Modeling volatility for the Swedish stock market

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Fall, 2015

Abstract
This thesis will investigate if adding an exogenous variable (implied volatility) to the variance equation will increase the performance for the GARCH(1,1) and EGARCH(1,1) models based on the OMXS30 index. These models are also compared with the implied volatility itself as a forecasting/modeling method. To evaluate the models the realized variance will be used as an unbiased estimator of the conditional variance. The findings suggest that adding implied volatility to the variance equation increase the overall performance.

Keywords: GARCH, Conditional variance, Realized variance, Implied volatility, Forecasting volatility, Heteroskedasticity, Time series

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1 Introduction

In finance one is interested in uncertainty for future events and their outcomes. The uncertainty is often called risk and is used for asset pricing such as derivatives or other types of financial instrument. It’s also used for portfolio management and in areas where uncertainty is of interest. Risk is generally measured with volatility i.e. the degree of variation in some asset. Due to the importance of volatility a broad interest has been to model the nature of it. One characteristic of financial time series, e.g. stock returns, is that they suffer from heteroscedasticity; the variance is not constant over time. (Engle 1982) introduced the ARCH model which captures this characteristic of heteroscedasticity by conditioning variance and (Bollerslev 1986) introduced the GARCH model which has been widely used in modeling and forecasting volatility (Bollerslev et al 1994). Since these publications there has been a boom in modified GARCH models that try to capture the nature of volatility. One model that has been brought to attention is the EGARCH model purposed by (Nelson 1991) which capture the leverage effect found in stock returns. The basics of these models are that they depend only on the information available on the time series itself and not on any exogenous information (exogenous variable).

In the last decades the use of financial derivatives has grown exponentially and market participants are using them frequently for different purpose and pricing them in general to market price equilibrium. This has led to the construction of implied volatility which measures the markets expectation on volatility. The CBOE Volatility Index (VIX) is the leading index measuring the expected volatility for the S&P 500 index and is often called the fear index. Due to globalization and interdependence, markets are integrated with each other so the VIX can be seen as a general fear index for the stock market in the western world (Europe and North America). Many listed stocks in European exchanges are also listed in the American exchanges. The VIX can therefore have relevant information when modeling financial volatility. The implied volatility for the OMXS30 index called SIXVX may also contain relevant information. (Kambouroudis and McMillan 2013) found evidence that, including the implied volatility to the variance equation, may increase the performance of the GARCH type models.

This thesis will model the volatility for the OMXS30 index which is a good index to represent the Swedish stock market as a whole. Six different GARCH models will be used, a plain vanilla GARCH(1,1) model,
a EGARCH(1,1) model and these two models with the VIX or SIXVX as exogenous variable in the variance equation. This is done to check if the performance of the models improves by adding the exogenous variable. The EGARCH model is chosen because it captures the basic characteristics of asset returns. Additionally the implied volatility will be used to investigate which method models and forecast volatility\(^1\) best for the omxs30 index, market expectation (implied volatility), the models or a combination of both.

\(^1\)In this thesis conditional variance and volatility will be used as the same metric where \(\sqrt{\text{conditional variance}}\) equal volatility
2 Economic Theory

2.1 Implied volatility

Implied volatility is a measure of the markets expectation for future volatility of some underlying asset. The way to calculate it is to use an option pricing model and then derive the volatility. Basic option pricing models compute the option price $C$ as

$$C = f(\sigma, \Omega),$$

(1)

where $\Omega$ denote all possible components the option pricing model use and are often given. $\sigma$ denote the volatility of the underlying asset. The implied volatility is then derived as

$$\sigma = g(C, \Omega),$$

(2)

where $C$ and $\sigma$ are one to one which means that if $C$ is high, the market expect turbulent condition in the future, in the underlying asset. The time horizon for the expected volatility is based on how the implied volatility is calculated, often measured for one month ahead expectation by weighing options with different expiration dates (Hull 2012).

2.2 Efficient-market hypothesis

The Efficient-market hypothesis (EMH) states that markets are information efficient. There are three types of EMH. The first one is called the weak form efficient which means that information in past stock prices are already incorporated in the current price of the stock. The second type is called semi-strong form efficiency which means that all the information available in public are already priced in the stock price. The third is called strong form efficiency which implies that all the information in past stock prices, private information and public information is incorporated in the current stock price. In this paper when referring to EMH the strong form efficiency will be used. You can’t beat the market because the price of the asset reflects all available information. All available information has been incorporated in the price of the asset (Ma 2004).

