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Mechanical torque ripple from a passive diode rectifier in a 12 kW vertical axis wind turbine

Morgan Rossander, Anders Goude, and Sandra Eriksson

Abstract—The influence of passive rectification on the mechanical torque of a permanent magnet generator for a directly driven vertical axis wind turbine has been studied. Passive diode rectification introduce electromagnetic torque ripple from the generator. The conversion of electromagnetic torque ripple into mechanical torque ripple and rotational speed ripple has been modelled, analytically evaluated and simulated. The simulations have been compared to measurements on an open site 12 kW prototype. A parameter study with the model illustrates the impact of shaft torsional spring constant, generator rotor inertia, generator inductance and dc-link capacitance. The results show that the shaft and generator rotor can be an effective filter of electromagnetic torque ripple from diode rectification. The measured mechanical torque ripple amplitude on the prototype is less than $\pm 0.9\%$ of nominal turbine torque. The measurements compare well with the simulations.

Index Terms—Wind power, vertical axis, passive rectification, diode rectification, permanent magnet generator torque ripple.

I. INTRODUCTION

WIND turbines with fixed blades require variable speed operation to maximize energy absorption from the wind. A directly driven permanent magnet synchronous generator (PMSG) provide high efficiency, but the output voltage and frequency varies with the speed. Since connection to the grid requires fixed frequency and controllability of the voltage it is common to rectify the generator output into a dc-voltage before further conversion and grid interface.

A well-known solution to convert an ac-voltage into dc is passive rectification using diodes. It is considered relatively robust and cheap compared to actively controlled rectification. The electromagnetic torque ripple caused by the rectification is considered one of the largest issues with diode rectifiers.

There are plenty of studies on various methods of reducing the rectifier torque ripple. The waveform can be altered with passive LC-filtering as in [1]. Introducing a boost circuit in the dc-link provide additional control of the power flow [2]. By designing the boost converter on the dc-link as a current source, the ripple is decreased according to [3]. Other solutions involve multiphase machines, such as five phases [4], six phases [5] or dual three phase windings, one Y-connected and one Δ -connected [6]. More examples of different passive and active rectifiers are given in [7]. Surprisingly few articles measure the actual mechanical torque ripple from synchronous machines [8]. In [5] the mechanical torque is measured, but with too low sample rate for any ripple analysis to be possible.

To our knowledge, no torque ripple measurements of a diode rectified PMSG have been published. As explained in [8] such measurements are difficult to perform since the mechanical properties of the rotating parts influence the measurement. In this paper, these mechanical influences are studied together with electrical properties of a passively rectified vertical axis wind turbine (VAWT) system. The study illustrates how the electromagnetic torque ripple caused by passive rectification propagates into mechanical ripple of torque and rotational speed.

The prototype used in this study is the 12 kW H-rotor VAWT prototype built by the Division of Electricity at Uppsala University in 2006. Design and construction of the turbine prototype is described in [9]–[11]. The generator is placed on the ground and is designed to handle the varying power and rotational speed of the turbine [12]. A low complexity electromechanical MATLAB Simulink simulation model of the turbine, generator and electric load is developed and presented in this paper. The model is verified with measurements on the prototype and a parametric study on the key electrical and mechanical properties influence on the ripple is performed. The measurements were obtained with the method described in [13]. Additionally, analytical transfer functions are presented for estimation of mechanical torque ripple, rotational speed ripple and angular deflection ripple from generator electrical measurements. The functions are verified by the simulations. Since frequency domain steady state analysis of diodes are difficult due to their switching behaviour, simulation is preferable [14]. No analytic representation of the diodes is presented here, however, there are fast approximative methods for analysis of systems with diode rectifiers, such as average-value modelling [15], [16].

There are other issues with passive rectification, such as decrease of generator efficiency and possible vibrations. The focus of this work is limited to how the electromagnetic torque ripple converts into rotational speed ripple of the generator rotor and mechanical torque ripple at the turbine hub.

II. THEORY

A. Generator model

There are a number of sources for generator torque ripple, such as reluctance cogging, oscillating load, winding design and placement of the magnets. The reluctance cogging is known to be small for the studied generator and is neglected in this study [12].

