Preprint

This is the submitted version of a paper published in *International Economic Review*.

Citation for the original published paper (version of record):

http://dx.doi.org/DOI: 10.1111/iere.12146

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:
http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-231539
ASYMMETRY OF REPUTATION LOSS AND RECOVERY UNDER ENDOGENOUS PARTNERSHIPS: THEORY AND EVIDENCE

BY TAKAKO FUJIWARA-GREVE, HENRICH R. GREVE, AND STEFAN JONSSON

Keio University, Japan; INSEAD, Singapore; Uppsala University, Sweden.

This paper is inspired by real-world phenomena that firms lose customers based on imprecise information and take a long time to recover. To rationalize the phenomena, we develop a new model of endogenously repeated Trust game with imperfect monitoring. In customer-efficient Markov equilibria, the dynamic path of customers at a firm exhibits the asymmetry of fast loss after a bad signal and slow recovery. Exit is systematic but formation of a new partnership is random. This factor of dynamic asymmetry has not been formally proved before. We also give empirical evidence of our equilibria at an individual-firm level.

1. INTRODUCTION

This paper is inspired by real-world phenomena where consumers punish firms based on imprecise information, and the dynamic path of the customer measure of a punished firm takes a certain form: fast loss after a bad signal, followed by slow recovery. To give a rationale to the phenomena, we make a new model of endogenously repeated games (e.g., Ghosh and Ray, 1996, Kranton, 1996, Carmichael and MacLeod, 1997, Rob and Yang, 2008, Fujiwara-Greve and Okuno-Fujiwara, 2009, and McAdams, 2011).

There are many real-world episodes of “undeserved loss” of customers that take a long time to recover. For example, Audi in the U.S. was badly hit by allegations of “unintended accelerations” in 1986. The bad press coverage continued intermittently until around 1989, when it was established that the incidents were mostly the result of driver mistakes. Yet, it took 15 years for the sales of Audi to gradually recover to the level of 1985 (see Figure 1). The average re-purchase cycle of regular

---

1 We are grateful to Kiminori Matsuyama, Pei-yu (Melody) Lo, Ichiro Obara, and Sergei Severinov for helpful comments. We also thank Premiepensionsmyndigheten (PPM) and especially Marcela Cohen Birman for generously providing data. Takako Fujiwara-Greve gratefully acknowledges the research fund from Keio Economic Society. Usual disclaimer applies. Corresponding author: Takako Fujiwara-Greve, Department of Economics, Keio University, 2-15-45 Mita, Minato-ku, Tokyo, 108-8345 JAPAN. E-mail: takakofg@econ.keio.ac.jp.

2 For more details, see Winter (2010).
passenger cars is 7.8 to 8.7 years,\textsuperscript{3} and thus 15 years is very long. It is natural to ask whether this is all “irrational”, emotional behavior of nervous customers or whether there is any rationale for it.

The literature of repeated games with imperfect monitoring (e.g., Green and Porter, 1984, Abreu et al., 1986, and Fudenberg et al., 1994) gives a partial answer that, in order to achieve (information-constrained) efficiency, players must punish after a bad signal is observed, even if the signal can be wrong. At the same time, efficiency and incentive for punishers require that the punishment phase must be designed to last just enough to cancel out possible deviation gains. However, there is no reason to gradually stop the punishment.\textsuperscript{5}

With a competitive market model of experience goods, Hörner (2002) shows that rational customers switch firms after an imperfect but bad signal of the firm’s effort, in order to discipline rational firms as well as to avoid inept ones. However, firms that had a bad outcome exit the market in his model (to get rid of inept firms), and thus there is no theory of recovery.

\textsuperscript{3}Aizcorbe, Starr and Hickman (2004).
\textsuperscript{4}Source: Audi press releases.
\textsuperscript{5}Another way to achieve (constrained) efficiency in repeated games with imperfect monitoring is a review strategy, which accumulates information for some periods before punishing (e.g., Radner, 1985). This type of strategy employed by all players clearly does not exhibit immediate punishment after a bad signal, nor gradual recovery.
We postulate that some consumers come to the affected firm, because they do not observe the bad signal or do not think it is important, which is also a rational behavior if the firm is in fact making effort. If the firm maintains its effort over time, such customers gradually accumulate to let the firm recover. To address this consumer behavior, we formulate a new model of endogenously repeated game and construct customer-efficient (which maximizes the total long-run payoff of all consumers) Markov equilibria that display the dynamic asymmetry.

The literature of endogenously repeated games so far concentrated on single-population, one-to-one matching models of a double moral hazard problem (Prisoners’ Dilemma). We extend the basic model to fit buyer-seller problems with imperfect monitoring. There are two populations (consumers/customers and firms) and many consumers can be matched with a firm simultaneously. The stage game is a market-wide Trust Game (Kreps, 1990) where consumers choose which firm to trust and then firms choose whether to make effort or not. At the end of each period, only an imperfect signal of each firm’s action is observed by its current customers. There is also a small turnover of consumers from the market for exogenous reasons and the same measure of newcomers enter the market with no information. After that, consumers choose firms, and firms choose effort levels. The game continues this way ad infinitum.

We give necessary and sufficient conditions for the existence of customer-efficient Markov equilibria, in which all firms make effort after any signal history. The equilibrium paths share the asymmetric change in customer measures, fast drop and slow recovery. The intuition is as follows. To enforce constant effort on firms, it is sufficient for consumers to adopt the Markov grim-trigger strategy: trust a firm as long as no bad signal is observed and move to a different firm as soon as a bad signal is observed.\(^7\) Hence a fast drop of customer measure occurs after a bad signal. Newcomers and consumers who decide to move to another firm choose a firm randomly (for the latter, among the firms that they had not trusted in the previous period). The random choice yields forgiveness by the society. If the

\(^6\)We distinguish the general term consumers from customers of a firm.

\(^7\)In an extension, we also construct equilibria in which only some customers move after a bad signal. This extension is closely related to the statement in Hirshman (1970) that competition (punishment by exit) is best served by a combination of inert and alert customers, in particular under noisy signals.
firm continues to make effort, as long as no more bad signals occur, the firm gradually accumulates newcomers and customers of other firms that get bad signals. Thus, customer loss is immediate and systematic, but recovery is slow and uncoordinated. All these are outcomes of rational behaviors.

Moreover, our equilibria do not require asymmetry in feasible actions or information at the time of decline and recovery, as postulated in macroeconomics literature (e.g., Hansen and Prescott, 2005, and Veldkamp, 2005). (For more discussion, see Concluding Remarks.) Thus our theory gives a new factor of asymmetric dynamics: strategic partner changes.

The new model needed a technical innovation in solving the firm-side optimization problem. Because consumers move among firms, a firm’s future measure of customers is dependent not only on its current customer measure and history but also on all other firms’ customer movements. In the language of dynamic programming, all firm’s value functions are interrelated, and an individual firm’s value function is not recursive in itself. We solved this convoluted dynamic optimization by concatenating all firms’ value functions and solving a vector equation.

Because our model and the equilibrium strategies are quite intuitive and testable, we also provide an empirical test. In the Swedish mutual fund market operated by the pension authority, the consumers/investors could freely change funds, and there was a firm that lost its reputation for a wrong signal. We show that the resulting consumer behavior was as predicted by our equilibria: although all funds were making effort, the affected brand of funds experienced a rapid loss of customers as a punishment, followed by a slow and uncoordinated recovery. The event was also an interesting case of (negative) “name stretching” (cf. Wernerfelt, 1988, Cabral, 2000, and Chen and Lai, 2010): firms can be punished by simply having a similar name to a scandalized one.

The paper is organized as follows. Section 2 introduces the new model of endogenously repeated Trust game and gives a characterization of the existence of customer-efficient Markov equilibria. Section 3 is the empirical analysis of the Swedish mutual fund market. In Section 4 we give concluding remarks of theoretical extensions and policy implications.
2. Theory

2.1 Model. Consider a continuum of homogeneous consumers (customers/principals) of measure 1 and a finite set \( \{1, 2, \ldots, N\} \) (where \( N > 1 \)) of ex-ante homogeneous firms (sellers/agents) playing the following infinite-horizon game. Time is discrete and denoted as \( t = 1, 2, \ldots \). At the beginning of the game, all consumers are newcomers who do not have information regarding any firm’s past behavior. At the end of each period, \( 1 - \delta \) fraction of the consumers (where \( 0 < \delta < 1 \)) leave the market for exogenous reasons, which we call “death.” Each dead consumer is replaced by a newcomer so that the population size of the consumers is the same over time.

At the beginning of \( t = 1 \), all (newcomer) consumers choose simultaneously one of the firms in \( \{1, 2, \ldots, N\} \) to Trust, with no prior information. After consumers’ firm choices, the distribution of consumers across firms, denoted as \( x(1) = (x_1(1), \ldots, x_N(1)) \in \Delta^{N-1} \), is revealed to all firms,\(^8\) and then firms simultaneously choose whether to make Effort or to Shirk. We assume that a firm cannot discriminate among its customers (those patronizing the relevant firm) and chooses the same action against all of its customers. This is the case, for example, if a firm manages a fund that pools all customers’ investment (as in our empirical part), or if a firm chooses the quality of a mass product.

