Numerical modeling and simulation of the deformation of wood under an applied indentation load

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Abstract

Mechanical wood pulping is widely used to gain fibers from a log of tree. The so-called stone grinding process to separate fibers, which are used for e.g. manufacturing of paper, is applied in the Swedish industry, but has not been extensively analyzed. In order to decrease the vast energy consumption of this process by using improved tools to wear the wood into pulp, a better understanding of the basic mechanics is needed. This work approaches this problem theoretically by using a high-resolution discrete finite element model, capable of capturing large-deformations and rotations, to describe the mechanical behavior at the micro-scale up to the macro-scale. Spherical indenting tools are pressed into modeled bodies and compared to experiments conducted in X-ray computed tomography equipment to judge the model. The numerical stress and strain fields, obtained in the finite element models, are analyzed in the region close to the indentation load, where cracks are anticipated to nucleate and grow. For ratios $r/R \rightarrow 0$, where $r$ is the average cell radius and $R$ is the indenter radius, the macroscopic stress and stress field approaches the classical Hertzian fields.
Chapter 1

Introduction

1.1 Contact mechanics

Contact mechanics is the study of the deformation of solids that touch each other at one or several points. The subject was first studied in 1882 by the German scientist Heinrich Hertz and was not really pursued for a long time. Until today researchers all around the globe try to understand the mechanics of contacts for several reasons and applications. Thanks to the increase in computational capacity is has become easier to test concepts and models which approach reality. Most work has focused on fairly isotropic and homogeneous materials such as glass or ceramics. In this thesis a finite element model is used to explore the contact mechanics of wood. The wood structure is discretely modelled as a two dimensional body, which is compressively loaded by a cylinder. The resulting forces, stresses, strains are of interest in order to optimize tools for mechanical fiber production. The motivation for improving those tools is to optimize the energy consumption of the mechanical fiber production in the highly energy consuming paper production process and this thesis is a first step into direction of understanding the impact of the geometry of the indenting tool on the wood.

1.2 Previous work on wood

In order to understand the mechanical properties of wood several hundreds of papers have been published [Ramskill (2002)]. The mechanical properties have been studied from macro to ultra structures with different methods.
Wood was compressed in all its different planes in the macro-structure and its stress and strains analysed [Gibson & Ashby (1997)]. Wood was exposed to lag screws, beam on foundations, indenting bolts, fracture propagation and more in experiments [Ramskill (2002)]. The properties were studied under different moisture conditions, chemical rework wood and wood with grains. [Ramskill (2002)] offers an extensive literature review. In the later years nanoindentation became more popular to investigate the mechanical properties of wood at cell, cell wall level and for single fibers [Wimmer et al. (1997), Konnerth et al. (2007), Lee et al. (2007), Wu et al. (2009), Wang et al. (2011), Tze et al. (2007), Brandt et al. (2010), Jaeger et al. (2011)]. Most commonly the Finite Element Method is used to describe the problem (For more insight [Mackerle (2005)] refers to 300 publication just the last 10 years). Modeling wood as a continuous continuum is the simplest way to approach the problem, however is regarding its heterogeneous structure not really successful. One of the closest relating projects to the presented one, is wood modeled as an orthotropic cylinder with hollow tubes as cells by [Gountsidou & Polatoglou (n.d.)]. A spherical indenter is pressed into the body in its axial-plane and the stress and strain fields are analyzed. Despite this extensive researched about wood, the mechanical phenomenon of fibers and their interaction in the micro-scale, especially under extreme conditions, i.e. separating process, is still not understood. Today the mechanical pulp production industry is working with two big spinning stones covered with nodes which are grinding the debarked wood into separated fiber bulks and fibers. This state of art was mostly developed by trial error and the industry’s experience, but has not been extensively analyzed. This project studies the instant when a wood sample consisting of fiber cells is confronted with a single node compressing into it, as in the grinding process, to approach the understanding of the bulked fiber mechanics under a compressive load.

1.3 Overview

The thesis will start with describing the structure and properties of wood. It will be followed up with the basics of stress and strain theory and a brief description of a Finite Element Model adapted for our problem. The results of the simulation are presented after defining the model and assumptions and are then compared to experimental results and classical Hertzian continuum theory to judge the model.
Chapter 2

Wood

2.1 Wood

Wood has been an essential material for human history and its importance is not diminished despite the development of new material and production methods. Due to its versatility and availability it is used in for example buildings, furnitures, fuels, instruments and paper. The intention of this thesis is to understand how wood deforms under a spherical contact having a radius of the same order of magnitude as the size of the wood cells. This is the first step in order to be able to design improved tools in the wood grinding process. The primary wood used in Swedish industry is spruce, the most common growing conifer in the boreal forest, the worlds grandest terrestrial ecosystem in the world.

2.2 Structure of wood

Wood as an organic material has a highly complex and heterogeneous structure. This complexity can be considered in different scales of magnitude and features can be defined in different hierarchical levels. This will be explored here with focus on the micro-structure level, which is the scale of the model.

