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Depletion rate analysis of fields and regions: a methodological foundation

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Abstract:

This paper presents a comprehensive mathematical framework for depletion rate analysis and ties it to the physics of depletion. Theory was compared with empirical data from 1036 fields and a number of regions. Strong agreement between theory and practice was found, indicating that the framework is plausible. Both single fields and entire regions exhibit similar depletion rate patterns, showing the generality of the approach. The maximum depletion rates for fields were found to be well described by a Weibull distribution.

Depletion rates were also found to strongly correlate with decline rates. In particular, the depletion rate at peak was shown to be useful for predicting the future decline rate. Studies of regions indicate that a depletion rate of remaining recoverable resources in the range of 2–3% is consistent with historical experience. This agrees well with earlier “*peak oil*” forecasts and indicates that they rest on a solid scientific ground.

Key words: depletion rate analysis, oil depletion, oil production modelling, peak oil, decline curve analysis

1. Introduction

Non-renewable fossil fuels supply more than 81% of the global primary energy supply. Oil remains the single largest fuel, satisfying 33% of the world's energy needs in 2009 [1]. Given the high reliance on oil, particularly within transportation and other important sectors, it is evident that policymakers and the public need reliable forecasts of future oil supply.

The most authoritative oil forecasts are those published annually by the International Energy Agency (IEA) and the Energy Information Administration (EIA) of the US Department of Energy. Policymakers and media often assume that the IEA's Reference Scenario represents the best available knowledge of the future oil production. However, recent studies uncovered a number of errors and unrealistic parameters in their models [2, 3]. The problem primarily lies in the depletion rate, i.e. the rate at which the oil can be extracted. The IEA assumed that available oil could be extracted faster than ever seen in history; using realistic values provided significantly less optimistic production outlooks [2]. Similarly, the EIA relied on a defective analogy for depletion rates that postponed the global production peak in their models. In this case, too, using historically realistic depletion rates indicated that oil production could start to decline well before 2030 [3].

Depletion rates have been studied for a long time in various forms and the oldest known studies go back to the late 1970s [4]. Many papers, studies and forecasts have been done using depletion rate analysis since then. However, the concept is still hard to grasp for many people. Analysts have used different definitions of depletion rates, inconsistent theory or perplexing terminology. Therefore, to determine how well depletion rate theory fits with reality, this study summarizes the topic, presents the theory of depletion rate analysis with more clarity and tests it against empirical data.

1.1 Data gathering and considerations

There is certainly an issue with obtaining good and openly available data sets. This study uses the latest updated version of the Uppsala giant oilfield database, originally described in more detail by Robelius [5] and later by Höök et al. [6, 7]. It contains over 300 giant oilfields worldwide accounting for over 1100 Gb of oil in 2P-terms (proven+probable reserves as of 2005 or later). Complementary data for hundreds of oilfields in Europe, the USA, China and other parts of the world has been combined with the giant oilfield data. In total, the analysis covers 1036 fields. The Middle East/OPEC is well represented with nearly 50% of the giant fields, but data for smaller fields were rarely available and most small fields in this study are from Western or Asian countries. Regardless, the data set is believed to give a good global picture because it contains fields of various sizes and types from different production strategies and socioeconomic conditions.

There are many possible reserve measures that can be used when analysing fields. This study primarily uses available 2P data to approximate the URR of each field. Bentley et al. [8] highlights how 2P data is close to true 50% probability estimates, and just as likely to see a decrease as an increase over the lifetime of the field. In a few cases, no 2P estimates were available and more traditional curve fits were used to estimate the URR. In the aggregated dataset some fields are surely overestimated, while others are just as likely to be underestimated. However, these effects tend to cancel each other out when aggregated.

A number of regions have also been included in the dataset. This is to see whether the regional behaviour echoes the mechanisms displayed at the field level. The URR estimates of the regions have been derived by various techniques such as field-size distributions [9] or curve fitting [10]. In some cases, URR estimates were derived from Campbell and Heapes [11] and BGR [12]. Here, generally high URR estimates have been chosen to account for potential reserve growth in the future and new discoveries.

Naturally, there are intrinsic shortcomings in the studied data. Different agencies and companies may use diverse terminology and definitions may simply have changed over time. Artifacts, such as terrorist strikes or major accidents, have severely influenced some fields, especially in Nigeria, but also events like Piper Alpha in the North Sea. Fields with severely disturbed behaviour were omitted from analysis.

Some fields exhibit a clear peak, commonly quite early in the field's life, followed a decline phase. Other fields can have long, possibly ranging for decades, plateau phases followed by the onset of decline. This study focuses on fields that are mature and have undergone most development stages. Consequently, only fields that have “*peaked*” and clearly reached the onset of decline have been included in the data set. For fields with a plateau, “*peaking*” was defined as the point where production lastingly leaves a 4% fluctuation band around the plateau level, just as earlier used by Höök et al. [7].

Figure 1. The size distribution of the 1036 studied fields. Only a small number of supergiants, i.e. fields larger than 10 Gb were available.

2. Defining depletion rates and depletion level

In principle, depletion is a fairly simple concept. Production of a non-renewable resource will always lead to depletion as there only is a limited amount available for recovery and production by definition will exhaust the available resource. Virtually any type of resource may show depletion behaviour, including forests or animals — despite their “*renewable nature*.” If the annual extraction is greater than the corresponding replenishment, the resource will be subject to depletion [13]. To fully understand depletion rate analysis, a solid mathematical framework is needed. A particularly important parameter is the remaining resource. It is defined as follows:

$$R_r = R_0 - Q_t + \sum r \quad (1)$$

Where R_r = remaining resource at time t , R_0 = base year resource estimate, Q_t = cumulative production at time t , r = subsequent annual revisions of the base year estimate at time t . The resource base may be fixed (where $r = 0$) or dynamic (i.e. changing over time). Introducing a time dependent estimate of the ultimately recoverable resources (URR) or estimated ultimate recovery (EUR) allows a simplification of Equation 1:

$$R_r = URR_t - Q_t \quad (2)$$

From this, the URR may be expressed as the remaining resource plus cumulative production at an arbitrary point in time. The next step is to define some kind of measure of how much of the resource that has been depleted. This may be called a *depletion level* and is here denoted $D_{URR,t}$:

$$D_{URR,t} = \frac{Q_t}{Q_t + R_r} = \frac{Q_t}{URR_t} \quad (4)$$

The depletion level parameter can vary from 0–100%, which gives a sound assessment of how much of the URR remains for production at a given time. For example, one may think of a cup that is half full (i.e. half empty) and note that it would have a depletion level of 50%.

