Fuel ion densities and distributions in fusion plasmas

Modeling and analysis for neutron emission spectrometry

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Abstract

Neutrons produced in fusion reactions in a magnetically confined plasma carry information about the distributions and densities of the fuel ions in the plasma. This thesis presents work where various theoretical models of different fuel ion distributions in the plasma are used to calculate modeled components of the neutron energy spectrum. The calculated components can then be compared with measured data, either to benchmark and validate the model or to derive various plasma parameters from the experimental data. Neutron spectra measured with the spectrometers TOFOR and the MPR, which are both installed at the JET tokamak in England, are used for this purpose. The thesis is based on three papers.

The first paper presents the analysis of TOFOR data from plasmas heated with neutral beams and radio frequency waves tuned to the third harmonic of the deuterium cyclotron frequency, which creates fast (supra thermal) ions in the MeV range. It is found that effects of the finite Larmor radii of the fast ions need to be included in the modeling in order to understand the data. These effects are important for fast ion measurements if there is a gradient in the fast ion distribution function with a scale length that is comparable to – or smaller than – the width of the field of view of the measuring instrument, and if this scale length is comparable to – or smaller than – the Larmor radii of the fast ions.

The second paper presents calculations of the neutron energy spectrum from the T(t,n)$^4$He reaction, for JET relevant fuel ion distributions. This is to form a starting point for the investigation of the possibility to obtain fast ion information from the t-t neutron spectrum, in a possible future deuterium-tritium campaign at JET. The t-t spectrum is more challenging to analyze than the d-d and d-t cases, since this reaction has three (rather than two) particles in the final state, which results in a broad continuum of neutron energies rather than a peak. However, the presence of various final state interactions – in particular between the neutron and the $^4$He – might still allow for spectrometry analysis.

Finally, in Paper III, a method to derive the fuel ion ratio, $n_t/n_d$, is presented and applied to MPR data from the JET d-t campaign in 1997. The trend in the results are consistent with Penning trap measurements of the fuel ion ratio at the plasma edge, but the absolute numbers are not the same. Measuring the fuel ion ratio in the core plasma is an important task for fusion research, and also a very complicated one. Future work should aim at measuring this quantity in several independent ways, which should then be cross checked against each other.
List of papers

This thesis is based on the following papers, which are referred to in the text by their roman numbers.

I Finite Larmor radii effects in fast ion measurements with neutron emission spectrometry
J Eriksson, C Hellesen, E Andersson Sundén, M Cecconello, S Conroy, G Ericsson, M Gatu Johnson, S D Pinches, S E Sharapov, M Weiszflog and JET EFDA contributors
Accepted for publication in Plasma Physics and Controlled Fusion.
My contribution: Developed the model that takes the finite Larmor radii of the fast ions into account when calculating neutron spectra, performed the data analysis and wrote the paper.

II Neutron emission from a tritium rich fusion plasma: simulations in view of a possible future d-t campaign at JET
J Eriksson, C Hellesen, S Conroy and G Ericsson
My contribution: Developed the code for calculating t-t neutron spectra from given fuel ion distributions, performed the simulations and wrote the paper.

III Fuel ion ratio measurements in NBI heated deuterium tritium fusion plasmas at JET using neutron emission spectroscopy
C Hellesen, J Eriksson, F Binda, S Conroy, G Ericsson, A Hjalmarsson, M Skiba, M Weiszflog and JET EFDA contributors
Manuscript
My contribution: Performed the TRANSP/NUBEAM simulations for half of the plasma discharges studied in the paper, contributed significantly to the data analysis and to the writing of the paper.
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Chapter 1

Some aspects of fusion energy research

1.1 Introduction

The earth is powered by energy from the sun. This energy, in turn, is released in fusion reactions primarily between hydrogen isotopes in the core of the sun. If these types of reactions could be exploited to produce energy in a controlled way on earth, it has the potential of becoming an important part of our energy supply. This is the goal of nuclear fusion research, and considerable effort has been put into achieving this goal for about 60 years [1].

Fusion energy has many attractive features. Fuel is abundant, the reaction products are not radioactive and the risk of a serious accident is relatively low. In particular, there is no such thing as a core meltdown in a fusion power plant. The plant would be a nuclear facility, though, and great care needs to be taken during construction, operation and decommissioning. After the end of its operation, the reactor construction materials would need to be stored for about 100 years in order for the neutron induced radioactivity to be reduced to non-harmful levels [2]. This chapter presents an overview of the basics of fusion energy research, with an emphasis on topics that are relevant for neutron diagnostics of fusion plasmas.

1.2 Fusion reactions

The main candidates for fueling a fusion reactor are hydrogen isotopes, primarily the isotopes deuterium (d) and tritium (t). The main reasons for this is:

1. The energy release from a fusion reaction is largest for reactions between light elements. This is due to the short range nature of the nuclear force, which binds light elements tighter together than heavy elements, and consequently the most energy is released when the lightest elements fuse and form heavier ones.

2. The energy required to make penetration of the Coulomb barrier probable, and make a fusion reaction possible, is lower for lighter elements, whose electric charge is smaller than for heavier elements.
CHAPTER 1. SOME ASPECTS OF FUSION ENERGY RESEARCH

<table>
<thead>
<tr>
<th>Reactants</th>
<th>Products</th>
<th>( E_{\text{fus}} ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d+d )</td>
<td>( ^3\text{He}+n )</td>
<td>3.27</td>
</tr>
<tr>
<td></td>
<td>( p+t )</td>
<td>4.04</td>
</tr>
<tr>
<td>( d+t )</td>
<td>( ^4\text{He}+n )</td>
<td>17.6</td>
</tr>
<tr>
<td>( t+t )</td>
<td>( ^4\text{He}+2n )</td>
<td>11.3</td>
</tr>
<tr>
<td>( p+t )</td>
<td>( ^3\text{He}+n )</td>
<td>-0.76</td>
</tr>
<tr>
<td>( d+^3\text{He} )</td>
<td>( p+^4\text{He} )</td>
<td>18.4</td>
</tr>
</tbody>
</table>

Table 1.1. Relevant fusion reactions and their corresponding energy release.

3. The energy loss due to radiation for a charged particle in motion scales as the square of the atomic number \( Z \), which makes heavier elements more difficult to heat to the temperatures required for fusion.

A summary of relevant fusion reactions is shown in table 1.1. Also shown is the energy \( E_{\text{fus}} \) that is released in the reaction. When two nuclei collide the probability for them to fuse is proportional to the product of their relative velocity \( v_{\text{rel}} \) and the cross section \( \sigma \) for the fusion reaction. Specifically, the number of reactions occurring per unit time when a beam of \( N_1 \) particles with velocity \( v_1 \) passes through a stationary target with particle density \( n_2 \) is

\[
R = N_1 n_2 v_1 \langle \sigma v \rangle. \tag{1.1}
\]

The cross sections for the fusion reactions in table 1.1 are shown in figure 1.1a. The \( d-t \) reaction has by far the largest cross section at lower energies, which is one of the reasons that this reaction is considered the most promising one for a fusion reactor.

The above discussion might suggest that a possible way to obtain a fusion reactor would be to simply fire a beam of deuterons into a block of tritium. However, even the \( d-t \) cross section is very small in comparison to other competing processes, such as Coulomb scattering. This means that the beam particles will lose their energy before a large enough fraction has taken part in a fusion reaction, making such an accelerator based fusion reactor impossible. One way to avoid this problem is to confine the fuel ions and heat them to high enough temperatures for the fusion reactions to take place. In such a situation the energy transferred in Coulomb collisions are not lost from the system, provided that the confinement is good enough. The number of reactions per unit volume from two populations of nucleons with densities \( n_1 \) and \( n_2 \) is given by

\[
R = n_1 n_2 \langle \sigma v \rangle, \tag{1.2}
\]

where the reactivity \( \langle \sigma v \rangle \) is given by the integral over the fuel ion distributions, \( f_1 \) and \( f_2 \), and the cross section, i.e.

\[
\langle \sigma v \rangle = \frac{1}{1 + \delta_{12}} \int_{v_1} \int_{v_2} f_1(v_1) f_2(v_2) v_{\text{rel}} \sigma(v_{\text{rel}}) dv_1 dv_2. \tag{1.3}
\]

Here, the Kronecker delta \( \delta_{12} \) is included in order to avoid double counting reactions for the case when particles 1 and 2 come from the same distribution. When the fuel ions of mass \( m \) are in thermal equilibrium at temperature \( T \) their velocities are distributed according to the
Figure 1.1. (a) Cross sections and (b) thermal reactivities for the fusion reactions in table 1.1.
Maxwellian velocity distribution. In this case the probability for a particle to have its speed between \( v \) and \( v + dv \) is

\[
f_M(v) dv = 4\pi v^2 \left( \frac{m}{2\pi k_B T} \right)^{3/2} \exp\left( -\frac{mv^2}{2k_B T} \right) dv,
\]

where \( k_B \) is the Boltzmann constant. The thermal reactivities for the fusion reactions in table 1.1 are shown in figure 1.1b. It is seen that the temperature needs to be of about 10-100 keV, i.e. 100-1000 million K, in order for the d-t reactivity to reach appreciable values. A fusion reactor must be able to confine the fuel and heat it to these temperatures. One of the most promising ways to confine and heat the plasma is offered by the tokamak reactor concept, which is described section 1.3 below. After this, the conditions that need to be met by an energy producing fusion reactor are examined in more detail in section 1.4.

