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Theoretical Studies of Hadronic Reactions with Vector Mesons

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Abstract

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Aiming at a systematic inclusion of pseudoscalar and vector mesons as active degrees of freedom in an effective Lagrangian, studies have been performed in this thesis concerning the foundations of such an effective Lagrangian as well as tree-level and beyond-tree-level calculations. Hereby, vector mesons are described by antisymmetric tensor fields.

First, an existing power counting scheme for both pseudoscalar and vector mesons is extended to include the pseudoscalar-meson singlet in a systematic way. Based on this, tree-level calculations are carried out which are in good agreement with the available experimental data and several processes are predicted. In particular, the ω-π^0 transition form factor is in better agreement with experimental data than the prediction done in the vector-meson-dominance model. Furthermore, a Lagrangian with vector mesons is used together with the leading contributions of chiral perturbation theory in order to calculate tree-level reactions in the sector of odd intrinsic parity. It turns out that both the Lagrangian with vector mesons and the Lagrangian of chiral perturbation theory are needed to describe experimental data.

Additionally, a feasibility check for one-loop calculations with pseudoscalar and vector mesons in the loop is performed. Thereby, only a limited number of interaction terms in the Lagrangian with vector mesons is used. The results are used to both renormalise the low-energy constants of chiral perturbation theory up to chiral order Q^4 and to determine the influence of loops with vector mesons on masses and decay constants of pseudoscalar mesons.

Keywords: Hadron physics, Vector mesons, Effective field theories, Chiral Lagrangian, Meson-meson interactionss

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List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

I **Electromagnetic transitions in an effective chiral Lagrangian with the η' and light vector mesons**
C. Terschlüsen, S. Leupold, and M.F.M. Lutz
Eur. Phys. J., A48:190, 2012

II **Reactions with pions and vector mesons in the sector of odd intrinsic parity**
C. Terschlüsen, B. Strandberg, S. Leupold, and F. Eichstädt
Eur. Phys. J., A49:116, 2013

III **Renormalisation of the low-energy constants of chiral perturbation theory from loops with dynamical vector mesons**
C. Terschlüsen and S. Leupold
Submitted to Phys. Rev. D, arXiv:1603.05524 [hep-ph]

IV **Contributions of loops with dynamical vector mesons to masses and decay constants of pseudoscalar mesons and their quark mass dependence**
C. Terschlüsen and S. Leupold
Submitted to Phys. Rev. D, arXiv:1604.01682 [hep-ph]

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List of other publications

The following publications are not included in this thesis.

V **Towards an effective field theory for vector mesons**
S. Leupold and C. Terschlüsen
In proceedings of the 50th International Winter Meeting on Nuclear Physics, Bormio, Italy, PoS BORMIO2012:024, 2012

VI **Exclusive measurements of the $\eta \to \pi^+\pi^-\gamma$ decay**
P. Adlarson *et.al.*
Phys. Lett., B707:243-249, 2012

VII **Electromagnetic transition form factors of mesons**
C. Terschlüsen and S. Leupold
In proceedings of the International School of Nuclear Physics, Erice, Italy, Prog. Part. Nucl. Phys., 67:401-405, 2012

VIII **Proceedings of the second International PrimeNet Workshop**
P. Adlarson *et.al.*
arXiv:1204.5509 [nucl-ex]

IX **Decays with vector mesons**
C. Terschlüsen
In proceedings of the MesonNet Workshop on Meson Transition Form Factors, arXiv:1207.6556 [hep-ph]

X **Radiative decays of vector and pseudoscalar nonets**
C. Terschlüsen, S. Leupold, and M.F.M. Lutz
In proceedings of MESON2012, Cracow, Poland, EPJ Web Conf., 37:05005, 2012

XI **Dynamics of the low-lying pseudoscalar and vector mesons**
C. Terschlüsen, S. Leupold, and M.F.M. Lutz
In proceedings of the 51st International Winter Meeting on Nuclear Physics, Bormio, Italy, PoS Bormio2013:046, 2013

XII **Photon-fusion reactions from the chiral Lagrangian with dynamical light vector mesons**
I.V. Danilkin, M.F.M. Lutz, S. Leupold, and C. Terschlüsen
Eur. Phys. J., C73:2358, 2013

XIII **Mini-Proceedings of MesonNet 2013 International Workshop**
K. Kampf *et.al.*
arXiv:1308.2575 [hep-ph]

XIV **Low-lying pseudoscalar and vector mesons and their dynamics: How to describe radiative reactions with an odd number of pions**
C. Terschlüsen, S. Leupold, and B. Strandberg
In proceedings of INPC2013, Florence, Italy, EPJ Web Conf., 66:06023, 2014

XV **Reactions with pions and vector mesons in the sector of odd intrinsic parity**
C. Terschlüsen, S. Leupold, and B. Strandberg
In proceedings of MENU2013, Rome, Italy, EPJ Web Conf., 73:07009, 2014

My contribution to the papers

Paper I: I performed all explicit calculations including the determination of open parameters. Furthermore, I was responsible for writing the section about transition form factors and parts of the appendix.

Paper II: This paper builds upon and extends the work done in my diploma thesis [1] and B. Strandberg's master thesis [2]. For the paper, the calculations from there were updated and extended with a modified Lagrangian, parameters were redetermined and all calculations were discussed from a common point of view. In addition to this extension of [1,2], the $\omega \to 3\pi$ decay was addressed including predictions for the not yet measured Dalitz-plot parameters. I performed all the calculations and wrote most of the article.

Paper III-IV: For these papers, I did all the calculations and most of the writing.

Contents

1. Introduction and motivation

This thesis deals with theoretical hadron physics in the low-energy regime, *i.e.*, the theoretical description of composite objects of elementary particles called *quarks*. In particular, mesons are considered which are minimally built up of a quark and an antiquark. For light mesons, theoretical calculations were performed in order to find a systematic and effective way to describe their reactions and dynamics. Thereby, the light pseudoscalar mesons pions, kaons, η and η' with spin zero as well as the light vector mesons ρ, ω, K^* and ϕ with spin one are considered.

One aim of particle physics is the identification of all elementary particles of nature and how these particles build up matter. All elementary particles identified so far, *i.e.*, quarks, leptons, the force-mediating particles for strong, electromagnetic and weak interactions and the Higgs-particle, are collected in the standard model of particle physics (SM) [3, 4] (cf. Fig. 1.1). This model is a quantum field theory with the elementary particles as degrees of freedom (DOF) describing besides many other aspects all dynamics and interactions of quarks and, thus, also of hadrons. The corresponding quantum field theory is split into two parts, quantum chromodynamics (QCD) as the theory for strong interactions with quarks and the force-mediating gluons as DOF and the Glashow-Salam-Weinberg electroweak theory (GSW) for both electromagnetic and weak interactions. While strong interactions are completely described by QCD, GSW is not sufficient to describe electromagnetic and weak interactions in a satisfactory way. In particular, the generation of masses for quarks and leptons cannot be claimed to be understood without being able to

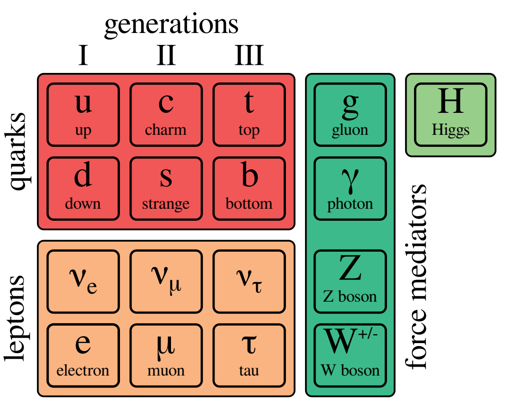


Figure 1.1. Elementary particles included in the SM.

explain the huge difference between the mass $m_e = 511\,\text{keV}$ of the electron as the lightest non-massless SM particle[1] and the mass $m_t \approx 170\,\text{GeV}$ of the top quark as the heaviest SM particle [5]. A probable reason for the shortcomings of GSW is the existence of additional elementary particle beyond the SM as, *e.g.*, dark-matter particles [3].

Searches for physics beyond the SM can be either carried out at very high energies where beyond-SM particles would be produced directly or at high-precision low-energy experiments. At the latter, the influence of new particles is tested on observables which can be well calculated in the SM. Thereby, it is very important to know the SM contributions to these observables with a high precision in order to be able to identify the possible influence of beyond-SM particles. Possible SM contributions are tied to hadronic reactions described by QCD at low energies. Such reactions are considered in this thesis. In section 1.1, the importance of low-energetic hadron physics for searches for physics beyond the SM is illustrated with help of the anomalous magnetic moment of the muon. In the section thereafter, the possible approaches to describe hadronic interactions at low energies are explained. Be aware that all calculations carried out within this thesis are based on QCD and, as explained above, do not contain beyond-SM particles.

In order to understand the structure of matter, not only the elementary particles of nature have to be identified. Additionally, structures of nucleons and other hadrons and of objects built up out of several nucleons or other hadrons are examined. Such objects could be, *e.g.*, atomic nuclei or neutron stars [6]. Hot and dense strongly interacting matter as, *e.g.*, a quark-gluon plasma and the phase structure of strong interactions are of interest as well [7]. All this has to be understood not only on a qualitative but also on a quantitative level. However, these questions are not in the focus of this thesis. Yet specific hadronic form factors are addressed here. Form factors parametrise the deviation from a point-like behaviour, *i.e.*, are related to the intrinsic structure of the considered object. However, the focus of this work and the selection of form factors is guided by the long-term aim of more precise SM calculations.

[1] Note that neutrinos are massless in the SM.

1.1 The importance of hadron physics at low energies for physics beyond the standard model

In this section, the importance of hadron physics for searches for physics beyond the SM is explained with help of the anomalous magnetic moment of the muon. As all particles with spin, the muon has an intrinsic magnetic moment. The intrinsic magnetic moment of a particle is antiproportional to its mass and proportional to its electric charge and spin operator. One forth of the proportionality factor is called *gyromagnetic ratio g*. For point-like particles, this ratio should be equal to two [8, 9]. However, experiments and quantum electrodynamics (QED) [10], the theory describing electromagnetic interactions, show that this is not the case for leptons, *i.e.*, for electrons, muons and tauons. Therefore, the *anomalous magnetic moment* a_l for a lepton l as the deviation from a gyromagnetic ratio of $g_l = 2$ is defined,

$$a_l := \frac{g_l - 2}{2}.$$

Theoretically, the anomalous magnetic moment a_l of a lepton can be determined by considering diagrams with a $2l$-γ interaction where γ denotes a photon. Such diagrams are depicted in Fig. 1.2 (cf., *e.g.*, [11] for contributing diagrams). A general $2l$-γ interaction illustrated by the dashed circle consists of several diagrams. The leading contribution is a *tree-level diagram*, *i.e.*, a diagram without loops. This diagram would give an anomalous moment of $a_l = 0$ and, hence, resembles the calculation for a point-like particle. Diagrams with loops can contain pure electromagnetic contributions as, *e.g*, the loop diagram depicted in Fig. 1.2 but also electroweak and hadronic contributions. These loop contributions yield an anomalous magnetic moment of $a_l \neq 0$ [10].

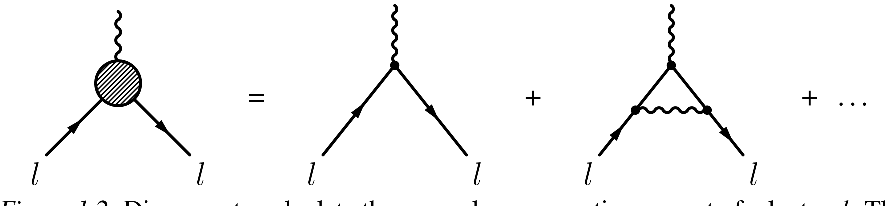
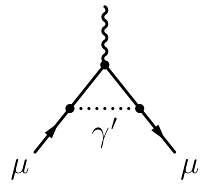


Figure 1.2. Diagrams to calculate the anomalous magnetic moment of a lepton l. The wiggled line denotes a photon, the dashed circle a general $2l$-γ interaction.

Both the magnetic moment of electron and muon are measured with a very high precision [11]. Also the theoretical calculations performed so far including (many-)loop diagrams are carried out with a high precision. One might wonder whether more investigations and measurements or theoretical calculations with even higher precision are needed. However, if there is any physics beyond the SM, the expected difference between experimental and theoretical (SM) result will be tiny and, thus, high precision results will be needed for both. For the magnetic moment of the muon, a discrepancy between experiment and theory of 3.2σ [11] was found which could be a hint of new physics[2]. Hence, the experimental measurements and theoretical calculations have to be improved in order to prove or disprove the discrepancy between the experimental and theoretical result for the magnetic moment of the muon. In Fig. 1.3, a one-loop diagram with a dark photon as an example for a possible beyond-SM contribution to the anomalous magnetic moment of the muon is shown.

Figure 1.3. One-loop diagram with a hypothetical dark photon γ' contributing to the anomalous magnetic moment of the muon [12].



It can be shown that a loop contribution to the magnetic moment is the more important, the lighter the particles in the loop are [13, 14]. Therefore, the contributions of light mesons as considered in this thesis are very important for the theoretical determination of the anomalous magnetic moment of the muon[3]. Its hadronic contributions are mainly given by hadronic vacuum polarisation (Fig. 1.4) and hadronic light-by-light scattering (Fig. 1.5) [11]. In Figs. 1.4 and 1.5, detailed examples for the hadronic contributions are shown as well.

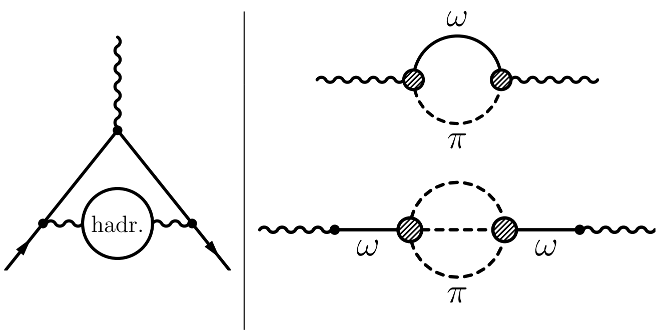


Figure 1.4. Hadronic vacuum polarisation (left-hand side) and examples for the hadronic contribution (right-hand side). Dashed circles denote a general interaction.

[2]For being called a discovery, the discrepancy has to be at least 5σ. However, a discrepancy of more than 3σ is definitely enough to justify further experimental and theoretical efforts.

[3]In addition, the heavier the lepton, the more sensitive it is to loop contributions [13, 14]. Therefore, no discrepancies have been found for the electron so far (cf. [11] and references therein). The tauon would be even more sensitive to loop contributions than the muon but has a too short lifetime for high precision measurements of spin flips in magnetic fields [15–17].