2.2.1 Macro-structure

Figure 2.1 shows cross section of a log as seen by the naked eye. Sections in the log are defined as different types of wood. The most obvious feature
are the annual rings, which consist of the early wood and late wood. Early wood grows in spring and the early summer; it is the brighter, thicker part of the growth ring. Late wood grows in summer and is denser and darker in color. For conifers in Sweden the growth rings have an approximate size of 1-10 mm. Between the stem and the bark in the so called cambium the wood grows radially by cell division. In softwood the inner 15-20 growths rings are specified as juvenile wood [Kyrkjeide (1990)], which is characterized by lower stiffness, greater longitudinal shrinkage, lower tangential shrinkage and lower density than mature wood. The outer growth rings are referred as sapwood, where the transportation of liquids longitudinal to the trees occurs. In between is the heartwood, chemical transformed wood, which can be considered dead, apart from some chemical substances stored to defend the tree against parasites, avoid decay, seal pores and make it less permeable. Trees are always exposed to exterior influences, which form imperfections as knots or compression wood.

Wood grows conically but can be approximated to grow cylindrically. A cylindrical coordinate system has three orthogonal planes of symmetry; radial, tangential and the axial (longitudinal). The longitudinal direction $L$ is along the pith from the root to the top. The radial direction $R$ is the direction of the growths. The tangential direction $T$ is perpendicular to the growths. At appropriate distance to the center of the tree trunk the curva-
2.2 Structure of wood

Figure 2.2: Cylindrical coordinate system for wood $LRT$ and slightly shifted one for fibers $L_0, R_0, T_0$ [Persson (2000)].

ture can be neglected. The mechanical properties can be specified by these directions. It has to be mentioned that the mechanical properties are related to the arrangement of fibers. The used coordinate planes $R$ and $T$ are in the cross section of the fibers. The longitudinal direction of the fiber is slightly shifted to the pith longitudinal direction. The shifted angle depends on the type of wood [Persson (2000)].

2.2.2 Micro-structure

Wood has different strength and stiffness properties in different directions. The great variation in the mechanical strength and stiffness of different wood kinds are related to the micro-structure of wood. Woods micro-structure is at the scale of millimeters. Mostly hexagonal cells are enclosing a lumen space and the relation of the varying thickness of the cell walls to the total volume, the so-called relative density is mainly governing the differences in strength and stiffness. According to [Dinwoodie (1981), Bodig & Jayne (1982), the features of micro-structures in wood can be described by [Gibson & Ashby (1997)]:

1. “Highly elongated cells which make the bulk of the wood, called tracheids in softwoods and fibres in hardwoods.” In this thesis tracheids
Figure 2.3: The micro-structure of a small part of a cross section of a log. The sample represents exact the extent of one growth ring in radial direction. The growth direction is upwards. The lumen volume of the cells are becoming smaller with the time of the year. The cell shape of the fibers in the tangential-radial plane is in between rectangular and hexagonal structures [Persson (2000)].
2.3 Mechanical properties and behavior of wood

will be referred by the term fibers as well, which is not strictly correct. In softwood the lengths of tracheids is roughly $2 - 7\, mm$. In the juvenile wood they are shorter than in mature wood, where they have approximately constant length. The thickness is about $25 - 80\, \mu m$ in tangential and $17 - 60\, \mu m$ in radial direction. The thickness is decreasing from early to late wood. The cell wall thickness itself is about $2 - 7\, \mu m$. The shape of these fibers in the tangential-radial plane can be described as hexagonal or even more rougher as rectangular (fig. 2.3).

2. “The rays, made up of radial arrays of smaller, more rectangular, parechyma cells.” They extend in radial direction as seen in figure 2.3.

3. “The sap channels, which are enlarged cells with thin walls and large pore spaces which conduct fluids up the tree.” They appear only in the sap wood.

2.2.3 Cell wall

At the scale of microns wood may be considered as a fiber-reinforced composite. As one can see in figure 2.4 the cell wall can be distinguished in 5 different layers in this scale. For the sake of keeping the model simple, the effective properties of the whole cell wall is considered as a composition of these 5 layers.

2.3 Mechanical properties and behavior of wood

The mechanical properties depend on the wood type and external factors. In most woods the composition and the layer orientation of the fiber-reinforced cell walls are very similar. The primary differences between woods are related to variation in cellular structure \cite{Gibson1997}. The external influences are small. For a general understanding, flaws such as knots are not considered in the model. The most important factor in describing cellular structure is the relative density, determined by the relative thickness of the cell wall. The density is highly dependent on the temperature, age, moisture content and the type of wood, i.e. early or late wood. The cell wall density
Figure 2.4: Wood at the scale of microns. Five different layers can be distinguished (Persson (2000)).