### 1.3 The tokamak fusion reactor

The fusion research today focuses on two main ways to approach the problem of confining the fusion fuel. One is called magnetic confinement, where the fuel is in the form of a plasma and confined by means of external magnetic fields. The other approach is called inertial confinement, where a laser pulse is used to compress a small fuel pellet, which is then confined by its own inertia. The work presented in this thesis is concerned exclusively with magnetic confinement, and in particular with a reactor concept known as the tokamak [3, 4], which is described in this section.

The magnetic confinement concept relies on the fact that charged particles in a plasma move under influence of the Lorenz force and will thereby follow the magnetic field lines. In order to confine a plasma with a magnetic field \( B \) the outward force from the plasma pressure gradient \( \nabla p \) must be balanced by the inward magnetic force from the interaction between \( B \) and the plasma current \( \mathbf{J} \),

\[
\mathbf{J} \times \mathbf{B} = \nabla p.
\]

This is the steady-state momentum equation in magneto-hydrodynamics (MHD) [5] and holds to a very good approximation for a Maxwellian or near-Maxwellian fusion plasma. An important consequence of this equation is that the magnetic field is everywhere perpendicular to the pressure gradient, i.e. \( \mathbf{B} \cdot \nabla \mathbf{p} = 0 \). This means that magnetic field lines in a plasma always have to lie on surfaces of constant pressure. In addition the magnetic field has to be divergence free, by Maxwell’s equations. It follows that the only way to create a spatially bounded magnetic field that fulfills equation (1.5) is to bend the field lines into the shape of a torus. However, a purely toroidal field is not sufficient to obtain equilibrium, due to the expanding forces induced by the toroidicity [6]. These forces can be balanced by adding a poloidal component to \( \mathbf{B} \).

In the tokamak, the toroidal field is created by coils outside the plasma and the poloidal field is created by running a toroidal current through the plasma, as shown in figure 1.2. The resulting helical magnetic field is to a good approximation toroidally symmetric and visualizations of tokamak equilibria are most often presented as projections on the poloidal plane, as exemplified in figure 1.3a. This plot shows contours of constant poloidal magnetic flux \( \psi_p \) inside the tokamak. Since the magnetic field lines lie on surfaces of constant pressure, as
remarked above, it follows that the poloidal flux is also constant on these surfaces. The contours of constant flux are therefore called "flux surfaces" and many plasma parameters can be represented as functions of the normalized flux coordinate

$$\rho = \sqrt{\frac{\psi_p - \psi_{p,0}}{\psi_{p,sep} - \psi_{p,0}}}, \quad (1.6)$$

where $\psi_{p,0}$ is the flux at the magnetic axis and $\psi_{p,sep}$ is the flux at the separatrix, which marks the edge of the plasma. For plasma parameters that are not accurately represented as flux surface quantities, it is common to use either a cylindrical coordinate system $(R, \phi, Z)$ or a toroidal coordinate system $(r, \phi, \theta)$, as illustrated in figure 1.3b.

The poloidal magnetic field is typically small compared to the toroidal field in a tokamak. This means that the magnitude of the magnetic field can be approximated by the toroidal field, which is inversely proportional to the major radius,

$$B = B_0 \frac{R_0}{R}, \quad (1.7)$$

where $B_0$ and $R_0$ are the magnetic field and radial coordinate at the magnetic axis (or any other reference position). Thus, the magnetic field is higher on the inboard side than on the outboard side in a tokamak. This affects the orbits of the confined particles, as described in the next section.

The neutron spectrometry measurements presented in this thesis were all done at the Joint European Torus (JET) tokamak [7, 8], located outside Abingdon in England. JET is considered to be a large aspect ratio tokamak, i.e. its major radius ($\sim 3$ m) is much larger than its
minor radius (∼1 m). It is the largest tokamak in the world and can operate with plasma volumes of 80-100 m$^3$, magnetic fields up to 4 T and plasma currents up to 5 MA. Also, JET is currently the only machine that is capable of operating with tritium and holds the world record of produced fusion power, 16 MW, set in 1997 [9].

The results from JET and other tokamaks around the world have laid the scientific and technological foundation for the next generation tokamak, ITER, which is currently under construction in Cadarache, France. This device, which is about ten times larger than JET, is meant to finally break the long sought barrier of more produced fusion power than externally applied heating power.

### 1.3.1 Particle orbits in a tokamak

The orbits traced out by the fuel ions – in particular fast ions, i.e. ions with supra-thermal energies – can have a great impact on neutron measurements, as described in chapter 3. The details of these orbits are also crucial for the understanding of the dynamics and performance of the external plasma heating systems [10], as well as the stability of the plasma [11]. An overview of some aspects of these particle orbits is presented in this section.

Charged particles with mass $m$ moving in the magnetic field of a tokamak are accelerated by the Lorentz force,

$$m \frac{d\mathbf{v}}{dt} = q \mathbf{v} \times \mathbf{B}, \quad (1.8)$$

in a direction perpendicular to the velocity $\mathbf{v}$. As a result, the plasma particles gyrate around the magnetic field lines with a frequency known as the cyclotron frequency

$$\omega_c = \frac{|q|B}{m}, \quad (1.9)$$
and a radius of gyration that is known as the Larmor radius

\[ r_L = \frac{mv_\perp}{q|B|}, \tag{1.10} \]

where \( v_\perp \) is the component of \( v \) perpendicular to the magnetic field. \( q \) is the charge of the particle, and hence the Larmor gyration will be in opposite direction for ions and electrons. It is common to separate the velocity into a parallel and a perpendicular component with respect to the magnetic field,

\[ v = v_\parallel + v_\perp. \tag{1.11} \]

The angle between the velocity and the magnetic field is called the pitch angle.

In the absence of forces parallel to \( v \) the kinetic energy of the particle,

\[ E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(v_\parallel^2 + v_\perp^2\right), \tag{1.12} \]

is a constant of motion. If, in addition, the temporal variation of \( B \) is slow compared to the gyro frequency and spatial variations are small on the scale of the Larmor radius, the magnetic moment

\[ \mu = \frac{mv_\perp^2}{2B} \tag{1.13} \]

is also conserved. Hence, the parallel velocity can be written as

\[ v_\parallel = \sqrt{\frac{2}{m}(E - \mu B)}, \tag{1.14} \]

from which it follows that when a particle moves from the low field side of the tokamak towards the high field side, \( v_\parallel \) decreases, i.e. the pitch angle increases. Depending on the initial value of the pitch angle the particle may lose all of its parallel velocity and be reflected back towards the high field region. This divides the plasma particles in a tokamak into two main classes, namely passing particles and trapped particles. Calculated orbits for one passing and one trapped particle in a JET magnetic field are shown in figure 1.4. It is seen that the orbit of a trapped particle resembles a banana when projected on the poloidal plane and therefore trapped orbits are commonly called “banana orbits”.

From the discussion above one might expect a particle to be locked to one field line in a given flux surface as it moves through the plasma. This is not the case, as seen from figure 1.4. The reason for this is that the gradient and curvature of the magnetic field cause the gyro-center of a particle to drift perpendicular to the field lines. This drift can be understood from the invariance of the canonical toroidal angular momentum, \( p_\phi \). It is obtained by differentiating the Lagrangian for a particle in an electromagnetic field,

\[ L = \frac{1}{2}m\left(v_\parallel^2 + v_\phi^2 + v_Z^2\right) - q\Phi + qA \cdot v, \tag{1.15} \]

with respect to the generalized toroidal velocity \( \dot{\phi} \left( = v_\phi/R \right) \),

\[ p_\phi = \frac{\partial L}{\partial \dot{\phi}} = mR^2 \dot{\phi} + qRA_\phi = mRv_\phi + q\psi_p. \tag{1.16} \]
Figure 1.4. Examples of (a) passing and (b) trapped 500 keV deuterons in a magnetic field at JET, shown as projections both in the poloidal (left) and toroidal (right) plane. In (a), the red orbit is co-passing and the blue orbit is counter-passing, with respect to the plasma current $I_P$. 
Φ and A are the electric and magnetic potentials, respectively, and for a toroidally symmetric field the toroidal component of A is related to the poloidal flux through \( \psi_p = RA_\phi \). Due to the toroidal symmetry of the tokamak \( \partial L / \partial \phi = 0 \), and consequently it follows from Lagrange’s equations that \( dp_\phi / dt = 0 \), i.e. \( p_\phi \) is a constant of motion.

The flux \( \psi_p \) is determined by the plasma current \( I_P \), and therefore the orbit of a particle depends on whether the motion is parallel or anti-parallel to \( I_P \). Consider a counter-passing particle, i.e. a particle moving in the direction opposite to \( I_P \), in a typical magnetic field at JET; \( I_P \) is normally in the direction of negative \( \phi \) at JET, which means that counter-passing particles have \( v_\phi > 0 \). If such a particle moves from the low field side towards the high field side of the plasma, its parallel velocity – which is approximately equal to \( |v_\phi| \) in a tokamak – is reduced in order to conserve the magnetic moment. Thus, the first term in equation (1.16) decreases and in order for \( p_\phi \) to be conserved the particle has to move towards higher values of the poloidal flux \( \psi_p \), i.e. outwards compared to the flux surface where it started. This is what happens to the blue orbit in figure 1.5a. The red orbit on the other hand, which is co-passing and thereby moves in the negative toroidal direction, must move towards lower values of \( \psi_p \) to conserve \( p_\phi \). A similar example is shown for a trapped particle in figure 1.5b. Note in particular that one consequence of \( p_\phi \)-conservation for trapped particles is that the particle always moves parallel to the plasma current on the outer leg of the banana orbit and anti-parallel on the inner leg.