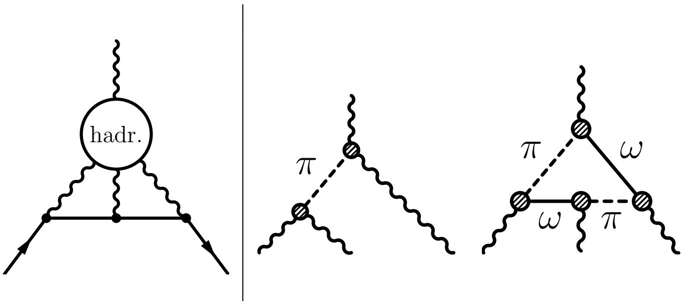


Figure 1.5. Same as Fig. 1.4 but for hadronic light-by-light scattering.

Determining the anomalous magnetic moment of the muon at very high precision is a research project many scientists are involved in. The experimental measurements are not only performed by one but by several collaborations at different facilities (cf. [11] for more details). Also theoretical calculations are not carried out by one group of scientists but split between several groups concentrating on different aspects as, *e.g.*, QED or hadronic contributions. In this thesis, no direct calculations of the anomalous magnetic moment of the moment are performed. However, the results presented here provide important input for these calculations.

For determining the anomalous magnetic moment of the muon, the intermediate photons and hadrons are virtual. However, any successful model or theory describing these reactions should also be able to describe reactions with real photons and/or real hadrons. Therefore, results from decay and scattering reactions can be used to cross check and/or constrain calculations for hadronic contributions to the anomalous magnetic moment. As an example for the connection between interactions with virtual and real photons and hadrons, the γ-γ transition via a virtual ω-meson and pion is shown on the left-hand side in Fig. 1.6. This transition contributes to the hadronic vacuum polarisation and is also depicted on the right-hand side in Fig. 1.4. Thereby, the dashed circles in these diagrams denote the ω-π transition form factor. This quantity also appears in the decay of an ω-meson into a pion and a dilepton shown on the

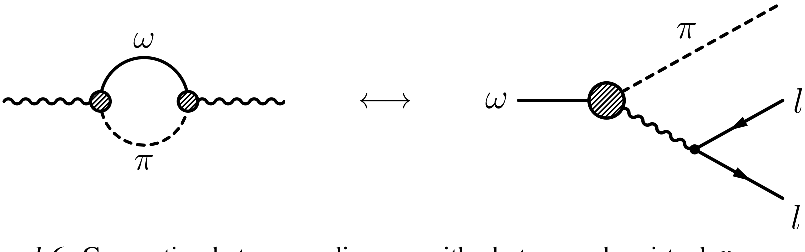


Figure 1.6. Connection between a diagram with photons and a virtual ω-meson and pion and the corresponding diagram with a real ω-meson and pion.

right-hand side in Fig. 1.6. Both the decay and the corresponding form factor are calculated in paper I. The ω-π transition form factor contributes to even more diagrams which are necessary for calculating the anomalous magnetic moment of the muon. It is, *e.g.*, involved in the diagram at the very right in Fig. 1.5 contributing to hadronic light-by-light scattering. Furthermore, the dashed circle in the lower diagram on the right-hand side in Fig. 1.4 is connected to the decay of an ω-meson into three pions and the electron-positron annihilation into three pions considered in paper II. The dashed circle in the middle diagram in Fig. 1.5, the π-γ transition form factor, is calculated in paper II as well. Both in Fig. 1.4 and in Fig. 1.5, examples for contributions to hadronic vacuum polarisation and hadronic light-by-light scattering are given which involve the lightest hadrons, the pseudoscalar pions. However, also the heavier pseudoscalar mesons η and η' are important for the calculation of the anomalous magnetic moment of the muon [11]. Therefore, not only flavour SU(2) is considered in all papers except paper II, *i.e.*, not only pions as pseudoscalar particles are considered. Instead, flavour SU(3) and therewith the η meson are taken into account in paper I and papers III-IV. In papers I and III, even the η' meson is involved.

1.2 How to describe hadronic reactions at low energies: Introduction to effective field theories

The aim of a quantum field theory as, *e.g.*, the SM is the description of physical processes by mathematical expressions. However, these expressions are often given as infinite series which are not applicable for numerical calculations. In order to perform numerical calculations, the infinite series have to be approximated by a finite expression. In this section, different methods for such approximations are discussed in particular for QCD at low energies.

Strong interactions can be described by QCD which covers interactions between quarks and gluons. The strong coupling constant is a running coupling constant, *i.e.*, its value depends on the momentum transfer[4]. Thereby, the higher the energy, the smaller is the strong coupling constant (cf. Fig. 1.7). Therefore, *perturbation theory* is applicable for a QCD reaction at high energies. In perturbative theories, results are expressed as a Taylor expansion in powers of the coupling constant of a considered kind of interaction. If the coupling constant is in fact small, a finite approximation will be possible since the higher the order, the less important the corresponding terms. However, perturbation theory is not applicable for a QCD reaction at low energies. Hence, it is necessary to develop alternative (finite) theoretical descriptions for QCD at low energies. Hereby, one distinguishes between phenomenological and

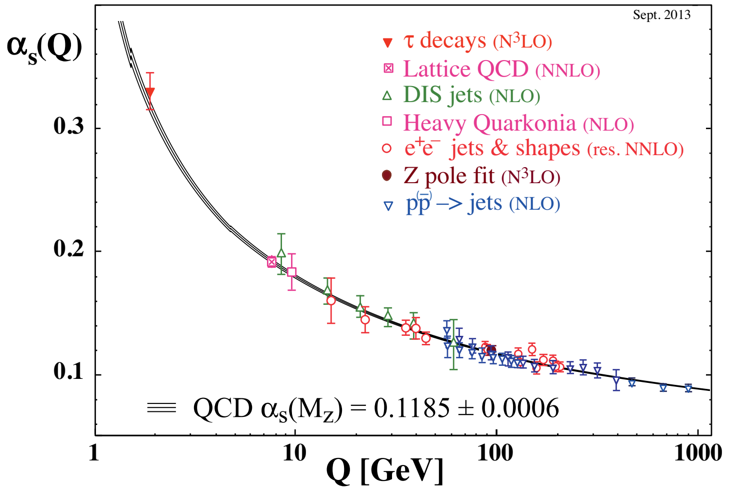


Figure 1.7. Experimental data for the strong coupling constant α_s as a function of the energy scale Q. The respective order of QCD perturbation theory used in the extraction of α_s is indicated in brackets. Picture taken from [5].

[4]Note that the coupling constant in QED is also running but for all accessible energies it depends much less on the energy in the system than the strong coupling constant.

systematic approaches. In a phenomenological approach, the dynamics of a system are described by a model. Such a model has no internal possibility to improve the accuracy in a systematic way and it is in particular not possible to quantify the model-intrinsic uncertainties.

An example for a phenomenological approach to describe interactions of hadrons with photons in the energy regime where light vector mesons become important is vector-meson dominance (VMD) [18]. In this approach, all interactions of hadrons with photons are assumed to be mediated by intermediate vector mesons. The decay $\eta \to \gamma l^+ l^-$ with a dilepton $l^+ l^-$, *e.g.*, would be completely described via a couple of intermediate ρ^0-, ω- or ϕ-mesons in VMD (Fig. 1.8). On the left-hand side in Fig. 1.9, the η-γ transition form factor is plotted in comparison to data taken by the NA60 collaboration [19] (cf., *e.g.*, paper I for a definition of the transition form factor). While VMD is able to describe this form factor and also experimental data for other processes very well, it fails to describe, *e.g.*, the ω-π^0 transition form factor plotted on the right-hand side in Fig. 1.9 in comparison to NA60 data [19]. Since VMD is a phenomenological approach, there is no systematic way to improve the calculation in order to be able to describe the ω-π^0 form factor better. In papers I and II, calculations are carried out with the approach which is used in this thesis to include vector mesons. These calculations are compared to VMD predictions (cf. sections 3.2 and 3.3).

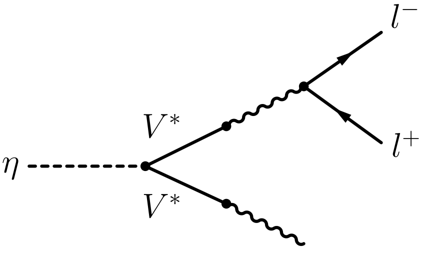


Figure 1.8. Diagram for the decay of an η-meson into a photon and a dilepton $l^+ l^-$ as predicted by VMD. V^* denotes a virtual ρ^0-, ω- or ϕ-meson.

The knowledge about theory-intrinsic uncertainties and possibilities for improvement is an important part of systematic approaches. An example for systematic approaches are perturbative theories. For a small coupling constant, a finite approximation of the Taylor expansion in powers of the coupling constant is possible and such an approximation can be indeed systematically improved by taking the next order in the coupling constant into account. Furthermore, the error due to truncating the Taylor expansion can be approximated by the following order whose contribution should be smaller than those of the orders taken into account. This is, *e.g.*, the case and also experimentally successfully tested for QED, the theory describing electromagnetic interactions [4].

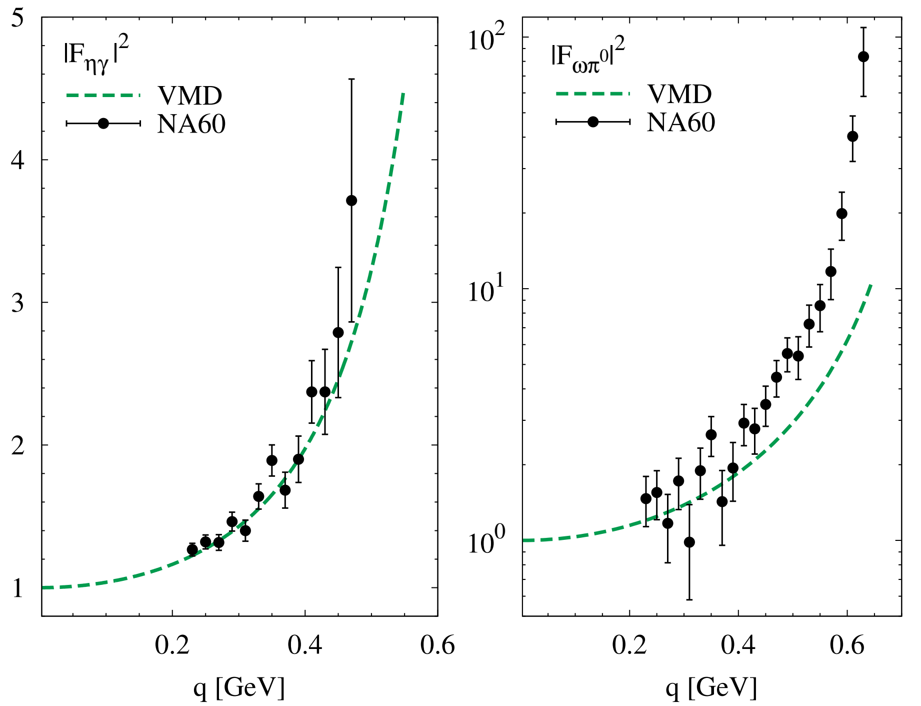


Figure 1.9. Transition form factors calculated with the VMD model in comparison to experimental data taken by the NA60 collaboration [19] for the decay $\eta \to \gamma\mu^+\mu^-$ (left-hand side) and for the decay $\omega \to \pi^0\mu^+\mu^-$ (right-hand side) as functions of the invariant dimuon mass q.

Another example for systematic approaches are effective field theories (EFTs) [20, 21]. Here, the importance of a given term in a Lagrangian is evaluated by comparing scales instead of expanding in orders of (small) coupling constants. Thereby, a scale describes the region in which the given EFT should be valid. Additionally, the DOF of an EFT might be different from the ones of the underlying microscopic theory. Only those DOF which are relevant at the scale of interest are used, *e.g.*, a rocket flying to the moon seen from the earth is not described as a composite object of atoms and molecules but a point-like particle with a total mass and a total energy. In the same spirit, the quark-gluon structure is not resolved (and not needed) for low-energy hadron physics. Furthermore, the scale of an EFT should be separated from energy regions which include possible DOF not used in the EFT. Often, the typical momentum or energy for a considered problem is taken as the scale of the EFT. Additionally, by comparing scales a power counting scheme for an EFT can be formulated which allows the ordering of terms in a Lagrangian by their importance.

Therewith, an EFT consists of the identification of three important parts: the scale, the relevant DOF, and the power counting scheme.

Due to confinement, quarks cannot be unbound at low energies but are bound in hadrons [22]. Therefore, it seems reasonable to take low-lying hadrons as relevant DOF in lower-energy regions of QCD instead of quarks and gluons. A successful EFT for low-energy QCD is chiral perturbation theory (χPT) which takes the lowest-lying hadrons, the pseudoscalar Goldstone bosons pion, kaons and the η-meson as relevant DOF [20, 23–25]. Their masses are small compared to typical masses of other hadrons. χPT is described in greater detail in chapter 2.

In principle, the Lagrangian of an EFT consists of an infinite number of interaction terms and therefore an infinite number of parameters. Counting rules are therefore essential in EFT in order to be able to estimate the relevance of a term in the Lagrangian. Without counting rules, an EFT cannot have predictive power. Since QCD is not solvable so far, the parameters of an EFT based on QCD are unknown and have to be determined by comparison with experimental data (up to a given order in the counting scheme). The number of unknown parameters is a disadvantage of an EFT in comparison with perturbative theories provided the latter is a renormalisable theory, which is the case for the SM. While in such perturbative theories only a small set of constants has to be determined, an EFT might have additional open parameters for each order included. Furthermore, the number of parameters might increase with the order taken into account. χPT, *e.g.*, has two parameters at leading order (LO) and already eight and ten, respectively, at next-to-leading order (NLO) depending on the number of flavours involved [23, 24][5]. The χPT parameters are also referred to as *low-energy constants*.

An EFT is only valid as long as the defining conditions hold. Therefore, it will be, at least, not valid anymore if DOF in addition to the considered ones become important. In χPT, kaons and the η-meson are relevant DOF. However, in reality their masses are not small in comparison to low-lying hadronic resonances as the σ-resonance and the light vector-meson nonet including ρ-, K^*-, ω- and ϕ-mesons [5]. Thus, χPT is hardly applicable for energies above approximately 500 MeV. On the other hand, vector mesons play an important role for interactions between pseudoscalar mesons and photons as, *e.g.*, VMD hypothesises. As seen in the previous section, these interactions provide important input for the calculation of the anomalous magnetic moment of the muon. Therefore, one would like to construct an EFT with DOF in addition to the DOF of χPT which would be applicable in a larger energy range up to

[5]The additional parameters h_i and H_i in [23, 24], respectively, are not present in any physical observable and, therefore, do not need to be determined.

about 1 GeV [6]. In this energy regime, the masses of the involved DOF are similar to the scale where loops become as important as tree-level contributions, $4\pi F_0 = 1.2\,\text{GeV}$. At least some of the loop contributions have to be resummed in order to solve this problem. Hence, a formalism with not only an extended number of DOF and a modified power counting compared to χPT but also a resummation scheme has to be developed and tested.