When it comes to defining a depletion rate one is with two choices. Should one base it on ultimate reserves or on the remaining recoverable resources? Secondly, one may also ask whether this makes a difference in practice. If q_t denotes annual production one can now define two different depletion rates. First one obtains a *depletion rate of URR*, denoted $d_{URR,t}$ (Equation 5). This parameter is useful if one has access to URR-estimates and reflects how much of the URR that is extracted annually:

$$d_{URR,t} = \frac{q_t}{Q_t + R_r} = \frac{q_t}{URR_t} \quad (5)$$

In contrast, the other choice gives a measure of how fast the remaining recoverable resources are becoming exhausted. This may be used to define a *depletion rate of remaining recoverable resources* at time t , here denoted $d_{RRR,t}$:

$$d_{RRR,t} = \frac{q_t}{R_r} = \frac{q_t}{URR_t - Q_t} \quad (6)$$

The $d_{RRR,t}$ -parameter has also been called *depletion rate of remaining reserves* and various similar things. In addition, it can also be expressed as a *production-to-reserve ratio* (P/R), since it is the reciprocal of the frequently used *reserves-to-production ratio* (R/P). Caution should be exercised not to mix values from the two different definitions. Jakobsson et al. [3] explored this further.

$$d_{RRR,t} = \frac{\text{production}}{\text{remaining recoverable resources}} = \frac{q_t}{R_r} = \frac{1}{\frac{R_r}{q_t}} \quad (7)$$

2.1 Depletion rates for regions

Any oil region, whether a single country or group of countries, consists of an arbitrary number of fields. First, we let $URR_{reg,t}$ denote the aggregated URR in a region with n fields. We also let Q_{reg} refer to the aggregated cumulative production and q_{reg} refer to the aggregated annual production in the same region at a given time. If $Q_{n,t}$ is the cumulative production of field n at time t , the depletion level of the URR of a region may now be expressed as follows:

$$D_{URR,t} = \frac{Q_{reg,t}}{URR_{reg}} = \sum_1^n \frac{Q_{n,t}}{URR_{reg,t}} \quad (8)$$

This shows that the depletion level of a region is dependent only on the current depletion level of its subparts, i.e. a weighted average of its subcomponents. The depletion rate of the URR in a region is the next thing to delineate. We let $q_{n,t}$ denote annual production of field n giving:

$$d_{URR,t} = \frac{q_{reg,t}}{URR_{reg,t}} = \sum_1^n \frac{q_{n,t}}{URR_{reg,t}} \quad (9)$$

Finally, the depletion rate of remaining reserves may be defined as follows:

$$d_{RRR,t} = \frac{q_{reg,t}}{URR_{reg,t} - Q_{reg,t}} = \sum_1^n \frac{q_{n,t}}{URR_{reg,t} - Q_{reg,t}} \quad (10)$$

It follows that regional depletion rates must lie somewhere between the smallest and largest depletion rates for individual fields, regardless of whether one uses depletion rates of the URR or remaining recoverable resources. The two principal factors that determine the regional value are:

1. The depletion rates of individual fields
2. The temporal distribution of field production profiles

The first factor is rather obvious since the regional depletion rate is a weighted average of the individual values. The temporal distribution refers to the extent to which field production profiles are clustered in time. The extreme case would be if all fields reached their maximum depletion rates simultaneously, in which case the regional value also would be at a

maximum. However, this is not likely to happen in reality as fields are usually at different stages of development at any given point in time. In contrast, more temporally dispersed fields will yield a lower regional depletion rate.

No suitable analytical formula for the temporal distribution of fields can be found. This comes from the fact that the timing of production from a particular field is determined by the year of discovery, available extraction technology, administrative barriers, macroeconomic circumstances, and other factors. Consequently, the estimation of regional depletion rates must chiefly rely on empirical experience of how regions generally develop.

3. The physics of depletion

Unlike many simple curve fitting models, like Hubbert Linearization, depletion can be connected to physics through all field development stages. This section gives a brief overview of the physics behind the depletion mechanism.

3.1 Oil reservoir fundamentals

An oilfield consists of one or several subsurface reservoirs that contain hydrocarbons. Hydrocarbons reside in the microscopic pore space of rocks, which are tiny void areas within the internal structure of the rocks. The situation is somewhat similar to a sponge soaked with water. The term *porosity* refers to the percentage of pore volume compared to the total bulk volume of a rock. High porosity means that the rock can contain more oil per volume unit. Any type of rock can be a reservoir as long as the pore space is large enough to store fluids and the pores are connected well enough to allow the oil to flow. However, sedimentary rocks such as sandstones and carbonates are the most frequently occurring reservoir rock types; sedimentary reservoirs dominate the world's known oilfields [14]. Porosities of more than 15% are deemed good or even excellent for oil reservoirs [15]. Porosity generally decreases with depth because the sediments become more compacted [16].

Oil, gas and water saturation levels are important factors and refer to the percentage of the pore volume that is occupied by oil or gas. Closer discussion of this can be found in Dake [17]. Most sedimentary rocks have their pores filled with water to some extent in normal circumstances [16]. Throughout development of the reservoir, the pore content change as production affects the reservoir. Normally this means that oil is being replaced by water.