The density is generally known by the proportion of mass $m$ divided by volume $V$.

$$\rho = \lim_{V \to 0} \frac{dm}{dV}$$  (2.1)

Since wood is a complex porous material, it isn’t trivial to explain its mechanical behavior. It can be said that for a small range of forces below the limit of material nonlinearity such as plastic, creep or damage the constitutive behavior can be approximated to be linear elastic, considering each of the three orthogonal planes (Gibson & Ashby (1997)). To describe behavior at the macroscopic scale it is beneficial to use the microscopic behavior and put the microscopic elements in relation to each other to naturally bridge scales in the structure. At the microscopic level, wood is a bulk of cells, mostly fibers consisting mainly of cell walls. The constitutive behavior of these cell walls can be considered linear elastic along and across the cell at small loads.
Chapter 3

Methods - Finite Element Method

To understand and predict physical phenomena, the mechanical behavior is analyzed with the finite element method (FEM). Wood is a highly complex anisotropic and heterogeneous material which makes it difficult to describe the physics mathematically with an approximation which holds over the entire sample. The idea of the FEM is to divide the geometry, i.e. the problem domain into a body of smaller elements, the so-called finite elements which are geometrically defined by nodal points. The approximations of these smaller elements become much simpler. Each element is solved approximately for itself and then all the elements are patched together with different rules and conditions to get a solution for the entire body. The approximations over those small FE between the nodal points is described by interpolations, mostly linear. It is possible to solve arbitrary partial differential equations with Finite Element Method, however, a deeper understanding is needed to estimate, whether the approach is actually valid. The following subchapters will explore the FEM further as applied to the wood model and present the Hertzian stress, which can be used for validating.

3.1 Finite Element

3.1.1 Strong form

The differential equation with the body force $b$, which expresses the stress equilibrium in the body, can be obtained is expressed by: $\text{Ottosen & Peters}$.
where $S$ is the 2. Piola-Kirchoff plane stress consisting of the normal stresses $\sigma_{xx}$, $\sigma_{yy}$ and the shear stress $\sigma_{xy}$.

\[
\nabla T = \begin{bmatrix} \delta & 0 & \delta \\ 0 & \delta & \delta \\ 0 & \delta & \delta \end{bmatrix} ; \quad S = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix}
\]  

### 3.1.2 Weak form

The weak formulation for the differential equations of equilibrium for this two-dimensional differential equation system, which holds irrespective of the constitutive relation for any solid material, can be derived by using natural boundary conditions of the traction vector $t$ and is expressed by

\[
\int_A (\nabla v)^T S dA = \oint_\zeta v^T t d\zeta + \int_A v^T btdA
\]

where $v$ is an arbitrary weight vector, $A$ the area, $\zeta$ the boundary path and $t$ the thickness.\cite{OttensoPetersson1991}

### 3.1.3 Finite element formulation

A 2-dimensional 4-node isoparametric element with 8 degrees of freedom is introduced. The displacement $u$ and the strain $\epsilon$ can be approximated by

\[
u = N^e a^e \quad \epsilon = B^e a^e
\]

where vector $a^e$ is the nodal displacement values and $N^e$ linear shape functions.\cite{OttensoPetersson1991}

\[
u = \begin{bmatrix} u_x \\ u_y \end{bmatrix} ; \quad \epsilon = \begin{bmatrix} u_{x1} & u_{y1} & u_{x2} & u_{y2} & u_{x3} & u_{y3} & u_{x4} & u_{y4} \end{bmatrix}^T
\]

\[
N^e = \begin{bmatrix} N_1^e & 0 & N_2^e & 0 & N_3^e & 0 & N_4^e & 0 \\ 0 & N_1^e & 0 & N_2^e & 0 & N_3^e & 0 & N_4^e \end{bmatrix}
\]
### 3.1 Finite Element

Nonlinear strain-displacement matrix

The nonlinear strain-displacement matrix \( \mathbf{B}^e \) in a total Lagrange formulation for a two-dimensional element in \( x \) - and \( y \) -coordinates is expressed by two parts \cite{Bathe1996}. The first part, which is linear, is described by:

\[
\mathbf{B}_0 = \begin{bmatrix}
\frac{\delta N_1^e}{\delta x} & 0 & \frac{\delta N_2^e}{\delta x} & 0 & \frac{\delta N_3^e}{\delta x} & 0 & \frac{\delta N_4^e}{\delta x} & 0 \\
0 & \frac{\delta N_1^e}{\delta y} & 0 & \frac{\delta N_2^e}{\delta y} & 0 & \frac{\delta N_3^e}{\delta y} & 0 & \frac{\delta N_4^e}{\delta y} \\
\frac{\delta N_1^e}{\delta x} & \frac{\delta N_1^e}{\delta y} & \frac{\delta N_2^e}{\delta x} & \frac{\delta N_2^e}{\delta y} & \frac{\delta N_3^e}{\delta x} & \frac{\delta N_3^e}{\delta y} & \frac{\delta N_4^e}{\delta x} & \frac{\delta N_4^e}{\delta y}
\end{bmatrix}
\tag{3.8}
\]

