Mean-Reverting Stochastic Models for the Electricity Spot Market

Viktor Edward

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Handledare och examinator: Johan Tysk
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Abstract

The liberalization of the electricity market has led to higher volatility and occasional price spikes. To protect both buyers and sellers, contracts are needed which requires good understanding of the underlying unit, the daily spot price.

A stochastic model of the spot price is suggested which is the exponential of the sum of a deterministic seasonal function, an Ornstein-Uhlenbeck stochastic process and a spike process. This model is suggested after observing strong seasonality with some mean-reverting stochastic behaviour and occasional spikes in historical data.

Calibrating the model is non-trivial since one function and two stochastic processes are estimated out of one data-set and a method for this is suggested. First the seasonal function is estimated, the Ornstein-Uhlenbeck process is then estimated from the deseasonalized data and then the spikes are estimated from extreme outliners.

Using the calibrated model the future spot price is predicated using a Monte Carlo method and some forward contracts are priced.
1 Introduction

In the early 1990s some countries started to liberalize their electricity markets. From being operated by the government where the price was set to reflect the marginal cost of production, the market changed to be run by the free market principle of supply and demand. The market became much more volatile compared to before which created a demand for contracts to protect from high prices but sacrificing opportunity to profit from low ones. Households do not buy their electricity from markets but big companies do. This thesis focuses on the NordPool market funded in Norway 1991.

Since the liberalization of the market started just about 20 years ago the field is quite new. But the concepts of options and derivatives are well researched by now in finance and the idea is to start off by seeing what theory can be recycled and the goal is to construct a stochastic model of the spot price. This thesis uses the model proposed by Tino Kluge [1] (University of Oxford) in his doctoral thesis from 2006 and this thesis is mainly written based on Kluge’s work and some theory from Björk [2]. The spot price market is modelled after a simple mean-reverting process with spikes and a seasonal component. Spikes usually happen when the demand is getting close to the maximum supply of the market and are unique for the electricity market.

This thesis starts off with a technical description of the Nordpool market in Chapter 2. In Chapter 3 a model of the spot price is proposed and calibrated after historical market data, which is the main topic of the thesis. Then some applications of the model are provided in Chapter 4 and future prices are predicted. Finally in Chapter 5 conclusions are drawn and future work suggested.

2 Electricity markets

The first goal of this chapter is to look at the differences between the electricity market and the stock market to determine what known theory from the well researched financial area can be recycled. Secondly a more firm description of the electricity spot and derivative market is given.
The assumption of no arbitrage is just like in the financial market the fundamental principle which all electricity market options will be based upon.

**Underlying unit:** The underlying are simply "units of electricity" and are mostly bought in units of 1 MWH.

**Production and Consumption:** In the share market the amount of shares remains constant (unless new shares are issued) which is not the case in the electrical market. In the electrical market "shares" are consumed when bought since storing electricity is not possible (maybe hypothetically but not practically). Based on microeconomic theories the long-term price should move towards the production cost. This is why mean-reverting models are mainly used to model the electricity price since the production cost doesn’t change much while in the share market some growth is assumed.

**Inability to store:** Electricity is considered a non-storable unit, also called a pure flow variable (measured in energy per time).

The inability to store electricity makes hedging not possible which leads to the market being automatically incomplete and independent of the stochastic process used to model the underlying. In other words the risk neutral probability measure $Q$ is not unique.

Since production and consumption have to be in balance all the time, to avoid power black outs, the price will experience seasonalities. In the event of a power plant failure the maximum supply could drop below the demand of the market and price spikes will occur. Due to this, supply and demand models are common but won’t be used in this thesis. Instead the stochastic model used will have a spike process included.

### 2.1 The spot market

NordPool was funded in Norway 1991 when the Parliament of Norway decided to deregulate the power trading market. In 1996 Sweden joined the market and thus NordPool became the world’s first international power exchange market. Eventually more countries also became a part of the NordPool market, Finland (1998), Denmark (Western in 1999, eastern 2000) and
in 2002 Nord Pool Spot were established. Nord Pool Spot also includes Estonia, Latvia and Lithuania. Since 2010 Nord Pool Spot is responsible for the day-ahead market (the spot market) and NASDAQ OMX for the forward contract market. More than 70% of all electrical energy in the Nordic countries are traded through NordPool as of today.

To be able to deliver the energy, transmission grids are provided by NordPool. These transmission grids are monopolistically operated and the prices are set to reflect the maintenance cost and the energy transportation loss. In Sweden Svenska Kraftnät, which is owned by the Swedish government, operates the transmission grid. Svenska Kraftnät originally owned 50% of NordPool and today own 20% of Nord Pool Spot. The transmission grid operating companies are responsible for ensuring that the same amount energy that is sold is delivered to the buyer and will have to buy the lost energy from the spot market to compensate.