Pores have two purposes in a reservoir. The first role is as a storage space for oil and other hydrocarbons and the second role is as a transmission network for fluid flows. Consequently, it is necessary that the pores are connected to allow movement of the hydrocarbons within the reservoir. This was first investigated by French engineer Henry Darcy [18] who studied fluid flows through a bed of packed sand. He derived a phenomenological expression to describe the behaviour and this is today known as Darcy's law (Equation 11). This law can also be directly derived from the Navier-Stokes equations [19]. The ability of a rock to permit fluid movement is called permeability, usually denoted k .

$$q = -\frac{kA}{\mu} \frac{\partial P}{\partial L} \quad (11)$$

Where q = volumetric flow rate, k = permeability, A = cross-sectional area, μ = fluid viscosity, and $\partial P/\partial L$ = pressure gradient over the length of the flow path. Permeability can differ in different directions, and generally horizontal permeability is greater than vertical permeability [16]. Good reservoirs are dependent on porosity and permeability. In general,

reservoir rocks do not seem to demonstrate a theoretical relationship between these properties, which makes empirical surveys important.

Fractures, cracks and rifts can transmit fluids well, thus partially bypassing permeability problems caused by the pore structure and this has been known to have a major influence on reservoir flows in certain reservoirs. Technology can also affect permeability. For example, fracturing techniques are used to enhance production in many Danish chalk reservoirs with low permeability [20].

3.2 Physical factors governing fluid flows

Within the oil reservoir the flow of fluids is the governing factor of the extraction process. Hydrocarbon fluids must reach the production wells to be extracted and consequently, the rock properties affecting fluid mobility will have a major influence on the amount that can be extracted and also on how fast it can be extracted. The movement of fluids in a reservoir depends on the following factors [21]:

- Depletion (leading to a decrease in reservoir pressure)
- Compressibility of the rock/fluid system
- Dissolution of the gas phase into the liquid
- Formation slope
- Capillary rise through microscopic pores
- Additional energy provided by an aquifer or gas cap
- External fluid injection
- Thermal, miscible or similar of manipulation of fluid properties

Viscous forces dominate the behaviour of fluids, both extracted and injected, in a reservoir. Under viscous conditions, flow rates are laminar and proportional to the pressure gradient that exists in the reservoir [21]. However, there are examples of tilted reservoirs and dipping formations where gravity drainage is the prime driving force. Capillary forces are a result of surface tension between the fluid phase and the pore walls, something that can form sealing conditions if the capillary entry pressure is high. Gravity and capillary forces act in the opposite directions and can be used to determine the initial distribution and saturation of oil, gas and water in any hydrocarbon-bearing porous structure [21].

In most reservoirs, more than one factor is responsible for the flow of fluids and local rock heterogeneities can greatly impact fluid flows. Some governing parameters can be affected by man-made measures while others cannot. The slope of the hydrocarbon-bearing formation is an example of a flow parameter that is fixed, while external fluid injection is dynamic and dependent on installed technology and the production strategy.

Initially, oil is recovered through the energy that a reservoir naturally contains (buoyancy energy, pressure energy, etc.), for instance via gas drive or water drive mechanisms. This is often called the primary recovery method and usually 10-30% of the oil in place can be recovered this way [22]. Major differences from field to field occur since individual reservoir properties often significantly influences recovery success.

Secondary recovery methods use injection of water and/or gas to maintain pressure thus feeding additional energy to the reservoir. About 30–50% of the oil in place can be recovered by use of primary and secondary recovery methods [22, 23]. Today, almost all oilfields suitable for secondary recovery methods are using the technique [23]. In other words, the easiest measures to increase recovery have already been taken.

Tertiary recovery methods, or enhanced oil recovery (EOR), include more complex methods such as injection of polymer solutions, surfactants, microbes, nitrogen or carbon

dioxide, that are capable of influencing rock and fluid properties. Only a small fraction of the world's oilfields are using EOR [23], which may be due to their high cost and the difficulty of implementing these advanced technologies.

3.3 Depletion-driven production decline

Oil production is subject to the natural laws that govern physical reality. More specifically, oil production is about fluid flows in the porous media that contain an oil accumulation. Depending on the number of details a flow model employs, such flows can be simulated with varying levels of complexity. However, the flow processes themselves generally exhibit complicated behaviour thus mathematical models must include statistical analysis, fractal and/or stochastic procedures [24].

Reservoir simulation models involve various numerical frameworks. One example is the ECLIPSE oil and gas simulator from Schlumberger Information Solutions [25]. Their simulator uses an implicit three dimensional finite difference approach to solve material and energy balance equations in multiphase fluid system with up to four components in a subsurface reservoir with complex geometry. Traditionally finite difference methods dominate but finite elements and streamlined numerical models are also used. Recently even more advanced computational techniques, such as neural networks and fuzzy logic [26] or algebraic multigrids [27], have been used to model reservoir flows.

Combining the reservoir flow models with drilling and development plans along with economic investment models for the field can result in reasonable descriptions of actual production and how it changes over time. However, precise prediction of fluid flows usually requires detailed data and knowledge of many important reservoir properties and parameters such as permeability, pressure and similar. Drilling plans, details around installations and development schemes are seldom openly available. In practical cases, much of the necessary data for accurate modelling is rarely available to outsiders since oil companies and producers do not release it. To mitigate this shortcoming, simplified models have been developed by various researchers and engineers. This section reviews some simple physical models and how depletion affects production flows.

3.3.1 Primary recovery

Initial pressure energy in the reservoir is the main driving force behind primary recovery. To conceptually understand how depletion affects fluid flows in such a case, a simplified example for gas entrapped in a reservoir can be considered. This allows the ideal gas law and related special cases to be used. One should also remember that behaviour of real gases deviates from ideal gases, notably at high pressures and temperatures. However, this can be handled with gas deviation factors [21].