The code used to calculate the orbits in the above examples was written during a diploma project [12] and has been further developed as a part of the work presented in this thesis. Orbits can be calculated either by specifying initial conditions for position and velocity, or by giving a set of constants of motion \((E, p_\phi, \Lambda, \sigma)\), where \( \Lambda \equiv \mu B_0 / E \) is the normalized magnetic moment and \( \sigma \) is a label that specifies if the particle is co-passing, counter-passing or trapped.

### 1.3.2 Heating the plasma

#### Self heating

The ability to heat the plasma to temperatures where the fusion reactivity is high enough is of great importance in magnetic confinement fusion research. In a future fusion reactor it is in practice required that most of the heating should come from the slowing down of the charged fusion products, i.e. mainly the \( \alpha \) particles from the d-t reaction, which are produced with an energy of 3.5 MeV. This is typically called self heating or \( \alpha \) particle heating. However, it is still necessary to develop other plasma heating techniques in order to be able to bring the plasma to the point where the self heating becomes large enough, as well as to be able to study the effect these fast ions have on confinement, stability, heat load on the walls etc. The three most common auxiliary heating systems used at JET – ohmic heating, neutral beam injection and ion cyclotron resonance heating – are briefly described below.

#### Ohmic heating

In a tokamak, one obvious heating mechanism is provided through the plasma current that generates the poloidal magnetic field. As the current flows through the plasma, the charges in the current will collide with other plasma particles and thereby heat the plasma. This is referred to as Ohmic heating, and it can be quantified in terms of the plasma resistivity, \( \eta \).
Figure 1.5. The orbits of two 500 keV deuterons in a magnetic field at JET, shown both on the poloidal (left) and toroidal (right) plane. The blue orbit starts with a positive \( v_\phi \) (i.e. anti-parallel to \( I_P \)) and must move into regions of higher poloidal flux in order to conserve \( p_\phi \), as \( v_\phi \) decreases in regions of higher magnetic field. The opposite happens for the red orbit, which starts in the direction parallel to \( I_P \) and therefore moves towards lower poloidal flux as the magnitude of \( v_\phi \) decreases. Examples are shown for a passing (a) and trapped (b) particle.
By Ohm’s law, the heating power from the current is \( P_\Omega = \eta J^2 \), where \( J \) is the current density. Unfortunately, an increase in temperature is inevitably associated with a decrease of the Coulomb cross section responsible for the resistivity. It can be shown that the resistivity is proportional to \( T_e^{-3/2} \), where \( T_e \) is the electron temperature. Hence, the efficiency of Ohmic heating is reduced at high temperatures, and in practice it cannot be used to heat the plasma above a few keV (at JET the typical Ohmic temperature is 2 keV). Alternative methods are therefore needed to reach the required temperatures.

**Neutral beam injection (NBI)**

Another way to heat the plasma is to inject energetic ions from an external source, i.e. an accelerator. The energetic ions are subsequently slowed down, transferring their energy through Coulomb collisions with the bulk plasma particles, much like the \( \alpha \) particles in the case of self heating. However, charged particles cannot penetrate the magnetic field to the center of the plasma and therefore the ions are neutralized as a last step before injection. Inside the plasma, the neutral atoms are ionized again, through charge exchange and ionization processes with the ions and electrons in the plasma.

JET is equipped with two neutral beam injector boxes, that can inject hydrogen, deuterium, tritium, \( ^3\)He or \( ^4\)He atoms into the plasma. The nominal injection energy is around 130 keV or 80 keV, but since some of the beam particles form molecules (e.g. \( \text{D}_2 \) and \( \text{D}_3 \)) there will typically also be a fraction of the beam particles with 1/2 and 1/3 of this energy. There are two different modes of injection at JET. One is the so called tangential injection, with an angle of about 60° to the magnetic field, and the other one is called normal injection and has a slightly larger angle. The injection is parallel to the plasma current. The total beam power available at JET today is about 35 MW for deuteron injection.

**Ion cyclotron resonance heating (ICRH)**

Radio-frequency (RF) waves can also be used to transfer energy to the plasma ions, by matching the RF to the ion cyclotron frequency. This heating scheme proceeds through three main steps. First, a system of RF generators and antennas are used to create an electromagnetic wave of the desired frequency. This wave then couples to the so called fast magnetosonic wave, which propagates towards the center of the plasma. When the wave with parallel wave number \( k_\parallel \) reaches a region where the resonance condition

\[
n\omega_k - \omega_{ct} - k_\parallel v_\parallel = 0, \quad n = 1, 2, \ldots
\]

is fulfilled for a given ion with parallel velocity \( v_\parallel \), energy may be transferred from the wave to this ion through ion cyclotron resonance interaction. This heating scheme is called ion cyclotron resonance heating (ICRH). Other types of wave particle interactions are also possible, such as electron Landau damping or transit time magnetic pumping, which transfer energy to the electrons rather than the ions [13].

It might be surprising that an ion can be accelerated not only at the fundamental (\( n = 1 \)) resonance but also at harmonics (\( n > 1 \)) of the cyclotron frequency. This is due to the non-uniform electric field seen by the particle during one gyro period. The strength of the interaction at harmonics of the cyclotron frequency is greater for more energetic ions, which
have larger Larmor radii and therefore see a bigger variation of the field during its gyration. The strength of the fundamental interaction, on the other hand, does not depend on the ion energy.

Due to the approximate $1/R$-dependence of the magnetic field in a tokamak, the resonance condition (1.17) will be fulfilled at a certain radial location, which in the cold plasma limit ($v || \to 0$) is given by

$$R_{\text{res}} = \frac{|q| B_0 R_0 n}{m \omega_{ci}}.$$  \hfill (1.18)

This allows for the possibility to control where the injected power is deposited.

The resonant ions are accelerated by the electric field of the wave. The electric field can be decomposed into a co-rotating ($E_+$) and a counter-rotating ($E_-$) circularly polarized component, with respect to the gyro motion of the ions. It is the $E_+$ component that gives rise to the acceleration. However, it turns out that if the plasma contains only one ion species, such as a d-d plasma which is the most common case for experiments at JET, $E_+$ becomes very close to zero at the fundamental cyclotron resonance [14]. This means that it is very inefficient to heat the majority ions in a plasma with fundamental ICRH. This problem can be solved by introducing a small minority population of another ion species, e.g. hydrogen in a deuterium plasma, and tune the ICRH to the minority cyclotron frequency. Another possibility is to heat the majority ions at a harmonic of the cyclotron frequency, e.g. second or third harmonic ICRH.

Since harmonic ICRH couples more efficiently to energetic ions, strong synergistic effects with NBI heating are expected. This is reported e.g. in [15] and Paper I, where the combined use of third harmonic ICRH and NBI gave rise very interesting neutron spectrometry data.

### 1.4 Burn criteria

The ultimate goal of the tokamak reactor – as well as of any other fusion energy experiment – is to create and maintain a situation where the produced fusion power exceeds the power that needs to be externally supplied to keep the fusion reactions going. In order to keep the tokamak plasma in steady state the power $P_{\text{loss}}$ that is lost from the plasma must be compensated by the $\alpha$ particle power $P_{\alpha}$ and the externally supplied heating power $P_{\text{ext}}$.

$$P_{\alpha} + P_{\text{ext}} = P_{\text{loss}}.$$  \hfill (1.19)

The number of fusion reactions per unit volume and time is given by the thermal reactivity multiplied by the reactant densities, as described in section 1.2. Considering only the d-t contribution, one obtains

$$P_{\alpha} = n_d n_t \langle \sigma v \rangle dt \frac{E_{\text{fus}}}{5} = n_{dt}^2 \frac{r}{(r+1)^2} \langle \sigma v \rangle dt \frac{E_{\text{fus}}}{5},$$  \hfill (1.20)

where $n_{dt} \equiv n_d + n_t$ is the particle density of the fuel ions and $r = n_t/n_d$ is the fuel ion ratio. $E_{\text{fus}}$ is the energy released per fusion reaction, i.e. 17.6 MeV for the d-t case, and the $\alpha$ particles carry 1/5 of this energy. The loss term can be quantified by the total thermal energy in the plasma, $3n_k B^2/2$, divided by the energy confinement time $\tau_E$, i.e. the characteristic time that energy can be kept in the reactor before it is lost to the surroundings due to radiation
or transport. The total density $n$ can be related to the electron density by the quasi-neutrality condition

$$n = n_e + n_{dt} + \sum_j Z_j n_j = 2n_e,$$  \hspace{1cm} (1.21)

where $n_e$ is the particle density of the electrons, and $n_j$ is the density of residual plasma ions with atomic number $Z_j$. In a tokamak plasma, these ions are typically helium "ash" from the fusion reactions, as well as impurities released from the reactor walls. Thus, the loss term becomes

$$P_{\text{loss}} = \frac{3n_e k_B T}{\tau_E}.$$  \hspace{1cm} (1.22)

Finally, it is common to relate the externally supplied power to the fusion power through the power gain factor $Q$, defined by

$$P_{\text{ext}} = \frac{P_{\text{fus}}}{Q} = \frac{5P_{\alpha}}{Q}.$$  \hspace{1cm} (1.23)

Obviously, it is required to have $Q \gg 1$ in an fusion power plant. Substituting equations (1.20), (1.22) and (1.23) into equation (1.19) gives

$$\frac{n_{dt}}{n_e} \frac{r}{(r+1)^2} \tau_E = \frac{3k_B T}{(\sigma v)_{\text{fus}} E_{\text{fus}} \left(\frac{1}{3} + \frac{1}{Q}\right)}.$$

The right hand side of this equation is called the "fusion product" in what follows. One important milestone on the way towards a fusion reactor is to reach "break even", which means that $Q = 1$ and the produced fusion power is equal to the externally supplied heating power. The ultimate goal is "ignition", i.e. when $Q \to \infty$ and the $\alpha$ particle power alone can compensate for the losses. The temperature dependence of the fusion product for break even and ignition is plotted in figure 1.6. The highest $Q$-value obtained to date is 0.67, achieved at JET in 1997 [9].