2.3 Leverage effect

The leverage effect means that stock returns are negatively correlated with volatility. Negative returns (bad news) tend to generate higher volatility.
than positive returns (good news) of the same magnitude. This hypothesis is based on how the firm’s capital structure buildup. If the firm’s equity market value falls, then the assets are financed with a larger proportion of debt which will generate higher volatility.

3 Statistical theory

3.1 Time series
A sequence of random variables is a stochastic process. The realization of the finite stochastic process is defined as the time series. To make statistical inference about the stochastic process the assumption of stationary is made, which means the probability structure do not change over the time. There are two types of stationary, strong and weak. The first one means that all random variables have the same mean, variance, distribution and covariance. Weak stationary has the properties of constant mean and same covariance structure as in strong stationary (Cryer and Chan 2008). The ARCHs and GARCHs processes are weak stationary.

3.2 Models
This thesis will model log returns conditional variance (volatility). Assuming the efficient markets hypothesis the expected return is zero. The conditional mean equation is then

$$r_t = E(r_t|\Omega_{t-1}) + \epsilon_t,$$

where \(\Omega_{t-1}\) indicate all past information. This implies,

$$r_t = \epsilon_t \quad \epsilon_t \sim IID(0,\sigma_t^2)$$

3.3 GARCH(q,p)
The GARCH model is defined as

$$\epsilon_t = z_t \sigma_t \quad z_t \sim IID(0,1),$$

where \(z_t\) is the innovation term. The conditional variance is given by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2.$$
\( \sigma_t^2 \) is nonnegative and weak stationary thus the required conditions are

\[
\alpha_0 > 0,
\]

\[
\beta_i \geq 0 \quad i = 1, 2, \ldots, p,
\]

\[
\alpha_j \geq 0 \quad j = 1, 2, \ldots, q,
\]

\[
\sum_{i=1}^{p} \beta_i + \sum_{j=1}^{q} \alpha_j < 1.
\]

Here \( p \) is the GARCH order and \( q \) the ARCH order. The GARCH coefficients measure how past values of the conditional variance affect the current conditional variance and the ARCH coefficients measure how squared residual (previous variance) affect the current conditional variance. If \( p=0 \) the model is an ARCH(q) model. Problems with this model is that \( t-1 \) squared residual sign have the same impact (symmetrical) on the conditional variance which sometimes may not be the case.

### 3.4 EGARCH(q,p)

The EGARCH model captures the basic characteristics of asset return by taking the leverage effect into account by estimating a parameter that capture the effect. The EGARCH model is given by

\[
\log \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \log \sigma_{t-i}^2 + \sum_{j=1}^{q} (\alpha_j z_{t-j} + \gamma_j (|z_{t-j}| - E|z_{t-j}|)), \quad (7)
\]

where the \( \alpha_j \) coefficient represent the asymmetry effect. This parameter is negative if the leverage effect occurs in the time serie.

### 3.5 GARCH(1,1)VIX

This model is a plain vanilla GARCH(1,1) model with the VIX as an exogenous variable in the variance equation. The model is given by

\[
\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 \epsilon_{t-1}^2 + \delta VIX_{t-1}.
\]

Conditions for non-negative conditional variance are
\[ \beta_1 \geq 0, \]
\[ \alpha_1 \geq 0, \]
\[ \beta_1 + \alpha_1 < 1, \]
\[ \alpha_0 + \delta > 0. \]

The parameter \( \delta \) will capture the effect of the exogenous variable VIX.

### 3.6 GARCH(1,1)SIXVX

This model is the same as the 3.5 model but with the SIXVX as an exogenous variable in the variance equation. The model is given by

\[
\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 \epsilon_{t-1}^2 + \delta \text{SIXVX}_{t-1}. \tag{9}
\]

It has the same conditions as in the 3.5 model.

### 3.7 EGARCH(1,1)VIX

This model is the EGARCH(1,1) with the exogenous variable VIX in the variance equation as

\[
\log \sigma_t^2 = \alpha_0 + \beta_1 \log \sigma_{t-1}^2 + (\alpha_1 z_{t-1} + \gamma \left( |z_{t-1}| - E |z_{t-1}| \right)) + \delta \text{VIX}_{t-1}. \tag{10}
\]

The parameter \( \delta \) represent the effect of the exogenous variable VIX on the conditional variance.