Boyles Law, first formulated by Robert Boyle in 1662 [28], describes the inverse proportionality of the absolute pressure and volume of a gas if the temperature is kept constant within an isolated system (Equation 12). This is often applicable in reservoirs since they are reasonably isolated and in thermal equilibrium with the surrounding bedrock, resulting in constant temperature.

$$Pressure * Volume = Constant \quad (12)$$

The law can also be rewritten into a relationship between pressures and specific volumes (volume occupied by a unit of mass) before and after a certain isothermal change:

$$p_1 v_1 = p_2 v_2 \leftrightarrow p_1 \frac{V_1}{m_1} = p_2 \frac{V_2}{m_2} \quad (13)$$

Where p_1, p_2 = pressure of gas at state 1 respectively 2, and v_1, v_2 = specific volume of gas at state 1 respectively 2. Gas extraction removes mass without changing the volume of the reservoir, i.e. $V_1 = V_2$. This makes the gas particles become further apart and exert less force per unit area of the container, i.e. a lowering of the pressure to maintain the balance. From Darcy's law (Equation 11) it follows that decreasing pressure leads to decreased flow rates, if all other things are equal. Consequently, extraction of gas from a reservoir will result in declining production over time, i.e. depletion-driven decline.

For oil extraction the situation is analogous to the emptying of a pressure bottle by opening a valve. In the primary recovery stage, oil is allowed to flow out of the wells under its own pressure with nothing re-injected into the reservoir. Under such circumstances the downhole pressure (the fluid pressure at the entry to a well pipe) drops as oil is produced [29]. This steady decrease in pressure will limit production rate according to Darcy's law (Equation 11) since flows are dependent on the pressure gradient between the downhole and the surface. Hence, depletion of the recoverable oil will also unconditionally lead to a depletion-driven decline in production rates under primary recovery.

3.3.2 Secondary recovery

In oilfields of significant size, much more oil is extracted during the secondary recovery stage than during the primary recovery stage [15, 30]. Water or other fluids are injected to maintain the reservoir pressure, which gives a fairly constant downhole pressure. In fields in which the production strategy is to maintain reservoir pressure, for instance by water or gas injection, the extracted volumes of oil and water will remain relatively constant through the life of the field, in agreement with the material balance equation [21]. This recovery strategy is preferable since it recovers a greater fraction of the oil-in-place and provides more control over the production rate.

Abrams and Weiner [29] developed a simple physical model to explain the depletion-driven decline in secondary recovery stages of oil production. It assumes that the oil reservoir is a sealed container filled with incompressible oil and other fluids at a downhole pressure, P_{dh} , much greater than the atmospheric pressure, P_{atm} . For simplicity, it may be assumed that a single pipe with the cross-sectional area $A(t)$ and a small volume compared to the volume of oil extends into the reservoir and equals the total area and volume of all active production wells in a more realistic case. In addition, $A(t)$ is assumed to be a continuous function of time, downhole fluids are well mixed and the downhole pressure and volume remain constant due to reinjection. Conservation of mass would require the oil production to satisfy:

$$q(t) = u(t)A(t)x(t) \quad (14)$$

Where $q(t)$ = total volume of oil exiting the well at *time* = t , $u(t)$ = velocity of the fluid leaving the wellhead at *time* = t , $x(t)$ = oil fraction of the output at *time* = t . The wellhead pressure (fluid pressure at the exit to a well pipe) can also be expressed as:

$$P_{wh} = P_{dh} - \rho gh - P_{pf} \quad (15)$$

Where ρ = density of the fluid, g = gravitational acceleration ($\sim 9.81 \text{ m/s}^2$), h = height of the column of fluid in the pipe, P_{pf} = pressure drop due to laminar or turbulent steady-state

flow. Assuming incompressible oil at the wellhead, the principle formulated by Bernoulli [31] now allows the velocity of the fluid exiting the pipe to be expressed as:

$$u(t)^2 = \frac{2(P_{wh} - P_{atm})}{\rho} \approx \frac{2 P_{wh}}{\rho} \quad (16)$$

The wellhead pressure determines the exit velocity. Constant downhole pressure from fluid reinjection implies that:

$$x(t) = x_0 \left(1 - \frac{V}{V_{rec}}\right) \quad (17)$$

Where x_0 = initial oil fraction, V_{rec} = total volume of recoverable oil. Inserting the expressions for $u(t)$ and $x(t)$ into Equation 14 now gives a functional model for production during secondary recovery

$$q(t) = x_0 \sqrt{\frac{2P_{wh}}{\rho}} A(t) \left(1 - \frac{V}{V_{rec}}\right) \quad (18)$$

The model has three input parameters (initial oil fraction, wellhead pressure, and total volume of recoverable oil) and those are commonly available or can be estimated from publicly accessible data for oilfields. This model also shows that oil production will fall and water production will rise as increasingly more injected water begins to reach the production wells. As the reservoir depletes, the well will eventually produce too much water to be economically viable despite the fact that reservoir pressure might still be high.

In real cases, the cross-section area of active wells varies with time as new wells are added and old ones are shut down. The number of active wells is a key determinant for production. Likewise, the time-varying area $A(t)$ must be specified in the model to be able to solve Equation 18. The number of active wells (and the cross-section area) increases and typically reaches a plateau as an oilfield matures. However, the economics of diminishing returns dictates that the number of active wells must eventually decrease to zero as production tails off and wells with low production are shut down.

Abrams and Wiener explored different drilling strategies and how they influence production [29]. By assuming various sigmoid shapes functions for $A(t)$, it was found that the production peak occurs when around 50% of the recoverable oil had been extracted. Another important result from this model is that the production flow, $q(t)$, always peaks before the maximum of $A(t)$. It was also concluded that even as the number of active wells is increasing, the production rate will begin decreasing [29]. As a result, drilling activity cannot be seen as a useful indicator because an increase in the number of wells, i.e. increasing $A(t)$, does not signify that production will rise.

3.4 Depletion rates after the onset of decline

Prior to the onset of decline, the depletion rate generally increases with increasing extraction. The depletion rate behaviour in the decline phase is another question and needs to be investigated in a more detailed manner. The decline phase of an oilfield may be described by decline curves, originally introduced by Arps [32]. Decline curves are more than just simple curve fits as they are strongly connected to physics. For example, the exponential decline

curve represents the solution to the physical flow equation of a well with constant pressure [33, 34].