After firms’ action choices, customers of firm \( j \) observe a common but imperfect signal of firm \( j \)’s action. Whether they observe signals of other firms is in fact irrelevant to the rest of our analysis, because we focus on equilibria in which leaving a firm is the only punishment (i.e., customers of other firms do not actively punish). Thus we assume that consumers do not observe the signals of firms other than their current partner firm. This assumption corresponds to the standard no-information-flow assumption of endogenous partnership games (e.g., Ghosh and Ray, 1996) as well as that of Hörner (2002). By assuming minimal information structure, we construct robust equilibria among information structures with more information to consumers.

The signal structure we consider is the following “imperfect relative reputation.”\(^9\) There are two

\(^8\)\( \Delta^{N-1} \) is the \( N - 1 \)-dimensional unit simplex. Because the total measure of consumers is 1, \( x_j \) is both the absolute measure of firm \( j \)’s customers as well as its market share.

\(^9\)The “reputation” in our paper means realized signals, or how consumers/principals feel about an outcome after
possible signals, Good and Bad. If firm \( j \) shirks while all others made effort, firm \( j \)’s performance is worse than other firms’, and thus it gets a Bad signal for sure and all others get a Good signal for sure. If all firms made effort, the “relative reputation” should be the same across firms. However, there is some imperfection, or an exogenous shock, to the relative reputation, so that with probability \( \epsilon \), one firm is randomly selected to obtain a Bad signal. The possibility of the “undeserved loss” of reputation is assumed to be symmetric, so that each firm has probability \( \epsilon/N \) of getting a Bad signal even when all firms are making effort. With probability \( 1 - \epsilon \), all firms get a Good signal. We allow any \( \epsilon \in [0, 1) \) which includes the perfect monitoring case. The stochastic structure of signals is assumed to be i.i.d. over time. Because we focus on equilibria where all firms make Effort, more than one firm shirking is not a relevant case and we can allow any signal structure for those cases.\(^\text{10}\)

To justify the relative reputation signal system, notice that all firms are ex ante homogeneous, and thus it is natural that consumers care about the relative standings. A possible cause of undeserved loss of reputation among homogeneous firms making homogeneous effort is an unfounded accusation as in the Audi example of the Introduction. Such accusations usually do not happen to multiple firms at the same time. Consumers want one scapegoat. Another possible cause of exogenous reputation loss is the opposite force of “reputation stretching” (e.g., Wernerfelt, 1988, Choi, 1998, Cabral, 2000, Bar-Isaac and Tadelis, 2008, Section 9.2, and Chen and Lai, 2010). Reputation stretching means that a new brand/firm can establish goodwill among consumers without its own history, if it can be connected to an existing brand/firm with good reputation. In empirical research, the reverse effect is also well documented: if consumers see bad news of a brand, they do not like “similar” brands either (e.g., the entire auditing industry lost its reputation by the scandal of a single office of Arthur Andersen, Huang and Li, 2009, and a bank run in India occurred after a different bank’s failure, Iyer and Puri, 2012). Therefore \( \epsilon/N \) can be interpreted as the probability that external bad news extends to a reputation loss of a firm in this market.

\(^\text{10}\)For completeness, one can assume that if two or more firms shirk, then the information is perfect so that exactly the shirking firms get Bad signals.
At the end of a period, exogenous death of consumers occurs and newcomer consumers enter the market. At the beginning of periods \( t \geq 2 \), all consumers simultaneously choose one of the firms to Trust. Consumer movements determine the customer distribution of the period, \( x(t) \in \Delta^N \), and after that, firms choose between Effort and Shirk which generates signals. The game continues this way. In summary, the game is an endogenously repeated Trust game (Kreps, 1990) with noisy signals. The outline of the game is depicted in Figure 2.

The one-shot payoff structure is as follows. Effort action is more costly to the firm than Shirk action: denote \( H > 0 \) as a firm’s one-shot payoff from a consumer when it chooses Shirk, and \( L \in (0, H) \) when it chooses Effort. In total, if firm \( j \) has \( x_j \in [0, 1] \) fraction of consumers in a period, its one-shot payoff is either \( Hx_j \) or \( Lx_j \).

Consumers receive a high one-shot utility \( h > 0 \) if a Good signal is realized at the firm they are currently trusting and a low utility \( -\ell \) (where \( \ell > 0 \)) if a Bad signal is realized. This utility structure is not only standard in imperfect-monitoring repeated games to warrant that consumers cannot infer the underlying action from the realized utility, but also incorporates the psychological effect of the reputation signal.

To make the model meaningful, we assume that (Trust, Effort) is efficient in the stage game, i.e., the sum of one-shot payoffs of a consumer and a firm is greater than that of (Trust, Shirk). The one-shot expected utility of a customer of a firm \( j \) when all firms are making effort is \( (1 - \epsilon + \sum_{k \neq j} \frac{\epsilon}{N})h - \frac{\epsilon}{N}\ell = (1 - \frac{\epsilon}{N})h - \frac{\epsilon}{N}\ell \). Hence the sum of the one-shot payoffs of a consumer and a firm when all firms choose Effort is \( (1 - \frac{\epsilon}{N})h - \frac{\epsilon}{N}\ell + L \), and the sum if this firm shirked is \( -\ell + H \).
ASSUMPTION 1. \((1 - \frac{1}{N})(h + \ell) > H - L\).

REMARK 1. Assumption 1 implies that \((1 - \frac{\epsilon}{N})h - \frac{\epsilon}{N}\ell + L > -\ell + H\) for any \(\epsilon \in [0, 1)\).

The probability \(\delta\) of staying in the game is the effective discount factor for each consumer. Firms are active in the game forever, and thus we assume that they discount payoffs by a common discount factor \(\beta \in (0, 1)\). The game is of complete information.

2.2 Customer-efficient, Pure-Markov Equilibria. We show the existence of customer-efficient equilibria, in which the sum of all consumers’ long-run expected payoff is maximized under the informational constraint. They can be constructed by simple (in fact belief-free) and plausible strategy combinations. To maximize all consumers’ long-run expected payoff, firms must make effort after any signal. If a firm starts shirking after a Bad signal, the payoff of newcomers at that firm will be reduced. Therefore we need to enforce the constant effort strategy of all firms, which chooses Effort after any signal history.

DEFINITION 1. The constant effort strategy of a firm \(j\) is a function \(s_j^E : \bigcup_{t=1}^{\infty} \{G, B\}^{t-1} \to \{\text{Effort, Shirk}\}\) such that \(s_j^E(h_{jt}) = \text{Effort}\) for any \(t = 1, 2, \ldots\) and any signal history \(h_{jt} \in \{G, B\}^{t-1}\) (where \(\{G, B\}^0 = \emptyset\)) of firm \(j\) itself.

In order to induce constant effort by firms, consumers must punish a firm with a Bad signal, and although consumers do not have to react immediately (they can for example react after two consecutive Bad signals), there are three reasons to focus on the following Markov grim-trigger strategy which leaves the firm as soon as a Bad signal is realized. First, the most stringent punishment enforces effort with the weakest condition on the payoffs and/or the signals. (We do need some additional conditions on the model to enforce all firms’ effort. See Propositions below.) Second, unlike the ordinary (simultaneous-move) repeated game with imperfect monitoring, the grim-trigger strategy in our model is not eternal punishment which causes efficiency loss (cf. Green and Porter, 1984). Rather, the stochastic Bad signal to other firms and inflow of newcomers make punishment eventually subside,
even though players use strategies with up to 1-memory (cf. Barlo et al., 2009). Third, although there
must be other efficient equilibria of our model, the Markov grim-trigger strategy by the consumers
together with the constant effort strategy by the firms require least complexity and are belief-free.

For each \( t = 1, 2, \ldots \), the signal history for a consumer \( i \) until \( t \)-th period is a sequence of pairs of
a firm and the private history at the firm:

\[
\begin{align*}
    h_{it} = \begin{cases}
    \emptyset & \text{if } t = 1, \\
    \{(j(i, \tau), z_{j(i, \tau)})_{\tau=1}^{t-1}\} & \text{if } t \geq 2,
\end{cases}
\end{align*}
\]

where \( j(i, \tau) \in \{1, 2, \ldots, N\} \) is the firm which consumer \( i \) trusted in period \( \tau \) and \( z_{j(i, \tau)} \in \{G, B\} \) is
the realized signal at firm \( j(i, \tau) \) in period \( \tau \).

**Definition 2.** A Markov grim-trigger strategy of consumer \( i \) is a function \( s^G_i \) such that

\[
    s^G_i(h_{it}) = \begin{cases}
    \text{Unif}(\{1, 2, \ldots, N\}) & \text{if } h_{it} = \emptyset, \\
    j(i, t-1) & \text{if } h_{it} = \{(j(i, 1), z_{j(i, 1)}), \ldots, (j(i, t-1), G)\}, \\
    \text{Unif}(\{1, 2, \ldots, N\} \setminus \{j(i, t-1)\}) & \text{if } h_{it} = \{(j(i, 1), z_{j(i, 1)}), \ldots, (j(i, t-1), B)\},
\end{cases}
\]

where \( \text{Unif}(X) \) is a (discrete) uniform distribution over the set \( X \).

