<http://www.diva-portal.org>

Postprint

This is the accepted version of a paper published in *International Journal of Electrical Power & Energy Systems*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Guo, W., Yang, J., Yang, W., Chen, J., Teng, Y. (2015)

Regulation quality for frequency response of turbine regulating system of isolated hydroelectric power plant with surge tank.

International Journal of Electrical Power & Energy Systems, 73: 528-538

<http://dx.doi.org/10.1016/j.ijepes.2015.05.043>

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-260775>

Regulation quality for frequency response of turbine regulating system of isolated hydroelectric power plant with surge tank

Wencheng Guo ^{a,*}, Jiandong Yang ^a, Weijia Yang ^b, Jieping Chen ^a, Yi Teng ^a

^a State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan, 430072, China

^b Department of Engineering Sciences, Uppsala University, Uppsala, SE-751 21, Sweden

Abstract: Aiming at the isolated hydroelectric power plant (HPP) with surge tank, this paper studies the regulation quality for frequency response of turbine regulating system under load disturbance. Firstly, the complete mathematical model of turbine regulating system is established and a fifth order frequency response under step load disturbance is derived. Then, the method of primary order reduction and secondary order reduction, for this complete fifth order system of frequency response, is proposed based on dominant poles. By this method, the complete fifth order system is solved and the regulation quality for frequency response is studied. The results indicate that the complete fifth order system always has a pair of dominant conjugate complex poles and three non-dominant poles. The primary fourth order equivalent system, which is obtained by primary order reduction, keeps the dominant poles almost unchanged, therefore it can represent and replace the complete fifth order system and it is obviously superior to other fourth order systems. The primary fourth order equivalent system is superimposed by two second-order subsystems, one of them is corresponding to two non-dominant real poles (i.e. head wave) and the other one is corresponding to a pair of dominant conjugate complex poles (i.e. tail wave), respectively. In the fluctuation process of frequency response, head wave decays very fast and works mainly in the beginning period while tail wave decays very slowly, fluctuates periodically and works throughout the period. The secondary order reduction of complete fifth order system can be conducted by using the second order system of tail wave, which is the main body of frequency response, to represent the fluctuation characteristics. The most important dynamic performance index that evaluates the regulation quality, i.e. settling time, is derived from the fluctuation equation of tail wave. The different characteristic parameters of turbine regulating system have different influences on the change rules of head wave, tail wave and settling time.

Keywords: hydroelectric power plant; surge tank; isolated operation; turbine regulating system; frequency response; regulation quality; dominant pole

1. Introduction

One of the main tasks of power system operation is to control grid frequency in the allowable variation range of rated frequency to ensure grid power quality when grid load changes, and load frequency control (LFC) is the primary measure to accomplish this task [1]. In modern power system, hydroelectric power plant (HPP) undertakes the major task of peak modulation (PM) and frequency modulation (FM) due to its characteristic of flexible operation. LFC of hydroelectric generating units is actualized by turbine control system (the core component is governor) [2].

Grid-connected operation and isolated operation are the main operation modes of hydroelectric generating units. The former is the normal and primary operation mode and the later serves in accidental and transient cases.

For grid-connected operation mode, LFC is implemented by the automatic generation control (AGC)

* Corresponding author. Tel.: +86 027 68772274; fax: +86 027 68772310.

E-mail address: wench@whu.edu.cn (W. Guo).

system of grid and AGC system of power plant on the turbine control system to accomplish primary frequency regulation (PFR), secondary frequency regulation (SFR) and exchange power control among regional grids, etc. There are considerable researches in this field [3-11]. Some advanced control strategies such as fuzzy control [6,7], robust control [8,9] and intelligent discontinuous control [10] were applied to the design of load frequency controller, and good control effects were achieved.

Under isolated operation mode, the operating conditions of governor are complicated due to the influence of the value of load change and the load characteristics of isolated grid et al. Hence, maintaining the grid frequency within a certain range is indeed a challenge. National standard of China [12] stipulates the limit value of decay rate for frequency response of isolated HPP under load disturbance. In practical cases, the frequency response is required to have good regulation quality on the premise of the satisfaction of stability. As to the issue of regulation quality of frequency response, references [13-15] carried out theoretical analysis, model test and numerical simulation on HPP without surge tank, and analyzed the setting and effect of governor parameters; however, there is little research on HPP with surge tank. It is well recognized that surge tank is indeed an important measure of pressure reduction. Since the influence of water-level fluctuation in surge tank, the wave form of frequency response shows the characteristic of head wave and tail wave [16], which is significantly different from the case without surge tank (shown in Fig. 1). The fluctuation characteristics of head wave and tail wave and the effect of system parameters on wave form were analyzed by numerical simulation [16-18]. By solving the fluctuation equation of tail wave, the fluctuation characteristics of tail wave and its relationship with regulation quality were studied [19,20]. It can be found that the researches in references [16-20] exist two major limitations. Firstly, most of the research techniques were numerical simulation. Due to the lack of theoretical analysis, the essence and change rule of regulation quality could not be revealed and recognized clearly. Secondly, in the process of theoretical analysis, the mathematical model of turbine regulating system was simplified too much to reduce the order of overall transfer function, because high order equations, especially fifth-order (or higher) equations, were difficult to analyze theoretically. Specifically, reference [19] obtained a second order overall transfer function by assuming that the water-level fluctuation in surge tank was a sinusoidal wave and neglecting the water inertia of penstock and the head loss of entire pipeline; reference [20] obtained a fourth order overall transfer function by neglecting the water inertia and head loss of penstock. The complete mathematical model was not established and the simplified models could not represent the original turbine regulating system accurately.

Fig. 1. Head wave and tail wave of time response of the frequency under load disturbance [16].