The electrical torque of a PMSG can be expressed as

$$\tau_{el} = \frac{\mathcal{E}_a i_a + \mathcal{E}_b i_b + \mathcal{E}_c i_c}{\Omega_g} \quad (1)$$

Division of Electricity, Department of Engineering Sciences, Uppsala University, Box 534, 751 21 Uppsala, Sweden e-mail: morgan.rossander@angstrom.uu.se.

where $\mathcal{E}_a$, $\mathcal{E}_b$ and $\mathcal{E}_c$ are the generated internal EMFs by the generator as line to neutral voltages, i_a , i_b , i_c are the currents and Ω_g is the rotational speed [17]. For a generator with non salient poles, the EMF of phase a can be determined as

$$\mathcal{E}_a = v_a + L \frac{di_a}{dt} + i_a R \quad (2)$$

where L is the equivalent series inductance and R is the equivalent series resistance in each phase of the generator winding and wires up to the voltage measurement point v_a , i.e. the terminal voltage of the generator. With a non salient rotor, the equivalent series inductance is assumed to be constant. Combining equations (1) and (2) gives the torque expressed in the abc frame as

$$\tau_{el} = \frac{1}{\Omega_g} (i_a v_a + i_b v_b + i_c v_c) + \frac{R}{\Omega_g} (i_a^2 + i_b^2 + i_c^2) + \frac{L}{\Omega_g} \left(i_a \frac{di_a}{dt} + i_b \frac{di_b}{dt} + i_c \frac{di_c}{dt} \right) \quad (3)$$

The equation can estimate the generator electrical torque based on measurements on the generator rotational speed, currents and the generator terminal voltages. The model described by equation (3) is easy to implement as a circuit simulation. In the case of a salient rotor design with non constant impedance, the dqo frame can be used for generator modelling (not used in this paper). The process and how to connect the dqo-model to the diode model is carefully explained in [14].

The winding design and placement of the magnets add harmonics to the generated EMFs of the generator. The 5th and 7th harmonic of the electric frequency both produce a 6th harmonic torque ripple. The 7th harmonic is in phase with the fundamental frequency, while the field of the 5th harmonic rotates in the opposite direction [18], [19]. Since the generator voltage reference is floating with respect to the load, the 3rd harmonic will be cancelled out.

The harmonic content and phase of the EMFs can be determined from frequency analysis of the no-load voltages of the generator. The resulting line to neutral voltage sources that model the internal EMFs, $\mathcal{E}_a$, $\mathcal{E}_b$ and $\mathcal{E}_c$ of the generator are

$$\mathcal{E}_n = k_g \Omega_g \sum_{m \in \{1,5,7\}} h_m \sin(m\alpha + \phi_{n,m}) \quad (4)$$

where index n is the electric phase (a , b and c), Ω_g is the rotational speed, α is the electric angle, and h_5 , h_7 are the 5th and 7th harmonic amplitude relative to the fundamental; $h_1 = 1$. The harmonic phase $\phi_{n,m}$ is the phase of harmonic m for electric phase n . For the fundamental; $\phi_{a,1} = 0$, $\phi_{b,1} = -2\pi/3$ and $\phi_{c,1} = 2\pi/3$. The generator voltage constant, k_g , is assumed to be constant over the studied operational range, i.e. any saturation in the magnetic circuit of the generator is not modelled.

B. Diode rectification

In this work, a full wave three phase bridge rectifier (six diodes) with a single smoothing capacitor (no middle neutral point) on the dc-link is considered. In every instant two diodes are conducting and the current flow is restrained to when the

absolute value of the line to line voltage is higher than the dc-link voltage. Hence, the instantaneous power will pulsate once for each of the rectified peaks, resulting in a power ripple at six times the electrical frequency. If the power is delivered by a generator, the power ripple will cause an electrical torque ripple. The shape and size of the ripple is affected by additional impedances in the system. While the switching behaviour of the diodes are difficult to study analytically, the diodes are easy to simulate as electrical components [14].

In mechanical systems the unit p can be used, which is the order frequency *once per revolution*. For a directly driven generator with 16 pole-pairs, the passive rectification ripple in the 6th electric harmonic produce a torque ripple with the frequency 96 p.

C. Turbine torque

For a wind turbine, the extracted torque from the wind, τ_t , is estimated using

$$\tau_t = \frac{C_p(\lambda) \rho A U^2 R_t}{2\lambda} \quad (5)$$

where R_t is the radius of the turbine, A is the projected turbine area, U is the wind speed and ρ is the air density. The power coefficient, C_p is a function of the tip speed ratio λ (TSR) and specifies the relative absorption of power from the wind by the turbine. The TSR is defined as

$$\lambda = \frac{\Omega_t R_t}{U} \Leftrightarrow \Omega_t = \frac{\lambda U}{R_t} \quad (6)$$

where Ω_t is the rotational speed of the turbine.

Assuming that the mechanical losses and the iron losses in the generator have little impact on the ripple and the operation point of the system, the average electrical generator torque simplifies to be about the same as the average turbine torque; $\langle \tau_t \rangle \approx \langle \tau_{el} \rangle$. For a given optimal TSR and an operational range of wind speeds, it is possible to determine the average rotational speed and average electrical torque using equations (5) and (6).