The “grim-trigger” means that consumers do not plan to stop punishment towards a firm. The
uniform probability to choose firms is natural given the homogeneity of firms.

**Proposition 1.** There exists \((\beta, \delta) \in (0,1)^2\) such that for any \((\beta, \delta) \geq (\beta, \delta)\), the (group-wise
symmetric) pure-strategy combination \( (s^G_i)_{i \in [0,1]}, (s^E_j)_{j \in \{1,2,\ldots,N\}} \) is a sequential equilibrium if and
only if one of the following conditions holds:

\[
    (A) \quad H - \frac{L}{N} \leq (N-1)L; \\
    (B) \quad \epsilon < \frac{N(N-1)L}{NH-L}.
\]

If the monitoring is perfect, i.e., \( \epsilon = 0 \), then (B) is satisfied. Thus the customer-efficient equilibrium
exists without an additional condition (other than high \( \beta \) and \( \delta \)), as in the Folk Theorem. The
equilibrium dynamic of the customer measure of a firm displays asymmetry of fast loss after a Bad
signal (by immediate move of customers) and slow recovery (by accumulation of newcomers and movers
from other firms). For details of the customer measure dynamics, see the end of subsection 2.3.
The formal proof is in Appendix, but we describe the outline of the proof. Because all firms have
the same strategy and generate the same long-run expected utility, consumers are indifferent between
punishing and not punishing after any signal, and hence $s_i^G$ is weakly optimal.

Each firm’s long-run optimization problem is heavily dependent on all other firms’ customer mea-
sures. To see this, notice that the customer measure of a firm $j$ has at most three components:
newcomers, stayers, and movers. Newcomers are consumers who entered the market this period and
randomly chose this firm. Stayers are the surviving customers from the previous period who decided
to trust this firm again. Movers are consumers who trusted another firm in the previous period but
decided to move and randomly chose this firm. Therefore a firm needs to know not only its own
customer measure but also other firms’ customer measures in order to calculate the expected measure
of future movers. However firms need not to know other firms’ signal histories.

Let $x_j(t)$ be the customer measure of firm $j$ and $x_{-j}(t) = (x_1(t), \ldots, x_{j-1}(t), x_{j+1}(t), \ldots, x_N(t))$
be the customer distribution of other firms at the beginning of period $t$. Firm $j$’s long-run expected
discounted payoff under the strategy combination $((s_i^G)_{i \in [0,1]}, (s_j^E)_{j \in \{1,2,\ldots,N\}})$ is formulated as

$$v(x_j(t); x_{-j}(t)) = L \cdot x_j(t) + \beta \{ (1 - \epsilon) \cdot v \left( \frac{\delta x_j(t) + 1 - \delta}{N} ; x_{-j}(t + 1) \right) + \frac{\epsilon}{N} \cdot v \left( \frac{1 - \delta}{N} ; x_{-j}(t + 1) \right) $$
$$+ \sum_{k \neq j} \frac{\epsilon}{N} \cdot v \left( \frac{\delta x_k(t) + 1 - \delta}{N - 1} + \frac{\delta x_j(t + 1)}{N} ; x_{-j}(t + 1) \right) \}. \tag{1}$$

To explain, the current period payoff is $L \cdot x_j(t)$ because firm $j$ makes Effort. Depending on the realized
signal in this period, the firm’s continuation value has $N + 1$ possible cases: all firms get a Good signal
(with probability $1 - \epsilon$), firm $j$ itself gets a Bad signal (with probability $\epsilon/N$), and one of other $N - 1$
firms gets a Bad signal. When all firms get a Good signal, firm $j$’s next period customer measure is
$x_j(t + 1) = \delta x_j(t) + (1 - \delta)/N$, the sum of surviving previous customers and $1/N$ of newcomers. If firm
$j$ got a Bad signal, $x_j(t + 1) = (1 - \delta)/N$ only. If one of the other firms, say firm $k \neq j$, with the
customer measure $x_k(t)$, got a Bad signal at the end of period $t$, then firm $j$’s customer measure in
period $t + 1$ will be the sum of stayers, newcomers, and movers:
\[
x_j(t + 1) = \delta x_j(t) + \frac{1 - \delta}{N} + \frac{\delta}{N - 1} x_k(t).
\]

Because of the mover measure, we cannot solve one firm’s long-run value function explicitly without solving the dynamics of all firms’ customer measures.\footnote{Hörner (2002) does not have this problem because the measure of movers is proportional to each firm’s own share, not dependent on other firms’ customer measure.} The key to solving this problem is to notice that the long-run payoff can be expressed as $L \times M_j(x_j(t))$, where $M_j(x_j(t))$ is the total discounted expected measure of customers firm $j$ has over the infinite horizon, starting with the measure $x_j(t)$. This is because the one-shot payoff is linear in the customer measure. Moreover, a vector equation of long-run discounted customer distribution $M(x)$, which is a concatenation of $M_j(x_j)$’s, turns out to be explicitly solvable. It is formulated as follows.

\[
M \left( \begin{array}{c}
x_1 \\
x_2 \\
\vdots \\
x_N
\end{array} \right) = \delta \left[ (1 - \epsilon) \cdot M \left( \begin{array}{c}
\delta x_1 + \frac{1 - \delta}{N} \\
\delta x_2 + \frac{1 - \delta}{N} \\
\vdots \\
\delta x_N + \frac{1 - \delta}{N}
\end{array} \right) \\
+ \sum_{k=1}^{N} \frac{\epsilon}{N} \cdot M \left( \begin{array}{c}
\delta x_k + \frac{1 - \delta}{N} + \frac{1}{N - 1} \delta x_k \\
\delta x_k + \frac{1 - \delta}{N} + \frac{1}{N - 1} \delta x_k \\
\vdots \\
\delta x_k + \frac{1 - \delta}{N} + \frac{1}{N - 1} \delta x_k
\end{array} \right) \right].
\]

As in (1), the first term of the RHS of (2) is the current period’s customer distribution, the second term is the continuation value when no firm got a Bad signal, and the third term is the sum of the continuation values when one firm gets a Bad signal. Using vector notation, (2) becomes\footnote{The notation $x'$ stands for the transpose of the row vector $x = (x_1, x_2, \ldots, x_N)$.}
\[
M(x') = x' + \beta \left[ (1 - \epsilon)M \left( \delta x' + \frac{1 - \delta}{N} e' \right) + \sum_{k=1}^{N} \frac{\epsilon}{N} M \left( A_k(\delta)x' + \frac{1 - \delta}{N} e' \right) \right],
\]
where $e' = (1, 1, \ldots, 1)'$ is the $N$-dimensional unit column vector, and, for any $k = 1, 2, \ldots, N$, the $N \times N$ matrix $A_k(\delta)$ (which describes the consumer re-distribution when firm $k$ got a Bad signal) is
defined as

\[
A_k(\delta) = \begin{pmatrix}
\delta & \cdots & 0 & \frac{\delta}{N-1} & 0 & \cdots & 0 \\
0 & \ddots & 0 & \vdots & 0 & \ddots & 0 \\
0 & \cdots & \delta & \frac{\delta}{N-1} & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \ddots & 0 & \cdots \\
0 & \cdots & 0 & \frac{\delta}{N-1} & \delta & \cdots & 0 \\
0 & \ddots & 0 & \vdots & 0 & \ddots & 0 \\
0 & \cdots & 0 & \frac{\delta}{N-1} & 0 & \cdots & \delta \\
\end{pmatrix},
\]

where the \(k\)-th row is the zero vector, and the \(k\)-th column is the mover distribution.

By the vector-matrix representation of (3), it should be clear that \(M(\cdot)\) is linear in \(x'\) and stationary. Using this fact, we can explicitly solve for \(M(x')\). Moreover, in the long run, the \(j\)-th coordinate of \(M(x')\) is independent from other firms’ initial measure of customers, because all firms get a Bad signal with uniform and i.i.d. probabilities over time, and thus the movers cancel out. Therefore we can separate the total discounted measure of customers of firm \(j\) as \(M_j(x_j)\). Once \(M_j(x_j)\) is explicitly computed, the derivation of the necessary and sufficient conditions (A) and (B) is straightforward.

The sufficient conditions are satisfied if there are sufficiently many firms in the market.

**Remark 2.** For any \(\epsilon \in (0, 1)\), there exists \(N > 1\) such that for any integer \(N \geq N\), (A) or (B) holds.

**Proof.** For (A), both sides are increasing in \(N\) but the LHS converges to \(H\) as \(N \to \infty\). The RHS keeps increasing. Thus there exists a lower bound to \(N\) above which (A) holds. (B) is equivalent to

\[
\frac{\epsilon}{N} < \frac{(N-1)L}{NH-\frac{L}{1-\beta}}.
\]

As \(N\) increases, the LHS converges to 0 and the RHS increases. \(\blacksquare\)

Therefore, the customer-efficient equilibrium exists for sufficiently large markets.

