<http://www.diva-portal.org>

Postprint

This is the accepted version of a paper published in *Energy Conversion and Management*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Della, R., Alhassan, E., Adoo, N., Bansah, Y., Nyarko, B. et al. (2013)
Stability analysis of the Ghana Research Reactor-1 (GHARR-1).
Energy Conversion and Management, 74: 587-593

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-208444>

STABILITY ANALYSIS OF THE GHANA RESEARCH REACTOR-1

(GHARR-1)

R. Della¹, E. Alhassan^{2*}, N.A. Adoo¹, C.Y. Bansah¹, B.J.B. Nyarko¹, E.H.K. Akaho¹

¹National Nuclear Research Institute, Ghana Atomic Energy Commission, P.O. Box LG 80,

Legon – Accra, Ghana.

²Division of Applied Nuclear Physics, Department of Physics and Astronomy, Uppsala

University, Box 516, 751 20, Uppsala, Sweden

Email: erwin.alhassan@physics.uu.se

Abstract

A theoretical model has been developed to study the stability of the Ghana Research Reactor one (GHARR-1). The closed-loop transfer function of GHARR-1 was established based on the model, which involved the neutronics and the thermal hydraulics transfer functions. The reactor kinetics was described by the point kinetics model for a single group of delayed neutrons, whilst the thermal hydraulics transfer function was based on the modified lumped parameter concept. The inherent internal feedback effect due to the fuel and the coolant was represented by the fuel temperature coefficient and the moderator temperature coefficient respectively. A computer code, RESA (REactor Stability Analysis), entirely in Java was developed based on the model for systems analysis. Stability analysis of the open-loop transfer function of GHARR-1 based on the Nyquist criterion and Bode diagrams using RESA, has shown that the closed-loop transfer function was marginally stable for variable operating power levels. The relative stability margins of GHARR-1 were also identified.

Keywords: Nyquist criterion, Bode plots, transfer function, RESA, lumped parameter concept

1. Introduction

In all physical systems there is always the need for control and stability. Nuclear reactors are not exceptions because of their peculiar nature. Reactor stability analysis can be considered in terms of the response of a steady state system to disturbances (perturbations) that can affect core reactivity [1]. These perturbations could be for example in the movement of control rods, changes in power demand, and variations in coolant flow rate etc. A reactor is clearly unstable if a reactivity perturbation can cause an unlimited increase in the neutron flux and hence the power output. The increase may occur continuously or it may result from a series of oscillations of increasing amplitude. In general, a reactor is stable if a perturbation of reactivity during steady state operation ultimately leads to another steady state. However, a system may be stable but its transient behaviour in going from one steady state to another may not be satisfactory; for example the power may undergo a series of oscillations of such large amplitudes before dying out such that, the safe operation of the reactor may be impossible. In Boiling Water Reactors (BWR), instabilities are of great importance as it affects the operation of a large number of commercial nuclear reactors worldwide. Two types of reactivity instabilities can be observed in commercial BWRs; these are core-wide reactivity instabilities in which all channels oscillate in-phase across the core and, the regional reactivity stability in which power or flow oscillating in two regions, are out-of-phase. A detailed review of the state of the art for coupled thermal hydraulic-neutronic instabilities in boiling water reactors can be found in Ref [14].

In this work, a preliminary attempt to understand the stability of GHARR-1 has been carried out using the closed-looped transfer function which describes the overall dynamic behavior of the reactor including the neutronics, thermal hydraulics and reactivity feedbacks [3]. The Ghana Research Reactor-1 (GHARR-1) is a commercial version of the Chinese constructed prototype Miniature Neutron Source Reactor (MNSR) and it is similar in design to the