This paper aims to overcome the above two limitations and further study the regulation quality for frequency response of turbine regulating system in isolated HPP with surge tank under load disturbance. **There are two motivations: (1) for isolated HPP with surge tank, reveal the relationship between the dominant / non-dominant poles and the different terms of denominator of overall transfer functions, and then establish the corresponding relation between the dominant / non-dominant poles and the**

head / tail wave of time response of the frequency; (2) propose the method of order reduction for high order system (fifth-order), and then work out the analytic formulae of fluctuation characteristic parameters which evaluate the control quality (the main motivation). The results obtained in this paper can provide a guidance for the improvement of control quality of hydroelectric generator unit with surge tank under isolated operation.

This article is organized as follows. In Section 2, the complete mathematical model of turbine regulating system that includes all subsystems (i.e. headrace tunnel, surge tank, penstock, turbine, generator and governor) is established. In Section 3, the overall transfer function is derived from complete mathematical model. In Section 4, the frequency response under step load disturbance is derived from overall transfer function. Then primary order reduction, which overcomes the problem of the solution of fifth order system, is proposed based on dominant poles. In Section 5, the fluctuation equations of head wave and tail wave of frequency response are solved from the equivalent system obtained by primary order reduction. Based on their fluctuation equations, the formation mechanisms, change rules and relationships with regulation quality of head wave and tail wave are revealed. Then secondary order reduction is proposed based on dominant poles, by using tail wave to represent the fluctuation characteristics of frequency response. According to the fluctuation equation of tail wave, the most important index of dynamic regulation quality, i.e. settling time, is obtained and the effects of influencing factors on settling time are investigated.

2. Mathematical model

The turbine regulating system of isolated HPP with surge tank is illustrated in Fig. 2, and its complete mathematical model that includes headrace tunnel, surge tank, penstock, turbine, generator and governor can be established as follows:

Fig. 2. Turbine regulating system of isolated grid operation hydroelectric power plant with surge tank.

(1) Controlled system [21-24]

Momentum equation of headrace tunnel:

$$h_F = -T_{wy} \frac{dq_y}{dt} - \frac{2h_{y0}}{H_0} q_y \quad (1)$$

Continuity equation of air cushion surge tank containing state characteristic of gas (conventional open-type surge tank is the special case of air cushion surge tank):

$$q_y = \frac{FH_0}{(1 + mFp_0 / \nabla_0)Q_0} \frac{dh_F}{dt} + q_t \quad (2)$$

Momentum equation of penstock:

$$h_F = T_{wt} \frac{dq_t}{dt} + \frac{2h_{t0}}{H_0} q_t + h \quad (3)$$

Moment equation and discharge equation of turbine:

$$m_t = e_h h + e_x x + e_y y \quad (4)$$

$$q_t = e_{qh} h + e_{qx} x + e_{qy} y \quad (5)$$

First derivative differential equation of generator:

$$T_a \frac{dx}{dt} = m_t - (m_g + e_g x) \quad (6)$$

(2) Turbine control system [21-24]

Equation of governor:

$$b_t T_d \frac{dy}{dt} = -(T_d \frac{dx}{dt} + x) \quad (7)$$

The nomenclatures in Eqs. (1)-(7) are presented in Appendix A.

3. Derivation of overall transfer function

For the situation of grid load disturbance, the block diagram of turbine regulating system is obtained from Eqs. (1)-(7) and shown in Fig. 3, where $G_s(s)=H(s)/Q_t(s)$ is the transfer function of pipeline system and it is derived from the Laplace transform of Eqs. (1)-(3). s is complex variable.

Fig. 3. Block diagram of turbine regulating system.

According to Fig. 3 and the Laplace transform of Eqs. (1)-(7), the following overall transfer function of turbine regulating system is obtained ($M_g(s)$ and $X(s)$ are the Laplace transforms of load disturbance m_g and frequency response x , respectively, and the former is input signal and the later is output signal.):

$$G(s) = \frac{X(s)}{M_g(s)} = -\frac{b_t T_d s (b_0 s^3 + b_1 s^2 + b_2 s + b_3)}{a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5} \quad (8)$$

The expressions of coefficients in Eq. (8) are presented in Appendix B.

4. Primary order reduction of turbine regulating system

The input signal for a step load disturbance can be computed from $M_g(s)=m_{g0}/s$, in which m_{g0} is relative value of the load step. Substitution of $M_g(s)=m_{g0}/s$ into Eq. (8) yields the following output signal for frequency response.

$$X(s) = -b_t T_d m_{g0} \frac{\sum_{i=0}^3 b_i s^{3-i}}{\sum_{i=0}^5 a_i s^{5-i}} \quad (9)$$

Eq. (9) shows that the frequency response derived from complete mathematical model of turbine regulating system under load disturbance is a fifth order system, which is called complete fifth order system in this paper. The denominator of this system is quintic (corresponding to the fifth order linear homogeneous differential equation) and cannot be factorized. Therefore it is not only impossible to solve the fluctuation equation of frequency response (i.e. $x=x(t)$) from complete fifth order system (Eq. (9)) directly, but also difficult to carry out theoretical analysis. According to Galois theory [25], fifth-order (or higher) equations have no extract roots formulas. Hence, the order reduction is the only method to solve Eq. (9). The order reduction of this paper is based on pole locations of system.

4.1. Pole analysis

Since the poles of complete fifth order system cannot be solved analytically, we can only analyze the pole locations for specific HPP by numerical calculation. Hence, three different types of HPPs with upstream air cushion surge tanks are selected (basic information are shown in Table A.1 of Appendix C), and the results of their pole locations are listed in Table 1-2, in which $n_f=F/F_{th}$ is amplification coefficient of sectional area of surge tank and F_{th} is critical stable sectional area.