D. Inertia and shaft dynamics

For analysis and isolation of the rectification ripple of the generator, all bearings are neglected, the shaft is approximated as a torsional spring and the turbine rotational speed is considered constant, despite the torque ripple. The last statement is reasonable since the turbine rotor inertia is assumed too large for the high frequency torque ripple to be able to alter the rotational speed of the turbine rotor. Under these assumptions, the mechanical system is reduced to a single rotating mass at the end of a rotational spring, where a fixed rotational speed enters one side of the spring and the generator rotor is the mass at the other end. The electromagnetic torque ripple drawn by the generator at time t can be expressed as

$$\tau_{el,R}(t) = \tau_{t,R}(t) - J_g \frac{d\Omega_{g,R}}{dt} \quad (7)$$

where J_g is the generator rotor inertia, $\Omega_{g,R}$ is the generator rotational speed ripple and $\tau_{t,R}(t)$ is the mechanical ripple propagating to the turbine as

$$\tau_{t,R}(t) = -\kappa \theta_R(t) \quad (8)$$

where κ is the shaft rotational spring constant and $\theta_R(t)$ the angular deflection of the shaft caused by the ripple. A positive θ_R is defined as in the rotational direction. A positive torque ripple from the generator is braking the rotor and the terms with θ_R and $\Omega_{g,R}$ in equations (7) and (8) are therefore negative. Realising that the generator rotational speed ripple is the time derivative of the angular deflection and inserting equation (8) into equation (7) give

$$\tau_{el,R}(t) = -\kappa\theta_R(t) - J_g \frac{d^2\theta_R(t)}{dt^2} \quad (9)$$

The Laplace transformations of equations (8) and (9) are

$$\tau_{t,R}(s) = -\kappa\theta_R(s) \Leftrightarrow \theta_R(s) = -\frac{\tau_{t,R}(s)}{\kappa} \quad (10)$$

$$\tau_{el,R}(s) = -\kappa\theta_R(s) - J_g s^2 \theta_R(s) + s\theta_R(t)|_{t=0} + \frac{d\theta_R(t)}{dt} \Big|_{t=0} \quad (11)$$

Assuming $\theta_R(t)|_{t=0} = 0$ and $\frac{d\theta_R(t)}{dt} \Big|_{t=0} = 0$ and solving equation (11) for the angular deflection gives

$$\theta_R(s) = \left(\frac{-1}{\kappa + s^2 J_g} \right) \tau_{el,R}(s) \quad (12)$$

From equation (12), the transfer function from electrical torque ripple to rotational speed ripple can be found by the derivative of θ_R :

$$\Omega_{g,R}(s) = s\theta_R(s) = \left(\frac{-s}{\kappa + s^2 J_g} \right) \tau_{el,R}(s) \quad (13)$$

Equations (10) and (12) provide the transfer function from generator electrical torque to turbine torque as

$$\tau_{t,R}(s) = \left(\frac{1}{1 + s^2 \frac{J_g}{\kappa}} \right) \tau_{el,R}(s) \quad (14)$$

Equations (12) to (14) can be used to estimate the resulting mechanical ripple from high frequency electric generator ripple using $s = i\omega$, where ω is the ripple frequency in rad/s. The functions can be applied to study the response of one frequency at the time, or on a full frequency domain signal. For the formulas to be valid, the frequency domain signal needs to be high pass filtered with a suitable cut-off frequency according to the assumptions in the beginning of this section. Additionally, the functions require the torsional spring constant of the shaft and the generator rotor inertia.

For a tubular shaft, the torsional spring constant can be determined by

$$\kappa = \frac{G\pi}{2\ell_{sh}} \left(r_{sh}^4 - (r_{sh} - t_{sh})^4 \right) \quad (15)$$

where G is the shear modulus, t_{sh} is the material thickness, r_{sh} is the outer radius and ℓ_{sh} is the length of the shaft.

E. Ripple definition

The peak-to-peak amplitude of a measured time signal is sensitive to noise and to estimate ripple amplitudes other definitions are more representative. One solution is the Mean Absolute Deviation (MAD) defined as

$$\text{mad}(x) = \frac{1}{N} \sum_{k=1}^N |x_k - \langle x \rangle| \quad (16)$$

where $\langle x \rangle$ is the average value. For a pure sinusoidal shaped ripple with any average, the peak-to-peak amplitude Δx can be estimated from the MAD by multiplication with π .

III. METHOD

A. Simulations

The simulations were performed in MATLAB Simulink. The model is illustrated in Fig. 1. No mechanical losses or dampers were implemented. The mechanical losses and iron losses (see Section II-C) were assumed to have little impact on the ripple amplitude and were therefore omitted in the model. Mechanical dampers were excluded to limit the amount of variables in the model. The series impedance of the capacitor bank was neglected, due to the low equivalent series resistance (ESR) of $<9\text{ m}\Omega$. The simulated time was 15 s with initial values of rotational speed and dc-voltage set to expected values. Data was saved for the last 5 s of the simulation and the time step size was 50 μs .