Canadian SLOWPOKE (Safe Low Power Critical Experiment) reactors [4]. In GHARR-1, instability can occur as a result of positive reactivity insertion due to either internal or external effects or both. If procedures are not correctly followed during operations such as the installation of a new core, a maximum of 6.71 mk could be inserted into the reactor core. Due to human error, the reactivity that could be inserted during the addition of incorrect thickness of top Be plates in the Al tray located on top of the core has been estimated to be 9 mk [2]. Reactivity induced transients could cause power to rise suddenly to levels beyond the capability of the reactor to remove heat, thereby increasing the risk of damage to reactor components. A theory was developed based on the neutronics and thermal hydraulics parameters of the reactor. The neutronics analyses were based on the formulation of the open-loop transfer function also known as the neutronics transfer function. This function was derived based on the point kinetic model using one group precursor approximation in which six groups of delayed neutrons and nine groups of prompt neutrons emitted from the beryllium reflectors were lumped together. The inherent internal feedback effects due to the fuel and the coolant were represented by the fuel and the moderator temperature coefficients. The thermal hydraulics transfer function was based on the modified lumped parameter concept. A computer code, RESA (REactor Stability Analysis) written in Java computer programming language was developed based on the theory for the analysis.

1.1 Ghana Research Reactor-1 (GHARR-1)

GHARR-1 is a 30 kW tank-in-pool type research reactor, which uses 90.2% enriched uranium fuel in aluminium alloy. It is a compact, safe and low power research reactor which is cooled by natural convection and can be operated at a maximum thermal neutron flux of 1×10^{12} n/cm²s. The diameter of the fuel is 4.3 mm and the thickness of the aluminium cladding material is 0.6 mm. The total length of the element is 248 mm and the active length

is 230 mm. The percentage of U in the UAl_4 dispersed in Al is 27.5% and the loading of U-235 in the core is 990.72 g. The fuel assembly, which is mounted on a 50 mm thick bottom-Be reflector, and surrounded by 100 mm thick annular-Be reflector in addition to a top-shim Be reflector with variable thickness, consists of 344 fuel pins, 4 tie rods and 6 dummy pins concentrically arranged in 10 rings. The core has a central guide tube through which a Cd control rod clad in stainless steel moves to cover the active length of 230 mm of the core. The single control rod is used for regulation of power, compensation of reactivity and for reactor shut down during normal and abnormal operations [4]. Fig. 1 shows a schematic diagram of the vertical cross-section of the reactor. The reactor complex contains 5 major components. These are the reactor assembly, control console, auxiliary systems, irradiation system and the pool. It is designed for use in universities, hospitals and research institutes mainly for neutron activation analysis, production of short lived radioisotopes, education and manpower development.

Fig. 1: A schematic diagram of the vertical cross-section of GHARR-1

Inherent safety features of GHARR-1 are the availability of highly negative moderator temperature coefficient of reactivity (approximately $-0.1 \text{ mk}/^\circ\text{C}$ at a temperature range of 20 to 45 $^\circ\text{C}$), and low critical mass [2]. These characteristics limit achievable peak power levels following an accidental insertion of reactivity and assure the safety of the reactor under all conceivable accident conditions [4]. In order to demonstrate the inherent safety characteristics of GHARR-1, dynamic experiments were carried out by Akaho et al. [5] by inserting step reactivity of 2.1 mk and ramp reactivity of the cold clean core excess reactivity of 4 mk. It was observed that the reactor power rose to peak values of 37.5 and 100.2 kW, respectively.

2. Theory consideration

2.1 Neutronics Transfer function

The neutronics (open-loop) transfer function was derived from the point kinetic equations. A detailed presentation of the point kinetic equations taking into consideration the delayed neutrons as well as photo neutron precursors can be found elsewhere [1, 6]. For the sake of simplicity, one representative precursor group was considered instead of the usual six delayed and nine photo neutron groups. This is because the reactivity and the prompt neutron generation time are not affected by the above assumption. As a result the equations can be written as:

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda C \quad (1)$$

$$\frac{dC}{dt} = \frac{\beta}{\Lambda} n - \lambda C \quad (2)$$

Where n is the neutron density, ρ is reactivity, β is the delayed neutron fraction, Λ is the prompt neutron generation time, λ is the delayed neutron concentration and C , the precursor neutron concentration. It was assumed that the reactor was initially in a steady state with n_o , c_o , ρ_o and allowed to undergo small perturbations in reactivity, concentration of precursors and neutron density; $\rho = \rho_o + \delta\rho$, $C = C_o + \delta C$, $n = n_o + \delta n$ about the unperturbed state. Substituting the perturbation forms into Eq. (1) and Eq. (2) and taking Laplace transformation results in eq. (3) and (4):

$$G_s = \frac{\delta n}{\delta \rho} = \frac{n_o}{\Lambda \left[s + \frac{s\beta}{\Lambda(s + \lambda)} \right]} \quad (3)$$

Where G_s is the zero-power transfer functions, δn and $\delta \rho$ is the change in neutron density and reactivity respectively and s is the Laplace transform variable, which is a complex variable.

Making $n_o = P_o$ where P_o is the steady state reactor power and after some algebra; the neutronics transfer function for GHARR-1, a low power light water reactor where delayed neutron effects are dominant becomes;

$$P_o G_s = \frac{1}{sA} \cdot \frac{s + \lambda}{s + \lambda + \beta / A} \quad (4)$$

But $\lambda \ll \beta / A$ for light water reactors and thus can be ignored. Letting $\beta / A = a$ and substituting a into Eq. (4) results into Eq. (5) which represents the zero-power transfer function. Eq. (5) is only valid for light water reactors with low power.

$$P_o G_s = \frac{s + \lambda}{As(s + a)} \quad (5)$$

2.2 Thermal Hydraulics Transfer function

The thermal hydraulic transfer function $H(s)$ was derived based on the lumped parameter concept taking into consideration the two dominant feedback effects; the moderator and the fuel temperature coefficients. The GHARR-1 core was treated by lumping the fuel together as one unit whiles the coolant flowing through all the ten concentric tubes was also considered as another unit. The physical model representing the modified lumped parameter concept for GHARR-1 is depicted in Fig 2. Thermal energy (T_f) produced from fission is transferred to the coolant via conduction, this energy is carried away by the moving coolant with temperature (T_c) flowing with a mass flow rate of w . Since GHARR-1 is cooled by natural convection, the heat transfer within the core can be represented by the following eq. (6) and (7);

Fig. 2: The physical Model

$$C_f M_f \frac{dT_f}{dt} = P - Ah(T_f - T_c) \quad (6)$$

$$C_c M_c \frac{dT_c}{dt} = Ah(T_f - T_c) - C_c w(T_c - T_{ci}) \quad (7)$$

Where $C_f M_f$ and $C_c M_c$ are heat capacity of the fuel and the coolant, T_f and T_c are the fuel and coolant temperatures, P is the reactor power, T_{ci} is the inlet coolant temperature, A is the area of the fuel can, w is the flow rate. The heat transfer coefficient h for GHARR-1 was determined from full scale simulation experiment and is expressed as [7];

$$h = \frac{k_f}{d_{eq}} n (G_{rf} P_{rf})^m \quad (8)$$

Where k_f is the thermal conductivity of the fuel and; $n = 0.68$, $m = 1/4$ $G_{rf} P_{rf} < 6 \times 10^6$ for laminar flow. The equivalent hydraulic diameter, d_{eq} which is four times the ratio of the cross-sectional area and the wetted perimeter is determined for the triangular lattice of GHARR-1 from the expression;

$$d_{eq} = 4 * \frac{\left(\frac{\sqrt{3}}{4} P_t^2 - \frac{1}{2} \left(\frac{\pi}{4} \right) d_0 \right)}{\frac{1}{2} \pi d_0} \quad (9)$$

Where P_t is the pitch, d_0 is the diameter of the fuel element and T_{fo} is the outlet fuel temperature. The flow rate is calculated from the correlation [13];

$$w = \frac{P}{C_c \left(7.04 T_{i-0.35} P(t)^{(0.59+0.001 T_1)} \right)} \quad (10)$$

