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Citation for the original published paper (version of record):

Niklasson, G A., Niklasson, M H. (2015)

Non-Gaussian distributions of melodic intervals in music: The Lévy-stable approximation.

Europhysics letters, 112(4): 40003

<http://dx.doi.org/10.1209/0295-5075/112/40003>

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Non-Gaussian distributions of melodic intervals in music: The Levy-stable approximation.

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Short title: Non-Gaussian distributions of melodic intervals in music.

PACS index: 05.40.Fb, 05.40.Ca, 05.45.Df

Abstract: The analysis of structural patterns in music is of interest in order to increase our fundamental understanding of music, as well as for devising algorithms for computer-generated music, so called algorithmic composition. Musical melodies can be analyzed in terms of a “music walk” between the pitches of successive tones in a notescript, in analogy with the “random walk” model commonly used in physics. We find that the distribution of melodic intervals between tones can be approximated with a Levy-stable distribution. Since music also exhibits self-affine scaling, we propose that the “music walk” should be modelled as a Lévy motion. We find that the Lévy motion model captures basic structural patterns in classical as well as in folk music.

Introduction - The fractal, or more precisely self-affine, structural properties of music have attracted interest since the pioneering work of Voss and Clarke [1]. The subject is of fundamental interest for understanding the patterns which characterize what we perceive as music. In addition, it is of importance for the field of algorithmic composition [2], i.e. computer generated music, which can be used as a tool for developing novel ideas in musical composition. Here we show that the distribution of melodic intervals, i.e. the increments (steps) between successive tones, in musical pieces is strongly non-Gaussian and can to a good approximation be fitted by Levy-stable [3] distributions. Earlier work has emphasized the similarity of music to “1/f noise” [1,4] or to fractional Brownian motion [5,6], thereby implicitly assuming a Gaussian distribution of melodic intervals. Our findings instead suggest that a basic pattern in the structure of music can be described as a Levy motion [7,8] and variations underlying this theme may be indicators of musical styles and composer preferences.

The statistical and scaling properties of music can be analyzed using different methodologies. Measurements of the spectral density of audio power and frequency fluctuations in audio recordings is a versatile and fast method that has often been used [1,4,9]. However, the results of

such a procedure will depend both on the musical composition and how it is actually performed. Here we take an alternative approach and analyze the actual musical scores, in order to emphasize the intention of the composer. It appears that Hsü and Hsü [10] were the first to analyze fractal properties of musical scores. They defined a fractal dimension of melodic intervals; a concept that has subsequently been strongly criticized [11]. However, the basic idea of digitizing a musical score to create a time series, by listing the pitch of each tone, is a sound one. Subsequently, Su and Wu suggested that musical scores should be analyzed as a so called “music walk” [6] between the pitch of successive tones, in analogy with the “random walk” model used in diverse areas of the physical sciences.

Fractal structures in music are well established, have attracted considerable attention and various analysis methods have been used [5,6,9,10], in order to determine the fractal dimension, D , or related quantities. In the case of fractional Brownian motion the square root of the variance of the increments scales as a power of the length of the considered time interval [5]. The scaling exponent is called the Hurst exponent, $H=2-D$ [5]. Methods based on the variance have been questioned in cases of non-Gaussian long-tailed distribution of increments [8,12]. However, it is very important to decouple the analysis of a fluctuating signal from underlying trends in the data. Detrended Fluctuation Analysis (DFA) [13,14] is a well-established method for advanced analysis of Hurst exponents and it has been found to perform better than a variety of other methods in comparisons using synthetic data [15,16].

In this paper we analyze the distribution of melodic intervals and find that they can be fitted to long-tailed Lévy-stable distributions. This leads us to introduce a new model for basic structures of music in terms of a Lévy motion.