In this thesis, the pseudoscalar and vector nonets are used as the relevant DOF. Thereby, other low-lying states as the broad σ-resonance are not regarded as quark-antiquark states but as meson-meson states that are dynamically created in the resummation scheme (cf. [26] and paper XII). The first purpose of the present work is to further develop and test a formalism at tree level for processes which are dominated by vector mesons and where resummation aspects are unimportant (cf. paper I and paper II discussed in section 3.2 and 3.3, respectively). Furthermore, first feasibility tests at one-loop level are performed (cf. paper III discussed in subsection 3.4.2). The influence of one-loop contributions with vector mesons on properties of pseudoscalar mesons is studied (cf. paper IV discussed in subsection 3.4.3).

[6]Be aware that the energy region where perturbative QCD is valid is not close to the region of χPT and cannot be applied at 1 GeV, either.

2. Introduction to Chiral Perturbation Theory

In this chapter, chiral perturbation theory (χPT) as the low-energy incarnation of the non-perturbative aspects of QCD is introduced. χPT is an effective field theory (cf. section 1.2 for general information on EFTs) based on QCD with the low-lying pseudoscalar-meson octet instead of quarks and gluons as relevant DOF. In the first section of this chapter, the choice of pseudoscalar mesons as relevant DOF is explained via Goldstone bosons and spontaneous symmetry breaking in QCD. Afterwards (section 2.2), the construction of χPT as an effective theory for Goldstone bosons is illustrated. In the last section of this chapter, the incorporation of the pseudoscalar singlet in χPT is discussed.

2.1 Goldstone bosons and spontaneous symmetry breaking in QCD

In the introduction, pseudoscalar mesons were already referred to as *Goldstone bosons*. In this section, a short introduction of Goldstone's Theorem and the concept of *spontaneous symmetry breaking* as well as its implications in QCD are discussed.

2.1.1 Goldstone's Theorem for a continuous, global symmetry

Goldstone's Theorem claims that if a symmetry of a Lagrangian is spontaneously broken, massless particles, so called *(Nambu-)Goldstone bosons* arise [27, 28]. Thereby, a symmetry is said to be spontaneously broken if the ground state $|0\rangle$ of a physical system, *i.e.*, the state minimising the energy, is not invariant under the full symmetry group of the Hamiltonian. In the following, the concept of spontaneous symmetry breaking and massless Goldstone bosons is explained.

Consider a Lagrangian with a continuous, global symmetry. Due to Noether's Theorem (cf., *e.g.*, [29, 30]), this symmetry yields a conserved current J_μ and a corresponding time-independent charge operator Q. If the symmetry is realised in the ordinary way, the charge operator can be chosen such that

$$Q\,|0\rangle = 0.$$

On the other hand, if the symmetry is spontaneously broken,

$$Q\,|0\rangle \neq 0.$$

The charge operator Q has to commute with the Hamiltonian $\hat{H}$ since it corresponds to a symmetry of the Lagrangian. Denoting the energy of the ground state as E_0 this yields

$$\hat{H}Q\,|0\rangle = Q\hat{H}\,|0\rangle = E_0 \cdot Q\,|0\rangle\,.$$

The system has a degenerate ground state because there exists more than one state minimising the energy, *i.e.*, more than one possible ground state.

Consider further an arbitrary field operator F coupling to one-particle states which is not invariant under the symmetry operation generated by Q. If $\{|n\rangle\}$ is a complete set of eigenstates of F, one can show (cf. [25,31] for details) that there exists at least one state $|n_0\rangle$ with mass $m_0 = 0$, the massless Goldstone particle. Additionally, this particle has to carry the same quantum numbers as the charge operator Q. Since the considered current J_μ is either a vector or an axial-vector current, the Goldstone particle $|n_0\rangle$ has to be a scalar or pseudoscalar boson. If the ground state of the physical system is not only spontaneously broken by one but by several charge operators, each of these operators will give rise to a massless Goldstone boson. The number of Goldstone bosons is equal to the difference between the number of generators of the full symmetry group of the Hamiltonian and the number of generators of the symmetry group of the ground state.

Additionally, the original fields in the Lagrangian can be expanded around an (arbitrarily) chosen ground state. Therewith, the Lagrangian can be rewritten in terms of these new fields, which represent both the massless Goldstone bosons and the massive particles of the theory. Represented in the new fields, the full symmetry of the Lagrangian is not obvious anymore but *hidden* in the new definition of the fields.

2.1.2 Goldstone bosons in QCD

Compared to masses of light hadrons as, *e.g.*, the ρ-meson with a mass of 775 MeV, the masses of the lightest quarks, the up-quark mass $m_u = 2\,\text{MeV}$ and down-quark mass $m_d = 5\,\text{MeV}$, are small [5]. Even the mass of the strange quark, $m_s = 95\,\text{MeV}$, is smaller than the mass of all hadrons [5]. Thus, it seems justified to consider QCD in the *chiral limit* with either massless up and down quarks or massless up, down and strange quarks. In this limit, QCD contains a spontaneously broken symmetry, which is discussed in the following. The corresponding Goldstone bosons are taken as the relevant DOF for χPT (cf. section 2.2).

In the chiral limit with two ($N = 2$) or three ($N = 3$) massless quarks, the QCD Lagrangian is invariant under the symmetry group (cf., *e.g.*, [25])

$$\mathrm{U}(1)_V \times \mathrm{U}(1)_A \times \mathrm{SU}(N)_V \times \mathrm{SU}(N)_A.$$

Here, the indices V and A denote vector and axial-vector symmetries, respectively. The corresponding group elements are of the form $\exp(i\theta^a t_a)$ for vector and $\exp(i\theta^a t_a \gamma_5)$ for axial-vector symmetries, respectively, with $\theta^a \in \mathbb{R}$ and the group generators t_a for the corresponding groups U(1) and SU(N). The matrix $\gamma_5 := i\gamma^0\gamma^1\gamma^2\gamma^3$ is defined via the Dirac matrices $\{\gamma^i\}$. QCD as the quantised theory of strong interactions is not invariant under $\mathrm{U}(1)_A$ anymore [32–35] such that the symmetry group reduces to

$$\mathrm{U}(1)_V \times \mathrm{SU}(N)_V \times \mathrm{SU}(N)_A.$$

Due to Noether's Theorem (cf., *e.g.*, [29, 30]), all subgroups of the symmetry group give rise to conserved currents and corresponding time-independent charges. While $\mathrm{U}(1)_V$ gives rise to baryon-number conservation, *i.e.*, conserved number of particles consisting of three quarks minus antiparticles consisting of three antiquarks, the currents from $\mathrm{SU}(N)_V$ and $\mathrm{SU}(N)_A$ yield conserved vector and axial-vector currents $V^{\mu,a}$ and $A^{\mu,a}$, respectively. For $N = 2$

there exist three vector- and three axial-vector currents while for $N = 3$ there exist eight currents each. Due to the $\mathrm{SU}(N)_V$ symmetry, particles can be organised in flavour multiplets [25, 31]. Within such a multiplet, all particles should have the same mass. In reality, the quark masses are non-zero and distinct and, thus, the masses of particles in the same multiplet are only approximately degenerated. For $N = 3$, both pseudoscalar and vector mesons can be organised in a singlet and an octet. Due to mixing it makes sense to combine the vector-meson singlet and octet to a nonet in the following.

Let Q_A^a be the time-independent charge corresponding to a given axial-vector current $A^{\mu,a}$. It can be shown that if Q_A^a does not break the symmetry spontaneously, *i.e.*, $Q_A^a |0\rangle = 0$, a degenerate state of negative parity[1] will exist for each state of positive parity existing in QCD and vice versa (cf., *e.g.*, [31] for details). Then, negative-parity states which are approximately degenerate with, *e.g.*, the ground-state baryon octet should be observable. Since no such states have been discovered, the hypothesis of no spontaneous symmetry breaking due to $\mathrm{SU}(N)_A$ is experimentally disproven. Thus, the axial-vector charges have to break the underlying symmetry spontaneously, *i.e.*, $Q_A^a |0\rangle \neq 0$. This spontaneous symmetry breaking of QCD yields three massless Goldstone bosons for $N = 2$ and eight for $N = 3$. These Goldstone bosons must have the same transformation behaviour as the axial-vector charges, *i.e.*, they have to transform under parity as

$$Q_A^a \mapsto -Q_A^a.$$

Therefore, the Goldstone bosons are pseudoscalars. Additionally, they have to transform under the non-broken symmetry group $\mathrm{SU}(N)_V$ as a triplet for $N = 2$ and as an octet for $N = 3$. The triplet is identified with the pseudoscalar pions, the octet with the low-lying pseudoscalar-meson octet. For the isospin symmetric case, the pseudoscalar octet states are collected in the matrix

$$\Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix}.$$

Note that chiral invariance and thus also spontaneous symmetry breaking of QCD due to chiral invariance is only approximate because the quark masses are non-zero in reality. Therefore, the masses of the pseudoscalar mesons as quasi-Goldstone bosons are not zero, either, but small compared to other hadrons. The quark-mass term is added to the chiral invariant QCD Lagrangian as an external source and is, of course, itself not chiral invariant. This is often referred to as *explicit symmetry breaking* of QCD in contrast to the spontaneous symmetry breaking in the chiral limit.

[1] Parity transformation describes the behaviour of a field with respect to a point reflection in space. A spinless invariant field has *positive parity*, a field transforming to its negative has *negative parity*.

2.2 χPT as an effective theory for Goldstone bosons

The aim of this section is to give a short overview about how to construct χPT, an effective theory for Goldstone bosons based on QCD. χPT was first suggested by Weinberg in 1979 [20] and further developed by Gasser and Leutwyler [23, 24][2].

As discussed in section 1.2, the DOF of a given theory are replaced by other, more effective DOF in an EFT. In χPT, these DOF are the Goldstone bosons associated with spontaneous symmetry breaking of QCD in the chiral limit as discussed in the previous section. According to whether the chiral limit in two flavours, *i.e.*, $m_u = m_d = 0$, or in three flavours with additionally $m_s = 0$ is chosen, χPT is defined for pions only or for the full pseudoscalar octet. This is referred to as SU(2) and SU(3) χPT, respectively. Since χPT is based on the QCD Lagrangian, it must have the same symmetry behaviour as QCD. In particular, χPT has to fulfil the following properties:

- The χPT-Lagrangian $\mathscr{L}_{\chi\text{PT}}$ has to be invariant under the symmetry group $\mathrm{U}(1)_V \times \mathrm{SU}(N)_V \times \mathrm{SU}(N)_A$ of the QCD Lagrangian. Thereby, the invariance under $\mathrm{U}(1)_V$ is fulfilled automatically because Goldstone bosons have baryon number zero and transform trivially under $\mathrm{U}(1)_V$. Invariance with respect to parity and charge conjugation has to be built in actively by not allowing non-invariant terms.
- Since the QCD-ground state is spontaneously broken with respect to $\mathrm{SU}(N)_A$, the χPT-ground state has to be broken with respect to the same subgroup.
- $\mathscr{L}_{\chi\text{PT}}$ has to contain those pseudoscalar DOF which resemble the Goldstone bosons of QCD and which transform according to $\mathrm{SU}(N)_V$.

Based on these properties, QCD can be reformulated in terms of Goldstone bosons defining the χPT-Lagrangian. However, this Lagrangian consists of an infinite number of interaction terms with an infinite number of unknown parameters[3]. In order to get a predictive Lagrangian, a power counting scheme is needed for evaluating the importance of a given term in the Lagrangian. This scheme is based on a scale separation. In the chiral limit, the pseudoscalar mesons are massless and, hence, there is a clear gap between them and all other hadrons. In reality, the masses are non-zero but still small. Especially for SU(2) χPT a significant gap exists[4]. Therefore, the dynamical (low-energy) scale of χPT is provided by the masses of the pseudoscalar Goldstone bosons. If the momenta of considered processes are of the order of the pseudoscalar masses, the scale separation will provide a clear-cut power counting scheme. All expansions are carried out around the formal limit where the considered

[2]For more detailed information on the construction of χPT cf., *e.g.*, [25], chapter 4.

[3]Recall from section 1.2 that these parameters are unknown as long as QCD is unsolved.

[4]See chapter 3 for a detailed discussion about the gap between pseudoscalar and vector mesons.

momenta vanish along with the pseudoscalar masses. Thereby, the masses vanish in the chiral limit. In other words, the power-counting scheme can be expressed as

$$m_P \in \mathscr{O}(Q),\ \partial_\mu \in \mathscr{O}(Q)$$

for the mass m_P of a pseudoscalar meson and a typical momentum Q.

Using the power-counting scheme given above, the χPT-Lagrangian in a given order can be constructed. It turns out that the LO and NLO Lagrangian are more symmetric than the underlying QCD Lagrangian. In contrast to the QCD Lagrangian, these two χPT-Lagrangians will be invariant under the transformation $\Phi \mapsto -\Phi$ if all external sources except masses are turned off. The Lagrangians contain only terms with an even number of Goldstone bosons. Reactions with an odd number as, *e.g.*, the decay $\pi^0 \to \gamma\gamma$ cannot be described[5]. Therefore, a term with an odd number of Goldstone bosons has to be added to the LO- and NLO-χPT Lagrangians. In contrast to the other terms in the LO- and NLO-χPT Lagrangians, this term does not result from the general construction principles for χPT but can be calculated from QCD directly. It is of $\mathscr{O}(Q^4)$ and can be determined following the construction done by Wess, Zumino and Witten [36, 37]. The Wess-Zumino-Witten Lagrangian includes only one open parameter n which can be restricted to integers by topological arguments. Its absolute value can be shown to be equal to the number of colours, $N_c = 3$, by matching to QCD. Additionally, the value $|n| = 3$ can be checked by comparing, *e.g.*, the theoretical result for the partial decay width for the decay $\pi^0 \to \gamma\gamma$ to experimental data [25][6]. The sign cannot be determined by comparing to experimental data since only the squared matrix element (and therewith only n^2) contributes to a physical observable. In other words, it is caused by the freedom to choose whether the field $+\Phi$ represents the pseudoscalar Goldstone bosons or its negative, $-\Phi$. This is discussed in more detail in paper II (cf. section 3.3).

[5]In next-to-next-to-leading-order χPT, this decay can actually be described. However, the calculated decay probability is so small that the π^0 would live about three orders of magnitude longer than in reality [21].

[6]This result can only be derived if the electric charges of up-, down- and strange-quark are fixed to be $+2/3$ and $-1/3$ times the positron charge, respectively. Since the quark hypercharges scale with $1/N_c$ in an anomaly-free standard model, the decay $\pi^0 \to \gamma\gamma$ does not yield any extra information on N_c. See [38] for more information.

2.3 χPT in the limit of a large number of colours

Spontaneous symmetry breaking and χPT were explained in the previous sections involving the light pseudoscalar octet as quasi-massless Goldstone bosons. There exists a ninth pseudoscalar meson, the η'-meson. It has, however, a mass of about 1 GeV which is significantly larger than the masses of the octet states and similar to the masses of the light vector mesons and other typical hadrons. In standard χPT, the octet state η_8 is identified with the η-meson and no calculations involving an η'-meson directly can be performed. Therefore, χPT in the limit of a large number N_c of colours is introduced in this section as a possible way to include the η'-meson as an additional degree of freedom of χPT [39].