For the parameter study, the properties of the original wind turbine were used as a baseline for the simulations, see Table I. For each parameter change, a set of operational conditions was simulated based on different wind speeds in the range of 3.9 m/s to 12.9 m/s. Assuming optimal TSR operation, the corresponding rotational speed and a matching average mechanical torque were determined using equations (5) and (6). The average electrical torque for each condition was assumed to be the same as the average mechanical torque according to Section II-C. The EMFs of the generator were determined using equation (4). The harmonics were determined from FFT of rotation synchronized measurements of the generator voltage at no-load. The relative harmonic amplitudes were derived as the average of the measurements and the three phases. The harmonic phases were taken as the average from the FFT rounded to integer multiples of $\pi/3$, since such agreement was good and expected.

The simulations were designed to only illustrate the influence of the passive diode rectification. The power converter on the dc-link was therefore simplified as a resistive load R_L , with a fixed value for each operation condition. To get correct average electrical torque, the simulation was repeated with adjustments to R_L for each condition using the iterative secant method. The simulations were repeated until the specified average electrical torque was obtained within $\pm 0.1\text{ Nm}$ for the last 5 s of the simulation.

B. Experimental set-up

For the experiments the 12 kW VAWT built by Uppsala University has been used. The system is illustrated in Fig. 2

and the key parameters are listed in Table I. The turbine blades are attached to support arms transferring the power to the hub and the shaft. The shaft is directly connected to the generator shaft via a coupling, i.e. there is no gearbox. On one of the blades, a force measurement assembly is installed in-between the support arm and the hub. The force measurement assembly is composed by four single axis load cells and the system allows for torque estimations [13]. The output currents, voltages and rotational speed of the generator are measured. The sampling of electric power and forces are synchronized and runs at a 2 kHz sample rate.

The passively rectified dc-voltage is inverted into three phase ac-voltage with 50 Hz by an SPWM-inverter (Sinusoidal Pulse Width Modulation) with a switching frequency of 19.5 kHz. The ac-power is filtered and transformed before dissipated in a Y-connected resistive load. A rotational speed PI-controller running at 10 Hz sets the reference output value of the SPWM-inverter. The generator is the same as in [12] and it has the same properties as the one in [20]. The amplitudes of the EMF harmonics were estimated from FFT of a FEM-simulation of the generator.

The diode rectifier consists of the reverse diodes of three IGBT-modules of type SEMiX-252GB126HDs. One module connects to each phase, and each module has two diodes (and two IGBTs). The dc-link capacitor is a bank of six electrolytic capacitors in parallel.

C. Measurement campaign and data treatment

Measurement data was collected on several occasions from September to December 2014. Data was selected from the measurements based on the Relative Standard Deviation (RSD) of 1 s averages of the rotational speed and the electrical torque. Data sets of 10 s where the RSD of the 1 s averages were within $\leq 1\%$ for the rotational speed and $\leq 5\%$ for the electrical torque were selected. Additionally, the datasets had targets for rotational speed and electrical torque from which there was an allowed deviation to get a spread in data. Once a suitable dataset had been identified, the measurement data of each set was extracted at the full 2 kHz sample rate.

The rotational speed was measured using hall switches on the rotor magnets. Some irregularities in the position of the sensors and magnets limited the speed measurement resolution [13]. The generator electrical torque was determined from the terminal voltages, line currents and the average rotational speed using equation (3). Since the rotor may have speed oscillations, the approximation to use the average rotational speed in equation (3) was justified by the large moment of inertia in the generator rotor, see Table I. Finally, only data from the last 5 turbine revolutions in the 10 s sets was used for the results. Integration of the average rotational speed was used to determine the number of revolutions.

The measured electrical and mechanical torques in all datasets were high pass filtered to isolate the impact of passive rectification of the generator. The rotational speed was used for estimating the cut-off frequency for each dataset to 90 p. The filters were implemented by FFT of the time series signals and removal of all frequency components corresponding to < 90 p in frequency domain. The signals were inverse transformed

TABLE I
SYSTEM PARAMETERS.

Power	12 kW
Nominal rotational speed	127 rpm
Nominal electrical frequency	33.9 Hz
Nominal torque	900 Nm
Number of generator pole-pairs	16
No load voltage, line to neutral	161 V
Relative EMF of 5 th harmonic, h_5	$7.6 \cdot 10^{-4}$
Relative EMF of 7 th harmonic, h_7	$3.6 \cdot 10^{-3}$
Generator inductance, L	1.8 mH/phase
Resistance, R	0.20 Ω /phase
Diode voltage drop	1.0 V
Diode series resistance	4.0 m Ω
dc-link capacitance	6×3.3 mF
dc-link capacitor ESR at 100 Hz	$\frac{49 \text{ m}\Omega}{6}$
Number of blades	3
Turbine radius	3.24 m
Shaft length, ℓ_{sh}	5.85 m
Shaft diameter	95 mm
Shaft material thickness, t_{sh}	3.6 mm
Shaft torsion spring constant, κ^*	29 300 Nm/rad
Inertia of generator rotor, J_g	16.9 kgm ²
Inertia of turbine	$\sim 15 J_g$
Optimal power coefficient, C_p^{**}	0.29
Optimal tip speed ratio, λ^{**}	3.3

* From equation (15), based on shaft material and dimensions.