**Corollary 1.** The equilibrium long-run payoff of a firm is independent of \(\epsilon\), and it is

\[
L \cdot M_j \left( \frac{1}{N} \right) = \frac{L}{N(1-\beta)}.
\]

**Proof.** By computation using Equation (7) in Appendix. \(\blacksquare\)

Corollary 1 shows that there is no loss of long-run payoff by the imperfect monitoring on the firms’ side. This is a sharp contrast to ordinary repeated games with imperfect monitoring, where the gain from other players’ bad signal is not considered.
2.3 General Markov Equilibria with Constant Effort. To complete the analysis of all customer-efficient Markov equilibria, we consider general Markov behavior strategies by consumers such that, after a Bad signal, each consumer mixes between moving to a randomly chosen different firm and staying at the previous firm.

**DEFINITION 3.** For each $\alpha \in (0, 1]$, a *Markov grim-trigger strategy* of consumer $i$ with staying probability $\alpha$ is a function $s_{i}^{G\alpha}$ such that

$$
\begin{align*}
   s_{i}^{G\alpha}(h_{it}) = \begin{cases} 
   \text{Unif}({1, 2, \ldots, N}) & \text{if } h_{it} = \emptyset \\
   j(i, t - 1) & \text{if } h_{it} = \{(j(i, 1), z_{j(i, 1)}), \ldots, (j(i, t - 1), G)\} \\
   \alpha j(i, t - 1) & \text{if } h_{it} = \{(j(i, 1), z_{j(i, 1)}), \ldots, (j(i, t - 1), B)\} \\
   +(1 - \alpha)\text{Unif}({1, 2, \ldots, N}\setminus \{j(i, t - 1)\}) & \text{if } h_{it} = \{(j(i, 1), z_{j(i, 1)}), \ldots, (j(i, t - 1), B)\}.
\end{cases}
\end{align*}
$$

This softer grim-trigger strategy played by all consumers is mathematically equivalent to an asymmetric strategy combination such that $1 - \alpha$ of the consumers use the Markov grim-trigger strategy and $\alpha$ of the consumers use the *inertia* strategy which stays with the first (randomly-chosen) firm after any private history. If firms play the constant effort strategy, not moving after a Bad signal is also optimal.

**DEFINITION 4.** The *inertia strategy* of consumer $i$ is a function $s_{i}^{I}$ such that

$$
\begin{align*}
   s_{i}^{I}(h_{it}) = \begin{cases} 
   \text{Unif}({1, 2, \ldots, N}) & \text{if } h_{it} = \emptyset \\
   j(i, t - 1) & \text{otherwise}.
\end{cases}
\end{align*}
$$

The softer grim-trigger strategy $s_{i}^{G\alpha}$ or an asymmetric strategy combination of the Markov grim-trigger strategy and the inertia strategy is not only important to complete our analysis of Markov customer-efficient equilibria but also to give grounds to the firms’ survival after a Bad signal. Hirshman (1970) argued that “For competition (i.e., exit by customers) to work as a mechanism of recuperation from performance lapses, it is generally best for a firm to have mixture of alert and inert customers. The alert customers provide the firm with a feedback mechanism which starts the effort at recuperation while the inert customers provide it with the time and dollar cushion needed for this effort to come to fruition.” Our asymmetric strategy distribution of consumers corresponds to this idea. Inert customers can provide income for affected firms, endogenizing firms’ infinite horizon.
Note that when \( \alpha = 1 \), the strategy combination reduces to the one in Proposition 1, and if \( \alpha = 0 \), firms do not follow the constant effort strategy.

**Proposition 2.** For any \( \alpha \in (0, 1] \), there exist \((\beta, \delta) \in (0, 1)^2\) such that for any \((\beta, \delta) \geq (\beta, \delta)\), the strategy combination \((s_i^1)_{i \in [0, \alpha]}, (s_i^G)_{i \in [\alpha, 1]}, (s_j^E)_{j \in \{1, 2, \ldots, N\}}\) (or, \((s_i^{G\alpha})_{i \in [0, 1]}, (s_j^E)_{j \in \{1, 2, \ldots, N\}}\)) is a sequential equilibrium if and only if (A) or (B) (in Proposition 1) holds.

The proof is in Appendix and is very similar to the one for Proposition 1. In fact \( \alpha \) does not matter for firms’ incentive to provide effort, as long as \( \alpha > 0 \). This is because the change in the fraction of movers both weakens the immediate punishment after a Bad signal and strengthens the long-term punishment by decreasing the future gain from other firms’ Bad signals. In the long-run, the effect of \( \alpha \) cancels out. However, the drop of customer measure after a Bad signal is smaller when \( \alpha < 1 \).

Let us examine how the measure of customers changes in our equilibria. At the beginning of the game, all firms receive the same share of customers \( \frac{1}{N} \). As long as no firm gets a Bad signal, the turnover of consumers does not change the symmetric customer distribution: \( x_j(t) = \frac{1}{N} \Rightarrow x_j(t+1) = \delta \frac{1}{N} + \frac{1-\delta}{N} = \frac{1}{N} \). When the first Bad signal hits a firm called \( j \), say at the end of period \( T \), the measure of customers of the firm decreases to \( x_j(T + 1) = \delta(1 - \alpha)x_j(T) + \frac{1-\delta}{N} = \delta(1 - \alpha)\frac{1}{N} + \frac{1-\delta}{N} \) in the next period, depending on the fraction of inert consumers, and only slowly increases over time after that, until another occurrence of a Bad signal.\(^\text{13}\) For example, if no more Bad signal obtains afterwards, the time-sequence of customers has two parts, one from surviving previous customers and one from the accumulation of newcomers, such as

\[
\begin{align*}
x_j(T + 2) &= \delta x_j(T + 1) + \frac{1-\delta}{N}, \\
x_j(T + 3) &= \delta x_j(T + 2) + \frac{1-\delta}{N} = \delta^2 x_j(T + 1) + \{1 + \delta\} \frac{1-\delta}{N}, \\
&\quad \ldots \\
x_j(T + t) &= \delta^{t-1} x_j(T + 1) + \{1 + \delta + \delta^2 + \ldots + \delta^{t-2}\} \frac{1-\delta}{N}. 
\end{align*}
\]

This shows that the recovery path is concave in the time since the last Bad signal, \( t \).

\(^\text{13}\)If another firm gets a Bad signal, this firm’s customer measure jumps up. If this firm gets a Bad signal, it drops.
It is also possible that Bad signals occur in consecutive periods, for example because of sustained media attention. (This depends on the definition of a “period” as well. In the empirical analysis, we consider both a real time unit, a week, as well as a two-part separation of “heated phase” and “post-heated phase”.) In this case the decline rate of customer measure depends on $\alpha$. See Figure 3 (of which the curve of $\alpha = 0.5$ resembles the Audi graph) for a numerical example in which Bad signals occur for 6 consecutive periods. Suppose that from (at the end of) $T$-th period until $T+k$-th period, Bad signals occurred to firm $j$ consecutively. Then the customer measure keeps dropping as follows.

$$x_j(T + 1) = \delta(1 - \alpha) \frac{1}{N} + \frac{1 - \delta}{N}$$

$$x_j(T + 2) = \delta(1 - \alpha)x_j(T + 1) + \frac{1 - \delta}{N} = \{\delta(1 - \alpha)\}^2 \frac{1}{N} + \{1 + \delta(1 - \alpha)\} \frac{1 - \delta}{N}$$

$$\cdots$$

$$x_j(T + k + 1) = \{\delta(1 - \alpha)\}^{k+1} \frac{1}{N} + \{1 + \delta(1 - \alpha) + \cdots + \{\delta(1 - \alpha)\}^k\} \frac{1 - \delta}{N}.$$
If all customers leave the affected firm immediately ($\alpha = 1$), then the firm has only newcomers $\frac{1-\delta}{N}$ for as long as the series of Bad signal lasts. If some customers remain even after each Bad signal, the customer measure drops less drastically but continuously until the sequence of Bad signal stops.

3. Evidence

To reinforce our theory, we provide new empirical evidence at the micro (individual firm) level that supports our model and the focal equilibria. We need a dataset with the following properties. Firms/sellers are comparable in performance and effort, but one of them has an unlucky loss of reputation. Consumers can freely move from a firm to another. A good dataset was found in Sweden where the government’s pension agency offers a menu of mutual funds with no cost of switching among them. Moreover, one fund management company, Skandia Fonder, was part of a firm group that had a scandal. Even though the mutual funds and the fund management company were not implicated in the scandal at all, investors could interpret the scandal news as a Bad signal. In this situation, our equilibria predict that investors leave the “scandal related” firm quickly and only gradually come back, but another rational behavior is to keep investing according to the fund performance. Therefore it is worthwhile to empirically test investor behavior.

3.1 Skandia Scandals. The scandals occurred at one of the largest Swedish companies called Skandia AB, which is a casualty insurance company. Its subsidiaries included firms offering unit-link savings (Skandia AFS), life insurance (Skandia Life), a bank (Skandia Banken), as well as an asset management firm (Skandia Asset Management, or SAM) and a mutual fund management firm (Skandia Fonder). The scandal had three distinct components. First, in January 2002, Skandia AB announced the sale of its asset management firm SAM to a Norwegian bank. This was considered to hurt the customers of the life insurance business of Skandia AB – Skandia Life. The worth of an asset management company is largely determined by its long-term asset management contracts, and it was estimated that two-thirds of the value of SAM came from a contract with the policy holders in Skandia Life. What upset many was that all proceeds from the sale went directly to Skandia AB,
rather than to Skandia Life.