As mentioned in subsection 2.1.2, the singlet axial-vector current A^μ is only conserved on the classical but not on the quantised level. In massless QCD, $\partial_\mu A^\mu \neq 0$ since there are non-zero terms referred to as anomaly [32–35]. Additionally, every axial-vector current has an explicit divergence due to the quark masses. However, the anomaly becomes zero in the large-N_c limit [39]. Thus, the singlet axial-vector current is conserved in the combined limit of large N_c and vanishing up-, down- and strange-quark masses. Furthermore, the corresponding symmetry is spontaneously broken in the same way as the octet axial-vector currents. Instead of eight pseudoscalar Goldstone bosons as in the pure chiral limit, there are nine pseudoscalar Goldstone bosons in the combined large-N_c and chiral limit. Thereby, the singlet Goldstone-boson state η_1 is collect in the singlet matrix,

$$\Phi_{\text{singlet}} = \sqrt{\frac{2}{3}}\eta_1 \cdot \mathbb{1}.$$

Now, χPT combined with large-N_c arguments can be formulated for non-zero quark masses where derivatives, quark masses and powers of $1/N_c$ are counted as small [39]. Defining δ as a small parameter, a reasonable counting scheme can be formulated as

$$m_P,\, \partial_\mu \in \mathscr{O}(\sqrt{\delta}),\; m_q \in \mathscr{O}(\delta),\; 1/N_c \in \mathscr{O}(\delta)$$

where m_q denotes a quark mass. For all considerations within this thesis, the relevant LO Lagrangian $\mathscr{L}^{\text{LO}}_{\chi\text{PT+large-}N_c}$ in large-N_c χPT is equal to the LO Lagrangian $\mathscr{L}^{\text{LO}}_{\chi\text{PT}}$ of standard χPT evaluated for the pseudoscalar nonet $\Phi_{\text{octet}} + \Phi_{\text{singlet}}$ plus an additional term depending only on the pseudoscalar singlet Φ_{singlet}. *I.e.*, the LO Lagrangian in large-N_c χPT is given by

$$\mathscr{L}^{\text{LO}}_{\chi\text{PT+large-}N_c} = \mathscr{L}^{\text{LO}}_{\chi\text{PT}}\Big|_{\Phi=\Phi_{\text{octet}}+\Phi_{\text{singlet}}} + \#\text{tr}\big[\big(\Phi_{\text{singlet}}\big)^2\big].$$

So far, the pseudoscalar nonet includes both the non-physical singlet state η_1

and the octet state η_8 instead of the physical fields η and η'. For the calculation of any physical observable the physical fields are defined in LO as

$$\begin{aligned} \eta &= -\eta_1 \sin\theta + \eta_8 \cos\theta, \\ \eta' &= \quad \eta_1 \cos\theta + \eta_8 \sin\theta \end{aligned}$$

with the η-η' mixing angle θ. As physical states, η and η' are unmixed and free-propagating states. Therewith, the η-η' mixing angle can be determined as a function of η, η', pion and kaon masses yielding a mixing angle of $\theta \approx -11^\circ$ [24]. Furthermore, including the singlet-Goldstone boson of the large-N_c limit does not only yield a redefinition of the physical η- and η'-states and extra terms in the χPT Lagrangian but also changes the values of the low-energy constants which are already present in standard χPT [39]. This is also discussed in paper III.

3. Towards an effective field theory for pseudoscalar and vector mesons

In this chapter, the main results of this thesis are discussed, *i.e.*, the testing and further development of an approach for an EFT with both light pseudoscalar and vector mesons. First, a general overview how vector mesons can be included into a χPT-like framework is given (section 3.1). Afterwards, the specific approach considered in this thesis is discussed by introducing papers I - IV which are the basis of this thesis (sections 3.2 - 3.4).

The framework used here is based on a power-counting and resummation scheme for both the pseudoscalar-meson octet and the vector-meson nonet proposed in [40]. Tree-level calculations [1, 40–42] and resummation calculations with tree-level input [26, 43] performed within this framework are in good agreement with the available experimental data and therefore justify additional investigations of this scheme. As a first step for further development of the approach from [40], the power counting scheme is improved including the pseudoscalar-meson singlet and therewith allowing for a systematic inclusion of both the η- and the η'-meson (paper I, section 3.2). The improved counting scheme is used in paper I for tree-level calculations of processes where resummation aspects are unimportant. While the calculations in paper I are performed for decays of or into vector mesons, *i.e.*, for reactions which cannot be described with χPT, reactions which can also be described with pure χPT only are discussed in paper II (section 3.3). These calculations enable in addition the discussion how a Lagrangian including vector mesons and the Lagrangian from pure χPT should be combined. After an approach for an EFT is successfully tested at tree level, it has to be discussed at NLO, *i.e.*, including loop diagrams. In paper III (subsection 3.4.2), a feasibility test for one-loop calculations with vector mesons is performed. The influence of these loops on the low-energy constants of pure χPT is discussed. Furthermore, one-loop calculations are used in order to estimate the influence of vector-meson loops on properties of pseudoscalar mesons as masses and decay constants (paper IV, subsection 3.4.3).

3.1 Introduction to effective field theories with pseudoscalar and vector mesons

As explained in the previous chapter, the low-lying pseudoscalar octet as the DOF of χPT is associated with the Goldstone bosons of spontaneous symmetry breaking of QCD. This requires massless up and down quarks for SU(2) χPT and massless up, down and strange quarks for SU(3) χPT. However, these quark masses are in reality indeed small but not vanishing such that the masses of the pseudoscalar-octet states are not vanishing, either [5]. Therefore, the gap between pseudoscalar mesons and other low-lying mesons as, *e.g.*, the light vector mesons is not that prominent anymore. The average mass of the K^*-mesons, *e.g.*, is equal to 892 MeV while the average mass of the pseudoscalar kaons is equal to 496 MeV [5]. This is much less than the difference between the pion mass (138 MeV) and the mass of the σ-resonance ((400 − 550) MeV) or the ρ-meson (775 MeV) [5]. The η'-meson included in large-N_c χPT (cf. section 2.3) has a mass of 958 MeV and is therewith even heavier than some vector mesons [5]. Hence, it seems reasonable to try to incorporate the light vector-meson nonet,

$$V = \begin{pmatrix} \rho^0 + \omega & \sqrt{2}\rho^+ & \sqrt{2}K^+ \\ \sqrt{2}\rho^- & -\rho^0 + \omega & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & \sqrt{2}\phi \end{pmatrix},$$

into a framework with pseudoscalar mesons. In this section, general consideration about an EFT with both pseudoscalar and vector mesons are presented.

There exist several attempts to formulate a framework which includes both light pseudoscalar and vector mesons as active DOF and allows for systematic improvements and determination of intrinsic errors (cf., *e.g.*, [44–68] and references therein). In a χPT-like framework with vector mesons as additional DOF, the χPT-power counting scheme, *i.e.*, the way how to classify the importance of terms in a Lagrangian, has to be adapted in order to include vector mesons. Furthermore, the resulting theory depends not only on the chosen power counting scheme but also on the representation used to describe the vector mesons as additional DOF. Within this thesis, vector mesons are represented by antisymmetric tensor fields [23, 69], *i.e.*, $V_{\mu\nu} = -V_{\nu\mu}$ for $\mu, \nu \in \{0, \ldots, 3\}$. Instead, vector mesons could be described by vector fields (cf. [65–67, 69] and references therein) or be included via a hidden local gauge mechanism [68]. The description of vector mesons as antisymmetric tensor fields turns out to be very efficient concerning reactions considered within the framework of this thesis (cf. paper II). The LO contribution according to the power counting scheme given in [40] of, *e.g.*, the decay $V \to P\gamma$ is described by two terms in the tensor representation[1],

$$\mathscr{L}_{\text{tensor}} = a_1\text{“}VA\text{”} + a_2\text{“}VV\Phi\text{”}$$

including the photon field A and two open parameters a_1 and a_2. The terms in quotation marks denote an interaction term following the symmetry and construction conditions for the given framework. If vector fields W_μ are used instead of tensor fields to describe vector mesons, the corresponding Lagrangian will yield three relevant terms for the decay $V \to P\gamma$ (cf., *e.g.*, [68]),

$$\mathscr{L}_{\text{vec.}} = b_1\text{“}WA\text{”} + b_2\text{“}WW\Phi\text{”} + b_3\text{“}W\Phi A\text{”}$$

with three unknown parameters b_1, b_2 and b_3. All three coupling constants have to be fit to experimental data and, hence, the vector representation has less predictive power than the tensor representation of [40] and paper II with only two unknown parameters. Note that with the Lagrangian in vector representation a V-P-γ interaction at LO is possible both directly and indirectly via an intermediate vector meson whereas only the indirect interaction is possible in the tensor-representation framework of [40] and paper II (Fig. 3.1).

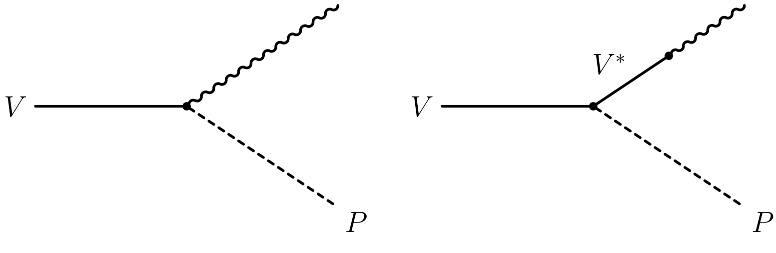


Figure 3.1. Tree-level diagrams for the decay of a vector meson V into a pseudoscalar meson P and a photon (wiggled line). The diagram on the left-hand side shows a direct decay, the one on the right-hand side an indirect decay via an intermediate vector meson V^*.

Irrespective of the chosen Lagrangian, *i.e.*, in particular irrespective of the chosen representation for the additional DOF and the chosen power counting scheme, the underlying physics is the same. However, all calculation performed within a given framework truncate an infinite series to a finite sum. This finite sum depends on the choices made to define the theory and, hence, also the (finite) calculations carried out within a given framework depend on it and might differ from framework to framework. In paper II the representation of vector mesons by antisymmetric tensor fields is compared to a vector representation, in paper III to calculations carried out within a hidden local gauge mechanism [68] (cf. sections 3.3 and 3.4). However, note that it is not

[1] In the framework of paper I which includes in addition the pseudoscalar singlet, an additional term for the singlet only has to be added (cf. section 3.2).

clear that there will exist a working power counting scheme at all if quarks and gluons are integrated out in QCD and a Lagrangian with pseudoscalar and vector mesons is generated. Thereby, a working power counting scheme is a way to order the infinite number of terms in the Lagrangian such that both the relevant physics is described correctly and the corresponding finite sums convert as fast as possible. It might be that no power counting scheme exists for the chosen representation of the involved DOF but that one exists for another representation. Yet, it might also be that this is not the case, either. Therefore, the calculations carried out within this thesis for an approach for an EFT with pseudoscalar and vector mesons do not only provide predictions for physical observables. Investigations are also necessary to examine if the suggested framework is an approach for an EFT at all.

3.2 How to include vector mesons in a χPT-like framework

In [40], a counting scheme for both the light pseudoscalar-meson octet and the light vector-meson nonet was suggested. Thereby, the pseudoscalar-meson octet contains only the octet state η_8 and not the singlet state η_1. One of the prime objectives of paper I is the systematic inclusion of the singlet state η_1 and therewith of the η'-meson in the framework of [40][2] (cf. section 2.3 for the connection between the η_1- and η_8-state and the η- and η'-meson and the inclusion of the η_1-state into χPT). Furthermore, the counting rules proposed in [40] are generalised and further systematised in paper I in order to make the assignment of the proper order in the power counting scheme for a given interaction more transparent.

As in [40], the power counting suggested in paper I depends on large-N_c considerations and uses the hadrogenesis conjecture as a possible justification for a mass gap between relevant and irrelevant DOF. Since the light pseudoscalar and vector mesons are taken as the only relevant DOF, the soft scale of the proposed EFT are the masses of these mesons. In the hadrogenesis conjecture, other low-lying mesons as, *e.g.*, axial-vector resonances are dynamically generated by interactions of pseudoscalar and vector mesons (cf. [26, 70] and paper XII). Furthermore, the mass gap between the DOF and other not dynamically generated mesons is assumed to be sufficiently large. In the large-N_c limit, all couplings are zero such that no dynamically generated particles exist and there is a gap between the included DOF and all other mesons. The counting scheme proposed in paper I relies on a mass gap connected to a hard scale $\Lambda_{\text{hard}} \geq (2-3)\,\text{GeV}$ and keeps that large-N_c behaviour which can be seen at the physical value N_c=3 as, *e.g.*, the suppression of many-body forces.

The proposed LO Lagrangian with the pseudoscalar and vector-meson nonets has additional terms compared to the one with the vector-meson nonet and pseudoscalar-meson octet only[3]. There is, *e.g.*, an additional term describing an interaction with an η_1-state, a vector meson and a photon whereas such a direct coupling is not possible for any octet state. Thus, electromagnetic transitions involving the η-singlet as considered in paper I do not only happen via intermediate vector mesons but can also happen directly at LO (cf. Fig. 3.1 for examples of direct and indirect decays). Other electromagnetic transitions not involving the η-singlet can only happen indirectly at LO. The possibility

[2]Calculations involving the η- and/or η'-meson were already carried out before in this framework. Yet, the η-meson was either approximated with the η_8-state [40, 42] or the description of η- and η'-meson was based on mixing of the η_1- and η_8-states only [1].

[3]Recall that this was already the case for the Lagrangian of large-N_c χPT compared to the one of standard χPT (cf. section 2.3).

of a direct decay is contrary to the VMD assumption that hadronic reactions involving photons are always mediated by intermediate vector mesons [18].

In paper I, electromagnetic transitions of light vector mesons to pseudoscalar mesons and vice versa are used as an LO test for the proposed counting scheme. As explained for large-N_c χPT, the physical fields η and η' needed to calculate any physical observable are given as functions of the (non-physical) singlet and octet states and the η-η' mixing angle θ. Again, the mixing angle can be determined as a function of the masses of the η-meson, the η'-meson, kaon and pion yielding $\theta \approx -11^\circ$. However, the masses of the η- and η'-meson will differ less than 5 % from the experimental values if the mixing angle is varied between $\pm 15^\circ$ and the squared sum of the masses is kept at its experimental value (cf. Fig. 3.2). Therefore, the mixing angle is taken as an open parameter in the calculations performed in paper I instead of using the fixed value from the physical η- and η'-masses. Its value is determined to be $\pm 2^\circ$ by using experimental data for decays of vector mesons into pseudoscalar mesons and real photons and vice versa.

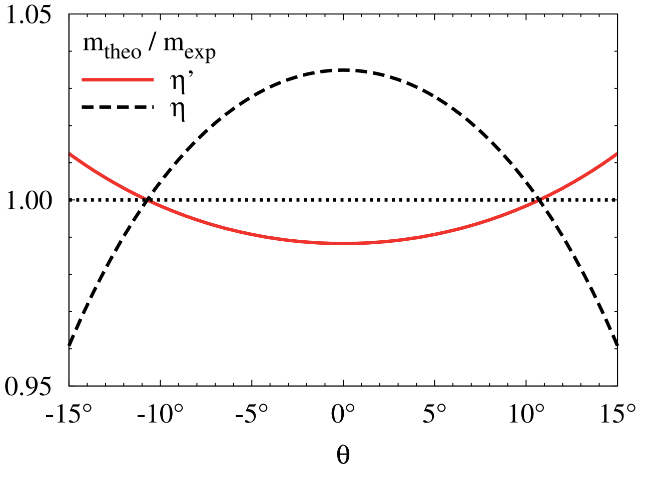


Figure 3.2. Ratio of theoretical calculated and experimental masses for the η-meson (solid red line) and η'-meson (dashed black line) as a function of the mixing angle θ. The squared sum of the η- and η'-masses is kept at its experimental value. See also paper I.