The second part of the \( \mathbf{B}^e \) matrix is nonlinear and is achieved by:

\[
\mathbf{B}_1 = \begin{bmatrix}
q_1 \frac{\delta N_1^e}{\delta x} & q_2 \frac{\delta N_2^e}{\delta x} & q_1 \frac{\delta N_1^e}{\delta y} & q_2 \frac{\delta N_2^e}{\delta y} \\
q_1 \frac{\delta N_1^e}{\delta x} & q_3 \frac{\delta N_1^e}{\delta y} & q_1 \frac{\delta N_1^e}{\delta x} & q_3 \frac{\delta N_1^e}{\delta y} \\
q_2 \frac{\delta N_2^e}{\delta x} & q_4 \frac{\delta N_2^e}{\delta y} & q_2 \frac{\delta N_2^e}{\delta x} & q_4 \frac{\delta N_2^e}{\delta y} \\
q_3 \frac{\delta N_1^e}{\delta y} & q_4 \frac{\delta N_1^e}{\delta y} & q_3 \frac{\delta N_1^e}{\delta y} & q_4 \frac{\delta N_1^e}{\delta y}
\end{bmatrix}
\tag{3.9}
\]

where

\[
q_1 = \sum_{i=1}^4 \frac{\delta N_i^e}{\delta x} u_{x,i}; \quad q_2 = \sum_{i=1}^4 \frac{\delta N_i^e}{\delta x} u_{y,i}; \quad q_3 = \sum_{i=1}^4 \frac{\delta N_i^e}{\delta y} u_{x,i}; \quad q_4 = \sum_{i=1}^4 \frac{\delta N_i^e}{\delta y} u_{y,i}
\tag{3.10}
\]

Adding these two expression gives the nonlinear strain-displacement matrix \( \mathbf{B}^e \)

\[
\mathbf{B}^e = \mathbf{B}_0 + \mathbf{B}_1
\tag{3.11}
\]

The Green-Lagrange strain increment, which is applied for large deformations, is received by multiplying the strain displacement matrix with the nodal displacement increment vector

\[
d\mathbf{\epsilon} = \mathbf{B}^e \cdot d\mathbf{a}^e
\tag{3.12}
\]

The plane strain consists of the normal strains \( \epsilon_{xx}, \epsilon_{yy} \) and the shear strain \( \gamma_{xy} \) for each node.

\[
\mathbf{\epsilon} = \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix}
\tag{3.13}
\]
Applying the linear constitutive relation gives the Piola-Kirchhoff stress increment, which is work conjugate to the Green Lagrange strain.

\[ dS = D \cdot d\varepsilon \]  \hspace{1cm} (3.14)

where \( D \) is the tangent stiffness tensor

\[
D = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix}
1 - \nu & \nu & 0 \\
\nu & 1 - \nu & 0 \\
0 & 0 & \frac{(1-2\nu)}{2}
\end{bmatrix}
\]  \hspace{1cm} (3.15)

with the two independent coefficients, the Young modulus \( E \) and Poisson’s ratio \( \nu \) for the stiffness tensor. The first two lines express the normal relation while the third line is the shear relation. The shear stress is independent of the normal stresses and normal strains and vice versa. The stresses and strains are updated after every load step according to \( S = S_0 + dS \) and \( \varepsilon = \varepsilon_0 + d\varepsilon \) where \( S_0 \) and \( \varepsilon_0 \) are the stress and strain at the previous load step.

### 3.1.4 Wood cell wall element

The standard finite element formulation is derived from the weak formulation by applying the shape functions [Ottosen & Petersson (1991)].

\[
K^e a^e = f^e = f^e_b + f^e_1
\]  \hspace{1cm} (3.16)

The stiffness matrix \( K^e \), the boundary vector, the boundary vector \( f^e_b \) and the load vector \( f^e_1 \) are expressed by

\[
K^e = \int_A B^e T D B^e t dA
\]  \hspace{1cm} (3.17)

\[
f^e_b = \int_{\xi_h} N^e T h t d\xi + \int_{\xi_o} N^e T t t d\xi
\]  \hspace{1cm} (3.18a)

\[
f^e_1 = \int_A N^e T b t dA
\]  \hspace{1cm} (3.18b)

where \( h \) is a pre described displacement vector and \( t \) a pre described traction vector.
3.2 Newton Rhapson

For contact problems it is necessary to find the equilibrium state for the body for the complete range of interest. The condition for the equilibrium is that the sum of the externally applied forces on the node $^t R$ is equal to the element node forces, which are the stresses in this configuration $^t F$.

$$^t R = ^t F$$  \hfill (3.22)
The Newton-Raphson technique is applied to obtain the equilibrium solution from the solution received after every increment in several iterations. The increment in displacement is calculated by

\[ t^{+\Delta t}K^{(i-1)}\Delta U^{(i)} = t^{+\Delta t}R - t^{+\Delta t}F^{(i-1)} \] (3.23)

where \( t^{+\Delta t}K^{(i-1)} \) is the current stiffness matrix

\[ t^{+\Delta t}K^{(i-1)} = \left[ \frac{\delta F}{\delta U} \right]_{t^{+\Delta t}u^{(i-1)}} \] (3.24)

and the improved displacement solution is

\[ t^{+\Delta t}U^{(i)} = t^{+\Delta t}U^{(i-1)} + \Delta U^{(i)} \] (3.25)

where \((1 - i)\) is the value of the previous iteration. The iteration is continued until appropriate convergence criteria are satisfied, i.e. the difference of the sum of all the forces at the nodes to zero is within a set error limit [Bathe (1996)].

