Debit Value Adjustment & Funding
Value Adjustment

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### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>BPS</td>
<td>Basis Points</td>
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<tr>
<td>CCP</td>
<td>Central Counterparty Clearing House</td>
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<td>CCR</td>
<td>Counterparty Credit Risk</td>
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<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<td>CDS</td>
<td>Credit Default Swap</td>
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<td>CSA</td>
<td>Credit Support Annex</td>
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<td>CVA</td>
<td>Credit Value Adjustment</td>
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<td>DVA</td>
<td>Debit Value Adjustment</td>
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<td>EAD</td>
<td>Exposure At Default</td>
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<td>EQD</td>
<td>Equity Derivatives</td>
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<td>EURIBOR</td>
<td>Euro Interbank Offered Rate</td>
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<td>FRA</td>
<td>Forward Rate Agreement</td>
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<td>FVA</td>
<td>Funding Value Adjustment</td>
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<td>IFRS</td>
<td>International Financial Reporting Standards</td>
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<td>IRS</td>
<td>Interest Rate Swap</td>
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<td>LGD</td>
<td>Loss Given Default</td>
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<td>LIBOR</td>
<td>London Interbank Offered Rate</td>
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<td>LVA</td>
<td>Liquidity Value Adjustment</td>
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<td>LTCM</td>
<td>Long Term Capital Management</td>
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<td>MtM</td>
<td>Mark-to-Market</td>
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<tr>
<td>OTC</td>
<td>Over The Counter</td>
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<td>P&amp;L</td>
<td>Profit and Loss Statement</td>
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<td>PD</td>
<td>Probability of Default</td>
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<td>PDF</td>
<td>Probability Density Function</td>
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<td>PV</td>
<td>Present Value</td>
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<td>R</td>
<td>Recovery Rate</td>
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<td>Repo</td>
<td>Repurchase Agreement</td>
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<td>STF</td>
<td>Structured Finance Transactions</td>
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Abstract

Recognizing the growing importance of the Debit Value Adjustment (DVA) and the Funding Value Adjustment (FVA), there are several challenges to implementing the DVA and the FVA, not least since there is no standard definition for these valuation adjustments as of today. Moreover, due to the fact that there is no commonly agreed method of how to price these, there are no market consensus nor regulatory guidelines on accurate approaches to compute the DVA and the FVA, which leaves the vast banks with this challenging and demanding task. This paper considers different approaches when pricing DVA and FVA, which stems from different authors in the financial industry. Furthermore, we will provide a comprehensive overview of both the implications and drawbacks for the valuation adjustments with respect to double counting, hedging strategies and its inclusion on a balance sheet. By not considering DVA and FVA would leave any bank behind the market and at a disadvantage to the banks that have adopted these valuation adjustments.
Acknowledgements

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Finally, we would also like to thank Jesus Christ and our families for giving us strength and faith in everything we do.
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Chapter 1

Introduction

1.1 Thesis demarcation

Recent financial crisis has lead to new behaviour of the financial system, norms that previously were neglected in the pre financial crisis has now been treated and adjusted in order to enhance the stability of the derivative markets. Banks and other financial participants on the Over-The-Counter (OTC) market usually thought, as they were too big to fail, which gave rise to persisted consequences. Furthermore, this has raised the awareness of mitigating the risk embedded in the financial market where financial regulators enforce stricter regulations, such as Basel II and Basel III. In order to avoid potential future losses it is essential for banks and other financial institutions to identify and quantify risk. Several Valuation Adjustments to the OTC derivative contracts have been established to be essential and more important than ever before in the credit crisis in the post Lehman Brothers crash for the whole financial industry.

Recognizing the growing importance of debit value adjustment (DVA) and funding value adjustment (FVA), this paper is designed to investigate the different approaches undertaken by the vast financial institutions to implement and price the DVA and the FVA. Currently there is no standard definition for these valuation adjustments as of yet and hence the many obstacles for financial institutions when calibrating DVA and FVA whilst carrying out the credit risk since some of them do not agree on how to manage these as of the several different calculations among different institutions.

We will do our best in providing a thorough and a comprehensive overview for the different concepts involved in the DVA and the FVA together with pricing techniques that previously have been developed and published by other researchers within this field of research and eventually summarize and compare the different methods established.
1.2 The Swap Market

When entering into a swap agreement the actual setup refers to letting two entities swap cash flows with one another. The swap does not take any initial monetary transactions into consideration. This makes it more desirable for both parties, as transaction fees do not apply, nor do limitations in terms of binding capital.

Swap agreements can involve any type of cash flow. The main purpose of entering into a swap is to exchange a floating cash flow with an inherent risk (high or low) for either a similar cash flow but with a different risk profile, or more likely a fixed cash flow.

It is important to bear in mind that, when entering into such a swap contract, both cash flows in the contractual agreement have a fair setup for both parties, i.e. the same expected net present value (NPV). All else being equal, the initial value of the contract will always be zero when being entered.

Nowadays it is unorthodox to trade swaps directly between two parties, unless both parties are financial institutions. Swaps are therefore most likely to be traded over the counter (OTC) through an intermediary. Generally, an intermediary is intended to be on the opposite side of the transaction of the swap agreement, while also finding peers to match and cover for the defaulting counterparties in such agreement. According to [6] the spread inherent in the swap agreement serves the function of covering for the default risk involved in the counterparties managed by the intermediary.

Commonly, the intermediary involved in such agreements will have an entire portfolio of entities that currently are in a swap agreement with one another. There may be several tools for mitigating the risk at the intermediary’s disposal arising from the event of default that any of the involved counterparties would suffer from on the financial intermediary’s liabilities.
1.2.1 Interest Rates Derivatives

The most common and most frequently-used type of traded swap is the interest rate swap (IRS), where the parties involved agree to exchange payment streams on a notional amount. There are several types of IRS, such as Fixed-to-Floating IRS and Fixed-to-Fixed IRS. However, the most liquid and commonly-used IRS by the financial market is the Fixed-to-Floating, which is also called the plain vanilla IRS.

The parties involved in such an agreement are either called the Receiver (the party who pays a floating interest rate to the other party) or the party called the Payer, i.e. whom ought to pay back the fixed interest rate in question to the Receiver.

The NPV of the fixed cash flow in a plain vanilla IRS is called the fixed leg, while the expected NPV of the floating cash flow is called the floating leg [13]. The floating leg of the IRS is typically linked to three- or six-month LIBOR rates, but can also follow any other interest rate index, e.g. three months EURIBOR.

1.2.2 Credit Default Swaps

Another type of swap playing in the majors when it comes to popularity and importance in the credit derivative market is the credit default swap (CDS), where the swap acts as an insurance policy in the event of default risk. It provides insurance against the default of an issuer (the reference credit) or on a specific underlying bond (the reference security). The standard approach in such agreement is illustrated in the figure below.

The protection buyer pays an annual or a semiannual premium until the event of either the expiry of the contract or default on the reference entity - whichever occurs first. In the event of a default, the protection seller compensates the protection buyer for the possible loss on the underlying bond.
Figure 1.1: An illustration of a CDS agreement in its most basic form between a protection buyer and a protection seller whereas the protection seller hedges against the credit risk inherent originated from the reference entity. Basis points are used as a measure to describe the percentage change in the value of interest rates, e.g. a decrease by 25 basis points means that the interest have decreased by 0.25%.

There is no doubt that CDS has in the last decade become one of the most important instruments, in fact this is mainly due to usefulness in assessing the credit risk of a company. The premium rate of such contracts is denoted by their respective CDS spreads, which are noted transparently and publicly for financial institutions and bigger corporations.

Subject to this assessment, we want to highlight a remarkable event in the historical data of CDS spreads, which were considered to be higher than normal prior to the financial crisis as shown below in Figure 1.2. The figure illustrates the CDS-spreads of five large Banks (Goldman Sachs, Bank of America, Société Générale, Lloyds and Deutsche), which had its first turbulent move in mid 2007 with a clear peak around March 2008 originated from the acquisition of Bear Stearns by JP Morgan.

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1http://www.isdacdsmarketplace.com/about_cds_market/how_cds_work
Furthermore, the stock market crash that took place in October 2008 gave rise to an enormous peak for Goldman Sachs. Therefore, the CDS market gave beforehand hints that something terribly was ahead of us and as far as we are concerned the financial market was aware of this possibly scenario before taking place.

Figure 1.2: CDS-spreads of the five large financial institutions \(^2\).

1.3 Introduction to Credit Risk

Most financial institutions devote considerable resources to the measurement and management of credit risk. For many years, regulators have required banks to keep capital to reflect the credit risk at their disposal. Credit risk arises from the possibility that borrowers and counterparties from different types of contract agreements may default, e.g. derivative transactions and mortgages. This occurs when an obligor fails to meet the agreements towards creditors - it could be for example when a firm goes bankrupt, or fails to pay a coupon in time on one of its issued bonds, or when a household fails to keep up with its amortization schedule.

Rating agencies, such as Moody’s, S&P, and Fitch, are in the business of providing ratings that describe the creditworthiness of corporate bonds. S&P and Moody’s classify their highest rating with AAA/Aaa. Bonds with this rating are classified as having almost no chance of default. Following that comes AA/Aa2, A/A2, BBB/Baa2, BB/Ba2, B/B2 and CCC/Caa2. These are also divided into

\(^2\)http://www.rbnz.govt.nz/research_and_publications/speeches/2012/4890923.html
subcategories by the rating agencies (such as A+, A, A-, or A1, A2, A3). Only bonds with ratings of BBB/Baa or above are considered to be in an investment grade. Each rating is related to the probability of default; a higher rating indicates a lower probability of default. The ratings describe the risk premium that is added to the interest rate of a loan or a bond issued by an entity [7]. Credit risk is something that is not static, since it can vary over time therefore should a company fall below a certain credit rating, its grade changes from investment quality to high yield status. High yield bonds are the debt of companies in some type of financial difficulty and due to their riskiness, they potentially have to offer much higher yields. The last statement tells us that all bonds are not by default inherently safer than regular stocks.

1.4 Counterparty Credit Risk

Counterparty credit risk (CCR) has gained substantial emphasis in recent years, mostly due to the credit crisis in 2007. Counterparty credit risk is the particular risk that a counterparty in a derivatives transaction will default prior to the maturity of a trade, and will not thus be able to fulfill its future obligations and payments, as required by the terms of the contract. Typically, the positions giving rise to CCR can be divided into two broad classes of financial products:

- OTC (over the counter) derivatives, e.g. interest rate swaps, FX forwards, credit default swaps
- STF (security financing transactions), e.g. repos, securities borrowing and lending.

The first of these is considered to be more risky than the latter, mainly due to the rapidly growth and size of the OTC derivative market in recent years and the diversity of complex OTC derivatives instruments [6].

Article [12] states that there are two features that differentiate counterparty credit risk from more traditional forms of credit risk. The first particularity of counterparty credit risk is the bilateral nature of the credit risk:

A derivate position is built in such way that it has both a positive market value for one party and a negative market value for the respective counterparty, but during the lifetime of the derivative contract, the market value of the contract can change such that the markets values to each party are now the opposite. Therefore, the presence of credit risk is now a factor for both sides of the contract.

Consider an IRS where both parties face credit risk. The contract has a positive market value for the fixed payer in the event that the floating rate is above the swap rate, and when the rates have an inverse relation the floating payer is said to have a positive value in his book. This is however not applicable to a bond,
due to the inability of the market value to change as the party holding a long position faces the entire risk of the issuer defaulting, and therefore bears the credit risk.

The second cause of counterparty credit risk is the variability in exposure. The exposure corresponds to how big a proportion of the capital is at risk. By determining the exposure, one can quantify the credit risk of holding a bond position, which is in fact the PV of the bond and also by weighting it with probability of the issuer defaulting. We will later on address, a more robust explanation to the concept of exposure, see section 3.2.
Chapter 2

Derivatives

In this chapter we will go through the fundamentals behind exchange-traded derivatives and OTC derivatives and describe the basic techniques of OTC derivatives. Furthermore, we will derive the classical pricing formula of a derivative written on an asset and sum up the chapter by displaying the process of a typical transaction schedule of an exchange traded derivative and when traded OTC.

2.1 Exchange-traded derivatives

An exchange-traded derivative is an instrument whose value is based on the value of another asset class and which is further traded on a regulated exchange. Due to its advantageous characteristics such as standardization and elimination of default risk, an exchange-traded derivative is hence not affected nor subject to counterparty risk since the exchange will most likely have a clearing entity to take on that role. Therefore, exchange-traded derivatives provide a market place where transparency is featured, and coupled with liquidity being facilitated [6]. As an example, if we consider trading a futures contract (an exchange-traded derivative) the only and tangible counterparty to the futures contract is the exchange itself. Thus, the underlying risk of not receiving the promised cash flows is quite low, since it depends on the survival of the exchange, and not that of a single counterparty [12]. Due to the need for customization and the demand for more complex structures of derivatives, a much more significant notional amount of derivatives are traded over the counter.
2.2 OTC Derivatives

The OTC derivative market is by far the largest market for derivatives, covering products such as exotic options, exotic derivatives and interest rate swaps, and it has grown substantially over the last few years. This is exemplified graphically in the figure below. One can see that the dramatic expansion has mainly been driven by interest rate and currency products. New markets have also been introduced, such as credit default swaps in 2001 and equity derivatives in 2002.

