Numerical option pricing without oscillations using flux limiters

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Abstract
A numerical method is developed for solution of Black-Scholes equation avoiding the oscillations that are common close to a discontinuity in the pay-off function. Part of the derivatives are evaluated explicitly and part of them are computed implicitly using operator splitting. The method is second order accurate in time and almost of second order in the asset price for smooth solutions and no system of nonlinear equations has to be solved. A flux limiter modifies the first derivative in the equation such that no oscillations occur in the solution in the numerical examples presented.
Keywords: Black-Scholes equation, option pricing, discontinuities, flux limiters
2010 MSC: 65M06

1. Introduction

In this note we consider the numerical solution of the Black-Scholes equation
\[ u_t(t, s) + \mathcal{L}u = u_t(t, s) + \mathcal{F}u + \mathcal{G}u = 0, \quad s \in \mathbb{R}^+, \quad 0 \leq t \leq T, \] (1)
where
\[
\begin{align*}
\mathcal{F}u &= -rsu_s(t, s), \\
\mathcal{G}u &= -\frac{1}{2}\sigma^2(t, s)s^2u_{ss}(t, s) + ru(t, s),
\end{align*}
\] (2)

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and a subscript $s$ or $t$ denotes derivation with respect to the variable. The solution $u$ to (1) and (2) determines the price of an option issued on the underlying asset $s$. The parameter $r \in \mathbb{R}^+$ is the short rate of interest and $\sigma \in \mathbb{R}^+$ determines the volatility of $s$. We have transformed the original final-value problem to an initial-value problem in forward time for simplicity of notation. The initial condition is given by

$$u(0, s) = \Phi(s),$$

where $\Phi(s)$ is the so-called pay-off function, i.e., the value of the option at time of maturity.

For many classes of problems, $\Phi$ and/or its derivatives are discontinuous. Standard time-integration schemes such as the Crank-Nicolson method [4] often fail to produce accurate numerical solutions due to oscillations arising from the discontinuities. For this reason, the so-called Rannacher scheme [11] is often employed [3], [10]. In this method a number of initial time-steps with the backward Euler method are preceding the Crank-Nicolson method in order to damp the oscillations occurring from the discontinuities in the pay-off function. However, for interest rate dominated problems, i.e., when $r$ is significantly larger than $\sigma$, also the Rannacher scheme gives rise to oscillations in the solution.

In the 1970’s, so-called flux limiters were introduced to handle shocks present in fluid flow problems, see e.g., [5], [7, p. 115]. This methodology has also been utilized for option pricing problems [9], [14]. Problems that are of convection-diffusion type like (1) often need to be discretized implicitly in time to avoid a severe restriction on the time step due to the second derivative in (1). The flux limiters that are effective for problems with discontinuities are nonlinear, including maxima, minima, and absolute values. A fully implicit discretization introduces an iteration in the scheme because of the nonlinearity. This complicates the solution procedure considerably since Newton iterations are not applicable.

To obtain a method that can handle discontinuities, yields stable discretizations in time, and does not introduce a nonlinear iteration in each time step,
we will use operator splitting in time \[12\]. It is of second order accuracy in time and almost of second order in the asset price and is parameter-free. The part $\mathcal{G}$ that includes second derivatives is treated implicitly in time for stability reasons while the first order part $\mathcal{F}$ is treated explicitly. This way, flux limiters can be used for $\mathcal{F}$ where it is needed to avoid oscillatory solutions, still not causing nonlinearities on the implicit side. The scheme can be extended to more dimensions in the asset price by dimensional splitting of $\mathcal{F}$ as is common in computational fluid dynamics \[7, \text{Ch. 19.5}\]. It can also be applied to other types of pay-off functions of low regularity than those tested here.

The outline of the rest of the paper is as follows. In Section \[2\] the discretization in space and time is presented. The method is analyzed with respect to accuracy and stability in Section \[3\] and numerical results for a European call option and a binary option are presented in Section \[4\]. Finally we discuss our method and draw conclusions in Section \[5\].