It is seen from figure 1.6 and equation (1.24) that the fundamental problem in fusion research is to achieve the following:

- **Heat the plasma to high temperatures.** The conditions for break even and ignition are least difficult to meet in the temperature region around 20-30 keV (about 200-300 million K), where the requirement on the fusion product is smallest. Various methods exist for this task, as described in section 1.3.2.

- **Create a plasma with high enough density and optimal fuel ion ratio.** The value of the fusion product increases with the fuel ion density $n_{dt}$ and the term $r/(r+1)^2$ is maximized for $r = 1$, i.e. $n_q = n_t$.

- **Create a low impurity plasma.** The fusion product is reduced if the fuel dilution, $n_{dt}/n_e$ is lowered due to the presence of impurities in the plasma.

- **Maximize the energy confinement time $\tau_E$.** One important aspect in order to have good confinement is the ability to understand and control the behavior of fast ions in the plasma [16]. These are ions with energies much higher than the thermal energies, e.g. charged fusion products and ions accelerated by the external heating systems. The need
for a low impurity plasma is also crucial for confinement, since the radiation losses due to Brehmsstralung increases quadratically with the charge of the plasma ions. Therefore, even a small number of heavy impurities could make it virtually impossible to reach ignition [17].

Neutron spectrometry can be used to obtain information about several of these issues, which is exemplified in this thesis. Paper I and section 3.4 are concerned with the analysis of fast ion measurements in deuterium plasmas at JET in 2008. Paper II and section 4.1 present a simulation study of the possibility to measure fast tritons in a possible future d-t campaign at JET. Finally, Paper III and section 4.2 present measurements of the fuel ion ratio using neutron spectrometry data from the d-t campaign at JET in 1997.

1.5 Modeling fuel ion distributions in the plasma

In chapter 3 it is described how the shape of the neutron energy spectrum is intimately connected to the velocity distributions of the fuel ions that produce the neutrons in the fusion reactions. The measured neutron energy spectrum can be analyzed to obtain information about these distributions. For this kind of analysis it is crucial to have models of the different fuel ion populations in the plasma. Such models can e.g. be compared and validated against experimental neutron data [15, 18]. Alternatively, given a model that is proved to be reliable, it is possible to calculate different components of the neutron emission which can be used to derive different plasma parameters from neutron spectrometry data, such as ion temperature [19] or the fuel ion ratio (Paper III). This section presents an overview of various ways to model the distributions of different fuel ion populations that arise in tokamak experiments.
CHAPTER 1. SOME ASPECTS OF FUSION ENERGY RESEARCH

Although the plasma as a whole is not in thermal equilibrium, it is typically assumed that the bulk plasma ions are everywhere distributed according to the Maxwellian distribution with a local temperature $T(r)$,

$$f_{\text{bulk}}(v, r) = 4\pi v^2 \left(\frac{m}{2\pi k_B T(r)}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T(r)}\right). \quad (1.25)$$

This distribution is isotropic in the cosine of the pitch angle, i.e. all directions of the velocity vector are equally probable. It is also frequently assumed that $T$ is a function of the normalized flux, $T = T(\rho)$.

In addition to the bulk plasma particles, the auxiliary heating systems can create fuel ion distributions which are very non-Maxwellian. The energy distribution function of fast particles created by NBI and/or ICRH can be modeled by solving a 1-dimensional Fokker-Planck equation, given by [20, 21]

$$\frac{\partial f}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \left[-\alpha v^2 f + \frac{1}{2} \frac{\partial}{\partial v}\left(\beta v^2 f\right) + \frac{1}{2} D_{\text{rf}} v^2 \frac{\partial^2 f}{\partial v^2}\right] + \frac{1}{v^2} \left(S(v) + L(v)\right). \quad (1.26)$$

Here $\alpha$ and $\beta$ are Coulomb diffusion coefficients derived by Spitzer [22], characterizing the slowing down process of energetic ions, and $D_{\text{rf}}$ is the ICRH diffusion coefficient, which is used to model the interaction between the ions and the ICRH wave field. It is given by

$$D_{\text{rf}} = K \left|E^+ J_{n-1} \left(\frac{k_{\perp} v_{\perp}}{\omega_k}\right) + E^- J_{n+1} \left(\frac{k_{\perp} v_{\perp}}{\omega_k}\right)\right|^2, \quad (1.27)$$

where $K$ is a numerical constant. $S(v)$ is a source term representing the particle injected with the NBI and $L(v)$ is a loss term that removes particles that reach thermal energies. At this point the particles are considered to belong to the thermal bulk plasma rather than to the slowing down distribution. The steady state ($\partial f/\partial t = 0$) energy distribution obtained from this equation was used for neutron spectrometry analysis e.g. in [15] and in Paper I. It was also used to calculate model distributions for the simulations of t-t neutron spectra in Paper II. Examples of calculated distributions for various heating scenarios are shown in figure 1.7.

The energy distribution obtained by solving equation (1.26) is not enough to calculate the neutron spectrum from a given ion population. The distribution of all three velocity components is needed, as described in chapter 3. Hence, in addition to the energy distribution, it is necessary to know the distribution of the pitch angle and the gyro angle of the particles. The gyro angle distribution is isotropic (as long as FLR effects can be neglected, see section 3.4 and Paper I). Depending on the level of accuracy required it can be sufficient to specify minimum and maximum values for the pitch angle, and consider the cosine of the pitch angle to be uniformly distributed within this range. This approach was followed in Paper I, where neutron spectra from plasmas heated with 3rd harmonic ICRH and NBI were studied. As described in detail in the paper, the ICRH accelerates the ions mainly in the perpendicular direction, which means that the pitch angles are driven towards $90^\circ$. Therefore, the pitch angles were assumed to be distributed in the range $90^\circ \pm 10^\circ$.

Several more sophisticated (and more computationally intensive) modeling codes exist. The slowing down of NBI particles can be modeled in realistic geometry with the NUBEAM code [23, 24]. This is a Monte Carlo code that self-consistently calculates the slowing down
distribution of energetic particles in a tokamak, taking both collisional and atomic physics effects into account. The output is a 4-dimensional distribution in energy, pitch angle and position in the poloidal plane, as exemplified in figure 1.8. The code is part of the plasma transport code TRANSP [25]. NUBEAM distributions were used to model the NBI contribution to the neutron emission for the fuel ion ratio measurements presented in Paper III.

ICRH heated plasmas can be modeled with the SELFO code [10], which self-consistently calculates the wave field and the ion distribution resulting from the wave particle interaction and collisions. The distribution function is given as a function of the constants of motion $(E, p_\phi, \Lambda, \sigma)$, described in section 1.3.1. An example of such a distribution, from [26], is shown in figure 1.9.
Figure 1.8. A NBI slowing down distribution at JET, calculated with the NUBEAM code. Panel (a) shows the fast ion density and panel (b) shows the energy ($E$) and pitch angle ($\xi$) distribution at the position indicated by the black cross in the $(R,Z)$-plane.

Figure 1.9. A SELFO distribution calculated for a JET plasma heated with 3rd harmonic ICRH and NBI.
Chapter 2

Measuring the neutron energy spectrum

A neutron from a fusion reaction carries information about the motion of the fuel ions that produced it. Therefore it is possible to extract information about the distributions and densities of different fuel ion populations in the plasma from the neutron energy spectrum and the relevant cross sections.

Energy cannot be directly observed. In order to measure the energy of neutrons emitted from a fusion plasma it is therefore necessary to measure some other physical quantity related to energy, such as the scintillation light resulting from a neutron interacting with nuclei in a detector, the flight time between two reference points or the deflection of charged secondary particles in a magnetic field. Here, two spectrometer systems based on the two latter principles will be briefly described. These are the TOFOR and MPR spectrometers, both installed at JET, which were used to obtain the data presented in this thesis.

2.1 The TOFOR spectrometer

The time-of-flight neutron spectrometer optimized for rate, named TOFOR [27] was installed in the roof laboratory above the JET tokamak in 2005. The viewing angle is close to perpendicular to the magnetic field lines and the distance from the spectrometer to the plasma mid-plane is around 19 meters.