### 3.8 EGARCH(1,1)SIXVX

This model is the EGARCH(1,1) with the exogenous variable SIXVX in the variance equation as

\[
\log \sigma_t^2 = \alpha_0 + \beta_1 \log \sigma_{t-1}^2 + (\alpha_1 z_{t-1} + \gamma \left( |z_{t-1}| - E |z_{t-1}| \right)) + \delta \text{SIXVX}_{t-1}. \tag{11}
\]
4 Data

4.1 OMXS30 index

The OMXS30 index is based on the 30 most traded securities on the Stockholm stock exchange and is revised every sixth month. The base value of the index was 500 in 1986 but due to a 4:1 split in 1998 the base value is now 125. The weight each security has in the portfolio is based on the market share of the security (Nasdaq 2014). The index gives a good representation on the market conditions in the Swedish equity market. The index open at 09:00am and close at 05:30pm (GMT+1).

Figure 1: Graph of OMXS30 index
4.2 CBOE volatility index

The CBOE volatility index (VIX) was created in 1993 to measure the implied volatility. The index is built on weighted put and call option with different strike prices with the Standard & Poor 500 (S&P 500) index as the underlying asset. The index measures the market expectation on the volatility for 30 days ahead (CBOE 2014). The index is expressed in annual volatility. The index open at 03:30pm and close at 10:00pm (GMT+1).

![Graph of CBOE volatility index (VIX)](image)

Figure 2: Graph of CBOE volatility index (VIX)
4.3 SIXVX volatility index

The SIXVX volatility index is calculated every trading day by SIX financial information. The index is based on weighted put and call options with the OMXS30 index as the underlying. The index gives a good representation of the expected volatility for the Swedish market 30 days ahead. The volatility is expressed in annualized term as in Figure 3 (SIX 2014). To convert it into daily variance the volatility is divided by the square root of 252, assuming that there are 252 trading days per year. Then the daily volatility is squared to get daily implied variance. The index opens at 09:00am and closes at 05:30pm (GMT+1).

Figure 3: Graph of SIXVX volatility index
5 Data analysis

5.1 OMXS30 return

The daily data used to estimate the models in this thesis is gathering from SIX EDGE database. For the OMXS30 series there are 1622 observations of the closing price for each trading day. The range is from 2009/01/05 to 2015/06/23. The log returns are calculated by

\[ r_t = 100 \cdot \ln \left( \frac{p_t}{p_{t-1}} \right), \tag{12} \]

where \( p_t \) denote the price of the portfolio at time \( t \). In Figure 4 the returns are plotted over time and the volatility tend to vary. Some periods have higher degree of volatility than other e.g. late 2011. On the vertical axis the histogram of the returns seems to have a bell shape distribution.

Figure 4: Graph of OMXS30 index returns with histogram
Returns are sometimes assumed to follow a normal distribution where the density function is given by

$$f(r) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(r - \mu)^2}{2\sigma^2}\right). \quad (13)$$

A common characteristic of asset returns is that they have fat-tailed distribution; they are skewed and suffer from leptokurtosis. The Jarque–Bera test (JB test) of normality as in the Appendix A gave statistical significant result (p-value 0.000) and the null is rejected of normal distribution. Table 1 show that the kurtosis is high thus the data seems to have a leptokurtosis distribution (fat-tailed). The skewness is negative, longer left tail and asymmetrical distribution. The largest return was 6.62% and the lowest return was -6.96%.

<table>
<thead>
<tr>
<th>Serie</th>
<th>Mean</th>
<th>Std.Dev</th>
<th>Max</th>
<th>Min</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>OMXS30</td>
<td>0.052319</td>
<td>1.324463</td>
<td>6.6235307</td>
<td>-6.968643</td>
<td>-0.145105</td>
<td>5.545124</td>
</tr>
</tbody>
</table>