Table 1

Pole locations for complete fifth order systems of different HPPs ($b_t=0.5$, $T_d=10s$, $n_f=1.2$).

HPP	Poles				
	s_1	s_2	s_3	s_4	s_5
A	-0.700109	-0.497736	-0.096566	-0.000297-0.010893i	-0.000297+0.010893i
B	-0.372004-0.252885i	-0.372004+0.252885i	-0.102115	-0.000713-0.008873i	-0.000713+0.008873i
C	-0.599470-0.145486i	-0.599470+0.145486i	-0.089744	-0.001166-0.017323i	-0.001166+0.017323i

Table 2

Pole locations for complete fifth order system of HPP A under different parameters.

Parameters	Poles				
$b_t/T_d(s)/n_f$	s_1	s_2	s_3	s_4	s_5
0.2/5/1.1	-0.240190	-0.210205-0.801875i	-0.210205+0.801875i	-0.000112-0.011491i	-0.000112+0.011491i
0.8/15/1.3	-1.144832	-0.259242	-0.048482	-0.000498-0.010323i	-0.000498+0.010323i

Table 1-2 show that:

(1) The real parts of five poles of each complete fifth order system are all negative. It indicates that the systems are stable under these selected parameters.

(2) There always exists a pair of conjugate complex poles (s_4 , s_5), and the absolute values of their real parts are much less than those of the other three poles (the latter is 77-1877 times larger than the former). It indicates that they are much closer to imaginary axis. Hence, this pair of conjugate complex poles are dominant poles [26]. The other three poles (s_1 , s_2 , s_3), either three real poles or one real pole and a pair of conjugate complex poles, are all non-dominant poles [26].

4.2. Primary order reduction

By deleting the leading term, i.e. a_0s^5 , the denominator of complete fifth order system becomes $\sum_{i=1}^5 a_i s^{5-i}$. Then pole locations are recalculated and listed in Table 3-4.

Table 3

Pole locations for complete fifth order systems of HPPs after deleting the leading term a_0s^5 ($b_t=0.5$, $T_d=10s$, $n_f=1.2$).

HPP	Poles				
	s_1	s_2	s_3	s_4	s_5
A	/	-0.257646	-0.100854	-0.000297-0.010893i	-0.000297+0.010893i
B	/	-0.215164	-0.113299	-0.000713-0.008873i	-0.000713+0.008873i
C	/	-0.285761	-0.092568	-0.001164-0.017329i	-0.001164+0.017329i

Table 4

Pole locations for complete fifth order system of HPP A under different parameters after deleting the leading term a_0s^5 .

Parameters	Poles				
	s_1	s_2	s_3	s_4	s_5
b_t/T_d (s) / n_f					
0.2/5/1.1	/	-0.922001	-0.270904	-0.000112-0.011491i	-0.000112+0.011491i
0.8/15/1.3	/	-0.202107	-0.048979	-0.000498-0.010323i	-0.000498+0.010323i

Table 3-4 show that:

- (1) The real parts of four poles of each system after deleting the leading term a_0s^5 are also all negative. This shows that the deletion of leading term a_0s^5 will not affect the stability of system.
- (2) There still exists a pair of dominant conjugate complex poles (s_4 , s_5), and their values almost remain the same as those before deleting the leading term (only dominant poles of HPP C change from $-0.001166\pm 0.017323i$ to $-0.001164\pm 0.017329i$, and all the rest remain unchanged.). This shows that the deletion of leading term a_0s^5 will not change the type, amount and the values of dominant poles. The other two non-dominant poles (s_2 , s_3) are real.

The dominant poles of closed-loop system have the greatest effect on transient response process, i.e. they determine the type of response and play a controlling role during the process. Hence, primary order reduction of complete fifth order system can be conducted based on the above pole analysis. Then the primary fourth order equivalent system for frequency response of turbine regulating system under load disturbance is obtained as follows:

$$X_E(s) = -b_t T_d m_{g0} \frac{\sum_{i=0}^3 b_i s^{3-i}}{\sum_{i=1}^5 a_i s^{5-i}} \quad (10)$$

The primary fourth order equivalent system (Eq. (10)) is applied to replace complete fifth order system (Eq. (9)), i.e. $X_E(s) \approx X(s)$. The three HPPs in Table A.1 are still taken as examples for comparative analysis of frequency responses of these two systems. The load disturbance is a 10% step reduction when the unit operates at rated power output, i.e. $m_{g0}=-0.1$. The comparisons of frequency responses are shown in Fig. 4 and the characteristic parameters of response curves are listed in Table 5.

Fig. 4. Comparisons of frequency responses of complete fifth order system, primary fourth order equivalent system and fourth order system in reference [20] under $m_{g0}=-0.1$.

Table 5

Characteristic parameters of response curves of complete fifth order system, primary fourth order equivalent system and fourth order system in reference [20] under $m_{g0}=-0.1$.

Types of frequency responses		Head wave		Tail wave	
		Maximum	Amplitude	Period (s)	Decay rate
HPP A ($b_r=0.5, T_d=10s, n_f=1.2$)	1	0.0316	0.0025	576.4385	0.0003
	2	0.0285	0.0025	576.4385	0.0003
	3	0.0255	0.0025	576.4385	0.0003
HPP A ($b_r=0.8, T_d=15s, n_f=1.3$)	1	0.0398	0.0054	610.0175	0.0005
	2	0.0379	0.0054	610.0175	0.0005
	3	0.0355	0.0052	610.0175	0.0005
HPP B ($b_r=0.5, T_d=10s, n_f=1.2$)	1	0.0358	0.0041	705.9753	0.0007
	2	0.0307	0.0041	705.9753	0.0007
	3	0.0252	0.0036	705.9753	0.0009
HPP C ($b_r=0.5, T_d=10s, n_f=1.2$)	1	0.0361	0.0071	363.1896	0.0012
	2	0.0326	0.0071	363.1884	0.0012
	3	0.0261	0.0054	356.9989	0.0016
	4	0.0295	0.0071	365.3012	0.0012
	5	0.0329	0.0054	354.9819	0.0016