Figure 2.1: The development of total outstanding notional amount of derivatives transactions covering the interest rate and currency instruments, credit default swaps and equity derivatives 1.

OTC derivatives are contingent claims that are traded and privately negotiated between counterparties, without the interference of an exchange or an intermediary. For these reasons they are subject to, and make up the vast part of, a firm’s counterparty risk. This is due to the fact that when trading with OTC derivatives, no third party exists to make sure that the obliged payments agreed upon are made. Thus the involved parties bear the entire credit risk - each counterparty is fully exposed to the risk that its counterparty will not be able to fulfill its obligations subject to the contract due to the possibility of default.

The OTC derivatives can be divided into several categories, which is shown in the figure below. Interest rate derivatives contribute to the vast share of total notional outstanding amount (approximately USD 350 trillion) followed by foreign exchange and credit default swaps at a rather slower pace. It is however

1http://www.bis.org/statistics/derstats.htm
of great importance to bear in mind that the reason why foreign exchange products contribute to such a small share is mainly due to the fact that they can establish a huge proportion of risk due to factors such as the impact of expiries dated over a longer periods and exchange of notional, e.g. on cross-currency swaps.

Figure 2.2: Products classified into OTC notional covering the first half of 2008.

2.3 Derivation of the pricing formula

Until now we have introduced the two most common types of derivatives in the market, i.e. exchange-traded and OTC derivatives. What follows, we derive the classical pricing formula of a derivative written on an asset given by the calculations below.

Similar to the standard Black-Scholes model with the following equation, which describes the price of the derivative over time accordingly

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0.
\]

the vast literature associated with financial mathematics assumes that the rate of return on the risk-less asset is constant and therefore seen as the risk-free interest rate, providing a particular hedger of a derivative to borrow and lend at the risk-free rate when enter into such transaction. This assumption can however be relaxed, let the short-term risk-free interest rate be denoted by \( r_t \) under the assumption that we are interested in pricing a derivative written on an asset \( S_t \) at time \( t \).

In practice, the short-term interest rate refers to the interest debt instruments and/or loan contracts such as Treasury bills and bank certificates of deposit.

\[\text{http://www.bis.org/statistics/derstats.htm}\]
having expiries of less than one year.

Moreover, the underlying asset pays continuous dividends $\delta_t$. Under the physical measure $\mathbb{P}$

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dW_t$$

where $\mu_t$ is the drift, $W_t$ a Brownian motion and $\sigma_t$ the volatility.

In the event of a replication portfolio, the corresponding replication formula for a derivative $V_t$ reads as,

$$V_t = \alpha_t S_t + B_t$$

(2.1)

whereas $\alpha_t$ tells us the number of purchased shares of $S_t$ and $B_t$ represents the value deposited in the risk-free bank account.

The price process $B$ is the price of a risk-free asset should it has the following dynamics.

$$dB_t = r_t B_t dt$$

where $r$ is any adapted process. The $B$-dynamics can be written as,

$$\frac{dB_t}{dt} = r_t B_t$$

(2.2)

Let $\mathbb{Q}$ be a risk-neutral measure equivalent to $\mathbb{P}$ and replacing the $\mathbb{P}$-drift term for $S_t$, that is $\mu_t$, by $(r_t - \delta_t)$, which is the $\mathbb{Q}$-drift term for $S_t$. Subsequently, applying Ito’s Lemma to equation (2.1)

$$\left( \frac{dV_t}{dt} + \frac{1}{2} \sigma_t^2 S_t^2 \frac{\partial^2 V_t}{\partial S_t^2} \right) dt + \frac{\partial V_t}{\partial S_t} dS_t = \alpha_t (dS_t + \delta_t S_t dt) + (-\alpha_t S_t + V_t) r_t dt$$

(2.3)

In the beginning of this section we claimed that the hedger of a particular derivative is assumed to borrow and lend at risk-free interest rate. Now let $\alpha$ constitute the hedge of the derivative, therefore in order to hedge the derivative $V_t$, we set

$$\alpha_t = \frac{\partial V_t}{\partial S_t}$$

Consequently, applying the hedging equation to (2.3) we obtain

$$\frac{\partial V_t}{\partial t} + (r_t - \delta_t)S_t \frac{\partial V_t}{\partial S_t} + \frac{1}{2} \sigma_t^2 S_t^2 \frac{\partial^2 V_t}{\partial S_t^2} = r_t V_t$$

(2.4)

where $r_t$ is the discount rate.
Under the assumption that $V_t$ has no cash flows until its expiry $T$, the solution of equation (2.4) with the terminal condition $V_T = g(S_T)$, where $g$ is the corresponding contract function for the derivative, becomes

$$\frac{V_t}{B_t} = E^Q \left[ \frac{V_T}{B_T} | \mathcal{F}_t \right]$$  \hspace{1cm} (2.5)

where $Q$ is the risk-neutral measure equivalent to $P$ such that $\mu^Q_t = r_t - \delta_t$ under which both $\frac{V_t}{B_t}$ and $\frac{S_t e^{\int_{s=t}^{s=T} \delta ds}}{B_t}$ are martingales and $\mathcal{F}_t$ contains all the information known at time $t$.

Thus, we have derived a simple pricing formula for a derivative written on an asset in its simplest case by applying Ito’s lemma to our pricing equation which further can be hedged by using $\alpha_t$.

### 2.4 Derivative Transactions

In the following section, we try to illustrate the difference between an OTC derivative transaction schedule and the corresponding one for an exchange traded derivative. The set-up between the two is somewhat different due to the differences for the two derivative classes which we pointed out earlier in this chapter.
2.4.1 OTC derivative schedule

Below we illustrate the governance of how a typical OTC derivative transaction looks like. Most likely, there are three parties involved in such transaction, i.e. a dealer, a counterparty and a hedging counterparty, whereas the counterparty commences the derivative transaction by entering a trade with the dealer in question, which further will try to hedge the same trade against the hedging party. The reason why the dealer tries to hedge the position is due to the willingness of being risk neutral to market risk, which makes the dealer to a market maker. In practice, a dealer would try to split the transactions into more than one part and thus hedge cash flows separately to different hedging counterparties. Below we display the OTC derivative transaction schedule when only considering one hedging counterparty.

Figure 2.3: The figure illustrates the idea behind the payment schedule and the ideal cash flow between the three involved parties when the counterparty commences an OTC derivative transaction. The counterparty makes a payment to the hedging counterparty through the dealer and the same manner goes for the payment stream made by the hedging counterparty to the counterparty via the dealer which is displayed by the arrows A and B accordingly. Subject to payment schedule above, the dealer is market risk neutral.
2.4.2 Exchange traded derivative schedule

Unlike the payment schedule and cash flows in an OTC transaction that we highlighted above, the operations of an exchange traded derivative are centralized to a clearing house, e.g. London Clearing House (LCH). The clearing house steps in between the involved parties as a CCP (central counterparty clearing house) to provide clearing for all outstanding trades and ensures among several other things, to set the characteristics of the derivative such as, notional amount and settlement dates. Most importantly, the clearing house carries and enforces margin requirements, in order to make sure that none of the involved parties would default on their own obligations.

This enables the clearing house to act as an independent third party to oblige the parties with losing positions to provide any additional cash to their margin accounts, which is seen a vital protection against a defaulting party. Hence, the main function of the clearing house is to net all the offsetting outstanding contracts of each party over and across all the other parties to ensure that each and involved party guaranteeing the fulfilment of each derivative contract.

Below we display the schedule of an exchange traded derivative transaction.

Figure 2.4: The figure illustrates the fact that in an exchange traded derivative, the dealer is instead facing the clearing house and not the counterparty in such derivative transaction, which by all means eliminates any potential default risk of the counterparty.

One should also note that, most often when derivatives transactions are not cleared by CCPs, the banks often charges the investor a so called margin, i.e. x bps when showing the price of the transaction to the client. They charge a margin in order to absorb the losses whenever and if the the other party should default by using collateral which has been posted by the defaulting party and thus protects the surviving party from enormous losses.

Also, the reader should keep in mind that, cleared transactions are also applicable for OTC derivatives as it applies for exchange traded derivatives, but in this context we prefer to assign the role of the CCP as a so-called exchange.
Chapter 3

Credit Risk Relationships

This chapter will review some important components of credit risk measurement as a foundation of the capital at banks disposal subject to the DVA and FVA. Capital is used to buffer banks against unexpected losses and changes in asset values. Nowadays banks and financial institutions take strategic risk in everyday operations, these risks are reflected in the volatility of the value of the banks assets and therefore presented commonly in finance under different circumstances in order to mitigate different types of risk. We start off by introducing two important factors when dealing with OTC derivatives, i.e. Exposure and Loss Given Default (LGD) and finish off by looking at their inclusion in different concepts of managing counterparty credit risk under Netting Agreements, ISDA Master Agreement, Collateralization and CSA Collateralization.

3.1 Loss Given Default

Loss Given Default (LGD) describes the percentage of loss when a bank’s counterparty goes default and moreover, the amount of funds that are lost by an investor when a borrower fails to make his repayments [4]. LGD is a core component of credit risk measures within the vast financial institutions and corporates, used to determine capital requirements and to assess and manage credit risk, i.e. the expected loss.

More specifically, the LGD is in bottom line the estimate of the proportion of Exposure at default (EAD), that will be lost in the event of a defaulting counterparty. Furthermore, EAD can in brief be described as the estimated amount that will be owed by an obligor at the point of such a default. The LGD is given by,

\[ LGD = 1 - R \]

whereas, the recovery rate, denoted by \( R \), is the ratio of the exposure that would be recovered in an event of default, which has a major role in the concept of calculating both the DVA and FVA, since it reflects the amount of losses a firm is exposed to.
The recovery rate is calculated as,

\[
R = \frac{\text{Net Recoveries}}{\text{EAD}} \times 100\%
\]

In order to enhance the methodology of LGD, we illustrate below a simple example.

At the point of a defaulting counterparty, a bank’s EAD is EUR 5 million, whereas the gross recoveries of EUR 750,000 are made and costs of EUR 75,000 are incurred. Therefore, on a non-discounted basis, the recovery rate can be calculated as follows,

\[
R = \frac{\text{EUR 750,000} - \text{EUR 75,000}}{\text{EUR 5 millions}} \times 100\% = 13.5\%
\]

Hence, the LGD value is,

\[
LGD = 1 - 13.5\% = 86.5\%
\]
3.2 Exposure

The exposure of a derivative contract represents the amount that would be lost should a counterparty default. The exposure depends on whether the contract is an asset or a liability to the investor. The very first thing a bank has to do, should the counterparty default, is to close out its positions with that counterparty. Furthermore, to determine the exposure as a consequence of the default, one usually assume that the bank enters into a similar derivative contract with a different counterparty mainly due to maintain its markets position [12]. In such a scenario, we can state that, the bank will end up with a net loss of zero since after entering into a similar contract with another counterparty, they will receive the market value of the contract and thus the bank ends up with having a net loss of zero.

On the other hand, should the derivative contract be positive from the banks perspective in the event of default, the bank closes out its position immediately, but this time without receiving anything by the defaulting party. Once again, the bank finds an another party to enter into a similar contract with and hence the bank pays the market value of the derivative contract and eventually the bank suffers from a net loss equal to the corresponding derivative contract’s market value.

Subsequently, with respect to the two scenarios above, we can conclude that the credit exposure of a bank, which only has one derivative contract with a counterparty, is the maximum of that contract’s market value and zero. Let the value of derivative contract \( i \) at time \( t \) be \( V_i(t) \), the contract level exposure is then given by

\[
E_i(t) = \max\{V_i(t), 0\}
\]  

(3.1)

We are aware of that the value of the contract may vary unpredictably over time depending on different market conditions and thus only the current exposure can be known with a certainty whilst the future exposure is unsettled [12]. As we mention earlier in this section, the contract can either be an asset or a liability from the banks perspective, which makes the counterparty risk bilateral between the bank and its counterparty in such derivative contract.

3.3 Netting

The word netting itself, refers to a type of a settlement of mutual obligations between two counterparties that processes the combined value of the involved transactions. Furthermore, netting has lately been a common practice in trading of options, foreign exchange and futures. Since netting is exclusively designed to lower the number of transactions required, let us therefore draw an example to this notion. For example, if Bank X is owe bank Y €10,000, whilst Bank Y is owe Bank X €2,500, then the value after the netting has taken place would be a €7,500 transfer from bank X to Bank Y.
According to [6] netting refers to the fact that single exposures to the overall transactions are non-additive and hence, the risk tends to be reduced significantly. Subsequently, the overall credit exposure in derivatives markets will grow at a slower pace in relation to the notional growth of the relevant market itself.

Netting is considered to be one of the risk mitigation methods, with the greatest impact on the structure of derivatives markets. In the absence of netting, liquidity would dry up and the size of the derivatives market would shrink drastically. The benefit of netting is thus the ability to hedge against underestimating or overestimating the underlying risk of the overall transactions. However, when not taking netting into account one can analyze the different transactions outstanding independently with the respective counterparty, as the exposures will stay additive.