Where P is steady state reactor power, C_c is heat capacity of the coolant, T_2 and T_1 are the outlet coolant and inlet coolant temperature respectively and $P(t)$ is reactor power at time t . Assuming small variations around the reactor power, coolant temperature and fuel temperature; $P = P_o + \Delta P$, $T_c = T_{ci} + \Delta T_c$, $T_f = T_{fo} + \Delta T_f$. Denoting $\lambda_f = Ah / C_f M_f$,

$\lambda_c = w/M_c$ and $\lambda_m = 1/C_m M_m$ and taking the Laplace transform of the Eq. (6) and (7) results;

$$sT_f = k_f P - \lambda_f (T_f - T_c) \quad (11)$$

$$sT_c = \lambda_c (T_f - T_c) - \lambda_w T_c \quad (12)$$

Solving for the fuel and coolant temperatures in eq. (11) and (12) gives;

$$T_c = \frac{\lambda_c T_f}{(s + \lambda_w + \lambda_c)} \quad (13)$$

$$T_f = \frac{k_f P (s + \lambda_w + \lambda_c)}{(s + \lambda_f)(s + \lambda_w + \lambda_c) - \lambda_c \lambda_f} \quad (14)$$

The total reactivity due to moderator and fuel can be expressed as;

$$\rho = \alpha_c T_c + \alpha_f T_f \quad (15)$$

Where α_c and α_f are the moderator and fuel temperature coefficients. In table 1, kinetic parameters of GHARR-1 are presented with the fuel and moderator coefficients of reactivity [2]. It should be noted from table 1 that, though the moderator temperature coefficient is a function of the fuel temperature coefficient, this will not violate the assumption of linearity when deriving the transfer functions.

Table 1: Kinetic Parameters of GHARR-1 [2]

Substituting T_c and T_f into Eq. (15) and simplifying results in Eq. (16), the thermal hydraulics transfer function;

$$H(s) = \frac{\rho}{P} = \frac{k_f \alpha_f \left[\left(\frac{\alpha_c}{\alpha_f} + 1 \right) \lambda_c + s + \lambda_w \right]}{s^2 + s(\lambda_w + \lambda_f + \lambda_c) + \lambda_f \lambda_w} \quad (16)$$

Denoting some terms in eq. (15) as $g = \lambda_f + \lambda_c + \lambda_w$, $q = \lambda_f \lambda_w$, $y = \left(\frac{\alpha_c}{\alpha_f} + 1 \right) \lambda_c + \lambda_w$, eq. (16), the thermal hydraulics transfer function reduces to the form;

$$Hs = \frac{k\alpha_f(s+y)}{s^2 + sg + q} \quad (17)$$

2.3 Open-loop Transfer function

The open-loop transfer function is the combine response of a system without incorporating the feedback component. Fig. 3 shows the open-loop block diagram for GHARR-1. Where θ_i and θ_o are the input and output signals respectively, $P_oG(s)$ is the neutronics transfer function and $H(s)$ is the thermal hydraulics transfer function with considering feedback.

Fig. 3: Open-Loop Block of GHARR-1

The equation describing the transfer function of the open-loop block diagram in fig. 3 is given by;

$$\frac{\theta_o}{\theta_i} = P_oG(s)H(s) \quad (18)$$

Simplifying Eq. (23) gives;

$$P_oG(s)H(s) = \frac{\alpha_f k_f (s + \lambda)(s + y)}{s\Lambda(s + a)(s^2 + sg + q)} \quad (19)$$

Since, s is a complex value in the form iw , separating the real components from the imaginary part gives eq. (20);

$$P_oG(iw)H(iw) = \frac{\beta(-w^2 + \lambda y) + \beta w(y + \lambda)i}{(w^4\Lambda + (-w^2\Lambda)(qag)) + (-w^3\Lambda(a + g) + waq\Lambda)i} \quad (20)$$

The equation as it stands is amenable to programming. In order to get a Nyquist plot, the real part of the equation is separated from the imaginary part and the frequency w , varied in the range $[-\infty, \infty]$.