1-complete fifth order system; 2-primary fourth order equivalent system; 3-fourth order system in reference [20]; 4-complete fifth order system, $T_{wt}=0$; 5-complete fifth order system, $h_0=0$

Fig. 4 and Table 5 show that:

(1) Deleting the leading term a_0s^5 leads to a slight difference in the beginning of frequency response curves (i.e. head wave) and almost no difference in the follow-up period (i.e. tail wave). Concretely, the reduction rates of maximums of head waves for four types of frequency responses are 9.83%, 4.82%, 14.31% and 9.62%, respectively. For tail waves, only the amplitude of HPP C changes from 363.1896s to 363.1884s while all the rest characteristic parameters remain unchanged. These results indicate that the leading term of the denominator of complete fifth order system has a small effect on head wave and almost no effect on tail wave.

(2) The tendency of frequency response curve of primary fourth order equivalent system is in accordance with that of complete fifth order system, and there is the satisfactory agreement between these

two curves. Therefore, primary fourth order equivalent system can represent the fluctuation characteristics of complete fifth order system accurately and it is feasible to replace the latter by the former.

4.3. Comparison of different fourth order systems

A fourth order system is obtained in reference [20], for the case in which the water inertia and head loss of penstock are both neglected. This system is the special case of complete fifth order system in condition of $T_{wt}=0$ and $h_{t0}=0$. For analyzing the difference between fourth order system in reference [20] and primary fourth order equivalent system and verifying the applicability of fourth order system in reference [20], the frequency responses of three HPPs in Table A.1 under $m_{g0}=-0.1$ are calculated and shown in Fig. 4 and Table 5. For comparison, the response curves of complete fifth order system in condition of $T_{wt}=0$ and that in condition of $h_{t0}=0$ are plotted in Fig. 4 (d).

It is shown in Fig. 4 and Table 5 that:

(1) For the HPPs with different net heads, the maximums of head waves of fourth order system in reference [20] are less than those of complete fifth order system. The reduction rates of four types of frequency responses are 19.28%, 10.75%, 29.61% and 27.66%, respectively, and these rates are greater than those of primary fourth order equivalent system.

(2) For the high head HPP (A), there is good agreement between the tail wave of fourth order system in reference [20] and that of complete fifth order system under different governor parameters and sectional areas of surge tank. Concretely, only the amplitude of tail wave under the combination of $b_r=0.8$, $T_d=15s$ and $n_f=1.3$ changes from 0.0054 to 0.0052, while all the rest characteristic parameters remain unchanged. For the middle and low heads HPPs (B and C), the agreement between the tail waves of fourth order system in reference [20] and that of complete fifth order system is poor, and the lower the head, the poorer the agreement. More exactly, the tail wave period of HPP B remains the same, however its amplitude and decay rate decrease by the rates of 12.19% and 28.57%, respectively; the amplitude and period of tail wave of HPP C decrease by the rates of 23.94% and 1.70%, respectively, and the decay rate increase by the rates of 33.33%. These results indicate that fourth order system in reference [20] cannot represent the fluctuation characteristics of complete fifth order system accurately.

(3) As pointed out above, fourth order system in reference [20] is the special case of complete fifth order system in condition of $T_{wt}=0$ and $h_{t0}=0$. This shows that the difference between these two systems is caused by the values of T_{wt} and h_{t0} . As shown in Fig. 4 (d), there is a close agreement of head waves between complete fifth order system in condition of $T_{wt}=0$ and fourth order system in reference [20], and a very close agreement of tail waves (only period increases by the rate of 0.58%) between complete fifth order system in condition of $T_{wt}=0$ and that in condition of $T_{wt}\neq 0$; there is a close agreement of head waves between complete fifth order system in condition of $h_{t0}=0$ and that in condition of $h_{t0}\neq 0$, and a very close agreement of tail waves (only period decreases by the rate of 0.56%) between complete fifth order system in condition of $h_{t0}=0$ and fourth order system in reference [20]. These results indicate that T_{wt} and h_{t0} have evident influences on the frequency response under load disturbance. T_{wt} mainly influences head wave while h_{t0} mainly affects tail wave, and the extent of influence will increase with the decrease of the net head.

In short, the fourth order system in reference [20], obtained by neglecting the water inertia and head loss of penstock, is only applicable to high head HPP. Hence, for the study of regulation quality for frequency response of turbine regulating system under load disturbance, primary fourth order equivalent system is more accurate and reasonable in comparison with fourth order system in reference [20].

5. Regulation quality for frequency response of turbine regulating system

In this section, the regulation quality for frequency response of turbine regulating system is analyzed by using primary fourth order equivalent system.

5.1. Fluctuation characteristics of frequency response

Firstly, the fluctuation characteristics of frequency response of primary fourth order equivalent system are analyzed. The poles of the system (i.e. Eq. (10)) are determined by the following equation:

$$s^4 + A_3s^3 + A_2s^2 + A_1s + A_0 = 0 \quad (11)$$

where $A_i = a_{5-i}/a_1$, $i=0, 1, 2, 3$.