Table 1: Table of descriptive statistics

5.2 Distribution of returns

To verify what an appropriate distribution for the unconditional returns can be, the returns population density function is estimated with kernel density estimator. The kernel function used in the estimation is the Epanechnikov. (Fregusson and Platen 2005) found empirical evidence that world stock indices returns follows the non-standardized Student’s t-distribution with about four degrees of freedom. The density function is given by

$$f(r) = \frac{\Gamma\left(\nu + \frac{1}{2}\right)}{\sigma \sqrt{\pi \nu \Gamma\left(\frac{1}{2}\right)}} \left(1 + \frac{1}{\nu} \left(\frac{r - \mu}{\sigma}\right)\right)^{-\nu + \frac{1}{2}}, \quad (14)$$

where $\Gamma(\cdot)$ is the gamma function and $\nu$ is the degrees of freedom. The parameter $\sigma$ is the overall scaling of the distribution. In Figure 5 the estimated density function of return and the theoretical t-distribution density function are plotted. The density functions seems to be the same.
To verify that the \( t \)-distribution holds a Kolmogorov-Smirnov test (KS-test) is performed as given in the Appendix A. The null of \( t \)-distribution is not rejected and the unconditional return is assumed to follow a \( t \)-distribution. If the returns are specified as in equation 4 and 5 this would imply that the innovation term in the GARCH models follows a \( t \)-distribution. A not so robust method to identify what distribution the innovation term has, is to compute the standardized residual and check what distribution they follow. The standardized residuals are given by rearranging equation 4 and 5 so

\[
\frac{r_t}{\sigma_t} = \hat{z}_t, \tag{15}
\]

then divide by the standard deviation to get the standardized residuals. This assumes that a model has been specified. Two GARCH(1,1) models are estimated using \( t \)- and normal distribution. Then the standardized residual are computed and a KS-test is performed as in the Appendix A to check the distribution against \( t \)- and normal distribution. Table 2 shows the p-values for respective KS-test. The null for normality are rejected but not when tested against \( t \)-distribution. This result is similar as (Sun and Zhou 2014), they study empirically what distribution the innovation term had and conclude that a \( t \)-distribution is preferable. By these findings the innovation
term is assumed to follow a non-standardized Student’s t-distribution as in equation 14.

<table>
<thead>
<tr>
<th>Test distribution</th>
<th>of GARCH(1,1)-N</th>
<th>of GARCH(1,1)-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-distribution</td>
<td>0,000</td>
<td>0,000</td>
</tr>
<tr>
<td>T-distribution</td>
<td>0,9047</td>
<td>0,8757</td>
</tr>
</tbody>
</table>

Table 2: Table of p-values from KS-test

5.3 Squared return (variance)

Given that the OMXS30 time series suffer from volatility clustering there should be serial correlation in the squared return (variance)\(^2\). In Figure 6 the squared returns seem to be varying (volatility clustering). The sample autocorrelation function (SACF) with ten lags shows that there are serial dependence and all the autocorrelation are outside the \(\pm \frac{2}{\sqrt{n}}\) bounds or the approximated 95 % confidence interval. To verify that the autocorrelation are different from zero a Ljung-box Q-test is performed for the ten lags as in the Appendix A. The null is rejected (p-value 0,000) and therefore the squared returns are assumed to have serial correlation.

![Graph of variance and SACF from OMXS30 returns](image)

Figure 6: Graph of variance and SACF from OMXS30 returns

\(^2\)For each random variable in the stochastic process there are only one observation to estimate the variance.
6 Estimation

In this thesis only low order GARCH model will be used where q=1 and p=1 for equation 6 and 7. When estimating the models parameters the program used is R with the rugarch package. When estimating the GARCH models one needs to assume a specific distribution for the innovation term. By the empirical findings the GARCH models will be estimated with the non-standardized Student’s t-distribution.

When estimating the models the observations for respective index most be in the same date. Due to the VIX are based on the S&P 500 index and there are trading days where the OMXS30 is closed and the S&P 500 open or vice versa, some days are missing. The total amount of observations after the correction is 1454. The $\alpha_0$ coefficient for the GARCH(1,1)-VIX and the GARCH(1,1)-SIXVX are set to be zero so the restrictions in section 3.5 are fulfilled, estimation without $\alpha_0 = 0$ gave large negative $\alpha_0$ which violated the restrictions.