2.4 Closed-loop Transfer function

The closed looped (Reactor) transfer function describes the overall dynamic behaviour of the reactor and represents the real dynamic response of the reactor system. It was formulated by taking into consideration the neutronics and the thermal hydraulics transfer functions. Fig. 4

represents a SISO (Single Input Single Output) system for the closed-loop transfer function of GHARR-1. Where $\rho(s)$ is the input signal to the system, $P_o G(s)$ is the neutronics transfer function, $H(s)$ is the thermal hydraulics (with feedback) transfer function, $P(s)$ is the output signal of the reactor power, and $\rho_T(s)$ is the output signal of the feedback.

Fig. 4: Closed-loop Transfer function block of GHARR-1

In order to determine the transfer function for the reactor, we make reference to the block diagram in Fig. 4. The output signal is expressed as:

$$P(s) = (\rho(s) - H(s)P(s))P_o G(s) \quad (21)$$

Simplifying eq. (21) leads to;

$$R(s) = \frac{P(s)}{\rho(s)} = \frac{P_o G(s)}{1 + P_o G(s)H(s)} \quad (22)$$

Substitution of the values for the variables in Eq. (22), letting $\beta = \alpha_f k$ and simplifying further results into the closed-loop transfer function;

$$R(s) = \frac{((s^2(g + \lambda) + \lambda q) + (s^3 + s(q + \lambda g)))}{((s^4 \Lambda + s^2(\beta + \Lambda(q + ag)) + \beta \lambda y) + (s^3 \Lambda(g + a) + s(\Lambda a q + \beta(\lambda + y)))} \quad (23)$$

Where Λ is the neutron generation time, λ is the half life of the neutron precursor, β is the delayed neutron fraction, and s is a complex variable. All other variables are the same as have been denoted in the text.

3. Stability Analysis

Stability of the closed-loop transfer function of GHARR-1 was analyzed by the application of the Nyquist criterion and Bode plot techniques. The Nyquist criterion is essentially a graphical method that allows for the establishment of the stability of the closed-loop system from the Nyquist plot of the open-loop transfer function whiles the Bode plot is based on calculating the gain and phase angle of the transfer function. For the estimation of the

stability limits of GHARR-1 closed-loop, the Routh–Hurwitz criterion was used by determining the locations of the roots of the characteristic equation of the system (the denominator of eq. (23)).

3.1 Bode stability criterion

The Bode diagram represents the systems (reactor) response in magnitude and phase to a sinusoidal input of any frequency through a log-log plot of the magnitude or phase verses the frequency [8]. It provides a sufficient condition for the closed-loop stability based on the properties of the open-loop transfer function [14]. On a Bode diagram, a closed-loop system is marginally stable if the bode curves crosses the critical point i.e. a phase (angle) of -180° and an amplitude of $1=0\text{dB}$. The phase margin is the difference between the -180° and the actual phase angle of the frequency response function measured at the frequency where the gain is 0dB (unity gain). The gain margin, on the other hand is the margin between the gain plot and the 0dB measured at the point where the phase angle reaches -180° crossing [15, 16]. The conclusions from the Bode plots were tested by simulating the system with MATLAB and by analysing the Nyquist plots. The gain and the phase were determined from eq. (24) and (25) respectively;

$$\text{gain (dB)} = 20\log|R(s)| \quad (24)$$

$$\text{phase} = \tan^{-1} \frac{\text{Im}(R(s))}{\text{Re}(R(s))} \quad (25)$$

Where $R(s)$ is the closed-loop (reactor) transfer function.

3.2 Nyquist stability criterion

Stability test for time invariant linear systems can be derived in the frequency domain using the Nyquist stability criterion [8]. The Nyquist stability is essentially a graphical method and allows the establishment of the stability of the closed loop system from the Nyquist plot of

the open loop transfer function. It is an alternative representation of the frequency response information; a polar plot of the frequency response produces a curve with frequency as a parameter relating the points on the polar plot to the appropriate points on the Bode plot [15,17]. Considering a system whose open loop transfer function $G(s)$ with a negative feedback $H(s)$, we recalled that the closed-loop transfer function was given in Eq. (22). The stability of the closed loop can be determined from the analysis of the right half-plane zeros of the characteristic equation (the denominator of Eq.(23)). The closed loop is stable if and only if the net number of anticlockwise encirclements of the $(-1,0)$ point by the Nyquist plot of the open loop transfer function $G(s)$ is equal to the number of poles of $G(s)$ in the right half-plane [17].