It is clear from references [27,28] that the four roots of Eq. (11) are the same as those of the following two equations:

$$s^2 + (A_3 + \sqrt{8s' + A_3^2 - 4A_2}) \frac{s}{2} + (s' + \frac{A_3s' - A_1}{\sqrt{8s' + A_3^2 - 4A_2}}) = 0 \quad (12)$$

$$s^2 + (A_3 - \sqrt{8s' + A_3^2 - 4A_2}) \frac{s}{2} + (s' - \frac{A_3s' - A_1}{\sqrt{8s' + A_3^2 - 4A_2}}) = 0 \quad (13)$$

where

$$s' = \sqrt[3]{-\frac{l}{2} + \sqrt{\frac{l^2}{4} + \frac{k^3}{27}}} + \sqrt[3]{-\frac{l}{2} - \sqrt{\frac{l^2}{4} + \frac{k^3}{27}}} + \frac{A_2}{6}, \quad k = \frac{A_1A_3}{4} - A_0 - \frac{A_2^2}{12},$$

$$l = -\frac{A_2^3}{108} + \frac{A_2(A_1A_3 - 4A_0)}{24} + \frac{A_0(4A_2 - A_3^2)}{8} - \frac{A_1^2}{8}.$$

Using Eqs. (12)-(13), primary fourth order equivalent system (Eq. (10)) is decomposed into the sum of two second order systems as follows:

$$X_E(s) = X_1(s) + X_2(s) \quad (14)$$

$$X_1(s) = \frac{C_5s + C_6}{s^2 + C_1s + C_2} \quad (15)$$

$$X_2(s) = \frac{C_7s + C_8}{s^2 + C_3s + C_4} \quad (16)$$

In Eqs. (15)-(16),

$$C_1 = \frac{1}{2}(A_3 + \sqrt{8s' + A_3^2 - 4A_2}), \quad C_2 = s' + \frac{A_3s' - A_1}{\sqrt{8s' + A_3^2 - 4A_2}},$$

$$C_3 = \frac{1}{2}(A_3 - \sqrt{8s' + A_3^2 - 4A_2}), \quad C_4 = s' - \frac{A_3s' - A_1}{\sqrt{8s' + A_3^2 - 4A_2}}.$$

C_5, C_6, C_7 and C_8 can be determined by solving the following system of linear equations:

$$\begin{cases} C_5 + C_7 = -b_0b_tT_d m_{g0} / a_1 \\ C_3C_5 + C_6 + C_1C_7 + C_8 = -b_1b_tT_d m_{g0} / a_1 \\ C_4C_5 + C_3C_6 + C_2C_7 + C_1C_8 = -b_2b_tT_d m_{g0} / a_1 \\ C_4C_6 + C_2C_8 = -b_3b_tT_d m_{g0} / a_1 \end{cases} \quad (17)$$

As found in Section 4.2, primary fourth order equivalent system has a pair of dominant conjugate complex poles and two non-dominant real poles. For stable system, the real parts of dominant conjugate complex poles are negative and their absolute values are minimum among all poles [21,26]. Then it is obtained that $C_1 > C_3 > 0$. Hence, the absolute values of real parts of poles determined by Eqs. (13) are less than those determined by Eqs. (12). Therefore, the dominant conjugate complex poles are determined by

Eqs. (13) and the two non-dominant real poles are determined by Eqs. (12), i.e. the second order system expressed by Eqs. (16) is the system which contains the dominant conjugate complex poles and the second order system expressed by Eqs. (15) is the system that contains the two non-dominant real poles.

According to the inverse Laplace transform of Eqs. (14)-(16), the following fluctuation equations for frequency response of primary fourth order equivalent system are obtained:

$$x_E(t) = x_1(t) + x_2(t) \quad (18)$$

$$x_1(t) = K_{11}e^{-\delta_{11}t} + K_{12}e^{-\delta_{12}t} \quad (19)$$

$$x_2(t) = K_2e^{-\delta_2t} \sin(\omega t + \varphi) \quad (20)$$

In Eqs. (19)-(20), $\delta_{11} = (-C_1 + \sqrt{C_1^2 - 4C_2})/2$ and $\delta_{12} = (-C_1 - \sqrt{C_1^2 - 4C_2})/2$ are the two non-dominant real poles; $\delta_2 = C_3/2$ and $\omega = \sqrt{4C_4 - C_3^2}/2$ are the inverse value of real part and the absolute value of imaginary part of dominant conjugate complex poles, respectively; Besides, $K_{11} = (C_5\delta_{11} - C_6)/(\delta_{11} - \delta_{12})$; $K_{12} = (C_5\delta_{12} - C_6)/(\delta_{12} - \delta_{11})$; $K_2 = \sqrt{C_7^2 + (C_7\delta_2 + C_8)^2}/\omega^2$; $\varphi = \arctan[C_7\omega/(C_7\delta_2 + C_8)]$.

Eq. (18) shows that the fluctuation of frequency response $x_E(t)$ is superimposed by two sub fluctuations (i.e. $x_1(t)$, $x_2(t)$), as shown in in Fig. 5. $x_1(t)$ is the sub fluctuation corresponding to the two non-dominant real poles, which decays fast and mainly affects the head wave. $x_2(t)$ is the sub fluctuation corresponding to dominant conjugate complex poles, which decays slowly, fluctuates periodically and determines the tail wave.

Fig. 5. Schematic diagram of head wave and tail wave of frequency response.

Taking HPP C as example, Fig. 6 shows the effects of T_{wy} , T_{wt} , n_f , b_t and T_d on the head wave and tail wave of frequency response.

Fig. 6. Effects of T_{wy} , T_{wt} , n_f , b_t and T_d on the head wave and tail wave of frequency response.

It is shown in Fig. 6 that:

(1) Head wave: T_{wy} has almost no influence on the crest of head wave. The values and occurrence time of crests are almost the same when T_{wy} takes different values. T_{wt} has an obvious influence on the crest. With the increase of T_{wt} , the value of crest increases and its occurrence time becomes earlier. n_f has a relatively large influence on the crest. When n_f increases, the value of crest increases, but the occurrence time remains almost stable. b_t and T_d have evident influences on the crest. With the rise of b_t and T_d , the crest becomes larger and occurs later.