** For the original design, [21].

Fig. 1. The electromechanical simulation set-up. The resistive load R_L models the power converter after the rectification step.

Fig. 2. The experimental set-up, V_g is the generator line to neutral terminal voltages, I_g is the generator currents, Ω_g is the generator rotational speed and V_{dc} is the dc-link voltage.

back into time domain. The ripple amplitudes of the measurements were all estimated as MADs of the filtered signals by equation (16).

To provide the analytic results, the derived transfer functions (equations (12) to (14)) of torque ripple, rotational speed ripple and angular deflection were applied to the filtered FFT of the measured electrical torque ripple. The resulting three signals were inverse transformed back into time domain and the MAD ripple estimated. Assuming the shape of all ripple to be fairly sinusoidal shaped, the peak to peak amplitude of each ripple can be estimated by multiplication with π .

The simulations designed for experimental comparison were performed using the same model as described in Section III-A. All parameters were set to the original configuration in Table I. The averaged experimental torques and rotational speeds were used as targets for the simulations. Again, the secant method was used to set R_L to obtain simulated average electrical torque within ± 0.1 Nm of the experimentally obtained average electrical torque. For easy comparison between simulations and measurements, the simulation ripple was also expressed as MADs.

IV. RESULTS AND DISCUSSION

A. Simulated parameter study

A parameter study was performed to identify how different parameters affect the 96 p ripple. For each parameter the full operational range was simulated assuming optimal TSR operation, see Section III-A.

The impact of shaft torsional spring constant and rotor inertia on the system is illustrated in Fig. 3. Changing the shaft torsional spring constant only affects the turbine torque ripple. A stiffer shaft has similar impact as a lighter generator rotor. A lowered generator rotor inertia also increases the rotational speed ripple. An interesting trend is that even if the electric ripple increase with increased electrical torque, the ripple on the turbine actually decreases. This is explained by equation (14), where the transfer of mechanical ripple decrease proportionally to Ω_g^{-2} .

The impact of dc-link capacitance and generator inductance is illustrated in Fig. 4. A larger capacitor lowers the dc-link voltage ripple, while the effects on the torque ripple, rotational speed ripple and turbine torque ripple is negligible. Therefore, the original capacitor can be considered large, in terms of mechanical ripple effects. As the capacitor size decreases, the dc-link voltage ripple increases and a small increase in electrical torque ripple is noticed.

The generator inductance has the most significant impact on the electrical torque ripple of all studied parameters, compare Figs. 3 and 4. The change of inductance is only $\pm 20\%$, while the other parameters were tested at 50% and 200% . The result is expected; adding inductance to a passively rectified system is known to lower the current ripple, and therefore reasonably also lower the torque ripple [1]. Interestingly, even if the impact of inductance is increasing at higher torques and higher rotational speeds, those are also the conditions where the transfer to mechanical torque ripple is low. Overall, the inductance therefore has a limited influence on the mechanical

ripple. The inductance also has a small influence on the rotational speed ripple.

For the voltage, speed and torque ripple a trend shift is noted in the range 130 Nm to 180 Nm ($\Omega_t = 48$ rpm to 57 rpm), see Fig. 4. The shift occurs at the rotational speed where the increase in electrical torque ripple goes from linear to a non linear shape. The position of the shift depends on the generator inductance, a lower inductance increases the linear region. For the original design the shift is at 5.5 rad/s (52.5 rpm). The current at points above the shift is continuous between the current "double-peaks" (as the current waveform in Fig. 5). For points below the shift, the current reach 0 between the double-peaks.

The simulations also allowed for a pure sinusoidal EMF to be tested. In that case, the electric MAD ripple amplitude increased by 6 Nm for an average torque > 180 Nm i.e. about 5 % of the total MAD ripple magnitude. The 5th and 7th voltage harmonics are actually cancelling some of the ripple from the diodes.

B. Verification of analytical solution

The transfer functions in Section II-D simplifies analysis of the system and makes it possible to evaluate mechanical impact of high frequency generator ripple from measurements on the electrical torque ripple. From measurements on generator terminal voltages and generator current, the electrical torque of the generator can be determined using equation (3). From the FFT of the high frequency electrical torque ripple, the mechanical torque ripple can be determined from equation (14) and rotational speed ripple of the generator rotor from equation (13).