The tree-level calculations for the electromagnetic transitions of pseudoscalar and vector mesons yield good agreement with the available experimental data. In particular, the ω-π^0 form factor is in very good agreement with data taken by the NA60 collaboration [19] (cf. Fig. 3.3). Recall from the introductory section 1.2 that this was a reaction where VMD fails to describe the experimental data.

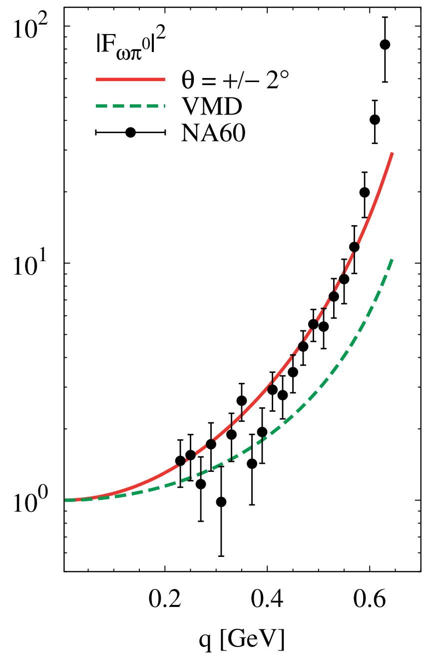


Figure 3.3. ω-π^0 transition form factor calculated with the approach discussed in this thesis (solid red line) and VMD (dashed green line) in comparison to data taken by the NA60 collaboration [19]. Cf. paper I for more explanation.

Since paper I was published, additional experimental data for reactions considered therein have been released. The transition form factor of the decay $\phi \to \eta e^+ e^-$ has been measured by the KLOE collaboration [71] for more values of the dielectron momentum q and with smaller error bars than the previous measurement taken at the VEPP-2M collider [72]. Here, there is a clear deviation between our calculation and the VMD prediction visible whereby VMD describes the data better (Fig. 3.4). An NLO calculation would be needed to investigate whether the description of this form factor can be improved within the framework suggested in paper I.

For the decay $\eta' \to \omega e^+ e^-$, data have been taken by the BESIII collaboration [73]. In Fig. 3.5, the number of events for different momenta q of the dielectron are plotted in comparison to calculations performed with the framework suggested in paper I and VMD. Both predictions are normalised to the experimental data (Fig. 3.5). There is no difference between our calculation and VMD visible; both calculations are in agreement with the data. The form factor of this decay could be used to determine the sign of the η-η' mixing angle and, if precise enough, help to determine the wave-function normalisation of the η'-meson as defined in paper I. Thereby, the measurement shown in Fig. 3.5 is equal to the form factor up to a QED factor and the normalisation.

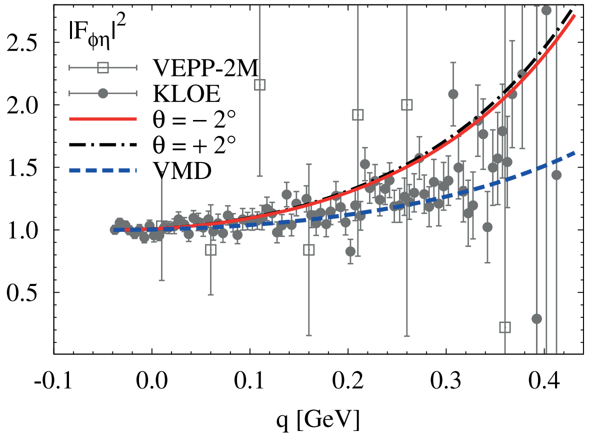


Figure 3.4. ϕ-η transition form factor calculated with the framework of paper I (solid red and dot-dashed black line) and with VMD (dashed blue line) in comparison to data taken at the VEPP-2M collider [72] (squares) and by the KLOE collaboration [71] (circles).

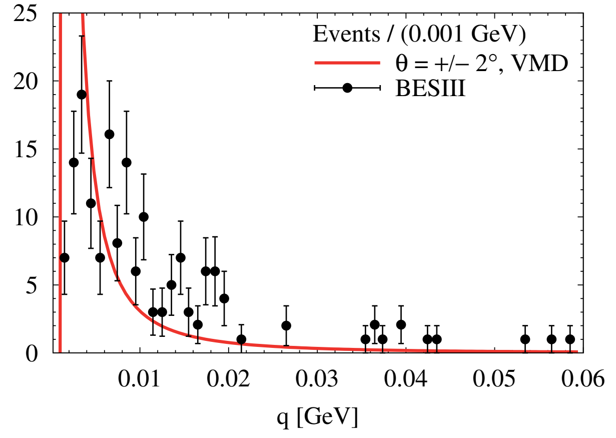


Figure 3.5. Events of the decay $\eta' \to \omega e^+e^-$ taken by the BESIII collaboration [73] in comparison to calculations performed within the framework of paper I and VMD calculations both normalised to the experimental data.

3.3 How to connect to other Lagrangians: Reactions with an odd number of pions

In the previous section, the suggested counting scheme was used for theoretical predictions at LO of reactions involving vector mesons as initial or final states (paper I). Such reactions cannot be described with pure χPT. However, there are also reactions which have both a pure χPT contribution and a contribution with intermediate vector mesons as, *e.g.*, the decay $\pi^0 \to \gamma\gamma$. The counting scheme presented in paper I generates both the pure χPT Lagrangian and the Lagrangian including vector mesons and, thus, reactions with both pure pseudoscalar-meson contributions and vector-meson contributions can be calculated on the basis of this counting scheme. In paper II, reactions with an odd number of pions, *i.e.*, in the sector of odd intrinsic parity[4], are considered. These reactions have both a pure χPT and a vector-meson contribution.

A Lagrangian for both the pseudoscalar-meson octet or nonet and the vector-meson nonet can be split into a pure pseudoscalar-meson part and a part including vector mesons whereby both parts contain a disjoint set of open parameters. Using the counting scheme proposed in paper I, the sub-Lagrangian for pseudoscalar mesons only is by construction the same as the χPT Lagrangian. Its parameters in LO and NLO are already determined [24]. Some of the LO parameters of the sub-Lagrangian including vector mesons are determined in paper I. However, only the absolute values of the open parameters and their relative signs within a sub-Lagrangian can be determined if both the χPT Lagrangian and the Lagrangian with vector mesons are discussed separately. In this way, it is not possible to fix the relative sign between the two sets of parameters. In other words, one cannot determine from separate discussions whether both the vector-meson field $+V$ and the pseudoscalar-meson field $+\Phi$ produce mesons, $-V$ and $+\Phi$ produce mesons or vice versa[5]. For illustration, consider a system of three Lagrangians

$$\mathscr{L}_1 = c_1\text{“}PGG\text{”},\ \mathscr{L}_2 = c_2\text{“}VG\text{”},\ \mathscr{L}_3 = c_3\text{“}PVV\text{”}$$

with P denoting a pseudoscalar-meson-like particle, G a photon-like particle, and V a vector-meson-like particle and three unknown parameters c_i. The absolute value $|c_1|$ can be determined from comparing calculations for, *e.g.*, the decay $P \to GG$ to experimental data. Hereby, the sign of c_1 is convention since either P or $-P$ can correspond to particle states. Additionally, either V or $-V$ can correspond to particle states such that the sign of c_2 is convention as well. Its absolute value can be determined from the interplay between V and

[4]For a meson, the *intrinsic parity* is defined as $P \cdot (-1)^J$ whereby P denotes the parity and J the spin of the meson. Hence, scalars and vectors have even (positive) intrinsic parity while pseudoscalar and axial-vectors have odd (negative) intrinsic parity.

[5]Note that the definitions of Φ and V are only questions of convention. For all physical observables, these definitions are not important as long as they are the same within all calculations.

electromagnetism, *e.g.*, via the decay $V \to G^* \to l^+l^-$ with a lepton pair l^+l^-. From the decay $V \to V'P$, the absolute value $|c_3|$ can be determined. However, the sign of c_3 relative to c_1 is not convention anymore. If the sign of c_1 is chosen, it will be fixed whether P or $-P$ corresponds to particle states. The same convention has to be used in $\mathscr{L}_3$ such that there is no freedom to choose the sign of c_3. Indeed, the sign of c_3 relative to c_1 has physical significance for processes with interference between $\mathscr{L}_1$ and $\mathscr{L}_3$. The decay $P \to GG$, *e.g.*, can happen both with a direct P-G-G interaction via $\mathscr{L}_1$ and with virtual vector mesons $V^*_{1/2}$ as a two-step process, $P \to V^*_1 V^*_2 \to GG$, involving $\mathscr{L}_2$ and $\mathscr{L}_3$. In practice, one might choose an arbitrary sign for c_3 as well and rephrase the question about the sign of c_3 as[6]: Does $\mathscr{L}_1 + \mathscr{L}_3$ or $\mathscr{L}_1 - \mathscr{L}_3$ describe the right physics?

Accordingly, the relative sign between the two parameter sets from pure χPT and from the Lagrangian with vector mesons will be important if interactions which can be described both with pure χPT and by including vector mesons are considered. Therefore, it is essential to know how to combine these two Lagrangians. Hereby, the relevant parameters only appear in the interactions parts of the pure χPT and vector-meson Lagrangian, $\mathscr{L}^{\text{int}}_{\chi\text{PT}}$ and $\mathscr{L}^{\text{int}}_{\text{vec}}$, respectively. In paper II, the sign of the parameter set for the Lagrangian with vector mesons is fixed in accordance with paper I. Hence, the question to be answered is: Does $\mathscr{L}^{\text{int}}_{\chi\text{PT}} + \mathscr{L}^{\text{int}}_{\text{vec}}$ or $\mathscr{L}^{\text{int}}_{\chi\text{PT}} - \mathscr{L}^{\text{int}}_{\text{vec}}$ describe the right physics?

This problem is discussed on the basis of reactions with an odd number of pions in paper II. However, if the counting scheme proposed in paper I is used and both the pure χPT Lagrangian and the Lagrangian with vector mesons are taken into account, it will not be possible to identify the LO Lagrangian for an energy regime around the vector-meson mass. The order of the Lagrangian with vector mesons differs for energies much smaller and of the order of the vector meson mass (cf. Tab. 3.1). In paper II, the respective LO-χPT Lagrangians in even and odd intrinsic parity are used. Thereby, the LO Lagrangian in odd intrinsic parity is the WZW term introduced in section 2.2. It is actually of $\mathscr{O}(Q^4)$ and not of $\mathscr{O}(Q^2)$ as the leading contribution in the sector of even intrinsic parity. For vector mesons, a Lagrangian coupling pseudoscalar mesons and external fields to vector mesons which is equivalent to χPT at NLO is used. This Lagrangian is a part of the (full) Lagrangian

	$\mathscr{L}_{\chi\text{PT}}$	$\mathscr{L}_{\text{vec.}}$
$Q^2 \ll m_V^2$	Q^4	Q^6
$Q^2 \approx m_V^2$	Q^4	Q^2

Table 3.1. *Orders of the pure χPT Lagrangian $\mathscr{L}_{\chi PT}$ and the Lagrangian $\mathscr{L}_{vec}$ including vector mesons in the sector of odd intrinsic parity. m_V denotes a vector-meson mass, Q a typical momentum.*

[6]For the considered decay $P \to GG$, the sign of c_2 is unimportant because it only appears quadratic.

presented in paper I. It describes an interaction of a vector meson V with three pions solely by a two-step process of $2V$-π and V-2π (left-hand side in Fig. 3.6) and the electromagnetic transition of V to π solely by $2V$-π and the transition V-γ (right-hand side in Fig. 3.6). The equivalence of the vector-meson Lagrangian to χPT at NLO is due to the essential saturation of the low-energy constants of χPT at $\mathcal{O}(Q^4)$ by vector-meson exchange for all channels where vector mesons can contribute [69].

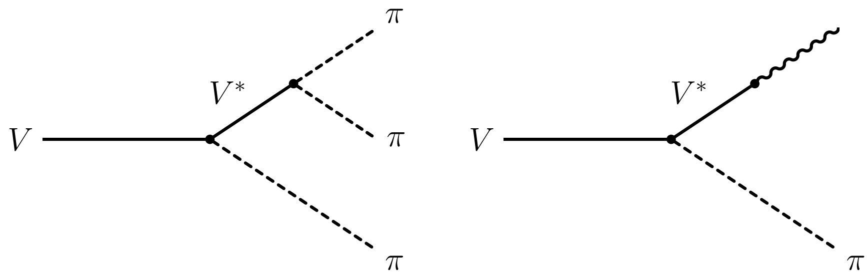


Figure 3.6. Diagrams for the decay of a vector meson V into three pions on the left-hand side and into a pion and a photon (wiggled line) on the right-hand side as described by the Lagrangian with vector mesons used in paper II.

It turns out that within this framework the reactions considered in paper II can neither be described with pure χPT nor with intermediate vector mesons only but both Lagrangians are needed. If only pure χPT at LO is considered, the form factor of the decay $\pi^0 \to \gamma e^+ e^-$, *e.g.*, will be equal to a constant and will therefore not describe the available experimental data (Fig. 3.7). However, if only the vector contribution is taken into account, the calculated π^0-γ form factor will not fulfil the condition to be normalisable to one at momentum square equal to zero. Thus, the results of paper II support the importance of using both the pure χPT Lagrangian and the Lagrangian including vector mesons. The π^0-γ form factor can even be used to determine how the two Lagrangian have to be combined as shown in Fig. 3.7. Furthermore, recall that reactions as discussed in paper II can only happen via intermediate vector mesons in the VMD model in contrast to pure χPT. However, even large deviations from VMD predictions are visible for some reactions calculated in paper II.