(2) Tail wave: T_{wy} , n_f , b_t and T_d all have important influences on tail wave, and the change of their values alters the amplitude, period, decay rate and initial phase of tail wave obviously. T_{wt} only has little influence on tail wave.

5.2. Secondary order reduction

The tail wave is corresponding to the dominant conjugate complex poles of primary fourth order equivalent system, and it is the main body of frequency response and the principal factor which determines the regulation quality. Hence, the regulation quality of frequency response can be analyzed based on tail wave.

According to the above analyses, secondary order reduction of complete fifth order system can be conducted based on tail wave, i.e. the second order system expressed by Eq. (16) is used to represent the fluctuation characteristics of frequency response. The fluctuation equation of tail wave corresponding to Eq. (16) is Eq. (20), and it is a damped vibration. In Eq. (20), K_2 , δ_2 , ω , φ and $T=2\pi/\omega$ are amplitude, decay rate, angular frequency, initial phase and period of damped vibration, respectively.

The most important dynamic performance index which evaluates the regulation quality of frequency response is settling time, which is the time from the occurrence of step load disturbance to reaching a new steady state. For engineering application, the settling time is defined as that the deviation between x and x_0 is no more than the allowable fluctuation bandwidth of frequency Δ [29]. The expression of settling time T_p [26] can be derived from Eq. (20) as follows:

$$T_p = \frac{1}{\delta_2} \ln \frac{K_2}{|\Delta|} \quad (21)$$

where the value of Δ is $\pm 0.2\%$ for strong grid and $\pm 0.4\%$ for small grid [29].

Eq. (21) shows that the value of T_p is determined by K_2 and δ_2 . It is worth noting that Eq. (21) is only an approximate formula, and T_p is actually influenced by T and φ as well.

5.3. Influencing factors analysis for regulation quality

Taking HPP C as an example, the effects of characteristic parameters of turbine regulating system

(including T_{wt} , n_f , p_0 , l_0 , m , b_t , T_d , e_g , T_a , m_{g0}) on settling time T_p are analyzed and shown in Fig. 7. In order to reflect the changing mechanism of T_p more clearly, Fig. 7 also gives the variation curves of characteristic parameters of damped vibration (including K_2 , δ_2 , T , φ). The default values of influencing factors are as follows: $T_{wy}=23.84s$, $T_{wt}=1.26s$, $p_0=200m$, $l_0=20m$, $m=1.4$, $n_f=1.2$, $b_t=0.5$, $T_d=10s$, $e_g=0$, $T_a=9.46s$, $m_{g0}=-0.1$, $|\Delta|=0.2\%$.

Fig. 7. Influencing factors analysis for regulation quality

Fig. 7 shows that:

(1) T_{wt} and T_a have small influences on settling time and characteristic parameters of damped vibration. With the rise of T_{wt} and T_a , T_p rises and the regulation quality becomes worse as a consequence. However, the deterioration of regulation quality is actualized by increasing K_2 and φ and decreasing δ_2 and T when T_{wt} increases while by increasing K_2 and T and decreasing δ_2 and φ when T_a increases. In addition, the variations of T_p , K_2 , δ_2 , T and φ with T_{wt} and T_a are approximate linearity.

(2) n_f , p_0 , l_0 , m , b_t , T_d and e_g have crucial influences on settling time and characteristic parameters of damped vibration. With the increase of n_f and l_0 and the decrease of p_0 and m , T_p reduces and the regulation quality becomes better as a consequence which is caused by increasing δ_2 and T and decreasing K_2 and φ . There exist critical values of n_f , p_0 and l_0 . In the right side of the critical values of n_f and l_0 and the left side of the critical value of p_0 , δ_2 is larger than zero, and T_p will tend towards infinite when n_f , p_0 and l_0 tend to their critical values; in the opposite sides of the critical values of n_f , p_0 and l_0 , δ_2 is less than zero which indicates the system is unstable. With the rise of b_t and T_d , K_2 , δ_2 and T increase and φ decreases, which lead to that T_p increases first and decreases afterward. As a result, the regulation quality becomes better firstly and worse afterward. With the rise of e_g , T_p reduces because of the increase of δ_2 and T and the decrease of K_2 and φ , which indicates that the regulation quality becomes better.

(3) m_{g0} is an extraneous influencing factor. Hence, its value has no influence on δ_2 , T and φ because they reflect the inherent characteristics of system. When the other parameters remain unchanged, φ between load reduction (i.e. $m_{g0} < 0$) and load increment (i.e. $m_{g0} > 0$) differs by π . With the increase of $|m_{g0}|$, T_p increases because of the rise of K_2 , and the regulation quality becomes worse as a consequence.

6. Conclusions

This paper proposes primary order reduction and secondary order reduction method for frequency response of turbine regulating system of isolated HPP with surge tank under load disturbance based on dominant poles. The complete fifth order system is solved and the regulation quality for frequency response is studied. The major conclusions are summarized as follows:

(1) The frequency response of complete mathematical model of turbine regulating system is a fifth order system. The complete fifth order system always has a pair of dominant conjugate complex poles, and the deletion of leading term a_0s^5 of the denominator will not change not only the type and amount but also the values of dominant poles. Primary order reduction of complete fifth order system can be conducted by deleting the leading term, and the primary fourth order equivalent system can represent and replace the complete fifth order system.

(2) The fourth order system in reference [20], obtained by neglecting the water inertia and head loss of penstock, is just applicable to high head HPP and not suitable to middle and low heads HPP. The lower the net head, the worse the agreement between fourth order system in reference [20] and complete fifth order system. The difference of head wave between these two systems is caused by T_{wt} , while the tail wave difference is generated by h_{t0} . Therefore, the primary fourth order equivalent system is more accurate and reasonable in comparison with fourth order system in reference [20].