In table 3 the estimated coefficients are presented and the corresponding p-value. $\alpha_0$ is the constant, $\alpha_1$ the ARCH coefficient, $\beta$ the GARCH coefficient, $\gamma$ the size effect parameter in the EGARCH models and $\delta$ the parameter that capture the exogenous variables effect on the conditional variance. All the parameter have statistical significant result on the 5 % level except the $\gamma$ coefficient for the EGARCH(1,1)-SIXVX model. The low p-values of the exogenous variable parameter indicate that the implied volatility have an effect on the conditional variance. All the restrictions are fulfilled that are presented in the statistical theory section.
<table>
<thead>
<tr>
<th>Models</th>
<th>$\alpha_0$</th>
<th>$\beta_1$</th>
<th>$\alpha_1$</th>
<th>$\delta$</th>
<th>$\gamma_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1,1)</td>
<td>0.022 (0.015)</td>
<td>0.908 (0.0000)</td>
<td>0.077 (0.0000)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>GARCH(1,1)-VIX</td>
<td>0.000 (-)</td>
<td>0.872 (0.0000)</td>
<td>0.086 (0.0000)</td>
<td>0.003 (0.0164)</td>
<td>-</td>
</tr>
<tr>
<td>GARCH(1,1)-SIXVX</td>
<td>0.000 (-)</td>
<td>0.832 (0.0000)</td>
<td>0.090 (0.0000)</td>
<td>0.0056 (0.0209)</td>
<td>-</td>
</tr>
<tr>
<td>EGARCH(1,1)</td>
<td>0.008 (0.0234)</td>
<td>0.976 (0.0000)</td>
<td>-0.144 (0.0000)</td>
<td>-</td>
<td>0.136 (0.0000)</td>
</tr>
<tr>
<td>EGARCH(1,1)-VIX</td>
<td>-0.213 (0.0001)</td>
<td>0.880 (0.0000)</td>
<td>-0.207 (0.0000)</td>
<td>0.011 (0.0006)</td>
<td>0.086 (0.0037)</td>
</tr>
<tr>
<td>EGARCH(1,1)-SIXVX</td>
<td>-0.46 (0.0059)</td>
<td>0.738 (0.0000)</td>
<td>-0.218 (0.0000)</td>
<td>0.026 (0.0050)</td>
<td>0.024 (0.5493)</td>
</tr>
</tbody>
</table>

Table 3: Table of estimated parameters and the p-value for respective GARCH model. P-values are in the parentheses.

7 Evaluation

One problem when evaluating conditional variance is the unobservable nature of it. The true value of the latent variable will never be observed. One way to overcome this problem is to estimate the unobservable variable. (Andersen and Bollerslev 1998) suggested the realized variance as an unbiased estimator of the conditional variance. The benchmark to compare the models will then be the realized variance.

To evaluate the models, two different methods will be used. The first is to evaluate the in-sample conditional variance and implied volatility by using Mincer Zarnowitz regression (MZ-regression) and the coefficient of determination. The second method is to evaluate the forecasted values by using mean squared error (MSE).

7.1 Realized variance

The daily realized variance is constructed by summing intraday squared returns for some time interval. Suppose there are n equally spaced squared returns as in equation 12 for a trading day e.g. 1 min squared returns. Then

$$RV_t = \lim_{n \to \infty} \sum_{i=1}^{n} r_i^2,$$

Here the implied volatility also will be evaluated with the realized variance.
where $RV_t$ is an estimate of the true variance of day $t$.

The OMXS30 data used to calculate the realized variance is gathered from the Esignal database in 10 min interval. The returns are computed as in equation 10 then squared and sum for each trading day using an algorithm. Each trading day contain 52 observation of 10 min closing price of the index. Due to Esignal only offer intraday data from 2009/05/29 the in-sample evaluation will start from that date.

7.2 Mincer Zarnowitz regression and $R^2$

When using MZ-regression to evaluate the models, one estimate the following OLS regression

$$RV_t = \alpha_0 + \beta_1 \hat{\sigma}_t^2 + \epsilon_t,$$

(17)

where $\hat{\sigma}_t^2$ is the conditional variance or implied volatility and $RV_t$ the realized variance. Then the following hypothesis are tested

$$H_0 : \alpha_0 = 0 \quad H_0 : \beta_1 = 1$$

$$H_1 : \alpha_0 \neq 0 \quad H_1 : \beta_1 \neq 1$$

If the intercept is equal to zero, i.e. not rejecting the null, the conditional variance is unbiased. If the slope parameter equals to one, then the realized variance is explained perfectly by the conditional variance.

To find out how much the in-sample conditional variance explain the variation of the realized variance the coefficient of determination will be used (non-adjusted $R^2$). The model with highest R-squared will be the model that performed best.