TOFOR consists of two sets of plastic scintillator detectors, S1 and S2, organized as shown in figure 2.1. S1 is placed in the beam of collimated neutrons and S2 (which is a ring shaped set of 32 detectors) is located a distance \( L \approx 1.2 \text{ m} \) from S1, at an angle \( \alpha = 30^\circ \) compared to the beam line. Some of the neutrons reaching the S1 detector will scatter elastically on the protons in the plastic scintillators, and the recoil protons are detected. If the neutron scatters at an angle close to \( \alpha \) it might also be detected in one of the S2 detectors. This is called a coincidence. The time-of-flight \( t_{\text{tof}} \) between the two interactions is related to the neutron energy \( E_n \) through

\[
E_n = \frac{2m_n r^2}{t_{\text{tof}}} ,
\]

(2.1)

where \( m_n \) is the mass of the neutron and \( r = 705 \text{ mm} \) is the radius of the constant time of
flight sphere (see figure 2.1b). The flight time for a scattered neutron with given initial energy $E_n$ from S1 to any point on this sphere is constant, independent of the scattering angle $\alpha$.

The interpretation of the measured time-of-flight spectrum is complicated by several factors. One is the difficulty to separate true coincidences from random coincidences when constructing the spectrum, i.e. to know which S1 event that corresponds to a given S2 event. These random coincidences show up as a flat background in the time-of-flight spectrum and it is possible to subtract it from the data by by looking at the unphysical, negative time-of-flight, region of the spectrum. However, the number of random coincidences increases quadratically with the count rate, which means that for a too high neutron flux the real neutron signal will be drowned by these false events. The present TOFOR system is projected to be capable of handling count rates up to 0.5 MHz [27], which is more than sufficient to handle the neutron rate from d-d plasmas, but not for d-t reactions in a 50/50 d-t plasma. In that case one has to limit the neutron flux with an adjustable pre-collimator.

Another complication is that a fusion neutron with one specific energy can give rise to a broad range of time of flights, depending on the details of how it scatters in the S1 detector on its way to S2. There is a broadening due to the finite dimensions of the detectors; the length of the flight path is not the same between all possible combinations of positions on the detectors. Furthermore, the neutron can lose some of its energy through multiple large angle scattering in S1, resulting in a longer time-of-flight. There is also the possibility that the neutron is scattered towards S2 through several small angle scattering events in S1. In this process less energy is lost than in one single interaction, which means that such a multi-scattered neutron gets a somewhat shorter flight time.

All these effects have been simulated in detail, using the particle transport code GEANT4 [28], and the results are contained in the response function of TOFOR, $R(E_n, t_{\text{tof}})^1$. In order

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1The response function of TOFOR also includes the effects of electronic broadening and voltage thresholds set in the data acquisition electronics, but the exact details are not important for the present discussion.
to obtain the TOFOR response $dN/dt_{tof}$ to a given neutron energy distribution $f_n(E_n)$ one integrates the product of $R$ with $f_n$ over all possible neutron energies,

$$
\frac{dN}{dt_{tof}} = \int R(E_n, t_{tof}) f_n(E_n) dE_n.
$$

(2.2)

As an example, the TOFOR spectra from several mono-energetic neutron energy distributions are shown in figure 2.2a. Detailed knowledge of the response function is crucial when analyzing neutron spectrometry data. Several examples of this are given in this thesis.

2.2 The MPR spectrometer

In the magnetic proton recoil (MPR) spectrometer [29] the neutrons transfer their energy to protons by elastic scattering in a polythene foil. The energy of a forward scattered proton produced in this way is very close to the original neutron energy. The energy distribution of these protons, and thereby the neutrons, is obtained by letting the protons pass through a magnetic spectrometer and measure the spatial distribution of protons along an array of scintillators (a hodoscope), placed in the focal plane of the magnetic system, as shown in figure 2.3. Different proton energies will give rise to different trajectories in the magnetic field and thereby different interaction positions in the hodoscope. The end result of a MPR measurement is a position histogram of the protons, $dN/dX_{pos}$.

As in the case of TOFOR, the response function of the MPR has been simulated with a Monte Carlo code, using a detailed model of the detector geometry. The MPR response for various mono energetic neutron energy distributions are shown in figure 2.2b.

The MPR was installed at JET in 1996. The original design was primarily made for detecting 14 MeV neutrons, but in 2005 the spectrometer was upgraded to allow for the detection of 2.5 MeV neutrons as well [30]. The upgraded spectrometer is called the MPRu. However, since the measurements in Paper III were performed with the original MPR spectrometer, the original name will be used throughout this thesis.
Figure 2.3. (a) Sketch of the MPR spectrometer and (b) its placement at JET.
Chapter 3

Modeling the neutron energy spectrum

Much of the work presented in this thesis relies on the calculations of the neutron energy spectrum from given fuel ion distributions. The calculated spectra can e.g. be compared with measured data in order to see if a certain model of the fuel ion distributions is compatible with the actual neutron emission. This section presents an overview of such reaction product spectra calculations.

3.1 Kinematics

When two particles of mass $m_a$ and $m_b$ interact and produce $n$ particles of mass $m_i \ (i = 1, 2, \ldots n)$, four-momentum conservation requires that

$$P \equiv P_a + P_b = \sum_{i=1}^{n} P_i,$$

(3.1)

where $P_j = (E_j, \mathbf{p}_j)$ is the four-momentum\(^1\) of particle $j$ and $P \equiv (E, \mathbf{p}) \equiv (E_a + E_b, \mathbf{p}_a + \mathbf{p}_b)$ is the total four momentum of the system. An equation for the energy of particle 1 can be obtained by subtracting $P_1$ from both sides of this equation and squaring, which gives

$$M^2 + m_1^2 - 2 (EE_1 - \mathbf{p} \cdot \mathbf{p}_1) = M^2_R.$$

(3.2)

Here,

$$M^2 = P^2 = (E_a + E_b)^2 - (\mathbf{p}_a + \mathbf{p}_b)^2$$

(3.3)

is the invariant mass of the reaction and $M^2_R = (\sum_{i=2}^{n} P_i)^2$ is the invariant mass of the residual particles. Writing $\mathbf{p}_1$ as $(E_1^2 - m_1^2)^{1/2} \mathbf{u}$, where $\mathbf{u} = p_1 / p_1$, and rearranging gives

$$E_1 - (E_1^2 - m_1^2)^{1/2} \frac{\mathbf{p}}{E} \cdot \mathbf{u} = \frac{M^2 + m_1^2 - M^2_R}{2E}.$$

(3.4)

This equation should be solved in order to obtain the energy of particle 1, emitted in direction $\mathbf{u}$.

\(^1\)Throughout this section, units are used in which $c = 1$.  

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In order to solve equation (3.4) one needs to know the value of \( M_R \). For most fusion relevant reactions there are only two particles in the final state, which means that \( M_R \) is simply equal to \( m_2 \), but if there are more particles \( M_R \) can take on any value as long as equation (3.1) is fulfilled. The case with three particles in the final state is discussed in more detail in section 4.1 and Paper II, where calculations of the neutron spectrum from the t-t fusion reaction is presented.

Further insight into the nature of equation (3.4) can be obtained by considering its solution in the center-of-momentum system (CMS), i.e. the reference frame where the total momentum \( p \) is zero and \( E = M \). The solution simply becomes

\[
E_1^* = \frac{M^2 + m_1^2 - M_R^2}{2M},
\]

where the asterisk is used to denote CMS quantities. Thus, in the CMS, the energy of particle 1 is independent of the emission direction. Furthermore, substituting the expression for \( E_1^* \) back into equation (3.4) and recognizing that \( p/E \) is the velocity of the CMS, \( \beta \), gives

\[
\gamma \left( E_1 - \left( E_1^2 - m_1^2 \right)^{1/2} \mathbf{u} \cdot \mathbf{\beta} \right) = E_1^*,
\]

where \( \gamma = \left( 1 - \beta^2 \right)^{1/2} \). This is nothing but the Lorentz transformation from our original reference frame to the CMS.

It is clear that \( E_1^* \) is uniquely determined if the invariant masses \( M \) and \( M_R \) have been specified. In any other reference frame however, \( E_1 \) depends on the emission direction \( \mathbf{u} \), and even for a given emission direction there may be two kinematically allowed values of \( E_1 \). This is the case whenever the speed of particle 1 in the CMS is smaller than \( \beta \), the speed of the CMS [31]. This condition is equivalent to

\[
\frac{M^2 + m_1^2 - M_R^2}{2m_1} < E.
\]

Whenever this inequality holds, there are generally two solutions for \( E_1 \) for a given \( \mathbf{u} \), and the emission direction is restricted to the forward hemisphere with respect to the velocity of the CMS. The double valued solution is not of great concern for fusion neutron spectrometry applications. This can be seen by noting that equation (3.3) implies that \( m_a + m_b \leq M \leq E \), which means that

\[
\frac{M^2 + m_1^2 - M_R^2}{2m_1} > \frac{(m_a + m_b)^2 + m_1^2 - M_R^2}{2m_1}.
\]

By substituting the relevant masses into the final expression it is seen that the total fuel ion kinetic energy, \( E - m_a - m_b \), needs to be about 70 MeV and 10 MeV, for d-t and d-d reactions respectively, in order for the second solution to come into play. Such high kinetic energies are very uncommon in a fusion plasma.