Analysis of music - We represent a musical score as a “music walk” using the method devised by Su and Wu [6]. Each note in the script is given an integer number characterizing the pitch and a number of time steps characterizing the duration of the note. The pitch is defined according to the twelve tone scale of Western music, in which each octave, which ranges from frequency f to $2f$, is divided into twelve intervals between tones. The music scores were digitized manually to obtain a time series of pitches, the so called music walk. There are twelve different notes, and hence twelve different values of pitch, in an octave. The pitch is related to the logarithm of the frequency of the tone and is represented by an integer. The frequencies in an octave are related to the pitches i by [6]

$$\frac{f_i}{f_0} = 2^{i/12} \quad (1)$$

where f_0 is the frequency of the base note in an octave. Since musical pieces span over a number of octaves, we number the pitches starting from the lowest note in the score. The shortest note in the score was taken to define the basic tonal time step. Hence a note may be assigned one or more time steps, depending on its duration. Pauses were assigned an appropriate number of time steps but no pitch, hence they were treated as interruptions of the music.

The melodic intervals were defined as the differences in pitch between adjacent time steps. A distribution of melodic intervals was generated from the differences in pitch between successive time steps. It is important to emphasize that we define this distribution in terms of differences

between time steps and not between successive notes, thus differing from the method used by Hsü and Hsü (10). We adopted our method in order to incorporate elements of both pitch and rhythm in the analysis of melodic intervals. The distributions were fitted to Levy-stable distributions using the maximum likelihood method of Nolan [17] by using the program “Stable”, which is also described in [17]. We used the S0 parametrization [18], which means that the characteristic function of the Levy-stable distribution is written as [17,18] (in the case $\alpha \neq 1$)

$$\varphi(t) = \exp \left\{ -\gamma^\alpha |\tau|^\alpha \left[1 + i\beta \left(\tan \frac{\pi\alpha}{2} \right) (\text{sign } \tau) \left(|\gamma\tau|^{1-\alpha} - 1 \right) \right] + i\delta\tau \right\}, \quad (2)$$

where α is the characteristic exponent ($0 < \alpha \leq 2$), β is the skewness ($-1 \leq \beta \leq 1$), γ is the scale parameter and δ is the location. The actual Levy-stable distribution is the Fourier transform of the characteristic function.

The Hurst exponent was determined by the DFA method, specifically we use a Matlab function due to Weron [15,19]. In the DFA method the time series is divided into non-overlapping bins of the same length, s . Next a trend-curve in each bin is computed by least-squares fitting and subsequently subtracted from the original data. We use a linear trend, but in principle one can also use higher-order polynomials [14] to describe the trend in each bin. Finally the variance of the detrended time series is calculated for each bin and averaged over all bins of the same size. This procedure is repeated for various bin sizes and the DFA estimator, which is proportional to the square root of the variance, is plotted on a double-logarithmic scale as a function of s . In the scaling range the DFA estimator is proportional to $\sim s^H$ [13-15].

Fig. 1. Pitch as a function of time step for (a) Bach’s Concerto for two violins in D minor, movement 2, and (b) Küchler’s Concertino in D major, Op. 15.

Results and discussion - Figure 1 shows two examples of generated “music walks”, specifically for Bach’s Concerto for two violins in D minor, movement 2, and for Kchler’s Concertino in D major, Op. 15. The patterns show qualitative similarities as well as differences. The music walks show a resemblance to fractional Brownian motion with dimension close to two [5], but this is only superficial, as shown below.

Figure 2 shows the distribution of melodic intervals for the two cases shown in figure 1, together with the best fits to Levy-stable distributions. It should be noted that we use a logarithmic ordinate scale in order to emphasize the tail regions. It is observed that the melodic intervals exhibit a distribution with a quite narrow peak around zero, but exhibiting pronounced tails. Overall the fits to Levy-stable distributions are satisfactory with characteristic exponents (see fig. 2) significantly smaller than the Gaussian value $\alpha=2$. However, we emphasize that the Lvy-stable distribution is only an approximate description of the distribution of melodic intervals. Upon close scrutiny it is observed that there are systematic differences from the Lvy-stable fit. For example, melodic intervals of ± 1 are clearly less prevalent than those with ± 2 , and the data exhibit large fluctuations from the Lvy fit in the tail regions. A smoothing procedure would bring the data close to a Lvy-stable distribution, but music data should not be tampered with in such a way, since they reflect the creativity of the composer and are unique to the studied music piece. Despite these reservations, we consider the Lvy stable distribution to offer a convenient parametrization of melodic interval distributions in music and we hypothesize that analyses of differences from the Levy-stable fit curve may give information on different styles and strategies used by composers.