2. Discretization

We consider a discretization of (1) in the asset price and time on a structured and equidistant grid $(t^n, s_j), t^n = n\Delta t, s_j = jh$. The operator $\mathcal{G}$ is approximated using centered second order finite differences

$$
\mathcal{G}u_j = -\frac{1}{2}\sigma_j s_j u_{j+1} - \frac{2u_j + u_{j-1}}{h^2} + ru_j.
$$

To avoid spurious oscillations we use flux limiters for the first order operator $\mathcal{F}$ as in \[7, \text{Ch. 6}\]

$$
\mathcal{F}u_j = -r s_j \chi \frac{\Delta_+ u_j - (f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}})}{h},
$$

with

$$
\Delta_+ u_j = u_{j+1} - u_j, \quad \chi = 1 + \frac{1}{2}\Delta tr,
$$

$$
f_{j-\frac{1}{2}} = \frac{1}{2} (1 - \frac{\Delta t}{h} r s_j \chi) \delta_{j-\frac{1}{2}}, \quad f_{j+\frac{1}{2}} = \frac{1}{2} (1 - \frac{\Delta t}{h} r s_j \chi) \delta_{j+\frac{1}{2}}.
$$

Here

$$
\delta_{j+\frac{1}{2}} = \phi(\Delta_+ u_{j+1}, \Delta_+ u_j),
$$

3
and \( \phi \) is the so called flux limiter function. We consider the minmod-limiter defined by

\[
\phi(a, b) = \begin{cases} 
0 & \text{if } ab \leq 0, \\
a & \text{if } |a| < |b|, \\
b & \text{if } |b| < |a|.
\end{cases}
\] (8)

In time we will employ a Strang operator splitting [12] that allows us to treat \( G \) implicitly in time and \( F \) explicitly. The implicit scheme to solve \( u_t + Gu = 0 \) from \( t^{n-1} \) to \( t^n \) is the TR-BDF2 [1], [13]

\[
\tilde{u}_n^j = u_{n-1}^j - \frac{\Delta t}{4} (G u_{n-1}^j + G \tilde{u}_n^j),
\]

\[
u_n^j = \frac{1}{3} (4\tilde{u}_n^j - u_{n-1}^j - \Delta t G u_n^j).
\] (9)

The explicit scheme to solve \( u_t + Fu = 0 \) from \( t^{n-1} \) to \( t^n \) is

\[
u_n^j = u_{n-1}^j - \Delta t F u_{n-1}^j.
\] (10)

A nonlinearity is introduced in \( F \) by the flux limiter [8] but it is applied explicitly in (10) to \( u_{n-1}^j \) with low additional computational complexity. The limiter will be active to avoid oscillatory behavior close to near discontinuities in the solution.

The solution to (1) is advanced one time step \( \Delta t \) with Strang splitting in two alternative ways. The first scheme \((GFG)\) is:

1. Compute \( u_{n-1/2}^j \) at \( t^{n-1/2} \) using (9) with time step \( \Delta t/2 \) and \( u_{n-1}^j \) as initial data.
2. Compute \( \tilde{u}_n^j \) at \( t^n \) using (10) with time step \( \Delta t \) in (5), (6), and (10) and \( u_{n-1/2}^j \) as initial data.
3. Compute \( u_n^j \) at \( t^n \) using (9) with time step \( \Delta t/2 \) and \( \tilde{u}_n^j \) as initial data.

The second scheme \((FGF)\) is:

1. Compute \( u_{n-1/2}^j \) at \( t^{n-1/2} \) using (10) with time step \( \Delta t/2 \) in (5), (6), and (10) and \( u_{n-1}^j \) as initial data.
2. Compute \( \tilde{u}_n^j \) at \( t^n \) using (9) with time step \( \Delta t \) and \( u_{n-1/2}^j \) as initial data.
3. Compute \( u^n_j \) at \( t^n \) using (10) with time step \( \Delta t/2 \) in (5), (6), and (10) and \( \hat{u}_j^n \) as initial data.

Both schemes are one step schemes where the solution \( u^n_j \) at \( t^{n-1} \) is integrated to \( u^n_j \) at \( t^n \). Two systems of linear equations are solved each time we use the TR-BDF2 in (9). In one \( s \)-dimension, the computational cost for solving these systems is low.