### 3.3 Hertzian stress

Contact mechanics originates to Hertz publication 1882 (On the contact of elastic solids). In a Hertzian contact problem a hard spherical indenter forms non-adhesive elastic contact with a continuous non-conforming surface. In the Hertzian theory it is assumed that the strains are small, within the elastic limit, each body can be considered as an elastic half-space and surfaces are frictionless. Several attempts were made to describe the stress fields of an Hertzian contact [Auerbach (1891), Huber (1904), Johnson (1985)]. The Westergaard stress functions can be used to get a full representation for a frictionless symmetric load.

\[ \sigma_x = Re[Z_1] - y[Z_1'] \] (3.26)
\[ \sigma_y = Re[Z_1] + y[Z_1'] \] (3.27)
\[ \sigma_{xy} = y[Z_1'] \] (3.28)

where \( Z_1 \) is

\[ Z_1 = -\frac{2p}{\pi a^2}(\sqrt{a^2 - (x + iy)^2} + i(x + iy)) \] (3.29)

where \( a \) is half the contact length, \( p \) the contact pressure and \( i = \sqrt{-1} \).

The Figures 3.2 and 3.3 show the theoretical stress and strain fields.
Figure 3.2: The graphics show the theoretical Hertzian result for the highest principal stress \( \sigma_1 \), the normal stresses \( \sigma_{xx} \) and the shear stress \( \tau_{xy} \) after the indention of a hemisphere where the body is a continuum.
Figure 3.3: The graphics show the theoretical Hertzian result for the highest principal strain $\varepsilon_1$, the normal strains $\varepsilon_{xx}$, $\varepsilon_{yy}$ and the shear strain $\gamma_{xy}$ after the indentation of a hemisphere where the body is a continuum.
Chapter 4

Modeling wood and deformation by application of a sphere-shaped load

The following chapters and sub chapters describe the parts of the model, the set up, boundary condition, indentation condition, the size relation between indenter and cell radius, the way the results are received and evaluated.

4.1 Modeling

The model consists of two different bodies with different shape and properties. The micro-structure of wood is modeled with finite elements, while the spherical indenter is modeled as a very stiff incompressible body. The study is limited to a two-dimensional problem for simplification and the behavior of the fibers between each other in the tangential-radial plane is analyzed.

4.1.1 The body

As seen in the Chapter 2.2.2 the micro-structure of wood in the tangential-radial plane consists mainly of hexagonal shaped cells which enclose a lumen. To simplify the model, it is assumed that there are no ray cells, no imperfection as knots and the variation of the cell size due to the seasonal growth is also neglected. To model wood with the FEM, the hexagonal shaped fibers are modeled by several 4 node quadrilateral elements. The fiber cell shape
4. Modeling wood and deformation by application of a sphere-shaped load

can be hexagonal, pentagonal or quadrilateral. All sides of these cell walls are constructed by one layer of multiple quadrilateral elements aligned in rows. The thickness and lengths are somewhat uniform, however due to the impact of the surrounding cells, variation occurs in the geometry of the elements. Because the length of the cell wall varies the number of elements varies as well between cell walls.

Fiber elements, which are created in equal amount in both direction of the coordinate system, form a body which resembles the micro-structure of wood. The shape of all these fibers are different and their size differ as well. The fibers are connected via their cell walls. Each cell wall consists of elements matched in number with the neighboring cell wall.

The lengths of the elements are very similar over the whole body. The thickness $t$ of the cell wall is held constant. An example body constructed of around 25 cells can be seen in Figure 4.1.

An algorithm creates a predetermined number of random Voronoi-cells, resulting in a random structure of cells with a random density. The algorithm iterates the thickness of the cell wall elements until the density of the body matches the arbitrary value 0.4. This would correspond to the relative density of an average softwood.

4.1.2 Indenter

For the simulation an indenter is needed to penetrate the body. The intending tool has the simple geometry of a cylinder. The material is considered very stiff in comparison to the stiffness of the cell walls.

4.2 Simulation

4.2.1 Set up

The indenting body is placed right above the middle of the wood body (Fig. 4.2).
4.2 Simulation

Figure 4.1: An example mesh of roughly 5x5 pores created by the mesh creating algorithm. The body resembles the micro-structure of wood in the tangential-radial plane.

Figure 4.2: Boundary conditions
4. Modeling wood and deformation by application of a sphere-shaped load

4.2.2 Boundary conditions

The boundaries of the body are described in Figure 4.2. The lower horizontal boundary is clamped, i.e. \( u_x = u_y = 0 \). All three outer surfaces, apart from the part of the boundary that is in contact with the cylinder, are stress free, i.e. \( \sigma_{xx} = \sigma_{yy} = \tau_{xy} = 0 \).