### 3.2 Integrating over reactant distributions

The results from the previous section can be used to find the energy of a neutron produced in a fusion reaction between two fuel ions with given velocities. However, in order to calculate the energy spectrum of the fusion neutrons, it is also necessary to integrate over the
The number of neutrons with energy \( E \), emitted in the direction \( u \) from position \( r \), is given by

\[
n(E, u, r) = \frac{1}{1 + \delta_{ab}} \int_{v_a} \int_{v_b} f_a(v_a, r) f_b(v_b, r) |v_a - v_b| \sigma(v_a, v_b, u) \delta(E - E_n(v_a, v_b)) \, dv_a dv_b,
\]

(3.9)

where \( f_a \) and \( f_b \) are the fuel ion velocity distributions and \( \sigma(v_a, v_b, u) \) is the cross section for the production of neutrons in the direction \( u \) by ions with velocities \( v_a \) and \( v_b \). Note the appearance of the Dirac delta function inside the integral, which is included in order to single out only the ions that produce neutrons of energy \( E \) when they fuse. The spectrum \( \Psi_n \) seen by a spectrometer is calculated by integrating equation (3.9) over the plasma volume in the field of view. The emission from each point in the plasma should then be weighted by the solid angle of the detector, \( \Delta \Omega(r) \), seen from that point, and only neutrons emitted in the direction of the detector, \( u_{\text{det}}(r) \), should be included in the spectrum, i.e.

\[
\Psi_n(E) = \int_r n(E, u, r) \Delta \Omega(r) \delta(u - u_{\text{det}}(r)) \, dr.
\]

(3.10)

Fusion cross sections are often tabulated as a function of the CMS energy \( M \) and the emission angle \( \theta \) relative to the relative velocity \( v_1 - v_2 \). Therefore, in the integral (3.9), it is necessary to transform the cross section \( \sigma(v_a, v_b, u) \) to \( \sigma(M, \theta) \). This is done by multiplying with the Jacobian \( \partial \Omega_{\text{CMS}} / \partial \Omega \),

\[
\sigma(v_a, v_b, u) = \sigma(M, \theta) \frac{\partial \Omega_{\text{CMS}}}{\partial \Omega},
\]

(3.11)

where the (relativistic) expression for the Jacobian can be written as [31]

\[
\frac{\partial \Omega_{\text{CMS}}}{\partial \Omega} = \frac{p_1^2}{E_p + E_n} \frac{p_1^2}{p_1^2 - E_1 u \cdot \beta},
\]

(3.12)

### 3.3 Thermal and beam-target spectra

In order to get a feeling for what a neutron spectrum might look like for various fuel ion distribution it is instructive to consider two special cases. Throughout this section the attention is restricted to fusion reactions producing one neutron and one residual particle, such as the d-d and d-t reactions. These types of reactions are the most relevant for neutron spectrometry.

Consider first the case when both fuel ion species are in thermal equilibrium. The integral (3.9) can then be performed analytically, provided that \( E_{\text{fus}} \gg k_B T \), which always holds in typical fusion plasmas. The resulting spectrum becomes [33]

\[
\Psi_{\text{th}} = \frac{n_1 n_2}{(1 + \delta_{12}) \sqrt{2 \pi m_n E_n}} \langle \sigma \rangle_{\text{th}} \left( \frac{m_1 + m_2}{2m_k B T} \right)^{1/2} \exp \left[ -\frac{m_1 + m_2}{m_n} \frac{(E_0 - \langle E_n \rangle)^2}{4k_B T \langle E_n \rangle} \right],
\]

(3.13)
The thermal neutron spectra from the d-d and d-t reaction for various temperatures are shown in figure 3.1a. The pitch angles of the beam particles are 90° (solid lines) and 20° (dashed lines). The spectrum is observed perpendicular to the magnetic field and the plasma temperature is 10 keV.

The other important special case is when one ion species is mono energetic and has a much higher energy than the other. In this case the total momentum $p$ becomes simply $p_a$, the momentum of the fast ion. Now consider a case where the neutron spectrum is observed at an angle perpendicular to the magnetic field. Due to the Larmor gyration, some fast ions will be moving away from the detector and some will be moving towards it. The energy of a fusion neutron emitted towards the detector depends on where on the Larmor orbit the reaction occurs. If a reaction occurs when the fast ion is moving towards the detector $p_a \cdot u = p_a = \sqrt{E_a^2 - m_a^2}$, which will correspond to a maximum positive Doppler shift of $E_n$. Similarly, in the opposite phase of the Larmor gyration there will be a maximum negative Doppler shift. The resulting spectra will have a characteristic “double humped” shape, as shown in figure 3.1b for mono energetic beams with different energies and pitch angles.

### 3.4 Finite Larmor radii effects (Paper I)

In practice, modeled ion distributions, $f_a$ and $f_b$, for use in equation (3.9) are typically not available as a function of all 6 dimensions $(v, r)$ of phase space. Even with sophisticated modeling codes 4-dimensional distributions are the typical output, as described in section 1.5. The gyro angle and the toroidal angle are the two most common coordinates to leave out, since the Larmor radius is typically small compared to plasma phenomena of interest and
the tokamak plasma is toroidally symmetric. Hence, an assumption on the gyro angle must be done when calculating the neutron spectrum. The most natural assumption is that the distributions are isotropic in the gyro phase, thus sampling this angle uniformly between 0 and $2\pi$ in the Monte Carlo calculation. Although this assumption works very well for almost all the experiments studied with TOFOR and the MPR, there are situations when it is not valid. This happens when the following two conditions are met:

1. There is a gradient in the spatial part of one of the ion distributions, with a scale length $L_\perp$ that is comparable to – or smaller than – the width of the field of view $\Gamma_{\text{inst}}$ of the measuring instrument.

2. $L_\perp$ is comparable to – or smaller than – the typical Larmor radii $r_L$ of the ions.

As reported in Paper I, this finite Larmor radius (FLR) effect was observed to affect TOFOR measurements of the neutron emission from plasmas heated with deuterium NBI and ICRH tuned to the 3rd harmonic of the deuterium cyclotron frequency, during a JET experiment in the autumn of 2008. This heating scheme created non-Maxwellian deuterium ions in the MeV range, with spatial distributions that were strongly peaked close to the ICRH resonance position $R_{\text{res}}$. In combination with a relatively low magnetic field of 2.25 T, this meant that $L_\perp$, $\Gamma_{\text{inst}}$ and $r_L$ were all of the order of a decimeter, and hence both conditions (1) and (2) above were fulfilled. Furthermore, $R_{\text{res}}$ was located close to the outboard edge of the field-of-view of TOFOR, which means that part of the Larmor gyration of a given fast ion may be invisible to the spectrometer.

In order to include the FLR effects in the modeling of the neutron spectra from these discharges it was assumed that the gyro centers of the fast ions were distributed only inside a limited region in the plasma, as illustrated by the shaded region in figure 3.2a. This region is called the "fast particle region" in Paper I and models the fact that the turning points of the ICRH accelerated ions are driven towards $R_{\text{res}}$. Inside the fast particle region the energy distribution is assumed to be given by the solution to the Fokker-Planck equation (1.26). The spectrum calculation is carried out by uniformly sampling the position of the gyro centers inside the fast particle region. The actual positions of the particles are then taken to be one Larmor radius away from their gyro center positions, in the direction given by randomly sampled gyro angles. Once this is done, the spectrum seen by TOFOR is calculated, and only reactions involving fast ions inside the field of view (indicated by the red dashed lines in figure 3.2a) are included in the calculations.

An example of a neutron spectrum calculated with and without taking FLR effects into account is shown in figure 3.2b. It is clear that the calculated spectrum describes the data better when FLR effects are included in the modeling. The low energy (long time-of-flight) side of the spectrum is overestimated by almost an order of magnitude if FLR effects are not taken into account. Similar results were seen for several time slices from all the discharges in the JET experiment under consideration. The agreement between experimental data and calculations were not always as good as in the example shown in figure 3.2b, probably due to limitations in the ICRH model, but for most of the time slices the TOFOR data can be understood in terms of the model presented above. Therefore, it is concluded that FLR effects can have great impact on fast ion measurements, provided that conditions 1 and 2 above are fulfilled. It is important to be aware of these effects, not only for the case of neutron spectrometry but also for other
fast ion diagnostic techniques with a collimated field of view, such as $\gamma$-ray spectroscopy and neutral particle analysis.

As a side note, it can also be mentioned that a short investigation of these FLR effects have been made also for the $\gamma$-camera at JET [35]. This was done as a part of the work presented in [36], where the redistribution of fast ions due to toroidal Alfvén eigenmodes was studied by comparing fast ion measurements (from the same JET experiment as in Paper III) against simulations with the HAGIS code [37]. The camera response was simulated for several trial ion distributions, given as functions of the constants of motion $(E, p_\theta, \Lambda, \sigma)$. The orbit code described in section 1.3.1 was used to calculate orbits from the distribution and the resulting camera response was calculated with a Monte Carlo simulation. Overall, the FLR effects were not very important for the camera measurements. The only appreciable effects could be seen in one of the vertical sight lines, which saw less counts when including the FLR effects. This sight line is very similar to the TOFOR one, and consequently views the same part of the plasma, with a lot of fast ions and a large spatial gradient. The other camera channels also see the fast ions but not the gradient. This also illustrates that both points (1) and (2) above need to be fulfilled in order for the FLR effects to come into play.
Chapter 4

Applications

4.1 Neutron spectra from the t-t reaction (Paper II)

The framework for calculating neutron energy spectra outlined in chapter 3 has been used to calculate the shape of the neutron spectrum from the T(t,n)\(^4\)He (t-t) reaction. These calculations were done in order to investigate the possibility to obtain fast ion information from t-t neutrons in a possible future d-t campaign at JET. One possible approach to a d-t campaign would be to start from a tritium plasma (after a period of hydrogen operations in order to clean out residual deuterium from the walls) and gradually increase the deuterium fraction to a 50/50 mixture. This is the opposite approach to the previous d-t campaign in 1997, when tritium was gradually added to a deuterium plasma. Therefore, this opposite approach would allow for the study of isotope effects on how transport, confinement and plasma stability are influenced by fast particles. It is therefore of interest to investigate to what extent neutron spectrometry could provide fast ion data when the t-t reaction is the main source of neutrons.