3.3 Routh-Hurwitz Stability Criterion

The Routh-Hurwitz criterion is a method for determining whether a linear system is stable or not by examining the locations of the roots of the characteristic equation (the denominator of Eq.(23)) of the system. A necessary condition for stability of the system is that all the coefficients (a_i) of the closed-loop characteristic equation are positive [18]. We can therefore conclude that, if at least one of the coefficients of the characteristic equation is negative, the system is unstable. The Routh-Hurwitz polynomial test was performed for variable reactor power to determine the coefficients of the characteristic equation $q(s)$ given in Eq. (26), using the Routh array.

$$q(s) = s^4 \Lambda + s^3 \Lambda(g + a) + s^2 (\beta + \Lambda(q + ag)) + s(\Lambda a q + \beta(\lambda + y)) + \beta \lambda y = 0 \quad (26)$$

Where all variables are the same as have been denoted earlier in the text. Applying the Routh array to Eq. (26) the Routh table was constructed and the coefficients of the characteristic equation $q(s)$ were determined. In table 4 we present the Routh-Hurwitz polynomial test for variable reactor power.

3.4 RESA (REactor Stability Analysis) code

RESA is an acronym for **RE**actor **S**tability **A**nalysis. It is written entirely in Java programming language based on the modified lumped parameter model as presented in the theory for stability analysis of the Ghana Research Reactor-1 (GHARR-1). It can be used for stability analysis of low power, light water moderated research reactors. In table 2, the thermal hydraulics and geometrical parameters of GHARR-1 used as inputs into the code are presented.

Table 2: Thermal hydraulics and geometrical parameters of GHARR-1 [4]

The conclusions from the Bode plots were compared with results generated with MATLAB and tested by analysing the Nyquist plots for the reactor. It was observed that the results obtained compared favourably as seen in Table 3.

4. Results and Discussions

The two widely used measures of relative stability of a system are the 1) Phase margin, which is defined as the angle that the frequency response would have to change to move to the -1 point. It is found by measuring the angular difference between the point on the frequency response at the unit circle crossing and -180° and 2) the gain margin, which on the other hand is the amount that the frequency would have to increase to move to the -1 point. It can be seen in Fig. 5 that the frequency response curve does not cross the -180° crossing point at all.

In Table 3, the relative stability margins of the GHARR-1 are compared with results simulated with MATLAB are presented. It can be seen from the table that the relative stability margins of GHARR-1 from RESA code compared favourably with results obtained with MATLAB.

Table 3: Comparison of Relative Stability Margins

The Routh-Hurwitz criterion has also been used to study the stability of GHARR-1. In table 4, we present results from the Routh-Hurwitz polynomial test, which requires that for a stable system the coefficients of the characteristic equation be positive. It can be observed from the table that all the coefficients positive for all variable power levels, implying that GHARR-1 is stable.

Table 4: Routh-Hurwitz polynomial test for variable reactor power

The Bode diagram for the open-loop transfer function at the maximum reactor power of 30kW is shown in Fig. 5, with the gain (decibels) and phase (degrees) of the transfer functions on the vertical axes and the frequencies on the horizontal axes respectively.

Fig. 5: Bode plot of GHARR-1

The curve is plotted for a power level of 30 kW, which is the maximum operating power level. The stability of the system is determined by the phase; the system is stable until the phase crosses the -180° . The frequency response curve shows a graph pattern of decreasing gain values with increasing frequencies with visible variations in the gain values around 10 to 20 dB. These decreasing gain values represent decreasing amplitudes of the power oscillations. At frequencies between 1 Hz and 10 Hz these oscillations become stable, however beyond 10 Hz the amplitudes continue to decrease. It can be observed from fig. 5 that, the closed-loop transfer function is stable since its phase does not cross the -180° line. This is not surprising as the GHARR-1 is designed with a negative moderator temperature feedback coefficient and a low critical mass [4]. The results obtained from the Bode diagrams compared favourably with results simulated with MATLAB. The results were further tested by analysing the Nyquist plot. Fig. 6 shows a plot of the Nyquist graph obtained from the RESA program with the imaginary values on the vertical axis and the real values on the horizontal axis.