4.2.3 Indentation conditions - contact algorithm

The indenter is penetrating the body from the upper side by 50\% of the sphere radius \( R \) in 8 iteration steps, to represent the path dependent reality and to obtain realistic results in an acceptable calculating time. The nodes with possible contact to the indenter, are found and moved to the indenter’s surface (Fig. 4.2 the contact area). This forced displacement results in the nodes having a force in \( F_x \) and \( F_y \) direction working on them. By the angle \( \alpha \) (the angle the node has to the hemisphere center), these forces at the surface of the hemisphere are transformed into normal \( F_n \) and tangential forces \( F_t \) according to:

\[
F_n = \cos(\alpha) \cdot F_x + \sin(\alpha) \cdot F_y \tag{4.1}
\]

\[
F_t = \sin(\alpha) \cdot F_x - \cos(\alpha) \cdot F_y \tag{4.2}
\]

The normal force \( F_n \) has to be negative, which means the normal force works against the indenter(center), i.e. compression is occurring. If this is not the case, the node is released. The indentation is assumed frictionless for simplification. This means there is no Coulomb friction \( F_t = \mu F_n \) and the tangential forces \( F_t \) of the nodes at the indenter surface have to be zero. To obtain this requirement the nodes with negative normal force are moved along the indenter surface for every iteration until their tangential forces are within an error limit for each indenting step.
4.2 Simulation

4.2.4 Size correlation

To find out more about the influence of the indenter, the simulation is executed with different sizes of the average cell radius $r$ in the body compared to the spherical tool radius $R$, which stays constant. The range between $R$ and $r$ is between $0 < r/R \leq 0.2$. In the limit $r/R \to 0$ the body is approaching a continuum.
4. Modeling wood and deformation by application of a sphere-shaped load
Chapter 5

Results

In this chapter the simulation results are presented and compared to experimental results. The experimental results and the contact mechanics of Hertz will be used to evaluate the model. The results are discussed and a conclusion is drawn. The chapter and this thesis is rounded up with a small outlook on the future showing the endless dimensions of possibilities to continue this study.

5.1 Simulation results

Deformation, stresses and strains are examined for seven different ratios $r/R$. Each ratio was simulated 20 times and the summed result distribution were mirrored at the body center axis, i.e. the axis where the center of the indenting sphere is moving along (see $y$-axis in Fig. 5.7). The reason for this procedure is to obtain a symmetric field and to make use of all the data-points effectively. The results are only shown for $r/R$ equal to 0.2 and 0.06. In Figure 5.1, four tangential-radial plane wood bodies after the indentation are shown. The simulation starts with the body surface located at $y = 0$. A small degree of deformation is observed; the surface around the indenter is suppressed and some bulking occurs directly beneath the indenter. This effect is more visible for decreasing average cell radius, i.e lower $r/R$. The stresses of the simulation are smoothed with an algorithm based on a penalized least square method, which allows fast smoothing by means of a discrete cosine transform (for more insight: Garcia (2009)) and shown in a contour plots (Fig. 5.2, 5.3). The plots are scaled with their maximum values and
Figure 5.1: The four graphics show the tangential-radial plane wood body set up including the indenter sphere for the later three graphics. For the first three the size ratio between the cell radius and indenter radius is approximately 0.2, while the last one has a ratio of approximately 0.06. The lower two are close ups. The graphics represent the body set up after the sphere was displaced 50 % of its radius in the $y$-direction. The indentation path is in negative $y$-direction.
the stresses are normalized with the contact pressure $p$. The first graphic shows the highest principal stress, which is defined as:

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \sqrt{0.25(\sigma_{xx} + \sigma_{yy})^2 + \sigma_{xy}^2}$$  \tag{5.1}$$

The second graphic shows the normal stress $\sigma_{xx}$, the third the normal stress $\sigma_{yy}$ and the fourth the shear stress $\tau_{xy}$. Directly beneath the indenter the normal stresses are compressive. The stress fields are drop-shaped and the stress gradient is highest directly below the contact circle. The normal stress in the $y$-direction (Graphic 3 in Fig.5.2) is mainly compressive and spreads through nearly the whole body. The normal stress in the $x$-direction deviates with increasing depth outwards (Graphic.2). The minimum of the principal stress (compressive) is located at the surface of the body outside of the contact circle (Graphic.1). The maximal tensile stress occurs at the same region, the contact circle, at the surface of the mesh. The shear stress maxima are located close but within the contact circle and below the surface with large deviation outwards. The deviation is deeper than the deviation of the normal stress in the $x$-direction. With decreasing average cell radius and decreasing ratio $r/R$ the stress field patterns retain the same shape, but change their magnitude and steepness. The gradient of the normal stresses below the indenter steepens, so the high stresses are more concentrated in the region below the indenter. The normal stress in the $y$-direction spreads higher up to meet the boundaries with decreasing radii ratio $r/R$. The maxima of the highest principal stress moves closer inwards and closer to the surface with decreasing radii ratio $r/R$.

In Figure 5.4 plots show the distribution of stresses at the axis beneath the indenter center and along the mesh surface for both ratios. The solid lines are the stresses for Hertzian solution. The normal stresses are negative and with increasing distance from their maximum value at the indentation center, the stresses converge to zero in $x$-and $y$-direction. With decreasing ratio $r/R \to 0$ the simulation result approaches more the Hertzian solution.