Paper II shows furthermore that representing vector mesons by antisymmetric tensor fields is very economic for reactions with an odd numbers of pions. In this representation, only one coupling constant in the vector-meson Lagrangian of odd intrinsic parity is needed to describe the low-energy reactions of pions, ρ- and ω-mesons in this sector at LO. If vector mesons are described by vector fields instead, often significantly more constants have to be used for those reactions in the sector of odd intrinsic parity which are discussed in pa-

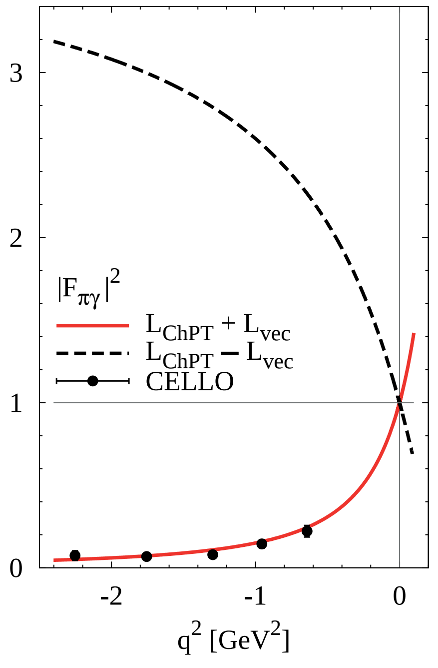


Figure 3.7. Squared π^0-γ form factor compared to space-like data taken by the CELLO Collaboration [74] for added (solid red line) and subtracted interaction Lagrangians (dashed black line). See also paper II.

per II [68, 75–78]. Three instead of one parameter, *e.g.*, are needed to describe these reactions in the hidden-gauge formalism [68, 75, 78].

3.4 How to go beyond tree level: One-loop calculations with vector mesons

In paper I and paper II discussed in the previous sections, a Lagrangian involving vector mesons is tested successfully in tree-level calculations for quantities where resummation aspects are unimportant. Further tree-level calculations are performed in [1, 40–42] and resummation calculations with tree-level input in [26, 43] and in paper XII. As a next step, calculations beyond tree level are necessary to verify the ansatz for an EFT with both pseudoscalar and vector mesons proposed in paper I. General considerations about calculations beyond tree level are discussed in subsection 3.4.1. In paper III (subsection 3.4.2), first feasibility tests at one-loop level are performed. The influence of one-loop diagrams with vector mesons on masses and decay constants of pseudoscalar mesons is studied in paper IV (subsection 3.4.3).

3.4.1 General considerations about loop calculations

In this subsection, loop calculations and renormalisation are discussed in general. To get an idea where loop diagrams contribute and of their influences on physical observables, the mass determination of a pseudoscalar meson as, *e.g.*, the η-meson is considered in the following. The mass of the η-meson can be experimentally determined, *e.g.*, in proton-proton collisions where it is given as the position of the corresponding peak in the dimuon spectrum as shown in Fig. 3.8.

Such a process can happen as illustrated by the (QCD-)tree-level diagram on the left-hand side in Fig. 3.9. The mass read of from this tree-level diagram corresponds to the prefactor of the mass term in the Lagrangian and is called

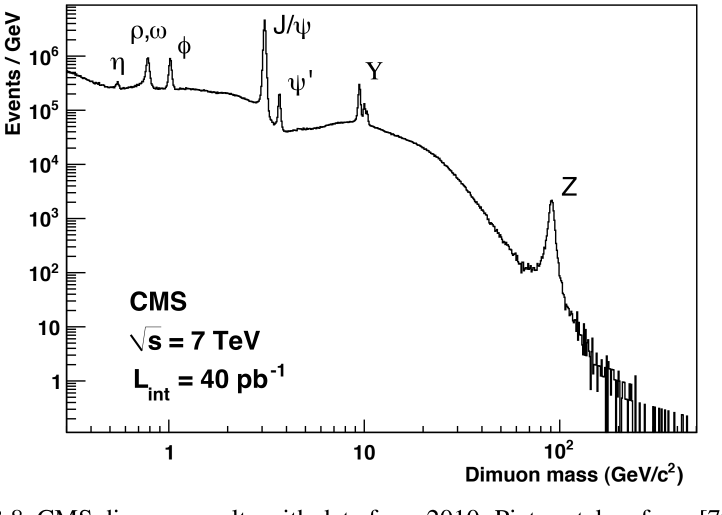


Figure 3.8. CMS dimuon results with data from 2010. Picture taken from [79].

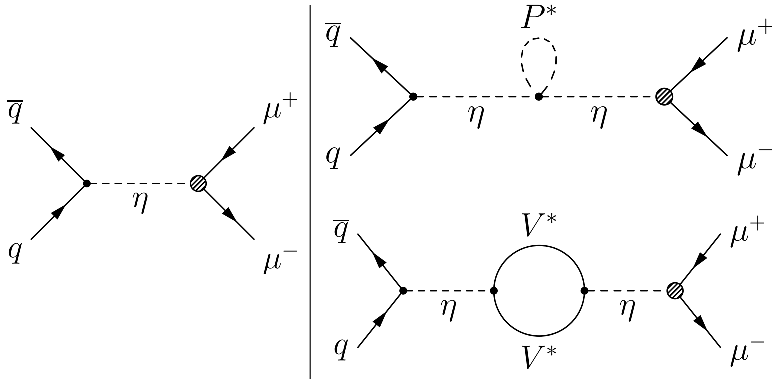


Figure 3.9. Tree-level diagram for the reaction $\bar{q}q \to \mu^+\mu^-$ via an intermediate η-meson (left-hand side) and two possible one-loop diagrams (right-hand side) with a virtual pseudoscalar meson P^* or virtual vector mesons V^* (upper and lower diagram, respectively). The dashed circle denotes a general η-μ^+-μ^- interaction.

bare mass m_{bare}. In addition, the reaction can happen via loops. Examples for one-loop diagrams with virtual pseudoscalar and vector mesons are given in Fig. 3.9 as well. Thus, the experimentally determined mass cannot be obtained from a tree-level calculation but has to include all possible loop diagrams. The real (measured) mass of the η-meson is decomposed as

$$m_\eta = \begin{pmatrix}\text{contributions from}\\ \text{tree-level diagrams}\end{pmatrix} + \begin{pmatrix}\text{contributions from}\\ \text{loop diagrams}\end{pmatrix} =: m_{\text{bare}} + m_{\text{loop}}.$$

Thereby, the loop contribution is infinite. Consider, *e.g.*, the upper right diagram in Fig. 3.9, a simple tadpole diagram with a particle of mass M and momentum k in the loop. Since it has to be integrated over all momenta which are not determined by energy-momentum conservation, the corresponding matrix element is proportional to the infinite integral

$$I_{\text{tadpole}} := \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 + M^2} = \infty.$$

Other loop diagrams yield infinite matrix elements as well. However, the mass itself as a physical observable cannot be infinite. Thus, the bare mass m_{bare} has to be infinite, too, such that m_η as the sum of the infinite contributions m_{bare} and m_{loop} is finite. The bare mass m_{bare} itself as a parameter in the Lagrangian is not a physical observable but only a mathematical construction and can therefore be infinite.

In section 1.2, it was discussed that an EFT is only valid in a limited energy range. χPT, *e.g.*, is only valid for low energies. In general, quantum field theories and in particular perturbative expansions of quantum field theories might only be valid in a limited energy regime [21]. They all might be seen as EFTs. However, if loop integrals as the integral I_{tadpole} given above are calculated, one has to integrate over all undetermined momenta including momenta at high energies. These momenta are far away from the momentum range where the EFT is valid. It would not fit into the understanding of an EFT if the knowledge about the remote-energy regime was needed for predictions in the energy regime where the EFT is valid.

To investigate the influence of the behaviour at high energies on low-energy observables, an infinite loop integral as the integral I_{tadpole} is split into a finite and an infinite part. Hereby, the infinite part includes the behaviour at high energies. First, a way has to be found to identify the infinite parts of the integral, *i.e.*, to give them a well-defined mathematical meaning (*regularisation*) (cf., *e.g.*, [4]). This procedure is not unique, there are several possible ways to identify infinite parts of an integral. A possibility is to introduce a so called *cut-off* Λ_{cut} and replace the infinite integral by

$$I_{\text{tadpole}} \mapsto \int_0^{\Lambda_{\text{cut}}} \frac{\mathrm{d}|k|}{16\pi^2} \frac{k^3}{k^2+M^2}\,.$$

At the very end, Λ_{cut} is sent to infinity to get back the original integral. After the regularisation by a finite cut-off, a *renormalisation point* μ is introduced and the integral is split into two parts,

$$I_{\text{tadpole}} \mapsto \int_0^{\mu} \frac{\mathrm{d}|k|}{16\pi^2} \frac{k^3}{k^2+M^2} + \int_{\mu}^{\Lambda_{\text{cut}}} \frac{\mathrm{d}|k|}{16\pi^2} \frac{k^3}{k^2+M^2} =: I_{\text{fin}} + I_{\text{inf}}.$$

Hereby, the first integral including momentum $|k|$ smaller than μ remains finite while the second integral with momentum $|k|$ larger than μ will become infinite when the cut-off Λ_{cut} is sent to infinity. In paper III and IV, the infinite part of an integral is identified by another way of regularisation which is explained in detail in subsection 3.4.2. Note that although the choice of the regularisation produces different results for the decomposition of a given integral into finite and infinite part the overall physical result has to be independent of this choice.

After identifying the infinite part of an integral, *e.g.*, via a cut-off Λ_{cut}, this infinite part can be added to the tree-level calculations corresponding to the calculations for the same observable. In the case discussed above, this would be the tree-level contribution for the mass. There should exist a matching term with the same structure in the tree-level contribution for each term in I_{inf}, a so called *counter term*. A given counter term includes a parameter of the underlying Lagrangian, in the case discussed here the bare mass m_{bare}. Since the mass of a particle has to be finite, the combination of the counter term with the bare mass and the corresponding loop contribution has to be finite.

The combined, *renormalised* mass is finite and defined as

$$m^r = m_{\text{bare}} + (\text{infinite loop contribution}) = (\text{finite}) .$$

Now, the cut-off Λ_{cut} can be sent to infinity. As already mentioned before, this implies that bare mass m_{bare} might become infinite. In contrast to this the renormalised parameter m^r remains finite and can be used to calculate the mass of the η-meson,

$$m_\eta = m^r + \left(\text{finite parts from } I_{\text{loop}}\right) . \tag{3.1}$$

Note that both quantities, m^r and the finite parts from I_{loop}, depend on the renormalisation point μ used to split up the integral. The physical quantity m_η does not.

Recall the problem discussed above for EFTs: The information from the remote-energy regime, *i.e.*, from the infinite parts of loop integrals are needed for predictions in the energy regime where an EFT is valid. Via renormalisation, all this information becomes part of renormalised parameters. However, if these parameters are matched with experimental data, the remote-energy regime will not influence physical observables anymore. After renormalising the parameters of an EFT once, one has full predictive power for observables without requiring any further knowledge about behaviour in the remote-energy regime. In the case of the example above, other observables which would have included the bare η-mass m_{bare} can be calculated using the renormalised mass m^r instead. The value for the renormalised mass can be determined by, *e.g.*, comparing the η-mass calculated by Eq. (3.1) with its experimental value. Then, all infinite contributions and therewith all high-energy information of the η-mass do not have to be taken into account again for calculating other observables.

As an intuitive example for this, consider again a rocket flying to the moon seen from earth as in section 1.2. In an effective theory, the rocket can be described as a point-like object with a total mass. To calculate this total mass, one needs to know the masses and binding energies of all atoms in the rocket. However, if instead the mass is measured once, no further knowledge about the atoms will be needed and one has predictive power for the flight to the moon and other observables that are only sensitive to a point-like rocket.

Infinite contributions do not only occur in mass calculations but can in principle occur in all calculations involving loop diagrams. However, it is not necessary to deal with this problem separately for each physical observable and each parameter in a Lagrangian but it can be considered on the level of the Lagrangian itself. In the following subsection, this is discussed for the χPT Lagrangian at LO and NLO involving loops with vector mesons (paper III). Be aware that in an EFT as χPT renormalisation is performed at a given order. Loops are calculated up to this order and the renormalised parameters for the Lagrangian up to this order are determined. Thus, renormalisation in an EFT (and in general in a quantum field theory) depends on the order taken into account and has to be redetermined if further orders shall be involved. In paper III one-loop calculations are performed up to chiral order Q^4 whereas in paper IV a full one-loop calculation is carried out. However, there are no divergent terms at $\mathscr{O}(Q^6)$ in this case.

3.4.2 Renormalisation of the low-energy constants of χPT from vector-meson loops

In paper III, the influence of vector-meson loops on the renormalisation of low-energy constants in χPT is discussed. The χPT-Lagrangian is considered up to NLO and one-loop diagrams with pseudoscalar and/or vector mesons up to chiral order Q^4 are taken into account. Thereby only pseudoscalar-meson fields are used as classical fields, vector mesons only show up in loops. The vector mesons are integrated out, *i.e.*, after integration the divergent terms of the one-loop diagrams with vector mesons must have the structure of terms with pseudoscalar-meson fields only. Hence, these diagrams renormalise the χPT but not the vector-meson Lagrangian. For this feasibility study not a full LO Lagrangian with vector mesons as in paper I is used but a simplified version including only (chiralised) 3-flavour versions of the phenomenologically well known ρ-2π and ρ-v interactions where v denotes an external source. These couplings describe the most prominent ways of interactions of vector mesons with pseudoscalar mesons and photons (cf. [18, 43] and paper II). The vector mesons are fully integrated out at the one-loop level and the infinities in the obtained effective action are sorted in powers of derivatives and quark masses. Thus, in this final sorting process the vector-meson mass is formally not treated as a soft scale. This would be misleading since, *e.g.*, an infinity proportional to $m_V^2 \partial^2$ is renormalised by the low-energy constant of the chiral Lagrangian of order Q^2 and not of order Q^4.

Contrary to the mass calculations discussed in subsection 3.4.1, not only one-loop diagrams contributing to one specific physical observable are calculated in paper III but all possible one-loop diagrams with pseudoscalar and/or vector mesons at $\mathscr{O}(Q^4)$. Therefore, the *effective action* Z is defined [4, 23, 24, 80]. It depends on the Lagrangian of the underlying theory and, thus, on all possible Feynman diagrams. Similar to the mass splitting discussed before, the effective action can be rewritten as

$$Z = \begin{pmatrix} \text{tree-level} \\ \text{diagrams} \end{pmatrix} + \begin{pmatrix} \text{diagrams} \\ \text{with loops} \end{pmatrix} =: Z_{\text{tree level}} + Z_{\text{loop}}.$$

The full action Z has to be finite but the contributing parts can be infinite.

As explained in subsection 3.4.1, it is necessary to give infinite parts of integrals a well-defined mathematical meaning in order to determine their influence on parameters and therewith on observables. Within paper III and IV, *dimensional regularisation* is used to identify infinite parts. In dimensional regularisation, the space-time dimension d is not constant but considered as a parameter $d = 4 + 2\varepsilon$ with an arbitrary $\varepsilon \in \mathbb{R}$. Therewith, an integral can be split into a part which is infinite for $\varepsilon = 0$, *i.e.*, for the physical space-time dimension $d = 4$, and a part which is finite for $\varepsilon = 0$. Furthermore, the infinite part can be formally modified by including specific terms of the finite part. Within this thesis, the infinities are defined by the *renormalisation scheme* of MS type which is slightly modified according to [24]. Hereby, MS is an abbreviation for *minimal subtraction*. In this renormalisation scheme, the infinite contribution of a given diagram consists of the part which is infinite for $\varepsilon = 0$ plus specific terms which are finite for $\varepsilon = 0$ but present for all calculated diagrams. The infinite contribution of the integral is defined as all terms proportional to

$$\frac{1}{16\pi^2}\left(\frac{1}{\varepsilon} - \gamma_E - 1 - \log(4\pi)\right)$$

with the *Euler-Mascheroni constant* $\gamma_E \approx 0.58$.