The cross section of the t-t reaction is similar to the d-d reaction, as seen in figure 1.1. However, since there are three reaction products rather than two,

\[
t + t \rightarrow n + n + ^4\text{He} + 11.3\text{MeV},
\]

the neutron spectrum calculations are a bit different compared to the d-d and d-t case. The residual invariant mass \(M_R\) in equation (3.4) can now take a distribution of values, rather than just one value as in the two product case. The possible values of \(M_R\) are determined from the kinematically allowed region in phase space which can be visualized with a Dalitz plot. This is a plot of \(M_{12}^2 \equiv (P_1 + P_2)^2\) versus \(M_{23}^2 \equiv (P_2 + P_3)^2\), i.e. the square of the invariant masses of two sub-systems of the reaction products. Note that if one of the neutrons is chosen to correspond to particle 1, then \(M_{23}\) is equal to \(M_R\). The minimum value of \(M_{12}\) is \(m_1 + m_2\) (particles 1 and 2 at rest in the CMS) and the maximum value is \(M - m_3\) (particle 3 at rest in the CMS). The range of \(M_{23}\) for a given value of \(M_{12}\) can be calculated from \(M_{12}\) and the particle masses, as described in [38]. An example of the boundaries of the Dalitz plot for the t-t reaction is shown in figure 4.1 for the limiting case of cold reactant, i.e. when both tritons are at rest (\(M = 2m_t\)).

If there are no interactions between the particles in the final state, the distribution of events inside the Dalitz boundaries will be uniform [38]. Thus, the neutron spectrum in the absence of
final state interactions can be calculated by uniformly sampling the Dalitz plot for each Monte Carlo event, thereby obtaining a value of $M_{23} = M_R$ that is used when solving equation (3.4) for the neutron energy. Such a spectrum is shown in figure 4.2a, for the limiting case of cold reactants.

However, accelerator experiments [39, 40] and inertial confinement fusion experiments [41] indicate that there are indeed final state interactions, which modify the shape of the neutron spectrum. The most significant change was the formation of a peak in the neutron spectrum due to the interaction of one of the neutrons with the $^4$He, forming a short lived $^5$He resonance in the ground state. The branching ratio for this reaction channel was observed to be 20% at a CMS energy of 250 keV [39]. At 110 keV the value 5% was found [40] and in [41] this reaction channel could not be observed. Modification of the neutron spectrum due to neutron-neutron interaction, as well as due to the formation of the $^5$He resonance in the 1st excited state, was also reported in [39], but the corresponding spectral features were not very prominent and were not considered in Paper II.

The invariant mass distributions of particle resonances can be described by the Breit-Wigner formula, derived e.g. in [42]. For a resonance with average mass $\hat{m}$ and decay width $\Gamma$ this distribution is given by

$$f(M_R) = \frac{1}{2\pi} \frac{\Gamma}{(M_R - \hat{m})^2 + \frac{\Gamma^2}{4}}.$$  \hspace{1cm} (4.2)

Measured values of $\hat{m}$ and $\Gamma$ for $^5$He are reported in [43], and are 4.67 GeV/c$^2$ and 648 keV, respectively. Using $M_R$-values from this distribution when calculating the neutron energies gives the spectrum in figure 4.2b, for cold reactants. For comparison it is also shown what the spectrum would look like if the $^5$He would be a stable particle with mass equal to $\hat{m}$. This spectrum is simply a peak at $E_n = 8.7$ MeV, and figure 4.2b clearly illustrates the broadening introduced by the decay width of the resonance.
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Figure 4.2. (a) Calculated t-t neutron spectrum for cold reactants when there are no final state interactions and (b) when one neutron and the $^4\text{He}$ interact to form a short lived $^5\text{He}$ resonance in the ground state. Also shown in part (b) is the peak that would be obtained if the $^5\text{He}$ would be stable (red dashed line).

Paper II presents calculation of these two contributions to the t-t neutron spectrum, for thermal plasmas at different temperatures, as well as for NBI heated plasmas and the combination of NBI and 3rd harmonic ICRH. The fast particle distributions were calculated from the 1-dimensional Fokker-Planck equation (1.26). The results are shown in figure 4.3. These graphs indicate that the three-body nature of the final state of the t-t reaction will complicate the attempts to extract fast ion information from a measured spectrum. The shapes of the spectra are not very sensitive to the underlying fuel ion distributions, which is likely to make a separation of e.g. the thermal and beam-thermal components more difficult than for the d-d or d-t reactions. This is particularly evident for the branch with no final state interactions (figure 4.3a) where the only difference shows up on the high energy tail of the neutron spectrum. The exception is the $^5\text{He}$ resonance spectrum from 3rd harmonic ICRH and NBI, which has a distinctly different shape compared to the thermal and NBI components.

These calculations could form a starting point for a more quantitative study about the possibilities for using neutron spectrometry as a fast ion diagnostic in tritium dominated plasmas. The next step would be to generate synthetic data for a given spectrometer response function and investigate how accurately different spectral features can be resolved.

4.2 Fuel ion ratio measurements (Paper III)

In section 1.4 it was shown that the total ion density, as well as the fuel ion ratio $n_t/n_d$ are key parameters for the development of an energy producing fusion reactor. Therefore, reliable measurements of these quantities in the plasma core are an important part in the development of a fusion reactor. Paper III presents a method to derive $n_t/n_d$ from neutron spectrometry measurements, using neutron spectra collected with the MPR spectrometer during the d-t campaign at JET in 1997.

The basic principle of the method is the fact that the fuel ion densities are related to the intensities of the different components of the neutron emission. For the MPR, which was set to detect neutrons in the d-t energy range ($E_n \sim 12 - 16$ MeV), the relevant contributions to the
Figure 4.3. Calculated neutron energy spectra from the t-t reaction from a thermal plasma at 10 keV (black solid line), a NBI heated plasma (blue dashed line) and a plasma heated with NBI and 3rd harmonic ICRH (red dash-dotted line). (a) No final state interactions. (b) The peak that is obtained when a $^3$He resonance is produced. The spectra have been normalized to the same peak intensity and are shown on both linear (top) and log scale (bottom).
neutron emission come from thermal and beam-thermal d-t reactions. Under the assumption that the fuel ion density profiles \( n_d(r) \) and \( n_t(r) \) are both proportional to the profile of the electron density \( n_e(r) \) in the region where the MPR has a non zero solid angle coverage \( \Omega(r) \), the intensities of the various contributions can be written as

\[
I_{\text{th,dt}} = \frac{n_d n_t}{n_e} \int n_e^2(r) \frac{\Omega(r)}{4\pi} dr
\]  
(4.3a)

\[
I_{\text{nb,dt}} = \frac{n_t}{n_e} \int n_e(r) n_{\text{ab,dt}}(r) \frac{\Omega(r)}{4\pi} dr
\]  
(4.3b)

\[
I_{\text{nb,td}} = \frac{n_d}{n_e} \int n_e(r) n_{\text{ab,td}}(r) \frac{\Omega(r)}{4\pi} dr
\]  
(4.3c)

where the subscript "nb,ab" is used to denote a beam of particles \( a \) reacting with a thermal population of particles \( b \). \( r \) is the "directed reactivity" for the different emission components, i.e. calculated according to equation (1.3), but considering only neutrons emitted in the direction of the detector. The integrals \( c_{\text{th,dt}}, c_{\text{nb,dt}} \) and \( c_{\text{nb,td}} \) appearing in the equations can be evaluated if the slowing down distribution of the beam is known, using LIDAR measurements of the electron density and charge exchange recombination spectroscopy measurements of the ion temperature profile (this is needed for the reactivity). In Paper III the code NUBEAM (see section 1.3.2) was used to model the beam distributions. Once these integrals are calculated the relative fuel ion densities \( n_d/n_e \) and \( n_t/n_e \) can be obtained from MPR measurements of \( I_{\text{th}}/I_{\text{nb}} \) and fission chamber measurements of the total neutron rate \( R_n \).

As an example of the method, consider JET pulse 42840, which was a high power d-t discharge heated with 10 MW tritium beams. The neutron rate peaked at \( 1.3 \times 10^{18} \) s\(^{-1} \), giving MPR data with high statistics. Time traces of relevant plasma parameters are shown in figure 4.4. The MPR spectrum for the time slice \( t = 14.0 - 14.25 \) s is shown in figure 4.5. The beam component resulting from the NUBEAM modeling has been fitted to the data together with a thermal component by minimizing the \( \chi^2 \), defined by

\[
\chi^2 = \sum_i \frac{(y_i - d_i)^2}{\sigma^2_{d,i}},
\]

where \( d_i \) is the experimental data in bin \( i \), \( y_i \) the model prediction of the signal in the same bin and \( \sigma d_i \) is the uncertainty of the corresponding data point. The free parameters for the fit are the intensities of the thermal and beam components \( (N_{\text{th,dt}} \) and \( N_{\text{nb,td}} \), the temperature of the thermal component \( (T) \), and the energy shift due to plasma rotation \( (\Delta E) \). The best fit values for these parameters are given in table 4.1a, along with their corresponding unconstrained uncertainties. The optimization and uncertainty estimation were done with a Monte Carlo routine sampling the 4-dimensional \( \chi^2 \) hyper surface.