The highest principal strains $\epsilon_1$ (Graphic 1 in Fig. 5.5) are most positive at the contact circle below the surface and deviate slowly outwards and deeper into the body. The normal strain in the $x$-direction $\epsilon_{xx}$ (Graphic 2 [5.5]) is highest below the indenter in a drop shape and decreases with increasing depth. The strain is most negative at the contact circle and deviates outwards while increasing. The normal strain in the $y$-direction $\epsilon_{yy}$ (Graphic
5. Results

$r/R = 0.2$

Figure 5.2: The graphics show the result for the highest principal stress $\sigma_1$, the normal stresses $\sigma_{xx}$, $\sigma_{yy}$ and the shear stress $\tau_{xy}$ after the indention of a sphere where its radius $R$ ratio to the cell ratio $r$ is $r/R=0.2$. The stresses were normalized with contact pressure $p$. 
5.1 Simulation results

$r / R = 0.06$

Figure 5.3: The graphics show the result for the highest principal stress $\sigma_1$, the normal stresses $\sigma_{xx}$, $\sigma_{yy}$ and the shear stress $\tau_{xy}$ after the indention of a sphere where its radius $R$ ratio to the cell ratio $r$ is $r / R = 0.06$. The stresses were normalized with contact pressure $p$. 
Figure 5.4: The left graphics show the distribution of the normal stresses beneath the indenter. The right graphics show the distribution of the stresses along the surface. The stresses in the upper graphics are for the radius ratio 0.2, while the lower ones are for 0.06. The solid drawn lines are the Hertzian stresses according to 3.29. The stresses are normalized with the contact pressure $p$. 
5.1 Simulation results

\[ r/R = 0.2 \]

Figure 5.5: The graphics show the result for the highest principal strain \( \epsilon_1 \), the normal strains \( \epsilon_{xx}, \epsilon_{yy} \) and the shear strain \( \gamma_{xy} \) after the indention of a sphere where its radius \( R \) ratio to the cell ratio \( r \) is \( r/R=0.2 \).
Figure 5.6: The graphics show the result for the highest principal strain $\epsilon_1$, the normal strains $\epsilon_{xx}$, $\epsilon_{yy}$ and the shear strain $\gamma_{xy}$ after the indentation of a sphere where its radius $R$ ratio to the cell ratio $r$ is $r/R=0.06$. 

$r/R = 0.06$
Figure 5.7: Schematic of Hertzian fracture with the highest normal stress at the surface \( x_0 \), the inclination of the highest stress contour \( \theta \).

3 in Fig 5.5) is only negative and highest directly beneath the indenter, decreasing in drop-like shape with increasing depth. The maxima of the shear stress (Graphic 4 in Fig 5.5) are within the contact circle below the surface and deviate outwards.

With decreasing average cell radius \( r \) and \( r/R \) the patterns of strains retain similar shape but change their magnitude and steepness. Comparing the results for the ratio 0.06 (Fig.5.6) to ratio 0.2 (Fig.5.5) the most noticeable change is the position of the compressive maximum for the normal strain in \( y \)-direction. It moves beneath the body surface and spreads much deeper and deviates more outwards.

In Figure 5.7 the variables \( x_0 \) and \( \theta \) are illustrated, which are evaluated for different radii ratios in the Figure 5.8. In the second graphic the position (distance from the indentation center) of the highest tensile stress on the surface is plotted versus the ratio of the average cell radius divided by the indenter ratio. With increasing cell radius the distance is slowly increasing linearly. In the first graphic, the inclination of the highest stress contour \( \theta \) when

\[
y \to 0; \quad x = x_0
\]

is plotted versus the radius ratio. With increasing cell radius the angle increases linearly.
Figure 5.8: Fig1. The inclination $\theta$ of the highest stress contour when $y \to 0$ and $x = x_0$ plotted versus the ratio of cell radius $r$ to the indenter radius $R \frac{r}{R}$. Fig2. The $x$ value $x_0$ for the highest normal stress on the body surface versus the radii ratio.

### 5.2 Experimental results

#### 5.2.1 Indentation and tomography

X-ray computed micro-tomography is a non-destructive technique to measure material density throughout a volume. It is ideally suited to obtain quantitative information of wood. The equipment used, a SkyScan-1172, has a resolution high enough to distinguish and measure individual fibers and fiber fragments. The instrument has a built-in compression stage, which was used to press a small indenter into the wood. The indenter, made of steel, was needle shaped with a 200 $\mu m$ radius spherical tip. The specimen was mounted on a slowly rotating holder that allows X-rays to enter from different directions. In this way, two-dimensional projection images were taken in a multitude of directions, which allows subsequent reconstruction of a 3D micro-structure. The penetration of the tip into the wood sample was step-wise increased from 0 to 50 $\mu m$ allowing for successive 3D images showing the complete deformation process when the indenter penetrates the wood. During each scan, 998 radio-graphs with an acquisition time of 8 s each were taken over 180 degrees with a pixel size of 2.44 $\mu m$. 
5.2.2 Full field strain analysis