As a verification of the analytic equations, the simulated electrical torque ripple from the original design was used to estimate the mechanical torque ripple, rotational speed ripple and angular deflection ripple. The MAD of the obtained values were compared to the MAD of the simulation. The maximum deviation between simulation and analytic evaluation from simulated electrical torque was $< 0.4\%$ of actual MAD-value for the mechanical torque ripple, rotational speed ripple and angular deflection ripple. The small deviation makes the analytic solutions as good as the simulation.

C. Simulation and experiment comparison

The experimental data was extracted from large data sets with aims to find similar operation points as for the original case in the parameters studies presented in Section IV-A and Figs. 3 and 4. Precise conditions are difficult to obtain from open site experiments and some spread in data was accepted, as described in Section III-C. Therefore, the results presented in this section have data points at similar average electrical torque, but with different rotational speeds.

An example of time series data is shown in Fig. 5. The voltage waveform clearly has good agreement with the experiment. The current waveform is very similar, but the simulated peaks are slightly higher, resulting in an overestimation of the electrical torque ripple by the simulation. The captured waveforms of voltage and current have similar shape and

Fig. 3. Simulation results with changes to the generator rotor inertia and the shaft spring constant.

amplitude, even if the peaks of the simulations in both cases are larger. The electrical torque ripple for different operating conditions is compared to simulations in Fig. 6. There is clearly a deviation between simulation and measurement that increases with average torque. The estimated maximum error of τ_{el} is ± 10 Nm based on variation in calibration values. The ripple estimation removes a large portion of the systematic errors so the maximum error of the MAD ripple is expected to be considerably lower. Consequently, the seen deviation has more sources. Besides the measurement error, a probable additional cause for the deviations is the estimation of inductance in the system. As seen by the parameter study, the electrical torque ripple is fairly sensitive to the harmonics in the generator EMF, and these harmonics should therefore also be considered a possible source of error.

The measured mechanical torque ripple trend is similar to the simulations, but at about twice the amplitude, see

Fig. 4. Simulation results with changes to the generator inductance and dc-link capacitance.

Fig. 6. The analytic results estimated from the measured electric torque almost coincide with the simulations. The slightly lower values is explained by the lower measured electrical torque ripple, compared to the simulations. There is a deviating cluster of five measurements in the range 250 Nm to 350 Nm where the MAD ripple is ~ 4.5 Nm. The maximum measurement error (propagated error) of the torque in this operation range is $< \pm 15$ Nm based on [13]. The ripple estimation removes a large portion of the systematic errors and an actual error in MAD is expected to be considerably lower. It is still reasonable to assume that the overall deviation is due to errors. However, for the cluster of five points, further analysis indicate the main common factor between the points to be the rotational speed. All five points have a rotational speed within 59.8 rpm to 60.7 rpm. The closest points outside the clustering are at rotational speeds of 51 rpm and 65 rpm. To further study this hypothesis, the time series data in Fig. 7 illustrates a comparison between one point from the cluster and another point at about the same average electrical torque,

Fig. 5. A comparison of simulated and measured time series for one case. The voltage and current are from one of the phases. The measured and simulated data are time synchronised based on a zero crossing of the voltage. The electric torque τ_{el} has been high pass filtered.

but at a different rotational speed, both marked in Fig. 6. Clearly, the deviating point has a larger magnitude in the mechanical torque ripple and a more distinct 96 p shape. The seen clustering may also be within the error span, but its consistency and clear 96 p pattern, compared to measurement points at other rotational speeds suggests that it is an actual phenomenon. It seems like the ripple is exciting some natural frequency in the mechanical system. However, the ripple frequency at 60 rpm is 96 Hz (see Section II-B), which is considered an unlikely mechanical natural frequency for this system. The simplicity of the simulation model should also be considered. The model lacks any dampers from shaft or bearings and the coupling between generator and the shaft is considered a stiff joint.

The rotational speed ripple and angular deflection ripple of the generator rotor were too small to be measured with the experimental set-up. However, they can be obtained from simulations or analytically by equations (12) and (13). Since the electrical torque MAD ripple is used as input, the formulas also return MAD ripple amplitudes. Assuming the ripple to be sinusoidal, the maximum peak to peak amplitude is estimated to $\Delta\Omega_g = 0.035 \text{ rad/s}$ (0.26 % of nominal). In similar fashion, the angular deflection ripple peaks at $\theta_R = 9 \cdot 10^{-5} \text{ rad}$. Clearly, the large electromagnetic ripple cause very small physical movement of the generator rotor, as discovered in the parameter study, see Section IV-A.