The most common types of netting used in the market are:

- **Payment netting** which covers a situation in which a financial institution ought to make and receive several payments during a given day. This first type of netting refers to an agreement to combine the embedded cash flows into a single net payment.

- **Close-out netting** is considered to be more significant to counterparty risk, as it lowers pre-settlement risk. This latter type of netting covers the netting of the value of a derivative contract in the event of a defaulting counterparty at a future date.

### 3.4 Netting Agreements

Most often, should there be several outstanding trades faced towards a defaulting party whilst leaving the exposed risk from the same counterparty unmitigated by any means, then one can express the prevailing maximum loss that the bank is exposed to as the sum of the contract-level credit exposures accordingly,

\[
E(t) = \sum_i E_i(t) = \sum_i \max\{V_i(t), 0\}
\]

(3.2)

Furthermore, the exposure that the bank is exposed to in such situation can be mitigated and further reduced by the notion of netting agreements. A netting agreement is in fact, a legally stipulated document between the two concerned and exposed parties, should one party default, which allows a type of aggregation of the outstanding transactions between the two parties to take place. Moreover, netting enables thus derivative transactions with negative value to be used to offset the transactions with positive value and thus only the net positive value designates credit exposure at the time of a potential default[12]. Therefore, the
total credit exposure arising from all derivative transactions in a netting set (solely those under the jurisdiction of the netting agreement) is reduced to the maximum of the net portfolio value and zero. This total credit exposure is given by

\[ E(t) = \max \left\{ \sum_i V_i(t), 0 \right\} \quad (3.3) \]

Moreover, there could also be several netting agreements with only one counterparty.

We sum up this section by looking at the case where there may be trades that are not covered by any netting agreement at all. Let \( NA_k \) represent the \( k \)th netting agreement with a counterparty. Consequently, we can express the counterparty-level exposure as,

\[ E(t) = \sum_k \max \left[ \sum_{i \in NA_k} V_i(t), 0 \right] + \sum_{i \notin \{NA\}} \max [V_i(t), 0] \quad (3.4) \]

where the inner sum in the first term sums the values of all trades that is only covered by the \( k \)th netting agreement, whilst the outer sum, sums up the exposures over all netting agreements. The latter term in the equation above is simply the sum of contract-level exposures of all trades that are not covered by any netting agreement [12].

Therefore, we can conclude that the netting agreement allows one to net the value of trades with the counterparty that will default before landing the actual claims and is hence, vital when recognizing the potential benefit of offsetting trades with a counterparty going into default.

### 3.5 Trading Relationship under ISDA Master Agreement

The International Swaps and Derivatives Association (ISDA) was founded in 1985 to ensure that the OTC derivatives markets operate in a safe and efficient manner. Moreover, they aim towards reducing the counterparty credit risk and increasing transparency within the derivatives markets while improving the industry’s operational infrastructure, building robust, stable financial markets together with a strong financial regulatory framework.

Published by the ISDA, the Master Agreement has a global scope designed to reduce and eliminate legal uncertainties and to provide tools in order to mitigate the counterparty risk for parties entering into OTC derivatives.

The Master Agreement contains a qualifying master netting agreement, corresponding to an agreement between two firms and outlines the contractual
obligations and standard terms that will eventually apply to all future outstanding transactions between the firms. Hence, all outstanding transactions between the two parties are handled by one agreement that enables both parties to collect the amounts due for each single trade and replace them with a net amount payable to one another. The convenience of a master agreement is mainly because it enables the counterparties to quickly negotiate upcoming future agreements or transactions, since both parties can rely on the contractual terms of the master agreement in order to avoid that the same terms will be repetitively negotiated once again and only to consider the deal-specific terms.

3.6 Trading Relationship under Collateralization and CSA

There are many ways to mitigate counterparty credit risk, including netting, collateralization (margining) and hedging. Although, collateral refers to assets offered by a borrower to a lender assets to hedge a loan. Should the borrower fail to repay the loan, the lender can then exercise the collateral to recover the losses. A simple example illustrating the methodology of collateral and its function can be found below.

Let us consider a lender A and a borrower B. The entity A lends an amount of money to B. In order to secure the transaction, B has to pledge some asset to A should he fail to pay his debt. In other words, a collateral is an agreement that limits the exposure of default towards a specific counterparty.

![Figure 3.1: The concept behind posting collateral in order to limit the exposure of default which is a very common obligation subject to derivative transactions.](image)

Figure 3.1: The concept behind posting collateral in order to limit the exposure of default which is a very common obligation subject to derivative transactions.

Both parties are motivated to mitigate the exposure to each other’s credit risk, which can be achieved by implementing a so-called Credit Support Annex (CSA), which follows from the ISDA Master Agreement. The CSA is a legal document that defines the terms under which collateral should be posted between two parties. The amounts that are posted are based on the current aggregated net present value (NPV) of all the outstanding trades between the two
parties. The party that has a negative present value (PV) in the outstanding trades (also called the Pledgor) is obligated to post collateral CSA. The party that receive the collateral is normally called the Secured Party [6].

Depending on the direction of the collateral as per the CSA agreement, it may either be bilateral or unilateral. The most common for a derivatives contract is a bilateral agreement, meaning that both can receive collateral and are expected to mitigate the counterparty risk for both parties. This sort of agreement is the most used within the OTC market. On the other hand, in a unilateral agreement only one of the parties has the obligation to post collateral to the other party.

We have seen different ways of how to mitigate the counterparty credit risk for OTC derivatives, but at end of the day it can never be completely covered, since there will always be an existing risk in derivative agreements and transactions. Below we try to illustrate a common situation in practice, that is a negotiated ISDA Master Agreement between a counterparty and a dealer coupled with a negotiated ISDA Master Agreement with CSA between the dealer and the hedging counterparty.

![Diagram](image)

Figure 3.2: The figure shows the current contractual trading relationship mentioned above, between counterparty, dealer and the hedging counterparty.
Furthermore, the consequence of that the dealer in such a transaction faces dissimilar contract with the counterparty and the hedging counterparty will eventually give rise to a possible mismatch of payment streams.

![Diagram showing payment streams](image)

Figure 3.3: The figure displays the payment streams between the counterparty, dealer, and the hedging counterparty contingent the relationship illustrated in figure 3.2. Hence, we can draw the conclusion that there will be a mismatch of payment streams for the dealer.

In a situation like this, the dealer ought to either post or receive collateral depending on how the current market conditions look like because of the underlying asset of the derivative. Therefore, the dealer is no more market risk neutral.

We wrap up this section by pointing out that, whenever there is a bilateral CSA in place between the two counterparties, one ought not to charge any CVA nor DVA from the client under any circumstances because, the collateral that is being posted per settled CSA covers up any potential losses caused by a defaulting counterparty.
Chapter 4

Valuation Adjustments

This chapter will present the underlying theory that is used as the basis for our thesis. We will try to provide a comprehensive description for the several valuation adjustments that play a major role today and that have to be taken into account when pricing OTC derivatives. These are the Credit Valuation Adjustment, Debit Valuation Adjustment and Funding Value Adjustment.

4.1 XVAs

In recent years there has been a profusion of adjustments to the risk-neutral price of an OTC derivative, often denoted as X-Value Adjustments (XVAs). This is the effect of the new trading environment, which is dominated to a great extent by credit, funding and capital costs. As a reaction to the 2008 financial crisis, financial institutions have become more aware of the adjustments that must be considered when valuing derivatives. Most if not all financial institutions have redeveloped their calculation models of the adjustments in the post-crisis period, having previously taken them for granted. In general, the financial markets have become more aware of counterparty credit risk and its importance, which has given rise to several types of valuation adjustments.

4.1.1 CVA – Credit Valuation Adjustment

In the aftermath of the recent financial crisis, it has become crucial to account for the risk of counterparty credit deterioration from the market perspective and further the default in pricing of OTC derivative transactions. The pricing component arising from this risk is the Credit Value Adjustment (CVA). In order to enable fair pricing when accessing the CCR, the CVA has been evolved to calculate the future risk for counterparties in the derivatives market. In brief, one can say that CVA is the difference between the risk-free value of a portfolio and the fair value of the same portfolio when taking the possible default of a counterparty into consideration. The CVA is an expected value incorporating
both exposure and the probability of default, and aiming towards achieving fair
pricing of derivatives.

CVA can be treated as either bilateral or unilateral whereas the latter is given
by the risk-neutral expectation of the discounted loss. In bottom line, CVA
is the amount in risk, which is subtracted from the mark-to-market value of
derivative position in order for investors to account for the losses they would
expect to suffer from a counterparty default. The discounted loss, \( L \) is given by,

\[
L = 1_{\{\tau \leq T\}}(1 - R)w(t)E(\tau)
\]

where \( \tau \) is a random variable, i.e. a stopping time that denotes the default
by the counterparty, \( T \) is the expiry of the longest transaction in the portfolio,
\( w(t) \) is the future value of one unit of the base currency invested today at the
prevailing interest for expiry \( t \) and \( 1_{\{t \leq T\}} \) is the indicator function accordingly.

Furthermore, we can define the unilateral (one-way) CVA in terms of risk-
neutral expectation as,

\[
CV A_{uni} = \mathbb{E}^Q[L] = \int_0^T (1 - R(t))\mathbb{E}^Q[w(t)E(t)|\tau = t]q(t)dt
\]

where \( q(t) \) is the probability density function of \( \tau \) with respect to the probability
measure \( Q \) and \( E(t) \) is the exposure at time \( t \). Thereto, the middle expression
is equal to the expression with conditional expectation because of mathemat-
ical properties of conditional expectations, which stems from the tower property.

Also bear in mind that when expressing the unilateral CVA in terms of condi-
tional expectation corresponds conveniently to the fact that the bank will only
suffer from potential expected losses if and only if, the other party defaults post
entering such derivative transaction [12].

The bilateral CVA refers thus to the counterparty credit risk that both par-
ties face in an OTC derivative contract, which in turn leads to the CCR of both
counterparties being affected by an OTC contract. This is mainly due to fact
that the OTC market is constructed so that both parties that are committed to
a contract will face a credit risk. This will therefore have an effect on the CCR
of both counterparties in a bilateral transaction, e.g. two parties entering into
an IRS transaction. Nowadays CVA has become a valuable instrument in the
derivatives market, mainly due to the substantial growth of the OTC deriva-
tives, whilst the frequency at which CVA is calculated has increased remarkably.
It is estimated on a daily basis and occasionally in real-time data [6].
4.1.2 DVA – Debit Valuation Adjustment

Until at the beginning of this century, large banks charged their corporate clients for the counterparty credit risk, where the unilateral CVA was taken into account. The CVA was adjusted according to the unilateral counterparty credit risk, which was constructed on the assumption that the counterparty had a credit risk and the investor as free of default risk. Prior to the financial crisis it was set that large banks were not default-charged. This has lately been criticized and large financial institutions have raised their awareness towards it. Banks have started to implement CVA between themselves and large corporates have also become aware that they face CCR arising from transactions with banks with a default risk. This has allowed the counterparties to charge each other with unilateral CVA, resulting in taking the DVA and bilateral CVA into account.

Debit Value Adjustment, (DVA) is more or less the opposite of CVA. It reflects the credit risk that the investor faces towards the other party. It defines the differences between the value of a derivative/financial-instrument, under the assumption that the bank is default risk-free and the default risk of the bank. When banks have changes in their own credit risk, it can result in changes to the DVA and also to the bilateral CVA against the counterparty. The DVA is sensitive to the bank’s creditworthiness (credit spreads and the probability of default) and changes that affect the expected exposure. The CVA and DVA have the opposite signs and while CVA decreases the value of a derivative, the DVA increases the value of the same derivative.

If a firm experiences a fall in its credit rating, it will cause an increase on MtM profits for the same firm. This is due to an increase in the probability of default and a depreciation of the credit rating; the firm will therefore have a decrease on the bilateral CVA. Let $CVA_{1Bilateral}$ and $CVA_{2Bilateral}$ denote the bilateral CVA before and after the firm faces this decline in credit rating - the formula below represents the bilateral CVA that is calculated as the difference between the unilateral CVA towards the counterparty and the DVA [6].

$$CVA_{1Bilateral} = CVA_{Unilateral} - DVA.$$

An increase in DVA will have a negative impact on CVA;

$$CVA_{2Bilateral} = CVA_{Unilateral} - (DVA + \Delta DVA) = CVA_{1Bilateral} - \Delta DVA.$$

Now if we were to apply the reasoning above to the practice of a firm, then this would mean that their current outstanding OTC derivative transactions have become less risky, as well as improving the MtM value of the derivative. Whether this is a realistic or a reasonable outcome is disputed. At this point
we can consider that DVA has a major impact when it comes to measuring counterparty credit risk and it will continue to do so, as the CCR framework continues to develop. With today’s new regulations it is a requirement of the respective bank to calculate DVA according to the report [6], *IFRS 13: Fair Value Measurement* ¹.