To be able to sell energy on the NordPool market a bond will have to be pledged to a security bank account. The bidding areas are divided into several regions, e.g. Sweden consists of four different regions, Luleå, Sundsvall, Stockholm and Malmö. The power companies will daily submit their hourly prices and quantity to be sold to NordPool for the upcoming day. NordPool then sets the hourly spot price for the following day using the supply-demand principle.

The daily averages, i.e. base load contracts are usually the underlying product in contracts. Even though the spot price varies a lot during the day, the daily average is mostly the underlying product for contracts. The maximum price of the day can be as high as twice of the minimum. The contracts sold on the Nasdaq OMX market are of the duration of a week, month, quarter or year and the delivery period is as long as four years for the yearlong contracts.

Due to the market being incomplete and hedging not possible one could think the market would be more risky for sellers compared to the stock market. But since producers produce electricity at an almost known future cost the risk is almost eliminated in exchange for the possibility for high profits with contracts.
In 2008 hydro power were 98.5% of Norway’s and 45% of Sweden’s energy production. This leads to there being a huge dependence on the weather. The model proposed however is solely based upon data without trying to understand the mechanics behind the price.

In Figure 1 the daily spot market price can be seen from 1 January 2012 to May 2014. First of all the spot price seems to lack any positive drift, so using a geometric Brownian motion (commonly used in the financial market) for modelling the spot price is not feasible. Instead a mean-reverting behaviour is shown and this property will be the base for the stochastic process used. A strong yearly seasonality is shown, which could be expected in cold Nordic countries where there are a high demand for heating during the winter. From Figure 2 a strong weekly seasonality is seen where the price goes down during weekends and peak during the working days, e.g. day 596 is a Sunday. Several spikes can also be seen, both positive and even some negative ones, in this thesis' model the negative ones will be ignored. The positive spikes are mostly explained by some power plant temporary being out of function and the supply and demand principle. Notice how fast the price during spikes reverts back to normal levels.
Figure 2: The daily spot price starting at first of January 2012 zoomed in to easier examine the weekly seasonality.

3 Spot price model and parameter estimation

There are two main approaches to model a stochastic process. The first one is trying to understand the underlying mechanics, how has the weather been, have any new laws been implemented etcetera. The other one, which will be used, is to simply look at the available data and try to fit a mathematical model to it.

Considering the observations made in the last chapter a natural model of the daily spot-price market would be of the form

\[
dX_t = -\alpha X_t dt + \sigma dW_t \\
S_t = \exp(f(t) + X_t)
\]  

(1)

where \( f(t) \) is a seasonal term and \( X_t \) an Ornstein-Uhlenbeck process. However this model won’t be able to deal with the spikes observed in figure 1. A
spike process will be added on the form suggested by Kluge and the model used will be of the form

\[
\begin{align*}
    dX_t &= -\alpha X_t dt + \sigma dW_t \\
    dY_t &= -\beta Y_t dt + J_t dN_t \\
    S_t &= \exp(f(t) + X_t + Y_t)
\end{align*}
\]

(2)

where \( J_t \) is an i.i.d. process representing the jump size, \( N_t \) the time in between the jumps and \( \beta \) a parameter describing how fast the spikes revert back to the normal level. \( W_t, N_t \) and \( J_t \) are assumed to be independent. For simplicity and due to only assuming positive spikes the jump sizes \( J_t \) will be assumed to be exponential and the jump frequency \( N_t \) to be a Poisson process.

Estimating \( f(t), X_t \) and \( Y_t \) from just knowing historical spot price values \( S_t \) leads to the estimation being non-trivial. However it feels natural to estimate the deterministic seasonal component first and by removing the seasonal component of \( S_t \) try to estimate the Ornstein-Uhlenbeck process. Finally using the estimate of \( X_t \) one could identify extreme outliers and use these to approximate spikes.

### 3.1 The seasonal part

For the seasonal part some assumptions of the function \( f(t) \) needs to be made. From the observations in chapter two there were strong weekly and yearly seasonality and the function is defined to be of the form

\[
f(t) = c + \sum_{i=1}^{6} a_i \cos(2\pi \gamma_i t) + b_i \sin(2\pi \gamma_i t),
\]

(3)

where \( \gamma_1 = 1, \gamma_2 = 2, \gamma_3 = 4, \gamma_4 = 365/7, \gamma_5 = 2\times365/7 \) and \( \gamma_6 = 4\times365/7 \) to represent weekly and yearly seasonality. To estimate the parameters the model is assumed to be of the form

\[
S_t = \exp(f(t))
\]

and a least-square method is used by minimizing

\[
\sum (\ln(S_t) - f(t))^2.
\]
Due to several spikes in the price an algorithm excluding outliers may be preferred and can be seen in Figure 3.