In decline curve models, it is assumed that the declining production starts at a given time t_0 , with initial production rate of r_0 and the initial cumulative production Q_0 . The production rate at time $t \geq t_0$ is denoted by $q(t)$ and the corresponding cumulative production at the same time is defined by the integral $Q(t) = \int_{t_0}^t q(u)du$.

The simplest decline curves are characterized by three parameters, the initial production rate $r_0 > 0$, the decline rate $\lambda > 0$ and the shape parameter $\beta \in [0,1]$. If production is allowed to continue without end and the integral $Q(t) = \int_{t_0}^t q(u)du$ converges as $t \rightarrow \infty$ it is possible to calculate the ultimate cumulative production of the decline phase, which can be summed with Q_0 to give the fields estimated URR. Other theorists have presented more general decline curves [20, 35–36]. The general case is a hyperbolic decline curve, while exponential decline is an important special case. Their key properties can be seen in Table 1.

Table 1. Key properties of hyperbolic and exponential decline curves of Arps type.

	Hyperbolic	Exponential
β	$\beta \in [0,1]$	$\beta = 0$
$q(t)$	$r_0[1 + \lambda\beta(t - t_0)]^{-1/\beta}$	$r_0 \exp(-\lambda(t - t_0))$
$Q(t)$	$Q_0 + \frac{r_0}{\lambda(1-\beta)} \left[1 - \left(1 + \lambda\beta(t - t_0) \right)^{1-\frac{1}{\beta}} \right]$	$Q_0 + \frac{r_0}{\lambda} (1 - \exp(-\lambda(t - t_0)))$
URR	$Q_0 + \frac{r_0}{\lambda(1-\beta)}$	$Q_0 + \frac{r_0}{\lambda}$

Beginning with the simple exponential case, it follows that the depletion rate of remaining resources equals the decline rate. Using expressions for production, cumulative production and URR for the exponential curve (Table 1) and inserting in the definition of d_t (Equation 6) gives:

$$d_t = \frac{r_0 \exp(-\lambda(t - t_0))}{\left(Q_0 + \frac{r_0}{\lambda}\right) - \left(Q_0 + \frac{r_0}{\lambda} (1 - \exp(-\lambda(t - t_0)))\right)} \quad (19)$$

This can now be simplified into:

$$d_t = \frac{\lambda * \exp(-\lambda(t - t_0))}{\exp(-\lambda(t - t_0))} = \lambda \quad (20)$$

Consequently, knowing the value of d_t before the onset of decline makes it possible to estimate the decline rate if the field will be following an exponential decline curve.

If one uses the definition of the depletion rate of URR (Equation 5), one obtains a more complicated expression that cannot be additionally simplified:

$$d_{URR,t} = \frac{r_0 \exp(-\lambda(t - t_0))}{\left(Q_0 + \frac{r_0}{\lambda}\right)} \quad (21)$$

Since r_0 , Q_0 and λ all are constants and the exponential expression is strictly decreasing, it follows that the depletion rate of the URR will decrease as the oilfield enters the decline phase. It also holds that the maximum value of $d_{URR,t}$ must occur before the field reaches the onset of decline.

It should also be mentioned that exponential behaviour is a simplified form of decline and that the general situation tends to be hyperbolic in nature. This makes it interesting to study how two depletion rates behave in a hyperbolic decline situation.

Once again, the definition of d_t (Equation 6) and the functions for production, URR and cumulative production in the hyperbolic case (Table 1) yield:

$$d_t = \frac{r_0[1 + \lambda\beta(t - t_0)]^{-1/\beta}}{\left(Q_0 + \frac{r_0}{\lambda(1 - \beta)}\right) - \left(Q_0 + \frac{r_0[1 - (1 + \lambda\beta(t - t_0))^{1-1/\beta}]}{\lambda(1 - \beta)}\right)} \quad (22)$$

Simplifying this gives an expression that is decreasing with time:

$$d_t = \frac{\lambda(1 - \beta)}{1 + \lambda\beta(t - t_0)} \quad (23)$$

In contrast, the definition of $d_{URR,t}$ (Equation 5) and the functions for production, URR and cumulative production in the hyperbolic case (Table 1) give:

$$d_{URR,t} = \frac{r_0[1 + \lambda\beta(t - t_0)]^{-1/\beta}}{\left(Q_0 + \frac{r_0}{\lambda(1 - \beta)}\right)} = \frac{1}{\left(Q_0 + \frac{r_0}{\lambda(1 - \beta)}\right)(r_0[1 + \lambda\beta(t - t_0)]^{1/\beta})} \quad (24)$$

One should note that $\beta \in [0,1]$, making the power generally negative. All together, the expression for $d_{URR,t}$ will also be strictly decreasing with time as $t \rightarrow \infty$.

The depletion rates of both remaining resources and ultimate reserves in the general hyperbolic case have very interesting properties, as they imply that a *maximum depletion rate* must be reached before the field reaches the onset of decline in the general case. Typically the maximum depletion rate occurs just before the onset of decline. It thus follows that the existence of a maximum depletion rate for individual fields indicates the existence of a maximum depletion rate on a regional or global scale.

3.5 The core of depletion rate analysis

Depletion-driven decline is a fundamental property of oil production derived from physics. It occurs when the recoverable resources become exhausted and production flow is affected. At some point, the production flows will simply diminish due to physical limitations. Depletion-driven decline is different from other forms of decline and much harder alleviate, since it can be eased only by expanding the recoverable reserves of the reservoir, which will ultimately be limited by the physical extent of the formation or other geological parameters.