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4.1.3 FVA – Funding Value Adjustment

Funding Value Adjustment (FVA) is, in the existence of an ISDA Master Agreement, an adjustment to the value of a derivatives portfolio which is designed to guarantee that the dealer of a contract recovers its average funding costs when it trades and hedges derivatives. When traders need to manage their trading positions, they need to gather cash in order to perform a number of operations (hedging positions and posting collateral). It is obtained from the treasury department or from the money market that has to be satisfied. Traders can also gather cash from its market position such as coupons, collateral and close-out payments. This will result to some revenue for the trader in which he is not ready to lend it for free and hence FVA goes in both ways, sometimes you gain from it and sometimes you lose depending on the circumstances of the trade.

Basically, FVA corresponds to a funding cost/benefit from borrowing or lending cash arising from day-to-day derivatives business operations, for example when posting and receiving collateral. Consider a situation where the dealer is to post cash collateral on the hedge, but does not receive that cash in return from its counterparty. The situation requires thus that the dealer has to raise the cash itself in order to cover the deficit caused by the counterparty.

Subject to the scenario above, many theoretical arguments claim that the dealer’s valuation should recover the total amount of its funding costs, while other dealers find these arguments unconvincing and therefore make the adjustment nonetheless. The raised issues involved in the FVA debate (whether products should be valued following cost prices or at market prices) are essential to many industries beyond the derivative market when thoroughly evaluating potential investments [8].

When taking the credit risk into account, pricing models such as the Black-Scholes-Merton play a crucial role for derivatives traders in the event of no-default value (NDV) of a derivative transaction. The NDV implies that whether both sides of a transaction will live up to their obligations, depends on the discount rate that is used in question. At the same time, if we assume that the risk-free rate is used, the resulting value is consistent both with theory and with market prices in the interdealer market as full collateralization is required.

According to [10], when adjusting for credit risk, the portfolio is given by

\[ \text{Portfolio Value} = \text{NDV} - \text{CVA} + \text{DVA}. \]  

(4.1)

Furthermore, in order to incorporate the dealer’s average funding costs for uncollateralized transaction, we implement the adjustment of FVA. One has to bear in mind that there is a difference between the NDV obtained when the risk-free rate is used for discounting and the NDV derived on the discounting at the dealer’s cost of funds. When incorporating FVA in equation (3.1), we
obtain the following equation

\[
\text{Portfolio Value} = \text{NDV} - \text{CVA} + \text{DVA} - \text{FVA}. \tag{4.2}
\]

However, one thing to be aware of is that several theoreticians claims that the FVA is not an adjustment for credit risk, since the CVA and the DVA take this into account and would otherwise imply double counting for the credit risk. The drawback of FVA is that, different market participants often have different estimates of the fair value, even though they often use identical models with the same market data.

Now if we consider the equation (3.1) - should the dealer and the counterparty have the same funding costs, this would then imply that the dealer’s FVA is equal in magnitude and contrary in sign to the FVA of the counterparty, which in turn will make them both still agree on the fair value which would not be the case in the event of different funding costs.

### 4.1.4 Double Counting of FVA and DVA

We have until now seen that the FVA concerns funding while the DVA treats a market participant’s own credit risk and thus they concern different perspectives of the uncollateralized derivatives market. This section will be based on the framework which stems from the article [10] on how these two adjustments interact and potentially overlapping one another.

It has been an enormous controversy regarding the relationship between these two valuation adjustments and whether the DVA should be ignored, which has raised the question of double counting. In order to examine the likelihood of a credit event to happen and whether the DVA is overflowed in the context of FVA, we will throughout this section introduce two distinguished types of characteristic functions, which cover the probability structure in the event of a credit event.

Now let \( DVA_d \) refer to the value to a bank that arises in the event of default on its own derivatives obligations whilst \( DVA_f \) reflects the value to the bank but this time in the event of default on its other liabilities, such as short-term debt, long-term debt, i.e. the DVA arising from the funding that is required for the derivatives portfolio.

Under the assumption that the whole of the credit spread is the compensation for default risk one can set the \( DVA_f \) equal to the FVA for a derivatives portfolio. This could be considered as valid since the PV of the expected excess of the bank’s funding for the derivatives portfolio under the risk-free rate is equal to the FVA. This in turn is also equal to the compensation the bank is providing to its lenders should the bank default and thus equivalent to the expected benefit to the bank in the scenario of defaulting on its own funding.
Consequently, FVA and $DVA_f$ will cancel out one another. It is also of great importance to bear in mind the fact that whenever a derivative needs funding, FVA is accounted as a cost whilst $DVA_f$ as a benefit. This applies for the other way around, so whenever a derivative provides funding the FVA is treated as a benefit and the DVA as a cost accordingly.

This will extend equation (3.2) to

$$Portfolio\ Value = NDV - CVA + DVAd + DVAf - FVA$$

$$= NDV - CVA + DVAd.$$  (4.3)

Here, both FVA and $DVA_f$ are additive across transactions whenever transactions being entirely uncollateralized. Therefore, the dealer’s FVA and $DVA_f$ are independent of other possible transactions entered into by the dealer for a derivative [10].

Now let us instead take the $DVAd$ under the scope. Equation (3.3) indicates and validates that it is appropriate to calculate this for a derivatives portfolio, which the dealer has with its counterparty.

Consider the case where a dealer has $n$ uncollateralized transactions with an end user. Moreover, we set up the definition for the value to an end user of the $i$th transaction at time $t$ as $v_i(t)$, whereas the dealer’s unconditional default rate at time is defined as $q(t)$.

In order to describe the likelihood of a default to occur we introduce three kinds of characteristic functions that describe the probability structure of such a credit event. The survival function $S(t)$ is the probability that a stopping time, $\tau$, occurs first after than any point in time, $t$,

$$S(t) = P[\tau > t] = 1 - Q(t)$$

where $Q(t)$ is the cumulative distribution function (CDF) of $\tau$.

Furthermore, $q(x)$ is the density function of the PD, meaning that $Q(x) = \int_{-\infty}^{x} q(s)ds$. Therefore, the density $q$ has the property that $\int_{0}^{\infty} q(s)ds = 1$, because $\tau \geq 0$.

Moreover,

$$q(t) = Q'(t) = -S'(t).$$

The relationship between $q$ and $S$ can also be described in terms of the hazard rate. The hazard rate $\lambda$ also called the survival analysis, originates from the mathematical insurance and actuarial science concept. It expresses the instantaneous conditional failure rate and can be defined using conditional probability.

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{P[t < \tau \leq t + \Delta t | \tau > t]}{\Delta t} = \frac{q(t)}{S(t)}.$$
In particular \( \lambda(t) = -(\log S(t))' \).

In this context,
\[
DV_A_d = \int_0^T w(t)q(t)[1 - R(t)]E\left[\max\left(\sum_{i=1}^n v_j(t), 0\right)\right]dt
\]

where \( T \) is the life of the longest existing transaction, \( R(t) \) the recovery rate at time \( t \), which in brief denotes the ratio of the exposure that would be recovered in an event of default, \( w(t) \) is the present value of $1 obtained at time \( t \) and \( E \) denotes the risk-neutral expectation.

Since the instantaneous forward credit spread at time \( t \) is \( q(t)[1 - R(t)] \), which reflects the default rate adjusted by the inclusion of the recovery rate, the \( FVA_b \) is given by,
\[
FVA_b = \int_0^T w(t)q(t)[1 - R(t)]E\left(\sum_{i=1}^n v_j(t)\right)dt
\]
where \( FVA_b \) is the benefit provided by FVA that is, \( FVA_b = -FVA \). It always holds that
\[
\max\left(\sum_{i=1}^n v_j(t), 0\right) \geq \sum_{i=1}^n v_j(t)
\]
Hence, \( DV_A_d \geq FVA_b \).

In order to reflect different errors when pricing options, take for instance the scenario when the end user purchases options from the dealer, i.e. positive \( v \)'s. This will result in \( DV_A_d = FVA_b \) and further imply that the \( DV_A_d \) is redundant due to the fact that \( DV_A_d \) and FVA now have the same effect and we have established some kind of overlapping. As such, in a situation where the dealer neglects the \( DV_A_d \) in his option pricing and merely takes the FVA into account in his pricing will be doing it correctly. In similar way, we obtain \( DV_A_d > FVA_b \) instead when not always having positive \( v \)'s.

Now instead we will examine the influence of adding up a new transaction to the already existing derivative portfolio of the client. Let the recently added transaction be worth \( \gamma(t) \) at time \( t \). The increasing \( FVA_b \) is given by
\[
\Delta FVA_b = \int_0^T w(t)q(t)[1 - R(t)]E[\gamma(t)]dt
\]

Moreover, the corresponding \( DV_A_d \) is given by,
\[
\Delta DV_A_d = \int_0^T w(t)q(t)[1 - R(t)]\left\{E\left[\max\left(\sum_{i=1}^n v_j(t) + \gamma(t), 0\right)\right]\right\} - E\left[\max\left(\sum_{i=1}^n v_j(t), 0\right)\right]
\]
\[\text{Equation (4.4)}\]
From previous calculations we have shown that the $DVA_d \geq FVA$. However $\Delta DVA_d$ can either be less than or greater than the $\Delta FVA_b$.

The controversy behind this conundrum of including both FVA and DVA in OTC transactions is mainly due to the fact that FVA has the disadvantage of creating arbitrage opportunities when prices tend to be favourable since a single price cannot serve the purpose of both reflecting the derivative trader’s funding costs and still be consistent with the prices regulated by the market. Therefore, in such an optimal scenario for an end user is thus to enter into a transaction with a bank facing high funding costs and enter into an offsetting transaction with a bank facing low funding costs in order to avoid both favourable and unfavourable prices when buying and selling options.
Chapter 5

DVA on the Balance Sheet

Currently there is a great debate of how to deal with and treat DVA with respect to a bank’s balance sheet. We will further address the accounting perspective of DVA, since we believe it adds value and will deepen the understanding of this spectrum. This particular section is mostly based on the [5], where the authors examines the different links and implications subject to Funding, Liquidity, Credit and Counterparty Risk. Furthermore, we will go under the scope with the essence of the DVA coupled with a quite robust conceptual framework to consistently encompass the DVA in a balance sheet of a financial institution. It is important to address the tie between DVA and its treatment in banks’ balance sheets since derivative contracts have an important impact by valuation adjustments on the same balance sheets which will eventually enable us to establish a thoroughly understanding of the correlation between these, which stems from the debate over this particular adjustment.

5.1 DVA’s treatment in banks’ balance sheets

Recent researches have provided several but not always satisfying, vindications for the reduction of the liabilities produced by the DVA. In what follows we will try to highlight and analyze DVA’s incorporation in a balance sheet and if it should be considered as applicable subject to banks’ balance sheet.

In brief, a balance sheet gives an comprehensive overview of a firm’s assets, liabilities and shareholders’ equities at a prospective time point \( t \). Furthermore, these three segments of a balance sheet gives investors an indication of how much of the firm assets and liabilities together with the total amount that has been invested by the shareholders. Under any circumstances the following formula for the balance sheet must hold

\[
Assets = Liabilities + Equities
\]
where the two sides should cancel out one another.

Now, let B and L denote two financial institutions as in a regular transaction, whereas the bank B enters into a transaction with bank L in terms of borrowing money from the latter one. In this particular case where B = borrower, L = lender, to keep things non-complex, we assume that there is a constant risk-free interest rate $r$ embedded in the transaction between the two parties similar to the one in the Black-Scholes model whereas each single financial institution pays a funding spread denoted $s_X$, $X \in \{B, L\}$ over the risk-free rate when borrowing money. The funding spread is simply the difference between the funding costs of a bank and the risk-free rate. The funding spread can be divided into two segments:

I) The premium that the lender charges the borrower to cover for the probability of default by the borrower which will be denoted as $\pi_X$, and the $LGD_X$ (indicated as a portion of the lent amount by the borrower).

II) The liquidity premium when applicable is denoted as $\gamma_X$.

Moreover, we assume that the borrower B is an institution with a balance sheet in its simplest form which is marked to market, corresponding to the fact that the accounting for the fair value of their assets or liabilities are based on the current market price. The stockholders determine to commence a transaction activity with an equity E until time of maturity T, whereas the amount E is deposited in a bank account $BA_1$ that is assumed to be risk-free. Furthermore, we assume that no premium is required over the risk-free rate, that makes it also to the hurdle rate to value investment projects whilst the borrower does not pay any liquidity premium, thus equivalent to $s_B = \pi_B$.

When commencing the transaction at time 0, the bank cuts a deal with a possible lender (institutional investor) to close a loan contract. The bank is not charging any funding costs when setting the fair amount to lend, and hence the lender does not need to pay any interest for the funding that they deploy in their business. The amount is deposited in a bank account $BA_2$, which is also assumed to be risk-free.
The balance sheet at time 0 is then given by,

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BA_1 = E$</td>
<td>$L = Ke^{-rT}$</td>
</tr>
<tr>
<td>$BA_2 = Ke^{(-r+s_B)T}$</td>
<td>$-DV_{AB}(0) = -e^{-rT}K(1 - e^{-s_BT}) = -e^{-rT}K(1 - e^{-s_BT})$</td>
</tr>
</tbody>
</table>

when considering the bank account $BA_2$, it seems that the lender has to discount positive cash flows received at maturity $T$ at a discount rate which include both the risk-free rate and the borrower’s funding spread. Also looking at the right hand side of the balance sheet, we have stated that the $-DV_{AB}(0) = -e^{-rT}K(1 - e^{-s_BT})$ which is simply the expected loss that the borrower will expose to the lender in the event of a default. We can easily check that the assets and liabilities balance since the $LHS = E + Ke^{(-r+s_B)T}$ and the $RHS = E + Ke^{-rT} - e^{-rT}K(1 - e^{-s_BT})$. Hence, the $DV_{AB}(0)$ is deducted from the risk-free present value of the loan paying back $K$ at maturity $T$, giving us an exact match for the PV of the loan and the amount deposited in $BA_2$ and is therefore not generating any P &L at time 0.