3.2 The Ornstein-Uhlenbeck process

Since an estimate of the seasonal function $f(t)$ is acquired the right hand side of
\[
\ln(S_t) - f(t) = X_t + Y_t.
\]
(4)
is left to be estimated. The idea is to ignore the spike process $Y_t$ for now and estimate the Ornstein-Uhlenbeck process $X_t$. If $X_t$ is estimated one can find the extreme outliers and use these to estimate $Y_t$. The deseasonalized-log values can be seen in Figure 4.
Figure 4: The log values of the spot price with seasonality removed.

**Definition 3.1** (Ornstein-Uhlenbeck process). A zero-mean Ornstein-Uhlenbeck process is a stochastic process satisfying the following Stochastic Differential Equation (SDE):

\[ dX_t = -\alpha X_t dt + \sigma dW_t, \quad (5) \]

where \( \alpha \) is the mean-reversion rate, \( \sigma \) the volatility and \( W_t \) is a standard Brownian motion.

To be able to estimate the parameters \( \alpha \) and \( \sigma \) one first need to know the solution of (5).

**Theorem 3.1** (Ito’s Lemma). For an Ito drift-diffusion process

\[ dX_t = \mu_t dt + \sigma_t dW_t \]

a twice differentiable function \( f(t, x) \) has the differential

\[ df(X_t, t) = \left( \frac{\partial f}{\partial t} + \mu_t \frac{\partial f}{\partial x} + \frac{1}{2} \sigma_t^2 \frac{\partial^2 f}{\partial x^2} \right) dt + \sigma_t \frac{\partial f}{\partial x} dW_t \]

10
Theorem 3.2. The solution to the SDE (5) defining the zero-mean Ornstein-Uhlenbeck process is given by

\[ X_t = X_0 e^{-\alpha t} + \int_0^t \sigma e^{\alpha(s-t)} dW_s, \]  

for \( 0 \leq s \leq t \).

Proof. By applying Ito’s lemma to the function \( f(X_t, t) = X_t e^{\alpha t} \) one get

\[ df(X_t, t) = \alpha X_t e^{\alpha t} dt + \sigma X_t e^{\alpha t} dW_t \]

assuming \( \mu = 0 \). Integration from 0 to \( t \) then gives

\[ X_t e^{\alpha t} = X_0 + \int_0^t \sigma e^{-\alpha t} dW_s \]

and hence

\[ X_t = X_0 e^{-\alpha t} + \int_0^t \sigma e^{\alpha(s-t)} dW_s. \]

The solution of the OU-process is later on used for simulations of the model. Using Theorem 3.2 and the following theorem about Ito integrals one can find a conditional distribution of change for an OU-process.

Theorem 3.3. [Ito integral] The integral

\[ \int_0^t f(t) dW_t \]

where \( W_t \) is a standard Brownian motion has expected value 0 and the variance

\[ \int_0^t f(t)^2 dt. \]
Proof. The definition of an Ito integral is
\[ I := \int_0^t f(t) dW_t = \sum_{\Delta t \to 0} f(t_i)(W_{t_{i+1}} - W_{t_i}). \] (7)

Brownian motion has the property that increments in time are independent with the expected value zero and the variance $\Delta t$. So the expected value of (7) is given by

\[ \mathbb{E}I = \mathbb{E} \sum_{\Delta t \to 0} f(t_i)(W_{t_{i+1}} - W_{t_i}) = \sum_{\Delta t \to 0} f(t_i)\mathbb{E}(W_{t_{i+1}} - W_{t_i}) = 0. \]

The variance is given by

\[ \mathbb{E}(I^2) - (\mathbb{E}I)^2 = \mathbb{E}(I^2) = \mathbb{E} \sum_{i,j} f(t_i)^2(W_{t_{i+1}} - W_{t_i})(W_{t_{j+1}} - W_{t_j}) = \sum_{\Delta t \to 0} f(t_i)^2\mathbb{E}(W_{t_{i+1}} - W_{t_i})^2 = \sum_{\Delta t \to 0} f(t_i)^2\Delta t = \int_0^t f(t)^2 dt. \]

Using that Brownian motion increments are independent with variance $t$. \hfill \Box