There is no theoretical limit to the depletion rate of an individual field or region. In theory, it would be possible to extract all the recoverable oil in a single year, but such a production strategy would hardly be feasible as it would more or less imply that the producers had to dig up the entire field to obtain all the oil in place — which is hardly profitable. Observed depletion rates are largely a result of various development strategies and limitations caused by investments or technology. In reality, one faces a number of tradeoffs

concerning economics, technology and geology parameters to find the optimal development strategy that maximizes oil recovery and/or return on investments, and different operators often optimize differently.

The advantage of depletion rate analysis comes from an underlying connection to physics and that all extraction is subjected to the physical laws that govern fluid movements in the reservoir. However, it is also dependent on installed technology, investments and other factors to some extent. In practice, it may be seen as a way to cluster variables aimed at reducing the number of unknown parameters in a field or region production behaviour. It cannot be seen as a replacement for the detailed and intricate reservoir simulators. It's best to view depletion rate analysis as a complimentary method that makes well-substantiated but rougher estimates.

Earlier studies have indicated that only a relatively narrow band of depletion rates are plausible at the onset of decline in giant oilfields [6]. This leads to the question about a maximum depletion rate and whether it can be useful for modelling. This concept is relatively simple and implies that at certain depletion rates, the depletion-driven decline caused by the extraction of the recoverable oil will begin to dominate over other production factors. At this point, the production of the entire field is forced into a stage of declining production. The connection between depletion rate and decline curve models also makes it possible to make an astute estimate of the actual decline rate. Also here, empirical data has indicated a strong linear correlation between high depletion rates and rapid declines [6, 20]. The author believes that further investigation of empirical data is necessary to establish a range for reasonable depletion rates on both field and region levels.

4. Empirical study of depletion rates

It is essential to have a good database to better understand depletion rate behaviour. The dataset used in this study has a broad distribution of field sizes, development strategies, geology and economic conditions. The number of fields is also large, which decreases the importance of individual oilfields and better exposes the typical patterns within this collection. For the sake of completeness, a number of regions have also been included. The regions are also very diverse in terms of economic and political conditions, as well as geology and size. Ultimate reserves of regions are a highly debated topic and such estimates can be done using many different methodologies [10]. First and foremost, one can see that there is a solid power relation between size and peak production level (Figure 2) in the data set. This stretches over several magnitudes and appears valid for both individual fields (ranging from tiny to supergiant) as well as regions of small to major size.

Figure 2. *The relation between field size and maximum production level.*

4.1 Depletion rate behaviour in oilfields

In this section we present the depletion rate of several fields. Official estimates of the ultimate recovery for these fields were taken from operators and relevant agencies. The methodology they used to derive the official numbers was not always clear, so decline curve analysis was also used to provide an alternative URR estimate. Figures 3–6 show that the maximum depletion rate, regardless of whether we use remaining recoverable resources (RRRD) or URR (URRD) to define the depletion rate, occurs at the peak production point or at the end of plateau and remains reasonably stable after that or decreases. This is in good agreement with the theoretical framework presented earlier.

The depletion rates and decline rates for 15 selected fields can be found in Table 2. The maximum depletion rates in this sample of fields are very different. The Norne and Claymore fields are approximately equally large but greatly differ in peak production

volumes. The corresponding average decline rates after the peak is 7.2% for Claymore and 16.1% for Norne, which is almost exactly the same as their depletion rates of remaining recoverable resources prior to peaking. The Daqing field declines at 3.1% while the remaining recoverable resources were depleted at 3.3% at the onset of decline. In contrast, the Maui oilfield declined at 17% and was depleted at around 20%. It appears that depletion rates could indeed be a vital pointer to the future average decline rate and this has already been used to forecast production for fields that have not reached the onset of decline yet [37].

In a greater perspective, all the 1036 analysed fields showed strong correlations between depletion rates at peak and the average post-peak decline rate. The linear correlation coefficient with mean decline rate was determined to be 0.79 for depletion rates of remaining recoverable resources and 0.77 for URR-depletion rates at peak, indicating the presence of a strong correlation. Consequently, accurate size estimations and depletion rate analysis can be a reasonably good determinant for future decline rate.

Table 2: *Depletion rates of remaining recoverable resources at peak (RRR-DAP) for selected fields in the database, along with the peak year, peak production levels, depletion level and average decline rate and the URR values used for calculations.*

Field	Country	Peak Year	Peak Prod. [kb/d]	RRR-DAP [%]	Dep. Lvl [%]	Decline rate [%]	Estimated URR [Gb]
Ankleshwar	India	1996	98	6.0	21.3	6.1	0.53
Beaulieu	UK	2001	10	25.3	25.3	30.9	0.02
Bonnie Glen	Canada	1976	85	10.5	49.5	16.0	0.53
Curlew	UK	1999	32	17.4	29.6	21	0.08
Emeraude	Congo	1982	35	8.4	46.2	6.7	0.26
Hankensbuttel	Germany	1992	11	6.7	50.0	7.1	0.12
Hod	Norway	1991	26	15.7	17.0	15.3	0.06
McKee	New Zealand	1989	11	11.5	37.7	12.9	0.05
Minas	Indonesia	1973	418	3.4	27.8	4.0	6.00
Prudhoe Bay	USA	1998	1533	7.5	46.7	8.1	13.0
Snorre	Norway	2003	234	10.0	48.9	10.8	1.50
Sporysheskoye	Russia	2003	114	14.3	32.2	15.5	0.37
Trap Springs	USA	1990	3	7.2	46.2	7.8	0.02
Tyra	Denmark	1994	30	8.7	36.1	7.8	0.18
Zhongyuan	China	1988	130	6.8	29.3	5.2	1.00

Figure 3. Production and depletion rate behaviour in the Norne giant oilfield in Norway. The official URR estimate (0.60 Gb) was taken from the Norwegian Petroleum Directorate fact pages, while the estimated URR (0.67 Gb) was obtained using decline curve fits.

Figure 4. Production and depletion rate behaviour in the Claymore giant oilfield in the UK. The official URR estimate (0.66 Gb) was taken from the operator Talisman [38], while the estimated URR (0.73 Gb) was obtained using decline curve fits omitting the disruptions following the Piper Alpha incident.