Second, in the April 2002 annual shareholders meeting, the management incentive programs of Skandia AB became intensely debated. A large institutional owner of Skandia AB published a debate article in one of the largest Swedish dailies stating their intention to vote against the suggested incentive program because they found it too expensive and not sufficiently related to the performance of the managers.

Third, in October 2002, there were media reports about ‘apartment dealings’ of the management of Skandia AB. Rental apartments in central Stockholm are extremely difficult to get and considered to be valuable because the market for such contracts has long been under government control. In this particular case, the luxury apartment of the financial director of Skandia AB was to be renovated at the expense of Skandia AB without a commensurate raise in the rental cost. While this specific renovation was called off due to the bad publicity it generated for Skandia AB, the media subsequently uncovered a number of other apartment dealings. The most frequent type involved top-level managers at Skandia AB providing their children with rental apartments in real estate owned by Skandia AB.

These three scandals led to a series of investigations and media disclosures that peaked at the end of December 2003. Shortly after, the chairman of the board resigned and an extraordinary shareholder meeting was called for January 2004. Skandia AB filed a lawsuit against its former CEO, finance director, and chairman of the board, and a public interest group filed a class action suit against the sale of SAM by Skandia AB. Skandia Life, after pressure from the consumers’ ombudsman, among others, decided to take Skandia AB to court for arbitration for SEK 2 billion of the 3.2 billion SAM sales proceeds. These events caused media attention to Skandia AB to be significantly higher than normal during 2002 – 2004, and public confidence in Skandia AB was seriously eroded. Partly as a consequence, Skandia AB was acquired by the South African insurance firm Old Mutual in 2005.

It is important to note that all the scandal events occurred around a few individuals at the helm of Skandia AB – the CEO, the CFO, and a few other managers. None of the board members or managers of the subsidiaries of Skandia AB, including Skandia Fonder, on which we are going to focus, were
implicated.

In the following, we investigate the transactions of mutual funds in the fund family “Skandia”, managed by Skandia Fonder, a subsidiary of Skandia AB, from January 2001 (before the Skandia scandals) until December 2007. For at least three reasons, such mutual funds have nothing to do with the above mentioned scandals. First, they were managed by an independent company, and the board members of Skandia Fonder do not overlap with those of Skandia AB. Second, a mutual fund in Sweden, unlike a life insurance or pension fund, is not allowed to own real estate. Thus, it is impossible to have the same type of apartment scandals at Skandia Fonder. Third, the mutual funds under the Skandia brand did not own disproportionately more of the stocks of the scandal-hit firm Skandia AB than other mutual funds. Hence even if the portfolio of the mutual funds may be affected by the scandals, the effect would be the same as other funds investing in Swedish stock. There is thus no rational reason to suspect that the scandals would affect the mutual funds’ performance.

Mutual funds map well to our theoretical model. They can be viewed as a Trust game. The one-shot payoff $H > 0$ of Shirk action can be interpreted as the fee of the fund management, and $H - L$ is the cost to make Effort. Likewise, a high utility $h > 0$ of consumers can be interpreted as the return of the fund, but bad news can decrease the utility to $-\ell$, due to consumers being concerned about holding a fund when its name has been tainted by a scandal.

In the Skandia scandals, it is possible that rational investors ignore the scandal news and only consider funds’ expected future performance to sell or buy. Alternatively, rational investors may adopt our equilibrium strategy\textsuperscript{14} and punish Skandia brand funds. Because it is an investment situation, there are other possible behaviors such as herding and speculation. We separate these possibilities and show that coordinated punishment and uncoordinated recovery occurred.

3.2 Data and Descriptive Analysis. We examine mutual fund transactions of Skandia brand under the Swedish public pension plan. We obtained data on mutual fund transactions from the Swedish 14Perhaps the reality is between the minimal information in our theoretical model (that only the current customers observe the signal) and the perfect information that all investors observe the signal, but recall that our equilibria are robust in increased information among consumers.
public pension authority *Premiepensions Myndigheten* (PPM). This was the governmental authority that managed the system of mandated individual pension savings that is a part of the new Swedish pension system.\textsuperscript{15} In 2000, the Swedish mandatory pension system was changed from pay-as-you-go where the state managed the entire pension to a defined contribution system where the individual is expected to actively manage a small part of the pension (Sundén, 1998, and Horngren, 2001). Two and a half percent of the annual income of every working Swede is put into an individual account, from which the person invests into a set of mutual funds. The PPM acts as a middleman between individual investors who control an account with accumulated pension rights and the mutual fund management firms that have funds registered within the PPM system. The pension rights of an individual must be fully invested at all times – there is no provision for directly holding cash in the system. As a result, all sales of funds in the PPM system are simultaneously purchases of shares in other funds.

Individual investors can change the funds they are currently investing in without paying a fee. The PPM accumulates the daily trades per fund, executes these as batch orders, and keeps records of the transactions. Thus the mutual fund management firm does not know the identity of each investor; only the PPM has this information.

PPM provided daily transaction data per mutual fund from October 2000 (when the system became operational) through December 2007 as well as the monthly yield of each mutual fund, its category, and its size at the start of each month. In order to focus the analysis on transactions initiated by individuals from existing funds, the data exclude transactions that were initiated by the PPM when new pension funds are allocated (PPM allocates new money according to an allocation key determined by each individual). The analysis omits the initial period of the PPM system (Oct.-Dec. 2000) and the first 90 days of any fund in order to eliminate the settling-in period effects. Except for these deletions, all available observations are used.

We aggregated the daily transaction data to weekly sums of fund sales (outflows), buys (inflows),

\textsuperscript{15}From January 2010, this role is with the Swedish Pensions Agency.
Table 1
DESCRIPTIVE STATISTICS

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly sales</td>
<td>11.343</td>
<td>2.385</td>
<td>0</td>
<td>16.383</td>
</tr>
<tr>
<td>Weekly buys</td>
<td>10.185</td>
<td>2.509</td>
<td>0</td>
<td>16.480</td>
</tr>
<tr>
<td>Weekly net flow</td>
<td>-5.421</td>
<td>10.022</td>
<td>-16.381</td>
<td>16.058</td>
</tr>
<tr>
<td>Fund type: Interest bearing</td>
<td>0.139</td>
<td>0.346</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Fund type: Mixed</td>
<td>0.202</td>
<td>0.402</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Fund focus: Europe</td>
<td>0.134</td>
<td>0.340</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Fund focus: Other region</td>
<td>0.415</td>
<td>0.493</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Fund focus: Industrial</td>
<td>0.069</td>
<td>0.253</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Monthly yield</td>
<td>0.002</td>
<td>0.045</td>
<td>-0.239</td>
<td>0.175</td>
</tr>
<tr>
<td>Fund value</td>
<td>18.147</td>
<td>1.560</td>
<td>11.609</td>
<td>20.540</td>
</tr>
<tr>
<td>Fund tenure</td>
<td>6.777</td>
<td>0.745</td>
<td>4.500</td>
<td>7.678</td>
</tr>
<tr>
<td>Scandals, lag 1</td>
<td>0.427</td>
<td>0.929</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Scandals, lag 2</td>
<td>0.433</td>
<td>0.933</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Heated phase</td>
<td>0.520</td>
<td>0.500</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Post-heated phase</td>
<td>0.193</td>
<td>0.394</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

and net flows (buys minus sales) in order to simplify the estimation of lagged effects of news.\textsuperscript{16}

Preliminary analysis showed that scandal news affected mutual fund flows for two weeks, which is captured by entering the first and second lag of the weekly number of scandal news in the weekly data. News in the same week are not entered because both news and investor reactions are summarized over the week, so investor movements in the same week as a news item may have occurred before the news. Possible reasons that news affect investor movements for two weeks rather than just one are discussed in the Method subsection. The natural logarithm of sales, buys, and net flows is used in order to reduce skew (for negative net flows, we use the logarithm of the absolute value of the flow).

The mutual fund data are supplemented by news reports from the most widely read daily business newspaper (Dagens Industri) and the most widely read daily general newspaper (Dagens Nyheter) from January 2001 through September 2006. A text search for Skandia and reading of each article

\textsuperscript{16}Daily effects are too irregular, particularly on Mondays, because all online applications over a weekend get registered on Mondays. Also, these are working people choosing pension funds when they have time to think, not active investors in stock markets. Thus weekly movements are sufficient to measure short-run decisions.

\textsuperscript{17}4164 observations, 16 funds, sample period of 299 periods.
was used to code each occurrence of the phrase ‘the Skandia scandal’, and these were counted for each week of the sample period.\textsuperscript{18}

Other variables in the regressions are selected for the potential relevance to buy and sell activity in mutual funds. Monthly yield is the most recent monthly yield, in percent, of the fund. Fund value is the logarithm of the fund size in Swedish Kronor. Fund tenure is the logarithm of the time in days that the fund has existed in the PPM system. The main analysis enters indicator variables for each fund to absorb idiosyncratic factors. We also did supplementary analyses with indicator variables for Fund type and Fund focus.\textsuperscript{19} Table 1 shows the descriptive statistics of the dataset.