Fig. 2. Distribution of melodic intervals for (a) Bach’s Concerto for two violins in D minor, movement 2, and (b) Kchler’s Concertino in D major, Op. 15. The best fits to a Levy-stable distribution ($\alpha=1.08$ in (a) and $\alpha=1.37$ in (b)) are shown by the curves.

Preliminary results indicate that there may be significant differences between classical and folk music. Data for two classical music pieces and two pieces of folk music are given in Table 1, which gives fit parameters of Levy-stable distributions, as obtained by the maximum likelihood method from the “Stable” software [17,18]. The statistical uncertainties are larger for the folk

music pieces because the data sets are smaller in those cases – folk music tunes are usually much shorter than classical works.

Table 1. Hurst exponent, H , and parameters of the Levy-stable distributions (characteristic exponent, α , skewness, β , location parameter, δ , and scale parameter, γ), that were fitted to the distribution of melodic intervals, for classical and folk music pieces. The error bars are given as 95 % confidence intervals, as computed by the “Stable” software.

	H	α	β	δ	γ
Classical music:					
Bach: Concerto for two violins in D minor	0.18	1.08 ± 0.08	-0.02 ± 0.10	-0.13 ± 0.10	1.19 ± 0.09
Köchler: Concertino in D major, Op. 15	0.27	1.37 ± 0.10	0.35 ± 0.15	-0.56 ± 0.14	1.33 ± 0.09
Folk music:					
Gärdeby tune	0.20	1.51 ± 0.19	0.33 ± 0.35	-0.30 ± 0.28	1.29 ± 0.17
Leksand “skänklåt”	0.50	1.56 ± 0.19	0.41 ± 0.38	-0.47 ± 0.25	1.15 ± 0.14

It is observed that there exists a trend that folk music pieces exhibit higher values of characteristic Levy exponent, although the error bars sometimes overlap. The location parameter, signifying the peak position, is close to zero as expected. The skewness of the distribution is also small and in two of the cases above not significantly different from zero.

Figure 3 shows the DFA fluctuation function as a function of bin size, i.e. time interval, for the two musical scores depicted in fig. 1. There clearly exist large scaling ranges where the DFA estimator displays a power law dependence on time interval. The DFA analysis in fig. 3 is almost free from the crossover effects, which frequently plague other methods [6] and make the determination of H uncertain. The slope of the scaling range gives Hurst exponents $H=0.18$ and 0.27 for the Bach and Köchler pieces, respectively, equivalent to fractal dimensions of 1.82 and 1.73 , respectively. The fractal dimensions are of similar magnitude to those found earlier from analysis of audio signals [9]. In addition, we find that $H < 1/\alpha$, which implies that the music walk is anti-persistent [8,20], i.e. positive increments tend to be followed by negative ones. Hurst exponents are tabulated in Table 1 and the uncertainties in H are estimated to be of the order of ± 0.03 .

Conclusion – We have found that distributions of melodic intervals of music compositions are strongly non-Gaussian and can be approximated and parametrized by long-tailed Levy-stable distributions. In addition the so called “music walks” exhibit self-affine fractal scaling. A Levy motion is defined by the requirements that the increments follow a Levy-stable distribution and that a range (i.e. variance) parameter exhibits self-affine scaling [8,20]. It is realized that these requirements are approximately fulfilled by the music pieces analyzed in this paper. Hence we suggest that a basic pattern in musical melodies, as analyzed by the “music walk” concept, can be modelled as a Levy motion. Differences in characteristic exponents of the Levy-stable fit distributions as well as departures from the basic Levy motion may reflect different music styles and genres. It is intriguing that concepts related to Levy motion models, previously applied in

physics [7,21] and geology [8,20], can also be applied to music, when the latter is analyzed by the music walk approach.

Fig. 3. Natural logarithm of DFA estimator as a function of logarithm of length of bin size (time interval) for (a) Bach's Concerto for two violins in D minor, movement 2, and (b) Küchler's Concertino in D major, Op. 15. The straight line fits give the indicated Hurst exponents.

Acknowledgments

We are grateful to Prof. Armin Bunde for suggesting us to use the Detrended Fluctuation Analysis method.

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