The component intensities \( N_{\text{th,dt}} \) and \( N_{\text{nb,td}} \) are not determined on an absolute scale. Therefore, only the ratio \( I_{\text{th,dt}}/I_{\text{nb,td}} \) \( (= N_{\text{th,dt}}/N_{\text{nb,td}}) \) can be deduced from the fitted parameters. By
Figure 4.4. Time traces of the total neutron rate, the NBI heating power, the ion temperature, the electron density and for JET pulse 42840.

Figure 4.5. MPR data for JET pulse 42840 at $t = 14.0 - 14.25$ s (points with error bars). Three components have been fitted to the data; a thermal component (solid black line), a beam component (black dashed line) and a low energy component, taking down scattered neutrons in the collimator into account (dotted line). There seemed to be a problem with the background correction for some of the channels (red dots). These channels were not included in the fit.
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>$N_{\text{th,dt}}$</td>
<td>$0.254 \pm 0.0052$ a.u.</td>
</tr>
<tr>
<td>$N_{\text{nb,td}}$</td>
<td>$0.0649 \pm 0.0053$ a.u.</td>
</tr>
<tr>
<td>$T$</td>
<td>$14.7 \pm 0.47$ keV</td>
</tr>
<tr>
<td>$\Delta E$</td>
<td>$150 \pm 2.7$ keV</td>
</tr>
</tbody>
</table>

(a) Values of the fitted parameters for the MPR data in figure 4.5.

<table>
<thead>
<tr>
<th>JET #</th>
<th>MPR $[n_t/n_d]$</th>
<th>KT5 $[n_t/n_d]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>42760</td>
<td>$0.61 \pm 0.07$</td>
<td>1.1</td>
</tr>
<tr>
<td>42780</td>
<td>$9.91 \pm 2.3$</td>
<td>4.4</td>
</tr>
<tr>
<td>42840</td>
<td>$10.2 \pm 1.9$</td>
<td>4.2</td>
</tr>
<tr>
<td>43011</td>
<td>$38.8 \pm 11.3$</td>
<td>12.1</td>
</tr>
</tbody>
</table>

(b) Average $n_t/n_d$ values derived from MPR data compared with Penning trap measurements (KT5) of $n_t/n_d$ at plasma edge.

Combining this measured ratio with equations (4.3a) and (4.3c) the relative tritium density can be obtained as

$$\frac{n_t}{n_e} = \frac{I_{\text{th},dt}}{I_{\text{nh},td}} c_{\text{th,td}}.$$  \hspace{1cm} (4.4)

However, yet another relation is needed for $n_d/n_e$. Such a relation can be obtained by using the total neutron emission rate $R_n$, which was measured by fission chambers in the present experiment. Neglecting contributions from the d-d and t-t reactions – which are small compared to the d-t contribution due to their much smaller cross sections – $R_n$ can be written as

$$R_n \approx R_{\text{th,dt}} + R_{\text{nh,td}},$$  \hspace{1cm} (4.5)

where

$$R_{\text{th,dt}} = \frac{n_d}{n_e} \frac{n_t}{n_e} \int n_e^2(r) \langle \sigma v \rangle_{\text{th,dt}} dr,$$  \hspace{1cm} (4.6a)

$$R_{\text{nh,td}} = \frac{n_d}{n_e} \int n_e(r) n_{\text{ab,td}}(r) \langle \sigma v \rangle_{\text{nh,td}} dr.$$  \hspace{1cm} (4.6b)

These equations are the same as (4.3a) and (4.3c), except that the directional reactivities $r$ have been replaced by the ordinary reactivities $\langle \sigma v \rangle$ and the integration is performed over the entire plasma volume rather than just the viewing cone of the spectrometer. Just as before, the integrals $C_{\text{th,dt}}$ and $C_{\text{nh,td}}$ can be evaluated by means of diagnostic data and the NUBEAM modeling. Solving equation (4.5) for $n_d/n_e$ gives

$$\frac{n_d}{n_e} = \frac{R_n}{C_{\text{th,dt}} \frac{n_t}{n_e} + C_{\text{nh,td}}}.$$  \hspace{1cm} (4.7)

Equations (4.4) and (4.7) gives the relative fuel ion densities from measurements of $I_{\text{th}}/I_{\text{nh}}, R_{\text{th}}$ and $n_e$ (for the NUBEAM modeling). In the example presented here one obtains $n_t/n_d = 10.1$. Analogous calculations can be done for deuterium beams or mixed beams.

Time traces of fuel ion ratios derived in this way are presented in Paper III for four JET d-t discharges, where the uncertainties in the derived fuel ion ratios are also discussed. Average
values of $n_t/n_d$, obtained both from the MPR measurements and from Penning trap measurements at the edge of the plasma, are presented in table 4.1b. It is seen that the trend in the derived fuel ion ratios is consistent with the Penning trap measurements but the absolute values are not the same. An absolute agreement is not necessarily expected though, since the two measurements are made in different parts of the plasma. For instance, the Penning trap measurement is likely to be more strongly influenced by the recycling of deuterium and tritium that have been retained in the reactor walls from previous discharges. This is discussed in more detail in the paper.

It would be desirable to further validate and benchmark the method presented here by applying it to more discharges and comparing it to other estimations of the fuel ion ratio. An excellent opportunity to do this would be in a possible future d-t campaign at JET. Since JET now has spectroscopic capabilities for both the d-d reaction (using TOFOR) and the d-t reaction (using the MPR) it would be possible to compare the method used in Paper III with the traditionally proposed method of determining $n_t/n_d$ from the ratio of the thermal d-t and d-d neutron emission intensities [44]. This was not possible to do in the d-t campaign in 1997, since TOFOR was not installed until 2005.
Chapter 5

Conclusions and outlook

“A learning experience is one of those things that say, ‘You know that thing you just did? Don’t do that.’”

Douglas Adams

This thesis, and the accompanying papers, give examples of how the neutron energy spectrum from a fusion plasma can be calculated from various theoretical models of the fuel ion distributions, as well as how these calculated spectra can be used to obtain information about said ion distributions.

In particular, it is reported in Paper I that it can be necessary to include the effect of the finite Larmor radii of the fast ions in the modeling, in order to correctly interpret neutron spectrometry measurements. This is the case when there is a spatial gradient in the fast ion distribution function, whose scale length is comparable to – or smaller than – the width of the field of view of the measuring instrument, and if the typical fast ion Larmor radius is comparable to – or larger than – this scale length. These effects were observed for TOFOR measurements in a JET experiment with 3rd harmonic ICRH and deuterium NBI. By including the effect of the Larmor gyration in the modeling of the neutron emission a good agreement between simulations and data was obtained for several time slices and discharges in the experiment. The agreement was not perfect for all time slices though, and it would be interesting to redo the analysis using a more sophisticated modeling tool for the fast ion distributions, such as the SELFO code. The FLR effect could also be further investigated in experiments e.g. by reversing the toroidal magnetic field at JET.

The results from Paper I, and the references therein, show that the fast ion distributions that are obtained by solving equation (1.26) capture the most important features that affect the shape of the neutron spectrum, at a comparatively low price in terms of computer time. Therefore, distributions calculated in this way were the natural choice for the simulations in Paper II, where neutron spectra from the t-t reaction were calculated. The main result from this study is the qualitative observation that fast ion measurements based on the t-t reaction might be more difficult than for reactions with two particle final states, such as the d-d and d-t reactions. A natural continuation of this investigation would be to quantify the results by generating synthetic data for a given spectrometer response function and investigate how well the different spectral features can be resolved. Another issue which could be addressed is the poorly known energy dependence of the branching ratio between the $^3$He resonance channel
and the three body continuum channel. This is one of the bigger sources of uncertainty in the modeling in Paper II. It would be interesting to investigate whether this energy dependence could be used as a free parameter in the modeling of the t-t neutron spectrum. Neutron spectrometry data could then potentially provide information about the branching ratio for energies from the keV range up to several MeV. But this is of course only speculations at this stage.

Finally in Paper III, 4-dimensional slowing down distributions from the code NUBEAM have been used together with the high resolution MPR spectrometer to estimate the fuel ion ratio in the plasma core. This is a complicated measurement, which relies not only on the accuracy of the spectrometry measurements, but also on several other diagnostics, such as the total neutron rate, the electron density and the ion temperature. One difficult aspect in the development of this method is the evaluation of the results. A subject for future work could be to do more measurements using the same principle, in plasma scenarios where the d-d intensity is high enough to allow the fuel ion ratio to be derived by comparing the d-d and d-t thermal components as well. This is a somewhat more direct measurement than what is done in Paper III, and it would be interesting to see how the two methods compare. Reliable measurements of the fuel ion ratio is of outermost importance for ITER and the demonstration of fusion as a possible energy source and therefore several independent techniques for measuring this quantity should be developed and cross checked against each other.
Acknowledgments

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My supervisors, Göran, Carl and Sean have spent much time helping me with just about everything, from details about fusion reaction kinematics, data analysis and programing, to tips and suggestions about writing papers and the overall plan for my PhD project. Your importance for the completion of this thesis cannot be emphasized enough.

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Bibliography