Fig. 6: Nyquist plot of GHARR-1

According to the Nyquist stability criterion, the closed loop system is marginally stable if the Nyquist curve goes through the critical point, which is the point $(-1, 0)$, the point where the phase angle is 180° and unity gain [16,17]. This is normally translated into an exponentially decaying behaviour of a system in the time domain. It can be seen from Fig. 6 that the frequency response curve does not go through the critical point $(-1, 0)$, implying that GHARR-1 is marginally stable.

5. Conclusion

The closed-loop transfer function of GHARR-1 was formulated using the modified lumped parameter concept with the approximation of one group of delayed neutrons. Analyses were done taking into consideration the reactivity feedbacks representing the fuel and moderator temperature coefficients of reactivity. Stability analysis of the open-loop transfer function based on the Nyquist stability criterion and Bode plots has shown that the MNSR is stable. Also, it was observed from the Routh-Hurwitz polynomial test for variable reactor power that GHARR-1 is stable. This work was carried out in an attempt to understand the stability of the GHARR-1 reactor. For future work, we plan to study the stability of the proposed Low Enriched Uranium core, taking into consideration the space dependency (both axial and radial), as part of international efforts to convert current HEU reactors to LEU [12].

Acknowledgements

The authors will like to acknowledge financial support received from the Department of Physics and Astronomy, Uppsala University for the corresponding author to attend the IYNC-2012 in Charlotte, USA.

References

- [1] S. Glasstone and A. Sesonske, Nuclear Reactor Engineering, third ed., Van Nostrand Reinhold Company, New York, 1981.
- [2] E.H.K Akaho and B.T. Maakuu, Simulation of Reactivity Transients in a Miniature Neutron Source Reactor Core. Nucl. Eng. and Des. 213 (2002) 31-42.
- [3] A. Hainoun, I. Khamis and G. Saba, Dynamic analysis of the closed-loop transfer function in the miniature neutron source reactor (MNSR). Nucl. Eng. Des. 232 (2004) 19-28.
- [4] SAR, The Ghana Research Reactor-1 Safety Analysis Report, Internal report (2003).
- [5] E.H.K Akaho, S. Anim-Sampong, B.T. Maaku and D.N.A. Dodoo-Amoo, Dynamic feedback characteristics of Ghana Research Reactor-1. Journal of Ghana Science Association, Vol 2, No 3 (2000) 200-208.
- [6] A. Hainoun and I. Khamis, Determination of neutron generation time in MNSR reactor by measurement of neutronics transfer function. Nucl. Eng. Des. 195 (2000) 299-305.
- [7] Z. Yongji, The Whole Simulated Heat Transfer Experiment and Calculations, MNSR Training Materials, China Institute of Atomic Energy, Beijing China, 1993.
- [8] D.L Hetrick, Dynamics of Nuclear Reactors, American Nuclear Society, 1993.
- [9] A. Hainoun, I. Khamis and G. Saba, Dynamic analysis of the closed-loop transfer function in the miniature neutron source reactor (MNSR). Nucl. Eng. Des. 232 (2004) 19-28.
- [10] J. Lewins, Nuclear Reactor Kinetics and Control, Pergamon press, Oxford, 1998.
- [11] J. Hahn, T. Edison, T. F. Edgar. A note on stability analysis using Bode plots. University of Texas at Austin, Texas, 2001.
- [12] S. Anim-Sampong, E.H.K. Akaho, B.T. Maakuu, J.K. Gbadago, A. Andam, J.J.R. Liaw and J.E. Matos, Neutronics Analysis for Conversion of the GHARR-1 using Monte Carlo Methods and UO₂ LEU Fuel; Proceedings of the IRRFM 2007/IGORR, Session VI - Safety, Operation and Research Reactor Conversion, (2007) 44-54.