V. CONCLUSIONS

It has been shown that the large electromagnetic torque ripple caused by passive rectification can be highly mechanically attenuated by the generator rotor inertia and the shaft as a torsional spring. No mechanical dampers are needed to achieve or model this effect. The configuration is realistic for a vertical axis wind turbine with a directly driven PMSG placed at ground level. A fairly simple electro-mechanical model and formulas have been derived to study the phenomenon.

Fig. 6. Simulation and experiment comparison. The time series of the circled points are presented in Fig. 7. Note that, the analytic and simulated results for τ_t are essentially overlapping.

Fig. 7. High pass filtered mechanical torque of the circled points in Fig. 6.

The simulation results predicted the mechanical torque ripple of the prototype to be $< \pm 0.25 \%$ of nominal torque. The experimental results confirm that the ripple is highly attenuated and in the same order of magnitude as the simulations. The measurements revealed an operational speed for which the rectification ripple could propagate less attenuated through the system. The presented model lacks the ability to predict this behaviour. The measured magnitude of the mechanical torque ripple for the special region was $< 0.6 \%$ of nominal. Still unverified, the reason may be excitation of natural frequencies.

As has been shown, the mechanical construction offers good mechanical filtering for the rectification ripple. The mechanical filtering has a stronger effect on the mechanical ripple than adding inductance to the electric circuit. Increased inductance has indeed a significant impact on the electrical torque ripple, still the mechanical ripple is only slightly changed. The angular twist of the shaft needed to perform the filtering is for the analysed construction about one 10000th of a degree. Even if the mechanical filtering from generator rotor and flexing shaft is effective at smoothing the torque ripple, other aspects must also be taken into account. The shaft is still subjected to a small angular twist at high frequency, which

may have mechanical implications. The natural frequencies of the shaft must harmonize with the rest of the design. The high oscillation of the electromagnetic forces may result in mechanical forces on the rotor magnets. Also, it should be kept in mind that even more mechanical aspects may be affected, for better or worse, by passive rectification. Examples could be rotor asymmetry, shaft resonance damping and fast load changes.

The model shows good agreement with measurements, both concerning statistical ripple estimation as well as waveforms. The presented model can therefore be useful for other studies of the system. The provided mechanical formulas can be used to estimate the high frequency mechanical effects from voltage and current measurements, given that the generator impedance is well known.