Figure 5. Production and depletion rate behaviour in the Daqing supergiant oilfield in China. The official URR estimate (23.23 Gb) was taken from Tang et al. [39], while the estimated URR (24.4 Gb) was obtained using decline curve fits.

Figure 6. Production and depletion rate behaviour in the Maui oilfield in New Zealand. The official URR estimate (163 Mb) was taken from Crown Minerals [40], while the estimated URR (191 Mb) was obtained using decline curve fits.

In Figure 6, we can see that the depletion rate of remaining recoverable resources for the Maui oilfield behaves strangely and spikes dramatically after the peak production point. This may be seen as an indication that the official reserves are underestimated and that the field likely will be able to produce more than 163 Mb. The URR estimate from a decline

curve fit gives more consistent depletion rate behaviour and may be seen as more realistic. Consequently, depletion rate behaviour can also be used to expose faulty reserve estimates. Unreasonably high depletion rates may indicate underestimations of the recoverable resources. In the same way, very low depletion rates at peak could imply that the field is overestimated (or possibly subjected to a cautious production strategy).

4.1.1 Probability distributions

Sadly, there is no theoretical way to derive the maximum depletion rate of an oilfield. The only way to obtain plausible maximum depletion rates is by empirical studies on real fields. Further analysis on the maximum depletion rate of remaining recoverable resources for just giant fields has been done by Höök et al. [6]. Figure 7 and 8 indicate that the maximum depletion rate, i.e. the depletion rate at peak or end of plateau, is dependent on size. Qualitatively, this can be explained by sensible development strategies for smaller fields as it would make little economic sense to install too much extraction equipment on a small field. For practical reasons, a stable and more lasting production profile is often preferable for producers to maximize return on investment.

Figure 7. The RRR depletion rate decreases with increasing size for both single fields and entire regions. One should also note that there is a significant spread and that the simple power fit is there only to depict the general trend.

Figure 8. The depletion rate of URR generally decreases with increasing size both for individual fields and entire regions.

Figure 9. Distribution of depletion rates of remaining recoverable resources together with a 3 parameter Weibull fit. The parameters were $\alpha = 1.67$, $\beta = 0.13$, $\gamma = 0.01$.

Figure 10. Distribution of depletion rates of URR together with a 3 parameter Weibull fit. The parameters were $\alpha = 1.41$, $\beta = 0.11$, $\gamma = 0.01$.

Quantitatively, none of the fields studied managed to reach a depletion rate of over 40%. Given the significant spread in the data, it is not trivial to define an average depletion rate at peak or plateau for this dataset. Such attempts have already been made for giant fields [6]. High depletion rates (over 20%) were only seen for either very small fields or those located offshore.

Closer studies on the distribution of the depletion rates for single oilfields seen in this dataset indicate that they seem to originate from a 3-parameter Weibull distribution (Figure 9–10). Kolmogorov-Smirnov, Anderson-Darling [41] and Chi-Square tests all showed that the Weibull distribution agreed better than other common distributions, including normal, lognormal, gamma, and exponential distributions. Typically log-logistic or lognormal distributions were second after Weibull in terms of goodness-of-fit.

The depletion rate of remaining recoverable resources at peak using the Weibull distribution gives a mean value of 13.2%, a mode of 9.0% and a standard deviation of 7.4%. The depletion rate of the URR at peak has a mean value of 10.8%, a mode of 5.5% and a standard deviation of 7.0%.

In principle, it would be possible to constrain the upper bound of the distribution to unity, since it is impossible to deplete more than 100% of the recoverable volumes. If such limits are used, the bounded Johnson distribution described by Slifker and Shapiro [42] will provide the best fit. However, the Weibull distribution is more widely known and probably easier to handle. Regardless of the exact nature of the statistical distribution found, it indicates that the high depletion rates will be rare events due to the rapidly declining probability as one gets further out in the tail region.

Interestingly, Weibull distributions also fit well to both the depletion level (Figure 11) and average post-peak decline rate (Figure 12). This consistency between frameworks indicates that this type of depletion rate analysis is theoretically solid.

Figure 11. Distribution of depletion levels together with a 2 parameter Weibull fit. The parameters were $\alpha = 1.9$, $\beta = 0.35$.

Figure 12. Distribution of decline rates together with a 2 parameter Weibull fit. The parameters were $\alpha = 1.44$, $\beta = 0.17$.

4.2 Depletion rates of regions

Regional depletion rate behaviour is also studied empirically. This has already been done by others, in particular Colin Campbell, who spearheaded the use of depletion rates of remaining recoverable resources in regional modelling in earlier studies (for example [11, 43-46]).

The depletion rate behaviour of both the North Sea and the USA can be seen in Figures 13 and 14. Despite the fact that these regions contain hundreds of individual oilfields, they exhibit behaviour similar to their subparts. The maximum depletion rate more or less coincides with the maximum production levels and will remain rather constant or decrease as production falls from the peak or plateau.

Figures 7 and 8 also show that smaller regions typically have larger depletion rates than larger regions. This is true regardless of how the depletion rate was defined. Naturally, scale (i.e. the size of a region) may become an obstacle to rapid depletion. Lack of knowledge and logistics tend to make it impossible to develop all subparts simultaneously. Typically, once frontier parts come on stream the firstly developed areas are already at the onset of decline. For the USA, new oilfields in Alaska or the Gulf of Mexico were discovered and developed too late to do anything other than slightly mitigate the decline from older oil regions within the Lower 48 states.

Among the studied region, countries with predominantly offshore production, such as Denmark, Norway, Mexico or the UK, were found to have the highest depletion rates. In contrast, regions with mostly land-based production tend to deplete their oil more slowly. This is as expected from the different financial conditions for on- and offshore developments.

Figure 13. Depletion rate behaviour of the North Sea based on an assumed URR of 80 Gb and production data from BP (2011). The maximum depletion rate of remaining recoverable resources is slightly more than 5%.