A common practice as of today, in order to include the DVA in banks’ balance sheets is to subtract the DVA from the current value of the risk-free PV of the liabilities [5]. But this way of practice could rather be seen as a bit counterintuitive mainly due to the fact that when for instance the creditworthiness of bank B worsens equivalent to when $\pi_B = s_B$ increases, then the PV of the liabilities drops.

The authors in [5] suggest that the DVA is the PV of the losses that the borrower is obliged to pay should he/she not be a risk-free economic operator rather than a type of reduction in the value of the liabilities subject to the credit risk of the borrower. Some financial institutions consider the DVA as the negative CVA and even though it still keeps it concept of compensation for the counterparty risk, this concept can only be seen as valid for the lender. Because if we move over to the borrower’s point of view, the negative of the CVA (i.e. the DVA), changes its nature from that of a compensation for a counterparty risk to that of a cost instead. However, the deduction from the liabilities can be vindicated by the compensation nature of the DVA, which cannot be supported since stockholders tend to not consider their bank’s default in the investments’ valuation process (most likely stockholders of a bank aim at making profits out of their investments and thus value projects on the base of the profits, costs and expected profit margin to be shared at the end of the bank’s activity) [5]. Now, if this statement holds, thus seeing the DVA as a cost, then it has to be moved
into the balance sheet to the reduction of the value of the net equity, i.e. the
difference between the fair market value of the bank’s assets and its liabilities,
rather than what was suggested earlier (risk-free PV of the debt). Therefore,
the balance sheet should be given by,

\[ BA_1 = E \\
BA_2 = Ke^{(-r+s_B)T} \]

\[ L = Ke^{-rT} \]

\[ E \]

\[ -DVA_B(0) = -Ke^{-rT}(1 - e^{s_BT}) = -Ke^{-rT}(1 - e^{-s_BT}) \]

similar to before, the RHS and the LHS cancel out one another and thus balancing,
but in comparison to our first balance sheet, this one does generate P&L
at time 0 in terms of a loss equal to the DVA. Consequently, we need show that
the DVA is actually the PV of the costs born by the borrower B upon maturity
of the loan transaction.

Now, if we instead examines what happens at maturity time T, obviously both
\( BA_1 \) and \( BA_2 \) have earn the risk-free rate embedded in this loan contract, which
is also the case for the risk-free value of the debt borne by the borrower and
therefore it follows that the \( DVA(T) \) declines to 0, meaning that there is no
longer a default risk involved (the likelihood of that the borrower will not be
able to fulfil his obligations towards the lender is zero) since the debt has expired
at time T. The balance sheet given at time T is hence given by,

\[ BA_1 = Ee^{rT} \]

\[ BA_2 = Ke^{s_BT} \]

\[ L = K \]

\[ E \]

the two sides do not cancel out one another and thus the balance sheet is not
balancing since no P&L is generated at maturity. Although we are missing
P&L, there is an income source \( (P_1) \) in terms of interest which stems from the
bank account \( BA_1 \) and losses \( (L_2) \) in terms of the funding spread that is the
difference between how much the bank account \( BA_2 \) is worth at maturity and
how much the borrower has paid back on the loan contract which is given by,

\[(Ee^{rT} - E) + (Ke^{sT} - K) = P_1 - L_2\]

if we add these profits and losses to the equity E and further consider the outflow of cash to be paid back on the loan (now we subtract K from \(BA_2\) on the LHS instead of having \(K = L\) on the RHS), then the balance is once again balancing and the two sides cancel out each other,

### Table 5.4: Balance sheet when outflow of cash is to be paid back on the loan

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>(BA_1 = Ee^{rT})</td>
<td>(L = 0)</td>
</tr>
<tr>
<td>(BA_2 = Ke^{sT} - K)</td>
<td>(E)</td>
</tr>
<tr>
<td></td>
<td>(P_1 - L_2)</td>
</tr>
</tbody>
</table>

hence the debt is equal to zero and the borrower’s loan transaction is upon completion at the lender’s disposal. Moreover, if would value its profitability by also including the hurdle rate then we would obtain,

\[(Ee^{rT} - E) + (Ke^{sT} - K) - E(e^{rT} - 1) = Ke^{sT} - K = L_2\]

thus we can conclude that as a whole, this loan contract has generated a loss in terms of \(L_2\) corresponding to the funding spread for the borrower on the amount K. When examining the last balance sheet it is clear to us that the authors were right in considering the DVA as the value of the losses suffered at maturity of the loan contract rather than as a reduction of the risk-free PV of the loan itself.

First and foremost, we can conclude that whenever the DVA is seen as the PV of a cost, then it simply corresponds to the reduction of the equity E that can be agreed upon at the launch of the loan contract which enables the fact that the DVA can be included in a marked to market balance sheet as a reduction of the equity E without any perverse effects should the creditworthiness of the borrower worsens. This is mainly due to the fact that if such scenario would take place, the PV of the costs would then increase whilst the net equity is diminishing accordingly. Therefore, the DVA should not be neglected into the balance sheet under any circumstances whatsoever.

Even though we have stipulated the importance of incorporating DVA on the balance sheet from an accounting point of view, still there are just a few institutions that record this particular adjustment. Personally, we believe that there are a number of reasons for not doing so; the counterintuitive impact of recording a gain in P&L as their own creditworthiness becomes progressively worse;
entities would most likely not be able to gain an economic benefit from its own credit gain upon close out of a derivative liability; the likelihood of an increase in the systematic risk that may arise from hedging DVA and most important, it is not mandatory to implement such an adjustment according to the accounting standards as of yet.
Chapter 6

Introduction to Funding Cost

Up to when the credit crisis took place the vast financial institutions borrowed money at the LIBOR rate, where the spread primarily between LIBORs with different tenor, i.e. the amount of time left for the repayment of a loan or contract were quite small. At that time and that environment, the LIBOR was treated as the risk-free rate due to the fact that this was the actual rate at which the most financial institutions too big to fail funded their business. The notation too big to fail stems from the fact that in particular banks are so large and so interconnected that their failure would be disastrous for the world’s economy, and they therefore must be supported by government in the event of a potential failure. In the aftermath of the Lehman era, this was no longer valid. It was also noted that the spread between the LIBOR and the overnight rate (OIS) reached levels of approximately 365 basis points in 2008, which could be compared to the 10 basis points before things started to escalate.

In order the settle the price of a derivative as of today we need to discount its corresponding future’s cash flows, the issue still remains though of which risk-free rate we must use for discounting now when we must not use the LIBOR rate as the risk-free rate.

6.1 Pricing of the Derivative subject to the funding cost

Here we combine two pricing processes, one for the stock and one for the derivative including their corresponding dynamics. We think of the funding cost as a "negative dividend", which is the difference between the risk-free interest rate and the rate of return. This "negative dividend" can be potentially different for the derivative we are considering and for the underlying asset, see [10]. Note that the risk neutral measure is a probability measure such that each share price is exactly equal to the discounted expectation of the share price under this measure. This is commonly used in the pricing of financial derivatives due to the fundamental theorem of asset pricing, which implies that in a complete market
A derivative’s price is the discounted expected value of the future pay-off under the unique risk-neutral measure.

The risk-free interest rate is the theoretical rate of return of an investment with no risk of financial loss. The risk-free rate represents the interest that an investor would expect from an absolutely risk-free investment over a given period of time. However, a risk-free rate can only be approximated in practice because even the safest investments carry a very small amount of risk. For instance, the interest rate on a three-month U.S Treasury bill can be used as the risk-free rate because of its low risk of investment. The risk-free interest rate can also be associated with the LIBOR rate, which is the benchmark for short-term interest rate that banks charge each other in the London interbank market. In a similar way the EURIBOR, Federal Funds Rate or national bond guaranteed by the government in stable economy market would be applicable.

The price process for the stock is defined as $dS_t$ and $dC_S$ is the negative dividend process for the stock. Therefore,

$$dS_t = S_t \mu_S dt + S_t \sigma_S d\bar{W}_t$$

$$dC_S = S_t (r - r_s) dt$$

where $\mu_S$ is the expected return on the stock, $\sigma_S$ is the stock’s volatility and $d\bar{W}_t$ is a Wiener process [10]. The $(r - r_s)$ represent the negative dividend rate for funding where $r_s$ is the funding rate of the stock.

The Derivative written on S with price process $\pi(t) = F(t, S(t))$, expiry at $T > 0$ and pay-off $\pi(T) = \Phi(T)$.

The price process for the derivative is defined as $dF$ and $dC_F$ is the negative dividend process for the derivative. Therefore,

$$dF = F \mu_F dt + F \sigma_F d\bar{W}$$

$$dC_F = F(r - r_d) dt$$

where $\mu_F$ is the expected return on the derivative, $\sigma_F$ is the derivative’s volatility. The $(r - r_d)$ represent the negative dividend rate for funding where $r_d$ is the funding rate for the derivative.

An example of funding cost could be when a bank search for new capital in order to reinvest it on other investments (e.g. stocks and derivatives) that can cover the interest rate of holding a debt in the event of issuing a bond. So the instrument should earn at least the interest rate of the debt rather than the risk free interest rate, $r$.

According to [1] we recall the following scheme when determining the arbitrage free price for a $T$-claim of the form $\Phi(S_T)$.
• Assume that the pricing function is of the form $F(t, S_t)$.
• Consider $\mu_S, \sigma_S, \Phi, F, r_s, r_d$ and $r$ are exogenously given.
• Use a self financed portfolio based on the derivative instrument and the underlying stock.
• Form a self-financed portfolio whose value process $V$ has a stochastic differential without any driving Wiener process, i.e. it is of the form

$$dV(t) = V(t)k(t)dt.$$  

• In the absence of arbitrage we must have $k = r$.
• The equation has a unique solution, thus giving us the unique pricing formula for the derivative, which is consistent with absence of arbitrage.

We combine the processes

$$dG_S = S_t(\mu_S + r - r_s)dt + \sigma_S S_t d\bar{W}$$
$$dG_F = F(\mu_F + r - r_d)dt + \sigma_F F d\bar{W}$$

Moreover, we build a portfolio from $G_S$ and $G_F$ so it is risk-free. Let $V(t)$ be the value of the portfolio and $u_S, u_F$ the portfolio coefficients.

$$dV = V \left\{ u_S \frac{dG_S}{S} + u_F \frac{dG_F}{F} \right\}$$

From Ito’s lemma we have:

$$\mu_F = \frac{1}{F} \left\{ \frac{\partial F}{\partial t} + \mu_S S \frac{\partial F}{\partial S} + \frac{1}{2} \sigma_S^2 S^2 \frac{\partial^2 F}{\partial S^2} \right\}$$
$$\sigma_F = \frac{1}{F} \sigma_S S \frac{\partial F}{\partial S}.$$  

We obtain

$$dV = V \left\{ u_S(\mu_S + r - r_s) + u_F(\mu_F + r - r_d) \right\} dt + \left\{ u_S \sigma_S + u_F \sigma_F \right\} d\bar{W}$$

we determine the portfolio weights by excluding the Wiener process in order to eliminate the risk and solve for $u_S$ and $u_F$ as the solution to the system

$$u_S \sigma + u_F \alpha_F = 0,$$
$$u_S + u_F = 1.$$  

The system has the following solution

$$u_S = \frac{\sigma_F}{\sigma_F - \sigma_S},$$
$$u_F = \frac{-\sigma_S}{\sigma_F - \sigma_S}.$$  

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Eventually, if we insert the values for $u_S$, $u_F$, $\mu_F$ and $\sigma_F$ into the equation $dV$ we end up with the following pricing equation:

\[
\begin{cases}
\frac{\partial F}{\partial t} + r_s S \frac{\partial F}{\partial s} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 F}{\partial s^2} - r_d F = 0, \\
F(T, s) = \Phi(s).
\end{cases}
\]

By the Feynman-Kac formula the pricing function of the derivative is given by

\[
F(t, s) = e^{-r_d(T-t)} E_{t,s} \Phi(X_T)
\]

where the dynamics of $X$ are given by

\[
dX_t = r_s X_t dt + \sigma X_t dW_t.
\]

where $X_0 = S_0$

For instance, see also [10], the funding adjusted price of a European call option for the stock with strike price $K$ and time to maturity $T$ is given by the formula $\Pi(t) = F(t, S(t))$, where

\[
F(t, s) = S_0 N(BA_1)e^{(r_s - r_d)T} - Ke^{-r_d T} N(BA_2)
\]

Here $N$ is the cumulative distribution function for the $N[0,1]$ distribution and

\[
BA_1 = \frac{\ln(S_0/K) + (r_s + \sigma^2/2)T}{\sigma \sqrt{T}}
\]

\[
BA_2 = BA_1 - \sigma \sqrt{T}
\]

In a similar way we can derive the price of a European put option on the stock with strike price $K$ and time to maturity $T$ accordingly

\[
F(t, s) = Ke^{-r_d T} N(-BA_2) - S_0 N(-BA_1)e^{(r_s - r_d)T}
\]
6.2 Pricing of non-collateralized derivatives

The existence of FVA, as we have seen, highly correlated to the recent financial crisis. Prior to the crisis, the bulge bracket banks used to disregard collateral and funding costs mostly since the funding for posting collateral can typically be tangible through a combination of low cost funding options but also due to this low cost was in beforehand offset by the interest rate (LIBOR) paid on collateral by the receiving party.