(3) The primary fourth order equivalent system is superimposed by two second-order subsystems, one of them is corresponding to two non-dominant real poles (i.e. head wave) and the other one is corresponding to a pair of dominant conjugate complex poles (i.e. tail wave), respectively. During the frequency fluctuation, head wave decays very fast and works mainly in the beginning period while tail wave decays very slow and fluctuates periodically and works throughout the period. T_{wy} , n_f , b_t and T_d have different influence on the fluctuation characteristics of head wave and tail wave.

(4) Tail wave is the main body of frequency response and the principal factor which determines the regulation quality. The secondary order reduction of complete fifth order system can be conducted by using the second-order system of tail wave to represent the characteristics of fluctuation frequency. The system of tail wave is a damped vibration. The characteristic parameters of turbine regulating system (including T_{wt} , n_f , p_0 , l_0 , m , b_t , T_d , e_g , T_a , m_{g0}) have different influence on settling time which evaluates the regulation quality of frequency response.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (Project No. 51379158).

Appendix A

Nomenclature:

Z	altitude difference between water level in surge tank and upstream reservoir (positive direction is downward)		
H_F	piezometric head at the bottom of surge tank	H	net head
Q_y	headrace tunnel discharge	Q_t	penstock discharge
n	unit frequency	Y	guide vane opening

M_t	kinetic moment	M_g	resisting moment
L_y	length of headrace tunnel	L_t	length of penstock
f_y	sectional area of headrace tunnel	f_t	sectional area of penstock
h_{y0}	head loss of headrace tunnel	h_{t0}	head loss of penstock
T_{wy}	water inertia time constant of headrace tunnel	T_{wt}	water inertia time constant of penstock
F	sectional area of surge tank	p_0	air absolute pressure in surge tank
∇_0	air absolute volume in surge tank	l_0	air chamber height
m	gas polytropic exponent	T_F	time constant of air cushion surge tank
e_h, e_x, e_y	moment transfer coefficients of turbine	e_{qh}, e_{qx}, e_{qy}	discharge transfer coefficients of turbine
T_a	unit inertia time constant	e_g	load self-regulation coefficient
b_t	temporary droop	T_d	damping device time constant

Note that:

(1) $z=(Z-Z_0)/Z_0$, $h_F=(H_F-H_{F0})/H_0$, $h=(H-H_0)/H_0$, $q_y=(Q_y-Q_0)/Q_0$, $q_t=(Q_t-Q_0)/Q_0$, $x=(n-n_0)/n_0$, $Y=(Y-Y_0)/Y_0$, $m_t=(M_t-M_{t0})/M_{t0}$, $m_g=(M_g-M_{g0})/M_{g0}$ are the relative deviations of corresponding variables. The subscript "0" refers to the initial value.

(2) $Q_0=Q_{y0}=Q_{t0}$, $l_0=\nabla_0/F$, $T_F=FH_0/[(1+mFp_0/\nabla_0)Q_0]$. We can get $T_F=FH_0/Q_0$ in the case of $\nabla_0 \rightarrow +\infty$, and it is the expression of time constant of open-type surge tank.

(3) m_g is actually equal to the relative deviation of load in the isolated operation. Hence, m_g is regarded as the load disturbance.

Appendix B

The expressions of coefficients in overall transfer function (Eq. (8)) are as follows:

$$\begin{aligned}
a_0 &= f_1 f_9 \\
a_2 &= f_1 f_{11} + f_2 f_{10} + f_3 f_9 + f_5 f_{13} + f_6 f_{12} \\
a_4 &= f_3 f_{11} + f_4 f_{10} + f_7 f_{13} + f_8 f_{12} \\
b_0 &= f_1 \\
b_2 &= f_3 \\
f_1 &= e_{qh} T_F T_{wy} T_{wt} \\
f_3 &= e_{qh} (T_{wy} + T_{wt}) + T_F \frac{2h_{y0}}{H_0} (1 + e_{qh} \frac{2h_{t0}}{H_0}) \\
f_5 &= T_F T_{wy} T_{wt} \\
f_7 &= T_{wy} + T_{wt} + T_F \frac{2h_{y0}}{H_0} \frac{2h_{t0}}{H_0} \\
f_9 &= b_t T_d T_a \\
f_{11} &= e_y \\
f_{13} &= -e_h e_{qy} \\
a_1 &= f_1 f_{10} + f_2 f_9 + f_5 f_{12} \\
a_3 &= f_2 f_{11} + f_3 f_{10} + f_4 f_9 + f_6 f_{13} + f_7 f_{12} \\
a_5 &= f_4 f_{11} + f_8 f_{13} \\
b_1 &= f_2 \\
b_3 &= f_4 \\
f_2 &= T_F \left[T_{wy} (1 + e_{qh} \frac{2h_{t0}}{H_0}) + T_{wt} e_{qh} \frac{2h_{y0}}{H_0} \right] \\
f_4 &= 1 + e_{qh} \frac{2(h_{y0} + h_{t0})}{H_0} \\
f_6 &= T_F (T_{wy} \frac{2h_{t0}}{H_0} + T_{wt} \frac{2h_{y0}}{H_0}) \\
f_8 &= \frac{2(h_{y0} + h_{t0})}{H_0} \\
f_{10} &= b_t T_d (e_g - e_x) + T_d e_y \\
f_{12} &= b_t T_d e_h e_{qx} - T_d e_h e_{qy}
\end{aligned}$$

Appendix C

Table A.1

Basic information of HPPs.