7.3 Mean Squared error (MSE)

The MSE measures the expected value of the squared error between the estimated (forecasted) variance and true variance. The formula is given by

$$MSE(\hat{\sigma}^2) = \mathbb{E}[(\hat{\sigma}^2 - \sigma^2)^2] = n^{-1} \sum_{i=1}^{n} (\hat{\sigma}_i^2 - \sigma_i^2)^2,$$

(18)

where $n$ denote the number of observation, $\sigma_i^2$ the realized variance for the $i$-th day and $\hat{\sigma}_i^2$ the estimated variance from the models. The model with lowest MSE will be preferable because on average the estimate will be closer
to the population parameter.

7.4 Forecasting

In this thesis only 1 day ahead forecast will be used. To illustrate the procedure of making a forecast, the GARCH(1,1) will be used as an example. The one day forecast is given by

\[ \sigma_{t+1}^2 = \alpha_0 + \beta_1 \sigma_t^2 + \alpha_1 \epsilon_t^2. \]  

(19)

The forecasts are done using a rolling window of 1424 observation and the parameters are re-estimated for each new window. The forecast procedure using the implied volatility is done by

\[ \sigma_{t+1}^2 = VIX_t^2. \]  

(20)

7.5 Evaluation time periods

When evaluating the models with the MZ-regression the time horizontal are from 2009/05/29 to 2015/05/07 including a total of 1424 observations. The forecasting period start from 2015/05/08 and ends 2015/06/23 with a total of 30 observations. When calculating the MSE for the implied volatility one period before is used so the time period used is from 2015/05/07 to 2015/05/22.
8 Results

In this section the result from the evaluation methods will be presented.

![Graph of estimated daily variances from respective model and implied volatility](image)

Figure 7: Graph of estimated daily variances from respective method

First the estimated in-sample conditional variances for respective model, realized variance and implied volatility \(^4\) are plotted in Figure 7 where the data has been transform to a 50 day average to remove the "noisy" lines and to get a clear plot. The smoothing is done with,

\(^4\)Here the implied volatility is squared to get it in variance
\[
\frac{1}{n} \sum_{i=1}^{n} \hat{\sigma}_i^2 \tag{21}
\]

where \(n\) is the number of observations and \(\hat{\sigma}_i\) the estimation for the \(i\)-th day. By this procedure the first 49 observations are missing. Most of the time the realized variance is under the solid lines which represent the models and implied volatility, this means that the models and implied volatility have overestimated the realized variance (thick line).

In Table 4 the result for respective evaluation method is presented. The R-square show that all the models and implied variance explain around 50% of the realized variance variation which is high. The EGARCH(1,1)-SIXVX model had the highest R-squared and explain 62% of the variation in realized variance. This model has a higher explanatory power than the implied volatility. The EGARCH(1,1) model had a R-square of 0.57 and GARCH(1,1) had the lowest with 0.43. The GARCH(1,1)-VIX and the GARCH(1,1)-SIXVX models performed better than the GARCH(1,1) model.

The results from the MZ-regression show that all the models except the EGARCH(1,1)-SIXVX have an intercept significantly different from zero. All the models have slope coefficient statistical significant different from one.

The results from the forecasts show that the model with the lowest MSE was the EGARCH(1,1) VIX model, this means that on average, the estimated conditional variance will have a square error of 0.20. The VIX had the lowest MSE (0.10) of all methods. The EGARCH(1,1) model had the worst MSE with 1.08.
<table>
<thead>
<tr>
<th>Model</th>
<th>$R^2$</th>
<th>MSE</th>
<th>$\alpha_0$</th>
<th>$\beta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1,1)</td>
<td>0.43</td>
<td>0.44</td>
<td>$-0.133 (0.0007)$</td>
<td>$0.657 (0.0000)$</td>
</tr>
<tr>
<td>GARCH(1,1)-VIX</td>
<td>0.46</td>
<td>0.34</td>
<td>$-0.220 (0.0000)$</td>
<td>$0.731 (0.0000)$</td>
</tr>
<tr>
<td>GARCH(1,1)-SIXVX</td>
<td>0.48</td>
<td>0.38</td>
<td>$-0.322 (0.0000)$</td>
<td>$0.820 (0.0000)$</td>
</tr>
<tr>
<td>EGARCH(1,1)</td>
<td>0.57</td>
<td>1.08</td>
<td>$-0.218 (0.0000)$</td>
<td>$0.717 (0.0000)$</td>
</tr>
<tr>
<td>EGARCH(1,1)-VIX</td>
<td>0.58</td>
<td>0.20</td>
<td>$-0.114 (0.0002)$</td>
<td>$0.637 (0.0000)$</td>
</tr>
<tr>
<td>EGARCH(1,1)-SIXVX</td>
<td>0.62</td>
<td>0.35</td>
<td>$0.002 (0.9520)$</td>
<td>$0.550 (0.0000)$</td>
</tr>
<tr>
<td>VIX</td>
<td>0.41</td>
<td>0.10</td>
<td>$-0.383 (0.0000)$</td>
<td>$0.728 (0.0000)$</td>
</tr>
<tr>
<td>SIXVX</td>
<td>0.55</td>
<td>0.40</td>
<td>$-0.332 (0.0000)$</td>
<td>$0.748 (0.0000)$</td>
</tr>
</tbody>
</table>