In the aftermath of the crisis, we cannot hide the fact that it has had an enormous impact in the role of funding in the derivatives markets as the funding became costly, and the interest rate paid on collateral no longer offsets the increasing cost [8]. The arising risk of credit lines being pulled is also impending as soon as the firm is perceived to going bankrupt. Therefore, the importance of value funding needs have become a vital part of the risk management strategy of financial institutions.

In this section we will treat the mechanism of pricing non-collateral derivatives with a different approach as in [6]. Let $V_t$ denote the price of the derivative written on an underlying asset $S_t$, which pays continuous dividend yield $\delta_t$.

The dynamics of the underlying asset under the physical measure is given by

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dW_t \tag{6.1}$$

Moreover, in order to price a non collateralized derivative we have to introduce some assumptions.

- $H_t := H(t, S_t)$: Derivative written on the underlying asset $S_t$ in accordance to avoid and to hedge against any substantial price movements of the derivative $V_t$ subject to its underlying.

- $f_t$: The rate that corresponds to whenever one is in need of capital and you borrow money in the market which comes with an additional interest rate.

- $i_t$: When borrowers need to borrow money in the market, you can charge them a rate at which you can lend it to them.

- $c_t$: Counterparties in a non-collateralized transaction most likely post collateral to one another, this rate is equivalent to the rate of the collateral paid on the posted collateral in such derivative transaction.

- $r_t$: Often one uses repo agreements for funding stocks, i.e. a party sells a government security to an investor usually on an overnight basis and pays them back in the future, which entitles him to an additional interest over and above the value of the securities. This repo rate is designated for the underlying stock $S_t$. 
Moreover, let us set our sights on the road map for the cash flows that would occur when considering an infinitesimal time interval, seen over \( t \) and \( t + dt \) in the event of funding and hedging any derivative. We provide such an overview in order to highlight the life-cycle of a non-collateralized derivative transaction strategy.

- From the Investor’s point of view, at time \( t \), this is where the actual derivative transaction takes place with three different parties entitled to different roles. First of there is a Hedger, who ought to enter into a derivative contract in the market, priced at \( V_t \). On the side of the transaction we have an Investor which is willing the undertake this derivative at his disposal. Furthermore, depending on the outcome of the price of the derivative \( V_t \), the Hedger will either fund the derivative should \( V_t > 0 \) or invest if \( V_t < 0 \) applicable which is quite logical to see since if the derivative face a positive value, it is in need of funding. Behind the scene of the transaction there is an interbank who is entitled as the Funding Counterparty. This is due since the Hedger wants to be risk-neutral in such transaction whereas the Interbank posses the inherent risk to the expense of obtaining posted collateral from the Hedger.

At maturity, the Investor will eventually pay back on the loan plus some additional interest arising, reflecting the time when entered into the transaction until it hits maturity. Subsequently, when thinking of it, in practice this could cause some arbitrage opportunities which is in line with the arguments of Hull & White [10]. This is a fact since the Hedger in question will be entitled to pay an amount corresponding to the rate at which he borrowed money \( f_t \) in a relative low rate from the bank’s treasury desk when in need of posting collateral and earn \( i_t \) for the money at which he lent for the same cause.

In practice, the arising cost will most likely passed along to the traders on the FVA desk, who reimburse it from investors through FVA. Therefore, the process of passing FVA on to investors will be acceptable as long as the bank’s funding cost remains at a relative low rate in comparison to others.

- As follows the Hedger will at time \( t \) either pay or receive the price at which the derivative is priced depending on the outcome of its value and further receive the value of the derivative at time upon the maturity date corresponding to \( t + dt \).

- It is optimal for the Hedger when entering such a derivative transaction to hedge against variations in the price of the derivative \( V_t \), e.g. to avoid ending up with a worthless derivative or an outcome which the Hedger is disadvantages by as the underlying stock \( S_t \) giving rise to some unexpected fluctuations, as such the Hedger agrees to enter into a cash-collateralized derivative denoted by \( H_t \) with corresponding notional amount \( \alpha_t \). This
is an orthodox approach of how to proceed in the environment of OTC-derivative in the aftermath of the crisis. We try to illustrate the non-collateralized derivative strategy for a financial institution’s P&L.

Figure 6.1: Strategy P&L for a non-collateralized derivative. Source: [6].

We are convinced that in the near future FVA will move disadvantageous financial institutions out of the range of pricing it consistent, further cancel out their profits and eventually a risk of pushing them out of business. This is mainly due to the fact that when not considering FVA, the risky value will solely concern the counterparty of the transaction and the issuer, whenever DVA is applicable, and the price stays consistent. Now, if we have a look at the replication strategy, it is quite obvious that if the funding costs for a particular replication strategy as in our case are included whereas the funding spread is non-zero then in most cases the client that was willing to buy this derivative, would almost surely purchase it from a buyer with a smaller FVA, i.e. a lower funding cost. As such the financial institution with the lower funding rate will stay more competitive on trades when in need of funding and thus "the law of one price" will no longer hold.

By denoting the gain of borrowing at one rate \( f_t \) and earn on lending at another \( i_t \), by \( \psi_t \) at time \( t \), the impact of all interfering cash flows in the sense of funding and/or hedging, we get the formula

\[
d\psi_t = -dV_t + \alpha_t \left[dH_t - H_t c_t dt\right] + \left[V_t^- f_t + V_t^+ i_t\right] dt \quad (6.2)
\]

In order to hedge from variations in derivative prices, i.e. to eliminate uncertainties, we choose the notional amount \( \alpha_t \) to be,

\[
\alpha_t = \frac{\partial V_t}{\partial H_t} \quad (6.3)
\]
furthermore, by applying Ito to both \( V_t \) and \( H_t \) in equation (6.2) we obtain the following dynamics

\[
dV_t = \frac{\partial V_t}{\partial t} dt + \frac{\partial V_t}{\partial S_t} dS_t + \frac{1}{2} \frac{\partial^2 V_t}{\partial S_t^2} \sigma_t^2 S_t^2 dt
\]

\[
dH_t = \frac{\partial H_t}{\partial t} dt + \frac{\partial H_t}{\partial S_t} dS_t + \frac{1}{2} \frac{\partial^2 H_t}{\partial S_t^2} \sigma_t^2 S_t^2 dt
\]

(6.4)

which yields

\[
\frac{\partial V_t}{\partial t} + \frac{1}{2} \frac{\partial^2 V_t}{\partial S_t^2} \sigma_t^2 S_t^2 - \alpha_t \left[ \frac{\partial H_t}{\partial t} + \frac{1}{2} \frac{\partial^2 H_t}{\partial S_t^2} \sigma_t^2 S_t^2 - H_t c_t \right] = V_t^- f_t + V_t^+ i_t
\]

(6.5)

where \[ \left[ \frac{\partial H_t}{\partial t} + \frac{1}{2} \frac{\partial^2 H_t}{\partial S_t^2} \sigma_t^2 S_t^2 - H_t c_t \right] = -(r_t - \delta_t)S_t \frac{\partial H_t}{\partial S_t} \]

and therefore equation (6.5) reads as,

\[
\frac{\partial V_t}{\partial t} + (r_t - \delta_t)S_t \frac{\partial V_t}{\partial S_t} + \frac{1}{2} \frac{\partial^2 V_t}{\partial S_t^2} \sigma_t^2 S_t^2 = V_t^- f_t + V_t^+ i_t
\]

(6.6)

Moreover, let the spreads over which the Hedger is posting collateral to the Funding Counterparty denote the funding spreads

\[
s_t^f = f_t - c_t, \quad s_t^i = i_t - c_t
\]

We can thus express equation (6.6) as,

\[
\frac{\partial V_t}{\partial t} + (r_t - \delta_t)S_t \frac{\partial V_t}{\partial S_t} + \frac{1}{2} \frac{\partial^2 V_t}{\partial S_t^2} \sigma_t^2 S_t^2 = c_t V_t + V_t^- s_t^f + V_t^+ s_t^i
\]

s.t \( V(T) = \psi \)

(6.7)

We have thus found a way of expressing the price of derivative \( V_t \) by the Feynman-Kac given by

\[
V_t = E_t^Q \left[ e^{-f_t^T c_{t^d} \psi T} - \int_t^T E_t^{Q_k} \left[ e^{f_u^T c_{u^d} s_u^k V_u^-} - \int_u^T E_t^{Q_{u^d}} \left[ e^{f_u^T c_{u^d} s_u^k V_u^+} \right] du \right] \right]
\]

(6.8)
Hence,

- Perfect Collateral Price = $E_t^Q \left[ e^{-\int_t^T r_s ds} \psi_T \right]$

- Funding Cost = $\int_t^T E_t^Q \left[ e^{\int_{t_u}^{t} r_s ds} s_u^i V_u^- \right] du$

- Funding Benefit = $\int_t^T E_t^Q \left[ e^{\int_{t_u}^{t} r_s ds} s_u^i V_u^+ \right] du$

where once again, $Q$ is the risk-neutral measure under which cash-collateralized deals grow at the pace of the collateral rate while $S_t$ grows at the repo rate. The perfect collateral price can be defined as the price given by collateralization in continuous time, with continuous mark-to-market of the investor’s portfolio should there be any events of default coupled with collateral account inclusive of margining fees/costs at any time during the lifetime of such derivative transaction.

We are now ready to make the following statements about FVA and Funding Cost:

We have throughout this section addressed the importance of incorporating FVA, but on the other hand what is then the opportunity cost of not considering FVA? It is now clear to us that funding cost cannot be ignored should a trade have positive P&L coupled with a high potential exposure since, if we would face disadvantageous market moves on the trade, which will put us out of the money, an enormous funding will be required forthwith to post collateral to the counterparty when in a situation where CSA is present.

The FVA can either be seen as a benefit or a cost from the firm’s point of view which arises from the fact that the same firm needs to raise money in order to fund a derivative until maturity.

Consequently, the cost arising by the need of funding will be considered as a benefit should the derivative generate positive cash flows which enables the Hedger to reduce our funding needs.

When pricing non-collateralized derivatives correctly, the Hedger of the transaction has to, without any loss of generality, discount the derivatives at his funding rate under the assumption that we are in the environment of having the same investment and funding rate.

We can hereby conclude that the FVA plays a major and a vital role in providing monetary incentive for the XVA desks to use less funding. This in turn can be tangible for instance by charging a higher rate in the event of borrowing from the treasury and afterwards paying for the posting money back to where the funding is originated initially, i.e. to the funding desk. This is why we think the FVA appears in most cases, represented by the sum of both the FCA (funding cost) and the FBA (funding benefit).
Chapter 7

First-Hitting Time

The prediction of default of a firm has become a broad topic for most financial institutions, which is not only due to the rapid growth of the credit derivatives and firms debt products. The default of a firm is often associated with the bankruptcy of the firm. But this is just one among several credit events though. The event of default can be predicted but only with a certain degree of probability. Even though it is desirable for most firms to have a probability of bankruptcy equal to zero, this can never be tangible since there will always exist a small portion of risk that a firm will declare bankruptcy during the lifetime of a contract. We are particularly interested in the credit event where the firm will fail to fulfil its obligation to honour the repayments of the debt.

In this study we are interested in how DVA can be embedded in the context of Probability of Default. As mentioned before, a higher PD would imply a decrease in the credit rating and hence an increase of the DVA. This could be compared in similar way to when debt of a firm increases and assets decrease. One could say that our PD represents the probability that the other party would go default, i.e. when the liabilities are greater than the assets and therefore strongly correlated with the DVA. One should keep in mind that DVA is not something one adds when a PD event occur, but instead something that two counterparties undertake in an OTC transaction. Therefore the PD can be interpreted as a part of the DVA.
7.1 Probability of Default - 1 dimensional

First-hitting-time models can be applied to the lifetime of a financial institution. Eventually, when the process reaches a threshold state corresponding to when the assets of a firm drops below the liabilities for the first time, the firm declares bankruptcy [12]. Merton’s model was developed in the 1970s and aims towards evaluating and examining the credit risk of a firm’s debt and has been a robust key determination in deciding company’s ability to recover from its debt and whether they are able to meet the standards and obligations subject to its financials but most and foremost to gauge the overall possibility to credit default [8].

Black and Cox extended Merton’s model in [2] by assuming that default could occur before maturity. Furthermore, default can happen when the level of asset value hits a lower boundary which is modelled as a deterministic function of time. The original approach of Black and Cox model where mainly focused on the possibility of a borrower violating its safety covenants that would cause a default before the time of maturity. First-passage time models estimates the probability and timing of the first time that a counterparty’s assets pass through the default point. Moreover, we are interested in the probability of default corresponding to when a firm’s assets are less than the liabilities, i.e. \( A_t < L_t \) under the time horizon when \( T > 0 \).