The generalization of the scheme to two \( s \)-dimensions \((s_1, s_2)\) is straightforward for a Cartesian grid. In the implicit part (9), the system of linear equations to be solved has a block-tridiagonal matrix with tridiagonal blocks. These systems can be solved either with a banded direct solver or a Krylov subspace iteration method. The approximation of the first derivatives is computed by applying \( \mathcal{F} \) to the solution at \( t^{n-1} \) first in one coordinate direction and then in the other coordinate direction. These contributions are then summed and multiplied by \( \Delta t \) to update the solution as in (10) \([7\text{, Ch. 19.3.2}]\). An operator splitting approach is possible also here by first advancing the solution in time by the \( s_1 \)-derivative and then by the \( s_2 \)-derivative either consecutively or using Strang splitting \([7\text{, Ch. 19.5}]\).

The accuracy and stability of the splitting methods are analyzed in the next section.

3. Accuracy and stability

The accuracy of (10) with \( \mathcal{F} \) in (2) is analyzed by considering the approximation of the first derivative. The solution of

\[
u_t - rsu_s = 0
\]

is advanced from \( t^{n-1} \) to \( t^n \) by expanding the solution in a Taylor series in time

\[
w^n = u^{n-1} + \Delta t u^n_t - 1 + \frac{1}{2} \Delta t^2 u^n_{tt} - 1 + \frac{1}{6} \Delta t^3 u^n_{ttt} - 1 + O(\Delta t^4).
\]

The terms \( u^n_t - 1 + \frac{1}{2} \Delta t u^n_{tt} - 1 \) approximate \( u_t \) at \( t^{n+\frac{1}{2}} \) in (12). Then \( u_t \) and \( u_{tt} \) in this expression are replaced by derivatives in \( s \) using (11). Introduce the
s-derivatives into (12) to obtain
\[ u^n = u^{n-1} + \Delta t s u_{s_{_{n-1}}} + \frac{1}{2} \Delta t^2 (r^2 s u_{s_{_{n-1}}} + (rs)^2 u_{ss_{_{n-1}}}) + \frac{1}{6} \Delta t^3 u_{sss_{_{n-1}}} + O(\Delta t^4) \]

\[ = u^{n-1} + \Delta t s (1 + \frac{1}{2} \Delta t r) u_{s_{_{n-1}}} + \frac{1}{2} (\Delta t s r) u_{ss_{_{n-1}}} + O(\Delta t^3) \]

\[ = u^{n-1} + \Delta t s (1 + \frac{1}{2} \Delta t r) u_{s_{_{n-1}}} + \frac{1}{2} (\Delta t s r (1 + \frac{1}{2} \Delta t r)) u_{ss_{_{n-1}}} + O(\Delta t^3) \]

\[ = u^{n-1} + \Delta t s (1 + \frac{1}{2} \Delta t r) u_{s_{_{n-1}}} + \frac{1}{2} (\Delta t s (1 + \frac{1}{2} \Delta t r))^2 u_{ss_{_{n-1}}} + O(\Delta t^3). \] (13)

The approximation of \( u^n \) is of formal second order in \( \Delta t \) in (13) and the same factor \( rs \chi \) is multiplying both \( u_s \) and \( u_{ss} \) as assumed in the derivation of the limiters in [7, Ch. 6]. It is noted in [7, pp. 163, 181] that ignoring the correction \( \chi \) for a variable coefficient in front of \( u_s \) formally decreases the order of accuracy but in practice the difference in the error is small. Then the two terms depending on \( u_{n-1}s \) and \( u_{n-1}ss \) are approximated as in (5).

The scheme (9) for the diffusive part of (1) is of second order accuracy in time [1]. Combining two second order accurate schemes in operator splitting as in GFG or FGF in Section 2 is also second order accurate in time [12]. Suppose that the solution is smooth such that the limiter in (7) is \( \delta_{j+\frac{1}{2}} = \Delta_+ u_j \) for \( j \) and \( j-1 \). Then \( F u_j^{n-1/2} \) in (10) is

\[ F u_j^{n-1/2} = -\frac{rs_j \chi}{2h} (u_{j+1} - u_{j-1}) - \frac{\Delta t (rs_j \chi)^2}{2h^2} (u_{j+1} - 2u_j + u_{j-1}). \] (14)

This approximation is of second order in the approximation of the s-derivatives in (13). The order of accuracy is lowered to one at extrema of the solution by the limiter [7, Ch. 8.4]. If we are interested only in \( u(T) \), then we can merge the last half step with the first half step in GFG or FGF into one full step with \( \Delta t \). This is possible for all inner time steps except for the first one at \( t = 0 \) and the last one to reach \( t = T \) [7, Ch. 17.4], [12, p. 511]. This procedure reduces the number of \( F \) or \( G \) steps by almost a factor 2 and is second order accurate in time at \( T \).