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REFERENCES

- [1] J. D. M. De Kooning, J. Van de Vyver, T. L. Vandoorn, B. Meersman, L. Vandeveld, J. D. M. D. Kooning, J. V. D. Vyver, T. L. Vandoorn, B. Meersman, and L. Vandeveld, "Joule losses and torque ripple caused by current waveforms in small and medium wind turbines," *EUROCON, 2013 IEEE*, no. July, pp. 889–896, 2013.
- [2] K. Amei, Y. Takayasu, T. Ohji, and M. Sakui, "A Maximum Power Control of Wind Generator System Using a Permanent Magnet Synchronous Generator and Boost Chopper Circuit," in *Proceedings of the Power Conversion Conference-Osaka 2002 (Cat. No.02TH8579)*. IEEE, 2002, pp. 1447–1452.
- [3] Y. Xia, K. H. Ahmed, and B. W. Williams, "Different Torque Ripple Reduction Methods for Wind Energy Conversion Systems Using Diode Rectifier and Boost Converter," in *Electric Machines Drives Conference (IEMDC), 2011 IEEE International*, 2011, pp. 729–734.
- [4] A. M. Mihai, S. Benelghali, A. Simion, R. Outbib, and L. Livadaru, "Design and FEM analysis of five-phase permanent magnet generators for gearless small-scale wind turbines," in *Proceedings - 2012 20th International Conference on Electrical Machines, IECM 2012*, 2012, pp. 150–156.
- [5] A. Di Gerlando, G. Foglia, M. F. Iacchetti, and R. Perini, "Analysis and test of diode rectifier solutions in grid-connected wind energy conversion systems employing modular permanent-magnet synchronous generators," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 5, pp. 2135–2146, 2012.
- [6] S. Kato, Y. Inui, and M. Michihira, "A comparative study of steady-state characteristics of permanentmagnet synchronous generator systems," in *Power Electronics and Applications, 2007 European Conference on*, 2007, pp. 1–10.
- [7] D. M. Vilathgamuwa and S. D. G. Jayasinghe, "Rectifier Systems for Variable Speed Wind Generation A Review," in *Industrial Electronics (ISIE), 2012 IEEE International Symposium on*, 2012, pp. 1058–1065.
- [8] G. Heins, M. Thiele, and T. Brown, "Accurate torque ripple measurement for PMSM," *IEEE Transactions on Instrumentation and Measurement*, vol. 60, no. 12, pp. 3868–3874, 2011.
- [9] A. Solum and P. Deglaire, "Design of a 12kW vertical axis wind turbine equipped with a direct driven PM synchronous generator," in *Proceedings of EWECE 2006 - European Wind Energy Conference & Exhibition, Athens*, 2006, pp. 1–9.
- [10] P. Deglaire, S. Eriksson, J. Kjellin, and H. Bernhoff, "Experimental results from a 12 kW vertical axis wind turbine with a direct driven PM synchronous generator," in *Proceedings of EWECE 2007 - European wind energy conference & exhibition, Milan*, 2007.
- [11] J. Kjellin, S. Eriksson, P. Deglaire, F. Bülow, and H. Bernhoff, "Progress of control system and measurement techniques for a 12 kW vertical axis wind turbine," in *Proceedings of EWECE 2008 - European wind energy conference & exhibition, Brussels*, 2008.
- [12] S. Eriksson, A. Solum, M. Leijon, and H. Bernhoff, "Simulations and experiments on a 12kW direct driven PM synchronous generator for wind power," *Renewable Energy*, vol. 33, no. 4, pp. 674–681, 2008.
- [13] M. Rossander, E. Dyachuk, S. Apelfröjd, K. Trolin, A. Goude, H. Bernhoff, and S. Eriksson, "Evaluation of a Blade Force Measurement System for a Vertical Axis Wind Turbine Using Load Cells," *Energies*, vol. 8, no. 6, pp. 5973–5996, June 2015.
- [14] A. A. Arkadan, T. M. J. Hijazi, N. A. Demerdash, J. G. Vaidya, and V. K. Maddali, "Theoretical development and experimental verification of a DC-AC electronically rectified load-generator system model compatible with common network analysis software packages," *IEEE Transactions on Energy Conversion*, vol. 3, no. 1, pp. 123–131, Mar. 1988.
- [15] J. Jatskevich, S. D. Pekarek, and A. Davoudi, "Fast procedure for constructing an accurate dynamic average-value model of synchronous machine-rectifier systems," *IEEE Transactions on Energy Conversion*, vol. 21, no. 1, pp. 435–441, 2006.
- [16] S. Chiniforoosh, H. Atighechi, A. Davoudi, J. Jatskevich, A. Yazdani, S. Filizadeh, M. Saeedifard, J. a. Martinez, V. Sood, K. Strunz, J. Mahseredjian, and V. Dinavahi, "Dynamic average modeling of front-end diode rectifier loads considering discontinuous conduction mode and unbalanced operation," *IEEE Transactions on Power Delivery*, vol. 27, no. 1, pp. 421–429, 2012.
- [17] P. Pillay and R. Krishnan, "Modeling of permanent magnet motor drives," *IEEE Transactions on Industrial Electronics*, vol. 35, no. 4, pp. 537–541, 1988.
- [18] V. Ruuskanen, "Design Aspects of Megawatt-Range Direct-Driven Permanent Magnet Wind Generators," Ph.D. dissertation, Lappeenranta University of Technology, Lappeenranta, Finland, 2011.
- [19] G. J. Wakileh, "Harmonics in rotating machines," *Electric Power Systems Research*, vol. 66, pp. 31–37, 2003.
- [20] F. Bülow, S. Eriksson, and H. Bernhoff, "No-load core loss prediction of PM generator at low electrical frequency," *Renewable Energy*, vol. 43, pp. 389–392, 2012.
- [21] J. Kjellin, F. Bülow, S. Eriksson, and P. Deglaire, "Power coefficient measurement on a 12 kW straight bladed vertical axis wind turbine," *Renewable Energy*, vol. 36, no. 11, pp. 3050–3053, 2011.

Morgan Rossander was born in Ockelbo, Sweden, in 1981. He finished his MSc in Engineering Physics at Uppsala University, Sweden in 2006 and worked as an electrical engineer in the automation industry. In 2013 he started his Ph.D. studies at the division of electricity, department of engineering at Uppsala University. Research interest are control strategies for vertical axis wind turbines and how the electrical system interacts with the turbine.

Anders Goude was born Stjärnsund, Sweden, in 1982. He finished his MSc in Engineering Physics at Uppsala University, Sweden in 2007 and studied towards a Ph.D. degree in engineering science with a specialization in science of electricity between 2007 and 2012. He currently holds a position as assistant professor at the division of electricity, department of engineering at Uppsala University. His research interest are development of simulation software for CFT applications and fluid mechanics of vertical axis turbines.

Sandra Eriksson was born in Eskilstuna, Sweden, in 1979. She finished her MSc in Engineering Physics at Uppsala University, Sweden in 2003 and studied towards a Ph.D. degree in engineering science with a specialization in science of electricity between 2004 and 2008. She currently holds a position as associate professor at the division of electricity, department of engineering at Uppsala University. Her main topics of interest are design of permanent magnet electrical machines, alternative permanent magnet materials as well as control strategies and electrical system for wind turbines.