Now let \( \tau \) represent the random time when the default occurs.

Let the assets be denoted as \( A_t \) and the liabilities as \( L_t \), where \( A_0 > L_0 \) at inception, we define the ratio process \( \{ C_t \} _{0 \leq t \leq T} \)

\[
C_t := \left(\frac{A_t}{L_t}\right), 0 \leq t \leq T
\]

The first passage time or hitting time for \( A_t \) to hit \( L_t \) is

\[
\tau = \inf\{0 \leq t \leq T : C_t \leq 1\}
\]

Let us consider the firm’s assets and liabilities with the following dynamics.

\[
dA_t = \mu_A A_t dt + \sigma_A A_t dW_t
\]

\[
dL_t = \mu_L L_t dt + \sigma_L L_t dW_t
\]

where \( W_t \) is a 1-dimensional standard Brownian motion, \( \mu_A, \mu_L \in \mathbb{R} \) are constants, \( \sigma_A, \sigma_L \in \mathbb{R} \) are constant vectors and \( A_0 > L_0 > 0 \) whereas the SDEs posses unique solutions [1]. The Geometric Brownian Motions (GBM) for the assets and the liabilities are thus given by

\[
A_t = A_0 \exp\left(\left(\mu_A - \frac{\sigma_A^2}{2}\right)t + \sigma_A W_t\right)
\]

\[
L_t = L_0 \exp\left(\left(\mu_L - \frac{\sigma_L^2}{2}\right)t + \sigma_L W_t\right).
\]
Now if we consider the ratio
\[ C_t = \frac{A_t}{L_t} \]
we are proving that division of one GBM by another GBM gives again a GBM accordingly
\[ \Rightarrow C_t = \frac{A_0}{L_0} \exp \left[ \left( \mu_A - \mu_L - \frac{\sigma_A^2}{2} + \frac{\sigma_L^2}{2} \right) t + (\sigma_A - \sigma_L)W_t \right] \]
It is now possible to find a drift coefficient $\mu_C$ and a volatility coefficient $\sigma_C$ compatible with the equation of a GBM.

We want to obtain the following GBM
\[ C_t = C_0 \exp[(\mu_C - \frac{\sigma_C^2}{2})t + \sigma_C W_t] \]
with $\sigma_C$ we do not really have a choice - it has to be $\sigma_C = \sigma_A - \sigma_L$.

Subsequently, this in turn yield the following formulas
\[
\begin{align*}
\mu_A - \mu_L - \frac{\sigma_A^2}{2} - \frac{\sigma_L^2}{2} & = \mu_C - \frac{\sigma_C^2}{2} \\
\mu_A - \mu_L - \frac{\sigma_A^2}{2} + \frac{\sigma_L^2}{2} & = \mu_C - \frac{1}{2} (\sigma_A^2 - \sigma_L^2)^2
\end{align*}
\]
whereas, the right hand side of the equation describes what we would like $\mu_C$ to solve, and hence leads to a viable formula for $\mu_C$.

\[ \mu_C = \mu_A - \mu_L + \frac{\sigma_A^2}{2} - \sigma_A \sigma_B \]
where the dynamic is given by
\[ dC_t = \mu_C C_t dt + \sigma_C C_t dW. \]
Furthermore, in order to get the first hitting time $\tau$ when $A_t \leq L_t$ one can simulate the probability density function numerically, more precisely we use Monte-Carlo simulation to find the probability density function (PDF) of $\tau$. The idea is to simulate the dynamics of our Assets and Liabilities and find the first time when $C_t < 1$ and simulate this $N = 5000$ times. Moreover, we plot a histogram which will reflect a sort of a density function for the first hitting time. This coupled with a function called allfitdist downloaded from Mathworks\(^1\) finds a pretty good fit of the density given some data.

\(^1\)http://www.mathworks.com/matlabcentral/fileexchange/34943-fit-all-valid-parametric-probability-distributions-to-data/content/allfitdist.m
Figure 7.1: The graph above illustrates the first time when the Liabilities are greater than the Assets.

Figure 7.2: The figure shows the simulated dynamics of the Assets and Liabilities and the first hitting time for when the ratio for the two GBMs, when $C_t < 1$. 
Figure 7.3: We plot a histogram that represents the density function for the first hitting time.

Figure 7.4: Here we have incorporated the function \texttt{allfitdist} which basically finds a pretty good fit of the density to our data. According to this function, the Weibull distribution is the best fit.


7.2 Probability of Default - Multidimensional

So far we have determined the PD only when considering a 1-dimensional process. Here we will conduct the PD in the multidimensional case for GBM. However, it is of great importance to embrace that the usefulness of simulation of \( A_t \) and \( L_t \) comes from the fact that these could be any types of processes and not only GBM or any similar. This is due to the fact that as of today there is no commonly agreed model for PD since it is extremely hard to estimate PD and therefore any model would be considered as good as long as it provides results.

Now in order to look at the multidimensional case we make sure to cover the case of both assets and liabilities being possibly correlated and driven by \( n \)-dimensional Brownian motion. The dynamics of the assets and the liabilities are given by

\[
dA_t = \mu_A A_t dt + \sum_{i=1}^{n} A_t \sigma^i_A dW^i_t \\
\]

\[
L_t = \mu_L L_t dt + \sum_{i=1}^{n} L_t \sigma^i_L dW^i_t \\
\]

Furthermore, by Itô’s formula

\[
d\log(A_t) = \mu_A dt + \frac{\sum_{i=1}^{n} \sigma^i_A dW^i_t}{A_t} - \frac{\sum_{i=1}^{n} \sigma^i_A^2 dt}{2A_t} \\
= \mu_A dt + \sum_{i=1}^{n} \sigma^i_A dW^i_t - \frac{\sum_{i=1}^{n} \sigma^i_A^2 dt}{2} \\
= \left( \mu_A - \frac{\sum_{i=1}^{n} \sigma^i_A^2}{2} \right) dt + \sum_{i=1}^{n} \sigma^i_A dW^i_t \\
\]

Analogously for the liabilities,

\[
d\log(L_t) = \left( \mu_L - \frac{\sum_{i=1}^{n} \sigma^i_L^2}{2} \right) dt + \sum_{i=1}^{n} \sigma^i_L dW^i_t \\
\]

Hence,

\[
A_t = A_0 \exp \left[ \left( \mu_A - \frac{\sum_{i=1}^{n} \sigma^i_A^2}{2} \right) t + \sum_{i=1}^{n} \sigma^i_A W^i_t \right] \\
L_t = L_0 \exp \left[ \left( \mu_L - \frac{\sum_{i=1}^{n} \sigma^i_L^2}{2} \right) t + \sum_{i=1}^{n} \sigma^i_L W^i_t \right] \\
\]

Now if we once again consider the ratio,

\[
C_t = \frac{A_t}{L_t} \\
\]

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we are proving that division of one GBM by another GBM gives again a GBM but this time when both assets and liabilities being possibly correlated and driven by n-dimensional Brownian motion

\[ C_t = \frac{A_0}{L_0} \exp \left( \left( \mu_A - \mu_L - \sum_{i=1}^{n} \frac{\sigma^2_{A}}{2} + \sum_{i=1}^{n} \frac{\sigma^2_{L}}{2} \right) t + \sum_{i=1}^{n} (\sigma_A - \sigma_L) W_t^i \right) \]

which follows from the fact that,

\[ \text{Cov} \left( \sum_{i=1}^{n} \sigma^i_A dW^i_t, \sum_{i=1}^{n} \sigma^i_L dW^i_t \right) = \sum_{i=1}^{n} \sigma^i_A \sigma^i_L dt, \sigma^i_A, \sigma^i_L > 0 \]

On the next page we simulate the stopping for the corresponding process that we just derived above.
Figure 7.5: The graph above illustrates the first time when the Liabilities are greater than the Assets for the n-dimensional case.

Figure 7.6: The first hitting time for the two GBMs, when $C_t < 1$. 
Figure 7.7: The plot illustrates a histogram which represents the density function for the first hitting time when both assets and liabilities being possibly correlated and driven by n-dimensional Brownian motion.

Figure 7.8: Here we apply the function `allfitdist` to our data but now for the n-dimensional case.
Chapter 8

Funding Strategies for XVAs

In this chapter we will derive a slightly different pricing technique for DVA and FVA from what we have seen earlier in this paper. We address this additional layer of complexity to pricing derivatives with respect to DVA and FVA by describing the authors Christoph Burgard and Mats Kjaer’s approach based on the article [3]. We will look at funding strategies in terms of when an issuer either holds or issues his own bonds. Such a strategy enables the issuer to hedge out a great share of its cash flows upon the issuer’s default. It is however important to bear in mind that, the value of derivative instruments does not solely depend on who is funding them, but how they are funded since the funding costs may for some reason result in shortfalls or windfalls to bond issuer should their firm go bankrupt and hence the derived adjustment such as DVA and FVA resulting from this depend on the funding strategy employed.

8.1 Set-up prior to semi-replication and pricing

Let us consider a simple derivative contract, with collateral possibly being posted, between an issuer B and a counterparty C. Furthermore, we denote the value of the derivative as $V$, which covers the probability of default of both the issuer and the counterparty coupled with any net funding costs that the issuer may run up against prior to his default. The use of a semi-replication strategy is as mentioned above, intended to enable the issuer B to deploy a strategy in order to hedge out market dependent factors and counterparty risk in terms of default which may not always provide an adequate hedge in the event of B’s own default. In this strategy we use a counterparty zero-coupon zero-recovery bond $P_C$, i.e. no coupon payments are made except the repaid face value at the time of maturity whereas no amount will be recovered should the counterparty go bankrupt. Also we have the issuer’s two own bonds denoted $P_1$ and $P_2$ of different seniors with different recoveries $R_1$ and $R_2$, and $S$ in order to hedge out the market factor of the underlying derivative contracts, e.g. an asset. The reader should note that, the description of a bond as "senior"
does not by itself mean that it is in any way secured and thus investors should not draw any conclusions as to the ranking of a bond solely on the basis of its description as "senior". The majority of bonds that are issued are senior unsecured obligations of the issuer.

Moreover, the dynamics for such instruments are given by,

\[ dS = \mu S dt + \sigma S dW \]  
(8.1)

\[ dP_C = r_C P_C^- dt - P_C^- dJ_C \]  
(8.2)

\[ dP_i = r_i P_i^- dt - (1 - R_i) P_i^- dJ_B \]  
(8.3)

where \( J_B \) and \( J_C \) indicate the default for the issuer and the counterparty, respectively, and \( P^-_{i/C} = P^-_{i/C}(t) \) represent the bond prices prior to any possible default. Furthermore, let \( P_1 \) designate the bond with the lower recovery rate, i.e. \( R_1 < R_2 \) and consequently \( r_1 > r_2 \). If we would end up in a case of zero basis points between bonds of different seniors it might be trivial to show that,

\[ r_i - r = (1 - R_i) \lambda_B, \]  
(8.4)

where \( r \) is the risk-free rate of such as case and \( \lambda_B \) represents the spread of a potentially zero-recovery zero-coupon bond of the issuer [3].

Moreover, let \( \hat{V}(t, S, J_B, J_C) \) denote the total value of the derivative instrument from the issuer’s point of view, whereas when considering general boundary conditions upon default of the counterparty or the issuer are given by,

\[ \hat{V}(t, S, 1, 0) = g_B(M_B, X) \]  
should B default first,

\[ \hat{V}(t, S, 0, 1) = g_C(M_C, X) \]  
should C default first,

where \( M_B \) and \( M_C \) are the amounts equivalent to when unwinding a transactions, i.e. resulting in close-out amounts and \( X \) represents the collateral posted between the parties. In [3] they like to refer these boundaries as "regular", if \( M_B = M_C = V \), where \( V \) is simply the price of the derivative when using the classical Black-Scholes formula, i.e. neglecting factors such as any involved counterparty, the likelihood of own default and funding costs. For instance, the regular bilateral boundary conditions with collateral are defined as,

\[ g_B = (V - X)^+ + R_B(V - X)^- + X g_C = R_C(V - X)^+ + (V - X)^- + X. \]  
(8.5)

### 8.2 Semi-replication and pricing

In order to conduct a semi-replication strategy we set up the following hedging portfolio \( \Pi \),

\[ \Pi = \delta(t) S(t) + \alpha_1(t) P_1(t) + \alpha_2(t) P_2(t) + \alpha_C(t) P_C(t) + \beta_S(t) + \beta_C(t) - X(t), \]  
(8.6)
with \( \delta(t) \) units of \( S \), \( \alpha_1(t) \), \( \alpha_2(t) \) and \( \alpha_C(t) \) represent the own and counterparty bonds, respectively, cash accounts \( \beta_S(t) \) and \( \beta_C(t) \) and the collateral account \( X(t) \). The cash accounts \( \beta_S \) and \( \beta_C \) are used to fund \( S \) and \( P_C \), therefore \( \alpha_CP_C + \beta_C = 0 \) and \( \delta S + \beta_S = 0 \), which assumed to pay net rates of \( (q_S - \gamma_S) \) and \( (q_C - \gamma_C) \), where \( q \) is the funding rate and \( \gamma \) is any potential dividend income [3]. Since the hedging position can either be collateralized or repo-ed, \( q_S \) and \( q_C \) may also be seen as collateral or repo-rates. Once again \( X \) is the collateral posted subject such a derivative transaction, which further pay the rate \( r_X \). Consequently, whenever \( X \) is positive, the counterparty has posted the amount \( X \) in collateral to the issuer.