Table 4: Table of the results from the evaluation methods. $\alpha_0$ and $\beta_1$ are the coefficients from the MZ-regression.

9 Conclusion

In this thesis the purpose was to investigate if adding the implied volatility to the variance equation would increase the performance of the different models. First the exogenous variable parameter $\delta$ in respective model have statistical significant results on the 5% level which indicate that the implied volatility have effect on the conditional variance. Also the coefficient of determination for the models with the implied volatility where overall higher than the models without. The model with highest R-square where EGARCH(1,1)-SIXVX that explain 62% of the variation of realized variance compared to the EGARCH(1,1) with R-square of 57%.

The results for the forecasted values shows that the models without implied volatility have higher MSE than the models with the exogenous variable. The model with the lowest MSE is the EGARCH(1,1)-VIX. One reason why this model has a lower MSE than the EGARCH(1,1)-SIXVX could be that the VIX index closes later than the SIXVX index and therefore would capture more relevant information for the forecast.

The MZ-regression results show that only the EGARCH(1,1)-SIXVX model produce conditional variance that is unbiased. All the models have statistically significant results on the slope parameter $\beta_1$ which implies that none
have a perfect linear relation with the realized variance. The best results for forecasting volatility was the VIX alone with MSE of 0.10 which may indicate on a better forecasting method than the models.
10 References


11 Appendix A

The significant tests will be presented in the general form here.

11.1 Jarque-Bera (JB) test of Normality

JB test if the random variable is normal distributed. A normal distributed random variable has a kurtosis of 3 and a skewness of 0. The JB test of normality test if the samples kurtosis and skewness are like the ones in a normal distribution. The statistic follows a chi-square distribution with 2 degrees of freedom.

\[ JB = n \left[ \frac{S^2}{6} + \frac{(K - 3)^2}{24} \right] \sim \chi^2_2 \]

where,

\[ S = \frac{\mathbb{E}(X - \mu)^3}{\sigma^3} \]

Here the \( \mu \) is the mean and the \( \sigma \) standard deviation and,

\[ S = \frac{\mathbb{E}(X - \mu)^4}{\left[ \mathbb{E}(X - \mu)^2 \right]^2} \]

The test hypothesis are,

\[ H_0 : X \sim Normal \]

\[ H_a : X \sim Not normal \]

11.2 Kolmogorov-Smirnov Test (KS test)

The KS test check if a sample comes from a specified distribution. Under the null hypothesis the sample comes from the specified distribution. From a sample \( x_1, x_2, \ldots, x_n \) of n iid observation you compute the empirical CDF \( \hat{F}_n(x) \) and then specify a CDF \( F(x) \). Define \( D_n \) as,

\[ D_n = \max_x |F(x) - \hat{F}_n(x)| \sim Kolmogorov distribution \]

Then if,

\[ D_n \geq D_{n, 1 - \alpha} \]
Reject the null where $D_n$ is the observed statistic and $\alpha$ is the significant level.

### 11.3 Ljung-Box Q-test

The Ljung-Box Q-test checks if all autocorrelation are together equal to zero or not. Under the null they are simultaneously equal to zero. The sample size $n$ should be large. The hypotheses are,

$$H_0: p_1 = p_2 = ... = p_k = 0$$

$$H_a: \text{at least one } p_j \neq 0 \quad j = 1, 2, ..., k$$

The test statistic are given by,

$$Q_{LB} = n(n + 2) \sum_{j=1}^{k} \frac{\hat{p}_j^2}{n - j} \sim \chi^2_{k \text{df}}$$

This statistic follows a chi-square distribution with $k$ degree of freedom. $k$ stands for the number of lags for the $p_j$ autocorrelation.

Reject $H_0$ when $\chi^2_{\text{obs}} > \chi^2_{1-\alpha, k}$