Next, the matching counter terms in the tree-level contribution $Z_{\text{tree level}}$ have to be identified for all infinite parts Z_{inf} of the loop contribution Z_{loop}. As for the bare mass m_{bare} in subsection 3.4.1, the combination of a counter term with parameter g_i and the corresponding loop contributions has to be finite because the full action Z is finite. The renormalised parameters are defined as

$$g_i^r := g_i + (\text{infinite loop contributions}) = (\text{finite})\,.$$

Again, this implies that the bare parameter g_i of the original Lagrangian might be infinite. Using the renormalised parameters, the effective action can be rewritten as

$$Z = Z^r_{\text{tree level}} + \left(\text{finite parts from } Z_{\text{loop}}\right)$$

whereby $Z^r_{\text{tree level}}$ contains the renormalised instead of the bare parameters. It is finite and can be used to calculate physical observables.

Paper III should not be understood as a full renormalisation of the low-energy constants of χPT including one-loop contributions with vector mesons. It is a feasibility check how to deal with loops emerging from a Lagrangian with vector mesons similar to the one proposed in paper I. Thereby, not only the method how to calculate the infinite contribution with vector mesons is discussed in paper III but also approaches which turned out to be not applicable. Paper III does not provide a plausibility check for the Lagrangian suggested in paper I. For such a check, further one-loop contributions have to be calculated. According to paper I, one-loop contributions are of NLO. Thus, all infinities appearing at one-loop order must find their counter terms from the LO $\mathcal{O}(Q^2)$ and NLO $\mathcal{O}(Q^4)$ Lagrangians. Recall that the vector mesons are counted as soft in paper I, *i.e.*, the vector-meson mass m_V^2 is of chiral order Q^2. Therewith, a one-loop diagram yielding a chiral structure as in the χPT Lagrangian of $\mathcal{O}(Q^6)$ but divided by m_V^2 is of importance Q^4 as well. Hence, the infinite part of such a diagram has to renormalise the NLO-χPT Lagrangian of $\mathcal{O}(Q^4)$. However, a chiral structure of $\mathcal{O}(Q^6)$ might not have a counter term in the NLO-χPT Lagrangian even if divided by m_V^2. Therefore, all infinite contributions with an $\mathcal{O}(Q^6)$ structure divided by m_V^2 either have to be zero directly or yield parameter combinations which must be set to zero for consistency reasons. If this was not the case, the counting scheme and the corresponding LO Lagrangian with vector mesons suggested in paper I would be implausible. Consistency conditions for the parameters on the other hand would be very useful due to the relative high number of free parameters in the LO Lagrangian proposed in paper I compared to the number of free parameters in LO χPT [24]. The work done in paper III provides the basis and identifies the applicable technique for a plausibility check of the Lagrangian proposed in paper I.

3.4.3 Contributions of vector-meson loops to masses and decay constants of pseudoscalar mesons

In paper IV, the influence of loops with vector mesons on masses and decay constants of pseudoscalar mesons is calculated. It is discussed how quantitatively different the effects from static versus dynamical vector mesons might be. Thereby, vector mesons will be referred to as *static* in a given framework if they do not appear as active DOF and as *dynamical* if they appear as active DOF. In this thesis, vector mesons have so far been treated as active DOF. However, they are no active DOF in pure χPT. Their influence (and the influence of all other non-active DOF) is encoded in the low-energy constants of χPT. The calculations performed in paper IV are therefore a study about the importance of extending pure χPT to a framework including in addition vector mesons as active DOF. Furthermore, for both one-loop calculations with static and dynamical vector mesons the dependence of pseudoscalar properties on quark masses is studied. Within paper IV, the conventions and the information about the influence of vector-meson loops on the renormalisation of the low-energy-χPT constants from paper III was used. The calculations in paper IV are performed for Feynman diagrams with two legs, *i.e.*, for two-point functions. This is a subclass of all possible diagrams at LO and NLO whereas the aim of paper III is the renormalisation of the full LO- and NLO-χPT Lagrangian.

For evaluating the influence of one-loop calculations with dynamical vector mesons on properties of pseudoscalar mesons, masses and decay constants are calculated in paper IV. The mass M of a particle is defined as the pole of its propagator Δ,

$$\Delta(p^2 = M^2)^{-1} \equiv 0,$$

whereby the propagator is a function of the squared momentum p^2 of the particle. In LO χPT, the pole of the propagator and therewith the LO mass of a pseudoscalar meson are given by the bare mass $\mathring{M}$. As already discussed in subsection 3.4.1, the propagator of a particle can be expressed as an infinite sum of diagrams if higher-order contributions and/or non-trivial LO contributions are taken into account. As shown in Fig. 3.10, this sum includes *one-particle irreducible* (1PI) diagrams. 1PI diagrams are diagrams which cannot be split into two complete diagrams by cutting a single propagator line. All 1PI diagrams contributing to the propagator are defined as the *self energy* Σ of a particle times $(-i)$ and depend on the squared momentum. Therewith, the full propagator is a sum over diagrams including the self energy and can be rewritten as an geometric series [4],

$$\begin{aligned} i\Delta(p^2) &= \frac{i}{p^2 - \mathring{M}^2 + i0^+} + \frac{i\left[-i\Sigma(p^2)\right]i}{(p^2 - \mathring{M}^2 + i0^+)^2} + \frac{i\left[-i\Sigma(p^2)\right]i\left[-i\Sigma(p^2)\right]i}{(p^2 - \mathring{M}^2 + i0^+)^2} + \ldots \\ &= \frac{i}{p^2 - \mathring{M}^2 - \Sigma(p^2) + i0^+}. \end{aligned}$$

Since the (full) mass M of a particle is given by the pole of the (full) propagator, it can then be determined via the mass equation

$$M^2 - \mathring{M}^2 - \Sigma(M^2) = 0.$$

In paper IV, the self energies and therewith the masses of pseudoscalar mesons are calculated in one-loop approximation for both pure χPT up to chiral order Q^4 and involving loops with vector mesons. The contributing diagrams are depicted in Fig. 3.11. Hereby, it turns out for loop diagrams involving vector mesons that only the last diagram in Fig. 3.11 will yield a non-vanishing contribution if the Lagrangian with vector mesons given in papers III-IV is used.

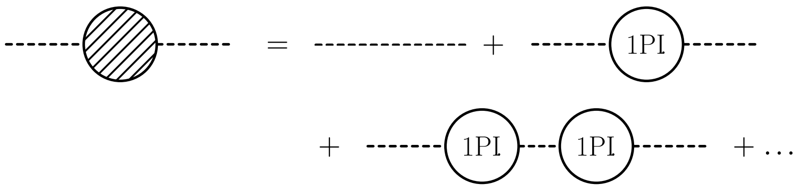


Figure 3.10. Propagator given as a sum of diagrams including 1PI diagrams. The dashed circle denotes the full contribution.

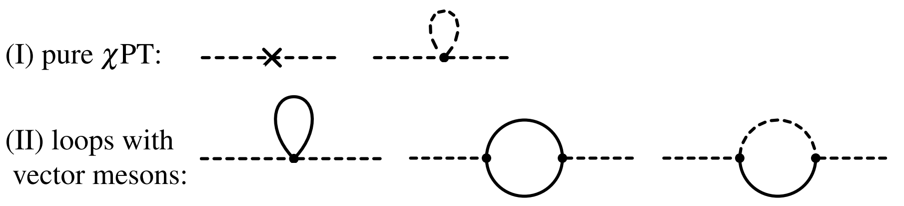


Figure 3.11. Contributing one-loop diagrams to the self energy $-i\Sigma$ from pure χPT at $\mathscr{O}(Q^4)$ (I) and from loops including vector mesons (II). A pseudoscalar meson is described by a dashed line, a vector meson by a solid line. The cross denotes an NLO vertex, the dot an LO vertex.

The decay constant of pseudoscalar mesons can be calculated using Feynman diagrams with an incoming weak field a_μ and an outgoing pseudoscalar meson as shown on the left-hand side in Fig. 3.12. These diagrams can be split into the product of 1PI diagrams with an incoming weak field and an outgoing meson and the diagram for the full meson propagator discussed before. The one-loop diagrams for χPT at chiral order Q^4 and with vector mesons which contribute to the 1PI diagrams with an incoming weak field are depicted in Fig. 3.13. As for the self energy, the diagram with both a pseudoscalar and a vector meson in the loop is the only non-vanishing one-loop diagram with vector mesons within the framework discussed in paper IV.

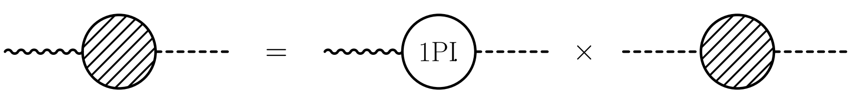


Figure 3.12. General diagram for calculating decay constants. The wiggled line denotes a weak field a_μ, the dashed circle a full contribution.

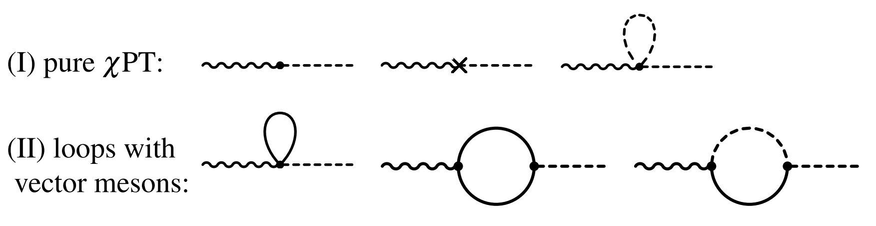


Figure 3.13. 1PI diagrams with an incoming weak field a_μ in pure χPT up to $\mathscr{O}(Q^4)$ (I) and with loops including vector mesons (II), respectively. A pseudoscalar meson is described by a dashed line, a vector meson by a solid line and the weak field by a wiggled line. The dot denotes an LO vertex, the cross an NLO vertex.

In paper IV, two comparisons for masses and decay constants of pseudoscalar mesons are performed. First, loops with static and loops with dynamical vector mesons are compared. Second, loops with dynamical vector mesons are compared to pure χPT loops. Both cases are studied as functions of the quark masses. Hereby, the quark mass dependence is encoded in the dependence on the bare pion mass. The square of the bare pion mass is proportional to the averaged up- and down-quark mass. For the first comparison, observe that loops with (static) vector mesons appear at next-to-next-to-leading order (N^{2}LO) in χPT [81], *i.e.*, at chiral order Q^6. Here, one-loop diagrams with NLO vertices contribute. The corresponding NLO vertices are in turn influenced by vector mesons [23, 82, 83] (and by other mesons not considered in this thesis). For the calculation with static vector mesons, the vector-meson propagator is shrank to a point for one-loop diagrams with vector mesons contributing to the masses and decay constants (cf. the last diagram in both Fig. 3.11 and Fig. 3.13). *I.e.*, the vector-meson propagator is approximated by $1/m_V^2$. This is the same approximation as done in pure χPT for momenta $q^2 \ll m_V^2$. The resulting diagram resembles a pure χPT-tadpole diagram[7] (cf. Fig. 3.14).

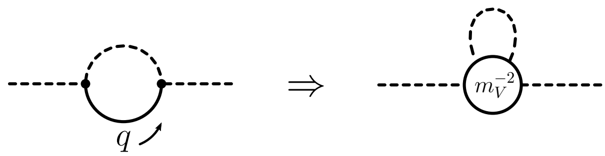


Figure 3.14. Transformation of a one-loop diagram with a vector meson to a tadpole diagram for a point-like vector-meson propagator. Note that the circle with the label m_V^{-2} should not be misinterpreted as a vector-meson loop. This circle represents a vertex.

For the calculation with dynamical vector mesons, the full vector-meson propagator is used. The differences between loop diagrams with point-like and with full propagators are considered as functions of the bare pion mass $\mathring{M}_\pi$ and in comparison to a reference point for $\mathring{M}_\pi$. In all calculations for masses and decay constants of pseudoscalar mesons, a difference between the calculations with static and dynamical vector mesons is already visible for bare masses above approximately 250 MeV. This indicates the importance of including dynamical vector mesons, *i.e.*, vector mesons as active DOF in a χPT-like framework. As an example, the difference between the calculation with a point-like and a full vector-meson propagator is shown in Fig. 3.15. Specifically, the vector-meson loop contributions to the (squared) mass of the pion are considered (not the full calculation for the mass). Hereby, ΔT and ΔI are

[7] Note that the result from the diagram with a point-like vector-meson propagator has to be the same as in N^{2}LO χPT with the low-energy constants determined in the resonance saturation picture [82]. Therewith, the results in paper IV can be checked. The N^{2}LO-χPT contributions to masses and decay constants of pseudoscalar mesons have already been calculated in [81].

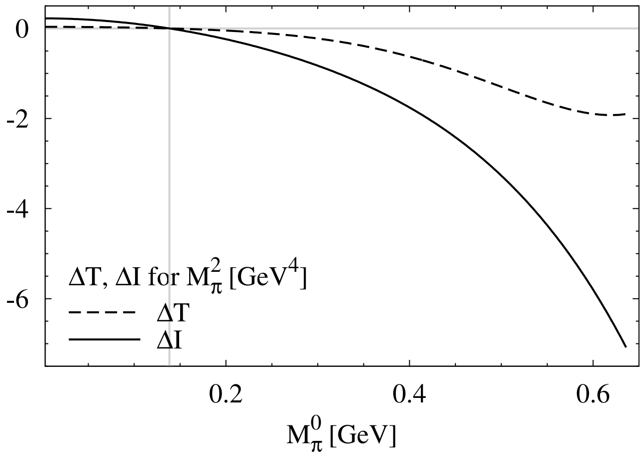


Figure 3.15. ΔT and ΔI for the squared mass of the pion as a function of the bare pion mass $\mathring{M}_\pi$. The vertical line represents the experimental pion mass $M_\pi^{\mathrm{exp}} = 138\,\mathrm{MeV}$. See also paper IV.

defined as the normalised difference between the calculation at a given bare pion mass $\mathring{M}_\pi$ and at the reference point $\mathring{M}_\pi = M_\pi^{\mathrm{exp}}$ for a point-like propagator (dashed line) and for the full propagator (solid line), respectively.