We would like to design the semi-replication strategy in such way that \( \hat{V} + \Pi = 0 \). Moreover, the issuer’s bond positions, which we here denote as \( \alpha_1P_1 \) and \( \alpha_2P_2 \) are used to fund and/or invest the outstanding cash that is not funded via the collateral, which gives us the following funding constraint,

\[
\hat{V} - X + \alpha_1P_1 + \alpha_2P_2 = 0 \tag{8.7}
\]

By combining the funding constraint above with the use of Itô’s lemma, the defined boundary conditions and the hedging ratios \( \delta \) and \( \alpha_C \), chosen in such proper way that the risks subject to the market and the default of the counterparty are hedged out, we can express the evolution of the total portfolio over \( dt \). We derive the above steps as follows.

Recall the hedging portfolio we set up earlier, i.e. equation (8.6) whereas this time we set up the dynamic of the same hedging portfolio which is given by,

\[
d\bar{\Pi} = \delta(t) dS(t) + \alpha_1(t) dP_1(t) + \alpha_2(t) dP_2(t) + \alpha_C(t) dP_C(t) + d\bar{\beta}_S(t) + d\bar{\beta}_C(t) - dX(t), \tag{8.8}
\]

where \( dS, dP_1, dP_2, \) and \( dP_C \) are given in equations (8.1) to (8.3) and \( d\bar{\beta}_S(t), d\bar{\beta}_C(t) \) and \( dX(t) \) are the changes in the cash and collateral accounts, \( \beta_{S,C} \) and \( X \) when neglecting any potential re-balancing of the two.

Moreover, we draw the assumption that the hedge account \( \beta_S \) is collateralized, with a funding rate designated by \( q_S \) coupled with a dividend yield \( \gamma_S \). In accordance with this, the position for the bond which is held by the counterparty is assumed to be set up through a repo transaction that costs the counterparty a repo rate of \( q_C \). The derivatives collateral account is assumed to cost a collateral rate \( r_X \) [3].

Now by excluding any potential re-balancing of these we would end up with the following increments in our cash and collateral accounts,

\[
d\bar{\beta}_S = \delta(\gamma_S - q_S)Sdt, \tag{8.9}
\]
\[
d\bar{\beta}_C = -\alpha_Cq_CP_Cdt, \tag{8.10}
\]
\[ d\bar{X} = -r_X X dt. \] (8.11)

The default value of the issuer bond position pre- and post any credit event associated with default is given by, \( P \equiv \alpha_1 P_1 + \alpha_2 P_2 \) and \( P_D \equiv \tilde{R}_1 \alpha_1 P_1 + \tilde{R}_2 \alpha_2 P_2 \). Furthermore, if we insert the equations (8.1) to (8.3) and the increments in our cash and collateral accounts, i.e. equations (8.9) to (8.11) into the dynamic of the hedging portfolio \( \Pi \) (8.8) we obtain the following portfolio,

\[ d\bar{\Pi} = (r_1 \alpha_1 P_1 + r_2 \alpha_2 P_2 + \lambda_C \alpha_C P_C + (\gamma - q)\delta S - r_X X) dt + (P_D - P) dJ_B - \alpha_C P_C dJ_C + \delta dS, \] (8.12)

where \( \lambda_C \equiv r_C - q_C \) designates by the spread of the yield of the zero-coupon bond \( P_C \) over its corresponding repo rate and thus is the rate which the counterparty hedging position is financed by upon default.

On the other hand, the way \( d\hat{V} \) of the derivative value evolve through its lifetime can be given by Ito’s lemma for jump diffusions as,

\[ d\hat{V} = \partial_t \hat{V} dt + \partial_S \hat{V} dS + \frac{1}{2} \sigma^2 S^2 \partial_S^2 \hat{V} dt + \Delta \hat{V}_B dJ_B + \Delta \hat{V}_C dJ_C, \] (8.13)

where

\[ \Delta \hat{V}_B = \hat{V}(t, S, 1, 0) - \hat{V}(t, S, 0, 0) = g_B - \hat{V} \] (8.14)

\[ \Delta \hat{V}_C = \hat{V}(t, S, 0, 1) - \hat{V}(t, S, 0, 0) = g_C - \hat{V}. \] (8.15)

Moreover, if we combine the way the derivative portfolio evolve in equation (8.13) with the hedging portfolio in equation (8.12) we get the following,

\[ d\hat{V} + d\bar{\Pi} = (\partial_t \hat{V} + \frac{1}{2} \sigma^2 S^2 \partial_S^2 \hat{V} + r_1 \alpha_1 P_1 + r_2 \alpha_2 P_2 + \lambda_C \alpha_C P_C + (\gamma - q)\delta S - r_X X) dt + (g_B + P_D - X) dJ_B + (\Delta \hat{V}_C - \alpha_C P_C) dJ_C + (\delta + \partial_S \hat{V}) dS, \] (8.16)

worth noting here is that the term in front of \( dJ_B \) follows from the fact that \( \hat{V} + P - X = 0 \) by the funding constraint in equation (8.7). By examining equation (8.16), it can be seen that we can get rid of the stock price and counterparty risks by properly choosing,

\[ \alpha_C P_C = \Delta V_C \] (8.17)

\[ \delta = -\partial_S \hat{V} \] (8.18)

which thus yields,

\[ d\hat{V} + d\bar{\Pi} = (\partial_t \hat{V} + A\hat{V} - r_X X + r_1 \alpha_1 P_1 + r_2 \alpha_2 P_2 + \lambda C \Delta \hat{V}_C) dt + (g_B + P_D - X) dJ_B, \] (8.19)

where \( A \equiv \frac{1}{2} \sigma^2 S^2 \partial_S^2 \hat{V} + (q_S - \gamma_S) S \partial_S \hat{V}. \)
Eventually by using the zero bond basis relation which is given by equation (8.4) and the funding constraint (8.7) together with the definition of $\Delta \tilde{V}_C = g_C - \tilde{V}$, in order to rewrite equation (8.19) and hence we are ready to express the evolution of the total portfolio over time step $dt$ by combining the funding constraint in (8.7) with Ito’s lemma, the boundary conditions and the hedging ratios as,

$$d\tilde{V} + d\Pi = (\partial_t \tilde{V} + A_t \tilde{V} - (r + \lambda_B + \lambda_C)\tilde{V} - s_X X + \lambda_C g_C + \lambda_B g_B - \epsilon_h \lambda_B)dt + \epsilon_h dJ_B,$$

(8.20)

where $s_X \equiv r_X - r \cdot P_D$.

When combining the derivative $\tilde{V}$ and the hedge portfolio $\Pi$, we can draw the conclusion that such as combination will stay risk-free as long as the issuer remains alive. The jump term, $\epsilon_h dJ_B$ reflects that upon the issuer’s default, this term gives rise to a hedge error of size $\epsilon_h$ whilst when considering no default and thus being alive, the issuer will incur a cost or gain of size $-\epsilon_h \lambda_B$ per unit of time whereas it sign and size depends on the post-value of the own bond portfolio and hence the funding strategy being employed. Moreover, the hedge error could either be a windfall or shortfall.  \[1\]

Now, if we would assume that the issuer would like to approach a similar strategy as of equation (8.20) with a self-financed portfolio while alive, this would then imply that the issuer would require a drift term of (8.20) equal to zero. This would further result in the following PDE for the risky value $\tilde{V}$ of the derivative

$$\partial_t \tilde{V} + A_t \tilde{V} + (r + \lambda_B + \lambda_C)\tilde{V} = s_X X - \lambda_C g_C - \lambda_B g_B + \lambda_B \epsilon_h$$

(8.21)

$$\tilde{V}(T, S) = H(S),$$

where $H(S)$ is the payout of the derivative at maturity.

Furthermore, we are particular interested in the correction $U = \tilde{V} - V$ to the the risk-free Black-Scholes price $V$. Now, by using the Black-Scholes PDE for $V$ we find the PDE for $U$ to be given by,

$$\partial_t U + A_t U + (r + \lambda_B + \lambda_C)U = s_X X - \lambda_C g_C - \lambda_B g_B - V + \lambda_B \epsilon_h$$

(8.22)

$$U(T, S) = 0,$$

Eventually, by applying Feynman-Kac to this PDE yields the following expressions for the $DVA$ and the $FCA$

$$DVA = - \int_t^T \lambda_B(u)D_{r+\lambda_B+\lambda_C}(t, u)\mathbb{E}_t[(V(u) - g_B(V(u), X(u)))du$$

(8.23)

\[1\] Windfall = An unexpected event that can results in a financial gain.

Shortfall = An event that does not reach up to the expectations.
\[ FCA = - \int_t^T \lambda_B(u) D_{r+\lambda_B+\lambda_C}(t, u) \mathbb{E}_t[\epsilon_h(u)] du \quad (8.24) \]

where \( D_{r}(t, u) \equiv \exp\left(-\int_t^u (\hat{r}(v) dv\right) \) is the discount factor between \( t \) and \( u \) for rate \( \hat{r} \). The measure of the expectations in these two equations is such that \( S \) is the drift term at rate \( (q_S - \gamma_S) \). Also, keep in mind that Hull & White [11] refer to the funding cost adjustment, FCA as the funding value adjustment, FVA which we determined previously in this paper and therefore FCA is treated here accordingly.

The FCA in this paper is simply the discounted survival probability weighted by the expected value of the hedge error \( \epsilon_h \) implied by our semi-replication. Unlike to the DVA, we can stress the fact that the FCA is not symmetric since \( \epsilon_h \) upon own default turns out to be different for the issuer and the counterparty. FCA is simply the cost for generating a windfall to the issuer’s bondholders should a default occur. Moreover, if the issuer for some reason would like to break even while being alive, then consequently an additional cost has to be adjusted and included to the derivative price charged by the issuer.

Upon completion of this chapter, it is clear to us that different derivatives funding strategies yield different funding costs to the issuer while alive against different windfalls/shortfalls should he default. We have covered and derived a general relationship between the two scenarios for an issuer whom required the hedged derivatives positions outstanding to be self-financing while alive, which in turn gives rise to different valuation adjustments that in most cases solely depend on what type of strategy one chooses to implement and deploy but also on the details when unwinding the outstanding derivative positions. This is mainly due to the fact that different issuers with different strategies and spreads over the repo rate for instance, may settle different values to the derivative positions.
Chapter 9

Conclusion

In this paper we have looked at researchers different views on the Debit Value Adjustment and the Funding Value Adjustment. Recognizing the difficulty to implementing the DVA and the FVA within the vast banks subject to the non standard and commonly agreed methodology of how to price these as of yet. We have throughout this thesis seen that there is a lack of including both DVA and FVA in a derivate transaction since FVA has the disadvantage of creating potential arbitrage opportunities when prices are favourable. The controversy behind this conundrum is mainly due to the fact that a single price should and cannot serve the purpose of both reflecting the derivative trader’s funding costs and still be consistent with market prices. It makes it also more difficult to factor in FVA subject to DVA since most financial institutions are highly aware of the potential underlying risk arising from miscalculating DVA and FVA.

Furthermore, from an accounting point of view, it is difficult to persuade accountants with the fact that FVA prices should be used as fair values since such prices can be arbitraging. The chance of accounting systems working in this way will be condemned, how could they otherwise endorse a behaviour such as letting a highly creditworthy end user buying options from a bank which happens to have high funding costs and eventually sell the options to another bank with considerably lower funding costs whereas all three parties involved in the transaction seem to be making profit. Meanwhile, there are just a few institutions that record the DVA even though we have highlighted the importance of incorporating the adjustment on the balance sheet.

When examining different replication strategies we must stress the fact of the importance of implementing effective hedging strategies with respect to DVA and FVA. Although we cannot withhold the fact that the lower funding rate a financial institution possesses, the more competitive on trades when in need of funding and thus the “the law of one price” does no longer apply. Therefore the FVA is essential in the context of providing monetary incentive for the institutions XVA desks to use less funding which could be achieved by charging
a higher rate when borrowing from its treasury and eventually pay back for the posting money back to the funding desk.

We have also come across the fact that the value of derivative instruments does not solely depend on who is funding them, but how they are funded due to the fact that the funding costs may yield potential shortfalls or windfalls to bond issuer in the event of a default and thus the derived adjustments such as DVA and FVA resulting from this may also depend on the funding strategy employed. Moreover, we also examined the scenario should a bond issuer show any incentives to break even while being alive, then consequently an additional cost has to be included to the derivative price charged by the issuer, which is in fact a so-called XVA release, meaning that an cost is to be put on the counterparty for the sake of changing the scheme of the derivative transaction.

The DVA and FVA are and will continue to be complex matters that will be heavily discussed in the financial industry and therefore we are thrilled to both embrace and follow any adoptions and adjustments which may be changed in due course for these regulations.
Bibliography