Theorem 3.4 (Conditional distribution of change for an OU-process). Given a zero-mean OU-process of the (5) the conditional distribution of change is given by

\[ X_{t+\Delta t} - X_t e^{-\alpha \Delta t} \sim N\left(0, \frac{\sigma^2}{2\alpha} (1 - e^{-2\alpha \Delta t})\right). \] (8)

Proof. From (6) the following holds

\[ X_{t+\Delta t} - X_t e^{-\alpha \Delta t} = \sigma e^{-\alpha \Delta t} \int_0^{t+\Delta t} e^{\alpha(s-t)} dW_s - \sigma e^{-\alpha \Delta t} \int_0^t e^{\alpha(s-t)} dW_s. \]

The result is then acquired from Theorem 3.3. \hfill \Box

From knowing the conditional distribution of change for the OU-process a Maximum likelihood estimate can be derived.
Theorem 3.5. likelihood for an OU-process] Given a set of observations $(S_0, S_1, ..., S_n)$, the MLE estimation for a zero-mean Ornstein-Uhlenbeck process of the form
\[dX_t = -\alpha X_t dt + \sigma dW_t\]
is given by
\[
\alpha = -\frac{1}{\delta} \log \left( \frac{S_{xy}}{S_{xx}} \right)
\]
\[
\hat{\sigma}^2 = \frac{1}{n} \left( S_{yy} - 2e^{-\alpha \delta} S_{xy} + e^{-2\alpha \delta} S_{xx} \right)
\]
\[
\sigma^2 = \hat{\sigma}^2 \frac{2\alpha}{1 - e^{-2\alpha \delta}}
\]
where $\delta$ is the time step and

\[S_{xx} = \sum_{i=1}^{n} S_{i-1}^2\]
\[S_{xy} = \sum_{i=1}^{n} S_{i-1} S_i\]
\[S_{yy} = \sum_{i=1}^{n} S_i^2.\]

Proof. A short version of the proof will be given. The conditional density function can be derived using the method used in Theorem 3.4 and gives
\[
f(S_{i+1}|S_i; \alpha, \hat{\sigma}) = \frac{1}{\sqrt{2\pi \hat{\sigma}^2}} \exp \left[ -\frac{(S_i - S_{i-1} e^{-\alpha \delta})^2}{2\hat{\sigma}^2} \right]
\]
which gives the log-likelihood function
\[
L(\alpha, \hat{\sigma}) = \sum_{i=1}^{n} \ln f(S_{i+1}|S_i; \alpha, \hat{\sigma}) =
\]
\[
-\frac{n}{2} \ln(2\pi) - n \ln(\hat{\sigma}) - \frac{1}{2\hat{\sigma}^2} \sum_{i=1}^{n} [S_i - S_{i-1} e^{-\alpha \delta}]^2.
\]
The maximum of (11) is found where the partials derivatives are zero, which
gives the MLE estimates of $\alpha$ and $\hat{\sigma}$:

$$\frac{\partial L(\alpha, \hat{\sigma})}{\partial \alpha} = -\frac{\delta e^{-\alpha \delta}}{\hat{\sigma}^2} \sum_{i=1}^{n} [S_i S_{i-1} - e^{-\alpha \delta} S_{i-1}^2] = 0$$  \hspace{1cm} (12)$$

and

$$\frac{\partial L(\alpha, \hat{\sigma})}{\partial \hat{\sigma}} = \frac{n}{\hat{\sigma}} - \frac{1}{\hat{\sigma}^3} \sum_{i=1}^{n} [S_i - e^{-\alpha \delta} S_{i-1}]^2 = 0$$  \hspace{1cm} (13)$$

which equals the equations in (9). For a rigorous proof see Sitmo [3].

Figure 5: The change each time step for the deseasonalized logarithmic price.

Using the Maximum Likelihood Estimation from Theorem 3.5 a first parameter estimation can be acquired. Extreme outliners are then excluded and a new estimation is done, this process is repeated three times. The changes in
Figure 6: The change each time step for the conditioned deseasonalized logarithmic price.

Table 1: Estimated parameter values for the Ornstein-Uhlenbeck process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>29.4568</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.4723</td>
</tr>
</tbody>
</table>

each step and a 99.7 % confidence interval can be seen in Figure 5 and the conditioned change in time can be seen in Figure 6 after one iteration. This method gives us the parameters in Table 1.

3.3 The Spike Process

From the algorithm used in the last chapter to estimate the Ornstein-Uhlenbeck process parameters, the outliners are saved and used to estimate the spike
process. The spike process $Y_t$ is defined to be

$$dY_t = -\beta Y_t \, dt + J_t \, dN_t$$

(14)

where $J_t$ is exponentially distributed, $N_t$ Poisson distributed and $\beta$ the mean reversion rate. The mean reversion rate decides how fast the spike diminishes and will be determined completely by some educated guessing. Here $\beta$ is set to $\beta = \frac{200}{365}$ which causes the spikes to have a half-life of one day.