The stability of GFG and FGF is evaluated by freezing the coefficients in (9), (10), (12), and (14) at \( s \) and assuming a periodic, smooth solution \( u_j^n = u(t^n, s_j) = z^n \exp(i\omega jh), j = 0, 1, \ldots, N - 1, \) in a finite interval \( s \in [0, \varsigma] \) for \( \omega = k \Delta \omega, k = 0, 1, \ldots, N - 1, \) with \( \Delta \omega = 2\pi/\varsigma \) and \( N = \varsigma/h \). Insert \( u_j^{n-1} \) into
Figure 1: The stability regions for the two alternatives $GFG$ (left) and $FGF$ (right) with $\Delta tr = 0.005$ and $\xi = \Delta tr \rho \chi / h$ and $\eta = \Delta t (\rho s / h)^2$ on the axes. The methods are stable to the left of the solid line.

To obtain the polynomial fulfilled by $z$, with

$$f_\omega(\Delta t) = -i \frac{\Delta r}{2} \rho s \chi \sin(\omega h) + 2 \left( \frac{\Delta r}{2} \rho s \chi \right)^2 \sin(\omega h / 2)^2,$$

$$g_\omega(\Delta t) = \left( 2 \left( \frac{\rho s}{h} \right)^2 \sin(\omega h / 2)^2 + r \right) \Delta t,$$

the growth factors for the explicit and the implicit steps are

$$F_\omega(\Delta t) = 1 - f_\omega(\Delta t), \quad G_\omega(\Delta t) = \frac{1}{1 + g_\omega(\Delta t) / 3} \left( 4 \frac{1 - g_\omega(\Delta t) / 4}{3} \frac{1}{1 + g_\omega(\Delta t) / 4} - \frac{1}{3} \right).$$

(15)

For stability, $z = F_\omega(\Delta t / 2)G_\omega(\Delta t)F_\omega(\Delta t / 2)$ for $FGF$ and $z = G_\omega(\Delta t / 2)F_\omega(\Delta t)G_\omega(\Delta t / 2)$ for $GFG$ must satisfy $|z| \leq 1$ for all $\omega$. A stability analysis with frozen coefficients is often indicative of the stability properties of the problem with variable coefficients.

The stability regions are shown in Figure 1 with $\xi = \Delta tr \rho \chi / h$ on the $x$-axis and $\eta = \Delta t (\rho s / h)^2$ on the $y$-axis. When $r = 0$ along the $y$-axis, the stability is given by the TR-BDF2 method, which is L-stable, i.e. it is stable for any $\eta \geq 0$ and $G_\omega(\Delta t) \to 0$ when $\eta \to \infty$ [1]. A method with this property will damp rapid spatial oscillations in the solution contrary to the Crank-Nicolson method. There is a CFL-condition on the length of $\Delta t$ for a constant $\eta$ as expected since the first derivative is integrated explicitly. When $\eta = 0$ the time step is restricted by $\xi \approx 1$ for $GFG$ and by $\xi \approx 2$ for $FGF$. The stability constraint on $\Delta t$ in 10 with 14 is $\xi \leq 1$. Since the time step is $\Delta t / 2$ in
FGF, the stability region in $\xi$ is expanded to the right compared to GFG. By increasing $\sigma_s$ we can make $|G_\omega|$ in (16) as small as we wish. Hence, for any $r_s$ there is a $\sigma_s$ sufficiently large such that $|z| \leq 1$ and the method is stable. The stability region is larger with FGF for small $\eta$ but is larger with GFG for larger $\eta$. The effect of changing $\Delta tr$ between 0 and 0.01 is small.

4. Numerical results

We consider two types of options, the European call option with $\Phi = \max(s - K, 0)$ and a binary option with $\Phi = 0$ for $s < K$ and $\Phi = 1$ for $s \geq K$. The parameters in (1) and (2) are $r = 0.1$, $K = 25$ and $T = 0.5$. Both $\sigma = 0.3$ and $\sigma = 0.03$ were used in order to evaluate the performance of the methods. At $s = 0$ and $s = s_{\text{max}} = 4K$ the solution $u$ was approximated to be linear. Five methods were compared:

- $FGF_\chi - FGF$ with $\chi$ defined in (6),
- $GFG_\chi - GFG$ with $\chi$ defined in (6),
- $FGF_1 - FGF$ with $\chi = 1$,
- $GFG_1 - GFG$ with $\chi = 1$,
- Rann - Rannacher time-stepping [11] with centered second order finite differences in space. First, four steps with the Euler backward method with $\Delta t/2$ are taken and then the solution is advanced by the Crank-Nicolson method with full time steps $\Delta t$.

