *Journal of African Earth Sciences*, **23**, 125–162, doi: [10.1016/S0899-5362\(96\)00059-0](https://doi.org/10.1016/S0899-5362(96)00059-0).
- Salminen, J., L. J. Pesonen, W. U. Reimold, F. Donadini, and R. L. Gibson, 2009, Paleomagnetic and rock magnetic study of the Vredefort impact structure and the Johannesburg Dome, Kaapvaal Craton, South Africa — Implications for the apparent polar wander path of the Kaapvaal Craton during the Mesoproterozoic: *Precambrian Research*, **168**, 167–184, doi: [10.1016/j.precamres.2008.09.005](https://doi.org/10.1016/j.precamres.2008.09.005).
- Stanley, J. M., and R. Green, 1976, Gravity gradients and the interpretation of the truncated plate: *Geophysics*, **41**, 1370–1376, doi: [10.1190/1.1440687](https://doi.org/10.1190/1.1440687).
- Stephens, D., 1990, The geology and gravity field in the central core of the Vredefort structure: *Tectonophysics*, **171**, 75–103, doi: [10.1016/0040-1951\(90\)90091-L](https://doi.org/10.1016/0040-1951(90)90091-L).
- Telford, W. M., L. P. Geldart, and R. E. Sheriff, 1990, *Applied geophysics*: Cambridge University press.
- Vasco, O. W., and C. Taylor, 1991, Inversion of airborne gravity gradient data, southwestern Oklahoma: *Geophysics*, **56**, 90–91, doi: [10.1190/1.1442961](https://doi.org/10.1190/1.1442961).
- Zhdanov, M. S., R. Ellis, and S. Mukherjee, 2004, Three-dimensional regularized focusing inversion of gravity gradient tensor component data: *Geophysics*, **69**, 925–937, doi: [10.1190/1.1778236](https://doi.org/10.1190/1.1778236).