Figure 4 shows the sum of weekly sales (left) and net flows (right) of Skandia brand funds expressed as a proportion of the size of the funds at the start of each month, with a count of scandal mentions in the press per week superimposed.\textsuperscript{20} It shows that there is sustained press attention to the Skandia

\textsuperscript{18}We also measured the tone of articles and tested with a variable of “negative news”. It turned out that the simple counting of the phrase ‘the Skandia scandal’ is just as good as the negative news variable.

\textsuperscript{19}Fund type is coded through the indicator variables Interest Bearing and Mixed (the omitted category is Equity). Fund focus was coded through the indicator variables Europe, Other Region, and Industrial (the omitted category is Sweden). See Table VI in the extended version of this paper.

\textsuperscript{20}To avoid effects of changes in the composition of funds, only funds that existed throughout the sample period are graphed.
scandals from October 2002 through February 2005. As we discussed in the Theory section, we can consider a week as a time unit with consecutive Bad signal occurrence, or separate the heated attention periods and other periods. For the latter, we use an indicator variable “heated phase” for the period from October 2002 through February 2005.

The graph of sales shows some peaks apparently unrelated to the scandals, but a lengthy run of sales during the heated phase followed by two smaller runs starting at the end of the heated phase and midway through the post-heated phase, respectively. These sales runs are matched by runs of negative net flows in the right graph, and comparison with the reference line (zero flow) shows that there is negative net flow for most of the time after the scandal broke. It appears like Skandia brand mutual funds suffered sales and negative net flows as a consequence of the scandals in Skandia AB.

Figure 5 shows that Skandia brand funds (two left-most lines) had similar performance compared to other funds in terms of excess return rates. The average 3-month excess daily returns per fund manager over the whole data period was normalized to 0 and the average excess return rates of selected
large fund families (Skandia, Handelsbanken, Storebrand, Folksam, and Robur) were plotted for the pre-peak period (Jan. 2001 - June 2004) and post-peak period (July 2004 - Dec. 2007) of the entire sample. The cut was made in the mid-point of the heated phase, to check whether fund managers shirked after the outbreak of the scandal. The mean and the 5% confidence intervals are shown. All large fund managers’ averages are statistically indistinguishable from the average return rate.\footnote{Table III in the extended version of this paper shows the excess returns for all fund managers in our sample. Some smaller (in PPM) or more specialized fund managers do have average excess returns that differ from zero either positively or negatively in this time period.}

In sum, we see in these figures that Skandia funds are negatively affected by a Bad signal even though their fund managers do not perform worse than others. For a more rigorous analysis, we next estimate regression models.

3.3 Method. Modeling mutual fund flow is often done through successive cross-sectional regressions with fund characteristics such as past performance, investment style, and fee as independent variables (e.g., Fama and MacBeth, 1973, and Berk and Green, 2004). Our question concerns the effect of new information (i.e., news about a scandal) on the flow of money to mutual funds rather than the average influence of, for example, fee or performance. This brings our modeling needs closer to another line of finance studies that investigates the effect of new information on investor behavior (e.g., Maheu and McCurdy, 2004). As in this literature, we need regression models that capture the effects of news items on the demand for a fund over short time periods.

We found that the time series of fund sales, buys, and net flows had persistence in the volatility term, which we model through a GARCH (1,1) specification\footnote{Thus the error term $\varepsilon_t$ has the following variance $\text{Var}(\varepsilon_t) = \sigma_t^2 = \gamma_0 + \omega_1 \varepsilon_{t-1}^2 + \omega_2 \sigma_{t-1}^2$.} (Bollerslev, 1986). The time series also have serial correlation in the expectation, which we model through an autoregressive model with first and second-degree terms (an AR(2) model). The AR(2) model fit better than AR(1) and no worse than AR(3), and also fit better than models with moving-average terms. We denote the control variables including fixed effects by the matrix $X$. News are denoted as $w$, the scandal period is $z$, and
each dependent variable (fund flow) is $y$. The final model is:

$$(4) \quad y_t = \beta X_{t-1} + \kappa_1 w_{t-1} + \kappa_2 w_{t-2} + \psi_1 z_{t-1} + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \varepsilon_t,$$

where $\rho_i$’s are the autocorrelation coefficients. The time unit is a week.

The serial correlation can have multiple causes. One possibility is that information other than the press mentions that we included in the model, such as word of mouth, is gradually spreading. Another is herding or learning, that investors wait for other investors to act and then react accordingly. Yet another possibility is that people are slow in reacting due to time constraints in thinking and acting on their pension investments. For example, although an online system for changing pension allocations was available and had high usage rates, mail-in changes were also possible and used by some investors. By using an autoregressive model, we isolate investor movements that directly respond to the scandal news from the serially correlated factors.

3.4 Empirical Analysis. The regression results are summarized in Table 2.\textsuperscript{23} We find evidence that investors are punishing Skandia brand funds, but some investors buy the funds as well, which contributes to slow recovery. Punishment sales should be distinguished from herd behavior or speculative behavior. As noted above, we have controlled for serial correlations to separate from herd behavior, and still sales are higher and net flows are lower after scandal mentions in the newspapers, in both Model 1 and 2. Speculative behavior is inconsistent with the continued sales and lower net flows after the heated phase, because once the worst time is over, there is no reason to keep selling to gain profit. Therefore we conclude that investors are punishing Skandia brand funds.

Let us go through the results in Table 2 one by one. In the first column, we see that scandal news increased sales in the next week and two weeks after. Sales also increased both during the heated phase and in the post-heated phase. The latter means that punishment goes on, but it is consistent with our prediction because the scandal mentions did not stop even after the heated phase. The softer punishment equilibrium fits with this observation because the fund sizes keep dropping as long as Bad

\textsuperscript{23}Indicator variables for individual funds are entered. Their coefficients and those of the control variables, except the monthly fund yield, are omitted. Full tables are in the extended version of this paper.
Table 2
REGRESSION PER SKANDIA BRAND FUND: GARCH(1,1) WITH AR(2)24

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Sales</th>
<th>Buys</th>
<th>Net flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\beta}$</td>
<td>$</td>
<td>z</td>
</tr>
<tr>
<td>Model 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monthly yield</td>
<td>-0.122</td>
<td>0.28</td>
<td>1.770</td>
</tr>
<tr>
<td>Scandals, lag 1 ($\kappa_1$)</td>
<td>0.034</td>
<td>2.78 **</td>
<td>0.067</td>
</tr>
<tr>
<td>Scandals, lag 2 ($\kappa_2$)</td>
<td>0.030</td>
<td>2.61 **</td>
<td>0.041</td>
</tr>
<tr>
<td>Heated phase ($\psi_1$)</td>
<td>0.594</td>
<td>4.20 ***</td>
<td>-0.092</td>
</tr>
<tr>
<td>Post-heated phase</td>
<td>0.795</td>
<td>4.41 ***</td>
<td>0.291</td>
</tr>
<tr>
<td>$y_1$ ($\rho_1$)</td>
<td>0.402</td>
<td>23.17 ***</td>
<td>0.328</td>
</tr>
<tr>
<td>$y_2$ ($\rho_2$)</td>
<td>0.209</td>
<td>11.93 ***</td>
<td>0.198</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-5869.23</td>
<td></td>
<td>-7755.83</td>
</tr>
<tr>
<td>Wald $\chi^2$ (23 d.f.)</td>
<td>3532.94</td>
<td></td>
<td>2180.02</td>
</tr>
<tr>
<td>Model 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monthly yield</td>
<td>-0.134</td>
<td>0.39</td>
<td>1.771</td>
</tr>
<tr>
<td>Scandals, lag 1 ($\kappa_1$)</td>
<td>0.033</td>
<td>2.64 ***</td>
<td>0.068</td>
</tr>
<tr>
<td>Scandals, lag 2 ($\kappa_2$)</td>
<td>0.029</td>
<td>2.53 *</td>
<td>0.042</td>
</tr>
<tr>
<td>Heated phase ($\psi_1$)</td>
<td>0.595</td>
<td>6.56 ***</td>
<td>-0.091</td>
</tr>
<tr>
<td>Post-heated phase</td>
<td>0.812</td>
<td>4.69 ***</td>
<td>0.230</td>
</tr>
<tr>
<td>Time since scandal</td>
<td>-0.006</td>
<td>0.63</td>
<td>0.016</td>
</tr>
<tr>
<td>$y_1$ ($\rho_1$)</td>
<td>0.403</td>
<td>23.45 ***</td>
<td>0.328</td>
</tr>
<tr>
<td>$y_2$ ($\rho_2$)</td>
<td>0.208</td>
<td>12.13 ***</td>
<td>0.199</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-5868.92</td>
<td></td>
<td>-7755.41</td>
</tr>
<tr>
<td>Wald $\chi^2$ (24 d.f.)</td>
<td>3542.79</td>
<td></td>
<td>2179.24</td>
</tr>
</tbody>
</table>

signals last. There is also positive serial correlation coefficients of the first ($\rho_1$) and second ($\rho_2$) order, which could be due to herding.