In Figures 2-5, the solution $u$ and $\Delta = \frac{\partial u}{\partial s}$ and $\Gamma = \frac{\partial^2 u}{\partial s^2}$ are displayed together with the respective errors at $t = T$ when comparing with the analytical formulas. The hedging parameters $\Delta$ and $\Gamma$ are approximated from the computed solution $u$ as $\Delta \approx \frac{(u_{j+1} - u_{j-1})}{2\Delta s}$ and $\Gamma \approx \frac{(u_{j+1} - 2u_j + u_{j-1})}{\Delta s^2}$. Here we have used $M = 100$ number of grid points in space and $N = M/10$ number of grid points in time with $\Delta t = T/N$. The convergence rates of the five methods are presented in Figures 6-9. The norm of the error was computed as
\[ \|u - u_{\text{analytical}}\|/\|u_{\text{analytical}}\| \] where both the 2-norm and the max-norm were considered. The analytical functions are given by

\[ u_{\text{analytical}} = N(d_1)s - N(d_2)Ke^{-rT}, \quad u_{\text{analytical}} = N(d_2)e^{-rT} \]

for the European call option and the binary option respectively. Here \( N(x) \) denotes the standard normal cumulative distribution function defined by

\[ N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz \]

and

\[ d_1 = \frac{\ln(s/K) + (r + \sigma^2/2)T}{\sigma \sqrt{T}}, \quad d_2 = \frac{\ln(s/K) + (r - \sigma^2/2)T}{\sigma \sqrt{T}}. \]

Similar analytical formulas are available for the hedging parameters \( \Delta \) and \( \Gamma \).

From Figures 2–9 we see that the accuracy using solution methods \( FGF_\chi, GFG_\chi \) and \( Rann \) are similar with the exception of the binary option with \( \sigma = 0.03 \) where the Rannacher scheme fails to produce an accurate solution. The solution computed with this method exhibits an overshoot at the strike-price in Figure 5 while the methods based on flux-limiters produce accurate, non-oscillatory solutions. For the European call option these three methods perform equally well. The parameter \( \Delta \) for the binary option shows a small overheat for all methods when \( \sigma = 0.03 \) since the flux-limiters that could avoid this are not acting on the derivatives of the solution but only on the solution itself. Moreover, it is clear that \( FGF_\chi \) and \( GFG_\chi \) yield second order convergence for the European call option while \( FGF_1 \) and \( GFG_1 \) in some cases fail to do that as predicted by theory in (13). Also the accuracy in \( u \) is deteriorating for increasing \( s \) with \( FGF_1 \) and \( GFG_1 \) in Figure 2. For the binary option the convergence order is restricted to 1 due to the discontinuity in \( u \) at \( t = 0 \).
Figure 2: The solution $u$ and hedging parameters $\Delta$ and $\Gamma$ for a European call option in the top row and the respective errors in the bottom row for $\sigma = 0.3$. $FGF_\chi$ is marked with $\triangledown$, $GFG_\chi$ with $\ast$, $FGF_1$ with $\square$, $GFG_1$ with $\star$, $Rann$ with $\circ$ and the analytical solution with a solid line.

Figure 3: The solution $u$ and hedging parameters $\Delta$ and $\Gamma$ for a European call option in the top row and the respective errors in the bottom row for $\sigma = 0.03$. $FGF_\chi$ is marked with $\triangledown$, $GFG_\chi$ with $\ast$, $FGF_1$ with $\square$, $GFG_1$ with $\star$, $Rann$ with $\circ$ and the analytical solution with a solid line.
Figure 4: The solution $u$ and hedging parameters $\Delta$ and $\Gamma$ for a binary option in the top row and the respective errors in the bottom row for $\sigma = 0.3$. $FGF_X$ is marked with $\triangledown$, $GFG_X$ with $\ast$, $FGF_1$ with $\Box$, $GFG_1$ with $\ast$, $Rann$ with $\circ$ and the analytical solution with a solid line.