HPP	Rated power output (MW)	Rated head (m)	Rated discharge (m ³ /s)	T_{wy} (s)	T_{wt} (s)	h_{y0} (m)	h_{r0} (m)
A	610.00	288.00	228.60	23.84	1.26	12.92	2.91
B	118.56	177.00	72.50	39.73	1.82	20.53	5.12
C	51.28	89.00	62.70	17.75	2.33	7.57	5.53

References

- [1] Kundur P. Power system stability and control. New York: McGraw-Hill Inc; 1994.
- [2] Wei SP, Wu YG, Lin JH. Hydro-turbine governor and grid load frequency control, Part 2: Simulation of grid load frequency control [in Chinese]. Hydropower Automation and Dam Monitoring 2006; 30(1): 18-22.
- [3] Bevrani H, Ghosh A, Ledwich G. Renewable energy sources and frequency regulation: survey and new perspectives. IET Renew Power Gener 2010; 4(5): 438-457.
- [4] Shayeghi H, Shayanfar HA, Jalili A. Load frequency control strategies: A state-of-the-art survey for the researcher. Energy Conversion and Management 2009; 50: 344-353.
- [5] Chaturvedi DK, Satsangi PS, Kalra PK. Load frequency control: a generalised neural network approach. Electrical Power and Energy Systems 1999; 21: 405-415.
- [6] Ertuğrul Çam. Application of fuzzy logic for load frequency control of hydroelectrical power plants. Energy Conversion and Management 2007; 48: 1281-1288.
- [7] Salhi I, Doubabi S. Fuzzy controller for frequency regulation and water energy save on microhydro electrical power plants. International Renewable Energy Congress, Sousse, Tunisia 2009; 106-112.
- [8] Stankovic AM, Tadmor G, Sakharuk TA. On robust control analysis and design for load frequency regulation. IEEE Transactions on Power Systems 1998; 13(2): 449-455.
- [9] Khodabakhshian A, Golbon N. Robust load frequency controller design for hydro power systems. IEEE Conference on Control Applications, Toronto, Canada 2005; 1510-1515.
- [10] Jing L, Ye LQ, Malik OP, Zeng Y. An intelligent discontinuous control strategy for hydroelectric generating unit. IEEE Transactions on Energy Conversion 1998; 13(1): 84-89.
- [11] Sheikh MRI, Mueen SM, Takahashi R, Murata T, Tamura J. Application of self-tuning FPIC to AGC for load frequency control in multi-area power system. IEEE Bucharest Power Tech Conference, Bucharest, Romania 2009; 1-7.
- [12] State Standard of the People's Republic of China. Test code of control systems for hydraulic turbines GB/T9652.1-2007 [in Chinese]. Beijing: China Standards Press; 2008.
- [13] Stein T. Frequency control under isolated network conditions. Water Power 1970; (9): 320-324.
- [14] Doolla S, Bhatti TS. Load frequency control of an isolated small-hydro power plant with reduced dump load. IEEE Transactions on Power Systems 2006; 21(4): 1912-1919.
- [15] Wei SP. Analysis and simulation on the primary frequency regulation and isolated grid operation characteristics of hydraulic turbine regulating systems [in Chinese]. Hydropower Automation and Dam Monitoring 2009; 33(6): 27-33.
- [16] Zhao GL, Yang JD, Yang AL. Study on regulation measures of regulation quality for the system of hydropower station with surge tank [in Chinese]. Yangtze River 2006; 37(7): 103-104, 110.
- [17] Fu L, Yang JD, Li JP, Bao HY. Frequency fluctuations features of hydropower stations with surge tank [in Chinese]. Engineering Journal of Wuhan University 2008; 41(5): 50-53.
- [18] Zhu KG, Zhou JX. Performance of tailrace surge chamber shared by two units [in Chinese]. East China Electric

- Power 2006; 34(12): 50-53.
- [19] Fu L, Yang JD, Li JP, Bao HY. Analysis on the governing quality of the hydropower station with surge tank [in Chinese]. Journal of Hydroelectric Engineering 2009; 28(2): 115-120.
 - [20] Fu L, Yang JD, Bao HY, Li JP. Generator unit frequency fluctuations of hydropower station with surge tank under load disturbance [in Chinese]. Journal of Hydraulic Engineering 2008; 39(11): 1190-1196.
 - [21] Liu QZ, Peng SZ. Surge tank of hydropower station [in Chinese]. Beijing: China Water Power Press; 1995.
 - [22] IEEE Working Group. Hydraulic turbine and turbine control models for system dynamic studies. IEEE Transactions on Power Systems 1992; 7(1): 167-179.
 - [23] Fang HQ, Chen L, Dlakavu N, Shen ZY. Basic modeling and simulation tool for analysis of hydraulic transients in hydroelectric power plants. IEEE Transactions on Energy Conversion 2008; 23(3): 834-841.
 - [24] Shen ZY. Turbine regulation. 3rd ed [in Chinese]. Beijing: China Water Power Press; 1997.
 - [25] Gallian JA. Contemporary abstract algebra. 7th ed. US: Cengage Learning; 2009.
 - [26] Franklin GF, Powell .D, Naeini AE. Feedback Control of Dynamic Systems. 6th ed. New Jersey: Prentice Hall; 2009.
 - [27] Zhang JB, Cheng BQ, Wang X. The exact solution of characteristic equation of flight disturbance motion [in Chinese]. Journal of Air Force Engineering University (Natural Science Edition) 2002; 3(8): 81-83.
 - [28] Compiling group of Mathematics Handbook. Mathematics Handbook [in Chinese]. Beijing: People's Education Press; 1979.
 - [29] Yang KL. Selection of turbine-generator inertia GD^2 and PID governor parameters [in Chinese]. Journal of Hydraulic Engineering 1998; (3): 1-8.