For the second comparison, both the pure χPT Lagrangian and the one containing (dynamical) vector mesons in addition are considered. In the spirit of EFTs, *i.e.*, assuming that there exists *the* EFT and not several ones, the difference between the two scenarios with and without vector mesons at a given chiral order can only reside in different values of the low-energy constants. Thus, in paper IV the low-energy constants of chiral orders Q^2 and Q^4 have been adjusted such that there is no difference between the two scenarios for physical observables up to (including) order Q^4. However, the one-loop contributions with vector mesons contain logarithms depending on the pseudoscalar and the vector-meson mass. Thus, there is a difference between a full one-loop contribution including these logarithms and the one obtained by adjusting the counter terms at order Q^4. This difference starts at order Q^6. If the one-loop contributions with vector mesons are not expanded in chiral orders but instead the full analytical structure is taken into account, the quantitative importance of the Q^6 contribution to a formal Q^4 calculation for masses and decay constants of pseudoscalar mesons can be explored. Therefore, the expression

$$\begin{pmatrix}\text{LO and NLO contr.}\\ \text{from pure } \chi\text{PT}\end{pmatrix} + \left\{ \begin{pmatrix}\text{full vector-}\\ \text{loop contr.}\end{pmatrix} - \begin{pmatrix}\text{vector-loop contr.}\\ \text{up to } \mathcal{O}(Q^4)\end{pmatrix} \right\}$$

for a mass or decay constant can be compared to the pure-χPT contribution at NLO. As an example for this comparison, the pion mass M_π as a function of the bare pion mass $\mathring{M}_\pi$ is shown in Fig. 3.16. Thereby, the different

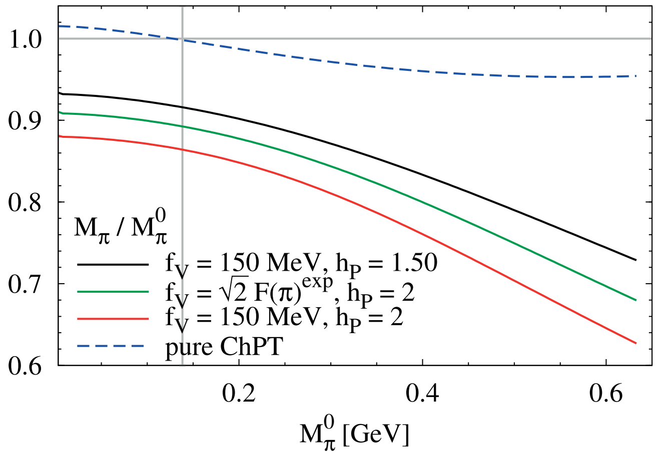


Figure 3.16. Normalised pion mass as a function of the bare pion mass for both a pure χPT calculation (dashed blue line) and a calculations with vector mesons (solid lines). The different colours for the calculation with vector mesons represent the results for the different values for f_V and h_P whereby they are in the same order as in the legend. The vertical line represents the experimental pion mass $M_\pi^{\text{exp}} = 138\,\text{MeV}$. See also paper IV.

colours for the calculations with vector mesons correspond to different values of the parameters f_V and h_P of the vector-meson Lagrangian (cf. paper IV for an explanation of these values). Both the pure χPT calculation and the calculations with vector mesons is normalised to the LO-χPT result, *i.e.*, to the bare pion mass. Therewith, the deviation of the pure χPT calculation from unity illustrates the difference between a calculation at chiral order Q^2 and at chiral order Q^4 and the deviation of the calculation with vector mesons from the pure χPT calculation illustrates a difference that is formally of order Q^6. If vector mesons were not important, the second deviation should be smaller than the first. If this were the case, the convergence of χPT should still be good for larger quark masses, *i.e.*, the difference of $\mathcal{O}(Q^6)$ should be smaller than the difference between calculations at $\mathcal{O}(Q^2)$ and at $\mathcal{O}(Q^4)$. However, this is neither the case for the pion mass nor for the other pseudoscalar properties studied in paper IV indicating that vector mesons are already important for small pion masses.

4. Summary and outlook

In this thesis, studies towards an effective field theory including both the light pseudoscalar-meson and vector-meson nonets are performed. Hereby, vector mesons are described by antisymmetric tensor fields.

As a first step, a power counting scheme including the pseudoscalar Goldstone singlet systematically and being based on the scheme suggested in [40] is formulated in paper I. Within this framework, tree-level calculations are performed for decays of vector mesons into pseudoscalar mesons and photons and vice versa which are in good agreement with the available experimental data. Thereby, the necessary parameters of the Lagrangian including the η-η' mixing angle are determined. In particular, the ω-π^0 transition form factor is described much better with our framework than with standard VMD which fails to describe the experimental data. A reduced Lagrangian with vector mesons is used in paper II to calculate tree-level reactions in the sector of odd intrinsic parity, *i.e.*, with an odd number of pions. Hereby, the relative sign between the parameter sets needed for the Lagrangian with vector mesons and the leading contribution of the χPT Lagrangian, respectively, is determined. Again, the calculations are in good agreement with the available experimental data.

After predicting physical observables at tree level, first calculations beyond tree level are carried out. A feasibility study is performed in paper III concerning one-loop calculations for both pseudoscalar and vector mesons in the loop using a Lagrangian with a limited number of interaction terms for vector mesons. Hereby, the vector mesons are integrated out completely and the infinities in the obtained effective action are sorted in powers of derivatives and quark masses up to chiral order Q^4. The infinities are used to renormalise the low-energy constants of LO and NLO χPT. Furthermore, the influence of loops with vector mesons on masses and decay constants of pseudoscalar mesons is studied in paper IV.

In the future, further calculations beyond tree level are especially of interest. This includes not only a one-loop plausibility check for the Lagrangian with vector mesons as discussed in subsection 3.4.2 but also calculations at NLO for physical observables. Thereby, vector mesons should not only be considered as particles in loops but also as external particles. For loop calculations with external vector mesons, at least a part of the Lagrangian with vector mesons at NLO is needed. If vector mesons are treated as external states, renormalisation calculations have to be performed as well for the parameters of the

Lagrangian with vector mesons. Additionally, the NLO Lagrangian with vector mesons must be used to calculate physical observable at NLO. This would, *e.g.*, be relevant for the ϕ-η transition form factor where the VMD prediction describes the available experimental data somewhat better than the LO calculation in our framework (cf. section 3.2). As in pure χPT, an NLO calculation would of course not only include loop diagrams but also tree-level diagrams with vertices of NLO (cf. Fig. 4.1). Be aware that it is reasonable to expect a larger number of parameters in an NLO Lagrangian with vector mesons compared to the LO Lagrangian. This is already the case for pure χPT [23, 24].

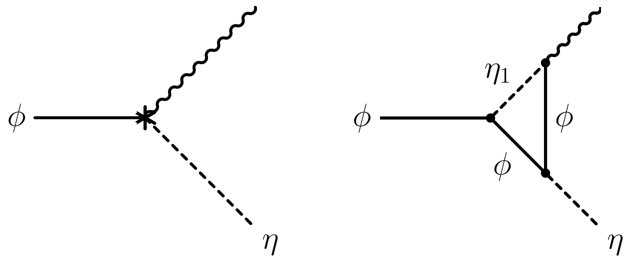


Figure 4.1. Example for a tree-level diagram (left-hand side) and a one-loop diagram (right-hand side) contributing to the decay $\phi \to \eta\gamma$ at NLO. The cross denotes an NLO vertex, the dot an LO vertex.

All in all, the good agreement of tree-level calculations performed in papers I and II discussed in this thesis but also in paper XII and in [1, 26, 40–43] justifies further investigations into the approach of an effective theory for both the pseudoscalar-meson and vector-meson nonet suggested in paper I. The feasibility test for the renormalisation of the low-energy constants of χPT from loops with vector mesons (paper III) and the corresponding one-loop calculations for pseudoscalar masses and decay constants (paper IV) provide the foundations for calculations beyond tree level.

Summary in Swedish

Avhandlingens titel:
Teoretiska studier av hadronreaktioner med vektormesoner

All materia i vår värld är uppbyggd av små byggstenar som kallas *elementarpartiklar*. Den så kallade *standardmodellen* för partikelfysik beskriver dessa partiklar och hur de växelverkar med varandra (se figur 4.2). Partiklarna i standardmodellen är antingen kvarkar, leptoner eller kraftförmedlande partiklar. Dessutom genererar Higgsfältet massorna hos elementarpartiklarna. Alla partiklar som är uppbyggda av kvarkar och/eller antikvarkar kallas *hadroner*. I denna avhandling ligger fokus på partiklar som är uppbyggda av en kvark och en antikvark: *mesoner*. Särskilt två typer av mesoner är av intresse: pseudoskalära mesoner och vektormesoner. De som behandlas i mitt arbete tillhör de allra lättaste partiklarna som består av ett kvark-antikvark-par.

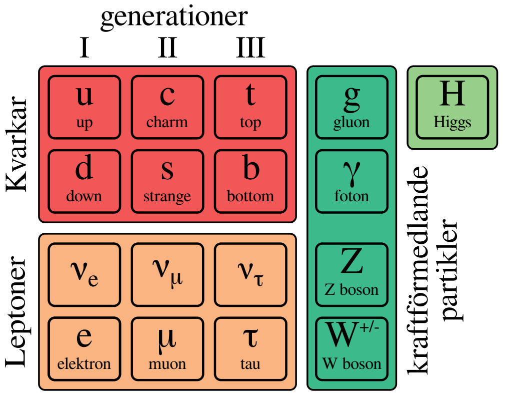


Figure 4.2. Standardmodellen för partikelfysik.

Syftet med teoretisk fysik är att beskriva fysikaliska fenomen med matematiska uttryck. För standardmodellen har sådana matematiska uttryck formulerats och testats. Tyvärr är de matematiska uttryck som beskriver fysiken ofta oändliga serier vilket gör det omöjligt att tillämpa dem på verkliga problem. Därför måste man hitta nya vägar och approximera de oändliga serierna med ändliga uttryck som kan användas för att förutsäga fysiska observabler. Huruvida en förutsägelse är användbar beror på om dess noggrannhet är bättre än noggrannheten hos motsvarande experimentella resultat. Därför måste man kunna avgöra hur stor osäkerheten i förutsägelserna är. Osäkerheten uppstår på grund av att man approximerar oändliga matematiska uttryck. Om det är möjligt att avgöra hur stora osäkerheterna är, kan teorin klassificeras som *systematisk*. Dessutom kan man förbättra en beräkning i en systematisk teori genom att göra den mer realistisk på ett kontrollerbart sätt. För växelverkan mellan kvarkar är en systematisk beskrivning möjlig för reaktioner vid höga energier. Vid låga energier däremot kan kvarkar inte existera som obundna partiklar utan bara som sammansatta system. Det finns metoder för att beskriva sammansatta system som om de vore elementarpartiklar vid låga energier. Lätta mesoner är exempel på sådana sammansatta system.

Lätta pseudoskalära mesoner är naturens lättaste partiklar som är uppbyggda av kvarkar och/eller antikvarkar. Det finns en framgångsrik strategi för en systematisk beskrivning av deras växelverkan vid låga energier som kallas *kiral störningsteori*. Syftet med denna avhandling är att undersöka huruvida man dessutom inkludera vektormesoner i kiral störningsteori.

Som ett första steg, har vi utvecklat en metod för en systematisk teori som beskriver samspelet mellan både pseudoskalära mesoner och vektormesoner (artikel I). Här är det avgörande att hitta ett sätt att bedöma hur viktig en specifik term i en beräkning är, det vill säga av vilken *ordning* termen är. När teorin är formulerad, kontrolleras ordningen i beräkningarna. Det finns två sätt att kontrollera en metod för en systematisk teori:

(a) Fysikaliska observabler beräknas och jämförs med experimentella data. Naturligtvis bör beräkningarna stämma överens med data.

(b) Teorin bygger på organisera termer. Rimlighetskontroller kan utföras så ordningen bevaras i alla beräkningar.

I artikel I och II beräknas fysiska observabler till lägsta ordningen. Till vänster i figur 4.3 illustreras den ledande ordningens bidrag till sönderfallet hos en ω-(vektor-)meson till en foton och en pseudoskalär pion med en så kallas *Feynmandiagram*. Ledande ordningens beräkningar i artikel I och II stämmer väl överrens med de experimentella data. Till höger i figur 4.3 sker ω-mesonsönderfallet via ett diagram med en sluten slinga. Detta betyder att vi har ett högre ordningens diagram. Det är även möjligt att ha flera slutna slingor i ett diagram. I artikel III och IV testar vi genomförbarheten hos beräkningar

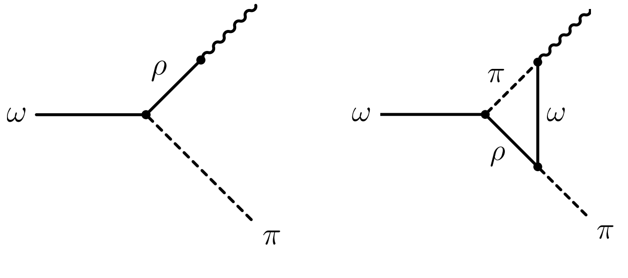


Figure 4.3. Ledande ordningens bidrag (till vänster) och ett exempel för ett diagram med en sluten slinga (till höger) som bidrar till ω-mesonsönderfallet till en foton och en neutral, pseudoskalär pion.

med en sluten slinga och testar hur diagram med en sluten slinga påverkar pseudoskalära mesoners egenskaper.

Ett exempel på en observabel för ω-mesonsönderfall till ett muon-antimuon-par och en neutral pion är den så kallade *formfaktorn*. Beräkningen av formfaktorn samt experimentella data visas i figur 4.4.

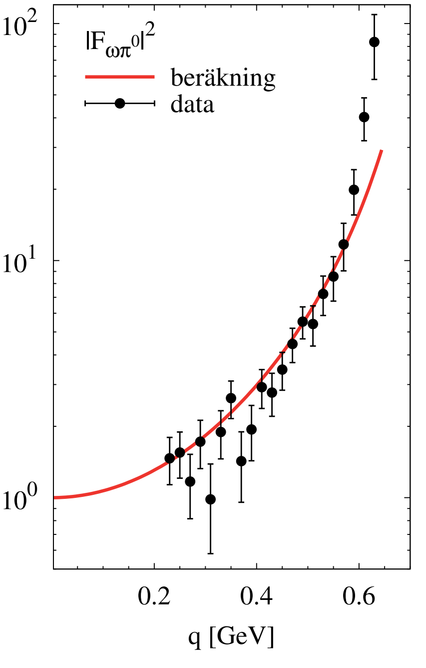


Figure 4.4. Formfaktorn för ω-mesonsönderfallet till ett muon-anitmuon-par med rörelsemängd q och en neutral pion samt experimentella data [19]. Se också artikel I.

Inte bara beräkningarna i artikel I and II, som diskuteras i denna avhandling, utan också beräkningarna till exempel i artikel XII stämmer bra överrens med experimentella data. Detta motiverar mer beräkningar i framtiden med strategin för en systematisk teori för både pseudoskalära mesoner och vektormesoner som föreslås i artikel I. Till exempel bör ytterligare beräkningar med slutna slingor utföras. Genomförbarhetsstudien för beräkningar med en slinga (artikel III) och hur de påverkar pseudoskalära mesoners egenskaper (artikel IV) bygger grunden för beräkningar med sluten slingor.

List of abbreviations

χPT	chiral perturbation theory
DOF	degrees of freedom
EFT	effective field theory
GSW	Glashow-Salam-Weinberg electroweak theory
LO	leading order
NLO	next-to-leading order
N^{2}LO	next-to-next-to-leading order
QED	quantum electrodynamics
QCD	quantum chromodynamics
SM	standard model of particle physics
VMD	vector-meson dominance
1PI	one-particle irreducible (diagram)

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A thesis is never the work of one person alone – and such is mine as well. All this work would not have been possible without the help and company of others.

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