The strain field in the indented wood was estimated using digital image correlation (DIC) techniques on cross-sections of reconstructed 3D XCT images. The cross-section picked was the radial-tangential plane, i.e. a plane showing the cross-section of fibers, in which the center of the indenter was moving (Fig. 5.9). Then, comparing cross-section images at different penetration depths, 2D coordinates of the deformed wood internal surfaces were calculated by a 2D digital image correlation algorithm from GOM Optical Measuring Techniques (n.d.). For most materials, a random or regular pattern must be applied to the surface of the specimen, which deforms along with the object. However, for wood materials there is no need for this thanks to the inherent pattern in the wood’s micro-structure. The random variations in density and thickness of cell walls are a sufficient reference pattern. The deformation of an approximately 2x2 \( \text{mm} \) domain in the indented wood was used for the calculations. The images are first divided into a number of sub-images (15x15 pixels), called macro-image facets. Using an image correlation algorithm, these facets are tracked in each successive image with sub-pixel accuracy. Then, from the correlation procedure, a complete displacement field is obtained. By numerical spatial differentiation of the displacement field an approximation of the strain field is determined. Fig. 5.9 shows the images of the analyzed reconstructed cross-sections from XCT. The white area in the upper part is the indenter having penetrated a small distance into the wood. Superimposed are contours of the estimated in-plane strain fields (normal strains \( \epsilon_{xx} \), \( \epsilon_{yy} \) and first principal strain \( \epsilon_1 \)).

5.3 Comparison - Discussion - Conclusion

Comparing the experimental (Fig. 5.9) and theoretical strains (Fig. 3.3) lend some interesting observations. The magnitudes of experimental strains are high even at relatively large distances from the tip (even though the penetration depth is low). The classical purely linearly elastic Hertzian strain fields have considerably higher gradients and the regions of high strain are located substantially closer to the tip. The important question for a model is always how accurately it approaches reality. Comparing the experiment strain results (Fig. 5.9) with the simulation results (Fig. 5.5, 5.6), a high resemblance can be seen. The normal strain in the \( x \)-direction e.g. shows at
Figure 5.9: The experimental indenting set up and results
the contact circle its positive maximum, while the strain directly below the indenter is negative in a drop-like shape. Comparison between the experimental data and the model poses numerous challenges, such as the difference in relative densities and the ratio of average cell size to indenter radius. Despite this, the similarity of the strain plots supports the validity of the model.

Another way to evaluate the model would be to compare the case where average cell radius approaches zero with the knowledge about Hertzian indentation results. If the radius of the average cells becomes zero, the body would become a continuum, i.e. the model would become a Hertzian contact problem. This means that the result for a very small $r$ should be similar to the Hertzian. The stress/strain fields (Fig. 5.2, 5.3, 5.5, 5.6) for the simulation in general look similar to the classical Hertz stress/strain fields (Fig. 3.2, 3.3) with their drop shaped zone beneath the contact circle, the maximum tensile stresses at the contact circle, falling off with increasing radial contact from the contact center and the stress gradient steepest to the contact circle \cite{Atkinson et al. 2012, Huber (1904)}. It can be said, that with decreasing average cell radius the resulting fields become more matching with the theoretical Hertz solution. Especially interesting is the development of the position $x_0$ of the highest normal stress on the body surface and the inclination $\theta$ of the highest stress contour at this position with decreasing average cell radius (Fig. 5.8). In this region cracking is expected to occur. The resulting values for very small $r$, especially the inclination are matching the results for the Hertzian solution well. Taking this into account the model seems to be fair.

According to the simulation results it can be said the geometry of a spherical indenter has an impact on the wood mesh while indenting. With increasing indenter radius the distance $x_0$ where the highest principal stress (tensile) occurs also increases, but with different gradients. The same applies for the inclination $\theta$ of the highest stress contour at the surface and at $x_0$. Interestingly, the ratio between the average cell radius and the indenter radius $r/R$ can be described as linearly dependent to $x_0/R$ and $\theta$. While it could be expected that the distances would change, the change in inclination is the most fascinating. It is also interesting that the stresses are higher in the sparse structure ($r/R = 0.2$) than in the dense ($r/R \rightarrow 0$). This is however not surprising, because less material have to carry load, i.e. higher stresses in the cell walls.
Aside from these relations, it can also be seen that the stress and strain patterns and especially steepness change with the length of the indenter radius if the body stays the same. This means that by choosing an indenter size, the distance at which the highest principal stress (tensile) occurs can be controlled. This determines where a crack is most likely to occur, and what inclination it will have. It is also possible to for example control how close to the surface the highest strain should occur. These features could be used to optimize the mechanical pulp production. For example with increasing \( \frac{r}{R} \) (i.e. cell radii become bigger compared to \( R \)) the angle of the contour with the highest stresses becomes steeper meaning that larger pieces may crack.

5.4 Outlook

It has been shown that the geometry of the indenter has an impact on the wood body considering the way and where the sample may crack, however till making an actual use of it in the industry, there is a way to go. Much work remains before practical application is possible. This study was limited to a small range of radii for one spherical indenter on a body, which could be deformed but not cracked in any kind of way. In order to better understand the mechanical properties of wood and optimize the mechanical pulp production, other indenter geometries and configurations should be explored. This study employed a single indenter. In a real application there would many indenting tools acting simultaneously, i.e. the used grinding stones has million of indenting nodes covering its surface. The model can also be improved to include fracturing, allowing the body to crack when a node is exposed at a threshold force. It would be interesting and essential for the fiber separating process to see how the crack spreads in the body and whether it would be along or across the cell walls under different indentions. Apart from this, different constitutive relations beyond linear elastic might give better results. A differentiation between relations along cell walls and across cell walls and using measured modulus values might further improve results. Much remains to be discovered about the contact mechanics of wood.
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