The regional dataset is less comprehensive than for individual oilfields, and regional URR estimates are more controversial. Consequently, calculating a statistical distribution and a reliable average value is challenging and probably less useful. Instead, depletion rate parameters for selected countries are shown in Table 2. Most of the world oil production comes from land-based sites and it is only reasonable to assume that the world will behave more like land-based regions. Therefore, the depletion rate of remaining recoverable resources at peak or plateau for the world can be expected to be around 2–3%. This is also in agreement with the numbers frequently used by Campbell [11, 43–46]. Using USGS data, Sorrell et al. [47] also found similar depletion rate values for regions and the world. Naturally, these values should not be seen as carved in stone and it is entirely possible that the entire world will peak at higher depletion rates that previously seen. However, this

requires deviation from the historical pattern. Those who argue that this will be the case must properly justify such statements and be prepared to carry the burden of proof.

From Table 3, one can also see that it is far more common to reach a production peak well before half of the ultimate reserves have been depleted. Only small and/or offshore regions come near a depletion level of 50%. This can also explain why regional production profiles tend to be asymmetric with a less rapid descent compared to the ascent.

Figure 14. Depletion rate behaviour of the USA based on an assumed URR of 300 Gb and production data from EIA (2011). The maximum depletion rate of remaining recoverable resources is only 2% here.

Table 3: Depletion rates at peak for selected countries and regions, along with the peak years, peak production levels and estimated URR values used in calculations.

Country	Peak Year	Peak Prod. [Mb/d]	RRR-DAP [%]	URR-DAP [%]	Depletion level [%]	Average decline [%]	Estimated URR [Gb]
Argentina	1998	0.89	2.5	1.6	37.0	2.6	20
Australia	2000	0.81	4.1	2.3	46.6	3.4	13
Austria	1955	0.07	3.0	2.6	17.9	1.9	1
Brunei	1979	0.26	2.2	1.6	27.8	2.3	6
Denmark	2004	0.39	7.0	4.1	46.3	7.2	3.5
Egypt	1993	0.94	2.4	1.6	30.0	1.4	20
Germany	1967	0.16	3.2	2.4	28.3	2.2	2.5
Indonesia	1991	1.52	3.4	1.6	54.9	3.5	35
Mexico	2004	5.5	5.2	2.3	46.6	4.2	60
North Sea	2000	6.4	5.0	2.9	44.3	5.2	80
Norway	2001	3.4	6.1	3.6	45.4	5.1	35
Romania	1977	0.31	1.7	1.0	38.5	3.7	11
Syria	1995	0.60	2.8	2.2	24.3	2.8	10
Tunisia	1986	0.11	1.2	1.0	17.5	0.9	4
UK	1999	2.9	6.9	3.0	53.6	6.8	35
USA	1970	9.6	1.7	1.2	31.5	1.4	300
Yemen	2002	0.46	4.9	3.3	34.5	6.6	5

5. Concluding discussion

Depletion rate analysis has been around a long time, although its underlying methodology has never been clearly presented. Confusion seems to surround the depletion rate methodology even though it is a simple concept once properly understood. This paper has hopefully helped to bring clarity and consistency while also establishing a rigorous mathematical framework.

Studying depletion rates is useful for understanding the production behaviour and what one may expect from the future. However, depletion rate analysis should not be used as a substitute for meticulous and highly accurate forecasting based on traditional techniques and methodologies. Instead it offers the possibility of making reasonable estimates about future production and test the accuracy of certain projections. Looking at analogies and studying the depletion rate behaviour of fields and regions of similar size is often a good start. However, differences within the reservoir or other local conditions can severely affect the production behaviour. An analyst must use his own judgment when applying this method.

The agreement between empirical data and the theoretical results is deemed good, indicating the soundness of the framework. Depletion rate analysis was also shown to be able to rule out unreasonable estimates on the ultimate recovery and predict the average decline rate prior to the onset of decline. High depletion rates were found to strongly correlate with high decline rates. The presence of a maximum depletion rate was shown both from a theoretical perspective as well as with empirical observations. A Weibull distribution was also found to describe the empirical reality with good fidelity and may be used to derive mean values and the most frequently occurring depletion rates.

For regions, similar behaviour was observed as exhibited by individual fields. The highest depletion rates were seen in predominantly offshore regions, while regions with land-based production depleted their oil at a lower rate. Based on analogies with previously peaked regions, one may expect that the maximum depletion rate of the world will be somewhere in the order of 2–3% based on remaining recoverable resources. This is in agreement with the numbers used by Campbell [11, 43–46]. Typical depletion levels seen at peak production were found to be significantly less than 50%. This implies that the maximum production levels typically are seen well before half of the recoverable oil has been extracted. Due to the large number of influencing factors (geologic, technological and economic) there is no universal maximum depletion rate that is applicable to all fields or all regions.

For example, Radetzki [48] harshly dismissed peak oil based on qualitative statements. Both Chapman [49] and Brecha [50] counter this standpoint by pinpointing that the critics of peak oil frequently use unreliable reserve data, optimistic assumptions, myths, vociferous denials and ideological polemicizing. In essence, the debate has been shifted away from the actual evidence for oil depletion and the reasonable production trajectories that may be expected. Empirical studies are important for the peak oil debate to render the debate scientific again.

The data presented in this study displays what kind of depletion rates one can find in practice and may expect from the future. Claims that future production will behave radically different (i.e. significantly higher depletion rates or levels, low decline rates, etc.) carries the burden of proof. For example, the IEA presented optimistic production outlooks that required depletion rates up to three times higher than the most extreme historical example without any closer explanation [2]. Proper conduct necessitates openness with data, assumptions as well as proper supporting evidence for quantitative claims about the future. Sadly, too much of the peak oil debate has been dominated by beliefs and astute rhetoric rather than hard facts. Oil depletion should be identified as part of a complex energy situation with the realisation that existing oil resources are continuously depleted with consequences and limitations for future production.

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