Table 2: Estimated parameter values for the spikes process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Exp(\mu)$</td>
<td>0.3122</td>
</tr>
<tr>
<td>$Po(\lambda)$</td>
<td>106.8000</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\frac{200}{365}$</td>
</tr>
</tbody>
</table>

Figure 7: A realization of the spike process with estimated parameter values.

In total 5 positive spikes were identified and it is easy to estimate the parameters $\mu$ and $\lambda$ for the exponential and Poisson distributions. The estimated
values can be seen in Table 2 and a realization of the spike model can be seen in Figure 7.

There is however one big flaw with the method used, only big spike values can be identified and with the exponential parameter obtained, spikes simulated can become bigger than realistically possible. To avoid unrealistic results an upper limit of three times the expected value can be set for the spikes to avoid bizarre predictions, this limit was set after running several simulations and observing the model behaviour.

4 The calibrated model

A full calibrated model has now been obtained. A quick way to see if the model is somehow realistic is to compare some realizations against the historical market data. A realizations can be seen in Figure 8.

![Figure 8: A realization of the calibrated model plotted against the market data.](image)
Obviously it cannot be said that the model is good from comparing one realization with the old data but at least one can say that the estimation methods seemingly are correct.

4.1 Monte Carlo predictions

To be able to get a good prediction of how the future spot price will develop a Monte Carlo method is used. Several realizations are made and the average will be the best prediction. Due to spikes being quite rare (occurring after a Poisson model with $\lambda = 106.8$) many iterations will be needed to give a fair representation of the spike processes. A rough estimation is that each day will have a spike every 100th simulation so to simulate them correctly around 100 000 iterations would be needed at least.

Figure 9: A predication made after one iteration of the calibrated model.

In Figure 9 and Figure 10 predictions after one and 100 000 Monte Carlo iterations can be seen. Due to the OU-process being mean-reverting with zero-mean one could expect the price to move towards the seasonal estimation
after a while and that is also what is happening. Due to the spike process the model prediction is however slightly above the seasonal.

4.2 Forward pricing

The standard formula for pricing forward contracts in the stock market is not applicable in the electrical market due to there being no price of risk and no possibility to store the underlying unit traded. Instead the market practice is to set the price to be the expected value of the spot price for that day. Forwards lasting a period longer than a day is priced to be the weighted average during the whole period.

In figure 11 some week and month long forward contracts are priced. The
forward prices are here set to

\[
F = \frac{1}{w} \sum_{i=0}^{n-1} w_i \mathbb{E}[S_{T+i}] \tag{15}
\]

where \(w_i\) is the weight of how much electricity is to be bought that day (measured in 1 MWH), \(w = \sum_{i=0}^{n-1} w_i\), \(T\) the delivery period and \(n\) is the amount of days the contract covers.

5 Discussion and conclusions

The model proposed in this thesis is not perfect but manages to mimic the main characteristics of the electricity market well, the seasonal part, the mean-reverting behaviour and the spikes are all accounted for. To improve the model some sort of weather prediction added would most likely cause
the best improvement since the Nordic countries, Norway and Sweden in particular, are very dependent on hydro-power hence downfall has a huge impact on the electricity production. Also warm days, especially during the weekends, can cause a drop in consumption from private consumers. Even if industry is responsible for most of the consumption a small drop in demand can cause a big drop in the price.

If an improvement to the model is desired without trying to understand the underlying mechanics adding either stochastic or seasonal dependent volatility might be the most natural next step. Also adding some randomness to the seasonality part or dependence between the stochastic processes, now assumed to be independent of each other, might give some improvements. The problem with this is that even if the historical data might fit better to our model, calibrating the model is becoming much harder and might not give better results.

As seen from the Monte Carlo predictions in Figure 10 the long term predictions moves towards the seasonal function. If the goal of the model is to simply price long term forward contracts an interpolation model might be to prefer and is also easier to calibrate. However information of the path of the process is then lost which is needed in the case of pricing Swing Options that are commonly used in the electricity market. Swing Options gives the buyer the possibility to change the amount of electricity bought several times and introduces a path-dependency.

The forward prices predicated from this model were slightly above the markets. This could be because the market prediction were more local (the price has been low the last month) or due to risk averse behaviour of the sellers. Also the model is calibrated after not even 2.5 years of data, in Kluge’s thesis 10 years of data were used. The reason for this is that to get access to more data from NordPool an expensive membership is required.
References