[13] S. Shuanki, Low Power Research Reactor Thermal Hydraulics, IAEA Workshop on Low Power Research Reactors , China Institute of Atomic Energy, Beijing, China, 1990.

[14] J. March-Leuba, & J. Rey, Coupled thermohydraulic-neutronic instabilities in boiling water nuclear reactors: A review of the state of the art. Nuclear Engineering and Design, 145(1-2), 97-111, 1993.

[15] D. E. Seborg, T. F. Edgar and D. A. Mellichamp, Process Dynamics and Control, Second Ed., John Wiley and Sons Inc.. 2004.

[16] Tutorial 7 – Stability Analysis. [online] Available at:

<<http://www.freestudy.co.uk/control/t7.pdf> > [Accessed 3 February 2013].

[17] V. Ougrinovski, Nyquist stability criterion, 2006. [online] Available at:

<<http://seit.unsw.adfa.edu.au/staff/sites/valu/teaching/ct2/nyquist.pdf> > [Accessed 2 February 2013].

[18] M. M. Aziz, Stability, 2010. [online] Available at:

<http://people.exeter.ac.uk/mmaziz/ecm2105/ecm2105_n6.pdf> Accessed 25 February 2013].

Fig. 1. A schematic diagram of the vertical cross-section of GHARR-1

Fig. 2. The physical Model

Fig. 3. Open-loop block of GHARR-1

Fig. 4. Closed-loop Transfer function block of GHARR-1

Fig. 5. Bode plot of GHARR-1

Fig. 6. Nyquist plot of GHARR-1

Table 1

Kinetic Parameters of GHARR-1 [2]

Prompt neutron generation time, Λ (s)	8.1E-05
Total effective delayed fraction, (β_{eff})	8.03E-03
Decay constant, λ (s^{-1})	9.62E-02
Moderator temperature coefficient (mk $^{\circ}C^{-1}$)	$\alpha_m = 2.6445E - 05 - 3.4752E - 06T_m$
Fuel temperature coefficient (mk $^{\circ}C^{-1}$)	$\alpha_f = 7.73E - 06 - 2.90E - 07T_m + 1.09T_f^2$

Table 2

Thermal hydraulics and geometrical parameters of GHARR-1 [4]

No.	Parameter	Description
1	Core shape	Cylinder
2	Core height	23 cm
3	Core diameter	23 cm
4	Fuel element shape	Thin rod
5	Number of fuel elements	344
6	Number of dummies	6
7	Control rod	1 located at the center
8	Max. radial form factor	2.1
9	Max. axial form factor	1.57
10	Clad outer diameter	5.5 mm
11	Fuel meat diameter	4.3 mm
12	Rod pitch	10.95 mm
13	Active height of fuel element	230 mm
14	Reactor cooling mode	Natural convection
15	Coolant inlet temperature	30 oC
16	Direction of flow	Upwards
17	Inlet orifice height	6 mm
18	Outlet orifice height	7 mm
19	Pressure at inlet	1 bar
20	Clad thickness	0.6 mm

Table 3

Comparison of Relative Stability Margins

Reactor power (kW)	RESA code		Matlab	
	Gain Margin (dB)	Phase Margin (°)	Gain Margin (dB)	Phase Margin (°)
3.00	+infinite	76.87	+infinite	78.50
15.00	+infinite	59.68	+infinite	61.00
30.00	+infinite	50.00	+infinite	52.50

Table 4

Hurwitz polynomial test for variable reactor power

Coefficients	Reactor Power		
	3 kW	15 kW	30 kW
a_0	0.00784	0.00790	0.00783
a_1	0.11946	0.14906	0.16711
a_2	0.34933	0.34887	0.34836
a_3	0.01146	0.01146	0.01145
a_4	8.100e-5	8.100e-5	8.100e-5