In the second column, buys also increased in a week after scandal mentions. This is a rational behavior because the fund performance was good and the news about a different Skandia branded firm can raise attention to the Skandia funds. However, the third column shows that the net flow is...
not changed significantly, suggesting that the buys did not exceed sales. In the second lag, the net flow effect was negative and (marginally) significant.\textsuperscript{25} Buys do not change in the longer span (heated phase and post-heated phase in the second column). Therefore, no coordinated come back is seen.

In Model 2, we added a variable “time since last scandal” that measures the weeks since the last press mention of a scandal, to do a detailed recovery analysis. The results are shown in the second half of Table 2. Again, buys are not significant, showing no coordination to come back as time passes since the last bad news. However, now the net flow has a statistically significant positive coefficient at the 5 percent level. This can be due to a joint reduction in sales and increase in buys, though neither effect is strong on its own. The net effect can be interpreted as evidence of recovery that is not the result of coordinated buys, which is consistent with our theory. The recovery is also slow because the positive coefficient of “time since last scandal” is two digits smaller than the negative coefficient of the “post-heated phase.” The time unit is weeks, implying more than three years for a full recovery to take place.\textsuperscript{26} Again, the sign of recovery implies that sales after the heated phase is not likely to be a speculative behavior but punishment.

In summary, we found evidence that, even though the Skandia brand funds are innocent, punishment and slow and unsystematic recovery occurred. There is possibly some herding behavior, but stronger effects are punishment sales and rational buys according to monthly yield.

4. Concluding Remarks

We conclude the paper with some possible extensions and policy implications. First, the asymmetric dynamic of customer loss and recovery can be generated even if newcomers do not choose firms with equal probability. The driving force of the asymmetric dynamic is (i) consumers punish after an imprecise signal, because there are equally good firms in the market, (ii) already matched consumers

\textsuperscript{25}We also tested with an indicator variable of fund manager changes. The results are similar.

\textsuperscript{26}We also checked the possibility of a concave recovery path by estimating a model including the squared “time since last scandal” variable. We found that the coefficient of the squared variable is negative for buys, suggesting a concave path but without significance. Because the squared specification is not exactly what the theory implies (the path will peak if it is quadratic), we cannot derive a meaningful conclusion from this analysis.
have no reason to come back to a punished firm, and (iii) recovery must rely on accumulation of newcomers and/or a bad signal of other firms, which are not systematic. Hence as long as newcomers come to the affected firm in a small scale, the recovery is slow. This includes the market-share allocation of newcomers, as in Hörner (2002), if the share is measured after surviving consumers moved around. (Then a firm with a Bad signal would have a very small market share.)

Second, the assumption of the relative reputation signal can be endogenized to make consumers choose to use such a measurement, instead of separate evaluation of each firm’s reputation. This is because the relative reputation signal will eventually cancel out the random loss of customers, but separate reputation signals may not. Thus the use of the relative signal is efficient in the long-run and still provides effort incentives of firms. This idea is related to the relative-performance pay mechanism in the contract theory (e.g., Lazear and Rosen, 1981).

Third, we can allow a consumer to be matched with multiple firms simultaneously without changing the nature of the model. Suppose that each consumer has multiple “portions” of money to spend/invest. If each portion is controlled by an “agent” of the original consumer, just like in an agent-form of an extensive form game (Selten, 1975), then the rest of the analysis goes through. With this extension, we can also add a different interpretation of newcomers. Existing consumers may put aside a (small) portion of money to experiment with a random firm. Then the movement of such experimental portions is the same as newcomers’ entry.

Fourth, the driving force of our asymmetric dynamics is quite different from existing theories of asymmetry, such as macro dynamics. There is abundant empirical evidence that economic variables such as unemployment rate (Neftçi, 1984), GDP (Jovanovic, 2006), and hours worked (Hansen and Prescott, 2005) display asymmetric dynamics of fast drop and slow recovery. Two important models that give rise to the asymmetry is asymmetric effects of constraints in feasible actions (for example, capacity constraint is binding in upturn but not in downturn, as discussed in Hansen and Prescott, 2005) and learning under endogenous information (e.g., Veldkamp, 2005). If we interpret the customer

---

27We thank Kiminori Matsuyama for drawing our attention to the macro literature.
measure as transaction quantity, its asymmetric rise and fall is not due to binding constraints, because consumers are free to move and firms can take all consumers who come to them. We do have endogenous information in that consumers only base their decisions on their current partner firm’s signal, and the choice of partner is endogenous. However this is different from learning models in which the amount and precision of information differs at upturn and downturn. Rather, our dynamic path is due to strategic partner changes.\(^{28}\)

Finally, we give two policy implications. Although we showed the existence of customer-efficient equilibria, it is better to remove drivers of undeserved loss of reputation. Some random loss of reputation is predictable, such as association with existing scandals. Associations are contingent on the nature of the scandal as well as geographical proximity to or similarity with the scandalized firm (e.g., Huang and Li, 2009 and Jonsson et al., 2009). As these contingencies are observable, steps could presumably be taken to relieve firms that are tainted by mere seeming relatedness to a scandal.

Business cycles may be magnified by the micro-level asymmetry of customer dynamics. In good times, customers can be reluctant to punish for imprecise signals, but in bad times they may be more sensitive to bad news. If that is the case, disciplining under imperfect monitoring is a de-stabilizing factor of the economy that amplifies the downturn of the market. Therefore for stability policies, it is also important to discourage over-disciplining.

APPENDIX

PROOF OF PROPOSITION 1. Consumer: As we argued in the text, consumers are ex-ante indifferent among firms when all firms follow the constant-effort strategy. Specifically, at any firm, a consumer’s ex-ante total expected payoff, denoted as \(U\), can be recursively formulated as follows.

\[
U = (1 - \frac{\epsilon}{N})(h + \delta U) + \frac{\epsilon}{N}(-\ell + \delta U)
\]

\(^{28}\)Recently, Liu (2011) showed a reputation dynamic of gradual building and sudden crash, in a repeated game model of long-run and short-run players with incomplete information. The driving force of his dynamic is the long-run firm’s manipulation of one-shot consumers’ belief. By contrast, our players are all long-run, firms have competing rivals, and there is no adverse selection.
To explain, with probability \((1 - \frac{\epsilon}{N})\) the current partner firm gets a Good signal and the consumer receives \(h\) this period and stays with this firm. The continuation payoff, starting from the next period, is the same as the total payoff starting from this period because the firm uses a stationary strategy with i.i.d. signal. With probability \(\frac{\epsilon}{N}\), the current firm gets a Bad signal and the consumer receives \(-\ell\) this period. Although she moves to another firm, all firms are ex-ante identical, so that the continuation payoff is again \(U\). It is then clear that moving and not moving after a Bad signal gives the same continuation payoff, thus it is (weakly) optimal to move after a Bad signal.

Firm: Let \(x(t) = (x_1(t), x_2(t), \ldots, x_N(t))\) be the customer distribution across firms \(\{1, 2, \ldots, N\}\) at the beginning of period \(t = 1, 2, \ldots\). By the initial random choice of firms, \(x(1) = (\frac{1}{N}, \ldots, \frac{1}{N})\). Let \(M((x_1, x_2, \ldots, x_N)')\) be the \(N\)-dimensional (column) vector of total discounted expected measure of customers of all firms, starting with the customer distribution \((x_1, x_2, \ldots, x_N)\) (where \(\sum_j x_j = 1\)). As we explained in the text, \(M(x')\) is recursive and linear in \(x'\) as in (2), which can be further arranged as follows.

\[
M(\begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_N 
\end{pmatrix}) = \begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_N 
\end{pmatrix} + \beta(1 - \epsilon) \left[ \delta M(\begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_N 
\end{pmatrix}) + (1 - \delta) M(\begin{pmatrix}
1/N \\
1/N \\
\vdots \\
1/N 
\end{pmatrix}) \right] \\
+ \beta \frac{\epsilon}{N} \left[ \delta M(\begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_N 
\end{pmatrix}) + (1 - \delta) M(\begin{pmatrix}
1/N \\
1/N \\
\vdots \\
1/N 
\end{pmatrix}) + \delta M(\begin{pmatrix}
-x_1 \\
-x_1/(N-1) \\
\vdots \\
-x_1/(N-1) 
\end{pmatrix}) \right] + \cdots \\
+ \beta \frac{\epsilon}{N} \left[ \delta M(\begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_N 
\end{pmatrix}) + (1 - \delta) M(\begin{pmatrix}
1/N \\
1/N \\
\vdots \\
1/N 
\end{pmatrix}) + \delta M(\begin{pmatrix}
x_N/(N-1) \\
x_N/(N-1) \\
\vdots \\
x_N 
\end{pmatrix}) \right],
\]
and therefore,

\[
M \left( \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \right) = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} + \beta \left[ \delta M \left( \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \right) + (1 - \delta) M \left( \begin{pmatrix} 1/N \\ 1/N \\ \vdots \\ 1/N \end{pmatrix} \right) \right] + \beta \frac{\epsilon}{N} \delta M \left( \begin{pmatrix} -x_1 + \sum_{j \neq 1} x_j/(N-1) \\ -x_2 + \sum_{j \neq 2} x_j/(N-1) \\ \vdots \\ -x_N + \sum_{j \neq N} x_j/(N-1) \end{pmatrix} \right).
\]

Using \( \sum_j x_j = 1 \), we have

\[
M \left( \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \right) = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} + \beta \left[ \delta M \left( \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \right) + (1 - \delta) M \left( \begin{pmatrix} 1/N \\ 1/N \\ \vdots \\ 1/N \end{pmatrix} \right) \right] + \beta \frac{\epsilon}{N} \delta M \left( \begin{pmatrix} -x_1 + (1-x_1)/(N-1) \\ -x_2 + (1-x_2)/(N-1) \\ \vdots \\ -x_N + (1-x_N)/(N-1) \end{pmatrix} \right).
\]

Hence the long-run measure of customers is independent from the initial measure of other firms’ customers. This is because all firms have the same random signal structure.