Figure 5: The solution $u$ and hedging parameters $\Delta$ and $\Gamma$ for a binary option in the top row and the respective errors in the bottom row for $\sigma = 0.03$. $FGF_X$ is marked with $\triangledown$, $GFG_X$ with $\ast$, $FGF_1$ with $\Box$, $GFG_1$ with $\ast$, $Rann$ with $\circ$ and the analytical solution with a solid line.
Figure 6: The error for increasing \( N \) in the solution \( u \) and the hedging parameters \( \Delta \) and \( \Gamma \) for a European call option in the 2-norm (top row) and the max-norm (bottom row) for \( \sigma = 0.3 \). \( FGF_{\chi} \) is marked with \( \bigtriangledown \), \( GFG_{\lambda} \) with \( * \), \( FGF_1 \) with \( \square \), \( GFG_1 \) with \( * \) and \( Rann \) with \( \circ \).

Figure 7: The error for increasing \( N \) in the solution \( u \) and the hedging parameters \( \Delta \) and \( \Gamma \) for a European call option in the 2-norm (top row) and the max-norm (bottom row) for \( \sigma = 0.03 \). \( FGF_{\chi} \) is marked with \( \bigtriangledown \), \( GFG_{\lambda} \) with \( * \), \( FGF_1 \) with \( \square \), \( GFG_1 \) with \( * \) and \( Rann \) with \( \circ \).
Figure 8: The error for increasing $N$ in the solution $u$ and the hedging parameters $\Delta$ and $\Gamma$ for a binary option in the 2-norm (top row) and the max-norm (bottom row) for $\sigma = 0.3$. $FGF_\chi$ is marked with ▽, $GFG_\chi$ with ★, $FGF_1$ with □, $GFG_1$ with ★ and $Rann$ with ◦.

Figure 9: The error for increasing $N$ in the solution $u$ and the hedging parameters $\Delta$ and $\Gamma$ for a binary option in the 2-norm (top row) and the max-norm (bottom row) for $\sigma = 0.03$. $FGF_\chi$ is marked with ▽, $GFG_\chi$ with ★, $FGF_1$ with □, $GFG_1$ with ★ and $Rann$ with ◦.
Finally in Figure 10 we show the 2-norm of the error in the computed solution at \( t = T \) for \( \sigma = 0 \) and \( N = 5 \). We vary \( M \) such that \( \xi = \Delta t rs\chi/h \) falls in the range 0 to 3 for \( s = K \), which is where the method first will be unstable. The plot verifies that for both the European call option and the binary option the stability criterion is \( \xi \leq 1 \) for \( GFG\chi \) and \( \xi \leq 2 \) for \( FGF\chi \).

![Figure 10: The error in the solution \( u \) for a European option (top) and binary option (bottom) when \( \sigma = 0.0 \). \( FGF\chi \) is marked with ▽ and \( GFG\chi \) with ∗.](image)

5. Conclusions

We have proposed and tested a numerical method without parameters for the Black-Scholes equation suitable for discontinuous pay-off functions. It is second order accurate in time \( t \) and almost of second order in the asset price \( s \) for smooth solutions. Part of the Black-Scholes operator is advanced in time explicitly and part of it implicitly in the operator splitting scheme due to Strang. Flux limiters are introduced to avoid oscillations in the solution close to discontinuities in the pay-off function. Two \((FGF)\) or four \((GFG)\) systems of linear equations are solved in every time step. This is reduced to two systems for both methods if we are satisfied with the solution only at the final time. The method is
applied to a European call option and a binary option. In some cases our schemes perform equally well as the commonly used Rannacher scheme but for the binary option and a small volatility $\sigma$ in Figures 5 and 9, which is the most difficult type of problem, our scheme is superior. Also, for a European option and $\sigma = 0.3$ the split method is slightly more accurate than the Rannacher method in Figures 2 and 6. The accuracy of the schemes $FGF_{\chi}$ and $GFG_{\chi}$ is almost indistinguishable in the numerical experiments but $FGF_{\chi}$ is stable for longer time steps with a small $\sigma$ than $GFG_{\chi}$ and requires only two solutions of linear systems in each step for a time series of solutions making it the preferred choice.

6. Acknowledgment

LvS was supported by the Swedish Research Council under contract number 621-2007-6388.

The authors are thankful to Master thesis student Wei Hu for his early contribution in [5].

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