By further rearrangements we get

\[
M \left( \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \right) = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} + \beta \delta M \left( \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \right) + \beta (1-\delta) M \left( \begin{pmatrix} 1/N \\ 1/N \\ \vdots \\ 1/N \end{pmatrix} \right) + \beta \frac{\epsilon}{N-1} M \left( \begin{pmatrix} 1/N \\ 1/N \\ \vdots \\ 1/N \end{pmatrix} \right) - \beta \frac{\epsilon}{N-1} M \left( \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \right)
\]

Finally, multiplying both sides with \((N - 1)\) and moving \(M((-x_1, \ldots, x_N)')\) to the LHS, we have an
explicit formula:

\[
(N-1)(1-\beta \delta) + \beta \delta \epsilon \}
\]

\[
M( \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} ) = (N-1) \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} + \{(N-1)\beta(1-\delta) + \beta \delta \epsilon \} M( \begin{pmatrix} 1/N \\ 1/N \\ \vdots \\ 1/N \end{pmatrix} ).
\]

In particular, when \( x_j = 1/N \) for all \( j \), (6) gives an explicit solution such that

\[
M( \begin{pmatrix} 1/N \\ 1/N \\ \vdots \\ 1/N \end{pmatrix} ) = \frac{1}{1-\beta} M( \begin{pmatrix} 1/N \\ 1/N \\ \vdots \\ 1/N \end{pmatrix} ).
\]

This makes sense, because starting from the same measure of customers and with the stationary and symmetric transition rule, all firms’ total expected measure of customers must be the same and on average \( 1/N \). Plugging (7) into (6), we have the total expected discounted measure of customers of firm \( j \), starting from an arbitrary distribution:

\[
M_j(x_j) = \frac{(N-1)x_j + \{(N-1)\beta(1-\delta) + \beta \delta \epsilon \} \frac{1}{(1-\beta)N}}{(N-1)(1-\beta \delta) + \beta \delta \epsilon}.
\]

For any firm \( j \) and any starting measure of customers \( x_j > 0 \), the firm does not deviate in one step if and only if

\[
L \cdot M_j(x_j) \geq H \cdot x_j + \beta L \cdot M_j(\frac{1-\delta}{N}).
\]

The RHS is the sum of the one-shot high payoff and the continuation value starting with the drop of customers to \( (1-\delta)/N \). By (8), the no-deviation condition (9) is equivalent to

\[
[L(N-1) - \{(N-1)(1-\beta \delta) + \beta \delta \epsilon \} H]x_j \geq -\frac{\beta \delta \epsilon L}{N}.
\]

When \( \beta \delta \to 1 \), the LHS of (10) converges to \( [L(N-1) - \epsilon H]x_j \) and the RHS to \( -\frac{\epsilon L}{N} \). Thus no firm deviates after any signal history, for sufficiently large \( (\beta, \delta) \), if and only if, for any \( x_j > 0 \),

\[
[L(N-1) - \epsilon H]x_j > -\frac{\epsilon L}{N}.
\]

For any (small) \( \epsilon \in [0, 1) \) such that \( L(N-1) - \epsilon H \geq 0 \), (11) holds for any \( x_j \in (0, 1) \). For (large) \( \epsilon \in [0, 1) \) such that \( L(N-1) - \epsilon H < 0 \), both sides of (11) are negative and it suffices to look at the
case of \( x_j = 1 \). The LHS of (11) at \( x_j = 1 \) is greater than the RHS if and only if

\[
L(N - 1) - \epsilon H > -\frac{\epsilon L}{N} \iff \epsilon < \frac{N(N - 1)L}{NH - L}.
\]

If (A), \((N - 1)L \geq H - \frac{L}{N}\), holds, the upper bound is at least 1. Hence, for any \( \epsilon < 1 \), (11) holds. Otherwise, (B) \( \epsilon < \frac{N(N - 1)L}{NH - L} \) warrants (11). (Note that, because \( \frac{L(N - 1)}{H} < \frac{N(N - 1)L}{NH - L} \), (B) includes the case of \( L(N - 1) - \epsilon H \geq 0 \).) □

PROOF OF PROPOSITION 2. Notice that the only change of \( M(\cdot) \) formulation under the new strategy combination is that, when firm \( k \) gets a Bad signal, \( \delta \alpha x_k \) is the measure of movers from firm \( k \), who are distributed among \( N - 1 \) other firms. Let us denote the new total discounted expected measure of customers as \( M^\alpha(\cdot) \). (5) now becomes

\[
M^\alpha\left(\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_N \end{array}\right) = \left(\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_N \end{array}\right) + \beta(1 - \epsilon)\left[ \delta M^\alpha\left(\begin{array}{c} 1/N \\ 1/N \\ \vdots \\ 1/N \end{array}\right) + (1 - \delta)M^\alpha\left(\begin{array}{c} -\alpha x_1 \\ \alpha x_1/(N - 1) \\ \vdots \\ \alpha x_1/(N - 1) \end{array}\right) \right] \\
+ \beta \frac{\epsilon}{N}\left[ \delta M^\alpha\left(\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_N \end{array}\right) + (1 - \delta)M^\alpha\left(\begin{array}{c} 1/N \\ 1/N \\ \vdots \\ 1/N \end{array}\right) + \delta M^\alpha\left(\begin{array}{c} -\alpha x_N \\ \alpha x_N/(N - 1) \\ \vdots \\ -\alpha x_N \end{array}\right) \right] + \cdots + \beta \frac{\epsilon}{N}\left[ \delta M^\alpha\left(\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_N \end{array}\right) + (1 - \delta)M^\alpha\left(\begin{array}{c} 1/N \\ 1/N \\ \vdots \\ 1/N \end{array}\right) + \delta M^\alpha\left(\begin{array}{c} -\alpha x_N \\ \alpha x_N/(N - 1) \\ \vdots \\ -\alpha x_N \end{array}\right) \right].
\]

By similar arrangements as in the proof of Proposition 1, we have an explicit formula:

\[
\{(N - 1)(1 - \beta) + \alpha \beta \delta \epsilon\} M^\alpha\left(\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_N \end{array}\right) = (N - 1)\left(\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_N \end{array}\right) + \{(N - 1)\beta(1 - \delta) + \alpha \beta \delta \epsilon\} M^\alpha\left(\begin{array}{c} 1/N \\ 1/N \\ \vdots \\ 1/N \end{array}\right).
\]

32
When \( x_j = 1/N \) for all \( j \), we again have
\[
M^\alpha \left( \begin{array}{c} 
 1/N \\
 1/N \\
 1/N \\
 1/N \\
\end{array} \right) = \frac{1}{1-\beta} \left( \begin{array}{c} 
 1/N \\
 1/N \\
 \vdots \\
 1/N \\
\end{array} \right),
\]
and thus
\[
M^\alpha_j (x_j) = \frac{(N-1)x_j + \{(N-1)\beta(1-\delta) + \alpha\beta\delta\epsilon\} \frac{1}{1-\beta N}}{(N-1)(1-\beta\delta) + \alpha\beta\delta\epsilon}.
\]
The condition that firm \( j \) does not deviate in one step is
\[
L \cdot M^\alpha_j (x_j) \geq H \cdot x_j + \beta L \cdot M^\alpha_j (\delta (1-\alpha) x_j + \frac{1-\delta}{N}).
\]
This is equivalent to
\[
(12) \quad [L(N-1) - \{(N-1)(1-\beta\delta) + \alpha\beta\delta\epsilon\}H - \beta\delta L(N-1)(1-\alpha)]x_j \geq -\frac{\alpha\beta\delta\epsilon L}{N}.
\]
In order for (12) to hold for sufficiently large \( \beta \) and \( \delta \), it suffices to have
\[
\alpha[L(N-1) - \epsilon H]x_j > -\frac{\alpha\epsilon L}{N}.
\]
For any \( \alpha \in (0,1] \), the above condition is equivalent to (11), hence the same necessary and sufficient condition obtains as in Proposition 1.

\[
\text{REFERENCES}
\]


